Percent, Ratio, and Rate

Just for Fun

One at a Time

Change TWO into TEN by changing one letter at a time. Each step must be a real word.

T	W	O
T	E	N

This one might look easy. But, can it be done? If you say no, show why not.

R	A	T	I	O
R	A	T	E	S

_____

_____

_____

_____

Decimal

Make as many words as you can from the letters of the word DECIMAL.

2-letter words: _____

3-letter words: _____

_____

4-letter words: _____

_____

_____

5-letter words: _____

6-letter words: _____

7-letter words: _____

_____

A Game for 4

Make a Pair

Get a deck of playing cards and remove the face cards.

Deal each player 6 cards. Put the remaining cards face down in a stack.

The first player turns over one card. If she holds a number that divides evenly into the number showing, that is a "pair." She takes the card and puts the pair face down.

The next player can take the card showing if the previous player could not make a pair, but he can. Then he turns over one more card from the stack for an extra turn.

Continue until one player is out of cards. The player who made the most pairs wins.

Variation: Use the face cards as well. A jack represents 12, a queen represents 15, and a king represents 20.

Activating Prior Knowledge

Relating Fractions, Decimals, and Percents

To write a fraction as a decimal, divide: $\frac{4}{5} = 4 \div 5 = 0.8$

To write a decimal as a percent, multiply by 100%: $0.8 \times 100\% = 80\%$

To write a percent as a decimal, divide by 100%: $80\% \div 100\% = 0.8$

1. Write each fraction as a decimal and a percent.

a) $\frac{3}{50} = 3 \div$ _____
$=$ _____
$=$ _____%

b) $\frac{7}{25} =$ _____
$=$ _____
$=$ _____%

c) $\frac{3}{5} =$ _____
$=$ _____
$=$ _____%

2. Write each percent as a decimal and as a fraction.

a) 36% = _____ = _____

b) 5% = _____ = _____

c) 44% = _____ = _____

d) 86% = _____ = _____

Finding Common Multiples

➤ To find the common multiples of 2, 4 and 5:

List the multiples of each number.

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, **20**, 22, 24, 26, 28, 30, 32, 34, 36, 38, **40**, ...

The multiples of 4 are 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, ...

The multiples of 5 are 5, 10, 15, **20**, 25, 30, 35, **40**, 45, 50, ...

The common multiples of 2, 4, and 5 are 20, 40, 60, ...

3. Find the first 5 multiples of each number.

a) 6: 6, 12, _____

b) 9: 9, _____

c) 15: _____

4. Find two common multiples for each set of numbers.

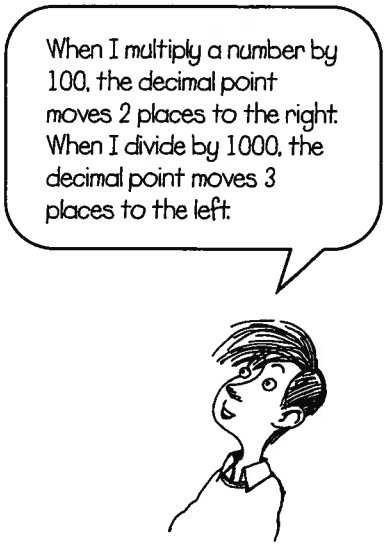
a) 15, 25

Multiples of 15 are _____

Multiples of 25 are _____

Two common multiples of 15 and 25 are _____ and _____.

b) 6, 10 _____ **c)** 4, 10, 15 _____



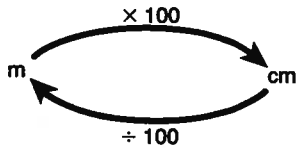
Converting between Metric Units

1 m = 100 cm 1 km = 1000 m

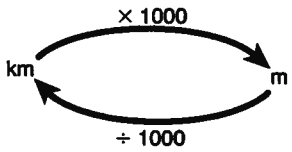
1 kg = 1000 g 1 L = 1000 mL

➤ To convert to a smaller unit, multiply.

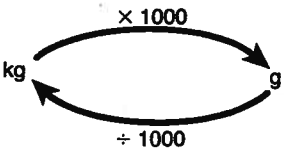
➤ To convert to a larger unit, divide.



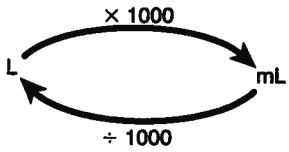
2.75 m = 2.75 × 100 cm
= 275 cm



2450 m = $\frac{2450}{1000}$ km
= 2.45 km



425 g = $\frac{425}{1000}$ kg
= 0.425 kg



3.4 L = 3.4 × 1000 mL
= 3400 mL

5. Convert. Show your work.

a) 3650 cm to metres

3650 cm = _____ m

= _____ m

b) 5260 mL to litres

5260 mL = _____ L

= _____ L

c) 17 kg to grams

17 kg = _____ g

= _____ g

d) 75 km to metres

75 km = _____ m

= _____ m

Quick Review

At Home

➤ Sophia and Jacob are in a basketball free-throw competition. Sophia makes 11 out of her 20 free throws and Jacob makes 10 out of 16 of his free throws. Who has the best free-throw percentage?

Sophia makes 11 out of 20 free throws, which can be expressed as the fraction $\frac{11}{20}$.

Percent means per hundred. To write the fraction as a percent, write the fraction with a denominator of 100: $\frac{11 \times 5}{20 \times 5} \times \frac{55}{100}$

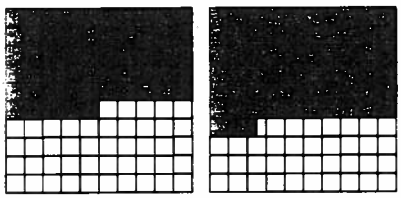
$\frac{55}{100}$ can be expressed as the decimal 0.55 or as the percent 55%.

Jacob made 10 out of 16 free throws, which can be expressed as the fraction $\frac{10}{16}$, or $\frac{5}{8}$.

Express $\frac{5}{8}$ with a denominator of 1000.

$$\frac{5 \times 125}{8 \times 125} = \frac{625}{1000}$$
$$= \frac{625 \div 10}{1000 \div 10}$$
$$= \frac{62.5}{100}$$
$$= 62.5\%$$

Shade hundred charts to represent Sophia's and Jacob's percentage of successful free throws. Sophia made 55% of her free throws, so 55 squares on the hundred chart are shaded. Jacob made 62.5% of his free throws, so $62\frac{1}{2}$ squares on the hundred chart are shaded. Jacob had the better free throw percentage.

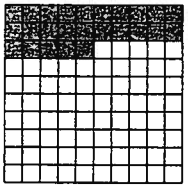


One small square on a hundred chart can be enlarged to show 100 squares. This is called a *hundredths chart.*

Each small square of a hundredths chart represents $\frac{1}{100}$ of 1%, or $\frac{1}{100}$%, or 0.01%.

$\frac{1}{4}$ of 1% or $\frac{1}{4}$% can be represented on the hundredths chart by shading $\frac{1}{4}$ of the hundredths chart, which is 25 squares.

$\frac{1}{4}\% = 0.25\%$

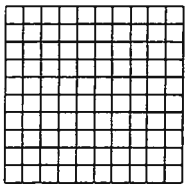


$\frac{1}{4}$ % can be written as a decimal.

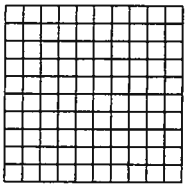
$\frac{1}{4}\% = \frac{0.25}{100} = \frac{25}{10\,000} = 0.0025$

1. Each hundred chart represents 100%. Shade the chart to represent the given percent.

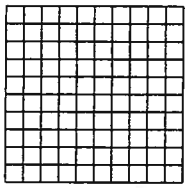
a) 30%



b) 7%



c) 86%



2. Write each percent as a fraction and as a decimal.

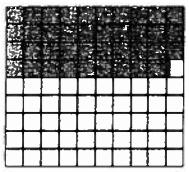
a) $6\% = \frac{\square}{100} = \_\_\_\_\_$

b) $87\% = \_\_\_\_\_ = \_\_\_\_\_$

c) $48\% = \_\_\_\_\_ = \_\_\_\_\_$

3. Each hundred chart represents 100%. What fraction is shaded? Write each fraction as a decimal and as a percent.

a)



$\frac{\square}{100} = \frac{\square}{10\,000} = \_\_\_\_\_ = \_\_\_\_\_$

b)

$\_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_ = \_\_\_\_\_$

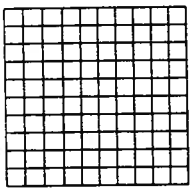
4. Write each percent as a fraction and as a decimal.

a) $48.5\% = \frac{\square}{100} = \frac{\square}{1000} = \_\_\_\_\_$

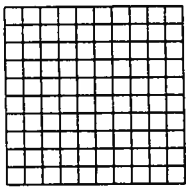
b) $10.75\% = \frac{\square}{100} = \frac{\square}{1000} = \frac{\square}{10\,000} = \_\_\_\_\_$

5. Use a hundred chart to represent 1%. Shade the chart to represent each percent.

a) 0.5%



b) 0.95%



6. Write each percent as a fraction and as a decimal.

a) $0.75\% = \frac{\square}{100} = \frac{\square}{10\,000} =$ _____

b) $0.4\% = \frac{\square}{100} = \frac{\square}{1000} =$ _____

7. Write each fraction as a decimal and as a percent. Use a calculator if necessary.

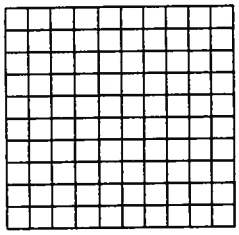
a) $\frac{10}{500} =$ _____ = _____ b) $\frac{15}{200} =$ _____ = _____ c) $\frac{17}{400} =$ _____ = _____

8. Use coloured pencils to shade the hundred chart.

Shade $\frac{3}{8}$ of the grid squares in the rectangle red.

Shade 25% of the grid squares green.

Shade 0.315 of the grid squares blue.



a) Explain how you decided on the number of squares to shade each colour.

b) What fraction of the hundred chart is not shaded? _____

c) Write the fraction that is not shaded as a decimal and as a percent.

_____

9. In a parking lot, 19 out of 40 cars are hybrids. Express the number of cars in the parking lot as a fraction and as a percent.

In the parking lot, _____ or _____ of the cars are hybrids.

10. Milo will pay $8\frac{1}{4}\%$ on a loan from a bank. Express the interest rate as a fraction and as a decimal.

$8\frac{1}{4}\%$ is the same as 8._____%.

The interest rate is _____ or _____.

Quick Review

At Home / At School

To calculate a percent of a quantity, first write the percent as a decimal.
Then calculate the decimal value of the quantity.

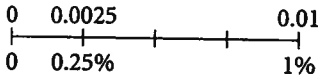
To find 140% of \$850, write 140% as a decimal. $140\% = \frac{140}{100} = 1.40$
Then, 140% of \$850 $= 1.40 \times \$850$
$= \$1190$

This answer can be illustrated on a number line.

Percents that are less than 1% can also be illustrated on a number line.
$1\% = \frac{1}{100} = 0.01$

Use this pattern:
100% = 1.0 10% = 0.10 1% = 0.01

0.25% = 0.0025

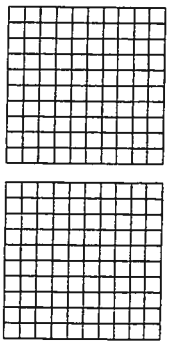


HINT

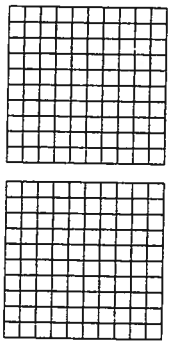
To change a percent to a decimal, move the decimal point 2 places to the left.

1. One hundred chart represents 100%. Shade hundred charts to show each percent.

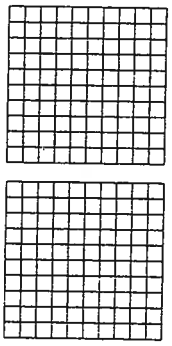
a) 125%



b) 150%



c) 200%



2. a) Write 175% as a decimal and draw a number line to show this percent.

175% = _____

b) Write 0.5% as a decimal and draw a number line to show this percent.

0.5% = _____

3. Write each percent as a decimal.

a) 230% _____ b) 185% _____ c) 324% _____

d) 0.74% _____ e) 0.7% _____ f) 0.09% _____

4. Write each fraction as a percent.

a) $\frac{1}{2}$ _____ b) $\frac{3}{2}$ _____ c) $\frac{5}{2}$ _____

d) $\frac{1}{100}$ _____ e) $\frac{1}{200}$ _____ f) $\frac{3}{200}$ _____

5. a) Draw a square with sides 1 cm long.

b) Redraw the square so the sides are 200% of the original length.
The new square has sides of length _____ cm.

6. a) Find the percent of each number. Draw number line to illustrate each answer.

i) 200% of 40

ii) 20% of 40

iii) 2% of 40

b) Describe the pattern in the answers to part a).

_____

c) Use the pattern in part a) to find each percent of 40.

i) 2000% of 40 = _____ ii) 0.2% of 40 = _____

> **Tip**
> *Extend the pattern both ways—increase and decrease by a factor of 10.*

7. A total of 45 412 runners participated in the Vancouver Sun Run.
Of these runners, 0.85% completed the run in under 40 min.

How many runners completed the run in under 40 min? _____

In fact, 0.13% of the runners completed the run in less than 34 min.

How many runners were in this group? _____

8. Which is the greater amount of money, 120% of 0.3% of $1000 or 120.3% of $1000?
Explain.

_____

Quick Review

At Home / At School

Several hundred students were surveyed. 160 students were from one school. These students represent 40% of those surveyed.

To find how many students were surveyed, follow these steps:
40% of those surveyed is 160.
1% of those surveyed is $\frac{160}{40} = 4$
100% of those surveyed is $4 \times 100 = 400$

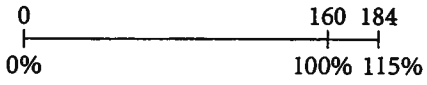
In the next survey, 15% more students were surveyed from the same school. To find the number of students surveyed, use the original number, 160, as 1 whole.

Method 1: The increase was 15%.
The new number is: 100% + 15% = 115%
115% of 160 = $1.15 \times 160 = 184$

> **Tip**
> *Choose the method you feel comfortable using.*

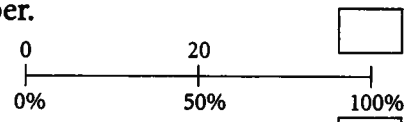
Method 2: The increase was 15%. 160 is 100%.
15% of 160 = $0.15 \times 160 = 24$
The new number is: 160 + 24 = 184

Both methods show that the new number of students surveyed is 184. This result can be illustrated on a number line.

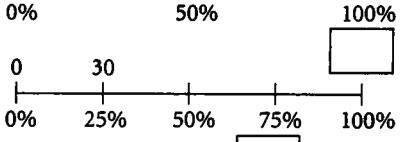


1. Use a number line to find each number.

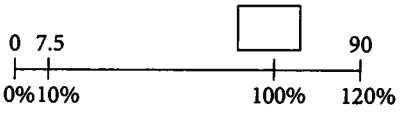
a) 50% of a number is 20



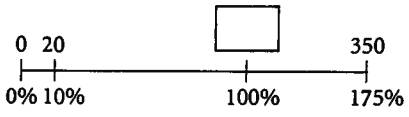
b) 25% of a number is 30



c) 120% of a number is 90



d) 175% of a number is 350



2. Find the number in each case.

a) 6% of a number is 9.

6% = 9

1% = _____

100% = _____

b) 28% of a number is 56.

28% = _____

1% = _____

100% = _____

c) 150% of a number is 36.

150% = _____

1% = _____

100% = _____

3. Write each increase as a percent. Illustrate each answer on a number line.

a) The width of the rectangle increased from 8 cm to 12 cm.

Increase = 12 cm – 8 cm = _____

Increase as a fraction of the original = _____ = _____

Percent increase = _____ × 100% = _____.

b) The price of a hotel room increased from $90.00 to $120.00.

Percent increase = _____

4. Write each decrease as a percent. Illustrate each answer on a number line.

a) The volume of water in the tank decreased from 40 L to 32 L.

Decrease = 40 L – 32 L = _____

Decrease as a fraction of the original = _____ = _____

Percent decrease = _____ × 100% = _____.

b) The number of students in the class decreased from 30 to 27.

Percent decrease = _____

5. In a batch of eggs, 3% were broken. There were 18 broken eggs.

How many eggs were there in the batch?

_____

Tip

Identify which number represents 1 whole, or 100%.

6. The prices for a day pass for skiing are:

Low Season: $52 High Season: $64 Spring Season: $58

a) Write the increase in cost from Low Season to High Season. _____
Illustrate the percent increase on a number line.

b) Write the decrease in cost from High Season to Spring Season. _____
Illustrate the percent decrease on a number line.

7. a) The rural population of Quebec is about 1 650 000.
This represents 22% of the population of Quebec. Estimate the population of Quebec.

_____

b) The population of Yukon Territory is about 31 400. Of these, 18 840 live in urban areas.
What percent of the population of Yukon Territory lives in rural areas?

_____

8. A fish tank contains 24 L of water. Water is added to increase the volume by 12.5%.
What is the new volume of water in the tank?

_____

9. Thirty-six percent of a number is 63. Find 124% of the number.

_____

10. A factory produces 900 items per week at a unit cost of $75. New equipment is installed that increases the productivity by 12% and reduces the unit cost by 16%.

a) What is the new production rate?

_____

b) What is the new unit cost?

_____

KEY TO SUCCESS

Problems can always be solved in more than one way. If you cannot solve a problem by one method, look at the problem from another view for an alternative method.

Sales Tax and Discount

Quick Review

At Home

When an item is sold at a reduced price, we say there is a **discount.**
In many provinces, taxes are added to the selling price.

Corey works in a shoe store. She has to calculate the cost of a pair of running shoes priced at $129 that is on sale at 20% off.

A discount of 20% means the sale price is:
100% – 20% = 80% of the regular price
80% of $129 = 0.8 × $129 = $103.20
The total sales tax in Corey's province is 13%. The tax is:
13% of $103.20 = 0.13 × $103.20 = $13.42
So, the cost of the running shoes is:
$103.20 + $13.42 = $116.62

Tip

Always round money amounts to the nearest hundredth of a dollar.

This can be calculated directly as: 113% of $103.20 = 1.13 × $103.20 = $116.62

1. Calculate a 14% tax on each item.

a) $288

_____ × $288

= _____

b) $36.50

_____ × _____

= _____

c) $149.99

_____ × _____

= _____

2. Calculate the cost, including 15% total sales tax, for each item.

a) $2.40

_____ × $2.40

= _____

b) $3428

_____ × _____

= _____

c) $128.79

_____ × _____

= _____

3. Calculate each discount and the sale price before tax.

a) $92 watch, 30% off

Discount: _____

Sale price: _____

b) $476 TV, 15% off

Discount: _____

Sale price: _____

4. Calculate the discount, sale price before taxes, and sale price including 13% total tax.

a) $28.95 book at 10% off

Discount: _____

Sale price: _____

13% tax: _____

Total cost: _____

b) $239 coat at 25% off

Discount: _____

Sale price: _____

13% tax: _____

Total cost: _____

5. The cost of a ticket for a CFL game 3 years ago was $36.00. The cost of the ticket has increased by 25%. Calculate the new cost of the ticket.

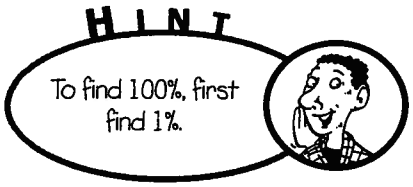
Increase in price: _____ Total cost: _____

6. Store A offers successive discounts of 10% one week and 20% the second week.
Store B offers a one-time discount of 25% the second week.

Which store offers the greater discount?

Store _____ offers the greater discount.

7. At a discount of 25%, skateboards are on sale for $135.
What is the original price?



The original price is _____.

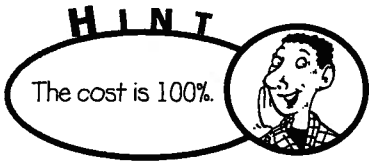
8. A TV set, regularly priced at $256, is offered for sale at 25% off. Sales tax is 15%.

a) Calculate the sale price at a 25% discount and then add 15% sales tax to it.

b) Add 15% tax to the original price and then calculate the sale price at a 25% discount.

Which calculation results in the greater discount? _____

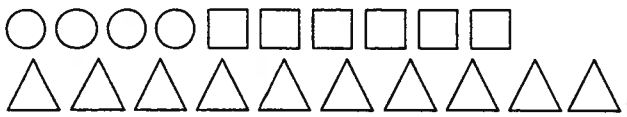
9. The sales tax in Ontario is 13%.
Janis pays a total of $32.77 for a fishing pole.
Find the cost of the fishing pole before sales tax.



Quick Review

At Home

The picture shows 4 circles, 6 squares, and 10 triangles.



Here are some ways you can use ratios, fractions, and percents to compare the shapes.

➤ **Part-to-Whole Ratios**

The ratio of circles to all of the shapes is 4 to 20 or 4:20.

This part-to-whole ratio can be written as the fraction $\frac{4}{20}$ or $\frac{1}{5}$.

It can also be written as a percent. $\frac{4}{20} = \frac{20}{100} = 20\%$

20% of the shapes are circles.

➤ **Part-to-Part Ratios**

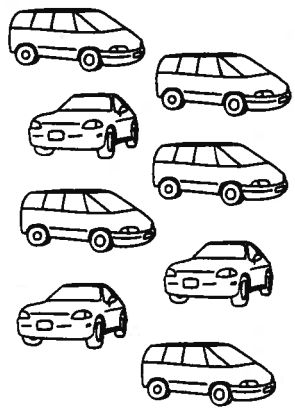
The ratio of circles to squares is 4 to 6 or 4:6. 4 and 6 are the terms of the ratio.

The ratio of circles to squares to triangles is 4 to 6 to 10 or 4:6:10.
This is called a three-term ratio.

A part-to-part ratio cannot be written in fraction or percent form, as it is not comparing one part to the whole.

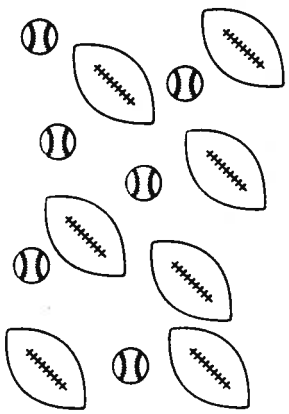
1. Write each ratio.

a) cars to vans



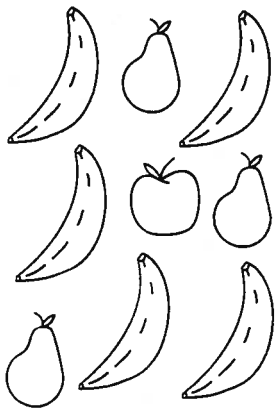
_____ : _____

b) footballs to baseballs



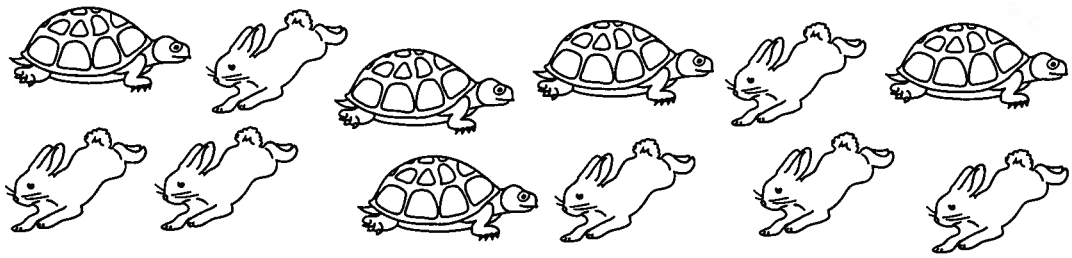
_____

c) bananas to fruit



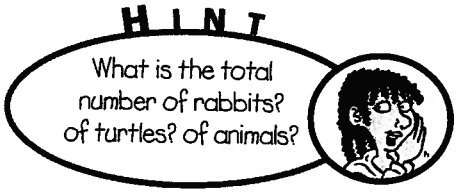
_____

2. Write each part-to-whole ratio as a ratio, a fraction, and a percent. Round percents to 2 decimal places.

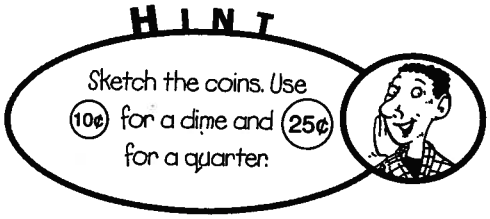


a) turtles to total animals ___ : ___, ___, _____

b) rabbits to total animals ___ : ___, ___, _____



3. Franny has only dimes and quarters in her pocket. The ratio of dimes to total coins is 8 to 11.



a) How many quarters might be in Franny's pocket? _____

b) What is the ratio of dimes to quarters? _____

c) What is the ratio of quarters to the total number of coins? _____

4. Make a sketch to show that the ratio of triangles to circles is 6:13.

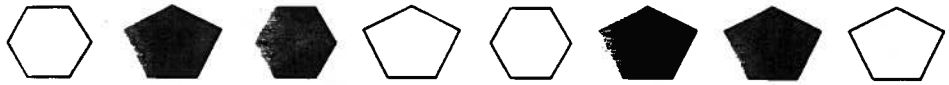
Write 3 ratios to compare the figures.

a) circles to triangles _____

b) circles to total figures _____

c) triangles to total figures _____

5. Write each ratio.



a) hexagons to pentagons _____

b) pentagons to hexagons _____

c) hexagons to total shapes _____

d) pentagons to total shapes _____

e) black figures to white shapes _____

f) white hexagons to black hexagons to white pentagons _____

6. What objects are being compared in each ratio?

a) 7:15 _____ to total vegetables

b) 2:7 tomatoes to _____

c) 2:7:6 _____ to carrots to _____

d) 6:7 _____ to _____

e) $\frac{2}{15}$ _____ to _____

f) $\frac{6}{15}$ _____ to _____

7. A pencil case contains 7 yellow, 3 red, 1 black, and 5 green pencil crayons.

a) Write each ratio.

- red:green _____
- yellow:red _____
- black:total pencil crayons _____
- yellow:total pencil crayons _____
- yellow:red:green _____

b) What is the ratio of yellow and red pencil crayons to total pencil crayons? _____

What percent of all the pencil crayons are red or yellow? _____

c) What is the ratio of green pencil crayons to black and red pencil crayons? _____

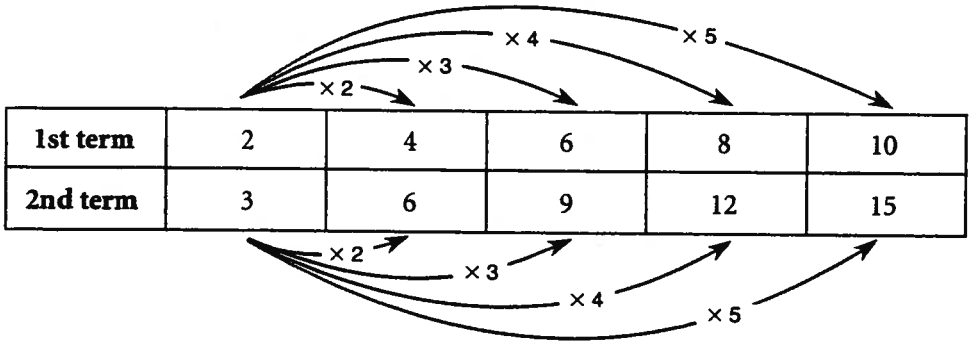
d) Suppose 2 yellow and 2 green pencil crayons are lost.
Rewrite the ratios in part a).

- red:green _____
- yellow:red _____
- black:total pencil crayons _____
- yellow:total pencil crayons _____
- yellow:red:green _____

5.6 Equivalent Ratios

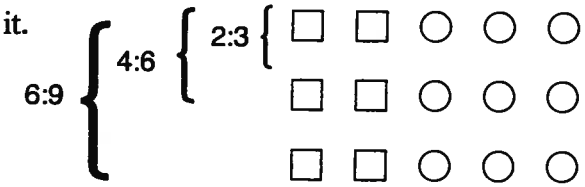
Quick Review

➤ You can find **equivalent ratios** by multiplying.
Multiply the terms by the same number.

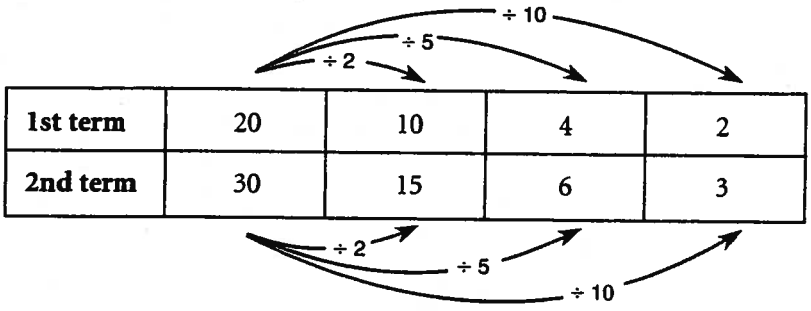


Four equivalent ratios of 2:3 are: 4:6, 6:9, 8:12, and 10:15.

Picture it.

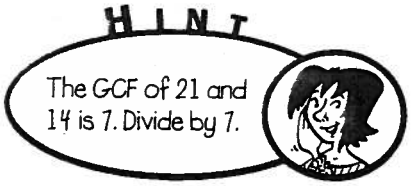


➤ You can also find equivalent ratios by dividing.
Divide the terms by the same number.



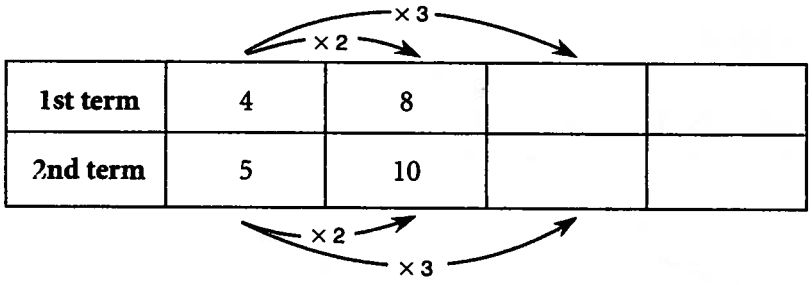
Three equivalent ratios of 20:30 are: 10:15, 4:6, and 2:3.

➤ To write a ratio in its simplest form, divide the terms by their GCF.
21:14 = (21 ÷ 7):(14 ÷ 7)
= 3:2



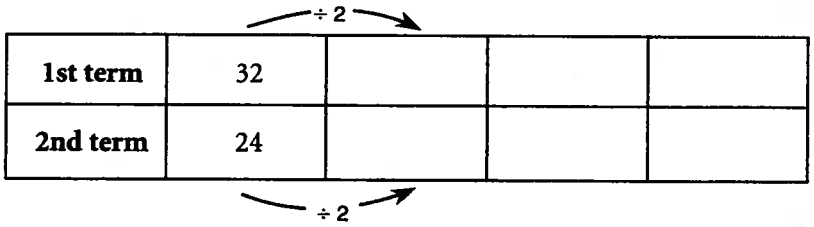
1. Write three ratios that are equivalent to each ratio.

a) 4:5



Three ratios equivalent to 4:5 are 8:10, _____, and _____.

b) 32:24



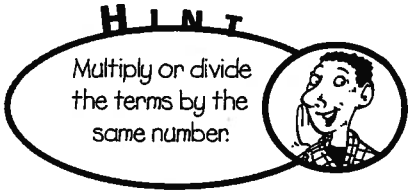
Three ratios equivalent to 32:24 are _____, _____, and _____.

c) 16:28

1st term				
2nd term				

_____

_____



2. Write two ratios that are equivalent to each ratio.

a) 8:5:2

_____

b) 24:16:12

_____

3. Write each ratio in simplest form.

a) 10:4

GCF of 10 and 4 is 2.

10:4 = (10 ÷ 2):(4 ÷ 2)

= 5: _____

b) 6:15

GCF of 6 and 15 is _____.

6:15 = (6 ÷ _____):(15 ÷ _____)

= _____ : _____

c) 14:28

_____

d) 25:10

_____

> **Tip**
> *Divide the terms by the GCF.*

4. **a)** Match the pairs of equivalent ratios.

i)			ii)		
5:6		1:2	1:8		1:9
18:3		15:18	3:27		1:3
9:18		8:40	12:36		9:1
4:20		6:1	18:2		2:16

b) How do you know that 12:36 and 1:3 are equivalent?

_____

_____

5. The ratio of cats to dogs at the animal shelter is 4 to 5.
How many cats could there be? How many dogs?
Write six different answers.

4 cats and 5 dogs

_____ cats and _____ dogs

_____

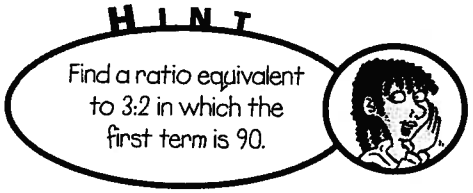
8 cats and _____ dogs

_____

_____

> **Tip**
> *Multiply each term by the same number.*

6. The length-to-width ratio of Colby's poster is 3:2.
The poster is 90 cm long. How wide is it?



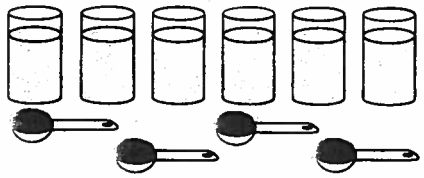
$3 : 2 = 90 : \_\_\_\_$ (× 30 on both terms)

The poster is _____ cm wide.

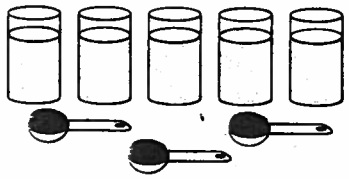
Quick Review

You can use equivalent ratios to compare ratios.

Joe and Petra make orange punch with different ratios of crystals to water.



Joe makes orange punch with 4 scoops of crystals and 6 cups of water.

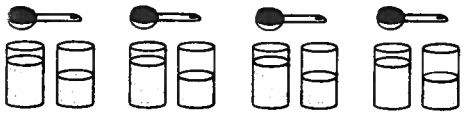


Petra makes orange punch with 3 scoops of crystals and 5 cups of water.

➤ **Method 1**

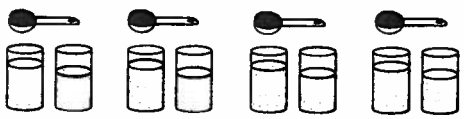
Draw a picture. Find out how much water for 1 scoop of orange crystals.

Joe



Orange crystals to water: $1:1\frac{1}{2}$

Petra



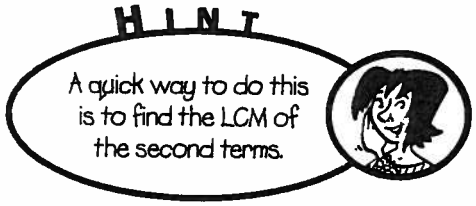
Orange crystals to water: $1:1\frac{2}{3}$

➤ **Method 2**

Equivalent Ratios

To find out whose orange punch is stronger:

- Write each mixture as a ratio.
- Write each ratio with the same second term.
- Compare the first terms.



Joe	Petra
4:6	3:5

The LCM of 6 and 5 is 30.

Use equivalent ratios.

$4:6 = (4 \times 5):(6 \times 5)$
$= 20:30$

Joe uses 20 scoops of crystals with 30 cups of water.

$3:5 = (3 \times 6):(5 \times 6)$
$= 18:30$

Petra uses 18 scoops of crystals with 30 cups of water.

$20 > 18$, so Joe's orange punch is stronger.

➤ **Method 3**

Write each ratio with a second term of 1.

For each ratio, divide each term by the second term.

Joe

$4:6 = \frac{4}{6} : \frac{6}{6}$

$= 0.\overline{6} : 1$

Petra

$3:5 = \frac{3}{5} : \frac{5}{5}$

$= 0.6 : 1$

Since $0.\overline{6} > 0.6$, Joe's orange punch is stronger.

➤ **Method 4**

Compare the part-to-whole ratios and change both ratios to percents.

	Joe	Petra
Ratio (part to whole)	4:10	3:8
Ratio expressed as a fraction	$\frac{4}{10}$	$\frac{3}{8}$
Percent	40%	37.5%

Joe has a higher percent of orange crystals in his punch, so his orange punch is stronger.

1.

A

B

Which mixture is stronger, A or B?

a) Draw a picture to show how much water is used for each scoop of powder in Mixture A and Mixture B.

A

B

b) Which mixture is stronger? Explain how you know.

Mixture _____ is stronger because _____.

2. Two cages contain white mice and brown mice.
In one cage, the ratio of white mice to brown mice is 2:3.
In the other cage, the ratio is 3:1.
The cages contain the same number of mice.

HINT

Multiply to find equivalent ratios. Add to find the total numbers of mice. Keep going until you get two totals that match.

a) What could the total number of mice be?

Cage A

White	Brown	Total
2	3	5
4	6	10
6		

Cage B

White	Brown	Total
3	1	
6		

The number of mice in each cage could be _____.

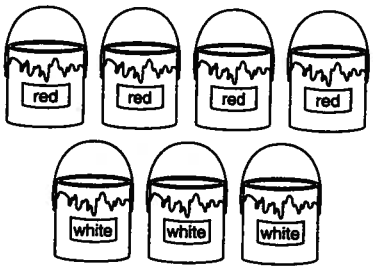
The total number of mice could be _____.

b) Which cage contains more white mice?

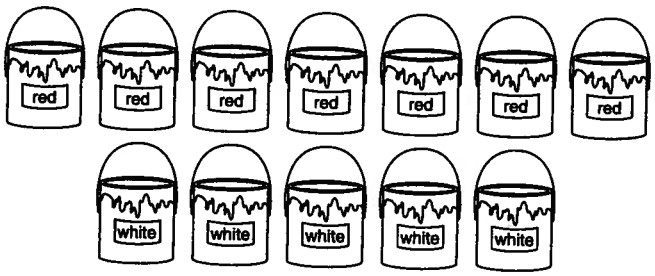
Number of white mice in A: _____ Number of white mice in B: _____

Cage _____ contains more white mice.

3.



A



B

The red paint and white paint in each picture will be mixed.

Write the ratio of red paint to white paint. A _____ B _____

Write each ratio with the same second term. A _____ B _____

Compare the first terms. _____

Which mixture will give a darker shade of red? _____

Tip

Use the LCM of the second terms to write equivalent ratios.

4. The ratio of computers to students at Jan's school is 3:5.
The ratio of computers to students at Karl's school is 2:3.
Both schools have the same number of students.
Which school has more computers? Show your work.

Jan's School	**Karl's School**
3:5	2:3
$= (3 \times 3):(5 \times \_\_\_\_\_)$	$= (2 \times 5):(3 \times \_\_\_\_\_)$
$= \_\_\_\_\_ : \_\_\_\_\_$	$= \_\_\_\_\_ : \_\_\_\_\_$
_____ computers to _____ students	_____ computers to _____ students

_____ school has more computers.

5. Hamid jogs 5 laps in 6 min.
Amelia jogs 8 laps in 11 min.
Which person jogs faster? Show your work.

6. The Rebels hockey team has won 9 of its first 15 games. No game was tied. The Sabres' record is 7 wins and 5 losses.

Which team has the better record? Show your work.

5.8 Solving Ratio Problems

Quick Review

At Home / At School

You can often solve a problem involving ratios by setting up a proportion.
A **proportion** is a statement that two ratios are equal.

In a box of red and blue marbles, the ratio of red marbles to blue marbles is 3:4.
If there are 48 blue marbles, you can find the number of red marbles using a proportion.

Let r represent the number of red marbles.
Then: $r:48 = 3:4$
In fraction form: $\frac{r}{48} = \frac{3}{4}$
To find the value of r, first isolate r by multiplying each side of the proportion by 48.

$$48 \times \frac{r}{48} = 48 \times \frac{3}{4}$$
$$r = \frac{144}{4}$$
$$= 36$$

There are 36 red marbles.

1. State the number you would multiply each side of the proportion by to isolate the variable.

a) $\frac{r}{6} = \frac{5}{6}$ _____ b) $\frac{t}{15} = \frac{2}{5}$ _____ c) $\frac{v}{3} = \frac{5}{6}$ _____

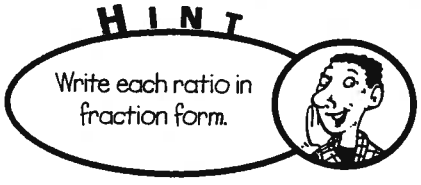
2. Find each missing term.

a) $p:4 = 9:12$

$\frac{p}{4} =$ _____

_____ $\times \frac{p}{4} =$ _____

$p =$ _____



b) $c:12 = 5:6$ _____ c) $3:14 = t:70$ _____

3. Find each missing term.

a) $\frac{f}{10} = \frac{4}{5}$ _____ b) $\frac{h}{8} = \frac{12}{3}$ _____ c) $\frac{w}{11} = \frac{6}{33}$ _____

d) $x:6 = 12:9$ _____ e) $m:4 = 9:6$ _____ f) $x:16 = 5:4$ _____

4. In a bag of coloured cubes, the ratio of green cubes to purple cubes is 5:7.
If there are 70 green cubes, how many purple cubes are there?

Let p represent the number of purple cubes. Write a proportion:

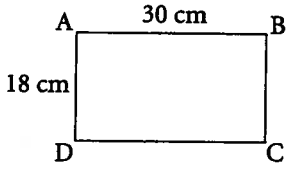
p: _____ = _____:_____

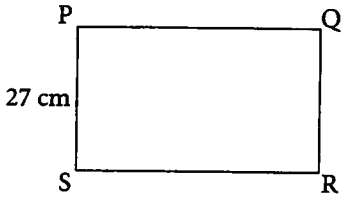
There are _____ purple cubes.

Tip

Writing the variable as the first term in the ratio makes it easier to solve the proportion.

5. Rectangles ABCD and PQRS have the same length-to-width ratio.
Calculate the length of rectangle PQRS.





The length of rectangle PQRS is _____.

6. The length of a bug is 6.4 cm in a drawing. The drawing was made using a scale of 4:1.
What is the actual length of the bug?
Let the actual length of the bug be l centimetres.
Length of bug:length of drawing = 1:4

$\frac{l}{6.4}$ = _____

_____ × $\frac{l}{6.4}$ = _____

l = _____

The actual length of the bug is _____ cm.

7. On a school trip, the ratio of teachers to students is 2:21. The ratio of boys to girls is 4:3.

If there are 18 girls on the trip, how many boys are there? _____

How many teachers? _____

Quick Review

➤ A **rate** is a comparison of two quantities measured in different units.

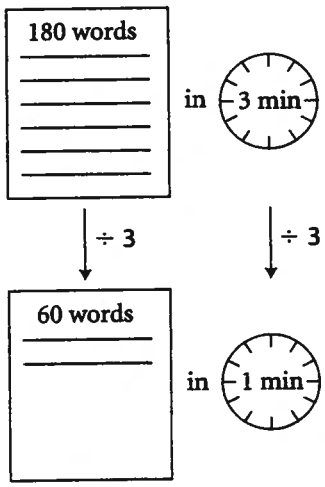
Leo types 180 words in 3 min.
180 words in 3 min is a **rate.**

This means Leo types 60 words in 1 min.
Leo's rate of typing is 60 words per minute.
You can write this as 60 words/min.

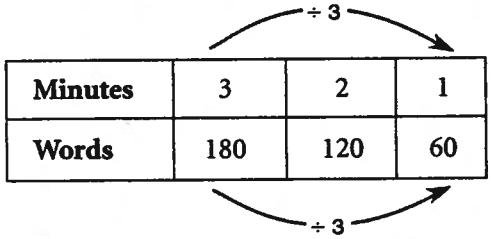
60 words/min is a **unit rate.**
It compares a quantity (60 words) to 1 unit (1 min).

➤ To find a unit rate, you can use a diagram, a table, or a graph.
In 3 min, Leo types 180 words.
In 1 min, Leo types 60 words.

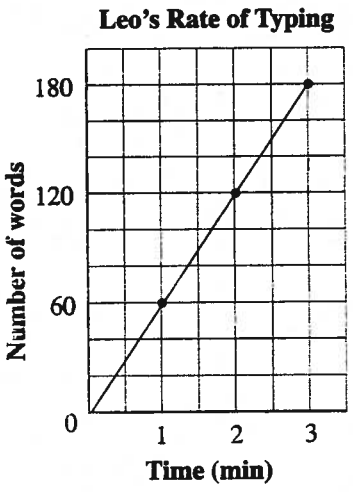
A Diagram



A Table



A Graph



Practice

1. Express as a unit rate.

a) Serena walks 4 km in 1 h. _____

b) Sanjit reads 3 books in 1 week. _____

c) The tap drips 25 drops in 1 min. _____

2. Express as a unit rate. Show your work.

a) Betty drives her car 150 km in 2 h.

> **Tip**
> *Express each rate in kilometres per hour (km/h).*

150 km ÷ 2 = _____ km

Betty's average driving speed is _____ km/h.

b) The helicopter travels 180 km in 3 h.

_____

_____

c) Gerald walks 1 km in 15 min.

Distance (km)	1	2		
Time (min)	15	30		

> **Tip**
> *1 h = 60 min*

Gerald's rate of walking is _____ km/h.

3. Determine whether the sentence expresses a ratio or a rate. Write the rate or ratio for each.

a) The cost of pecans is $10.89 for each kilogram. ratio/rate _____ per kg

b) Three out of every seven people are wearing glasses. ratio/rate _____

c) Mr. Thompson travelled 620 km in 6 h. ratio/rate _____

d) Each block of a quilt has 5 red patches, 4 yellow patches, and 6 blue patches.
ratio/rate _____

e) In 7 games, the team scored a total of 23 points. ratio/rate _____

4. Maria charges $15 for 3 h of babysitting.

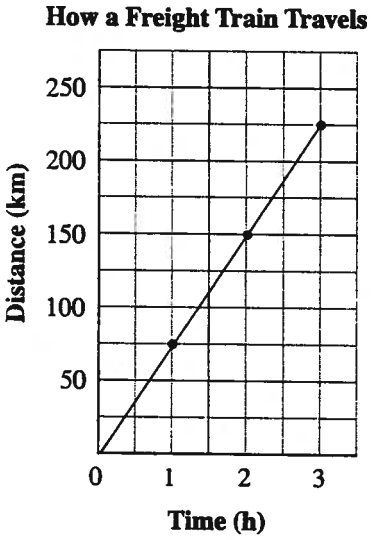
a) What is Maria's rate per hour? _____

b) How much does Maria charge for 5 h of babysitting? _____

c) How many hours does Maria have to babysit to earn $50?

Maria has to babysit _____ to earn $50.

5. The graph shows the distance a freight train travels in 3 h.



a) How far does the train travel in 1 h?

_____

b) What is the average speed of the train?

_____

6. Frozen fruit bars cost $3.95 for 5 bars.
Find how many you can buy with $12. Show your work.

_____

7. Terence came to Canada shopping on a long weekend.
The exchange rate for his US money was $1.00 US to $1.05 Canadian.

a) How many Canadian dollars would Terence get for $500.00 US?

_____

b) Terence spent $504.90 Canadian altogether during his 3-day stay in Canada.
What was his average spending per day?

_____

c) A jacket he purchased cost $39.95 Canadian.
What is this value in US dollars?

_____

5.10 Comparing Rates

Quick Review

At Home / At School

➤ To compare different rates, you need to calculate their unit rates.

A case of 12 cartons of juice costs $11.76.
A packet of 3 cartons of the same juice costs $2.88.
To find which juice is the better buy, compare the unit costs of the 2 packages.
The unit cost of the case of 12 cartons is: $11.76 ÷ 12 = $0.98
The unit cost of the packet of 3 cartons is: $2.88 ÷ 3 = $0.96
So, the packet of 3 cartons is the better buy.

The unit cost can be written as a unit rate:
The unit rate of the better-buy juice is $0.96/carton.

To find the unit rate of a 450-g packet of cereal that costs $3.96, use the cost of 100 g as the unit cost.
The cost of 100 g of the 450-g packet is: $\frac{\$3.96}{450} \times 100 = \$0.88$
So, the unit rate of the 450-g packet of cereal is $0.88/100 g.

Practice

1. Write a unit rate for each.

a) 6 bottles of juice for $3.96

$\frac{\$3.96}{6}$ = _____

b) 840 words typed in 12 min

_____

c) $564 earned in 4 weeks

_____

d) 130 mL of toothpaste for $1.69

_____

Tip

A unit rate can be a rate for a quantity greater than 1.

2. Which is the better buy? Explain.

a) 475 g of cereal for $3.80 or 750 g for $6.30

_____

b) 385 mL of shampoo for $5.39 or 400 mL for $5.72

_____

3. Find the average speed of each.

a) 242 km in 4 h _____

b) 372 km in 6 h _____

c) 309 km in 5 h _____

Which is the greatest average speed? _____

4. Shamar types 279 words in 4.5 min, Tasha types 320 words in 5 min, and Cody types 341 words in 5.5 min. Who has the greatest average typing speed?

_____ has the greatest average typing speed.

5. In the first 6 games of the basketball season, Lucinda scored 87 points.

a) What was her average number of points scored per game? _____

b) At this rate, how many points will Lucinda score in 26 games? _____

6. Which is the better buy?
Twelve 710-mL bottles of water for $6.60 or twenty-four 500-mL bottles for $9.18

_____

7. Population density is defined as the average number of people per square kilometre. The population density of Canada is approximately 3.5 people/km^2.
Use the data in the table. Name a province or territory that has:

a) A population density closest to that of Canada _____

b) A population density about half of that of Canada _____

c) A population density about 4 times that of Canada _____

d) A population density about 300 times that of Nunavut _____

Province/Territory	Population	Area (km^2)
Ontario	12 393 000	917 700
Saskatchewan	995 000	591 700
British Columbia	4 196 000	925 200
Nunavut	29 600	1 939 000

In Your Words unit 5

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

discount *the amount that a price is reduced due to a sale*

sales tax

ratio

equivalent ratios

rate

proportion

List other mathematical words you need to know:

Unit Review

LESSON

5.1 **1.** Write each decimal as a fraction and as a percent.

a) $0.15 = \frac{\square}{100} =$ _____

b) $0.4 =$ _____ $=$ _____

c) $0.875 = \frac{\square}{1000} = \frac{\square}{100} =$ _____

d) $0.003 =$ _____ $=$ _____

2. In Carmela's class, 61% of the students are girls, while in Analise's class, 20 out of 32 students are girls. Which class has a greater ratio of girls to students? Explain how you found out.

_____

5.2 **3.** Write each percent as a fraction and as a decimal.

a) 85% = _____ = _____

b) 0.7% = _____ = _____ = _____

c) 139% = _____ = _____

d) 412% = _____ = _____

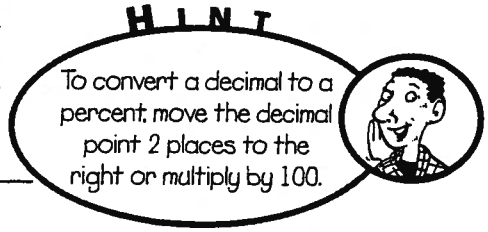
4. Write each fraction as a decimal and as a percent.

a) $\frac{4}{5} =$ _____ $=$ _____

b) $\frac{8}{5} =$ _____ $=$ _____

c) $\frac{3}{1000} =$ _____ $=$ _____

d) $\frac{15}{6000} =$ _____ $=$ _____



5. In 1895, the population of a small town was 2120.
By 1905, the population increased to 115% of the 1895 figure.

a) What was the population in 1905?

_____

b) Find the increase in population from 1895 to 1905.

_____

LESSON

5.3 **6.** Find the amount in each case.

a) 8% is 56 kg. _____

b) 125% is 85 cm. _____

c) 0.48% is 84 L. _____

7. In a sponsored walk for charity, 560 students participated. Of these, 0.72% completed the 15-km walk. How many students completed this distance?

_____

8. Write each increase or decrease as a percent.

a) The price of gasoline rose from 132.5¢/L to 137.8¢/L.

Percent increase = _____

b) The number of trucks crossing the border fell from 3240 to 2673.

Percent decrease = _____

9. A water tank is filled with 1500 L of water. In 1 h, the tank loses 5.4% of the water due to leakage. What is the volume of water in the tank after 1 h?

_____

5.4 **10.** The tax rate is 12%. Calculate the selling price of each item before and after tax.

a) $125 item at 10% off

Before: _____

After: _____

b) $1820 item at 25% off

Before: _____

After: _____

c) $6.80 item at 15% off

Before: _____

After: _____

11. The sale price of a computer at 15% off is $746.30. What is the regular price?

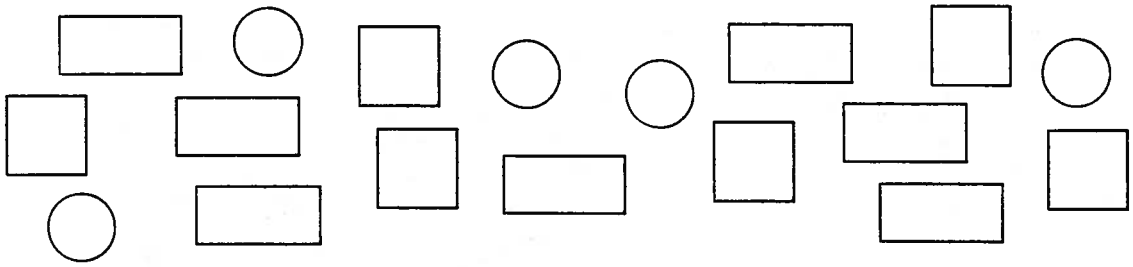
_____

12. A store owner buys coats for $56 each. She adds 30% to the cost and sells the coats at 15% off. Find the selling price of each coat.

_____

LESSON

5.5 **13.** Write each ratio.

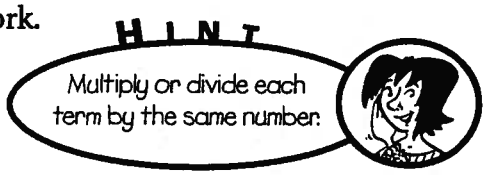


a) squares to circles _____

b) rectangles and circles to squares _____

c) circles to total figures _____

5.6 **14. a)** Write three ratios equivalent to 2:5. Show your work.



2:5 = (2 × ____):(5 × ____) 2:5 = _____ 2:5 = _____

= ____ : ____ = _____ = _____

b) Write three ratios equivalent to 36:18. Show your work.

36:18 = _____

_____

36:18 = _____

_____

36:18 = _____

_____

LESSON

15. Write each ratio in simplest form.

a) 25:15 = (25 ÷ _____):(15 ÷ _____)

= _____ : _____

b) 28:35 = _____

= _____

c) 45:72 = _____

= _____

5.7 **16.** Class 8B has 3 globes for every 7 students.
Class 8D has 2 globes for every 5 students.
Each class has the same number of students.
Which class has more globes? Explain.

Tip
Write each ratio with the same second term.

_____

_____

5.8 **17.** At a summer camp, for every 3 students who sailed, 5 kayaked.
Forty-five students kayaked. How many students sailed?

Let s be the number of students who sailed. Write a proportion.

Tip
Writing the variable as the first term in the ratio makes it easier to solve the proportion.

s: _____ = _____:_____

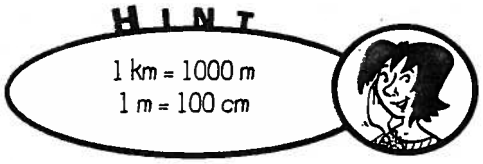
_____ students sailed.

18. In a bag of coloured cubes, the ratio of red cubes to total number of cubes is 5:7.
If there are 105 cubes in the bag, how many cubes are red?

_____ cubes are red.

LESSON

19. The scale of a map is 1:6 000 000.



a) The distance between 2 towns on the map is 8.7 cm. What is the actual distance?

1 cm on the map represents _____ cm of actual distance.

The actual distance between the 2 towns is:

_____ × _____ cm = _____ cm = _____ km

b) The distance between 2 other towns is 1248 km. What is the distance on the map?

_____

5.9 **20.** Express as a unit rate.

a) The van travels 280 km in 4 h.

Tip
Express in kilometres per hour (km/h).

_____

b) Mikki jogs 2 km in 20 min.

_____

5.10 **21.** Which is the better buy?

2.9 L of detergent for $4.56 or 3.8 L for $5.78

_____

22. A cruise ship travelled 84 km in 3.5 h.
At this rate, how long will it take to travel 1050 km?

_____

23. Which country has the greater population density? Write its population density.
The United Kingdom has about 60 million people and an area of 244 800 km^2, and China has about 1806 million people and an area of 9 590 000 km^2.

_____