Chapter 9

Linear Relations and Linear Equations

GOAL

You will be able to

- represent a linear relation using a table of values and a graph
- describe the relationship between two variables in a table or on a graph
- write an equation you can use to solve a problem
- solve equations that involve integers, and verify the solutions

◀ How is the number of arch-shaped windows along the wings of the Saskatchewan Legislature related to the number of rectangular windows?

Chapter 9

Getting Started

YOU WILL NEED
- grid paper
- a ruler
- cubes and counters

Number Tricks

Ivan created this number trick.

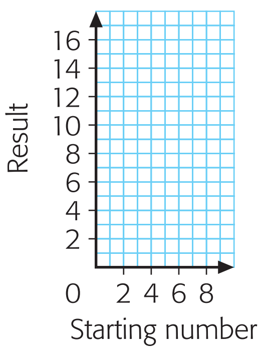
- Choose a number.
- Add 5.
- Multiply by 4.
- Subtract the number you chose.
- Subtract 2.
- Divide by 3.
- Tell me your result, and I can tell you what number you chose!

? How can you create your own number trick?

A. The table shows the results of Ivan's trick with two different starting numbers. Copy and extend the chart and test the trick by starting with 3, 4, 5 and 6.

Starting number (n)	Add 5	Multiply by 4	Subtract the original number	Subtract 2	Divide by 3 (result)
1	6	24	23	21	**7**
2	7	28	26	24	**8**

Number Tricks



B. What rule do you think Ivan uses to predict the starting number?

C. Draw a graph to show how the starting number relates to the result.

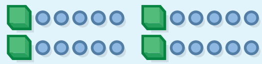
D. If someone started with 84, what would the result be?

E. If the result is 243, what was the starting number?

Communication | *Tip*

When you multiply a **variable** by a number or another variable, omit the multiplication sign. For example, write $4n$ instead of $4 \times n$, and write ab instead of $a \times b$.

F. Ivan began making this chart to show how the trick works. Copy and complete the chart to describe the rest of the steps.

Steps in the trick	How it works	Expression
Choose a number.		n
Add 5.		$n + 5$
Multiply by 4.		$4n + 20$

G. How does the model in step F help you understand the number trick?

H. Make up your own number trick. Use models, graphs, or **algebraic expressions** to show your classmates how your trick works.

algebraic expression

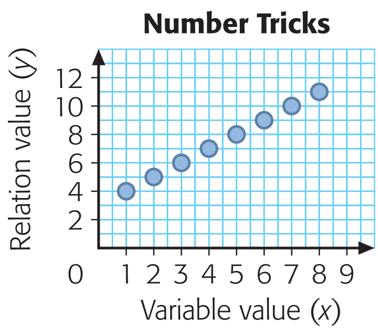
the result of applying arithmetic operations to numbers and variables; for example, $3x$ or $5x + 2$. Sometimes this is just called an expression.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decisions.

1. Whenever you use a table of values to describe how two variables are related, you can describe the relationship with a pattern rule.

2. The point (1, 4) is on this graph, so the pattern rule that was used to make the graph must be $y = 4x$.



3. Another way to write the algebraic expression $5(n + 4)$ is $5n + 4$.

4. If two different **equations** use the same letter for the variable, the variable does not have to represent the same amount.

9.1 Making a Table of Values

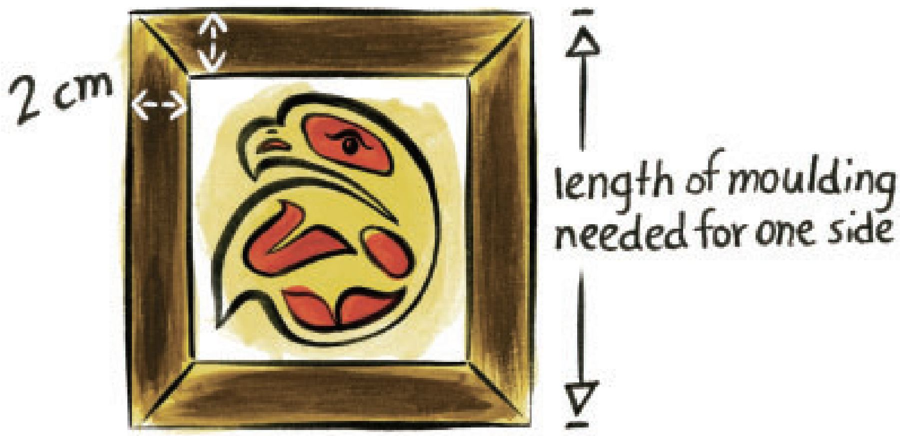
GOAL

Create a table of values for a given linear relation.

LEARN ABOUT the Math

Taira painted different-sized square tiles to sell at the Winnipeg Folk Festival. She will also sell wooden moulding so that people who buy her tiles can make frames. The moulding is 2 cm wide.

Taira wants to make a table of values that will help her customers decide what length of moulding they need to buy for any size tile.



? How can Taira figure out the length of moulding that someone would need to buy to frame each of her tiles?

relation

a rule that allows you to use one number to get information about another number

A. Use words to describe the relation between the length of one side of the tile and the length of one side of the frame.

B. Write an algebraic expression to describe the relation from part A. Use a variable to represent the length of one side of the square tile.

Length of one side of tile (cm)	Total length of moulding needed (cm)
11	
15	
20	
30	
33	
40	
43	
45	

C. What equation can Taira use to determine the total length of moulding required for all four sides of a frame for a square tile of any size?

D. Use your equation to determine the total length of moulding needed to frame a square tile with sides that are 11 cm long.

E. Copy and complete the table of values to show what length of moulding customers will need to buy to frame each of the other tile sizes.

Reflecting

F. Why is making a table of values a good strategy when there are many different-sized squares?

WORK WITH *the Math*

Example 1 | Creating a table of values

Renée runs a dog-walking service. She charges a fee of \$3 per walk, plus \$1 for every 10 min of the walk. Renée wants to send out flyers for her business. How can she determine sample rates for her flyers?

Renée's Solution

I used t to represent the time I spend walking a dog and I used c to represent the total cost of the walk.

I wrote an equation I can use to figure out the cost of a walk if I know how long it lasts.

$$\frac{t}{10} + 3 = c$$

It costs \$1 for every 10 min of walking, so the cost of the walking time in dollars is $t \div 10$ (or $\frac{t}{10}$, which means the same thing). I added 3 for the extra \$3 charge.

Time spent walking, t (min)	Cost of walk, c (\$)
15	4.50
30	6.00
45	7.50
60	9.00

I used my equation to make a table of values for every 15 min. I substituted the times for *t* to figure out the costs. I decided to stop at 60 min because most walks would be shorter than that.

A Checking

1. Create a table of values for each equation by substituting values from 1 to 6 for n.

   a) $c = 4(n + 3)$ **b)** $t = \frac{n}{2} + 7$

2. **a)** Create an equation for each statement.

Statement	Equation
k is equal to half of n added to four times n	
t is equal to five times the sum of four and n	

   b) Create a table of values for $n = 4$, $n = 6$, and $n = 8$ for each equation you wrote in part a).

B Practising

3. Write an equation you can use to make a table of values for each situation.

   a) Figure out the cost, c, of n tickets to a rock concert at \$35 per ticket.

   b) Figure out the cost, c, of n cookies if the price is 2 cookies for \$1.

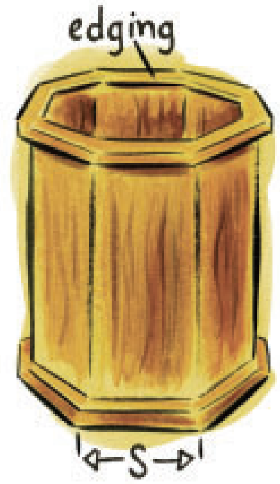
   c) Figure out the cost, c, to rent a car for t days at \$27 per day, if you have a coupon for \$15 off the total cost.

4. Choose one situation from question 3. Create a table of values using any five values you choose for one of the variables.

5. Sierra is planning a party. The food will cost \$8 per guest and the cake will cost \$24.

   a) Write an equation someone can use to figure out the cost of the party for any number of guests.

   b) Create a table of values to show the cost of the party for 5 to 10 guests.



Reading Strategy

Summarizing

Summarize the steps you will take to solve the problem.

6. Nick and Thao are trying to determine the total length of wooden edging needed to frame cedar planters. The planters are octagonal, but come in different sizes. Each piece of edging is 5 cm longer than one side, s. Nick thinks they should use $L = 16s + 80$. Thao thinks they should use $L = 16(s + 5)$.

a) Create a table of values for either Thao's equation or Nick's equation. Include rows for these side lengths: 15 cm, 20 cm, and 25 cm.

b) Compare your table with one that a classmate wrote for the other equation. What do you notice? Why did this happen?

7. Four friends run a lawn care business. They agree to split the profits equally every day, and each friend also agrees to donate $10 of their earnings to a local charity.

a) Write an equation they can use to determine how much money each friend will receive, after donating to charity, if the business earns t dollars in a day.

b) Suppose the business earns $320. How much will each friend receive?

8. A rectangle is 2 cm longer than it is wide.

a) Write an equation you can use to determine the width when you know the length.

b) Write an equation you can use to determine the perimeter when you know the length.

c) Create a table of values showing the perimeters of rectangles with these lengths: 10 cm, 12 cm, 14 cm, 16 cm, 18 cm, and 20 cm.

d) Determine the perimeter of a rectangle with a length of 36 cm. Explain what you did.

9. a) What information can you get from a table of values that you cannot get from the related equation? Give an example.

b) What information can you get from an equation that you cannot get from the related table of values? Give an example.

9.2 Graphing Linear Relations

YOU WILL NEED
- grid paper
- a ruler

GOAL

Construct a graph from the equation of a given linear relation.

LEARN ABOUT the Math

Angèle plays on a team that is going to an Ultimate Flying Disc tournament.

Six players will travel together and share the cost. Each player will pay an entry fee of $40.

The total travel cost for all six, not counting the entry fees, will be between $300 and $800.

? How can Angèle figure out how much she might have to pay for the trip?

Total travel cost (t)	Angèle's cost (a)
300	90
360	
420	

A. Write an equation Angèle can use to calculate her share of the travel cost plus the entry fee. Check your equation using the first pair of values from the table at the left.

B. Continue the table of values to show how Angèle's share of the cost will change as the total travel cost increases from $300 to $800.

Communication | *Tip*

In a table of values, changes to one variable depend on what happens to the other variable. For example, you cannot determine Angèle's cost until you know the total cost of the trip. In a table of values, the amount you need to know first usually goes in the left column. In an ordered pair, it is the first coordinate. On a graph, it is plotted along the horizontal axis.

C. Angèle plans to make a graph to estimate her share of the cost for any total amount from $300 to $800. Which of these points will be on her graph: (300, 90) or (90, 300)? Explain how you know.

D. Draw the axes for Angèle's graph. Choose a scale for each axis that will help you graph all the data from the table, and that will also help you use your graph to determine other possible costs. Give your graph a title.

E. Plot the points from your table of values on the graph. Then place a ruler along the points you plotted. Look for another set of coordinates that could describe an amount Angèle might have to pay.

F. Estimate the amount Angèle would pay if the total cost were $750.

G. Check your answer to part F by substituting the coordinates of the $750 point into your equation from part A.

Reflecting

H. How did you decide what scale to use on each axis?

I. Look at your graph. How does it show that the relation between the total cost and the cost for Angèle is a **linear relation**?

linear relation

a relation whose plotted points lie on a straight line

WORK WITH the Math

Example 1 | Graphing and analyzing data

Complete the table for the equation $y = x - 3$.
Graph the points from your table.
Determine the value of x when $y = -7$ using your graph.

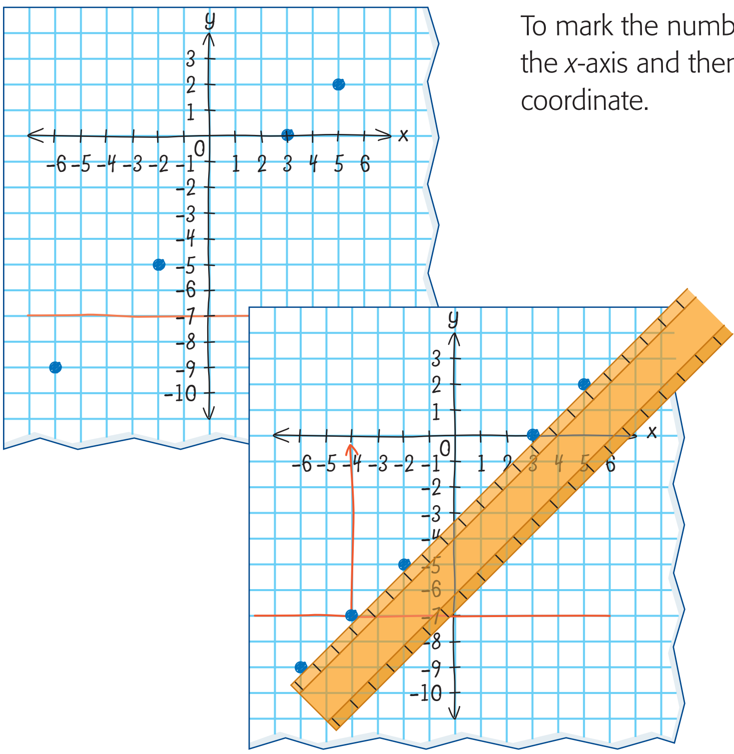
x	y
−6	−9
−2	
3	
5	

Holly's Solution

x	y
-6	-9
-2	-5
3	0
5	2

The equation was $y = x - 3$, so I subtracted 3 from each value for x to get the matching value for y.

Some numbers in my table were negative, so I drew a grid with positive and negative numbers. I called the horizontal axis x and the vertical axis y. To mark the number, I found each x-coordinate on the x-axis and then counted up or down to the y-coordinate.



When $y = -7$, $x = -4$.

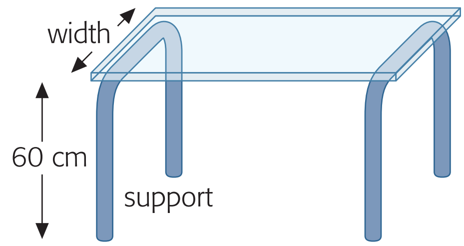
I wanted to know which point that was in line with the rest would have a y-coordinate of −7, so I drew a horizontal line at $y = -7$.

Then I put my ruler along the points and drew a new point where the line met my ruler. Then I drew a line up from the new point to the x-axis. The line met the

Example 2 | Solving a problem by graphing

Elinor makes custom tables. Each table has two supports, made from metal tubing, as shown. The height of each support is 60 cm, and the width matches the table width.

Elinor has 380 cm of metal tubing. What is the widest table she can make? Use a graph.



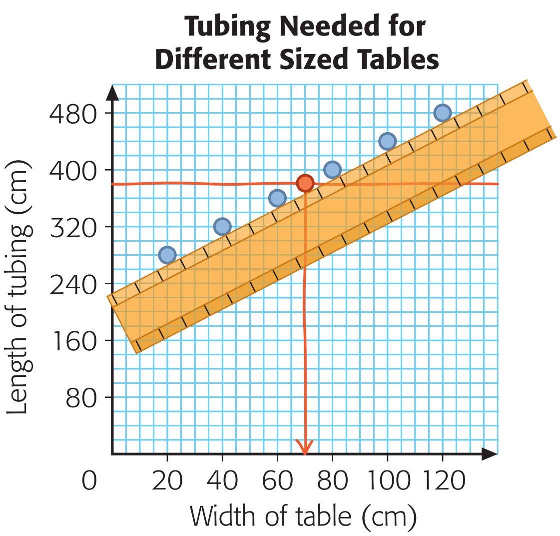
John's Solution

My equation is t = 2(120 + w), where t is the total length of tubing and w is the table width.

The length of tubing in one support is 60 cm + 60 cm + width. A shorter way to write that is 120 + w. Each table has two supports so I multiplied 120 + w by 2.

w	t
20	280
40	320
60	360
80	400
100	440
120	480

I used a spreadsheet to make a table of values. I decided that a table would probably not be less than 20 cm wide or more than 120 cm wide, so I used these amounts to start and end my spreadsheet.



I used my spreadsheet to make a **scatter plot**.

I used 1 square = 5 cm for my horizontal scale so I could show widths from 20 cm up to 120 cm (1.2 m).

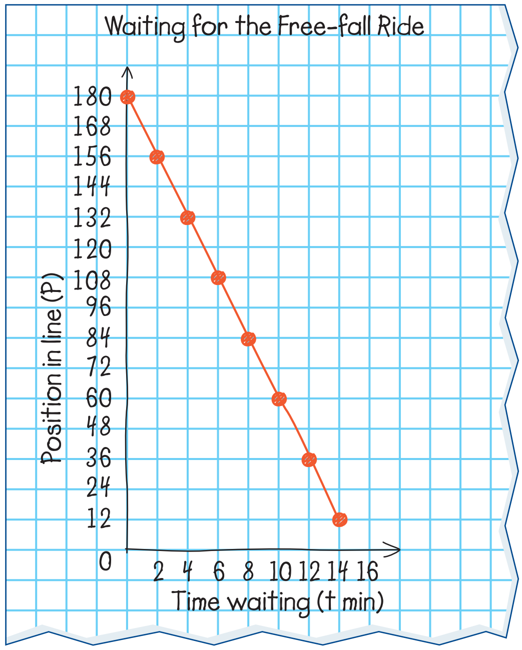
I used 1 square = 20 cm for my vertical scale because I figured out I would not need too many squares to show the greatest number, 480 cm.

I printed my graph and drew a line across from 380 cm on the length axis. Then I put my ruler along the points on my graph and drew a new point where the line met my ruler. Then I drew a line down from the new point to the width axis. The line met the width axis at 70, so (70, 380) is a point on the graph.

Elinor can make a table 70 cm wide from 380 cm of tubing.

Example 3 | Describing a relationship between variables

Tuyet manages an amusement park and is checking on the wait times for different rides. She made this graph to show how a customer moves forward in line while waiting for the Free-fall Ride. What does Tuyet's graph show about the relationship between the time spent waiting and the customer's position in the line?



Lam's Solution

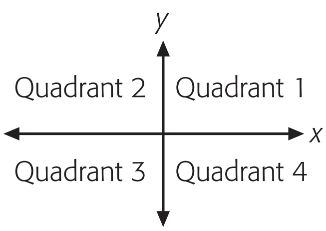
Tuyet's graph shows that the person moves up 24 positions in line every 2 min.

The first point on the graph is (0, 180), so a person who just arrived has waited 0 min and is 180th in line. The next point is (2, 156). After 2 min, the person has moved from 180th to 156th. 180 – 156 = 24, so the person has moved up 24 spaces.

The next point is (4, 132), so after another 2 min, the person has moved up 24 more spaces.

The rest of the points are lined up, so the person keeps moving up 24 spaces every 2 min.

A Checking



1. a) Create a table of values for the equation $y = 8 - 2x$.

b) Graph the points from your table on a Cartesian coordinate system.

c) Examine your graph. Do you think it is possible that the graph of this equation will ever contain points in Quadrant 3? Explain your reasoning.

B Practising

2. Treena is going to the fair. Admission to the fairgrounds is \$15, and the rides are \$5 each.

a) Create a table of values to show how much Treena would pay for admission and for 1, 4, and 10 rides.

b) Graph the data from your table of values. Determine Treena's total cost, including admission, if she goes on 7 rides.

c) If Treena has \$75 to spend on rides and admission, how many rides can she go on?

3. a) Create a table of values for the equation $y = \frac{x}{4} + 2$. Choose x-values that are multiples of 4.

b) Graph the points from your table.

c) When $x = 10$, will y be an integer? How do you know?

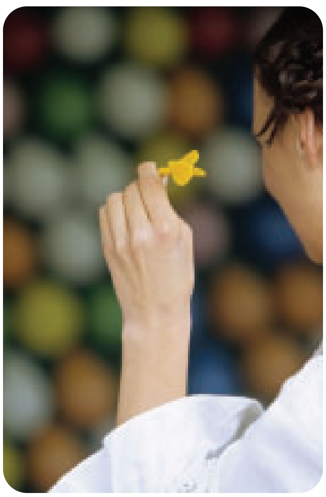
d) Estimate the value of y when $x = 10$. Use your graph.

4. a) Graph the equation $y = -2(x + 3)$

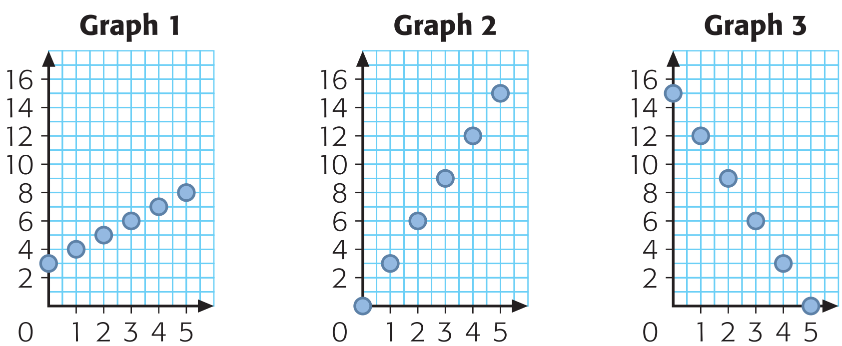
b) Describe what happens to y each time x increases by 1.

c) When $y = 0$, is x positive or negative?

d) At what point on the graph do the x-values change from negative to positive?



5. At a Pop-a-Balloon game at the fair, you pay \$1 and get 3 darts to throw at the balloons. Which graph below could you use to figure out how many darts you could throw for different prices in dollars? Explain how you know.



6. Sparkly Clean Window Cleaners charges $4 per window to wash windows in a home. This week, they are offering a $15 discount to all customers with 5 or more windows.

a) Make a graph to show the window-cleaning cost for homes with 5 to 20 windows. What happens to the cost each time the number of windows increases by 5?

b) Use your graph to estimate the cost for 17 windows.

c) Use your graph to estimate the number of windows that could be cleaned for $29.

d) Why do you think the discount does not apply if a home has fewer than 5 windows?

7. A class at Elmwood School is planning a three-day ski trip. The bus costs $850 and lift tickets cost $75 per person.

a) Make a graph to show the total cost for 5, 10, 15, and 20 skiers.

b) What is the total cost for 18 skiers?

c) What would it cost per student if 14 students went on the trip and shared the cost equally?

8. The large pool at the Pan-Am Pool complex in Winnipeg holds 4700 kL (1 kL = 1000 L). The pool drains at a rate of 40 kL/h.

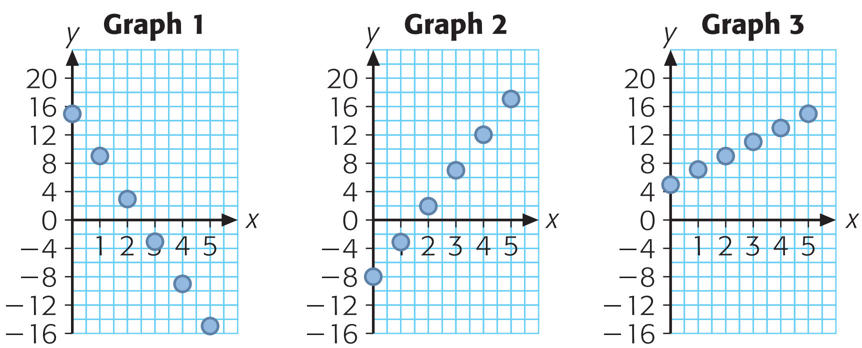
a) Graph the relation between the amount of water left in the pool and the number of hours it has been draining.

b) How much water is left in the pool after 60 h?

c) Estimate how long the pool will take to drain completely.

9. Match each equation to its graph. Explain your reasoning.

a) $y = 2x + 5$ **b)** $y = 5x - 8$ **c)** $y = -6x + 15$



10. What can a graph show you about a situation that you cannot see on a table of values? Give an example from this lesson.

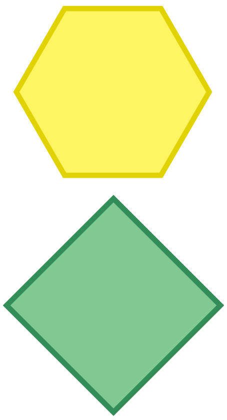
9.3 Exploring Possible Values

YOU WILL NEED
- pattern blocks

GOAL

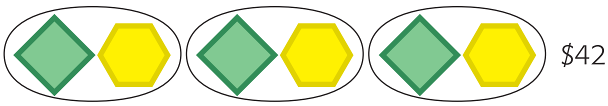
Explore possible values for variables in a given equation.

EXPLORE the Math

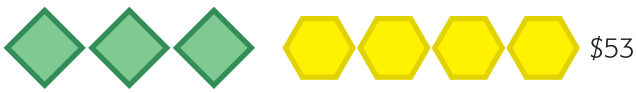


John is competing in the school Think Bowl event.
He will win a point for his team if he can figure out how much each of these special coins is worth. John can use two or more of the clues to help him solve the problem.

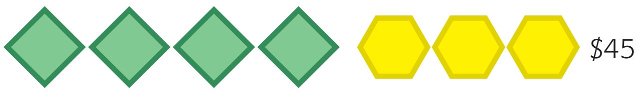
Clue 1: If you add the coins and multiply by 3, the sum is $42.



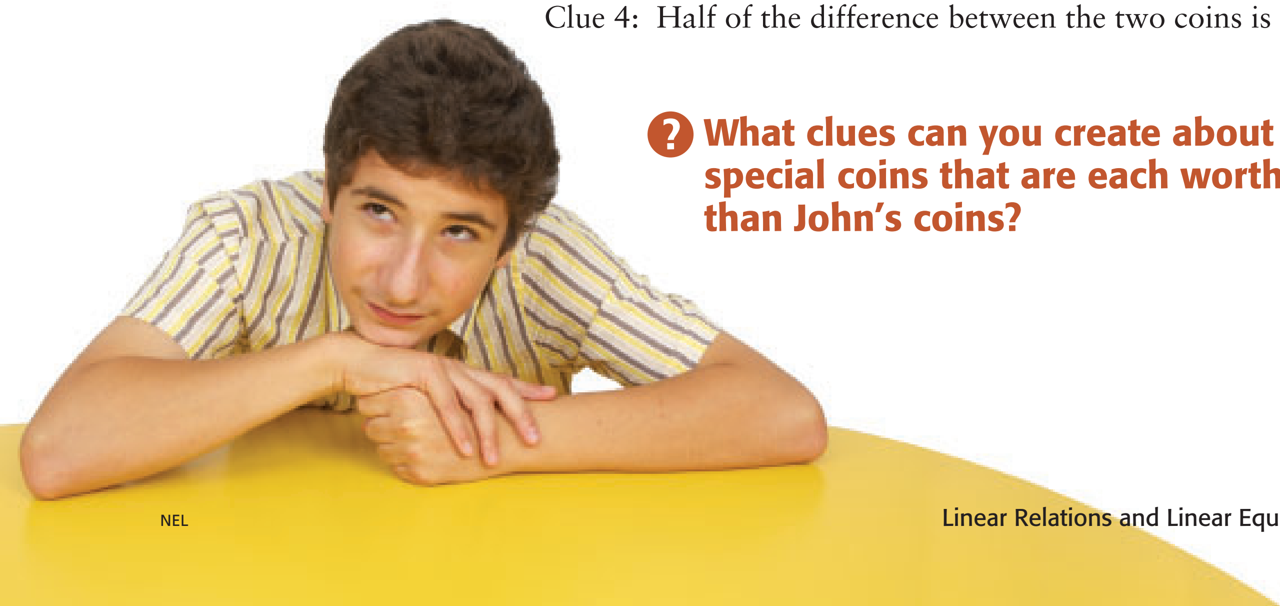
Clue 2: Three green coins and four yellow coins are worth $53.



Clue 3: Four green coins and three yellow coins are worth $45.



Clue 4: Half of the difference between the two coins is $4.



? What clues can you create about two special coins that are each worth more than John's coins?

9.4 Drawing Diagrams to Represent Equations

GOAL

Draw a diagram to determine the missing value in an ordered pair.

LEARN ABOUT the Math

Holly's mother sells hand-made chocolates. She charges \$1 for every three chocolates, plus \$5 for the tin they come in. While Holly is working at the store, a customer arrives with \$16 to spend on a tin of chocolates.

Holly knows this equation can help her calculate the cost for any number of chocolates:
$c = \frac{n}{3} + 5$ where c is the cost and n is the number of chocolates.

? How many chocolates can the customer buy?

Example 1 | Drawing a diagram

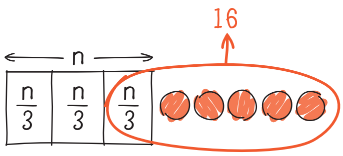
I used a diagram to solve the equation $c = \frac{n}{3} + 5$ when c was 16.

Holly's Solution

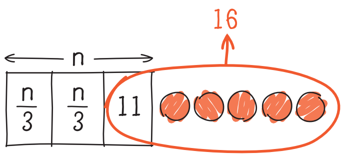
$c = \frac{n}{3} + 5$

$16 = \frac{n}{3} + 5$

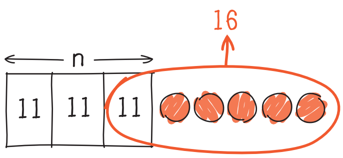
I had to figure out what n is when c is 16. I rewrote the equation by substituting 16 for c.



$16 = \frac{n}{3} + 5$ means that if you divide n into 3 equal parts and then you add 5 to one part, you will have 16. I drew a rectangle to show n. Then I divided the rectangle into 3 equal parts and labelled each part $\frac{n}{3}$. I added 5 to one part to make $\frac{n}{3} + 5$.



$\frac{n}{3} = 11$ because 11 is what is left if you take 5 away from 16.



Each $\frac{n}{3}$ part must be 11 because the parts are equal.

$\frac{n}{3} = 11$

$n = 11 \times 3$

$n = 33$

The customer can buy 33 chocolates for $16.

If $\frac{n}{3} = 11$, then n must be 33.

Reflecting

A. How can Holly use her diagram to verify her solution?

B. What are some advantages and disadvantages of Holly's solution?

WORK WITH the Math

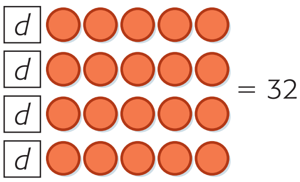
Example 2 | Solving a more complex equation

Solve the equation $4(d + 5) = m$ when $m = 32$.

Solution A: Dividing both sides by the same amount

$4(d + 5) = m$
$4(d + 5) = 32$

Rewrite the equation with 32 instead of *m*.



Draw a picture to show that $4(d + 5)$ means "4 groups of $(d + 5)$." You can use a box to represent the variable because you do not know what amount it represents. You can draw red counters to represent the constant amount (+ 5).

4 groups of $(d + 5)$ equals 32,
so $(d + 5)$ must equal $32 \div 4 = 8$.

Figure out the value of each group of $d + 5$.

Since $d + 5 = 8$, then *d* must equal 3.

Then figure out the value of *d*.

Check:

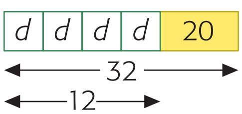
Left side	Right side
$4(d + 5)$	32
$4(3 + 5)$	
$4(8)$	
32	

Verify by substituting 3 for *d* in the original equation.

Solution B: Writing the multiplication differently

$4(d + 5) = m$
$4(d + 5) = 32$
$4d + 20 = 32$

Rewrite the equation with 32 instead of *m*.
Multiply each part of the $d + 5$ by 4.



$4d + 20 = 32$ means that if you have 4 groups of d and you add 20, the result is 32. Draw a rectangle to represent 32 and divide it into parts that show $d + d + d + d + 20$.

$4d = 12$
$d = 3$

The yellow part of the rectangle is worth 20, so the green part must be worth 32 – 20 or 12. Each *d* must be worth 3 because $4(3) = 12$.

Example 3 | Solving an equation involving subtraction

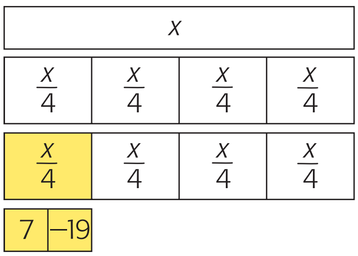
Solve the equation $\frac{x}{4} - 7 = y$ when $y = -19$.

Solution

$\frac{x}{4} - 7 = y$

$\frac{x}{4} - 7 = -19$

Rewrite the equation with -19 instead of y. This equation means, "If you divide x into 4 equal parts and then subtract 7 from one of the parts, the answer is -19."



Draw a rectangle to represent x, the starting amount.

Divide x into 4 equal parts to show $\frac{x}{4}$.

The equation says that, if you subtract 7 from $\frac{x}{4}$, there will be -19 left. The yellow part of the picture shows this. It also shows that $\frac{x}{4} = 7 + (-19)$, so $\frac{x}{4} = -12$.

$7 + (-19) = -12,$

$$\text{so } \frac{x}{4} = -12$$
$$4\left(\frac{x}{4}\right) = 4(-12)$$
$$x = -48$$

There are 4 groups of $\frac{x}{4}$ in x, so if $\frac{x}{4} = -12$, then $x = -48$.

Check:	Left side	Right side
	$\frac{x}{4} - 7$	-19
	$\frac{-48}{4} - 7$	
	$(-12) - 7$	
	-19	

Verify the solution by substituting -48 for the variable x in the original equation.

A Checking

1. Solve each equation using a diagram.

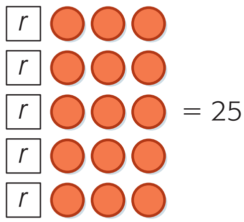
a) $t = 5(n + 1)$, when $t = 30$ **b)** $p = \frac{x}{3} - 2$, when $p = -5$

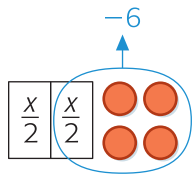
B Practising

2. Solve each equation.

a) $\frac{x}{3} + 23 = y$, when $y = 41$ **b)** $3x + 9 = y$, when $y = 36$

3. Each diagram is the beginning of a solution to an equation. Write the equation.

a) 

b) 

4. Continue each solution in question 3.

5. Solve each equation.

a) $6x + 5 = y$, when $y = 41$

b) $3x - 24 = y$, when $y = 12$

c) $\frac{x}{2} + 9 = y$, when $y = -13$

d) $4(x + 7) = y$, when $y = 20$

6. a) Create a table of values for $y = 3x - 4$.

b) Graph the points from your table on a Cartesian coordinate system.

c) Use your graph to determine the values of x when $y = 11$ and $y = -16$.

d) Why is a graph more useful than a diagram when you have to solve an equation for more than one value of the variable?

7. Bowling costs $4 per game plus $2 for shoe rental.

a) Write an equation you can use to determine the cost of bowling for any number of games.

b) During a tournament, Max spent $30. How many games did he bowl?

8. Which of these equations would you solve by drawing a diagram? Explain your choice(s) and show the diagrams you would use.

A. $4x = y$, when $y = 24$

B. $4x + 320 = y$, when $y = 424$

C. $n - 5 = d$, when $d = 0$

D. $\frac{x}{3} - 2 = y$, when $y = 3$

Mid-Chapter Review

Frequently Asked Questions

Q: How can you graph a linear relation?

n	c
40	1200
80	2000
120	2800
160	3600
200	4400
240	5200

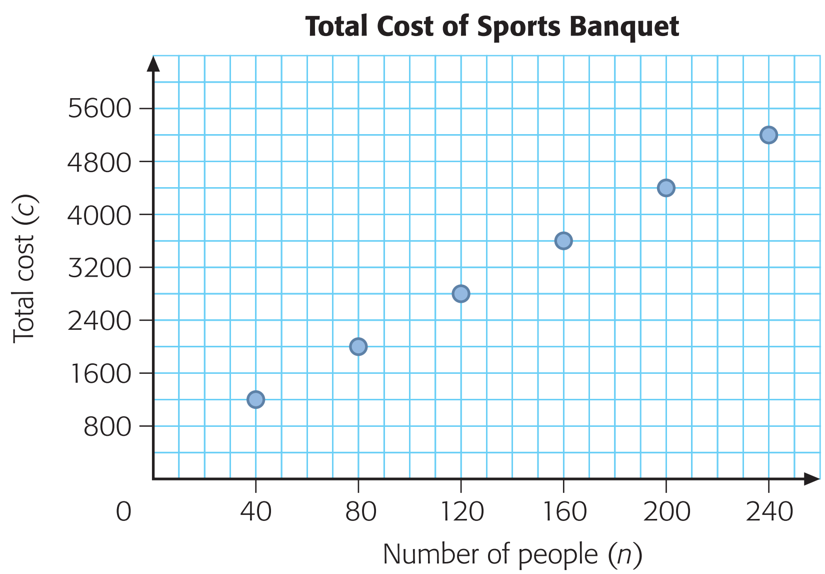
A: You can create a table of values using the relation. For example, suppose a sports banquet for up to 240 people costs \$20 for each person plus \$400. The relation is $20n + \$400 = c$. Decide on reasonable values for one of the variables and create the table.

You need to know the number of people to get the cost, so n goes on the horizontal axis.

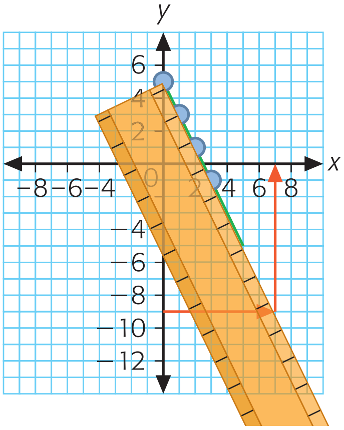
The n-axis will go from 0 to at least 240. You can use a scale of 1 square for 20 people.

The c-axis will go from \$0 to at least \$5200. You can use a scale of 1 square for \$400.

Plot points on the scatter plot using the table of values.



Q: **How can you solve an equation using a graph?**



A: For example, use the graph of $-2x + 5 = y$ to solve $-2x + 5 = -9$.

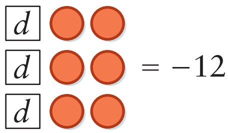
Step 1:
Place a ruler along the plotted points.
Look for a point on the line formed by the ruler where $y = -9$.

Step 2:
Draw a vertical line from that point to the horizontal axis to find the x-value for that y-value. That point is the value of x that solves the equation.

For $-2x + 5 = -9$, $x = 7$.

Q: **How can you solve an equation by drawing a diagram?**

A: For example, solve $3(d + 2) = t$ when $t = -12$ by drawing a picture to figure out what the equation means.



3 groups of  equal −12.

$3(-4) = -12$, so each group of  must equal −4.

If $d + 2 = -4$, then d must be −6.

Check.

Left side	Right side
$3((-6) + 2)$	-12
$= 3(-4)$	
$= -12$	

Practice

Lesson 9.1

$25n + 50 = c$	
n	c
3	
	225
15	
	550

1. Complete the table of values for $25n + 50 = c$.

2. A digital television provider charges $30 a month for the basic package, plus $2 for each additional theme group.
   a) Write an equation for the monthly cost of various television packages.
   b) Create a table of values to show the cost of the basic package plus 0, 2, 4, 6, 8, and 10 theme groups.
   c) How much would a basic package plus 7 theme groups cost?
   d) Suppose Jerry's bill is $52. How many theme groups does he get?

Lesson 9.2

3. Graph each relation on the same set of axes.
   a) $y = 4x - 5$ b) $y = -3(x - 4)$ c) $y = \frac{x}{2} + 1$

4. Darius is a long-distance truck driver. His average speed is 80 km/h and he takes a 1 h break on each trip. A relation for the number of hours he is on the road, t, is $t = \frac{d}{80} + 1$, where d is the distance he drives. Graph this relation.

5. Use your graph from question 4 to determine how many hours Darius takes to drive each distance.
   a) 400 km b) 640 km c) 880 km

6. Use your graph from question 4 to determine how far Darius drives in each period of time.
   a) 5 h b) 8 h c) 10 h

Lesson 9.4

7. Solve each equation by drawing a diagram.
   a) $4x + 3 = y$, when $y = 23$
   b) $2(x + 5) = y$, when $y = -8$
   c) $\frac{x}{5} + 2 = y$, when $y = 6$
   d) $\frac{x}{4} - 5 = y$, when $y = 4$

9.5 Solving Equations with Counter Models

YOU WILL NEED
- red and blue cubes
- red and blue counters
- Balance Mat

GOAL

Model and solve linear equations concretely.

LEARN ABOUT the Math

The National Hockey League (NHL) is made up of 30 teams in Canada and the United States. Before the NHL expanded in 1967, there were far fewer teams.

If you start with the number of teams before the expansion, add 4, and then multiply by 3, the result is 30, which is the current number of teams in the NHL.

? How many teams were in the National Hockey League before it expanded?

Example 1 | Modelling to solve an equation

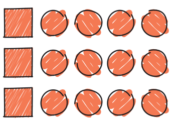
I wrote an equation to describe the number of teams and then solved the equation by modelling with cubes and counters on a balance.

Lam's Solution

I used t to represent the number of teams before the 1967 expansion of the NHL.

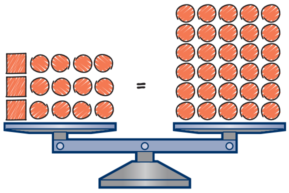
$3(t + 4) = 30$

I wrote an equation to show the clues: "Add 4 to t and then multiply by 3."
$3(t + 4)$ means the same as $3 \times (t + 4)$.



$3(t + 4)$

I used a red cube to represent t.
I added 4 round counters to show $t + 4$.
To show $3(t + 4)$, I made 3 rows of $t + 4$.

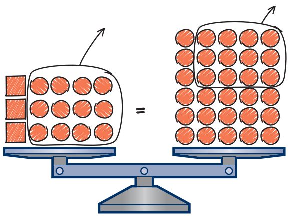


3(t + 4) = 30

3t + 12 = 30

I still had to figure out how much t was worth. $3(t + 4) = 30$, so I thought of a pan balance with $3(t + 4)$ on the left side and 30 on the right side. I made a model with cubes and counters.

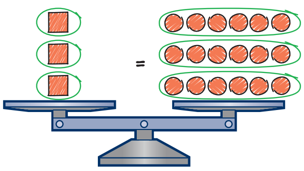
$3(t + 4)$ was the same as $3t + 12$ because there were 3 t-cubes and 12 counters.



3t + 12 – 12 = 30 – 12

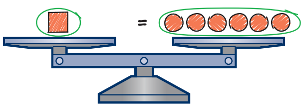
3t = 18

I subtracted 12 counters from each side. The amounts stayed balanced but the 3 t-cubes were alone on one side. That helped me figure out that 3 t-cubes were worth 18.



$$\frac{3t}{3} = \frac{18}{3}$$

I noticed that I could make 3 groups of t on the left side and 3 groups of 6 on the right side. This is like dividing both sides by 3.



t = 6

I took away two of the groups on each side. Since the groups were equal, the amounts that were left stayed balanced. This let me see the value of one red cube, or t.
Then I knew that each t was worth 6.

There were 6 teams in the NHL before the 1967 expansion.

Check:	Left side	Right side
	3(t + 4)	30
	= 3(6 + 4)	
	= 3(10)	
	= 30 ✔	

I verified my solution by substituting 6 for the variable t in my equation. The left side was equal to the right side, so my solution was correct.

isolate

to show the same equation in an equivalent way so the variable is alone on one side

Reflecting

A. Whenever you add, subtract, multiply, or divide on one side of the equation, you need to make the same change on the other side. Why is it important do this?

B. How did isolating the three red cubes on one side help Lam solve the problem?

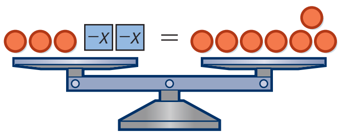
WORK WITH the Math

Example 2 | Modelling equations with negative numbers

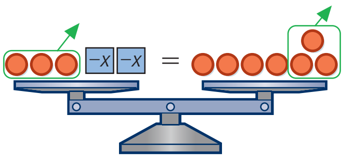
Solve the equation $3 - 2x = 7$.

Solution A: Solve by balancing

$$3 - 2x = 7$$
$$3 + (-2x) = 7$$

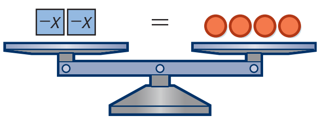


Model the equation with cubes and counters. Use red for positive and blue for negative. You cannot show $3 - 2x$ on the left side, but you can show $3 + (-2x)$, which means the same thing.



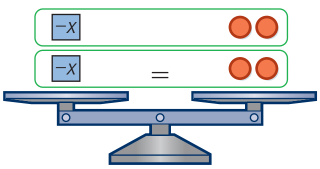
Isolate the cubes on one side by subtracting 3 counters from each side.

$$3 - 3 + (-2x) = 7 - 3$$



The cubes are alone and the equation still balances.

$$(-2x) = 4$$



Make two equal groups on each side to determine the value of $-x$.

$$\frac{-2x}{2} = \frac{4}{2}$$
$$-x = 2$$
$$x = -2$$

If $-x = 2$, then x must equal the opposite of 2, or -2.

Check: Left side | Right side

$3 - 2x$ | 7

$= 3 - 2(-2)$

$= 3 - (-4)$

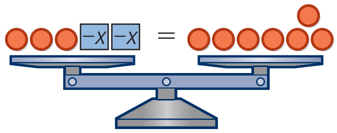
$= 3 + 4$

$= 7$ ✔

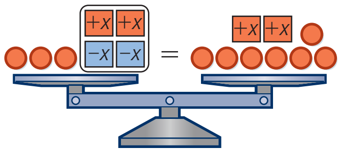
Verify the solution by substituting 2 for x in the original equation. The left and right sides are equal, so the solution is correct.

Solution B: Solve using the zero principle

$$3 - 2x = 7$$

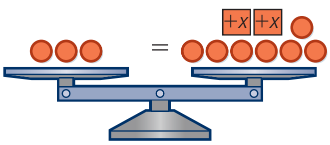


Model the equation with cubes and counters.



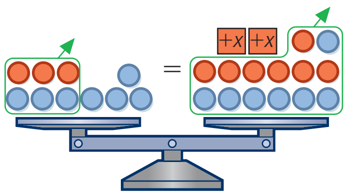
$$3 + (-2x) + 2x = 7 + 2x$$
$$3 + (-2x) + 2x = 7 + 2x$$
$$3 + 0 = 7 + 2x$$

Remove the negative cubes by using the zero principle. Add $2x$ to both sides. The circled cubes are opposites, so their value is 0.



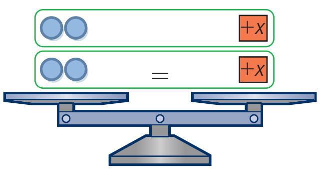
$$3 = 7 + 2x$$

Now only positive *x*-cubes are left, so it will be easier to find the value of *x*.



$$3 + (-7) = 7 + (-7) + 2x$$
$$-4 = 0 + 2x$$
$$-4 = 2x$$

Isolate the cubes on one side by adding −7 to both sides.



$$\frac{-4}{2} = \frac{2x}{2}$$
$$-2 = x$$

2 *x*-cubes are worth 4 negative counters. Each *x*-cube must be worth 2 negative counters, so $x = -2$.

This is the same thing as dividing both sides of the equation by 2.

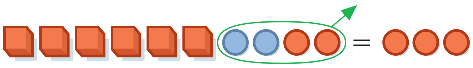
Example 3 | Modelling equations with fraction solutions

Model and solve the equation $6x - 2 = 1$.

Solution

$6x - 2 = 1$

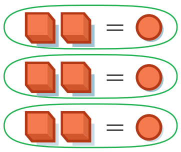
Model the equation with cubes and counters. Use 2 negative counters to represent -2.



$6x + (-2) + 2 = 1 + 2$

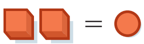
$6x = 3$

Isolate the cubes on the left side using the zero principle. Add 2 red counters to each side to remove the 2 blue counters. The circled counters are opposites, so their value is 0.



$$\frac{6x}{3} = \frac{3}{3}$$

There are only 3 red counters on the right side of the equation, so divide each side into 3 equal groups.



$2x = 1$

2 red cubes have the same value as 1 red counter.

$$\frac{2x}{2} = \frac{1}{2}$$

To find the value of one red cube, imagine the red counter split in half.

$$x = \frac{1}{2}$$

Two cubes are equal to two $\frac{1}{2}$ counters, so one cube is equal to one $\frac{1}{2}$ counter.

The solution to the equation $6x - 2 = 1$ is $x = \frac{1}{2}$.

Check:

Left side	Right side
$6x - 2$	1
$= 6\left(\frac{1}{2}\right) - 2$	
$= 3 - 2$	
$= 1$ ✔	

Verify the solution by substituting $\frac{1}{2}$ for x in the original equation. The left and right sides are equal, so the solution is correct.

A Checking

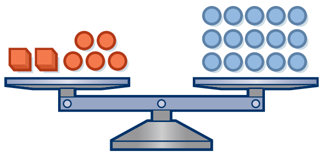
1. Model and solve each equation.

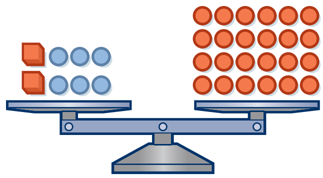
   a) $2(z + 3) = 14$
   b) $35 = 5(k - 4)$
   c) $-6 = 5x + 4$
   d) $-2d + 3 = 9$

B Practising

2. a) Solve $3x + 5 = 26$ using a model. Record your steps.
   b) Explain how your model helped you solve the equation.

3. Write an equation for each model, then solve your equations.

   a) 

   b) 

4. a) Solve $4x - 2 = 5$ using models or a diagram.
   b) Why would it be difficult to complete the solution using only cubes and counters?

5. Model and solve each equation.

   a) $4g = -24$
   b) $2h - 5 = 11$
   c) $4(m - 2) = 20$
   d) $5a = 13$
   e) $4a + 3 = -3$
   f) $3(y + 2) = -12$

6. Write each sentence as an equation, then solve the equation.
   a) Seven more than five times a number is 22.
   b) Three less than four times a number is 21.
   c) Four times two more than a number is 32.

7. Solve each equation. Show what you did.
   a) $-3d + 5 = -4$
   b) $-10 = 6x - 4$
   c) $-2(z - 3) = -8$

8. a) Draw a picture to solve the equation $\frac{x}{5} - 4 = 8$.
   b) Why would it be difficult to use cubes and counters to solve an equation where the variable is divided by a number?

9. Jack Nicklaus is one of the greatest golfers of all time. Over his career, he had 20 wins in major tournaments.

a) If you multiply the number of times he finished third by 2 and then add 2, the result is the number of times he finished first. How many times did he finish third?

b) If you divide the number of times he finished in the top three by 4 and then subtract 3, you get the number of third-place finishes. How many top-three finishes did Nicklaus have? How many times did he place second?

c) The Masters Tournament is the most prestigious tournament in golf. Four times one less than Nicklaus's number of Masters wins is equal to his total number of first-place wins. How many times did he win the Masters?

10. Create a balance problem for a classmate to solve. Use small paper bags with heavy counters in them to represent variables. (Make sure to put the same number of counters in each bag.) See if your classmate can write an equation to describe your problem, and then solve it to figure out how many counters you put in each bag.

11. If you are modelling an equation on a real pan balance, what is the reason you cannot use one cube to represent a variable?

12. When you make a model to solve an equation, why does it help to use different materials to represent the variables and the **constants**?

9.6 Solving Equations Symbolically

GOAL

Solve a linear equation symbolically.

LEARN ABOUT the Math

The world's largest indoor triple-loop roller coaster is the Mindbender at Galaxyland Amusement Park in the West Edmonton Mall.

Thirty-one people were waiting to get on the Mindbender ride. After they boarded, two trains were full and the third train had five empty seats.

? How many seats are in each train on the Mindbender?

Communication | *Tip*

- This solution shows how to write each step in a calculation directly under the previous step. Lining up the equal signs makes the calculation easier to read and check.

Example 1 | Using symbols

Determine the number of seats on each train by writing a symbolic solution.

Renée's Solution

I used n to represent the number of seats in each train.

$$n + n + (n - 5) = 31$$

Two full trains had *n* people in seats, and one train had $n - 5$ people in seats. There were 31 people altogether.

$$3n - 5 = 31$$

I replaced $n + n + n$ with $3n$ to simplify my equation.

$$\begin{aligned} 3n - 5 &= 31 \\ 3n - 5 + 5 &= 31 + 5 \\ 3n &= 36 \end{aligned}$$

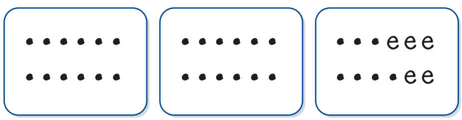
I wanted to isolate $3n$ on one side so I could see how much it was worth and figure out *n*. To get $3n$ alone, I used the zero principle and added 5 to both sides.

$$\begin{aligned} \frac{3n}{3} &= \frac{36}{3} \\ n &= 12 \end{aligned}$$

There are 12 seats on each train.

Now I knew 3 full trains had 36 seats. To get *n*, the number of seats on one train, I divided both sides by 3. This kept the equation balanced.

Check:



$12 + 12 + 7 = 31$ ✔

To check, I drew each train and showed the full and empty seats. My picture matched what it said in the problem, so my answer was correct.

Example 2 | Working backwards

Determine the number of seats on each train by working backwards through the operations.

John's Solution

$3n - 5 = 31$

Forward	Backwards
Start with n.	Result is n.
Multiply by 3.	Divide by 3.
Subtract 5.	Add 5.
Result is 31.	Start with 31.

I wrote the operations that happen to the variable in the equation. Then I worked backwards to figure out what operations I would have to do to change 31 back to *n*.

Start with 31.

Add 5: $31 + 5 = 36$

Divide by 3: $\frac{36}{3} = 12$

Result is n: $n = 12$

I started with 31 and followed my backwards operation steps to figure out *n*.

Check:

Left side	Right side
$3n - 5$	31
$= 3(12) - 5$	
$= 36 - 5$	
$= 31$ ✔	

I verified my solution by substituting 12 for *n* in the equation. The left side was equal to the right side, so my solution was correct.

Reflecting

A. How is Renée's solution similar to solving equations with cubes and counters?

B. How do you think John knew which operations to use? Could he have done the same operations in a different order?

C. How are John's and Renée's solutions alike?

Example 3 | Solving equations involving division

Solve $\frac{c}{4} + 1 = 6$.

Solution

$$\frac{c}{4} + 1 = 6$$

$$\frac{c}{4} + 1 - 1 = 6 - 1$$

$$\frac{c}{4} = 5$$

To get c alone on one side of the equation, start by subtracting 1 from both sides.

$$4\left(\frac{c}{4}\right) = 4(5)$$

$$c = 20$$

$\frac{c}{4}$ means $c \div 4$, so you can figure out what c is by multiplying $\frac{c}{4} \times 4$.
Multiply the right side by 4 to keep the equation balanced.

Check:

$$\frac{c}{4} + 1 = 6$$

You can verify the solution by using a model. $\frac{c}{4} + 1 = 6$ can mean that, if you divide c counters into 4 groups and then add 1 to one of the groups, that group will have 6 in it.

$\frac{c}{4} \rightarrow$ 

Since $c = 20$, start with 20 counters. Divide the 20 counters into 4 groups.

$\frac{c}{4} + 1 \rightarrow$ 

$\frac{c}{4} + 1$ ✔

Add 1 counter to one of the groups. Now there are 6 counters in that group, so $\frac{c}{4} + 1 = 6$.

The solution is correct.

Example 4 | Solving equations with integers

Solve the equation $12 = -6(y - 4)$.

Solution A

$$12 = -6(y - 4)$$
$$12 = -6y + 24$$
$$12 - 24 = -6y + 24 - 24$$
$$-12 = -6y$$
$$\frac{-12}{-6} = \frac{-6y}{-6}$$
$$2 = y$$

Multiply each term in the brackets by −6.

Subtract 24 from both sides to isolate −6y.

Divide both sides by −6 to isolate y.

Check: Left side 12

Right side
$-6(y - 4)$
$= -6(2 - 4)$
$= -6(-2)$
$= 12$ ✔

Verify by substituting 2 for y in the equation. The left and right sides are equal, so the solutions are correct.

Solution B

$$12 = -6(y - 4)$$
$$\frac{12}{-6} = \frac{-6}{-6}(y - 4)$$
$$-2 = (y - 4)$$
$$-2 = y - 4$$
$$-2 + 4 = y - 4 + 4$$
$$2 = y$$

Divide both sides by −6 to simplify the equation.

Add 4 to both sides to isolate y.

A Checking

1. Solve each equation and record your steps.
   a) $a + 12 = 60$
   b) $39 = b - 13$
   c) $4s = 72$
   d) $17 = 4a + 5$
   e) $2(n - 4) = 6$
   f) $\frac{w}{5} - 3 = 17$

B Practising

2. Solve each equation and record your steps. Show one way to verify each solution.

a) $q - 17 = -31$

b) $-117 = -9s$

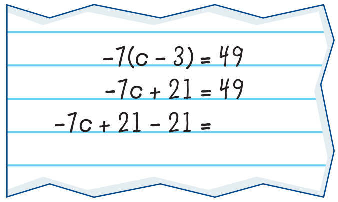
c) $\frac{z}{6} = -5$

d) $7d - 11 = 66$

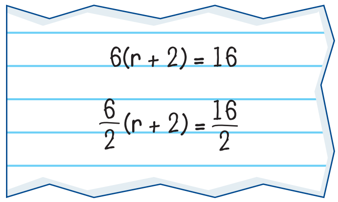
e) $14(n + 5)42$

f) $8 = \frac{t}{4} - 9$

3. Mei began solving the equation $-7(c - 3) = 49$. Copy and complete her solution.



4. Sam began solving the equation $6(r + 2) = 16$. Copy and complete his solution.



5. Solve $-3(x - 3) = -18$ in two different ways. Show what you did.

6. Solve each equation and record your steps. Show one way to verify each solution.

a) $11 = 5x - 4$

b) $-3m + 7 = 40$

c) $\frac{z}{5} = -7$

d) $-3(z - 8) = 6$

e) $9 = 2(x + 3)$

f) $9 - 7x = -12$

7. Alex says to Callie, "I am thinking of a number. If you divide my number by 8 and then subtract 12, the result is 14." Write and solve an equation to determine Alex's number.

8. a) Carina bought three cafeteria lunches—one for herself and one for each of her two friends. Each lunch had a veggie sub and a dessert. The desserts cost \$2 each and Carina paid \$18 altogether. Write an equation you can use to determine the cost of each sub.

b) Solve the equation. How much did each sub cost?

9. Solve each equation and record your steps. Show one way to verify each solution.

a) $8t - 1 = 5$

b) $\frac{k}{3} - 5 = 7$

c) $-4(f - 2) = 12$

d) $\frac{m}{6} + 9 = 5$

10. A Grade 8 class held a charity car wash and earned $368. They gave $200 to an animal shelter. They shared the rest of the money equally among four other charities. How much money did each charity receive?

11. Emily and her friends spent the day at the West Edmonton Mall. Each person paid $35 for admission to the Waterpark, and $25 to get into Galaxyland for the evening. The group also paid $40 to rent a cabana at the Waterpark. They spent $700 altogether. How many people were in the group?

12. Six friends took a trip to the Canadian Rockies. They swam in Banff Upper Hot Springs, rode a gondola at Lake Louise, and went on a glacier tour at the Columbia Icefields. Visiting all three attractions cost a total of $426 for the group. The gondola ride was $24 per person, and admission to the hot springs was $8 per person. How much did the glacier tour cost?

13. Aaron said, "When I solve an equation symbolically, it helps if I think about cubes and counters on a pan balance." Do you agree with Aaron? Explain.

9.7 Correcting Errors in Solutions

YOU WILL NEED

- a coin
- grid paper

GOAL

Verify solutions to linear equations and identify and correct any errors.

LEARN ABOUT the Math

Taira played four rounds in a golf tournament. Her final score was 288. She has a 3 handicap. This means that, to calculate her final score for each round, she subtracted her handicap, 3, from the number of strokes she took.

Here are three solutions that try to calculate Taira's mean score for each round before her handicap was applied. The expression $4(s - 3)$ was used to express her total score before the handicap was applied, where s represents the mean score for one round.

? Which one of these solutions is correct?

Solution 1	Solution 2	Solution 3
$4(s - 3) = 288$ $4(s - 3) = 288$ $4s - 12 = 288$ $4s - 12 - 12 = 288 - 12$ $4s = 276$ $\frac{4s}{4} = \frac{276}{4}$ $s = 69$ The mean score for each round without the handicap was 69.	$4(s - 3) = 288$ $\frac{4}{4}(s - 3) = \frac{288}{4}$ $s - 3 = 72$ $s - 3 + 3 = 72 + 3$ $s = 75$ The mean score for each round without the handicap was 75.	$4(s - 3) = 288$ $4s - 3 + 3 = 288 + 3$ $4s = 291$ $\frac{4s}{4} = \frac{291}{4}$ $s = 72\frac{3}{4}$ The mean score for each round without the handicap was $72\frac{3}{4}$.

A. Which solution is correct?

B. Describe the error in each incorrect solution.

Reflecting

C. How did you verify each solution and identify any errors?

D. Why is it important to verify the solution to an equation?

WORK WITH the Math

Example 1 | Identifying and correcting errors

Identify and correct the errors in this solution.

$$\begin{aligned} -2(x - 1) &= -22 \\ -2x - 2 &= -22 \\ -2x - 2 + 2 &= -22 + 2 \\ -2x &= -20 \\ \frac{-2x}{-2} &= \frac{-20}{-2} \\ x &= 10 \end{aligned}$$

Taira's Solution

$$\begin{aligned} -2(x - 1) &= -22 \\ -2x + 2 &= -22 \\ -2x + 2 - 2 &= -22 - 2 \\ -2x &= -24 \\ \frac{-2x}{-2} &= \frac{-24}{-2} \\ x &= 12 \end{aligned}$$

I did not have to verify this solution because the problem told me it was incorrect.
I found the errors by checking each step. I noticed a problem in the second line where the person multiplied −2 by −1 and got −2 instead of +2.
I fixed the problem and then finished the solution.

Check:

Left side	Right side
$-2(x - 1)$	-22
$= -2(12 - 1)$	
$= -2(11)$	
$= -22$ ✔	

I verified my solution, and it was right because both sides were equal.

A Checking

1. Verify each solution. Identify and correct any errors.

a)
$$\begin{aligned} 4x &= 24 \\ 4x - 4 &= 24 - 4 \\ x &= 20 \end{aligned}$$

b)
$$\begin{aligned} x + 4 &= 3 \\ x + 4 - 4 &= 3 + 4 \\ x &= 7 \end{aligned}$$

c)
$$\begin{aligned} 5(x + 2) &= 35 \\ 5(x + 2) \div 5 &= 35 \div 5 \\ x + 2 &= 7 \\ x + 2 - 2 &= 7 - 2 \\ x &= 5 \end{aligned}$$

B Practising

2. Solve each equation and verify your solution.

a) $5x + 3 = 38$ **b)** $\frac{x}{3} + 5 = -4$ **c)** $4(x - 2) = 20$

3. Verify each solution. Identify and correct any errors.

a)
$$\begin{aligned} 4(x + 3) &= 24 \\ 4(3x) &= 24 \\ 12x &= 24 \\ \frac{12x}{12} &= \frac{24}{12} \\ x &= 2 \end{aligned}$$

b)
$$\begin{aligned} 5(x + 2) &= 45 \\ 5x + 10 &= 45 \\ 15x &= 45 \\ \frac{15x}{15} &= \frac{45}{15} \\ x &= 3 \end{aligned}$$

c)
$$\begin{aligned} 5 + 4x &= 13 \\ 5 + 4x - 4x &= 13 - 4x \\ 5 &= 9x \\ \frac{5}{9} &= \frac{9x}{9} \\ \frac{5}{9} &= x \end{aligned}$$

4. Compare the errors you identified in question 3. What do you notice?

5. Solve each equation and verify your solution.
   a) $18 = 3(d - 1)$
   b) $3 - 2k = -9$
   c) $7 = \frac{s}{2} + 5$

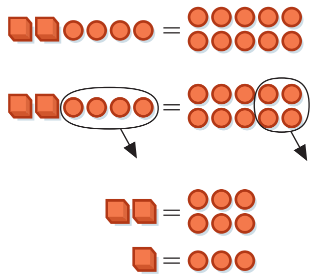
6. Verify each solution. Identify and correct any errors.

a)
$$\begin{aligned} 56 &= 8(x + 3) \\ 56 &= 8x + 24 \\ 56 - 24 &= 8x + 24 - 24 \\ 32 &= 8x \\ \frac{32}{32} &= \frac{8x}{32} \\ x &= \frac{8}{32} \\ x &= \frac{1}{4} \end{aligned}$$

b)
$$\begin{aligned} 7x - 2 &= -16 \\ 7x - 2 + 2 &= -16 + 2 \\ 7x &= -14 \\ x &= -2 \end{aligned}$$

c)
$$\begin{aligned} \frac{s}{6} + 3 &= 11 \\ 6\left(\frac{s}{6}\right) + 3 &= 6(11) \\ s + 3 &= 66 \\ s + 3 - 3 &= 66 - 3 \\ s &= 63 \end{aligned}$$

d) $2(d + 4) = 10$



$d = 3$

7. Katie's mother is buying school supplies for Katie and her four sisters. In one store she spends \$20. She buys a ruler for each child and a package of pens that costs \$3. Write and solve an equation to determine the cost of each ruler. Verify your solution.

8. When you solve an equation, why is it important to record your solution steps as well as the answer?

Math GAME

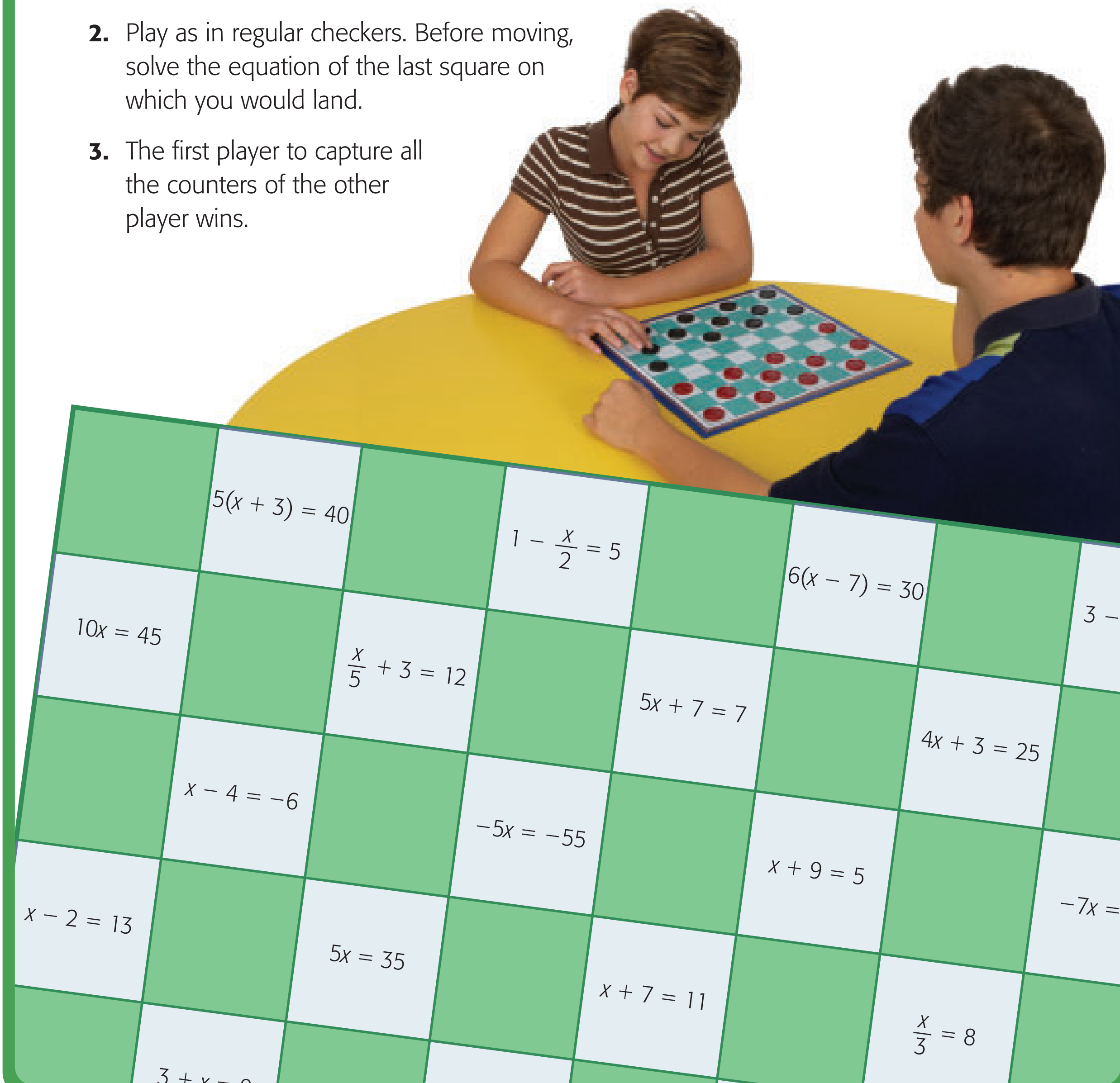
Equation Checkers

Number of players: 2

YOU WILL NEED
- a checkerboard with equations attached as shown
- 12 checkers for each player

How to Play

1. Set up the counters on the squares with equations, as in a regular game of checkers.
2. Play as in regular checkers. Before moving, solve the equation of the last square on which you would land.
3. The first player to capture all the counters of the other player wins.



9.8 Solve Problems Using Logical Reasoning

GOAL

Solve problems that involve equations using logical reasoning.

LEARN ABOUT the Math

? How can Holly use logical reasoning to determine Angèle's target heart rates?

Example 1 | Using logical reasoning

Determine Angèle's minimum and maximum target heart rates.

Holly's Solution

1. Understand the Problem

I know Angèle's resting heart rate. I can use that to determine the other rates.

2. Make a Plan

I will relate each pair of heart rates with equations and then solve them.

3. Carry Out the Plan

$r + 39 = m$

I wrote an equation to express Angèle's minimum target heart rate, *m*, in relation to her resting target heart rate, *r*.

$60 + 39 = 99$

Angèle's minimum target heart rate is 99 beats per minute.

I substituted her resting heart rate to evaluate her minimum target heart rate.

$2m - 36 = x$

I wrote an equation to relate the maximum rate, *x*, to the minimum rate.

$2(99) - 36 = x$

$198 - 36 = x$

$162 = x$

Angèle's maximum target heart rate is 162 beats per minute.

I substituted her minimum target heart rate to solve for her maximum heart rate.

4. Look Back

Angèle's minimum heart rate is 39 more than her resting rate.

$60 + 39 = 99$

Angèle's maximum heart rate is 36 beats less than twice her minimum rate.

$2 \times 99 - 36 = 162$

I verified my answers.
My answers matched what it said in the problem.

Reflecting

A. How did Holly use logical reasoning to solve the problem?

WORK WITH the Math

Example 2 | Using logical reasoning and equations

A senior admission at the Royal Tyrrell Museum is $2 less than an adult admission. A youth admission is $2 more than $\frac{1}{2}$ a senior admission. The youth admission is $6. What are the senior and adult admissions?

Holly's Solution

1. Understand the Problem

I know the youth admission. I can use that to determine the senior admission. Then I can use the senior admission to determine the adult admission.

2. Make a Plan

I will relate each pair of admissions with equations and then solve them.

3. Carry Out the Plan

$$\frac{s}{2} + 2 = y$$

I wrote an equation to express the youth admission, *y*, in relation to the senior admission, *s*.

$$\frac{s}{2} + 2 = 6$$

I substituted the youth admission to solve for the senior admission.

$$\frac{s}{2} + 2 - 2 = 6 - 2$$

$$\frac{s}{2} = 4$$

$$s = 8$$

The senior admission is $8.

$$a - 2 = s$$

I wrote an equation to express the adult admission, *a*, in relation to the senior admission, *s*.

$$a - 2 = 8$$

I substituted the senior admission to solve for the adult admission.

$$a - 2 + 2 = 8 + 2$$

$$a = \$10$$

The adult admission is $10.

4. Look Back

The senior admission is $8. Half of that is $4.
The youth admission should be $2 more, or $6.
The adult admission is $10. The senior admission is $2 less, or $8. ✔

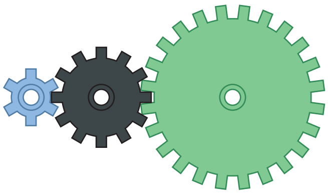
I verified my answers.
My answers matched what it said in the problem.

A Checking

1. Solve this problem using logical reasoning and an equation. Jie has $36 in the bank. She deposits the same amount of money each week. Three weeks later, she has $162. How much money did she deposit each week?

B Practising

2. Vanessa is a Winnipegger who is planning a vacation to Los Angeles. The return fare on Airline A is $65 more than the bus fare. The bus fare is $177 more than one-half of the fare on Airline B. The air fare on Airline A is $464. Determine the Airline B fare and the bus fare.



3. Jasper put together a set of gears in science class. The circumference of the green gear is 24 cm. The circumference of the black gear is 4 cm more than one-third of the green gear. The circumference of the blue gear is half of the black gear. Determine the circumference of the black gear and blue gear.

4. Alexis likes whitewater rafting. A two-day rafting trip on the Elaho and Squamish rivers near Whistler, BC, is $300, while the cost of a one-day trip on the Tatshenshini River in the Yukon is $125.

a) Write a problem about the cost of the two trips.

b) Exchange your problem with a classmate. Have your classmate solve your problem to see if it is set up correctly.

5. The students in Mr. Hegel's math class are trying to guess his age. He tells them that his daughter is four years younger than one-third of his age. When asked about his daughter, he tells the class that she is two years older than half of Meg's age. Meg, a student in the class, is 14. How old is Mr. Hegel? Explain your answer.

Reading Strategy

Inferring

What clues in the text help you better understand the problem?

6. How can using equations make it easier for you to solve problems logically?

CURIOUS Math

A Winning Formula for Billiards

YOU WILL NEED
- a ruler
- a protractor

How many rebounds will a billiard ball make on its way to the corner pocket?

Number of rebounds = *length of table* + *width of table* – 2

This winning formula only works when:
- The greatest common factor of the length and width is not 1. (Express the ratio of length to width in lowest terms. For example, a table 6 units long and 4 units wide has the same rebound formula as a table 3 units long and 2 units wide. Write 3 + 2 – 2.)
- The table has only corner pockets.
- The length and width of the table are whole numbers.
- The ball is hit from a corner at a 45° angle.
- The rebounds do not include the original hit or the sinking of the ball in the corner pocket.

Number of rebounds $= l + w - 2$
$= 4 + 3 - 2$
$= 7 - 2$
$= 5$

1. Draw models of billiard tables on grid paper with these dimensions: 5 by 3, 6 by 4, 6 by 5, and 12 by 8.

2. For each table, predict (using the formula) the number of rebounds a ball will take to reach a corner pocket.

Size of table	5 by 3	6 by 4	6 by 5	12 by 8
Number of rebounds				

3. Check your predictions by drawing the path of the ball on each table using a ruler and a protractor.

Chapter Self-Test

1. a) Create a table of values for $y = -3(x - 2)$.

b) Graph the points. Use your graph to identify four more points to add to your table.

2. Solve. Record your steps and verify your solution.

a) $3a = 21$ **c)** $2c - 7 = -2$ **e)** $-5 = \frac{t}{4} - 3$

b) $\frac{x}{6} = -4$ **d)** $-20 = 4(z - 1)$ **f)** $-2(z - 3) = 12$

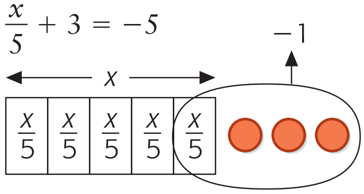
3. Allison is flying to Québec in 7 weeks. She has \$90 and the return airfare is \$405. How much must she save each week? Use an equation.

4. Verify each solution. If the solution is incorrect, identify the error and write the correct solution.

a)

$$2(a - 5) = 24$$
$$2(a - 5 + 5) = 24 + 5$$
$$2(a) = 29$$
$$\frac{2a}{2} = \frac{29}{2}$$
$$a = 14\frac{1}{2}$$

b) $\frac{x}{5} + 3 = -5$



$\frac{x}{5}$ and 3 more $= -1$

$$\frac{x}{5} = -4$$
$$x = -\frac{4}{5}$$

5. Patrick and John are planning a trip to the Northwest Territories. Trip A lasts for three days. Trip B lasts for six days. Trip C costs \$2750 for eight days. Trip C is twice the cost of Trip B, minus \$1650. Trip B is twice the cost of Trip A, minus \$100. How much is Trip A?

What Do You Think Now?

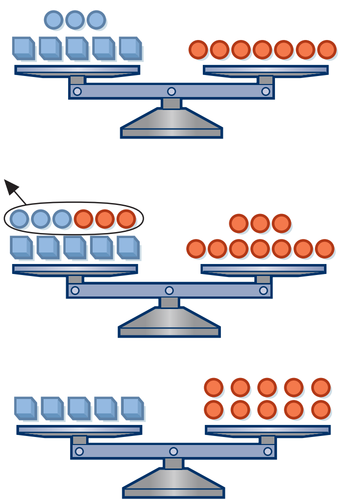
Revisit What Do You Think? on page 365. How have your answers and explanations changed?

Chapter 9

Chapter Review

Frequently Asked Questions

Q: How can you solve an equation by balancing?



A: An equation is like a level pan balance because the two sides are equal. As you solve an equation, you must add, subtract, multiply, or divide the same way on both sides to keep the balance. For example, solve $-5x - 3 = 7$.

$$\begin{aligned} -5x - 3 &= 7 \\ -5x - 3 + 3 &= 7 + 3 \\ -5x &= 10 \\ \frac{-5x}{5} &= \frac{10}{5} \\ -x &= 2 \\ x &= -2 \end{aligned}$$

Add 3 to both sides, to balance.

Divide each side by 5 to determine the value of $-x$.

If $-x = 2$, then $x =$ the opposite, -2.

The solution is $x = -2$.

Q: How do you solve an equation with an expression in brackets?

A1: Sometimes you can isolate the variable by dividing both sides by the multiplier and then adding or subtracting to isolate further. For example,

$$\begin{aligned} -2(x + 5) &= -8 && \text{Divide both sides by } -2. \\ x + 5 &= 4 && \text{Add } -5 \text{ to both sides.} \\ x + 5 - 5 &= 4 - 5 \\ x &= -1 \end{aligned}$$

A2: Sometimes you multiply through and then solve the equation. For example,

$$\begin{aligned} -2(x + 5) &= -9 && \text{Multiply the terms inside the brackets by } -2. \\ -2x + (-10) &= -9 \\ -2x + (-10) + 10 &= -9 + 10 && \text{Add 10 to both sides.} \\ -2x &= +1 && \text{Divide both sides by } -2. \\ x &= -\frac{1}{2} \end{aligned}$$

Q: How can you verify and correct the solution to an equation?

A: Substitute the solution into the original equation to see whether the left side and right side are equal. If the solution is incorrect, carefully work through each line to find the error.

For example, verify this solution.

$$
\begin{aligned}
3 - 4x &= 15 \\
3 - 3 - 4x &= 15 - 3 \\
4x &= 12 \\
\frac{4x}{4} &= \frac{12}{4} \\
x &= 3
\end{aligned}
$$

Check: Left side | Right side

$3 - 4x$ | 15

$= 3 - 4(3)$

$= 3 - 12$

$= -9$ ✗

The solution is incorrect. When 3 was subtracted, the left side became $-4x$, not $4x$.

New Solution

$$
\begin{aligned}
3 - 4x &= 15 \\
3 - 3 - 4x &= 15 - 3 \\
-4x &= 12 \\
\frac{-4x}{-4} &= \frac{12}{-4} \\
x &= -3
\end{aligned}
$$

Verify the new solution. Check:

Left side | Right side

$3 - 4x$ | 15

$= 3 - 4(-3)$

$= 3 - (-12)$

$= 15$ ✔

The new solution is correct.

Practice

Lesson 9.1

1. Janelle is filling a swimming pool at a rate of 120 L/min. When she begins, 4000 L of water are already in the pool.

a) Write an equation for the amount of water in the pool after t min.

b) Janelle is tracking water use for a conservation study. Create a table of values for the amount of water in the pool after 15, 30, 45, and 60 min.

Lesson 9.2

2. Janek has \$480 in his bank account. Each week, he takes out \$24 for karate lessons.

a) Make a graph to show how the amount in Janek's account changes each week.

b) How much money is in Janek's account after 7 weeks?

c) How many karate lessons has Janek taken if he has \$264 in his account?

Lesson 9.4

3. Solve each equation by drawing a diagram.

a) $3(x + 4) = y$, when $y = 27$ **b)** $\frac{x}{3} - 7 = y$, when $y = 2$

Lesson 9.5

4. Model and solve each equation.

a) $3t - 4 = 8$ **b)** $11 = 8z + 5$ **c)** $5 - 3x = 26$

Lesson 9.6

5. Solve each equation.

a) $-6(a + 7) = 54$ **b)** $16 = \frac{t}{8} - 32$ **c)** $-5x - 9 = -7$

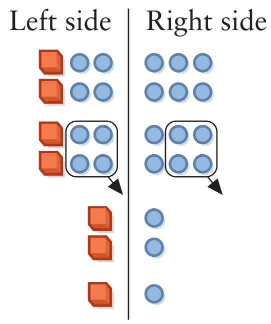
Lesson 9.7

6. Verify the solution. Identify and correct any errors.

$$19 - 7x = -2$$
$$19 - 19 - 7x = -2 - 19$$
$$-7x = -21$$
$$\frac{-7x}{7} = \frac{-21}{7}$$
$$x = -3$$

7. Verify each solution. Identify and correct any errors.

a) $2(d - 2) = -6$



$d = -1$

b)

$$\begin{aligned} -3(p + 6) &= -15 \\ -3(p + 6 - 6) &= -15 - 6 \\ -3p &= -21 \\ \frac{-3p}{-3} &= \frac{-21}{-3} \\ p &= 7 \end{aligned}$$

Lesson 9.8

8. A whitewater rafting company offers three different tours. The beginners' tour is \$19 more than half the cost of the intermediate tour. The advanced tour is \$25 less than double the cost of the beginners' tour. The advanced tour is \$85. How much do the other two tours cost?

Chapter 9

Chapter **Task**

Task | *Checklist*

- ✔ Did you show all your calculations?
- ✔ Did you verify your solutions?
- ✔ Did you explain your thinking?

Planning a Dragon Boat Festival

Many Canadian cities host dragon boat festivals, such as this one in Kelowna.

One festival wants to raise money for charity.

- There will be 600 volunteers and between 140 and 180 teams.
- Each team will have 25 members and pay $1600 to enter.
- Each team will compete in three races.
- Each race will have five teams and take 10 min.
- The festival has raised $252 000 from sponsors.
- The racers and volunteers will all celebrate at a dinner that will cost the festival $20 for each person.
- All profits go to charity.

? How much money will the festival be able to donate to charity?

Prepare a report for the festival. Support your conclusions with equations, tables of data, and graphs. State any assumptions.

A. Make a graph the schedulers can use to determine the number of races that will be held depending on how many teams there are.

B. Determine the minimum and maximum number of hours the races will take.

C. Determine the minimum and maximum cost of the food for team members and volunteers.

D. You hope to raise $500 000, including the money from sponsors. How many teams need to enter to meet your goal?

E. Additional expenses are $38 000. If you do raise $500 000, how much money will you be able to donate to charity?