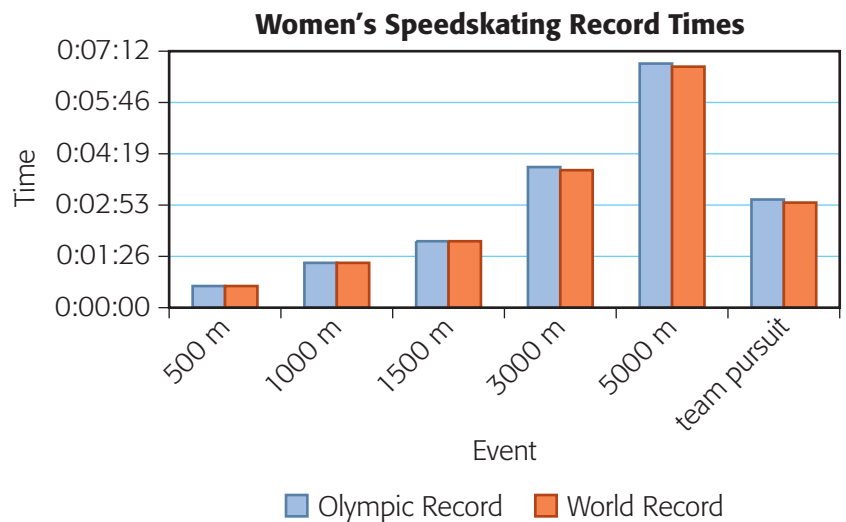






Chapter 8

Data Analysis

GOAL

You will be able to

- compare the strengths and limitations of different types of graphs
- communicate about the choice and format of a graph
- interpret graphs and identify inappropriate conclusions
- identify ways that graphs can misrepresent data

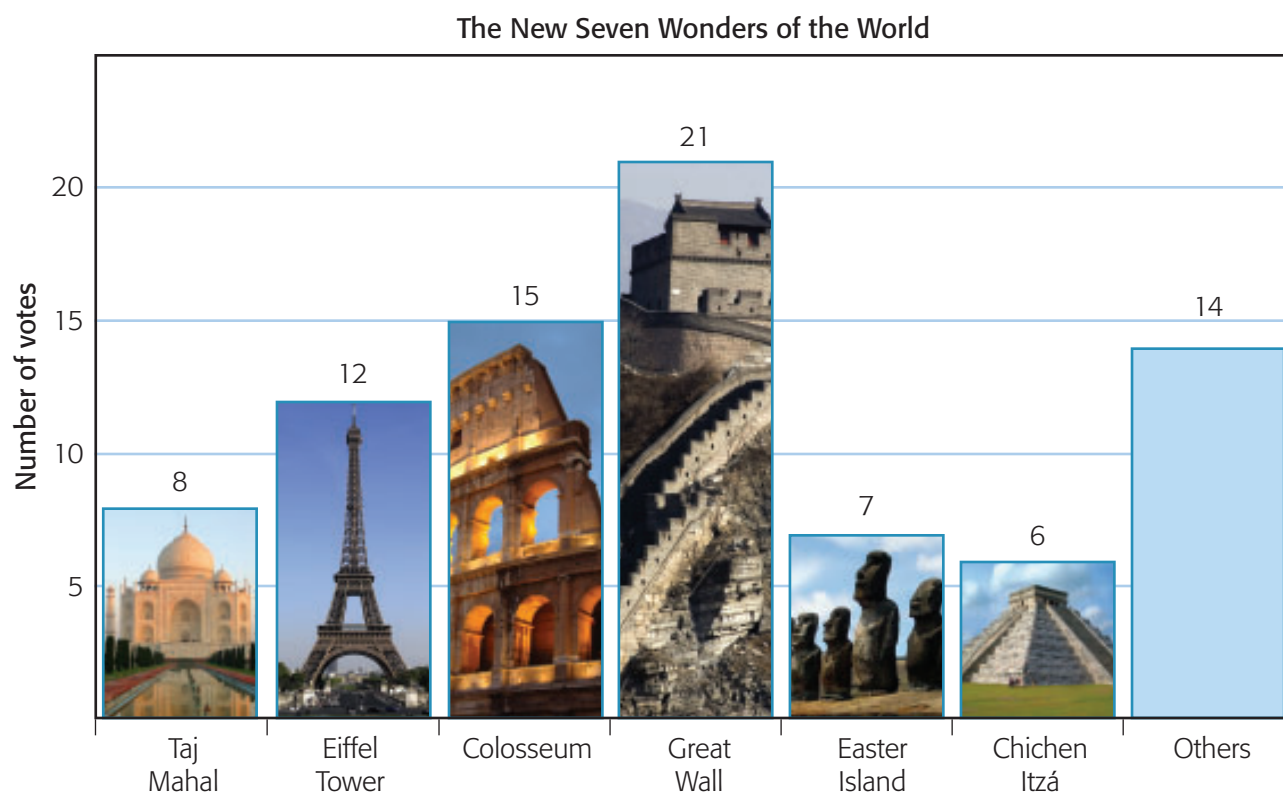
- ◀ In 2007, Canadian speed skater Cindy Klassen held the world records for 1000 m, 1500 m, and 3000 m. What conclusions about world and Olympic record times can you draw from the graph?

YOU WILL NEED

- grid paper

Different Graphs

Elena and Mark read about an online vote to determine the New Seven Wonders of the World. They decided to conduct their own survey.



? What information can a graph give you?

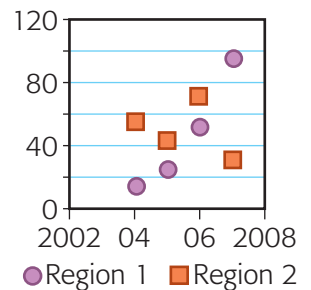
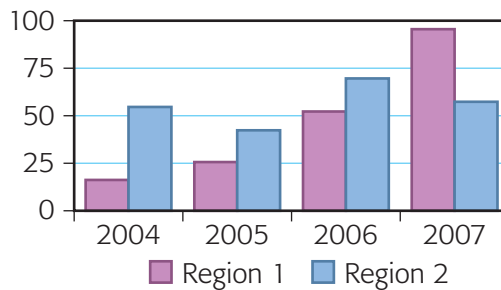
- Can the graph be used to decide what the New Seven Wonders of the World should be? Why or why not?
- Can you use a pictograph to display the data? Explain.
- Can you use a line graph to display the data? Explain.

- D.** If you used a circle graph to display the data, what would the whole represent? How would you decide how big to make each part?
- E.** Construct a line graph, circle graph, or pictograph to display the data. Why did you choose that type of graph?

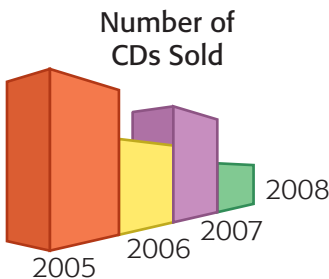
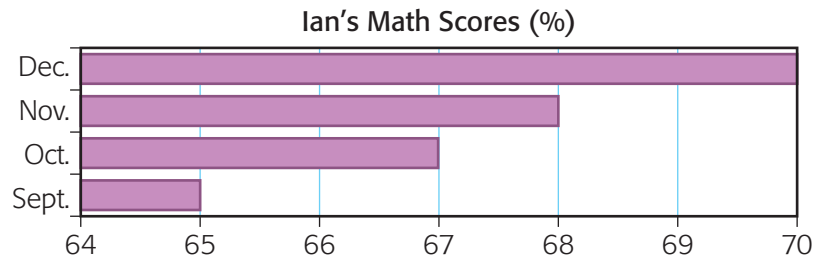
What Do You Think?

Decide whether you agree or disagree with each statement.
Be ready to explain your decision.

- 1.** These graphs probably show the same data.

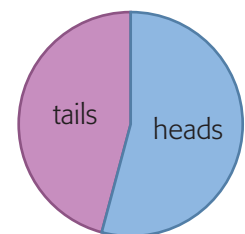


- 2.** The graph shows that Ian's monthly math scores are increasing quickly.



- 3.** The graph shows that sales of CDs were greatest in 2007.
- 4.** You can use the circle graph to determine how many times heads was tossed.

Results of 1000 Coin Tosses



8.1

Choosing a Graph to Display Data

YOU WILL NEED

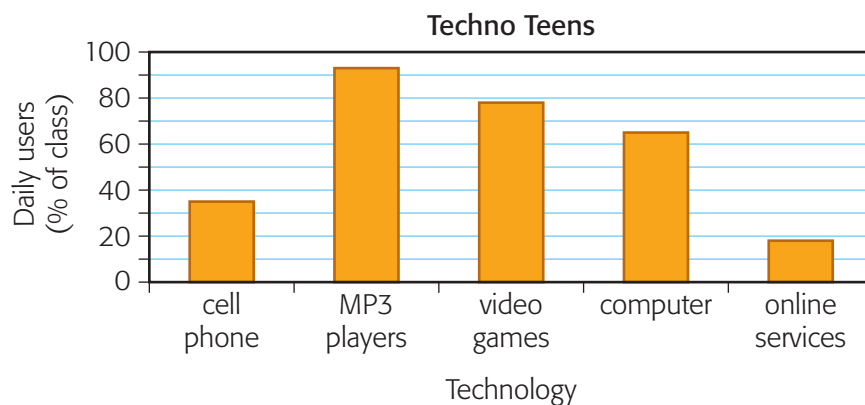
- grid paper

GOAL

Justify the choice of a graph to display data.

EXPLORE the Math

Joseph read that MP3 players are the most popular form of technology used by people under 20. He wondered if the results were the same for his class, so he conducted a survey. He decided to use a bar graph to display his results.



? What are the advantages and limitations of displaying the results using different types of graphs?

8.2

Changing the Format of a Graph

YOU WILL NEED

- grid paper
- a ruler
- pencil crayons

GOAL

Compare how different types of graphs present information.

LEARN ABOUT *the Math*

Kaitlyn wants to know how fit her class is. She recorded resting heart rates for all 30 students. She wants to display the data in a graph.

? How will the format of the graph affect interpretation of the data?



Resting Heart Rates (beats per minute)

70	60	62	53	76	84	67	62	80	74
70	84	60	76	68	70	72	78	56	64
68	72	70	60	78	80	67	76	66	54

Example 1

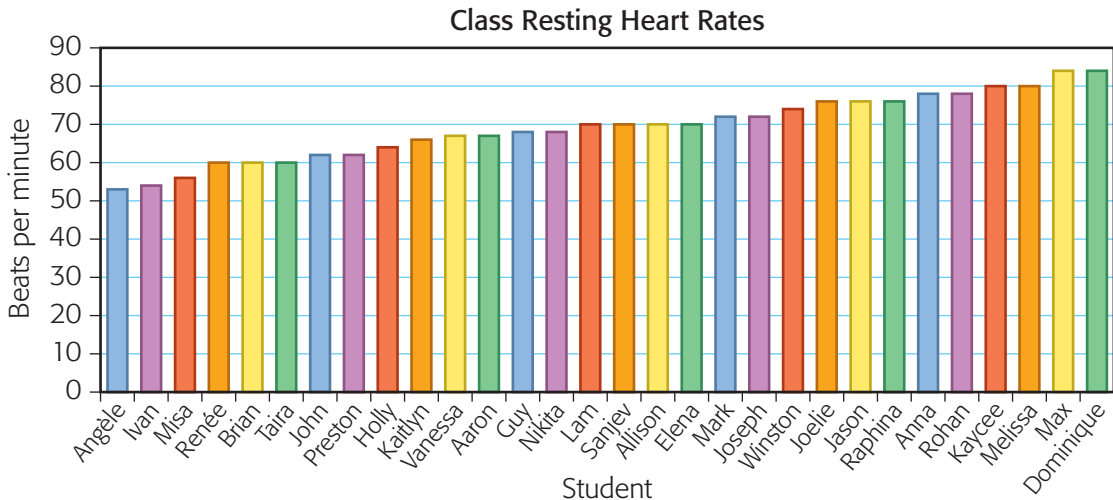
Creating a bar graph

I displayed the data in a bar graph.

Kaitlyn's Solution

53 54 56 60 60 60 60 62 62 64 66 67 67 68 68 70
70 70 70 72 72 74 76 76 76 78 78 80 80 84 84

First, I ordered the data from the smallest number to largest.



I used a bar for each person. The height of the bar tells you the heart rate of that person.

My bar graph contains each individual piece of data. I can compare the heart rates of individuals. To talk about the whole class is difficult because I have to look at many bars at the same time.

Example 2

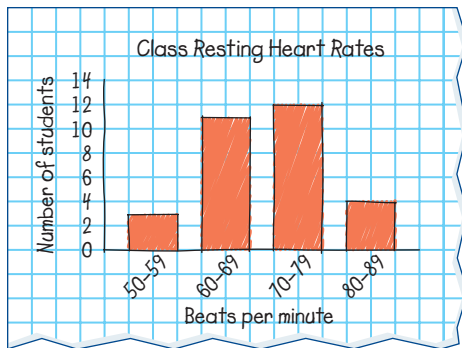
Creating a different bar graph

I grouped the data, then I constructed a bar graph.

Sanjev's Solution

50 – 59: 53, 56, 54
60 – 69: 60, 62, 67, 62, 60, 68, 64, 68, 60, 67, 66
70 – 79: 70, 76, 74, 70, 76, 70, 72, 78, 72, 70, 78, 76
80 – 89: 84, 80, 84, 80

There were many data values, so I organized the data into groups based on the 10s digits.



Each bar represents the number of people with heart rates in that group.

I have fewer bars to look at, so it is easier to say things about the whole class. I can see that 23 of 30 heart rates fall in the two middle groups. I do not see which students are in each group though.

Example 3 | Choosing a different graph

I used a circle graph to display the data.

Joseph's Solution

52-59: 53, 54, 56

60-67: 60, 62, 67, 62, 60, 64, 60, 67, 66

68-75: 68, 68, 70, 74, 70, 70, 72, 72, 70

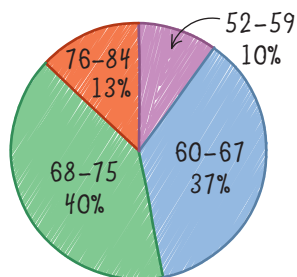
76-84: 76, 76, 78, 78, 76, 84, 80, 84, 80

Group size	3	11	12	4
Percent	10	36.7	40	13.3

The range of data is $84 - 53 = 31$. That is about 32, so I used 4 groups of 8 possible data values.

I calculated each group size as a percent of 30 (the total number of students).

Class Resting Heart Rates
(beats per minute)



Each wedge represents the percent of the class that is in that group.

I can see that 77% of the class have heart rates between 60 and 75 beats per minute, but I cannot see how many people are in each group.

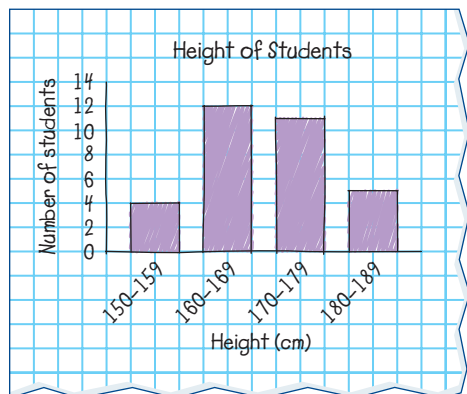
Reflecting

- How are the three graphs alike and how are they different?
- Create questions you could answer best using each of the graphs.

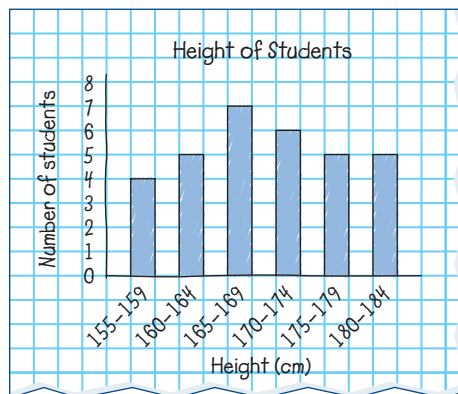
WORK WITH the Math

Example 4 Interpreting graphs

Vanessa and Elena graphed the heights of the students before the class trip to the amusement park.



Vanessa's Graph



Elena's Graph

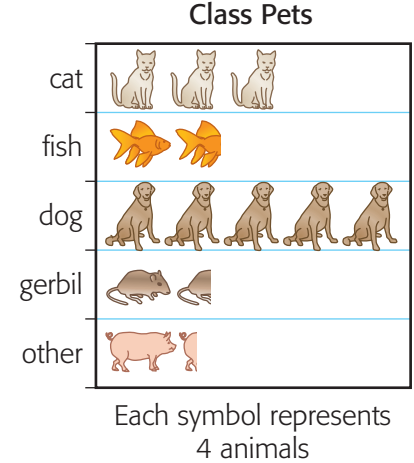
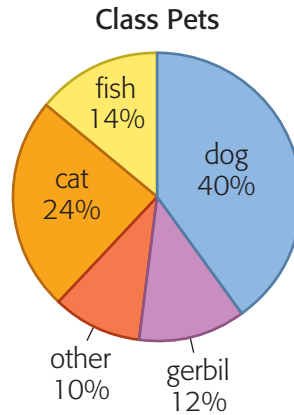
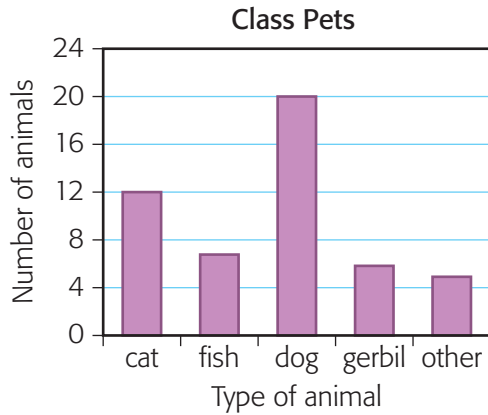
- How are the graphs different?
- What conclusions can you make about the heights of the students?
- Which graph would you use to determine the number of students who could go on a roller coaster ride if you must be at least 162 cm tall? Explain.

Solution

- The groups for the two graphs are different. In Vanessa's graph there are four groups for data values, each with 10 possible heights. In Elena's graph there are seven groups for data values, each with 5 possible heights.
- Vanessa's graph shows that 12 out of 32 students are from 160 to 169 cm tall. Elena's graph shows that 7 of those 12 students are from 165 to 169 cm tall. Elena's graph gives more precise information. Her graph also shows that no one is shorter than 155 cm or taller than 184 cm. You cannot tell that from Vanessa's graph.
- You can only estimate from the graphs. Elena's graph shows that at least 23 students can go on the roller coaster. Vanessa's graph shows that at least 16 students can go on the roller coaster. Elena's graph gives a better estimate.

A Checking

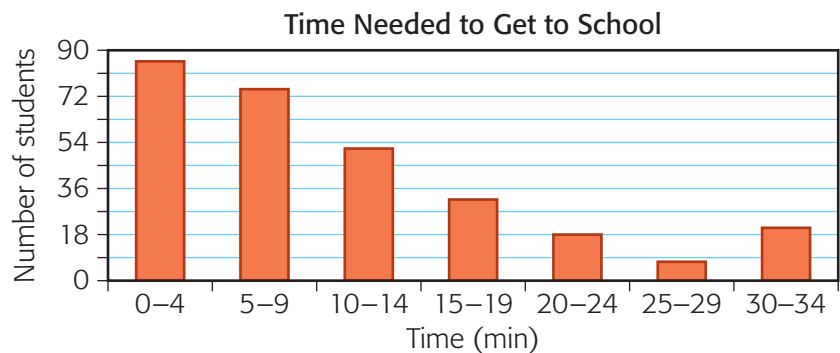
- Jason surveyed the Grade 8 classes about their pets. He drew three different graphs to display the data.



- How are all three graphs the same? How are they different?
- How are the impressions the graphs give different?
- Is one graph better at presenting the data than another? Explain.

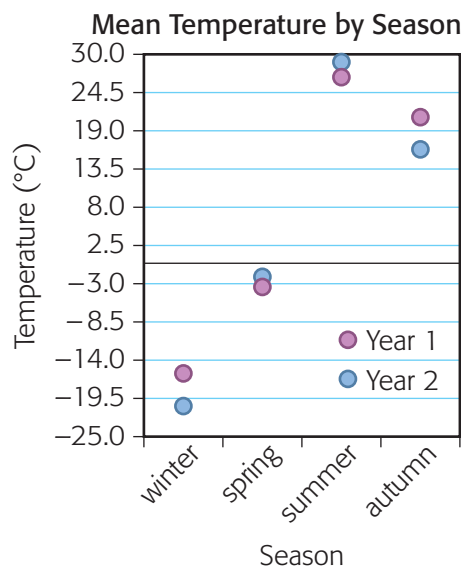
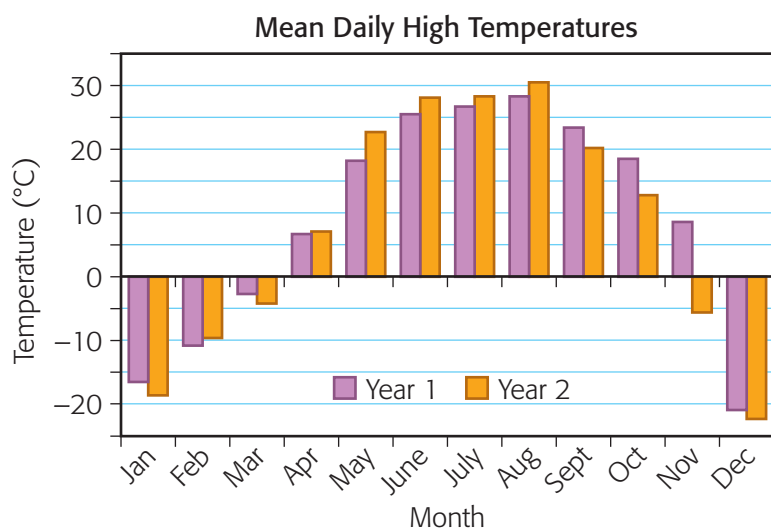
B Practising

2.



- What question is easy to answer using this graph?
- What question about the time to get to school is not as easy to answer using this graph?

3. Gail recorded the daily high temperature for her town for two years. She created two graphs to display the data.



Reading Strategy

Inferring

Look at the graphs before reading question 3. What inferences can you make about the data?

- Which graph could you use to decide the year that was generally warmer? Do both graphs give you the same conclusion?
 - What information is missing in the scatterplot?
4. Manon and Juan recorded the running times for 100 movies. Manon displayed the data in a bar graph that listed each movie individually. Juan grouped the data and created a circle graph.
- Could Juan have used Manon's bar graph to create his circle graph? How?
 - Why could Manon not use Juan's circle graph to create his bar graph?
5. The following times, in seconds, were recorded for a 100 m race.
- | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|
| 13.9 | 14.3 | 14.4 | 13.7 | 15.2 | 15.4 | 14.8 | 13.9 | 13.9 |
| 14.5 | 14.7 | 14.4 | 13.8 | 14.2 | 13.1 | 13.8 | 12.4 | 13.8 |
| 12.7 | 13.4 | 14.4 | 13.9 | 14.0 | 14.4 | 14.3 | 14.5 | 11.8 |
| 14.8 | 12.9 | 12.3 | 12.8 | 13.7 | 13.1 | 15.0 | | |
- Sort the data into groups in two different ways.
 - Create two graphs using your answers from part a).
 - Write a statement about the data that each graph supports.
6. What happens to the bars in a bar graph when you change the size of the groups?

8.3

Communicate about Choosing a Graph

GOAL

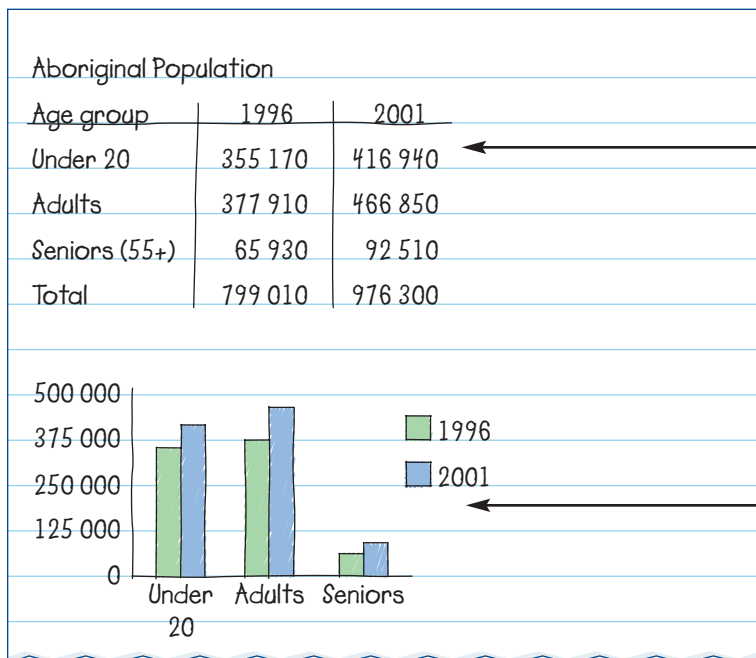
Explain how to choose an appropriate graph for a set of data.

LEARN ABOUT *the Math*

Sanjev used census data to compare the number of Aboriginal people in Canada in different age groups in 1996 and in 2001. His report included a table and graph.

? How can Sanjev improve his report?

Sanjev's Report



Vanessa's Questions

Where did you get your information?

Would titles and labels make the bar graph easier to read?

I chose a double bar graph to show the number of Aboriginal people in each age group for each year. With this graph, I can compare both the number in each age group and the changes in number from 1996 to 2001.

Each blue bar is higher than the green bar, so the graph shows that each age category grew from 1996 to 2001.

Why did you not use other types of graphs?

Which category changed the most?
What else does the graph show?

Communication Checklist

- ✓ Did you include all of the important details?
- ✓ Did you justify your choice of graph?
- ✓ Did you make reasonable conclusions?
- ✓ Did you justify your conclusions?

A. Use Vanessa's questions and the Communication Checklist to rewrite Sanjev's report.

Reflecting

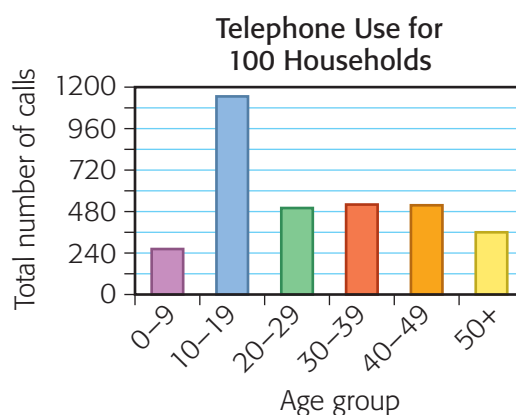
- B.** What parts of the Communication Checklist did Sanjev cover well?
- C.** How did Vanessa's questions help you improve Sanjev's communication about the populations?

WORK WITH the Math

Example 1 Using graphs to make decisions

Kaitlyn surveyed 100 households and displayed the results in a bar graph.

How can Kaitlyn use the bar graph to convince her parents she needs her own phone line?



Solution

The graph shows that the 10–19 age group receives many more calls than other age groups. This means that other members of the family probably answer a lot of calls that are not for them. Having a second phone line would make it easier for the whole family.

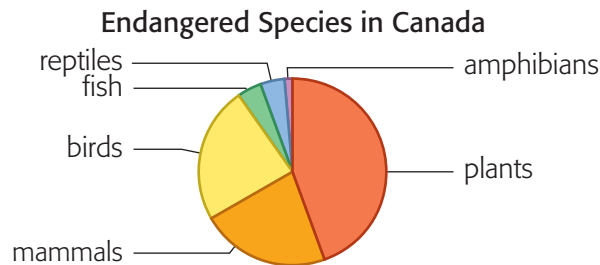
- How do you budget your money?
- How tall are your classmates?
- How do you spend your day?
- Which four banks give the highest rates of interest?
- How do three magazines compare for number of copies sold in one year?

A Checking

1. Explain why you would use a circle graph to display information about some of the topics on the left, but not others. Use the Communication Checklist to improve your explanations.
2. How might a track and field team use graphs to display their records? What would be the advantage in using each type of graph? Use the Communication Checklist to help write your answer.

B Practising

3. The circle graph shows the breakdown of 55 endangered species in Canada.



- a) Use the graph and the Communication Checklist to write a report about endangered species.
 - b) Could another type of graph be used to display this data? Explain.
4. Exchange reports from question 3 with a classmate. Make suggestions to help them improve their report.
 5. How do the data you collect and the purpose of the graph help you decide which kind of graph to use? Use examples to help you explain.

Frequently Asked Questions

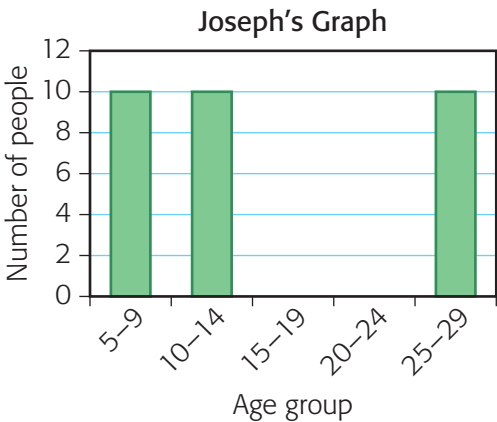
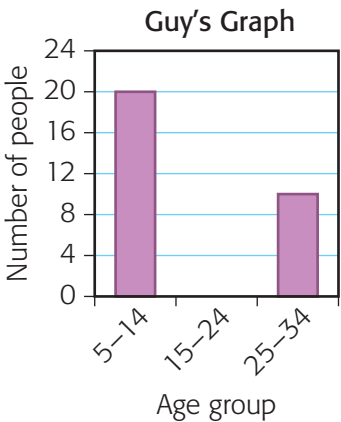
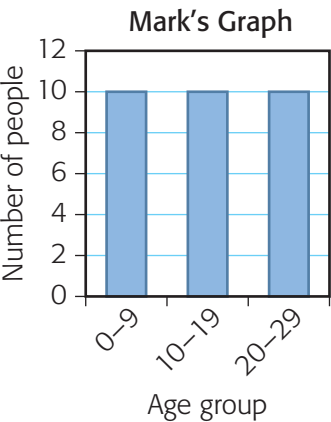
Q: How do you decide which type of graph you should use to represent data?

A: The type of graph depends on what you want the graph to show.

Type of graph	What the graph shows
line graph	trends
circle graph	how a whole is divided
scatterplot	if there is a relationship between two sets of data
bar graph/pictograph	compare numbers of things in different categories

Q: How does the choice of groups affect the appearance of a graph?

A: Choosing different groups can change the group that a data value belongs to. For example, these bar graphs display the same data but are very different in appearance:



You can often get more information from graphs with smaller groups. For example, Joseph's graph shows there are no people in the 15 to 19 or 20 to 24 age groups. You cannot tell that from Mark's graph.

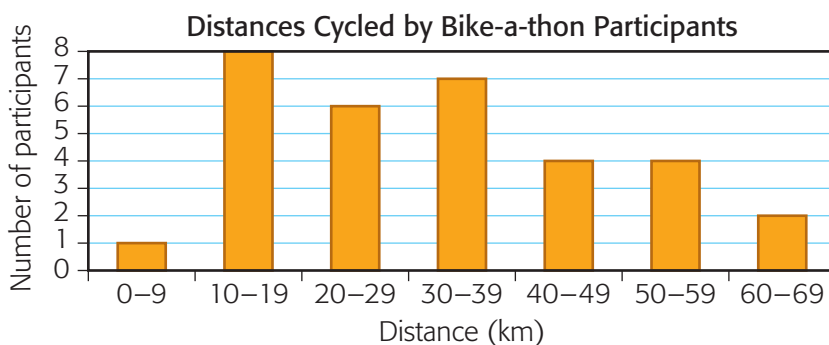
Practice

Lesson 8.2

1.

Distance (km) Cycled by Bike-a-thon Participants

17	20	53	27	29	33	36	19
61	13	28	27	16	54	19	42
56	45	8	15	36	25	47	44
32	12	10	65	31	52	35	30



- Which group of distances was most popular? Use the graph to help you answer.
- Create another bar graph with different groupings of the data. Use your new graph to decide the most popular group of distances.
- Compare your answers to parts a) and b). Did using different groups change the interpretation of the graph? Explain.

Lesson 8.3

- Display this movie data in a graph. Why did you choose that type of graph?
 - Compare your graph with a classmate's. Discuss each of the following:
 - Did you choose the same type of graph?
 - What impressions do you get from each graph?
 - Which graph appears to be the most effective in conveying the information?
 - Write a report for a movie rental company about how they should stock their shelves. Does your graph support your recommendations? Would your classmate's graph support the same recommendations?

Top Earning Movies of 2006 (in millions)

- Pirates of the Caribbean: Dead Man's Chest* (\$423.3)
- Cars* (\$244.1)
- X-Men: The Last Stand* (\$234.4)
- The Da Vinci Code* (\$217.5)
- Ice Age: The Meltdown* (\$200.1)

8.4

Changing the Scale of a Graph

YOU WILL NEED

- pencil crayons
- a ruler
- grid paper or graphing software

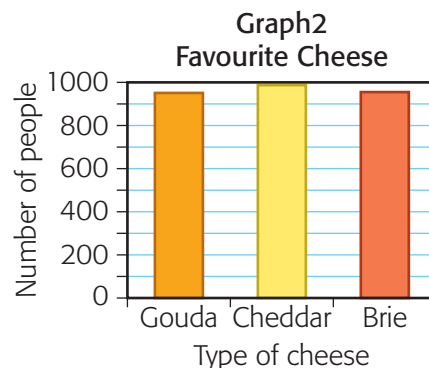
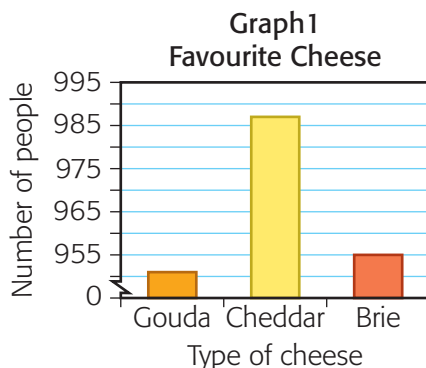
GOAL

Explain how changing the scale of a graph can affect its appearance.

LEARN ABOUT the Math

Joseph made two graphs to show the results of a survey about favourite types of cheese for sale at a food fair.

Favourite Cheese	
Cheese	Number of people
Gouda	951
Cheddar	987
Brie	955



? How does changing the scale of a graph affect its appearance?

- What is the meaning of the symbol $\nearrow$ on the vertical axis of the graph? Explain how you know this.
- Do the graphs use the same data? Explain how you know.
- How do the scales of the two graphs differ?
- How does changing the scale of the graph affect the conclusions you might draw?

Reflecting

- Why might Joseph choose one graph over the other?

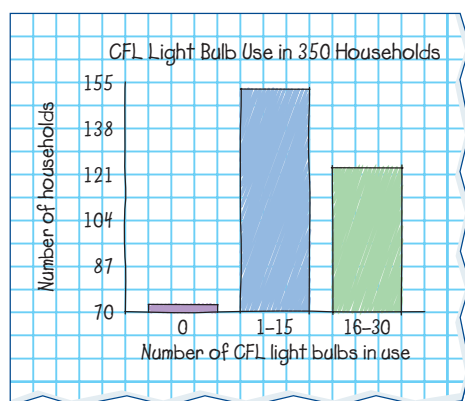
WORK WITH the Math

Example 1 Changing the scale of a graph

Elena is doing a project on energy conservation. She surveyed 350 households to see how many energy-saving compact fluorescent lamp (CFL) bulbs they were using. What conclusions can she make from the data?

Number of CFL bulbs in use	Number of households
0	73
1-15	153
16-30	124

Elena's Solution

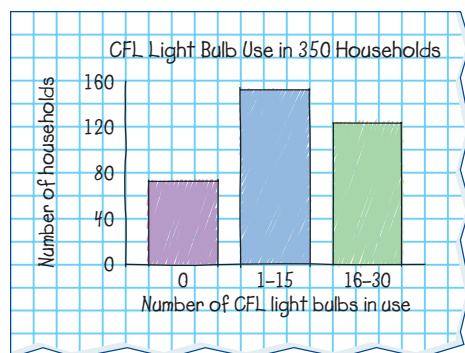


Almost all of the households use CFL bulbs.

I chose a bar graph so that it was easy to compare the number of households in each group.

Since the least value is 73 and the greatest is 153, I let the vertical axis go from 70 to 155.

Kaitlyn's Solution



Most households are using energy-saving light bulbs, but about $\frac{1}{5}$ of them are not.

I also chose a bar graph, but my vertical scale started at 0. I think this gives a more accurate impression because I can see better how the whole is divided.

The 0 bar is about half the size of each of the other bars, so it is about $\frac{1}{5}$ of the total.

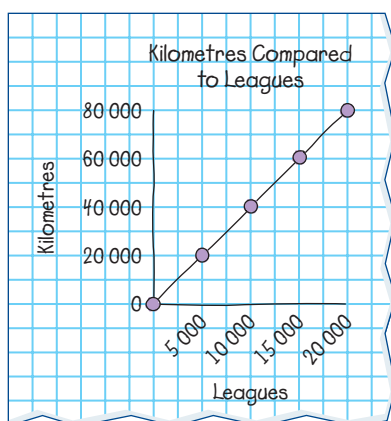
Example 2 | Changing the scale of a line graph

When Jules Verne wrote the science fiction classic *20 000 Leagues Under the Sea*, one league was about 4 km. How can readers compare leagues to kilometres as they read?

Sanjev's Solution

Leagues	Kilometres
0	0
5 000	20 000
10 000	40 000
15 000	60 000
20 000	80 000

I made a table showing leagues and kilometres.

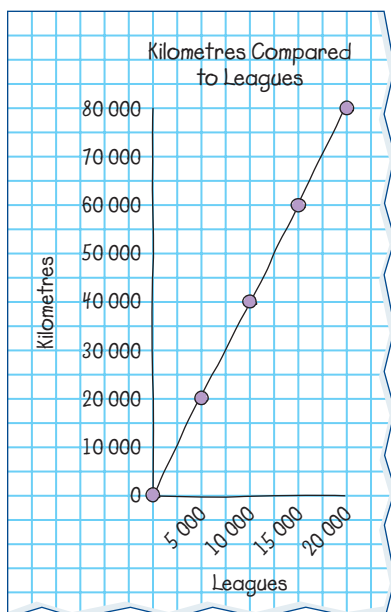


I used the table to create a line graph. Two squares represent 5 000 leagues on the horizontal axis and 20 000 kilometres on the vertical axis.

My graph shows that there is a steady rise in the number of kilometres as the number of leagues increases.



Vanessa's Solution



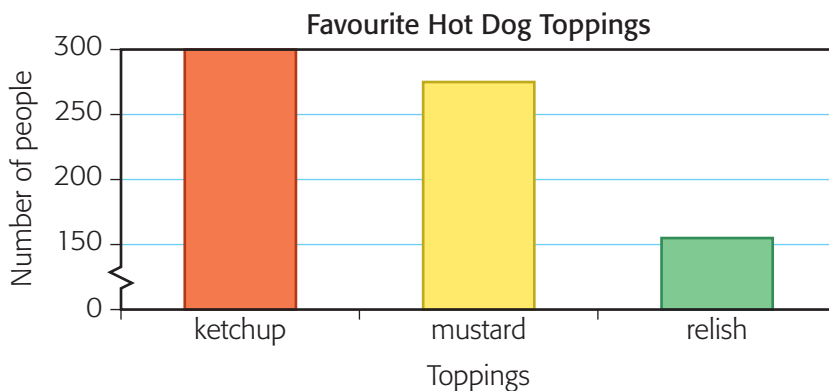
I used Sanjev's table of leagues and kilometres, but I let two squares on the vertical axis represent 10 000 kilometres.

My graph shows a sharp rise in the number of kilometres as the number of leagues increases.

A Checking

1. A vendor at a stadium collected data about favourite hot dog toppings. She used the data to create a bar graph.

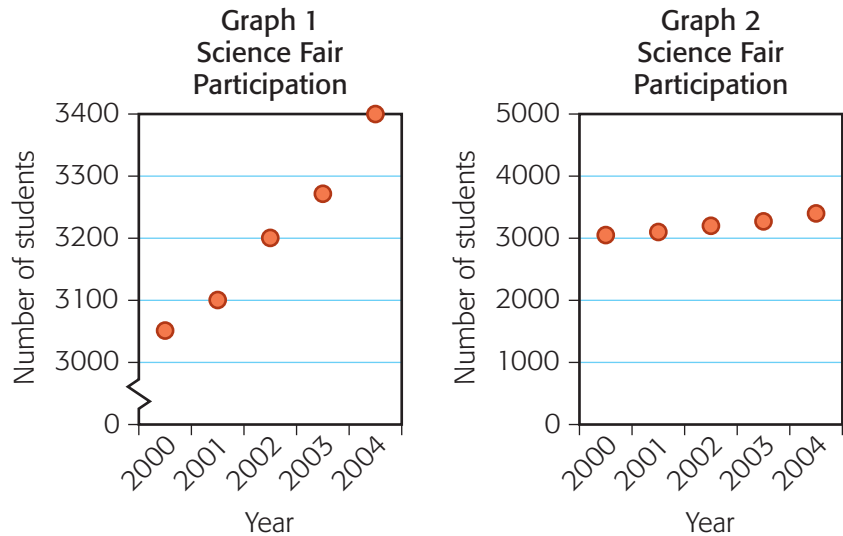
Favourite Hot Dog Toppings	
ketchup	300
mustard	275
relish	155
hot peppers	42



- a) What does the graph appear to show about the popularity of the toppings?
- b) Make a bar graph with a different scale to show less of a difference between the bars.

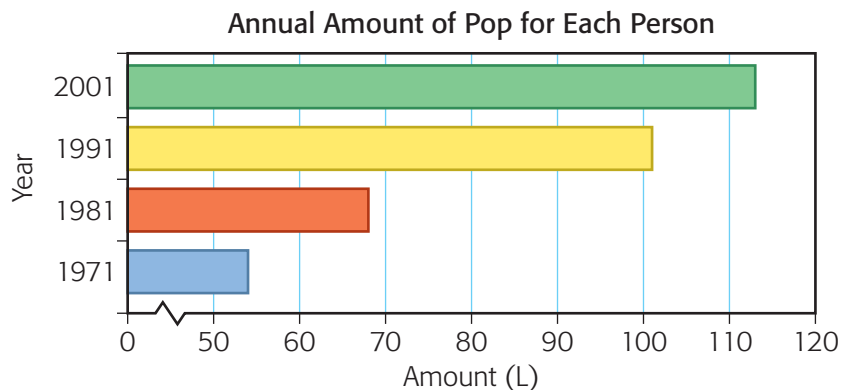
B Practising

2. These two graphs show the number of students participating in provincial science fairs over several years.

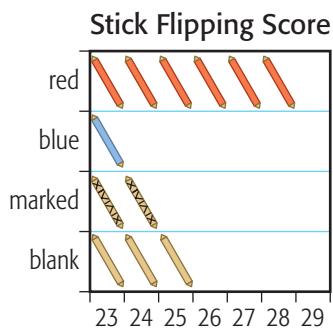


- Which graph seems to show a greater increase in attendance?
 - Do the graphs appear to show the same data? Explain your reasoning.
 - Which graph might the organizers of the science fair prefer to show parents? Explain your choice.
3. Sagitha used data about the amount of pop the average Canadian drinks in a year to make a bar graph.

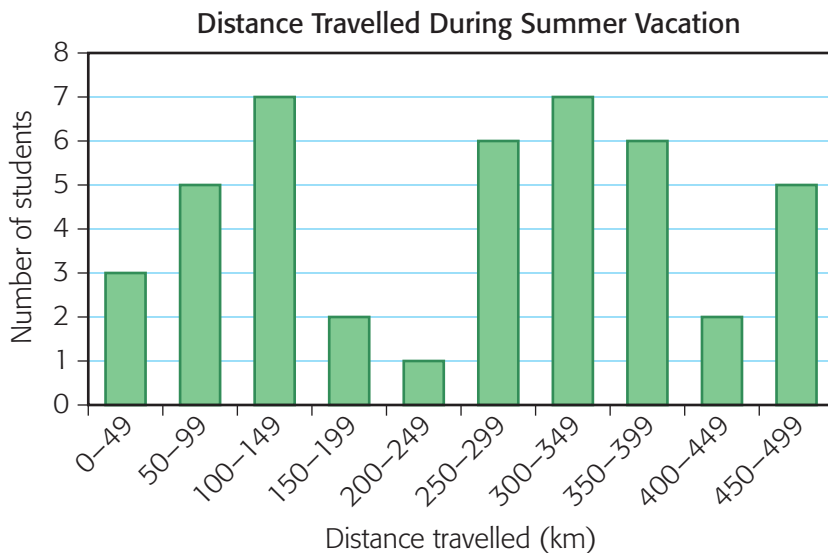
Annual Amount of Pop for Each Person	
Year	Amount(L)
1971	54
1981	68
1991	101
2001	113



- Draw another graph that appears to show smaller increases from decade to decade.
- Describe how your scale differs from Sagitha's scale.

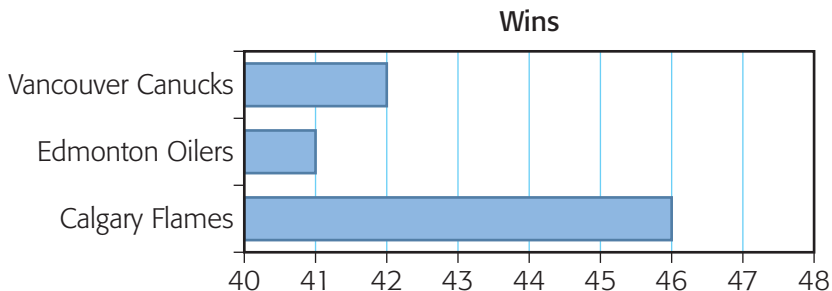


4. Your score in a stick-flipping game depends on which side lands face up. Trevor and his friends made a graph showing the number of times each side landed face up during one game.
 - a) Can you conclude that the red side lands face up much more frequently than the blue? Explain.
 - b) How would you change the graph to better show the results?
5. Students were asked how far they travelled during their summer vacations. The results are shown in this graph.



Create a new graph to show that the majority of the students travelled less than 300 km.

6. How can you change the scale on a graph to make the differences between categories appear less or greater than they really are? Use this graph to help you explain.



8.5

Recognizing Misleading Graphs

YOU WILL NEED

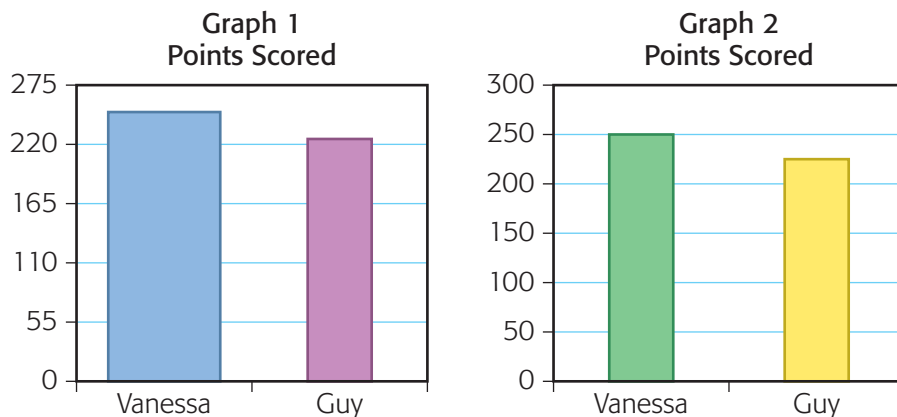
- grid paper

GOAL

Describe how to analyze graphs to be sure the data are represented fairly.

LEARN ABOUT *the Math*

Vanessa and Guy were contestants in a trivia game show. The graphs show their results.



? How does one of the graphs create a false impression?

- Does each graph display the same data? How do you know?
- What is it that makes the two graphs appear different?
- Which graph shows that both students scored a similar number of points?
- What impression does the other graph give?
- Do you think one of the graphs is misleading? Why or why not?

Reflecting

- F. Which graph should be used to compare the results fairly?
- G. When might someone want to use the other graph?

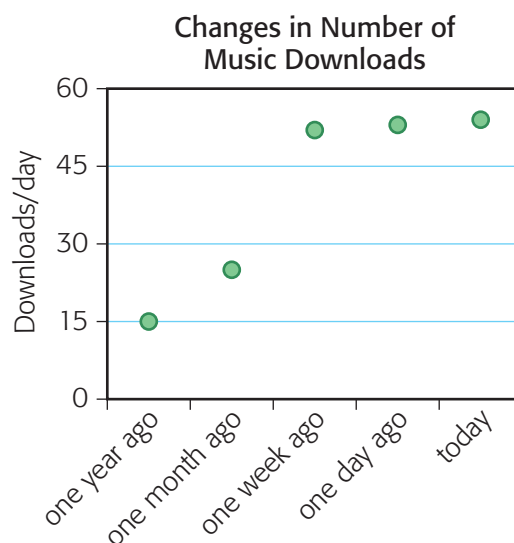
WORK WITH the Math

Example 1

Analyzing graphs for inconsistencies

Mark drew a graph to show how the number of music downloads from an online vendor changed over time.

- a) What impression does the graph give about the change in the number of downloads?
- b) How is this graph misleading?



Elena's Solution

- a) Looking at the points, I can conclude that downloads increased very sharply at first, then tapered off. They still increased, but not at such a great rate as before.
- b) The horizontal scale is evenly spaced, even though the dates it represents are not. The data value for one year ago is probably a mean for the whole year. The data for one day ago and today are not means, they are for only one day. Maybe those two days are outliers. When the mean for this month is calculated, the data could show a big increase, a big decrease, or no change from a year ago or a month ago.

The graph is misleading because it appears that you can use it to predict downloads will increase slowly in the future, but you cannot predict at all using this graph.

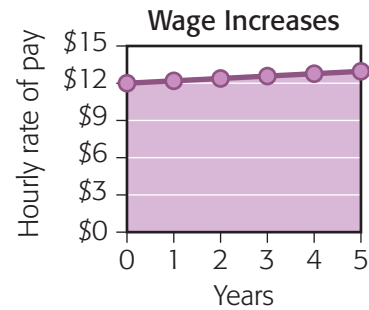
The rate of increase is shown by the change in the heights of the dots.

When you read the labels, it is possible to figure out what the graph is showing, but Mark could have created a less misleading graph using the means for each month.

Example 2

Identifying inconsistent conclusions

A company put this graph in a newspaper ad to show how quickly employee pay is rising. Would you want to work for this company?



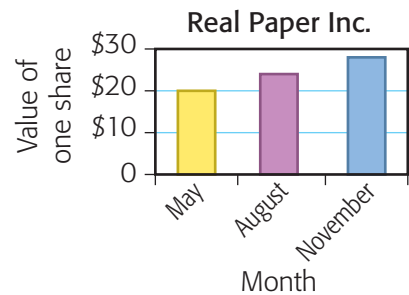
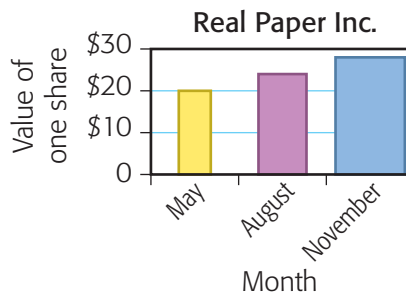
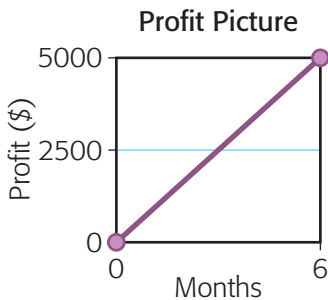
Kaitlyn's Solution

The graph makes it look like the rates go up a lot, but they do not. I would not want to work for this company.

At first I thought the large shaded area represented the increase in the rate of pay. I looked closely at the graph and realized that the increase in pay is the change in the height, which is tiny.

A Checking

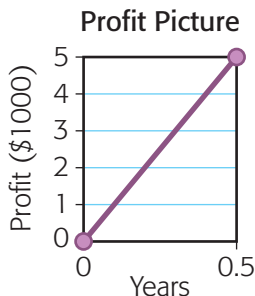
- These two graphs show the same data.



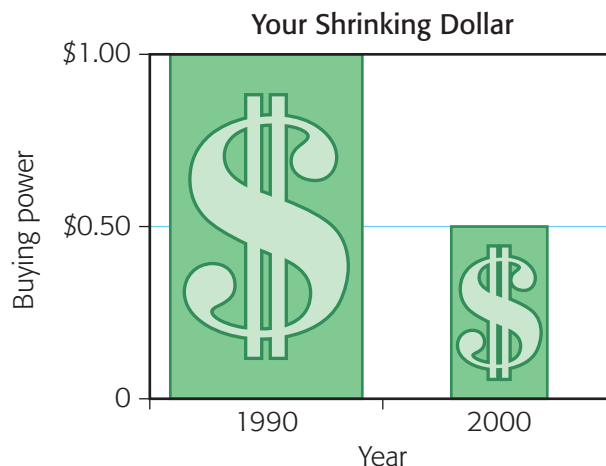
- Which graph creates a false impression?
- What false impression is created? Explain.

B Practising

- A company asked two artists to draw a graph of the company's profits for a six-month period.
 - How are the graphs alike? How are they different?
 - If you were the president of the company, which graph would you choose to report profits? Explain.
 - Predict the company's profit for the seventh month. What assumption did you make to choose your answer?



3.



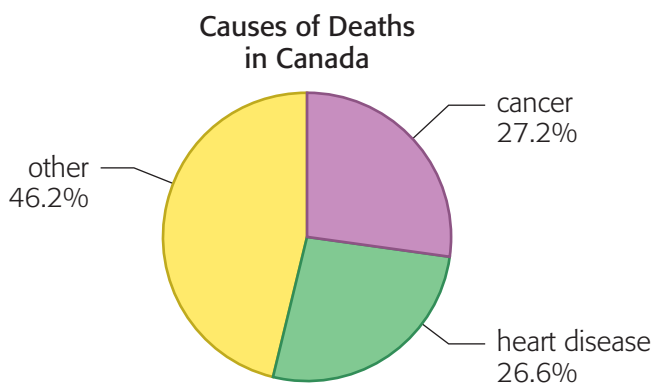
- Measure and compare the areas of the bars. How do the areas of the symbols compare?
- Explain how the graph misrepresents the change in the buying power of a dollar.
- Draw a less misleading graph.

Reading Strategy

Questioning

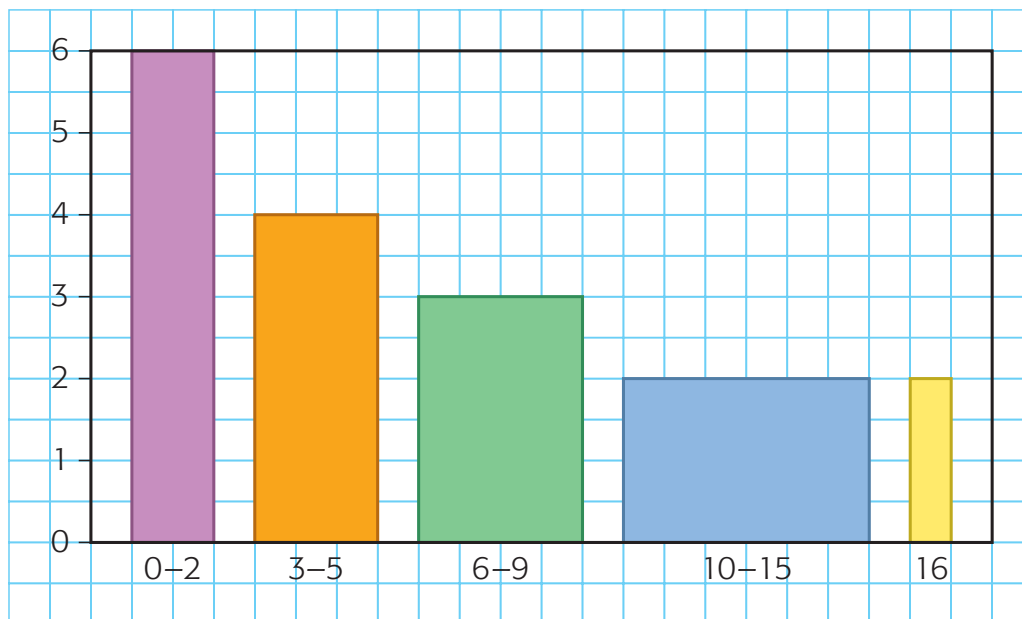
Look at the graph. What questions can you ask about the graph? Where might you find answers about those questions?

- A news report said that cancer is the leading cause of death in Canada. Marissa looked up data online and made a circle graph to show that it is not. Should she write to the news station to tell them about their mistake?



- When you look at a visual representation of data, what can you look for to be sure the data are represented fairly?

Tricking the Eye



When the bars in a graph have different widths, your eye tends to compare areas instead of heights. Just as in a circle graph, the area of each bar can represent the percentage of all the data that falls in that category.

1. Suppose the height of each bar tells you how many data values are in that group.
 - a) How many data values altogether are represented by this graph?
 - b) How many bars represent the same number of data values?
2. Suppose the area of each bar tells you how many data values are in that group.
 - a) How many data values are represented in this graph altogether?
 - b) How many bars represent the same number of data values?
3. Which interpretation, height or area, do you think is a more accurate way to represent the data? Why?

Data Matching

YOU WILL NEED

- game cards

The goal of this game is to match graphs that could be displaying the same data.

Number of players: 2 to 4

How to Play

1. Lay the cards out face down.
2. On your turn, flip any two cards over. If the graphs are displaying the same data, keep the cards and score a point. If the graphs are not displaying the same data, turn them face down again. Your turn is over.
3. When all the matches have been made, the player with the highest score wins.



Chapter Self-Test

1. Anne recorded the colours of 100 cars in the parking lot of a supermarket. These are her results.

Colour of car	Number of cars of each colour
white	20
silver	32
black	18
red	12
blue	18

What type of graph would you use to display Anne's data? Explain.

2. A biologist who is studying gulls measured and recorded these wingspans (rounded to the nearest centimetre).

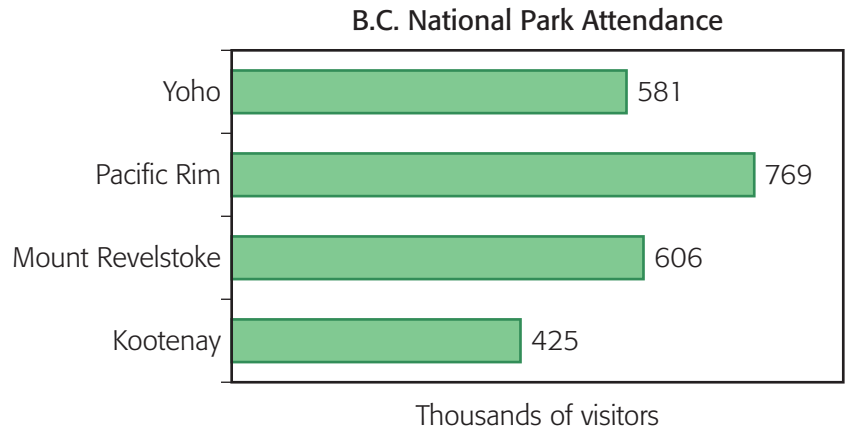
132 145 162 135 142 122 138 124
 135 140 128 122 145 138 139 122
 146 150 167 128 134 147 151 122

- a) Construct two bar graphs, using different groupings for the data.
 - b) Could you get different impressions of the wingspans from the different graphs you created? Explain.
3. Canadian Western Natural Gas Company reported that prices have dropped since 2001.

Cost of Natural Gas per Gigajoule					
2001	2002	2003	2004	2005	2006
\$14.00	\$4.00	\$5.50	\$6.50	\$7.50	\$13.00

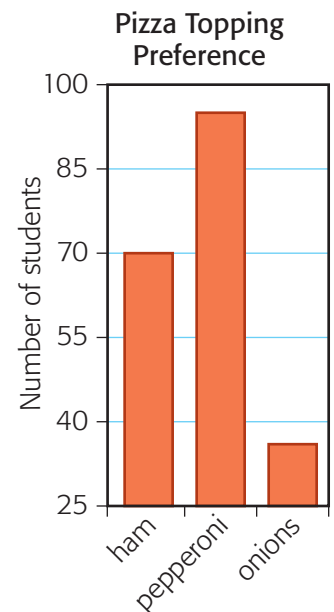
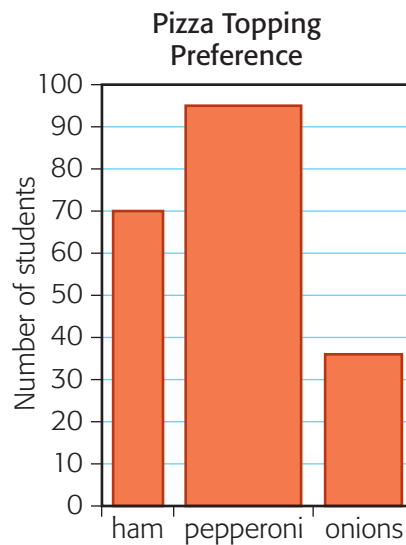
- a) What type of graph might you choose to display this data? Explain.
- b) What information will your graph show?
- c) What other type of graph would be suitable to display this data?

4.



How could you change the scale of the graph to make the number of Pacific Rim Park visitors look greater? Construct the new version.

5.



- How are the graphs the same?
- How are the graphs different?
- What impression does each graph give about the students' preferences?
- Which graph do you feel is presented in a misleading way?

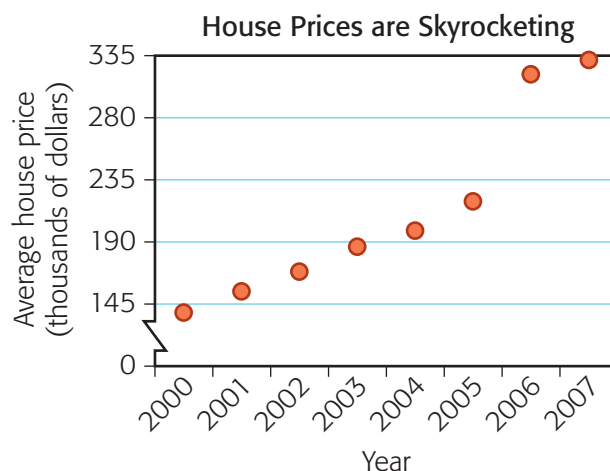
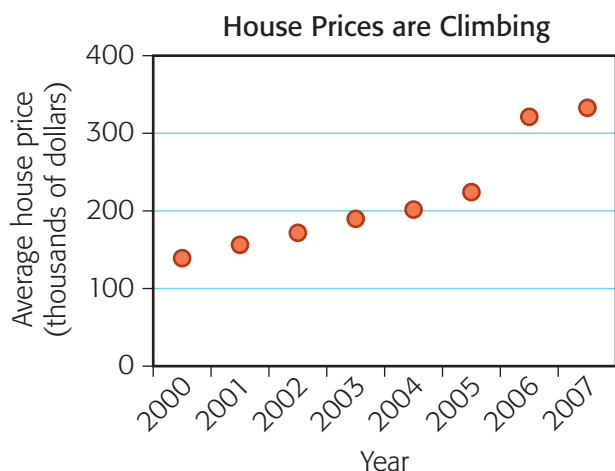
What Do You Think Now?

Look back at What Do You Think? on page 331. How have your answers and explanations changed?

Frequently Asked Questions

Q: How can you change the appearance of a graph?

A: You can change the scale of the graph. This can cause people to have a different impression of the data. For example, these graphs show the same data, but the scale in the one on the right greatly exaggerates the increase in housing prices.

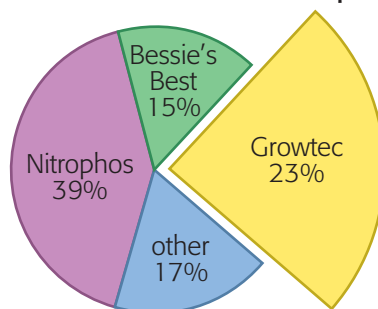


Q: How can a graph misrepresent data?

A: A graph can display data in such a way that a person looking at the graph gets the wrong impression.

For example, enlarging the sector for Growtec in the circle graph makes it appear that they have a larger market share than they really do.

Market Share of Fertilizer Companies



Practice

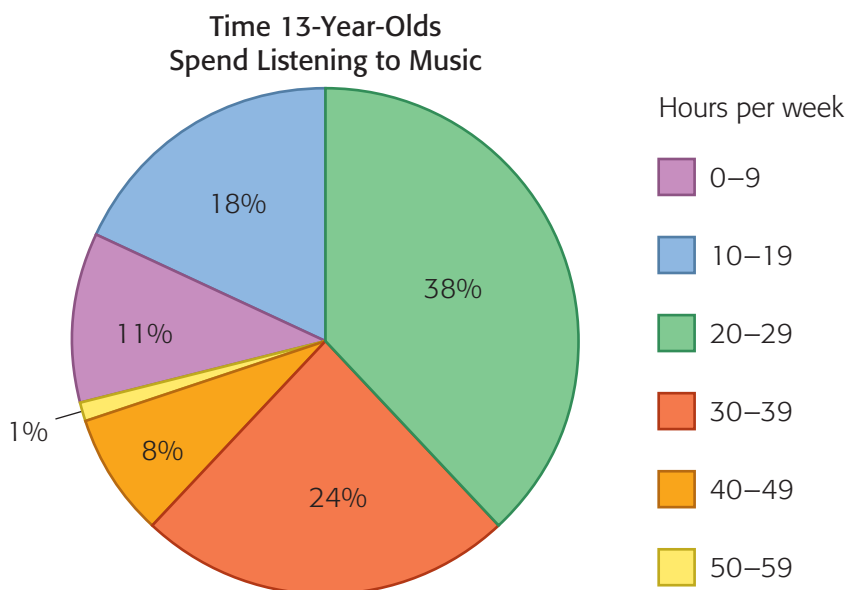
Lesson 8.2

1. The data show the life expectancy for various countries.
 - a) Organize the data shown into groups. Construct a bar graph.
 - b) Repeat part a) using different groups. How did your bar graph change?

Lesson 8.3

2. a) Explain why a bar graph is a good choice for the data in question 1.
b) What other graphs would be appropriate? Explain.
3. Rohan conducted a survey and presented his findings in a graph. He concluded that music is very important to 13-year-olds.

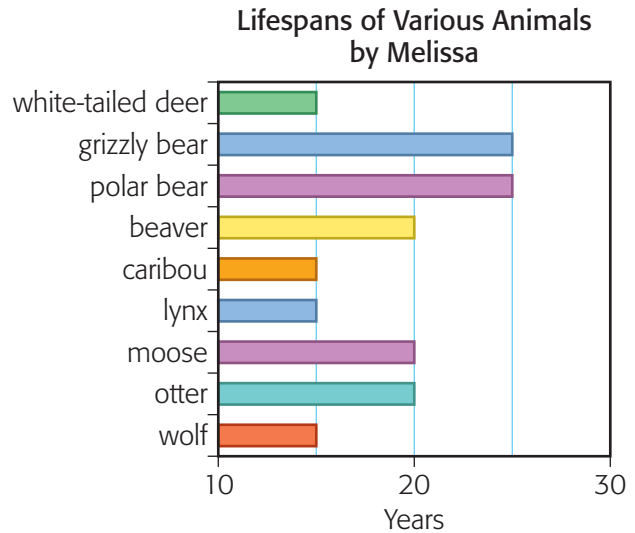
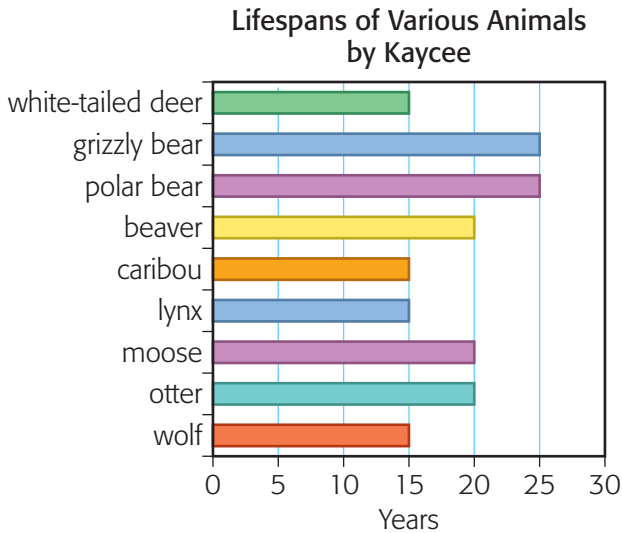
Country	Life expectancy (years)
Bangladesh	57
Brazil	66
Canada	78
China	70
Ethiopia	50
Iceland	79
India	59
Japan	80
Mexico	73
Norway	78



- a) Why is a circle graph a good choice for this purpose?
- b) What details might be a good idea to include?
- c) How does Rohan's graph support his conclusion?

Lesson 8.4

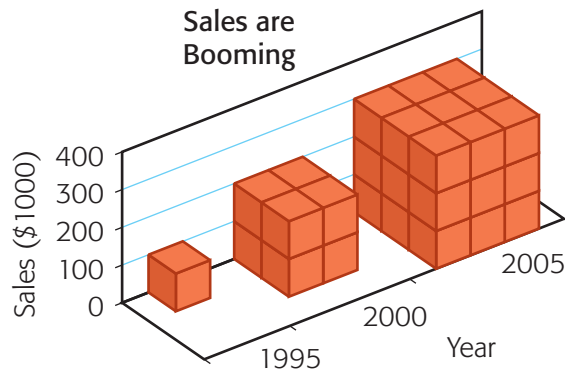
4. Kaycee and Melissa used the same software to create a graph showing the same data.



- How are the graphs different?
- What does each graph imply about the lifespan of the polar bear compared to the wolf?
- Describe what would happen if the scale on Melissa's graph started at 15.

Lesson 8.5

5.



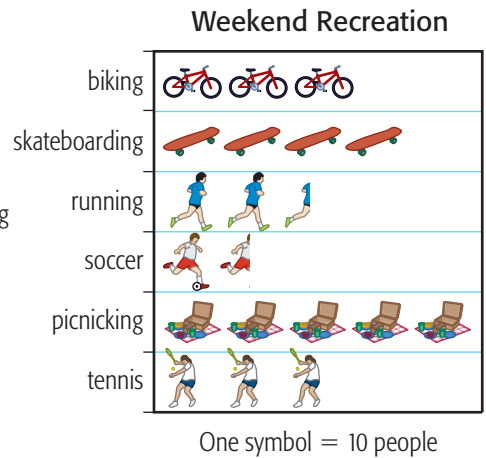
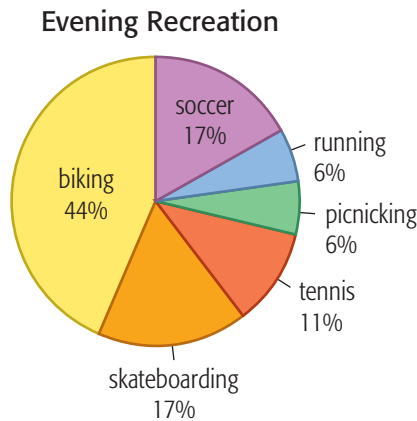
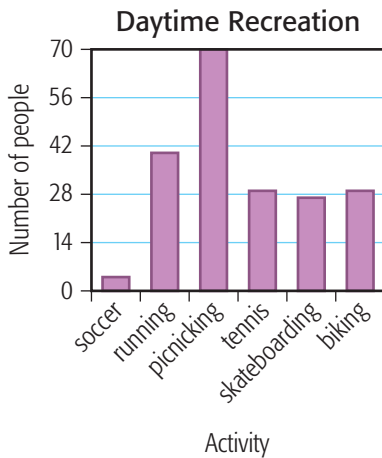
- How many cubes represent the sales for each year?
 - Does the graph accurately represent the change in sales? Explain.
 - How could the graph be drawn to show the data more fairly?
6. How can you make graphs that display data well, and do not misrepresent the data?

Task Checklist

- ✓ Do the graphs support your argument?
- ✓ Did you clearly explain your recommendations?

Convincing Statistics

The town is deciding what facilities to build in the new recreational area. You and your friends have been asked to make recommendations. Studies done so far have produced these graphs:



? How can you use the graphs to make recommendations?

- A. Are the graphs misleading in any way?
- B. Which graphs support the argument that picnic areas should be built? Explain.
- C. How could you create a new graph to make the case for picnic areas more convincing?
- D. How can you use the graphs to make the argument for biking trails convincing?
- E. Based on the graphs, what three recommendations would you make? Explain your thinking.