



Chapter 4

Percents

GOAL

You will be able to

- interpret, represent, and use percents greater than 100%
- interpret, represent, and use percents between 0% and 1%
- relate percents to fractions and decimals
- solve problems involving percents

- ◀ What percent of Canadians do you think use the Internet regularly?

YOU WILL NEED

- a coin
- grid paper

Hitchhiker's Thumb

About 25% of people have hitchhiker's thumb.



Hitchhiker's thumb



Not hitchhiker's thumb

? Do more students in your class have hitchhiker's thumb than would be expected?

- A. What fraction does 25% represent? Draw a picture to show why.
- B. In your class, how many people would you expect to have hitchhiker's thumb?
- C. What multiplication could you do to answer part B?
- D. What division could you do to answer part B?
- E. About how many students in a school of 600 would you expect to have hitchhiker's thumb? Explain your thinking.
- F. Find out how many students in your class have hitchhiker's thumb. Compare that number with your answer to part B. Do more students have hitchhiker's thumb than expected or not?

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

- 1. A percent of a number is always less than that number.
- 2. Every number is some percent of every other number.

4 and 16
4 is 25% of 16

9 and 20
9 is 45% of 20

- 3. 0.5% means one half.
- 4. To get 15% of a number, you can take 30% of the number's double.

4.1

Percents Greater than 100%

YOU WILL NEED

- 10×10 Grids
- a calculator

GOAL

Represent and interpret percents greater than 100%.

LEARN ABOUT *the Math*

Ivan is 160 cm tall. Taira is 152 cm tall.
Both Ivan and Taira are 13 years old.

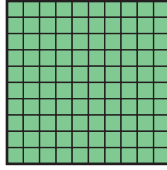


An adult's height is normally 107% of his or her height at age 13.

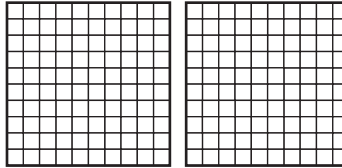
? How tall are Ivan and Taira likely to be as adults?



- A.** What percent of this grid is shaded?



- B.** Shade the grids to show 107%. Circle the part that represents the 100%.



- C.** Suppose the grid in part A represents Ivan's present height. How many centimetres does each small square in the grid represent?
- D.** Use your answer to part C to figure out what 107% of Ivan's present height is, to the nearest tenth of a centimetre.
- E.** Repeat parts C and D for Taira.
- F.** How tall do you predict that Ivan and Taira will be?

Reflecting

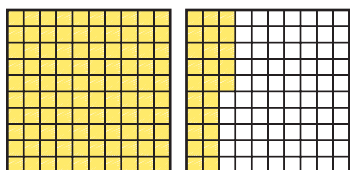
- G.** Why did you use more than one 10-by-10 grid to represent 107%?
- H.** Why did you have to decide that the first grid represented 100% to interpret the percent you showed in part B?



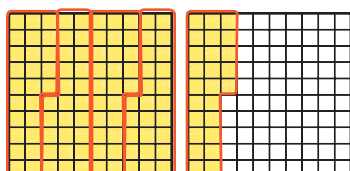
Example 1 Using a grid to solve a percent problem

Renée's CD collection is 125% the size of Angèle's collection.
Renée has 75 CDs. How many CDs does Angèle have?

Renée's Solution



$$125\% = 100\% + 25\%$$



15 15 15 15 15

$$125\% = 5 \text{ groups of } 25\%$$

$$75 \div 5 = 15$$

Each of the 5 groups represents 15 CDs.

$$4 \times 15 = 60$$

If 125% is 75, then 100% is 60.

Angèle has 60 CDs.

I drew a 10-by-10 grid to represent Angèle's collection. I thought of that amount as 100%.

To show 125% for my collection, I needed to use part of a second grid.

There were 125 squares to represent the 75 CDs in my collection.

I divided the 125 into 5 equal sections. I did that because I wanted to create sections I could add to make up 100%.

Each section represented $125\% \div 5$ and also represented $75 \div 5$ CDs.

Angèle's collection is the full first grid. It has 4 sections of 15.

Example 2**Using reasoning to solve a percent problem**

A bacon double cheeseburger, king-size fries, and a medium milkshake provide a Grade 8 student with 390% of the recommended daily grams of fat allowance for a person that age.

How many grams of fat are in the meal if the recommended daily allowance is 20 g?

Lam's Solution

$$100\% \text{ of } 20 \text{ g} = 20 \text{ g}$$

$$400\% \text{ of } 20 \text{ g} = 4 \times 20 \text{ g} = 80 \text{ g}$$

$$10\% \text{ of } 20 \text{ g} = (20 \div 10) \text{ g} = 2 \text{ g}$$

$$390\% \text{ of } 20 \text{ g} = (80 - 2) \text{ g} = 78 \text{ g}$$

The meal has 78 g of fat.

100% of something is the whole thing.

$$390\% = 400\% - 10\%$$

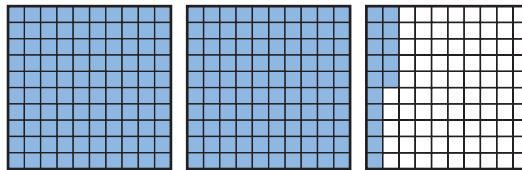
400% is four times 100%.

To get 10%, I divided 100% by 10.

I calculated 390% by subtracting 10% from 400%.

A Checking

1. What percent does the diagram show? One full grid represents 100%.

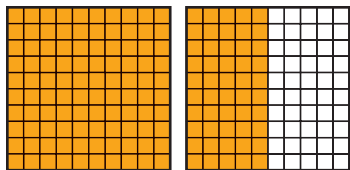


2. Represent 167% using 10-by-10 grids. Use one full grid to represent 100%.
3. A girl usually grows to be 125% of the height she was at age 9. If a girl is 132 cm tall at age 9, what will her adult height likely be?

B Practising

4. Represent each percent using 10-by-10 grids. Use one full grid to represent 100%.

a) 135% b) 250% c) 310%



5. Paul says that the grids show 150%. Rebecca says that they show 75%. How could each be correct?

6. Solve.

a) 120% of 40 = ■ c) 110% of 48 = ■

b) 130% of 200 = ■ d) 220% of ■ = 99

7. A fast food meal contains 70 g of fat. What percent of the recommended daily allowance of 20 g is this?



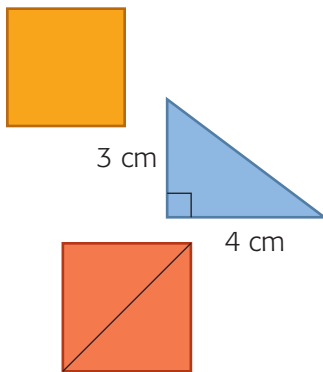
8. Yanir has \$50 in pennies. Use grids to model and calculate each amount.

a) 90% of \$50 b) 310% of \$50

9. A faucet is dripping at a rate of 1 L/h. Why would you not use a percent to describe the rate?



- 10.** The population of a town is 600. Use grids to model and calculate each percent of that population.
- a)** 185% **b)** 225%
- 11.** Last year, 800 students were enrolled at Susan's school. This is 250% of the enrolment in the school 15 years ago.
- a)** Use a diagram to help you calculate the enrolment 15 years ago.
- b)** What percent of the current enrolment is your answer to part a)?
- 12. a)** What percent of the side length of a square is its perimeter?
b) What percent of the shortest side length of this triangle is its perimeter?
c) Estimate. What percent of the side length of the square is its diagonal?
d) Create and solve your own question involving measurements where the answer is a percent greater than 100%.
- 13.** A download speed using high-speed Internet was 1316 KB/s. An upload speed on the same network was 327 KB/s.
- a)** Estimate the percent of the upload speed that represents the download speed.
- b)** Estimate the percent of the download speed that represents the upload speed.
- 14.** One number is 500% of another.
- a)** What could the numbers be?
- b)** What percent is the lower number of the higher one? How do you know?



4.2

Fractional Percents

YOU WILL NEED

- 10×10 Grids
- Thousandths Grids
- a calculator

GOAL

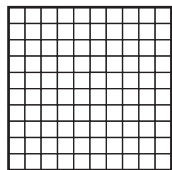
Represent and interpret percents between 0% and 1%.

LEARN ABOUT *the Math*

You can taste sweetness if 0.5% of a sugar-and-water mixture is sugar.



? What is the least amount of sugar that must be in a 250 g sugar-and-water mixture for it to taste sweet?



- Why should 0.5% be less than 1%?
- How could you represent 0.5% on this grid?
- Suppose the full grid represents 250 g of a sugar-and-water mixture. What does your answer to part B represent?
- What is the least amount of sugar, in grams, that is in the mixture if it tastes sweet? Explain.

Communication | Tip

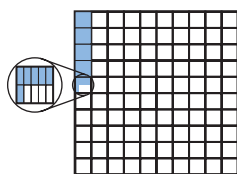
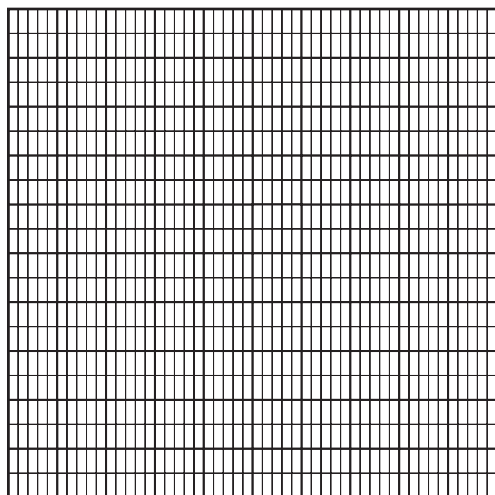
You can read fractional percents like 0.5%, for example, as five-tenths of a percent, or 0.23% as twenty-three hundredths of a percent.

Reflecting

- E.** How would you represent 2.5% using the grid you used in part B? How do you know?
- F.** Why is 0.5% not equal to $\frac{1}{2}$?

WORK WITH the Math**Example 1****Representing percents less than 1%**

How could you use a thousandths grid to show 0.6% and 4.6%?

John's Solution

1% is one hundredth, so 0.1% is one tenth of one hundredth.

$$0.1\% = \frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$$

There are 1000 thousandths in the full grid. Each of the small rectangles is $\frac{1}{1000}$, or 0.1%.

$$0.6\% = 6 \times 0.1\%$$

$$4.6\% = 4\% + 0.6\%$$

Example 2**Solving a problem involving percents**

About 0.9% of the Canadian population is Sikh. If Canada's population is about 34 million, about how many people are Sikh?

Holly's Solution

$$1\% \text{ of } 34\,000\,000 = 340\,000$$

I know that 0.1% is $\frac{1}{10}$ of 1%.

$$\begin{aligned} 0.1\% \text{ of } 34\,000\,000 &= 340\,000 \div 10 \\ &= 34\,000 \end{aligned}$$

$$0.9\% \text{ of } 34\,000\,000 = 340\,000 - 34\,000 = 306\,000$$

To calculate 0.9%, I calculated 1% and then subtracted 0.1%.

There are about 306 000 Sikhs in Canada.

Ivan's Solution

$$0.9\% = \frac{0.9}{100}$$

I wrote 0.9% as a decimal and then wrote it as a fraction.

Multiply numerator and denominator by 10 to get an equivalent fraction.

$$\begin{aligned} \frac{0.9}{100} &= \frac{9}{1000} \\ &= 0.009 \end{aligned}$$

$$0.009 \times 34\,000\,000 = 306\,000$$

I knew that one way to calculate percents is to multiply by the equivalent decimal.

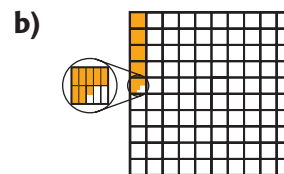
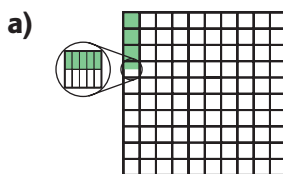
There are about 306 000 Sikhs in Canada.

A Checking

- Use a thousandths grid to represent each of these percents.
a) 0.75% **b)** 1.4% **c)** 4.9%
- How many grams of sugar would you need to make a 1 kg sugar-and-water mixture that is 0.5% sugar?

B Practising

3. What percent does each of these grids represent? The full grid is 100%.



4. How can knowing that 5% of a mass is 25 g help you to calculate each of these?
- a) 1% b) 0.1% c) 2.5%
5. a) Explain how you might estimate the value of 0.3% of 630.
 b) Represent 0.3% on a 10-by-10 grid.
 c) Calculate 0.3% of 630. Explain your strategy.
6. You can taste saltiness if 0.25% of a mixture is salt. At least how many grams of salt would there have to be in 1 kg of salt water to taste the salt?
7. Air contains 0.93% argon and 0.03% carbon dioxide. In 1 L of air, how much of each would there be?
- a) argon b) carbon dioxide
8. About 0.8% of Canada's exports go to Germany. For each million dollars in exports, how many dollars' worth goes to Germany?
9. a) How do you know that 1 mm is 0.1% of 1 m?
 b) What percent of 1 m is 3.2 mm?
10. Which of these ways of calculating 2.5% of a number is correct? Explain.
- a) Calculate 5% and then divide by 2
 b) Calculate 25% and then divide by 10
 c) Divide by 4 and then divide by 10
 d) Divide by 4 and then divide by 100
11. When is 0.1% of a number a whole number?
12. Is 5.1% of a number always very close to 5% of the number? Explain using examples.



4.3

Relating Percents to Decimals and Fractions

YOU WILL NEED

- 10×10 Grids
- Thousandths Grids

GOAL

Express a percent as an equivalent decimal or fraction, or a decimal or fraction as an equivalent percent.

LEARN ABOUT *the Math*

One pair of skis costs 150% of the cost of another pair of skis.

? What fraction of the price of the cheaper skis is the price of the more expensive skis?

- A full 10-by-10 grid represents 100%. Use a decimal and a mixed number or improper fraction to write the number of grids you would shade to represent 150%.
- Why does the ratio 150:100 compare the costs of the two pairs of skis?
- What fraction and decimal of the price of the cheaper skis is the price of the more expensive skis?



Reflecting

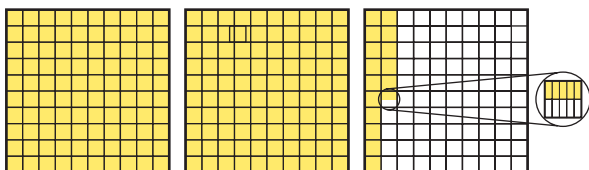
- D. How could you have predicted that the fraction in part C would be a mixed number or improper fraction and that the decimal would be greater than 1?
- E. How are the ratio in part B and the fraction and decimal in part C related?

WORK WITH the Math

Example 1

Relating fractions, decimals, and percents

Use a fraction, a decimal, and a percent to describe the shaded area.
Use one full 10-by-10 grid to represent 100%.



Ivan's Solution

Percent:

$$100\% + 100\% + 15\% + 0.5\% = 215.5\%$$

Each full grid is 100%.

Decimal:

$$1 + 1 + 0.15 + 0.005 = 2.155$$

Each full grid is 1.

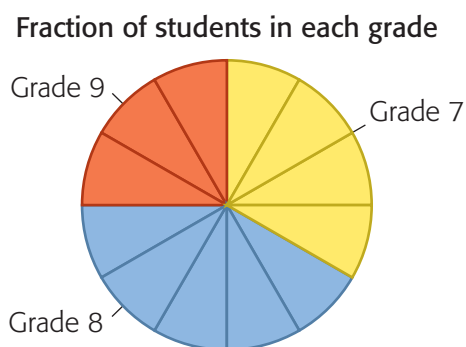
Fraction:

$$\begin{aligned} 2 + \frac{15}{100} + \frac{5}{1000} &= \frac{2000}{1000} + \frac{150}{1000} + \frac{5}{1000} \\ &= \frac{2155}{1000} \end{aligned}$$

There are 2 full grids and another $\frac{15}{100} + \frac{5}{1000}$ of a third grid.

Example 2**Relating a circle graph to percents**

This circle graph shows what fraction of the students in a school is in each grade. What percent of the students are in Grade 8?

**Taira's Solution**

$\frac{5}{12}$ are in Grade 8.

$$\frac{5}{12} = 5 \div 12$$

$$5 \div 12 \approx 0.417$$

$$0.417 = 41.7\%$$

Each section of the graph represents $\frac{1}{12}$.

The first two decimal places tell the whole-number percent. The third decimal place is tenths of a percent.

Example 3**Writing a fraction as a percent**

A group sponsoring a contest says that 1 out of 16 tickets wins a prize. What percent of the tickets win a prize?

Angèle's Solution

$$1 \div 16 = \frac{1}{4} \text{ of } \frac{1}{4}$$

$$\frac{1}{4} = 0.25$$

$$0.25 \div 4 = 0.0625$$

$$0.0625 = 6.25\%$$

I could divide 1 by 16 to write $\frac{1}{16}$ as a decimal.

I knew $\frac{1}{16}$ is $\frac{1}{4}$ of $\frac{1}{4}$.

First I thought of the decimal for $\frac{1}{4}$. Then I took $\frac{1}{4}$ of that by dividing by 4.

Then I wrote the percent by multiplying the decimal by 100.

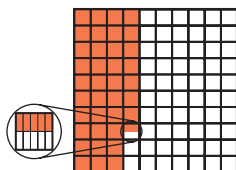
A Checking

- Shade in each fraction of a grid. Use one full grid to represent 1. Write the percent for the fraction.
 - $\frac{3}{4}$
 - $1\frac{1}{5}$
- 1.1% of Canadians are Jewish.
 - Write the percent as a decimal.
 - Write it as a fraction.

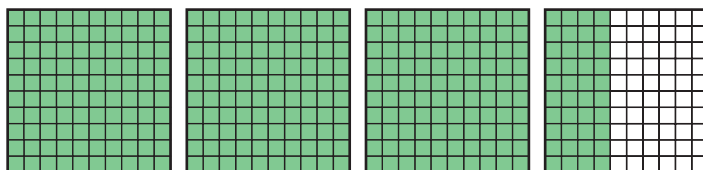
B Practising

- Use a fraction, a decimal, and a percent to describe each shaded area. A single full grid represents 100%.

a)

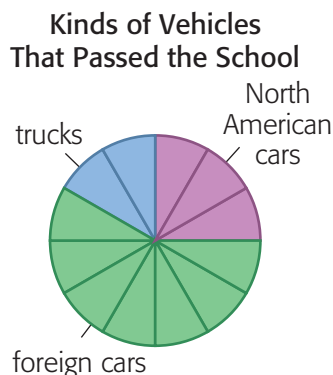


b)



- Complete the chart.

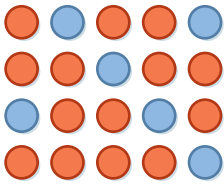
	Percent	Equivalent fraction	Equivalent decimal
a)	3.2%		
b)		$\frac{5}{4}$	
c)			0.064



- The population of Abbotsford, BC, is 136% of the population of Kamloops, BC. Write the percent as a fraction and as a decimal.
- Joel's class did a traffic survey and drew a circle graph to show what kinds of vehicles passed the school on a particular morning.
 - What percent of the traffic was trucks?
 - What percent was foreign cars?
 - What percent was North American cars?



7. You are downloading a file. The progress bar looks like this.
- Estimate the percent of the file that has been downloaded.
 - Test your estimate by measuring.
 - Use part b) to write the percent as a decimal and as a fraction.
8. The average Canadian spends about 0.09 of a 24 h day watching television.
- What fraction of a day is this?
 - What percent of a day is this?
 - About how many minutes is this?
9. The percent of people with blood type A is 410% of the fraction of people with blood type B. Write this percent as a fraction and a decimal.
10. The fraction of people with blood type O is $\frac{9}{2}$ the number of people with blood type B. Write this as a percent.
11. a) Write the number of red counters as a fraction and as a percent of the number of blue counters.
b) Remove 5 counters so that there are 400% as many red counters as blue ones. How many of each colour of counter did you remove?
12. Franca knew that 20 was 2.5% of a number. Explain why you can use each of these methods to calculate the number.
- Divide 20 by 2.5 and then multiply by 100.
 - Divide 20 by 0.025.
13. Use what you know about fractions to calculate 50% of 200%.
14. Why is it usually easier to express a decimal as a percent than a fraction as a percent? Why is it not always easier?



4.4

Solving Problems Using a Proportion

YOU WILL NEED

- a ruler (optional)
- a calculator

GOAL

Solve a percent problem using an equivalent ratio.

LEARN ABOUT *the Math*

Lam has a mass of 62.0 kg. After a season of lacrosse, his body fat was reduced from 18% of his total mass to 12.5% of his total mass, but his total mass did not change.

? How much body fat did Lam lose?



Calculate the mass of fat loss.

Lam's Solution

18% - 12.5% is about 5%

$$10\% \text{ of } 62 = 6.2$$

$$5\% \text{ of } 62 = 6.2 \div 2 \\ = 3.1$$

I lost about 3.1 kg of fat.

$$18\% - 12.5\% = 5.5\%$$

$$5.5\% = \frac{5.5}{100} \\ = \frac{55}{1000}$$

$$\frac{55}{1000} = \frac{\square}{62}$$

$$\frac{55 \times 62}{1000 \times 62} = \frac{\square \times 1000}{62 \times 1000}$$

$$\frac{3410}{62\,000} = \frac{1000 \times \square}{62\,000}$$

$$3410 = 1000 \times \square$$

$$3410 \div 1000 = \square$$

$$3.410 = \square$$

I lost 3.4 kg of fat.

First I estimated.

The answer is about 5% of 62.

That is half of 10%.

I calculated the percent change.

Then I set up a proportion to solve the problem.

I know my answer is reasonable, because it is close to my estimate.

Reflecting

- A.** How did Lam choose the values for the proportion?
- B.** Why did solving the proportion solve Lam's problem?

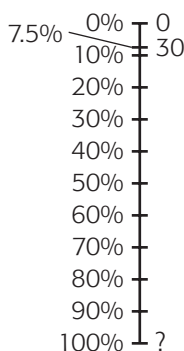
WORK WITH the Math

Example 2

Using a visual model to set up a proportion

7.5% of the boys in Joe's school play lacrosse. This is 30 boys.
How many boys are in the school?

Solution



$$\begin{array}{l} \frac{7.5}{100} = \frac{30}{\blacksquare} \\ \quad \times 2 \\ \frac{7.5}{100} = \frac{15}{200} \\ \quad \times 2 \\ \frac{15}{200} = \frac{30}{\blacksquare} \\ \quad \times 2 \\ \blacksquare = 200 \times 2 \\ \quad = 400 \end{array}$$

There are 400 boys in the school.

Think of the problem as figuring out the answer to the question: "30 is 7.5% of what number?"

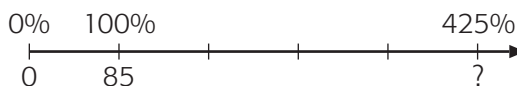
That means you know what 7.5% is, but you want to know what 100% is.

Draw a diagram to visualize the proportion. Place 7.5% close to, but above, 10%. You can see that 100% should be a lot more than 30. Use equivalent fractions to make it easier to solve the proportion.

Since the numerator was multiplied by 2, the same must be true for the denominator.

A Checking

- How does this diagram show that 425% of 85 is more than 4×85 ?



- The body mass for muscle should be about 310% of the mass for fat. Luc's fat mass is 10.4 kg. What should his muscle mass be?

B Practising

- Solve.

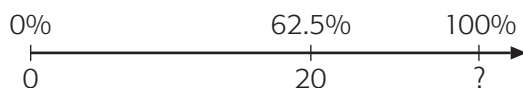
a) $\blacksquare = 225\%$ of 48

b) $\blacksquare = 37.5\%$ of 480

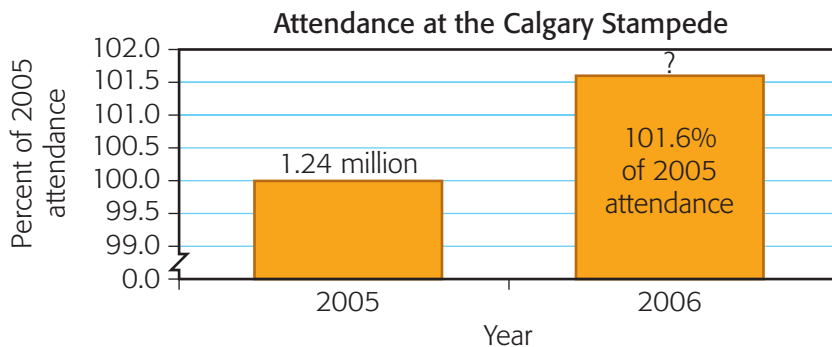
c) $78 = 325\%$ of $\blacksquare$

d) $84.7 = 770\%$ of $\blacksquare$

4. Explain how to use the diagram to estimate the solution to 62.5% of $\blacksquare = 20$



5. Draw a diagram to show each.
- 20% of 115 is 23.
 - If 40 is 80% of $\blacksquare$, then $\blacksquare$ must be 50.
6. A popular music download site reported these statistics:
In April 2007, there were 5.6 million downloads a day. This was 0.2% of all downloads from that site since it started. How many downloads were there from the site from when it started until April 2007?
7. **a)** The ratio 5:1000 describes the scale on a map. Write the ratio as a fraction.
b) What percent describes the distance on the map compared to the actual distance?
c) What percent describes the actual distance compared to the map distance?
8. The population of Alberta in 2006 was 110.6% of its population in 2001. The population in 2001 was 2 974 807. Estimate the population in 2006.
9. Use the information in the graph to estimate the attendance at the Calgary Stampede in 2006.



10. How does knowing how to create equivalent ratios help you to calculate the percent of a number?

4.5

Solving Percent Problems Using Decimals

YOU WILL NEED

- a calculator

GOAL

Use the decimal representation of a percent to solve a problem.

LEARN ABOUT *the Math*

In Canada, more and more people are living in towns and cities. In January 2007, about 13.5% of Saskatchewan's population of 987 939 was Aboriginal. About 46.7% of the Aboriginal people were living in towns and cities.

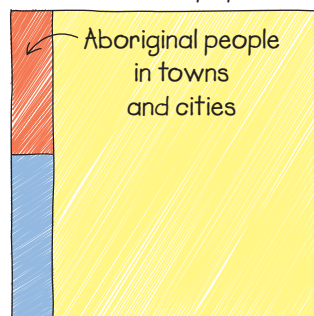
? About how many Aboriginal people in Saskatchewan live in towns and cities?



The question said “about,” so I decided to estimate.

Angèle’s Solution

Saskatchewan’s population



13.5% is close to 10%

46.7% is close to 50%

$$10\% \text{ of } 50\% = 0.1 \times 0.5 \\ = 0.05$$

987 939 is close to 1 000 000

$$0.05 \times 1\,000\,000 = 50\,000$$

About 50 000 Aboriginal people in Saskatchewan live in towns and cities.

I drew a diagram to help me figure out what to do.

I realized I had to calculate 46.7% of 13.5% of 987 939.

I estimated 13.5% as 10%.

Since I rounded 13.5% down, I rounded 46.7% up to 50% to estimate.

I needed 50% of 10%, so I multiplied equivalent decimals.

Then I multiplied by an estimate of the population. I used 1 million for that estimate.

Reflecting

- A.** Why could you not have just calculated 46.7% of 1 000 000 directly to solve the problem?
- B.** Angèle rewrote the percents as decimals to solve the problem. How else could you have solved the problem?

WORK WITH the Math

Example 2 Solving a problem using decimal division

Online sales in Canada in 2006 were 139.8% of online sales in 2005. The value of the sales in 2006 was \$49.98 billion. What was the value of the sales in 2005?

Solution

$$139.8\% = 1.398$$

Write 139.8% as a decimal.

$$1.398 \times 2005 \text{ sales} = 2006 \text{ sales}$$

Write the equation relating the sales for 2005 and 2006.

$$1.398 \times 2005 \text{ sales} = \$49.98 \text{ billion}$$

Divide both sides by 1.398.

$$\begin{aligned} 2005 \text{ sales} &= \$49.98 \text{ billion} \div 1.398 \\ &= \$35.75 \text{ billion} \end{aligned}$$

Use a calculator to do the division.

Sales in 2005 were \$35.75 billion.

A Checking

- Rewrite these equations with decimals you could use to solve each, then solve them.
 - 15.2% of 35 =
 - 124% of 18 =
 - 5.5% of = 40
 - 160% of = 30
- In November, the number of visitors to the school blog rose to 112% of the number in October. There were 500 visitors to the blog in October. How many visitors were there in November?

October: 500 visitors to the school blog

November: 112% of the number of October visitors

B Practising

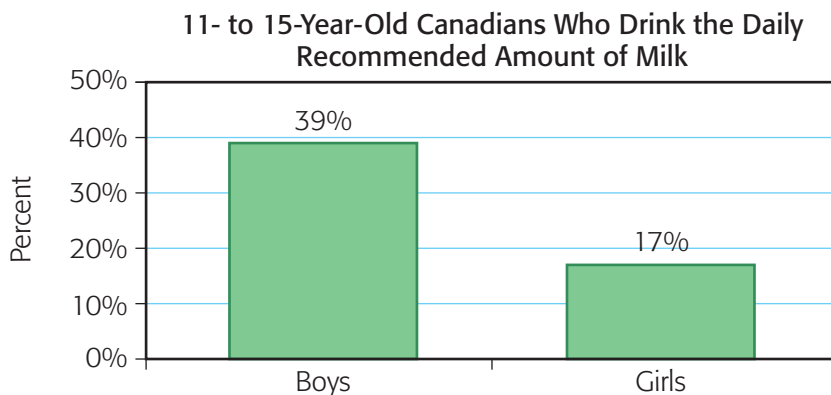
3. Solve each by using a decimal equivalent for the percent.
 - a) 1.4% of 500 = ■
 - b) 0.45% of 250 = ■
 - c) $560 = 350\%$ of ■
 - d) $24 = 0.8\%$ of ■
4. What percent question is Ellen solving when she performs each computation? For example, a question for the calculation $40 \div 0.2$ could be, “40 is 20% of a number. What is the number?”
 - a) 0.45×36
 - b) 1.2×45
 - c) 0.004×180
 - d) $56 \div 0.07$
 - e) $36 \div 1.8$
 - f) $90 \div 0.005$
5. The cost of an item in Alberta is 105% of the listed price to include the GST. What is the cost of each of these items with tax included?



6. Jeff’s parents bought new flooring for his room. There was a sale, so they only had to pay 80% of the regular cost. If they paid \$400, what was the regular price?
7. The chart below shows the most popular computer screen resolutions in Canada in 2007.
In a school where 400 students had computers, about how many would be using a screen resolution of 800×600 ?

Screen resolution	Percent of users
1024×768	54.31%
800×600	19.94%
1280×1024	12.06%
1280×800	3.93%
1152×864	3.75%

8. Refer to the graph. In a school with 480 boys aged 11 to 15, how many boys drink the amount of milk they should?

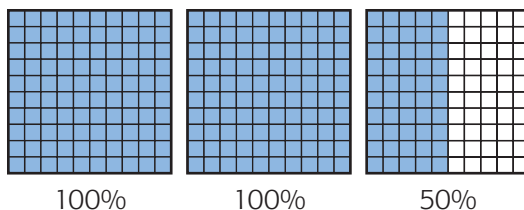


9. It is predicted that Aboriginal people will make up 32.5% of Saskatchewan's population in 2045. They made up 13.3% of the population in 1995. Why is the population increase not $32.5\% - 13.3\% = 19.2\%$ of the 2045 population?
10. The population of China is divided into 56 different ethnic groups. The population of the Han group is 90.56% of the Chinese population. Among the 55 other groups, the Dai people has the least population, which is 1.12% of the population of those 55 other groups. If the Chinese population is 1.6 billion, what is the Dai population?
11. Manuel is saving for a new mountain bike that costs 212% of the amount currently in his savings bank. The bike costs \$349. How much has he saved?
12. In a survey, 365 girls and 345 boys in Grade 8 were asked, "What is your favourite weekend activity?" If 7.4% of the girls and 10.1% of the boys chose watching TV and videos, how many more boys than girls chose this activity?
13. Describe a percent question you would solve using each of these calculations.
- a)** 1.25×400 **b)** $400 \div 1.25$ **c)** 0.035×400

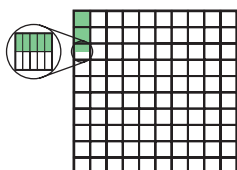
Frequently Asked Questions

Q: How can you represent percents greater than 100%?

A: You have to say what you mean by 100%. Then you can represent the percent greater than 100% based on that. For example, you can represent 250% on grids.



You can also represent the percent as the decimal 2.5 ($1 + 1 + 0.5$) or the fraction $\frac{5}{2}$ or mixed number $2\frac{1}{2}$.



Q: How can you represent percents that involve parts of 1%?

A: You can divide 1% into parts. For example, you can represent 2.5% as $2\% + 0.5\%$. 0.5% is half of 1%.

Then you can represent that on a grid.

You can also write 2.5% as a decimal or fraction.

$$2\% = 0.02 \text{ and } 0.5\% = 0.005$$

$$2\% + 0.5\% = 0.02 + 0.005 = 0.025$$

$$2.5\% = \frac{2.5}{100} \xrightarrow{\times 2} \frac{5}{200} \xrightarrow{\div 5} \frac{1}{40}$$

Q: How can you solve a percent problem?

A: You can use a proportion or you can multiply or divide by a decimal.

For example, suppose you know that 30 is 150% of a number and you want to figure out that number.

You could set up the proportion $\frac{30}{\blacksquare} = \frac{150}{100}$. You notice that $150 = 5 \times 30$, so $5 \times \blacksquare = 100$, and $\blacksquare = 20$.

Or, you can write 150% as 1.5. If $1.5 \times \blacksquare = 30$, then multiply both sides by 2.

$$3 \times \blacksquare = 60$$

$$\blacksquare = 20$$

Practice

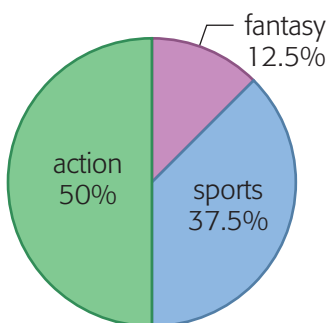
Lesson 4.1

1. Represent each percent. Use a 10-by-10 grid to represent 100%.
a) 140% **b)** 315% **c)** 284%

Lesson 4.2

2. Represent each percent. Use a thousandths grid.
a) 0.8% **b)** 3.7% **c)** 15.5%
3. In a survey, 40 students were asked this question: What type of video game do you prefer? The circle graph shows their responses. Use a hundredths grid to calculate the number of students who preferred each type of game.

Favourite Video Games



Lesson 4.3

4. Estimate the equivalent percent for each fraction. Explain your reasoning.
a) $\frac{135}{95}$ **b)** $\frac{29}{26}$ **c)** $\frac{3}{640}$

Lesson 4.4

5. On a multiple-choice science test, Marcus answered 67.5% of the questions correctly. If there were 40 questions on the test, how many did he answer correctly?

Lesson 4.5

6. There are 15 girls in Daniel's school choir. 37.5% of the students in the choir are girls. How many students are in the choir?
7. When water freezes, its volume increases by 10.1%.
a) If 150 L of water freezes, what is the increase in volume?
b) Estimate the original volume of water if the increase in volume is 22 L.

4.6

Solve Problems by Changing Your Point of View

YOU WILL NEED

- a calculator

GOAL

Solve problems by looking at situations in different ways.

LEARN ABOUT *the Math*

Holly lives in British Columbia, where the PST is 7%. She wants to buy a new guitar. She finds the guitar she wants on sale for 25% off the regular price of \$329.98.

? How can Holly calculate the cost of the guitar, including taxes?



Example 1 Solving a problem using related percents

What is the cost of the guitar, including taxes?

Holly's Solution

1. Understand the Problem

The cost has two parts.

Cost = discounted price + taxes

2. Make a Plan

First I will calculate the discounted price = original price – 25% discount.

Then I will add 7% for the PST.

Then I will add 5% for the GST.



3. Carry Out the Plan

$$\text{Original price} = \$329.98$$

$$\begin{aligned} 25\% \text{ of original price} &= \$329.98 \div 4 \\ &= \$82.50 \end{aligned}$$

$$\begin{aligned} \text{Discounted price} &= \$329.98 - \$82.50 \\ &= \$247.48 \end{aligned}$$

$$\begin{aligned} \text{PST} &= 7\% \text{ of } \$247.48 \\ &= 0.07 \times \$247.48 \\ &= \$17.32 \end{aligned}$$

$$\begin{aligned} \text{GST} &= 5\% \text{ of } \$247.48 \\ &= \frac{1}{2} \text{ of } 10\% \text{ of } \$247.48 \\ &= \frac{1}{2} \text{ of } \$24.75 \\ &= \$12.38 \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= \$247.48 + \$17.32 + \$12.38 \\ &= \$277.18 \end{aligned}$$

4. Look Back

I realized I could have thought about the problem differently and it would have been a lot easier.

The discounted price = 75% of the original price.

Adding 7% and then 5% to the discounted price is the same as taking 112% (100% + 12%) of the discounted price.

I could have calculated:

$$\begin{aligned} \text{Total cost} &= 112\% \text{ of } 75\% \text{ of } \$329.98 \\ &= 1.12 \times 0.75 \times \$329.98 \end{aligned}$$

$$= \text{277.1832}$$

Reflecting

- A. How did Holly change her point of view when she looked back?
- B. Why was changing her point of view useful?

WORK WITH the Math

Example 2

Solving a percent problem using a ratio table

Ivan made a poster by enlarging a 10 cm by 5 cm picture to 380% of its size. What is the area of the poster?

Ivan's Solution

1. Understand the Problem

I have to calculate the area of the poster.

2. Make a Plan

I can calculate the area of the picture and then figure out 380% of that area.

3. Carry Out the Plan

area of the picture = $10\text{ cm} \times 5\text{ cm}$
 $= 50\text{ cm}^2$

I set up a ratio table. The top row is the area and the bottom row is the enlargement percent.

Area	50	200	10	190
%	100%	400%	20%	380%

Diagram annotations:
A blue arrow labeled $\times 4$ points from 50 to 200 in the Area row.
A red arrow labeled $\div 5$ points from 200 to 10 in the Area row.
A black arrow points from 400% to 20% in the % row.
A black arrow points from 20% to 380% in the % row.

To get 380%, I calculated 400% and subtracted 20%.
The area of the poster is 190 cm^2 .

4. Look Back

I can estimate; 380% is about 400%. So $50\text{ cm}^2 \times 4 = 200\text{ cm}^2$, which is close.

I could have written the percent as a decimal and then multiplied it by the area of the picture.

A Checking

1. For each, write the single multiplication that will give you the necessary information.
 - a) the price of an item on sale for 20% off if you know the regular price
 - b) the total cost, with 5% tax, of an item if you know the price without tax
2. Describe two ways to calculate 50% of a number if you know the value of 20% of the number.



B Practising

3. Daniel buys a video game, which is on sale for 30% off the regular price of \$69.98. In Alberta, he pays 5% GST. How much does Daniel pay?
4. A used kayak sells for \$450. The combined taxes are 13%. What is the purchase price?
5. A picture for a school yearbook has an area of 80 cm^2 and a perimeter of 42 cm. The picture was reduced by 20% to fit into the available space. What is the area of the reduced picture?
6. Alan missed 20% of the number of days of school that Richard did. Richard missed 150% as many days as Bella did. How many days could they each have missed? Give two possible answers.
7. Use two different ratio tables to solve this problem:
After working at a part-time job, Rhea has 450% as much money saved as she had before. She had \$120 before. How much does she have now?
8. Why is solving a percent problem using a ratio table a way of solving a problem by changing your point of view?

Reading Strategy

Predicting

Use the Activate, Predict, Read, and Connect Chart to predict the solution.

4.7

Solving Percent Problems Using Fractions

YOU WILL NEED

- a coin
- grid paper

GOAL

Create and solve a percent problem using fractions.

EXPLORE the Math

Problem

12 boys were in a class.
They made up 40% of the
class. How big was the
class?

To solve the problem on the card, Angèle divided 12 by $\frac{2}{5}$.

? What problems involving percent can you create that could be solved by taking $\frac{5}{8}$ of a number?



4.8

Combining Percents

YOU WILL NEED

- a calculator

GOAL

Use percents to solve problems involving two percentages.

LEARN ABOUT the Math

John wants to buy an MP3 player. In a newspaper, he sees a player that regularly sells for \$119.95. It is advertised at 20% off, but, because he lives in British Columbia, he has to pay 5% GST and 7% PST. He has saved \$115 from babysitting.

? Does John have enough money to buy the MP3 player?

Example 1

Working with discounts and sales tax

I needed to calculate the total cost of the player.

John's Solution

$$0.2 \times \$119.95 = \$23.99$$

$$\begin{aligned}\text{sale price} &= \text{regular price} - \text{discount} \\ &= \$119.95 - \$23.99 \\ &= \$95.96\end{aligned}$$

$$5\% + 7\% = 12\%$$

$$0.12 \times \$95.96 = \$11.5152$$

The tax would be \$11.52.

The total price would be

$$\$95.96 + \$11.52 = \$107.48.$$

I have enough money.

The discount is 20% of \$119.95. I wrote 20% as the decimal 0.2.

I had to calculate the sale price before I used the GST and PST. The two tax percents could be added since they are both percents for the same amount.

I wrote 12% as a decimal. I used a calculator to multiply. I rounded the decimal to the nearest hundredth.

I added the tax to the price to get the total cost.

I compared it to the \$115 I had saved.



Reflecting

- A. Why could you have calculated 80% of the regular price instead of subtracting 20% from the regular price?
- B. Why could you have multiplied the sale price by 1.12 instead of adding the tax to the sale price?
- C. Why might you have calculated the total cost this way:
 $0.8 \times 1.12 \times 119.95$?

WORK WITH the Math

Example 2 | Calculating interest

Miranda took out a loan to buy a computer. The computer cost \$1299. The interest rate on the loan is 8.25% of the original price each year. Calculate the amount of interest Miranda will pay over the two years.

Solution

$$\begin{aligned}\text{Yearly interest} &= \text{amount of loan} \times \text{annual interest rate} \\ &= \$1299 \times 8.25\% \\ &= \$1299 \times 0.0825 \\ &= \$107.1675\end{aligned}$$

Calculate the interest on the loan for one year

The interest for one year is \$107.17.

$$\begin{aligned}\text{Total interest} &= \text{number of years} \times \text{yearly interest} \\ &= 2 \times \$107.17 \\ &= \$214.34\end{aligned}$$

Calculate the interest for 2 years.

Miranda will pay \$214.34 in interest.

A Checking

1. A television is on sale for 25% off the regular price of \$339.95. Calculate the discount and the final cost if the tax is 5% in Alberta, where there is no PST.

B Practising

2. Mikael's father bought a new car for \$35 500. The car decreased in value by 20% after one year. What was the value of the car after the one year?
3. Calculate the total tax in Manitoba for each item (7% PST and 5% GST).

a)



b)



c)



4. Jake purchased these items in Yukon, where there is no PST but there is 5% GST.

a) Calculate the sale price for each item before taxes.

i)



ii)



iii)



b) Calculate the price for each item with taxes.

5. Lawrence added the taxes to the price of an item before taking off the discount. Tina took off the discount and then added the taxes. Will they get the same purchase price? Explain.
6. Calculate the interest on a deposit of \$500 that pays 3.5% per year over five years.
7. Miriam wants to buy a pair of inline skates. One store is selling the skates at 15% off the regular price of \$149.95. Another store is selling the skates for \$139.95, with 10% off. Which store has the better price?
8. Complete.
 - a) $6\% \text{ of } 100 + 8\% \text{ of } 100 = \square\% \text{ of } 100$
 - b) $6\% \text{ of } 100 + 8\% \text{ of } 120 = \square\% \text{ of } 100$
9. The price of a \$150 item is increased by 25%. After a couple of weeks, it is reduced by 25%. Why is the final price not \$150?

Greatest Number

YOU WILL NEED

- a standard deck of cards

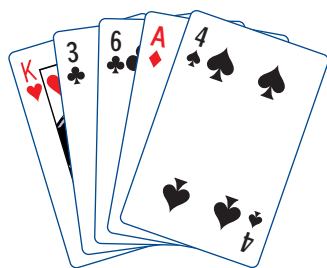
The goal of the game is to end up with the greatest value possible.

Number of players: 2 to 5

How to Play

1. Shuffle the cards. Deal five cards to each player.
2. The aces count as 1, the face cards count as 0, and numbered cards count as their face values.
3. Each player chooses three cards to form a three-digit number that represents a percent and the remaining two cards form a two-digit number.
4. Calculate the percent of the number.
5. Players compare their results. The one with the greatest value wins.

Renée's Turn



The digits I can use are 0, 3, 6, 1, and 4.

I will take 630% of 41.

$$6.30 \times 41 = 258.3$$

My result is 258.3.



4.9

Percent Change

YOU WILL NEED

- a calculator

GOAL

Solve problems involving changes described as percents.

LEARN ABOUT *the Math*

In 2005, the number of movie tickets sold in Canada increased 0.5% to 120.3 million.

Suppose it increased another 0.5% in 2006.

? How many tickets would have been sold in 2006?

- A. Why can you describe the ticket sales in 2005 as 100.5% of the sales in 2004?
- B. How many tickets were sold in 2004?
- C. How many tickets would have been sold in 2006?

Reflecting

- D. Why are the ticket sales in 2006 not 101% of the sales in 2004?
- E. How could you have calculated the number of tickets for 2004 if you knew the percent increases from 2004 to 2005 and from 2005 to 2006, and the number of tickets sold in 2006?
- F. If a percent increase is 10%, is the old value 90% of the new one? Explain.



WORK WITH the Math

Example 1 | Calculating a percent increase

This year, Jasleen's song library increased by 40%. She has 420 tunes in it now. How many songs did she have before?

Holly's Solution

	$\div 2$	$\div 7$	$\times 3$		
%	140%	70%	10%	30%	100%
Number of tunes	420	210	30	90	300

She used to have 300 tunes.

If the song library increased 40%, now it is 140% of what it was before.

I used a ratio table. I tried to get an equivalent ratio where the percent was 100% instead of 140%. First, I got to 70% and then I tried to find a way to get 30% so I could add the two columns to get the 100%.

Lam's Solution

$$140\% = 1.40$$

$$1.4 \times \square = 420$$

$$10 \times 1.4 \times \square = 10 \times 420$$

$$14 \times \square = 4200$$

$$\square = 4200 \div 14$$

$$= 300$$

She used to have 300 tunes.

I know that 420 tunes is 140% of the old number of tunes.

I wrote 140% as a decimal.

Then I wrote an equation to relate the old number of tunes to 420.

I decided to multiply both sides of the equation by 10 to get rid of the decimal.

I divided both sides by 14.

Example 2**Calculating a percent decrease**

Ellen had \$800 in her bank account. She withdrew \$80 to buy a gift for her friend.

- a) By what percent did the balance decrease?
- b) What percent of the old balance is the new balance?

Taira's Solution

$$\begin{aligned}\text{a) } \frac{80}{800} &= \frac{1}{10} \\ \frac{1}{10} &= 10\%\end{aligned}$$

The percent decrease is 10%.

To calculate the percent, I had to compare the amount withdrawn to the original balance, not the new balance, using a ratio or fraction. Then I wrote it as a percent.

$$\text{b) } 100\% - 10\% = 90\%.$$

The new balance is 90% of the old one.

I checked by comparing 720 to 800.

$$\begin{aligned}\frac{720}{800} &= \frac{9}{10} \\ &= 0.9 \\ &= 90\%\end{aligned}$$

I had to subtract from 100% to find the amount that remained.

A Checking

- 1. Calculate.
 - a) a 30% increase from 50
 - b) a 150% increase from 50
 - c) a 20% decrease from 50
 - d) a 0.5% decrease from 50
- 2. The population of a town with 8500 people increased 8% last year.
 - a) How do you know that the increase was less than 850 people?
 - b) What percent of 8500 is the new population?
 - c) What is the new population?
 - d) What percent of the new population is the old population?

B Practising

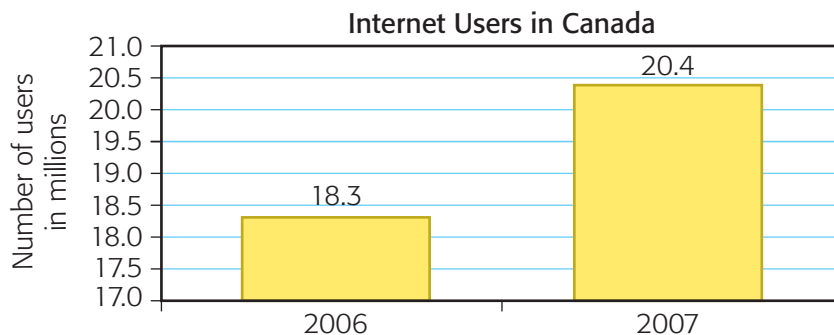
- 3. Calculate the percent increase or decrease.
 - a) from 200 to 100
 - b) from 80 to 90
 - c) from 50 to 200
 - d) from 500 to 450



4. Exports of wood to China from Canada increased by 150% from 2000 to 2005. What percent describes the amount of wood exported in 2005 compared to the year 2000?
5. A car dealer reported a 4.5% drop in car sales to 520 cars. What percent of the original car sales was the new total?
6. Sam increased the savings in his bank account by 200% when he added a birthday gift from his grandmother. Now he has \$330. How much was the gift?
7. In 2001, the population of Nunavut was 26 745. In 2006, it was 30 782.
 - a) What was the percent increase in population? Explain your thinking.
 - b) If the increase continues at the same rate, what population would you expect in 2011?



8. The graph shows the number of Internet users in Canada in January 2006 and January 2007.



- a) What was the percent increase?
- b) What is the January 2007 value as a percent of the January 2006 value?

9.
 - a) In April 2007, home sales in Calgary dropped 11.01% from the sales in March 2007. There were 3505 homes sold in April. How many homes were sold in March?
 - b) The 3505 homes represent a 3.88% increase in home sales from April 2006. How many homes were sold in April 2006?
10. Kendra said that her amount of homework increased 400% when it went from one half-hour of work to 2 h of work. Do you agree? Explain.
11. The number of students attending francophone schools in Alberta increased from 1600 in 1994 to 3800 in 2004. What was the percent increase?
12. A child's mass increased from 30.0 kg to 40.0 kg in two years. Skin makes up about 16% of the mass of a body. About how many kilograms of skin did the child gain during the two years?
13. The growth in visitors to a community website from 2006 to 2007 was 117%. The number of visitors was 8.9 million in 2006. How many visitors were there in 2007?
14. Canadian digital download sales increased 122% from 2005 to 2006. The growth rate was much higher than in the United States or Europe. There were 14.9 million downloads in 2006. How many downloads were there in 2005?
15.
 - a) Gasoline prices increased from 114.9¢/L to 118.9¢/L in one month. What was the percent increase, to the nearest tenth of a percent?
 - b) If the price for a container of gourmet chocolate-covered potato chips increased at the same rate as in part a), what would be the new price of a \$7.50 container?
16. The price of a computer decreased by 25%. Which of these procedures would give the new price? Explain.
 - A. multiply current price by 1.25
 - B. multiply current price by 0.25 and subtract from the present price
 - C. multiply current price by 0.75
 - D. take $\frac{3}{4}$ of the current price

Reading Strategy

Monitoring Comprehension

Identify the signal words in these questions. How can understanding these words help you solve math problems?

Double Your Money

You might think it would take 10 years for an amount to double if it increases by 10% each year.

You would be wrong! You can calculate how long it takes.

1. Imagine you have \$1. Calculate its value after a year if it increases by 10% in that year.
2. Use the new value to calculate the value after a 10% increase on that new value.

Year	Value at start of year	Value at end of year
1		
2		

3. Repeat until you get to \$2. How long did it take?
4. Repeat steps 1 to 3 for an increase of 15% each year. How long did it take?
5. Determine the percent that would allow you to double your money in two years.





1. Use a grid to model and calculate each.
 - a) 110% of 70
 - b) 37.5% of 180
2. Andrea calculated 0.035×50 to determine a certain percent of 50. What percent was it?
3. An addition to a house increases the floor area from 275 m^2 to 300 m^2 . By what percent was the original floor area increased?
4. The Mackenzie River is the longest river in Canada. It is 4241 km long, but the Nile River is about 158% as long. About how long is the Nile River?
5. The number of students who bought lunch in a school cafeteria increased 0.8% from January to February. If 125 students bought lunch in February, how many bought lunch in January?
6. A pair of jeans purchased in Manitoba cost \$44.99 before taxes. They are on sale for 15% off.
 - a) If PST is 7% and GST is 5%, how much would the jeans cost after taxes?
 - b) What percent of the original regular price is the price with taxes?
7. The number of new homes on a street increased by 300% from January to March and by 100% from March to July. By what percent had the number of houses increased from January to July?

What Do You Think Now?

Revisit What Do You Think? on page 143. How have your answers and explanations changed?

Frequently Asked Questions

Q: How can you solve percent problems using fractions?

A: You can relate the percent to an equivalent fraction and multiply or divide by that fraction.

For example, to calculate 125% of 48, you can write

125% as $\frac{125}{100} = \frac{5}{4}$ and multiply 48 by $\frac{5}{4}$.

$$\begin{aligned}\frac{5}{4} \times 48 &= 5 \times 48 \div 4 \\ &= 5 \times 12 \\ &= 60\end{aligned}$$

Q: How and when can you combine percents?

A: When you are adding, subtracting, multiplying, or dividing two percents of the same number, you can perform the calculation with the percent values and then apply them to the number.

For example, to calculate the GST and PST on an item, you can add the two percents and then multiply by the price.

When you are considering percents of two different numbers, you must calculate each value separately and then compute.

For example, 20% of 50 + 10% of 40 is not 30% of either 50 or 40; 20% of 50 + 10% of 40 = 10 + 4 = 14; 14 is 28% of 50 and it is 35% of 40.

Q: How do you calculate percent change?

A: When an amount increases or decreases, you can describe the percent change by relating the increase or decrease to that amount.

For example, if you increase 100 to 105, the increase of 5 is 5% of the original amount of 100. The final amount, 105, is $5\% + 100\% = 105\%$ of the original amount.

If you decrease 100 to 95, the decrease of 5 is 5% of the original amount of 100. The final amount is 95% of the original amount.

Practice

Lesson 4.1

- Use grids to model and calculate each amount. One full grid represents 100%.
a) 205% **b)** 140% **c)** 330% **d)** 118%
- Describe a situation where you might use 200%.
- a)** Write 12:5 as a percent.
b) Why would you not write the rate 12 L in 4 min as a percent?

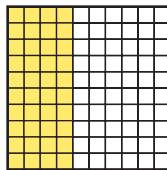
Lesson 4.2

- Use a thousandths grid to represent each percent.
a) 0.2% **b)** 4.1% **c)** 10.9%
- Rick's class is 5.2% of the number of students in the school. If there are 32 students in his class, how many students are in the school?

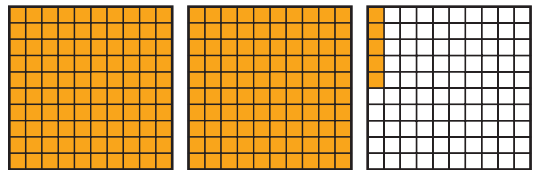
Lesson 4.3

- Use a fraction, a decimal, and a percent to describe each shaded area. One full grid represents 100%.

a)



b)



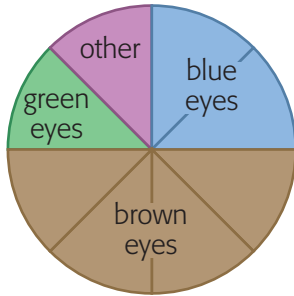
- Describe each as a percent.

a) $\frac{5}{2}$

b) 0.004

c) 1.58

Eye Colour Among Canadian School Children



8. A Canadian census showed that the eye colour among Canadian school children could be described by the circle graph. What percent of the students had green eyes?

Lesson 4.4

9. Solve.

- a) 15% of $\blacksquare = 6$ c) $\frac{45}{18} = \blacksquare\%$
 b) 32% of $65 = \blacksquare$ d) 0.8% of $2\,500\,000 = \blacksquare$

10. A sugar-and-water mixture of 250 g contains 8 g of sugar. What percent of the mixture is sugar?

Lesson 4.5

11. Calculate.

- a) 14% of 80 b) 118% of 20 c) 1.5% of 3000

12. In Alain's class, 15 students play in the local soccer league. They make up 6% of the league. How many students are in the league?

Lesson 4.6

13. Write each amount as a percent of the regular price of the jeans.
 a) the sale price with 35% off
 b) the cost with 5% GST only



Lesson 4.8

14. Luke bought a hockey sweater with a regular price of \$68.95. The sweater was on sale for 35% off, and the taxes were 12%. Determine each amount.
 a) the discount c) the taxes
 b) the sale price d) the purchase price

Lesson 4.9

15. Calculate the percent increase or decrease.
 a) from 50 to 200 c) from 300 to 3000
 b) from 80 to 60 d) from 1000 to 100
16. A population increased by 15% from 1996 to 2001 and by 22% from 2001 to 2006. What percent is the increase from 1996 to 2006?

YOU WILL NEED

- a calculator
- a measuring tape or ruler

Task | Checklist

- ✓ Did you use the different types of percents required?
- ✓ Did you write a fraction as a percent and include percent increases or decreases?
- ✓ Are your calculations clear and easy to follow?
- ✓ Are your descriptions clear and easy to understand?

137%, 6% + 7%, 0.21%



All About You

You can describe your life using many different numbers and measurements.

? How could you describe yourself using percents?

- You must use at least 10 percent values.
- Some percents have to be greater than 100% and some have to be less than 1%.
- Some percents have to describe an increase or decrease.
- Some percents have to involve combining percents.
- Some descriptions have to involve starting with a fraction and then rewriting it as a percent.

A. Think about your height.

- What could you compare it to so the percent describing it is greater than 100%?
- What could you compare it to so the percent describing it is less than 1%?

B. Think about the number of people in your family.

- How could you describe yourself in relation to your family with a percent greater than 100?
- Why would you probably not use a percent less than 1%?

C. Think about the length of your foot compared to the lengths of your fingers. What percents could you use to compare them?

D. Imagine that your adult height is 107% of your current height. If your arms were also 107% as long, how long would they be?

E. Complete the description of yourself following the rules above. Show your calculations.