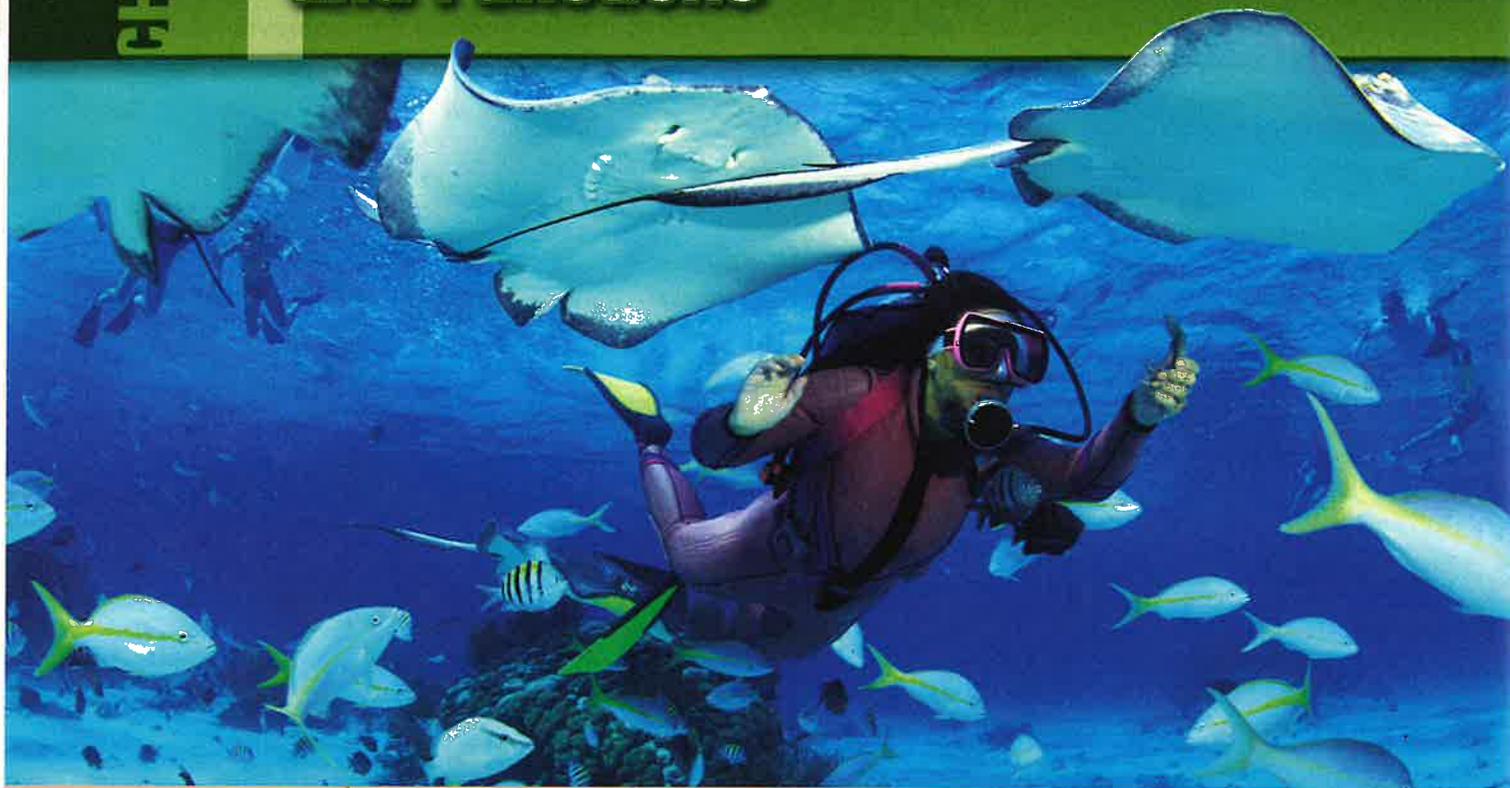# Expressions, Equations, and Functions

·· Then

○ You have learned how to perform operations on whole numbers.

·· Now

○ In this chapter, you will:

- Write algebraic expressions.

- Use the order of operations.

- Solve equations.

- Represent and interpret relations and functions.

- Use function notation.

- Interpret the graphs of functions.

·· Why? ▲

○ **SCUBA DIVING** A scuba diving store rents air tanks and wet suits. An algebraic expression can be written to represent the total cost to rent this equipment. This expression can be evaluated to determine the total cost for a group of people to rent the equipment.

Expressions, Equations, and Functions
Activity

Verbal Phrases for Operations

Addition Key Words
- sum
- in all
- total
- altogether
- increased by
- more than
- combined
- together
- added to

Subtraction Key Words
- difference
- less
- left
- take away
- decreased by
- minus
- difference between
- less than, fewer than

Multiplication Key Words
- product
- of (especially with a fraction)
- times
- multiplied by
- repeated addition

Division Key Words
- per
- out of
- ratio
- quotient
- percent (divide by 100)
- divided or split equally

connectED.mcgraw-hill.com **Your Digital Math Portal**

| Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets |

Diagnose Readiness | You have two options for checking prerequisite skills.

1 **Textbook Option** Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck	**Quick**Review

QuickCheck

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplest form.*

1. $\frac{24}{36}$ 2. $\frac{34}{85}$ 3. $\frac{36}{12}$

4. $\frac{27}{45}$ 5. $\frac{11}{18}$ 6. $\frac{5}{65}$

7. $\frac{19}{1}$ 8. $\frac{16}{44}$ 9. $\frac{64}{88}$

10. **ICE CREAM** Fifty-four out of 180 customers said that cookie dough ice cream was their favorite flavor. What fraction of customers was this?

Find the perimeter of each figure.

11.

3.2 cm   3.2 cm

1.8 cm

12.

$6\frac{1}{2}$ in.

$2\frac{3}{4}$ in.
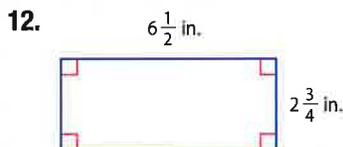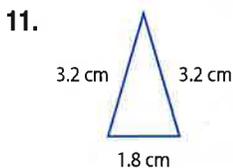
13. **FENCING** Jolon needs to fence a rectangular garden. The dimensions of the garden are 6 meters by 4 meters. How much fencing does Jolon need to purchase?

Evaluate.

14. $6 \cdot \frac{2}{3}$ 15. $4.2 \cdot 8.1$ 16. $\frac{3}{8} \div \frac{1}{4}$

17. $5.13 \div 2.7$ 18. $3\frac{1}{5} \cdot \frac{3}{4}$ 19. $2.8 \cdot 0.2$

20. **CONSTRUCTION** A board measuring 7.2 feet must be cut into three equal pieces. Find the length of each piece.

QuickReview

Example 1

Write $\frac{24}{40}$ in simplest form.

Find the greatest common factor (GCF) of 24 and 40.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

The GCF of 24 and 40 is 8.

$\frac{24 \div 8}{40 \div 8} = \frac{3}{5}$ Divide the numerator and denominator by their GCF, 8.

Example 2

Find the perimeter.

12.8 ft

5.3 ft

$P = 2\ell + 2w$
$\quad = 2(12.8) + 2(5.3)$ $\ell = 12.8$ and $w = 5.3$
$\quad = 25.6 + 10.6$ or 36.2 Simplify.

The perimeter is 36.2 feet.

Example 3

Find $2\frac{1}{4} \div 1\frac{1}{2}$.

$2\frac{1}{4} \div 1\frac{1}{2} = \frac{9}{4} \div \frac{3}{2}$ Write mixed numbers as improper fractions.

$\quad = \frac{9}{4}\left(\frac{2}{3}\right)$ Multiply by the reciprocal.

$\quad = \frac{18}{12}$ or $1\frac{1}{2}$ Simplify.

2 **Online Option** Take an online self-check Chapter Readiness Quiz at <u>connectED.mcgraw-hill.com</u>.

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 1. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

Expressions, Equations, and Functions Make this Foldable to help you organize your Chapter 1 notes about expressions, equations, and functions. Begin with five sheets of plain paper.

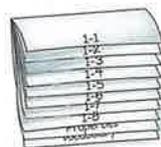**1** **Fold** the sheets of paper in half along the width. Then cut along the crease.
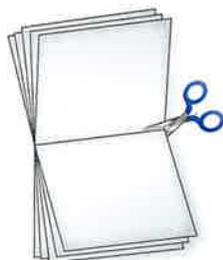
2 **Staple** the ten half-sheets together to form a booklet.

3 **Cut** nine centimeters from the bottom of the top sheet, eight centimeters from the second sheet, and so on.

4 **Label** each of the tabs with a lesson number. The ninth tab is for Properties and the last tab is for Vocabulary.

NewVocabulary

| English		Español
algebraic expression	p. 5	expression algebraica
variable	p. 5	variable
term	p. 5	término
power	p. 5	potencia
coefficient	p. 28	coeficiente
equation	p. 33	ecuación
solution	p. 33	solución
identity	p. 35	identidad
relation	p. 40	relacíon
domain	p. 40	domino
range	p. 40	rango
independent variable	p. 42	variable independiente
dependent variable	p. 42	variable dependiente
function	p. 47	función
intercept	p. 56	intersección
line symmetry	p. 57	simetría
end behavior	p. 57	comportamiento final

ReviewVocabulary

additive inverse inverso aditivo a number and its opposite

multiplicative inverse inverso multiplicativo two numbers with a product of 1

perimeter perímetro the distance around a geometric figure
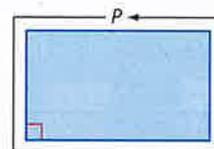
Variables and Expressions

: Then	**: Now**	**: Why?**
• You performed operations on integers.	**1** Write verbal expressions for algebraic expressions. **2** Write algebraic expressions for verbal expressions.	• Cassie and her friends are at a baseball game. The stadium is running a promotion where hot dogs are $0.10 each. Suppose d represents the number of hot dogs Cassie and her friends eat. Then $0.10d$ represents the cost of the hot dogs they eat.

New Vocabulary

algebraic expression
variable
term
factor
product
power
exponent
base

Common Core State Standards

Content Standards
A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Mathematical Practices
4 Model with mathematics.

1 **Write Verbal Expressions** An **algebraic expression** consists of sums and/or products of numbers and variables. In the algebraic expression $0.10d$, the letter d is called a variable. In algebra, **variables** are symbols used to represent unspecified numbers or values. Any letter may be used as a variable.

$$0.10d \qquad 2x + 4 \qquad 3 + \frac{z}{6} \qquad p \cdot q \qquad 4cd \div 3mn$$

A **term** of an expression may be a number, a variable, or a product or quotient of numbers and variables. For example, $0.10d$, $2x$ and 4 are each terms.

| The term that contains x or other letters is sometimes referred to as the *variable term*.	$\rightarrow 2x + 4 \leftarrow$	A term that does not have a variable is a *constant term*.

In a multiplication expression, the quantities being multiplied are **factors**, and the result is the **product**. A raised dot or set of parentheses are often used to indicate a product. Here are several ways to represent the product of x and y.

$$xy \qquad x \cdot y \qquad x(y) \qquad (x)y \qquad (x)(y)$$

An expression like x^n is called a **power**. The word *power* can also refer to the exponent. The **exponent** indicates the number of times the base is used as a factor. In an expression of the form x^n, the **base** is x. The expression x^n is read "x to the nth power." When no exponent is shown, it is understood to be 1. For example, $a = a^1$.

$$x^n \quad \text{base} \quad \text{exponent}$$
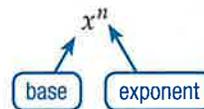
Example 1 Write Verbal Expressions

Write a verbal expression for each algebraic expression.

a. $3x^4$

three times x to the fourth power

b. $5z^2 + 16$

5 times z to the second power plus sixteen

▸ **Guided**Practice

1A. $16u^2 - 3$

1B. $\frac{1}{2}a + \frac{6b}{7}$

2 Write Algebraic Expressions Another important skill is translating verbal expressions into algebraic expressions.

KeyConcept Translating Verbal to Algebraic Expressions

Operation	Verbal Phrases
Addition	more than, sum, plus, increased by, added to
Subtraction	less than, subtracted from, difference, decreased by, minus
Multiplication	product of, multiplied by, times, of
Division	quotient of, divided by

Example 2 Write Algebraic Expressions

Write an algebraic expression for each verbal expression.

a. a number t more than 6

The words *more than* suggest addition.
Thus, the algebraic expression is $6 + t$ or $t + 6$.

b. 10 less than the product of 7 and f

Less than implies subtraction, and *product* suggests multiplication.
So the expression is written as $7f - 10$.

c. two thirds of the volume v

The word *of* with a fraction implies that you should multiply.
The expression could be written as $\frac{2}{3}v$ or $\frac{2v}{3}$.

▶ **Guided**Practice

2A. the product of p and 6   **2B.** one third of the area a

Variables can represent quantities that are known and quantities that are unknown. They are also used in formulas, expressions, and equations.

🌐 **Real-World Example 3** Write an Expression

SPORTS MARKETING Mr. Martinez orders 250 key chains printed with his athletic team's logo and 500 pencils printed with their Web address. Write an algebraic expression that represents the cost of the order.

Let k be the cost of each key chain and p be the cost of each pencil. Then the cost of the key chains is $250k$ and the cost of the pencils is $500p$. The cost of the order is represented by $250k + 500p$.

▶ **Guided**Practice

3. COFFEE SHOP Katie bakes 40 pastries and makes coffee for 200 people. Write an algebraic expression to represent this situation.

Example 1   Write a verbal expression for each algebraic expression.

1. $2m$ **2.** $\frac{2}{3}r^4$ **3.** $a^2 - 18b$

Example 2   Write an algebraic expression for each verbal expression.

4. the sum of a number and 14   **5.** 6 less a number t

6. 7 more than 11 times a number   **7.** 1 minus the quotient of r and 7

8. two fifths of the square of a number j **9.** n cubed increased by 5

Example 3   **10. GROCERIES** Mr. Bailey purchased some groceries that cost d dollars. He paid with a $50 bill. Write an expression for the amount of change he will receive.

Practice and Problem Solving

Example 1   Write a verbal expression for each algebraic expression.

11. $4q$ **12.** $\frac{1}{8}y$ **13.** $15 + r$ **14.** $w - 24$

15. $3x^2$ **16.** $\frac{r^4}{9}$ **(17)** $2a + 6$ **18.** $r^4 \cdot t^3$

Example 2   Write an algebraic expression for each verbal expression.

19. x more than 7   **20.** a number less 35

21. 5 times a number   **22.** one third of a number

23. f divided by 10   **24.** the quotient of 45 and r
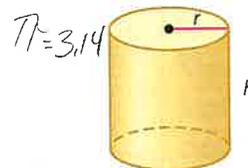
25. three times a number plus 16   **26.** 18 decreased by 3 times d

27. k squared minus 11   **28.** 20 divided by t to the fifth power

Example 3   **29. GEOMETRY** The volume of a cylinder is π times the radius r squared multiplied by the height h. Write an expression for the volume.

$\pi = 3.14$

h

30. FINANCIAL LITERACY Jocelyn makes x dollars per hour working at the grocery store and n dollars per hour babysitting. Write an expression that describes her earnings if she babysat for 25 hours and worked at the grocery store for 15 hours.

Write a verbal expression for each algebraic expression.

31. $25 + 6x^2$ **32.** $6f^2 + 5f$ **33.** $\frac{3a^5}{2}$

34. CCSS SENSE-MAKING A certain smartphone family plan costs $55 per month plus additional usage costs. If x is the number of cell phone minutes used above the plan amount and y is the number of megabytes of data used above the plan amount, interpret the following expressions.

a. $0.25x$

b. $2y$

c. $0.25x + 2y + 55$

Masterfile

35 **DREAMS** It is believed that about $\frac{3}{4}$ of our dreams involve people that we know.

    a. Write an expression to describe the number of dreams that feature people you know if you have d dreams.

    b. Use the expression you wrote to predict the number of dreams that include people you know out of 28 dreams.

36. SPORTS In football, a touchdown is awarded 6 points and the team can then try for a point after a touchdown.

    a. Write an expression that describes the number of points scored on touchdowns T and points after touchdowns p by one team in a game.

    b. If a team wins a football game 27-0, write an equation to represent the possible number of touchdowns and points after touchdowns by the winning team.

    c. If a team wins a football game 21-7, how many possible number of touchdowns and points after touchdowns were scored during the game by both teams?

37. ⚃ **MULTIPLE REPRESENTATIONS** In this problem, you will explore the multiplication of powers with like bases.

    a. Tabular Copy and complete the table.

10^2	×	10^1	=	$10 \times 10 \times 10$	=	10^3
10^2	×	10^2	=	$10 \times 10 \times 10 \times 10$	=	10^4
10^2	×	10^3	=	$10 \times 10 \times 10 \times 10 \times 10$	=	?
10^2	×	10^4	=	?	=	?

    b. Algebraic Write an equation for the pattern in the table.

    c. Verbal Make a conjecture about the exponent of the product of two powers with like bases.

H.O.T. Problems   Use Higher-Order Thinking Skills

38. REASONING Explain the differences between an algebraic expression and a verbal expression.

39. OPEN ENDED Define a variable to represent a real-life quantity, such as time in minutes or distance in feet. Then use the variable to write an algebraic expression to represent one of your daily activities. Describe in words what your expression represents, and explain your reasoning.

40. **CCSS** **CRITIQUE** Consuelo and James are writing an algebraic expression for *three times the sum of n squared and 3*. Is either of them correct? Explain your reasoning.

> **Consuelo**
> $3(n^2 + 3)$

> **James**
> $3n^2 + 3$

41. CHALLENGE For the cube, x represents a positive whole number. Find the value of x such that the volume of the cube and 6 times the area of one of its faces have the same value.

x

42. WRITING IN MATH Describe how to write an algebraic expression from a real-world situation. Include a definition of algebraic expression in your own words.
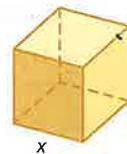
43. Which expression best represents the volume of the cube?

- **A** the product of three and five
- **B** three to the fifth power
- **C** three squared
- **D** three cubed
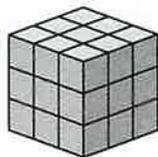
44. Which expression best represents the perimeter of the rectangle?
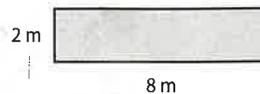
- **F** $2\ell w$
- **G** $\ell + w$
- **H** $2\ell + 2w$
- **J** $4(\ell + w)$

45. SHORT RESPONSE The yards of fabric needed to make curtains is 3 times the length of a window in inches, divided by 36. Write an expression that represents the yards of fabric needed in terms of the length of the window ℓ.

46. GEOMETRY Find the area of the rectangle.

- **A** 14 square meters
- **B** 16 square meters
- **C** 50 square meters
- **D** 60 square meters

2 m
8 m

47. AMUSEMENT PARKS A roller coaster enthusiast club took a poll to see what each member's favorite ride was. Make a bar graph of the results. (Lesson 0-13)

Our Favorite Rides						
Ride	Big Plunge	Twisting Time	The Shiner	Raging Bull	The Bat	Teaser	The Adventure
Number of Votes	5	22	16	9	25	6	12

48. SPORTS The results for an annual 5K race are shown at the right. Make a box-and-whisker plot for the data. Write a sentence describing what the length of the box-and-whisker plot tells about the times for the race. (Lesson 0–13)

Annual 5K Race Results			
Joe	14:48	Carissa	19:58
Jessica	19:27	Jordan	14:58
Lupe	15:06	Taylor	20:47
Dante	20:39	Mi-Ling	15:48
Tia	15:54	Winona	21:35
Amber	20:49	Angel	16:10
Amanda	16:30	Catalina	20:21

Find the mean, median, and mode for each set of data. (Lesson 0–12)

49. $\{7, 6, 5, 7, 4, 8, 2, 2, 7, 8\}$

50. $\{-1, 0, 5, 2, -2, 0, -1, 2, -1, 0\}$

51. $\{17, 24, 16, 3, 12, 11, 24, 15\}$

52. SPORTS Lisa has a rectangular trampoline that is 6 feet long and 12 feet wide. What is the area of her trampoline in square feet? (Lesson 0–8)

Find each product or quotient. (Lesson 0–5)

53. $\frac{3}{5} \cdot \frac{7}{11}$

54. $\frac{4}{3} \div \frac{7}{6} = \frac{24}{18} \quad 1\frac{1}{2}$

55. $\frac{5}{6} \cdot \frac{8}{3}$

Evaluate each expression.

56. $\frac{3}{5} + \frac{4}{9}$

57. $5.67 - 4.21$

58. $\frac{5}{6} - \frac{8}{3}$

59. $10.34 + 14.27$

60. $\frac{11}{12} + \frac{5}{36}$

61. $37.02 - 15.86$

Order of Operations

·· Then

● You expressed algebraic expressions verbally.

·· Now

● **1** Evaluate numerical expressions by using the order of operations.

2 Evaluate algebraic expressions by using the order of operations.

·· Why?

● The admission prices for SeaWorld Adventure Park in Orlando, Florida, are shown in the table. If four adults and three children go to the park, the expression below represents the cost of admission for the group.

$4(78.95) + 3(68.95)$

Ticket	Price ($)
Adult	78.95
Child	68.95
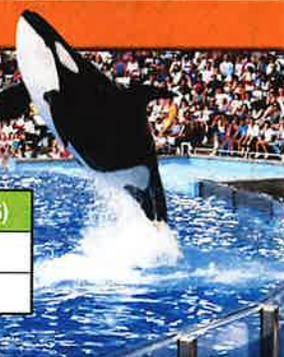
NewVocabulary
evaluate
order of operations

Common Core State Standards

Content Standards
A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Mathematical Practices
7 Look for and make use of structure.

1 **Evaluate Numerical Expressions** To find the cost of admission, the expression $4(78.95) + 3(68.95)$ must be evaluated. To **evaluate** an expression means to find its value.

Example 1 Evaluate Expressions

Evaluate 3^5.

$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ Use 3 as a factor 5 times.
  $= 243$ Multiply.

▶ **Guided**Practice

1A. 2^4 **1B.** 4^5 **1C.** 7^3

The numerical expression that represents the cost of admission contains more than one operation. The rule that lets you know which operation to perform first is called the **order of operations**.

KeyConcept Order of Operations

Step 1 Evaluate expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Multiply and/or divide from left to right.

Step 4 Add and/or subtract from left to right.

Example 2 Order of Operations

Evaluate $16 - 8 \div 2^2 + 14$.

$16 - 8 \div 2^2 + 14 = 16 - 8 \div 4 + 14$ Evaluate powers.
  $= 16 - 2 + 14$ Divide 8 by 4.
  $= 14 + 14$ Subtract 2 from 16.
  $= 28$ Add 14 and 14.

▶ **Guided**Practice

2A. $3 + 42 \cdot 2 - 5$ **2B.** $20 - 7 + 8^2 - 7 \cdot 11$

When one or more grouping symbols are used, evaluate within the innermost grouping symbols first.

Example 3 Expressions with Grouping Symbols

Evaluate each expression.

a. $4 \div 2 + 5(10 - 6)$

$$
\begin{aligned}
4 \div 2 + 5(10 - 6) &= 4 \div 2 + 5(4) &&\text{Evaluate inside parentheses.} \\
&= 2 + 5(4) &&\text{Divide 4 by 2.} \\
&= 2 + 20 &&\text{Multiply 5 by 4.} \\
&= 22 &&\text{Add 2 to 20.}
\end{aligned}
$$

b. $6\left[32 - (2 + 3)^2\right]$

$$
\begin{aligned}
6\left[32 - (2 + 3)^2\right] &= 6\left[32 - (5)^2\right] &&\text{Evaluate innermost expression first.} \\
&= 6[32 - 25] &&\text{Evaluate power.} \\
&= 6[7] &&\text{Subtract 25 from 32.} \\
&= 42 &&\text{Multiply.}
\end{aligned}
$$

c. $\dfrac{2^3 - 5}{15 + 9}$

$$
\begin{aligned}
\dfrac{2^3 - 5}{15 + 9} &= \dfrac{8 - 5}{15 + 9} &&\text{Evaluate the power in the numerator.} \\[2mm]
&= \dfrac{3}{15 + 9} &&\text{Subtract 5 from 8 in the numerator.} \\[2mm]
&= \dfrac{3}{24} \text{ or } \dfrac{1}{8} &&\text{Add 15 and 9 in denominator, and simplify.}
\end{aligned}
$$

▶ **Guided**Practice

3A. $5 \cdot 4(10 - 8) + 20$ **3B.** $15 - \left[10 + (3 - 2)^2\right] + 6$ **3C.** $\dfrac{(4 + 5)^2}{3(7 - 4)}$

2 Evaluate Algebraic Expressions To evaluate an algebraic expression, replace the variables with their values. Then find the value of the numerical expression using the order of operations.

Example 4 Evaluate an Algebraic Expression

Evaluate $3x^2 + \left(2y + z^3\right)$ if $x = 4, y = 5, z = 3$.

$$
\begin{aligned}
3x^2 + \left(2y + z^3\right) \\
&= 3(4)^2 + (2 \cdot 5 + 3^3) &&\text{Replace } x \text{ with 4, } y \text{ with 5, and } z \text{ with 3.} \\
&= 3(4)^2 + (2 \cdot 5 + 27) &&\text{Evaluate } 3^3. \\
&= 3(4)^2 + (10 + 27) &&\text{Multiply 2 by 5.} \\
&= 3(4)^2 + (37) &&\text{Add 10 to 27.} \\
&= 3(16) + 37 &&\text{Evaluate } 4^2. \\
&= 48 + 37 &&\text{Multiply 3 by 16.} \\
&= 85 &&\text{Add 48 to 37.}
\end{aligned}
$$

▶ **Guided**Practice

Evaluate each expression.

4A. $a^2(3b + 5) \div c$ if $a = 2, b = 6, c = 4$ **4B.** $5d + (6f - g)$ if $d = 4, f = 3, g = 12$

ENVIRONMENTAL STUDIES Science on a Sphere (SOS)® demonstrates the effects of atmospheric storms, climate changes, and ocean temperature on the environment. The volume of a sphere is four thirds of π multiplied by the radius r to the third power.

a. Write an expression that represents the volume of a sphere.

Words	four thirds	of	π multiplied by radius to the third power
Variable	Let r = radius.		
Equation	$\frac{4}{3}$	$\times$	πr^3 or $\frac{4}{3}\pi r^3$

b. Find the volume of the 3-foot radius sphere used for SOS.

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$= \frac{4}{3}\pi(3)^3$ Replace r with 3.

$= \left(\frac{4}{3}\right)\pi(27)$ Evaluate $3^3 = 27$.

$= 36\pi$ Multiply $\frac{4}{3}$ by 27.

The volume of the sphere is 36π cubic feet.

▶ **Guided**Practice

5. FOREST FIRES According to the California Department of Forestry, an average of 539.2 fires each year are started by burning debris, while campfires are responsible for an average of 129.1 each year.

A. Write an algebraic expression that represents the number of fires, on average, in d years of debris burning and c years of campfires.

B. How many fires would there be in 5 years?

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–3 Evaluate each expression.

1. 9^2

2. 4^4

3. 3^5

4. $30 - 14 \div 2$

5. $5 \cdot 5 - 1 \cdot 3$

6. $(2 + 5)4$

7. $[8(2) - 4^2] + 7(4)$

8. $\frac{11 - 8}{1 + 7 \cdot 2}$

9. $\frac{(4 \cdot 3)^2}{9 + 3}$

Example 4 Evaluate each expression if $a = 4$, $b = 6$, and $c = 8$.

10. $8b - a$

11. $2a + (b^2 \div 3)$

12. $\frac{b(9 - c)}{a^2}$

Example 5 **13. BOOKS** Akira bought one new book for $20 and three used books for $4.95 each. Write and evaluate an expression to find how much money the books cost.

14. **CCSS REASONING** Koto purchased food for herself and her friends. She bought 4 cheeseburgers for $2.25 each, 3 French fries for $1.25 each, and 4 drinks for $4.00. Write and evaluate an expression to find how much the food cost.

Examples 1–3 Evaluate each expression.

15. 7^2

16. 14^3

17. 2^6

18. $35 - 3 \cdot 8$

19. $18 \div 9 + 2 \cdot 6$

20. $10 + 8^3 \div 16$

21. $24 \div 6 + 2^3 \cdot 4$

22. $(11 \cdot 7) - 9 \cdot 8$

23. $29 - 3(9 - 4)$

24. $(12 - 6) \cdot 5^2$

25. $3^5 - (1 + 10^2)$

26. $108 \div [3(9 + 3^2)]$

27. $[(6^3 - 9) \div 23]4$

28. $\dfrac{8 + 3^3}{12 - 7}$

29. $\dfrac{(1 + 6)9}{5^2 - 4}$

Example 4 Evaluate each expression if $g = 2$, $r = 3$, and $t = 11$.
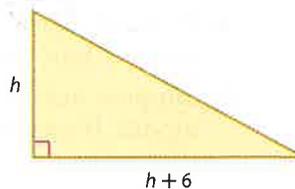
30. $g + 6t$

31. $7 - gr$

32. $r^2 + (g^3 - 8)^5$

(33) $(2t + 3g) \div 4$

34. $t^2 + 8rt + r^2$

35. $3g(g + r)^2 - 1$

Example 5 **36. GEOMETRY** Write an algebraic expression to represent the area of the triangle. Then evaluate it to find the area when $h = 12$ inches.

37. AMUSEMENT PARKS In 2004, there were 3344 amusement parks and arcades. This decreased by 148 by 2009. Write and evaluate an expression to find the number of amusement parks and arcades in 2009.

38. CCSS STRUCTURE Kamilah sells tickets at Duke University's athletic ticket office. If p represents a preferred season ticket, b represents a blue zone ticket, and g represents a general admission ticket, interpret and then evaluate the following expressions.

a. $45b$

b. $15p + 35g$

c. $6p + 11b + 22g$

Duke University Football Ticket Prices	
Preferred Season Ticket	$100
Blue Zone	$80
General Admission	$70

Source: Duke University

Evaluate each expression.

39. 4^2

40. 12^3

41. 3^6

42. 11^5

43. $(3 - 4^2)^2 + 8$

44. $23 - 2(17 + 3^3)$

45. $3[4 - 8 + 4^2(2 + 5)]$

46. $\dfrac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}$

47. $25 + \left[(16 - 3 \cdot 5) + \dfrac{12 + 3}{5} \right]$

48. $7^3 - \dfrac{2}{3}(13 \cdot 6 + 9)4$

Evaluate each expression if $a = 8$, $b = 4$, and $c = 16$.

49. $a^2bc - b^2$

50. $\dfrac{c^2}{b^2} + \dfrac{b^2}{a^2}$

51. $\dfrac{2b + 3c^2}{4a^2 - 2b}$

52. $\dfrac{3ab + c^2}{a}$

53. $\left(\dfrac{a}{b} \right)^2 - \dfrac{c}{a - b}$

54. $\dfrac{2a - b^2}{ab} + \dfrac{c - a}{b^2}$

55. SALES One day, 28 small and 12 large merchant spaces were rented. Another day, 30 small and 15 large spaces were rented. Write and evaluate an expression to show the total rent collected.

THE FLEA MARKET
MERCHANT SPACE RENTALS
Small space $7.00/day
Large space $9.75/day
Open Daily from 9:00–6:00

56. SHOPPING Evelina is shopping for back-to-school clothes. She bought 3 skirts, 2 pairs of jeans, and 4 sweaters. Write and evaluate an expression to find how much she spent, without including sales tax.

Clothing	
skirt	$25.99
jeans	$39.99
sweater	$22.99

57 PYRAMIDS The pyramid at the Louvre has a square base with a side of 35.42 meters and a height of 21.64 meters. The Great Pyramid in Egypt has a square base with a side of 230 meters and a height of 146.5 meters. The expression for the volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height.

  a. Draw both pyramids and label the dimensions.

  b. Write a verbal expression for the difference in volume of the two pyramids.

  c. Write an algebraic expression for the difference in volume of the two pyramids. Find the difference in volume.

58. FINANCIAL LITERACY A sales representative receives an annual salary s, an average commission each month c, and a bonus b for each sales goal that she reaches.

  a. Write an algebraic expression to represent her total earnings in one year if she receives four equal bonuses.

  b. Suppose her annual salary is $52,000 and her average commission is $1225 per month. If each of the four bonuses equals $1150, what does she earn annually?

H.O.T. Problems   Use Higher-Order Thinking Skills

59. ERROR ANALYSIS Tara and Curtis are simplifying $[4(10) - 3^2] + 6(4)$. Is either of them correct? Explain your reasoning.

Tara

$[4(10) - 3^2] + 6(4)$

$= [4(10) - 9] + 6(4)$

$= 4(1) + 6(4)$

$= 4 + 6(4)$

$= 4 + 24$

$= 28$

Curtis

$[4(10) - 3^2] + 6(4)$

$= [4(10) - 9] + 6(4)$

$= (40 - 9) + 6(4)$

$= 31 + 6(4)$

$= 31 + 24$

$= 55$

60. REASONING Explain how to evaluate $a[(b - c) \div d] - f$ if you were given values for $a, b, c, d,$ and f. How would you evaluate the expression differently if the expression was $a \cdot b - c \div d - f$?

61. CCSS PERSEVERANCE Write an expression using the whole numbers 1 to 5 using all five digits and addition and/or subtraction to create a numeric expression with a value of 3.

62. OPEN ENDED Write an expression that uses exponents, at least three different operations, and two sets of parentheses. Explain the steps you would take to evaluate the expression.

63. WRITING IN MATH Choose a geometric formula and explain how the order of operations applies when using the formula.

64. WRITING IN MATH Equivalent expressions have the same value. Are the expressions $(30 + 17) \times 10$ and $10 \times 30 + 10 \times 17$ equivalent? Explain why or why not.

65. Let m represent the number of miles. Which algebraic expression represents the number of feet in m miles?

A $5280m$

B $\frac{5280}{m}$

C $m + 5280$

D $5280 - m$

66. SHORT RESPONSE

Simplify: $\left[10 + 15(2^3)\right] \div \left[7(2^2) - 2\right]$
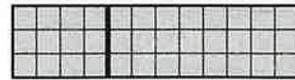
Step 1 $[10 + 15(8)] \div [7(4) - 2]$

Step 2 $[10 + 120] \div [28 - 2]$

Step 3 $130 \div 26$

Step 4 $\frac{1}{5}$

Which is the first *incorrect* step? Explain the error.

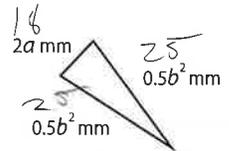**67. EXTENDED RESPONSE** Consider the rectangle below.

Part A Which expression models the area of the rectangle?

F $4 + 3 \times 8$ H $3 \times 4 + 8$

G $3 \times (4 + 8)$ J $3^2 + 8^2$

Part B Draw one or more rectangles to model each other expression.

68. GEOMETRY What is the perimeter of the triangle if $a = 9$ and $b = 10$?

A 164 mm   C 28 mm

B 118 mm   D 4 mm

Write a verbal expression for each algebraic expression. (Lesson 1-1)
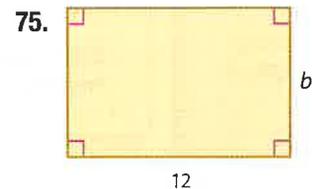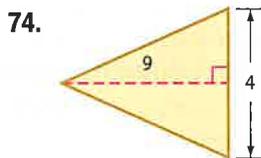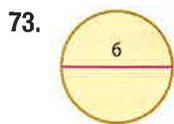
69. $14 - 9c$

70. $k^3 + 13$

71. $\frac{4 - v}{w}$

72. MONEY Destiny earns \$8 per hour babysitting and \$15 for each lawn she mows. Write an expression to show the amount of money she earns babysitting h hours and mowing m lawns. (Lesson 1-1)

Find the area of each figure. (Lesson 0-8)

73.

74.

75.

76. SCHOOL Aaron correctly answered 27 out of 30 questions on his last biology test. What percent of the questions did he answer correctly? (Lesson 0-6)

Find the value of each expression.

77. $5.65 - 3.08$

78. $6 \div \frac{4}{5}$

79. $4.85(2.72)$

80. $1\frac{1}{12} + 3\frac{2}{3}$

81. $\frac{4}{9} \cdot \frac{3}{2}$

82. $7\frac{3}{4} - 4\frac{7}{10}$

Properties of Numbers

- You used the order of operations to simplify expressions.

1 Recognize the properties of equality and identity.

2 Recognize the Commutative and Associative Properties.

- Natalie lives 32 miles away from the mall. The distance from her house to the mall is the same as the distance from the mall to her house. This is an example of the Reflexive Property.

NewVocabulary
equivalent expressions
additive identity
multiplicative identity
multiplicative inverse
reciprocal

Common Core State Standards

Content Standards
A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Mathematical Practices
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

1 **Properties of Equality and Identity** The expressions $4k + 8k$ and $12k$ are called **equivalent expressions** because they represent the same number. The properties below allow you to write an equivalent expression for a given expression.

KeyConcept Properties of Equality

Property	Words	Symbols	Examples
Reflexive Property	Any quantity is equal to itself.	For any number a, $a = a$.	$5 = 5$ $4 + 7 = 4 + 7$
Symmetric Property	If one quantity equals a second quantity, then the second quantity equals the first.	For any numbers a and b, if $a = b$, then $b = a$.	If $8 = 2 + 6$, then $2 + 6 = 8$.
Transitive Property	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $6 + 9 = 3 + 12$ and $3 + 12 = 15$, then $6 + 9 = 15$.
Substitution Property	A quantity may be substituted for its equal in any expression.	If $a = b$, then a may be replaced by b in any expression.	If $n = 11$, then $4n = 4 \cdot 11$

The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

KeyConcept Addition Properties

Property	Words	Symbols	Examples
Additive Identity	For any number a, the sum of a and 0 is a.	$a + 0 = 0 + a = a$	$2 + 0 = 2$ $0 + 2 = 2$
Additive Inverse	A number and its opposite are additive inverses of each other.	$a + (-a) = 0$	$3 + (-3) = 0$ $4 - 4 = 0$

There are also special properties associated with multiplication. Consider the following equations.

$$4 \cdot n = 4$$

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

$$6 \cdot m = 0$$

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

KeyConcept Multiplication Properties

Property	Words	Symbols	Examples
Multiplicative Identity	For any number a, the product of a and 1 is a.	$a \cdot 1 = a$ $1 \cdot a = a$	$14 \cdot 1 = 14$ $1 \cdot 14 = 14$
Multiplicative Property of Zero	For any number a, the product of a and 0 is 0.	$a \cdot 0 = 0$ $0 \cdot a = 0$	$9 \cdot 0 = 0$ $0 \cdot 9 = 0$
Multiplicative Inverse	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.	$\frac{a}{b} \cdot \frac{b}{a} = 1$ $\frac{b}{a} \cdot \frac{a}{b} = 1$	$\frac{4}{5} \cdot \frac{5}{4} = \frac{20}{20}$ or 1 $\frac{5}{4} \cdot \frac{4}{5} = \frac{20}{20}$ or 1

PT

Example 1 Evaluate Using Properties

Evaluate $7(4 - 3) - 1 + 5 \cdot \frac{1}{5}$. Name the property used in each step.

$$7(4 - 3) - 1 + 5 \cdot \frac{1}{5} = 7(1) - 1 + 5 \cdot \frac{1}{5} \qquad \text{Substitution: } 4 - 3 = 1$$

$$= 7 - 1 + 5 \cdot \frac{1}{5} \qquad \text{Multiplicative Identity: } 7 \cdot 1 = 7$$

$$= 7 - 1 + 1 \qquad \text{Multiplicative Inverse: } 5 \cdot \frac{1}{5} = 1$$

$$= 6 + 1 \qquad \text{Substitution: } 7 - 1 = 6$$

$$= 7 \qquad \text{Substitution: } 6 + 1 = 7$$

Guided Practice

Name the property used in each step.

1A. $2 \cdot 3 + (4 \cdot 2 - 8)$
$= 2 \cdot 3 + (8 - 8)$ ___?___
$= 2 \cdot 3 + (0)$ ___?___
$= 6 + 0$ ___?___
$= 6$ ___?___

1B. $7 \cdot \frac{1}{7} + 6(15 \div 3 - 5)$
$= 7 \cdot \frac{1}{7} + 6(5 - 5)$ ___?___
$= 7 \cdot \frac{1}{7} + 6(0)$ ___?___
$= 1 + 6(0)$ ___?___
$= 1 + 0$ ___?___
$= 1$ ___?___

2 **Use Commutative and Associative Properties** Nikki walks 2 blocks to her friend Sierra's house. They walk another 4 blocks to school. At the end of the day, Nikki and Sierra walk back to Sierra's house, and then Nikki walks home.

The distance from Nikki's house to school	equals	the distance from the school to Nikki's house.
2 + 4	=	4 + 2

This is an example of the **Commutative Property** for addition.

Key Concept Commutative Property

Words	The order in which you add or multiply numbers does not change their sum or product.
Symbols	For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
Examples	$4 + 8 = 8 + 4$ $7 \cdot 11 = 11 \cdot 7$

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

Key Concept Associative Property

Words	The way you group three or more numbers when adding or multiplying does not change their sum or product.
Symbols	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.
Examples	$(3 + 5) + 7 = 3 + (5 + 7)$ $(2 \cdot 6) \cdot 9 = 2 \cdot (6 \cdot 9)$

Real-World Example 2 Apply Properties of Numbers

PARTY PLANNING Eric makes a list of items that he needs to buy for a party and their costs. Find the total cost of these items.

Party Supplies	
Item	Cost ($)
balloons	6.75
decorations	14.00
food	23.25
beverages	20.50

Balloons		Decorations		Food		Beverages
6.75	+	14.00	+	23.25	+	20.50

$= 6.75 + 23.25 + 14.00 + 20.50$ — Commutative (+)
$= (6.75 + 23.25) + (14.00 + 20.50)$ — Associative (+)
$= 30.00 + 34.50$ — Substitution
$= 64.50$ — Substitution

The total cost is $64.50.

GuidedPractice

2. **FURNITURE** Rafael is buying furnishings for his first apartment. He buys a couch for $300, lamps for $30.50, a rug for $25.50, and a table for $50. Find the total cost of these items.

Example 3 Use Multiplication Properties

Evaluate $5 \cdot 7 \cdot 4 \cdot 2$ using the properties of numbers. Name the property used in each step.

$$5 \cdot 7 \cdot 4 \cdot 2 = 5 \cdot 2 \cdot 7 \cdot 4 \qquad \text{Commutative } (\times)$$
$$= (5 \cdot 2) \cdot (7 \cdot 4) \qquad \text{Associative } (\times)$$
$$= 10 \cdot 28 \qquad \text{Substitution}$$
$$= 280 \qquad \text{Substitution}$$

▶ **Guided**Practice

Evaluate each expression using the properties of numbers. Name the property used in each step.

3A. $2.9 \cdot 4 \cdot 10$

3B. $\frac{5}{3} \cdot 25 \cdot 3 \cdot 2$

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1   Evaluate each expression. Name the property used in each step.

1. $(1 \div 5)5 \cdot 14$

2. $6 + 4(19 - 15)$

3. $5(14 - 5) + 6(3 + 7)$

4. FINANCIAL LITERACY Carolyn has 9 quarters, 4 dimes, 7 nickels, and 2 pennies, which can be represented as $9(25) + 4(10) + 7(5) + 2$. Evaluate the expression to find how much money she has. Name the property used in each step.

Examples 2–3 Evaluate each expression using the properties of numbers. Name the property used in each step.

5. $23 + 42 + 37$

6. $2.75 + 3.5 + 4.25 + 1.5$

7. $3 \cdot 7 \cdot 10 \cdot 2$

8. $\frac{1}{4} \cdot 24 \cdot \frac{2}{3}$

Practice and Problem Solving

Extra Practice is on page R1.

Example 1   Evaluate each expression. Name the property used in each step.

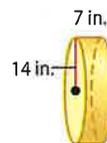**9** $3(22 - 3 \cdot 7)$

10. $7 + (9 - 3^2)$

11. $\frac{3}{4}[4 \div (7 - 4)]$

12. $[3 \div (2 \cdot 1)]\frac{2}{3}$

13. $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$

14. $6 \cdot \frac{1}{6} + 5(12 \div 4 - 3)$

Example 2   **15. GEOMETRY** The expression $2 \cdot \frac{22}{7} \cdot 14^2 + 2 \cdot \frac{22}{7} \cdot 14 \cdot 7$ represents the approximate surface area of the cylinder at the right. Evaluate this expression to find the approximate surface area. Name the property used in each step.

7 in.

14 in.

16. CCSS REASONING A traveler checks into a hotel on Friday and checks out the following Tuesday morning. Use the table to find the total cost of the room including tax.

Hotel Rates Per Day		
Day	Room Charge	Sales Tax
Monday–Friday	$72	$5.40
Saturday–Sunday	$63	$5.10

Examples 2–3 Evaluate each expression using properties of numbers. Name the property used in each step.

17. $25 + 14 + 15 + 36$

18. $11 + 7 + 5 + 13$

19. $3\frac{2}{3} + 4 + 5\frac{1}{3}$

20. $4\frac{4}{9} + 7\frac{2}{9}$

21. $4.3 + 2.4 + 3.6 + 9.7$

22. $3.25 + 2.2 + 5.4 + 10.75$

23. $12 \cdot 2 \cdot 6 \cdot 5$

24. $2 \cdot 8 \cdot 10 \cdot 2$

25. $0.2 \cdot 4.6 \cdot 5$

26. $3.5 \cdot 3 \cdot 6$

27. $1\frac{5}{6} \cdot 24 \cdot 3\frac{1}{11}$

28. $2\frac{3}{4} \cdot 1\frac{1}{8} \cdot 32$

29. SCUBA DIVING The sign shows the equipment rented or sold by a scuba diving store.

    a. Write two expressions to represent the total sales to rent 2 wet suits, 3 air tanks, 2 dive flags, and selling 5 underwater cameras.

    b. What are the total sales?

30. COOKIES Bobby baked 2 dozen chocolate chip cookies, 3 dozen sugar cookies, and a dozen oatmeal raisin cookies. How many total cookies did he bake?

THE DEEP
SCUBA SUPPLIES

SPECIALS
Underwater
Camera $18.99

RENTALS
Air Tanks $ 7.50
Wet Suit $10.95
Dive Flag $ 5.00

Evaluate each expression if $a = -1$, $b = 4$, and $c = 6$.

(31) $4a + 9b - 2c$

32. $-10c + 3a + a$

33. $a - b + 5a - 2b$

34. $8a + 5b - 11a - 7b$

35. $3c^2 + 2c + 2c^2$

36. $3a - 4a^2 + 2a$

37. FOOTBALL A football team is on the 35-yard line. The quarterback is sacked at the line of scrimmage. The team gains 0 yards, so they are still at the 35-yard line. Which identity or property does this represent? Explain.

Find the value of x. Then name the property used.

38. $8 = 8 + x$

39. $3.2 + x = 3.2$

40. $10x = 10$

41. $\frac{1}{2} \cdot x = \frac{1}{2} \cdot 7$

42. $x + 0 = 5$
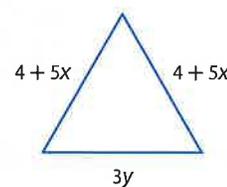
43. $1 \cdot x = 3$

44. $5 \cdot \frac{1}{5} = x$

45. $2 + 8 = 8 + x$

46. $x + \frac{3}{4} = 3 + \frac{3}{4}$

47. $\frac{1}{3} \cdot x = 1$

48. GEOMETRY Write an expression to represent the perimeter of the triangle. Then find the perimeter if $x = 2$ and $y = 7$.

$4 + 5x$ $4 + 5x$ $3y$

49. SPORTS Tickets to a baseball game cost $25 each plus a $4.50 handling charge per ticket. If Sharon has a coupon for $10 off and orders 4 tickets, how much will she be charged?

50. CCSS PRECISION The table shows prices on children's clothing.

    a. Interpret the expression $5(8.99) + 2(2.99) + 7(5.99)$.

    b. Write and evaluate three different expressions that represent 8 pairs of shorts and 8 tops.

    c. If you buy 8 shorts and 8 tops, you receive a discount of 15%. Find the greatest and least amount of money you can spend on the 16 items at the sale.

| Shorts	Shirts	Tank Tops
$7.99	$8.99	$6.99
$5.99	$4.99	$2.99

51. GEOMETRY A regular octagon measures $(3x + 5)$ units on each side. What is the perimeter if $x = 2$?

52. ⚡ **MULTIPLE REPRESENTATIONS** You can use *algebra tiles* to model and explore algebraic expressions. The rectangular tile has an area of x, with dimensions 1 by x. The small square tile has an area of 1, with dimensions 1 by 1.

$x + 2$

4
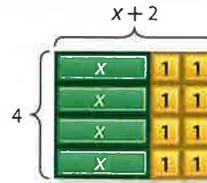
a. Concrete Make a rectangle with algebra tiles to model the expression $4(x + 2)$ as shown above. What are the dimensions of this rectangle? What is its area?

b. Analytical What are the areas of the green region and of the yellow region?

c. Verbal Complete this statement: $4(x + 2) = \underline{?}$. Write a convincing argument to justify your statement.

53 GEOMETRY A **proof** is a logical argument in which each statement you make is supported by a statement that is accepted as true. It is given that $\overline{AB} \cong \overline{CD}$, $\overline{AB} \cong \overline{BD}$, and $\overline{AB} \cong \overline{AC}$. Pedro wants to prove $\triangle ADB \cong \triangle ADC$. To do this, he must show that $\overline{AD} \cong \overline{AD}$, $\overline{AB} \cong \overline{DC}$ and $\overline{BD} \cong \overline{AC}$.
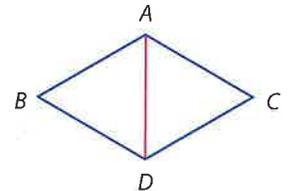
a. Copy the figure and label $\overline{AB} \cong \overline{CD}$, $\overline{AB} \cong \overline{BD}$, and $\overline{AB} \cong \overline{AC}$.

b. Explain how he can use the Reflexive and Transitive Properties to prove $\triangle ADB \cong \triangle ADC$.

c. If AC is x centimeters, write an equation for the perimeter of $ACDB$.

H.O.T. Problems   Use Higher-Order Thinking Skills

54. OPEN ENDED Write two equations showing the Transitive Property of Equality. Justify your reasoning.

55. CCSS ARGUMENTS Explain why 0 has no multiplicative inverse.

56. REASONING The sum of any two whole numbers is always a whole number. So, the set of whole numbers $\{0, 1, 2, 3, 4, \dots\}$ is said to be closed under addition. This is an example of the **Closure Property**. State whether each statement is *true* or *false*. If false, justify your reasoning.

a. The set of whole numbers is closed under subtraction.

b. The set of whole numbers is closed under multiplication.

c. The set of whole numbers is closed under division.

57. CHALLENGE Does the Commutative Property *sometimes*, *always* or *never* hold for subtraction? Explain your reasoning.

58. REASONING Explain whether 1 can be an additive identity. Give an example to justify your answer.

59. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong with the other three. Explain your reasoning.

| $x + 12 = 12 + x$ | $7h = h \cdot 7$ | $1 + a = a + 1$ | $(2j)k = 2(jk)$ |

60. WRITING IN MATH Determine whether the Commutative Property applies to division. Justify your answer.

61. A deck is shaped like a rectangle with a width of 12 feet and a length of 15 feet. What is the area of the deck?

A 3 ft²

B 27 ft²

C 108 ft²

D 180 ft²

62. GEOMETRY A box in the shape of a rectangular prism has a volume of 56 cubic inches. If the length of each side is multiplied by 2, what will be the approximate volume of the box?

F 112 in³   **H** 336 in³

G 224 in³   **J** 448 in³

63. $27 \div 3 + (12 - 4) =$

A $\dfrac{-11}{5}$ **C** 17

B $\dfrac{27}{11}$ **D** 25

64. GRIDDED RESPONSE Ms. Beal had 1 bran muffin, 16 ounces of orange juice, 3 ounces of sunflower seeds, 2 slices of turkey, and half a cup of spinach. Find the total number of grams of protein she consumed.

Protein Content	
Food	**Protein (g)**
bran muffin (1)	3
orange juice (8 oz)	2
sunflower seeds (1 oz)	2
turkey (1 slice)	12
spinach (1 c)	5

Spiral Review

Evaluate each expression. (Lesson 1-2)

65. $3 \cdot 5 + 1 - 2$

66. $14 \div 2 \cdot 6 - 5^2$

67. $\dfrac{3 \cdot 9^2 - 3^2 \cdot 9}{3 \cdot 9}$

68. GEOMETRY Write an expression for the perimeter of the figure. (Lesson 1-1)
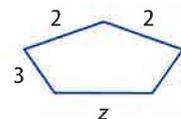
Find the perimeter and area of each figure. (Lessons 0-7 and 0-8)

69. a rectangle with length 5 feet and width 8 feet

70. a square with length 4.5 inches

71. SURVEY Andrew took a survey of his friends to find out their favorite type of music. Of the 34 friends surveyed, 22 said they liked rock music the best. What percent like rock music the best? (Lesson 0-6)

Name the reciprocal of each number. (Lesson 0-5)

72. $\dfrac{6}{17}$

73. $\dfrac{2}{23}$

74. $3\dfrac{4}{5}$

Skills Review

Find each product. Express in simplest form.

75. $\dfrac{12}{15} \cdot \dfrac{3}{14}$

76. $\dfrac{5}{7} \cdot \left(-\dfrac{4}{5}\right)$

77. $\dfrac{10}{11} \cdot \dfrac{21}{35}$

78. $\dfrac{63}{65} \cdot \dfrac{120}{126}$

79. $-\dfrac{4}{3} \cdot \left(-\dfrac{9}{2}\right)$

80. $\dfrac{1}{3} \cdot \dfrac{2}{5}$

1-3 Algebra Lab
Accuracy

All measurements taken in the real world are approximations. The greater the care with which a measurement is taken, the more accurate it will be. **Accuracy** refers to how close a measured value comes to the actual or desired value. For example, a fraction is more accurate than a rounded decimal.

CCSS **Common Core State Standards**
Content Standards
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Mathematical Practices
6 Attend to precision.

Activity 1 When Is Close Good Enough?

Measure the length of your desktop. Record your results in centimeters, in meters, and in millimeters.

Analyze the Results

1. Did you round to the nearest whole measure? If so, when?
2. Did you round to the nearest half, tenth, or smaller? If so, when?
3. Which unit of measure was the most appropriate for this task?
4. Which unit of measure was the most accurate?

Deciding where to round a measurement depends on how the measurement will be used. But calculations should not be carried out to greater accuracy than that of the original data.

Activity 2 Decide Where to Round

a. **Elan has $13 that he wants to divide among his 6 nephews. When he types 13 ÷ 6 into his calculator, the number that appears is 2.166666667. Where should Elan round?**

Since Elan is rounding money, the smallest increment is a penny, so round to the hundredths place. This will give him 2.17, and $2.17 × 6 = $13.02. Elan will be two pennies short, so round to $2.16. Since $2.16 × 6 = $12.96, Elan can give each of his nephews $2.16.

b. **Dante's mother brings him a dozen cookies, but before she leaves she eats one and tells Dante he has to share with his two sisters. Dante types 11 ÷ 3 into his calculator and gets 3.666666667. Where should Dante round?**

After each sibling receives 3 cookies, there are two cookies left. In this case, it is more accurate to convert the decimal portion to a fraction and give each sibling $\frac{2}{3}$ of a cookie.

c. **Eva measures the dimensions of a box as 8.7, 9.52, and 3.16 inches. She multiplies these three numbers to find the measure of the volume. The result shown on her calculator is 261.72384. Where should Eva round?**

Eva should round to the tenths place, 261.7, because she was only accurate to the tenths place with one of her measures.

Exercises

5. Jessica wants to divide $23 six ways. Her calculator shows 3.833333333. Where should she round?

6. Ms. Harris wants to share 2 pizzas among 6 people. Her calculator shows 0.3333333333. Where should she round?

7. The measurements of an aquarium are 12.9, 7.67, and 4.11 inches. The measure of the volume is given by the product 406.65573. Where should the number be rounded?

For most real-world measurements, a decision must be made on the level of accuracy needed or desired.

Activity 3   Find an Appropriate Level of Accuracy

a. **Jon needs to buy a shade for the window opening shown, but the shades are only available in whole inch increments. What size shade should he buy?**

He should buy the 27-inch shade because it will be enough to cover the glass.

27.5 in.

b. **Tom is buying flea medicine for his dog. The amount of medicine depends on the dog's weight. The medicine is available in packages that vary by 10 dog pounds. How accurate does Tom need to be to buy the correct medicine?**

He needs to be accurate to within 10 pounds.

c. **Tyrone is building a jet engine. How accurate do you think he needs to be with his measurements?**

He needs to be very accurate, perhaps to the thousandth of an inch.

Exercises

8. Matt's table is missing a leg. He wants to cut a piece of wood to replace the leg. How accurate do you think he needs to be with his measurements?

For each situation, determine where the rounding should occur and give the rounded answer.

9. Sam wants to divide $111 seven ways. His calculator shows 15.85714286.

10. Kiri wants to share 3 pies among 11 people. Her calculator shows 0.2727272727.

11. Evan's calculator gives him the volume of his soccer ball as 137.2582774. Evan measured the radius of the ball to be 3.2 inches.

For each situation, determine the level of accuracy needed. Explain.

12. You are estimating the length of your school's basketball court. Which unit of measure should you use: 1 foot, 1 inch, or $\frac{1}{16}$ inch?

13. You are estimating the height of a small child. Which unit of measure should you use: 1 foot, 1 inch, or $\frac{1}{16}$ inch?

14. **TRAVEL** Curt is measuring the driving distance from one city to another. How accurate do you think he needs to be with his measurement?

15. **MEDICINE** A nurse is administering medicine to a patient based on his weight. How accurate do you think she needs to be with her measurements?

The Distributive Property

- You explored Associative and Commutative Properties.

- **1** Use the Distributive Property to evaluate expressions.

- **2** Use the Distributive Property to simplify expressions.

- John burns approximately 420 Calories per hour by inline skating. The chart below shows the time he spent inline skating in one week.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Time (h)	1	$\frac{1}{2}$	0	1	0	2	$2\frac{1}{2}$

To determine the total number of Calories that he burned inline skating that week, you can use the Distributive Property.

New Vocabulary
like terms
simplest form
coefficient

1 Evaluate Expressions There are two methods you could use to calculate the number of Calories John burned inline skating. You could find the total time spent inline skating and then multiply by the Calories burned per hour. Or you could find the number of Calories burned each day and then add to find the total.

Method 1 Rate Times Total Time

$$420\left(1 + \frac{1}{2} + 1 + 2 + 2\frac{1}{2}\right)$$
$$= 420(7)$$
$$= 2940$$

Method 2 Sum of Daily Calories Burned

$$420(1) + 420\left(\frac{1}{2}\right) + 420(1) + 420(2) + 420\left(2\frac{1}{2}\right)$$
$$= 420 + 210 + 420 + 840 + 1050$$
$$= 2940$$

Either method gives the same total of 2940 Calories burned. This is an example of the **Distributive Property**.

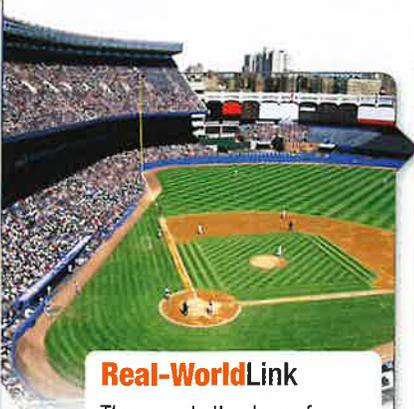**KeyConcept** Distributive Property

Symbol	For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.
Examples	$3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$ $\qquad$ $4(9 - 7) = 4 \cdot 9 - 4 \cdot 7$ $\quad\; 3(7) = 6 + 15$ $\qquad\qquad\qquad\;\; 4(2) = 36 - 28$ $\qquad\; 21 = 21$ $\qquad\qquad\qquad\qquad\quad\; 8 = 8$

The Symmetric Property of Equality allows the Distributive Property to be written as follows.

$$\text{If } a(b + c) = ab + ac, \text{ then } ab + ac = a(b + c).$$

Real-World Example 1 Distribute Over Addition

SPORTS A group of 7 adults and 6 children are going to a University of South Florida Bulls baseball game. Use the Distributive Property to write and evaluate an expression for the total ticket cost.

USF Bulls Baseball Tickets	
Ticket	Cost ($)
Adult Single Game	5
Children Single Game (12 and under)	3
Groups of 10 or more Single Game	2
Senior Single Game (65 and over)	3

Source: USF

Understand You need to find the cost of each ticket and then find the total cost.

Plan $7 + 6$ or 13 people are going to the game, so the tickets are $2 each.

Solve Write an expression that shows the product of the cost of each ticket and the sum of adult tickets and children's tickets.

$$2(7 + 6) = 2(7) + 2(6) \qquad \text{Distributive Property}$$
$$= 14 + 12 \qquad \text{Multiply.}$$
$$= 26 \qquad \text{Add.}$$

The total cost is $26.

Check The total number of tickets needed is 13 and they cost $2 each. Multiply 13 by 2 to get 26. Therefore, the total cost of tickets is $26.

GuidedPractice

1. **SPORTS** A group of 3 adults, an 11-year old, and 2 children under 10 years old are going to a baseball game. Write and evaluate an expression to determine the cost of tickets for the group.

You can use the Distributive Property to make mental math easier.

Example 2 Mental Math

Use the Distributive Property to rewrite $7 \cdot 49$. Then evaluate.

$$7 \cdot 49 = 7(50 - 1) \qquad \text{Think: } 49 = 50 - 1$$
$$= 7(50) - 7(1) \qquad \text{Distributive Property}$$
$$= 350 - 7 \qquad \text{Multiply.}$$
$$= 343 \qquad \text{Subtract.}$$

GuidedPractice

Use the Distributive Property to rewrite each expression. Then evaluate.

2A. $304(15)$ **2B.** $44 \cdot 2\frac{1}{2}$
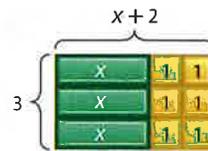
2C. $210(5)$ **2D.** $52(17)$

2 Simplify Expressions You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.

The rectangle at the right has 3 x-tiles and 6 1-tiles. The area of the rectangle is $x + 1 + 1 + x + 1 + 1 + x + 1 + 1$ or $3x + 6$. Therefore, $3(x + 2) = 3x + 6$.

Example 3 Algebraic Expressions

Rewrite each expression using the Distributive Property. Then simplify.

a. $7(3w - 5)$

$$7(3w - 5) = 7 \cdot 3w - 7 \cdot 5 \qquad \text{Distributive Property}$$
$$= 21w - 35 \qquad \text{Multiply.}$$

b. $(6v^2 + v - 3)4$

$$(6v^2 + v - 3)4 = 6v^2(4) + v(4) - 3(4) \qquad \text{Distributive Property}$$
$$= 24v^2 + 4v - 12 \qquad \text{Multiply.}$$

▶ **Guided**Practice

3A. $(8 + 4n)2$ **3B.** $-6(r + 3g - t)$

3C. $(2 - 5q)(-3)$ **3D.** $-4(-8 - 3m)$

Like terms are terms that contain the same variables, with corresponding variables having the same power.

$$5x^2 + 2x - 4 \qquad\qquad 6a^2 + a^2 + 2a$$

three terms   like terms   unlike terms
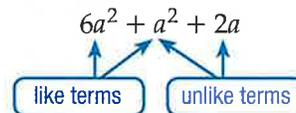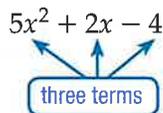
The Distributive Property and the properties of equality can be used to show that $4k + 8k = 12k$. In this expression, $4k$ and $8k$ are like terms.

$$4k + 8k = (4 + 8)k \qquad \text{Distributive Property}$$
$$= 12k \qquad \text{Substitution}$$

An expression is in **simplest form** when it contains no like terms or parentheses.

Example 4 Combine Like Terms

a. Simplify $17u + 25u$.

$$17u + 25u = (17 + 25)u \qquad \text{Distributive Property}$$
$$= 42u \qquad \text{Substitution}$$

b. Simplify $6t^2 + 3t - t$.

$$6t^2 + 3t - t = 6t^2 + (3 - 1)t \qquad \text{Distributive Property}$$
$$= 6t^2 + 2t \qquad \text{Substitution}$$

▶ **Guided**Practice

Simplify each expression. If not possible, write *simplified*.

4A. $6n - 4n$ **4B.** $b^2 + 13b + 13$

4C. $4y^3 + 2y - 8y + 5$ **4D.** $7a + 4 - 6a^2 - 2a$

Example 5 Write and Simplify Expressions

Use the expression *twice the difference of 3x and y increased by five times the sum of x and 2y.*

a. Write an algebraic expression for the verbal expression.

Words	twice the difference of $3x$ and y	increased by	five times the sum of x and $2y$

▼

Variables   Let x and y represent the numbers.

▼

Expression	$2(3x - y)$	$+$	$5(x + 2y)$

b. Simplify the expression, and indicate the properties used.

$2(3x - y) + 5(x + 2y) = 2(3x) - 2(y) + 5(x) + 5(2y)$ Distributive Property

$\qquad\qquad\qquad\qquad = 6x - 2y + 5x + 10y$ Multiply.

$\qquad\qquad\qquad\qquad = 6x + 5x - 2y + 10y$ Commutative ($+$)

$\qquad\qquad\qquad\qquad = (6 + 5)x + (-2 + 10)y$ Distributive Property

$\qquad\qquad\qquad\qquad = 11x + 8y$ Substitution

▶ **Guided**Practice

5. Use the expression *5 times the difference of q squared and r plus 8 times the sum of 3q and 2r.*

  A. Write an algebraic expression for the verbal expression.

  B. Simplify the expression, and indicate the properties used.

The **coefficient** of a term is the numerical factor. For example, in $6ab$, the coefficient is 6, and in $\frac{x^2}{3}$, the coefficient is $\frac{1}{3}$. In the term y, the coefficient is 1 since $1 \cdot y = y$ by the Multiplicative Identity Property.

ConceptSummary Properties of Numbers

The following properties are true for any numbers a, b, and c.

Properties	Addition	Multiplication
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	0 is the identity. $a + 0 = 0 + a = a$	1 is the identity. $a \cdot 1 = 1 \cdot a = a$
Zero	—	$a \cdot 0 = 0 \cdot a = 0$
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Substitution	If $a = b$, then a may be substituted for b.	

Math History Link

Kambei Mori
(c. 1600–1628)
Kambei Mori was a Japanese scholar who popularized the abacus. He changed the focus of mathematics from philosophy to computation.

Example 1

1. **PILOT** A pilot at an air show charges $25 per passenger for rides. If 12 adults and 15 children ride in one day, write and evaluate an expression to describe the situation.

Example 2

Use the Distributive Property to rewrite each expression. Then evaluate.

2. $14(51)$

3. $6\frac{1}{9}(9)$

Example 3

Use the Distributive Property to rewrite each expression. Then simplify.

4. $2(4 + t)$

5. $(g - 9)5$

Example 4

Simplify each expression. If not possible, write *simplified*.

6. $15m + m$

7. $3x^3 + 5y^3 + 14$

8. $(5m + 2m)10$

Example 5

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

9. 4 times the sum of 2 times x and six

10. one half of 4 times y plus the quantity of y and 3

Practice and Problem Solving

Extra Practice is on page R1.

Example 1

11. **TIME MANAGEMENT** Margo uses dots to track her activities on a calendar. Red dots represent homework, yellow dots represent work, and green dots represent track practice. In a typical week, she uses 5 red dots, 3 yellow dots, and 4 green dots. How many activities does Margo do in 4 weeks?

12. **CCSS REASONING** The Red Cross is holding blood drives in two locations. In one day, Center 1 collected 715 pints and Center 2 collected 1035 pints. Write and evaluate an expression to estimate the total number of pints of blood donated over a 3-day period.

Example 2

Use the Distributive Property to rewrite each expression. Then evaluate.

13. $(4 + 5)6$

14. $7(13 + 12)$

15. $6(6 - 1)$

16. $(3 + 8)15$

17. $14(8 - 5)$

18. $(9 - 4)19$

19. $4(7 - 2)$

20. $7(2 + 1)$

21. $7 \cdot 497$

22. $6(525)$

23. $36 \cdot 3\frac{1}{4}$

24. $\left(4\frac{2}{9}\right)21$

Example 3

Use the Distributive Property to rewrite each expression. Then simplify.

25. $2(x + 4)$

26. $(5 + n)3$

27. $(4 - 3m)8$

28. $-3(2x - 6)$

Example 4

Simplify each expression. If not possible, write *simplified*.

29. $13r + 5r$

30. $3x^3 - 2x^2$

31. $7m + 7 - 5m$

32. $5z^2 + 3z + 8z^2$

33. $(2 - 4n)17$

34. $11(4d + 6)$

35. $7m + 2m + 5p + 4m$

36. $3x + 7(3x + 4)$

37. $4(fg + 3g) + 5g$

Example 5

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

38. the product of 5 and m squared, increased by the sum of the square of m and 5

39. 7 times the sum of a squared and b minus 4 times the sum of a squared and b

40. GEOMETRY Find the perimeter of an isosceles triangle with side lengths of $5 + x$, $5 + x$, and xy. Write in simplest form.

41 GEOMETRY A regular hexagon measures $3x + 5$ units on each side. What is the perimeter in simplest form?

Simplify each expression.

42. $6x + 4y + 5x$

43. $3m + 5g + 6g + 11m$

44. $4a + 5a^2 + 2a^2 + a^2$

45. $5k + 3k^3 + 7k + 9k^3$

46. $6d + 4(3d + 5)$

47. $2(6x + 4) + 7x$

48. FOOD Kenji is picking up take-out food for his study group.

   a. Interpret the expression $4(2.49) + 3(1.29) + 3(0.99) + 5(1.49)$.

   b. How much would it cost if Kenji bought four of each item on the menu?

Use the Distributive Property to rewrite each expression. Then simplify.

Menu	
Item	**Cost ($)**
sandwich	2.49
cup of soup	1.29
side salad	0.99
drink	1.49

49. $\left(\frac{1}{3} - 2b\right)27$

50. $4(8p + 4q - 7r)$

51. $6(2c - cd^2 + d)$

Simplify each expression. If not possible, write *simplified*.

52. $6x^2 + 14x - 9x$

53. $4y^3 + 3y^3 + y^4$

54. $a + \frac{a}{5} + \frac{2}{5}a$

55. ⚏ **MULTIPLE REPRESENTATIONS** The area of the model is $2(x - 4)$ or $2x - 8$. The expression $2(x - 4)$ is in *factored form*.
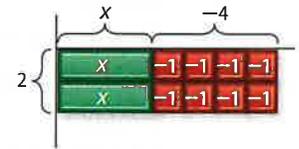
   a. Geometric Use algebra tiles to form a rectangle with area $2x + 6$. Use the result to write $2x + 6$ in factored form.

   b. Tabular Use algebra tiles to form rectangles to represent each area in the table. Record the factored form of each expression.

   c. Verbal Explain how you could find the factored form of an expression.

Area	Factored Form
$2x + 6$	
$3x + 3$	
$3x - 12$	
$5x + 10$	

H.O.T. Problems   Use Higher-Order Thinking Skills

56. **CCSS PERSEVERANCE** Use the Distributive Property to simplify $6x^2[(3x - 4) + (4x + 2)]$.

57. REASONING Should the Distributive Property be a property of multiplication, addition, or both? Explain your answer.

58. 📝 **WRITING IN MATH** Why is it helpful to represent verbal expressions algebraically?

59. WRITING IN MATH Use the data about skating on page 25 to explain how the Distributive Property can be used to calculate quickly. Also, compare the two methods of finding the total Calories burned.

60. Which illustrates the Symmetric Property of Equality?

  A If $a = b$, then $b = a$.

  B If $a = b$, and $b = c$, then $a = c$.

  C If $a = b$, then $b = c$.

  D If $a = a$, then $a + 0 = a$.

61. Anna is three years younger than her sister Emily. Which expression represents Anna's age if we express Emily's age as y years?

  F $y + 3$ **H** $3y$

  G $y - 3$ **J** $\dfrac{3}{y}$

62. Which property is used below?
If $4xy^2 = 8y^2$ and $8y^2 = 72$, then $4xy^2 = 72$.

  A Reflexive Property

  B Substitution Property

  C Symmetric Property

  D Transitive Property

63. SHORT RESPONSE A drawer contains the socks in the chart. What is the probability that a randomly chosen sock is blue?

Color	Number
white	16
blue	12
black	8

Evaluate each expression. Name the property used in each step. (Lesson 1-3)

64. $14 + 23 + 8 + 15$

65. $0.24 \cdot 8 \cdot 7.05$

66. $1\frac{1}{4} \cdot 9 \cdot \frac{5}{6}$

67. SPORTS Braden runs 6 times a week for 30 minutes and lifts weights 3 times a week for 20 minutes. Write and evaluate an expression for the number of hours Braden works out in 4 weeks. (Lesson 1-2)

SPORTS Refer to the table showing Blanca's cross-country times for the first 8 meets of the season. Round answers to the nearest second. (Lesson 0-12)

Cross Country	
Meet	Time
1	22:31
2	22:21
3	21:48
4	22:01
5	21:48
6	20:56
7	20:34
8	20:15

68. Find the mean of the data.

69. Find the median of the data.

70. Find the mode of the data.

71. SURFACE AREA What is the surface area of the cube? (Lesson 0-10)

8 in.

Evaluate each expression.

72. $12(7 + 2)$

73. $11(5) - 8(5)$

74. $(13 - 9) \cdot 4$

75. $3(6) + 7(6)$

76. $(1 + 19) \cdot 8$

77. $16(5 + 7)$

Write a verbal expression for each algebraic expression.
(Lesson 1-1)

1. $21 - x^3$
2. $3m^5 + 9$

Write an algebraic expression for each verbal expression.
(Lesson 1-1)

3. five more than s squared

4. four times y to the fourth power

5. **CAR RENTAL** The XYZ Car Rental Agency charges a flat rate of $29 per day plus $0.32 per mile driven. Write an algebraic expression for the rental cost of a car for x days that is driven y miles. (Lesson 1-1)

Evaluate each expression. (Lesson 1-2)

6. $24 \div 3 - 2 \cdot 3$

7. $5 + 2^2$

8. $4(3 + 9)$

9. $36 - 2(1 + 3)^2$

10. $\dfrac{40 - 2^3}{4 + 3(2^2)}$

11. **AMUSEMENT PARK** The costs of tickets to a local amusement park are shown. Write and evaluate an expression to find the total cost for 5 adults and 8 children. (Lesson 1-2)

Adult $45
Children $25
ADMIT ONE ADMIT ONE

12. **MULTIPLE CHOICE** Write an algebraic expression to represent the perimeter of the rectangle shown below. Then evaluate it to find the perimeter when $w = 8$ cm. (Lesson 1-2)

w
$4w - 3$
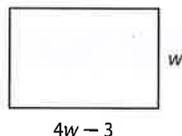
A 37 cm
B 232 cm
C 74 cm
D 45 cm

Evaluate each expression. Name the property used in each step. (Lesson 1-3)

13. $(8 - 2^3) + 21$

14. $3(1 \div 3) \cdot 9$

15. $[5 \div (3 \cdot 1)]\dfrac{3}{5}$

16. $18 + 35 + 32 + 15$

17. $0.25 \cdot 7 \cdot 4$

Use the Distributive Property to rewrite each expression. Then evaluate. (Lesson 1-4)

18. $3(5 + 2)$

19. $(9 - 6)12$

20. $8(7 - 4)$

Use the Distributive Property to rewrite each expression. Then simplify. (Lesson 1-4)

21. $4(x + 3)$

22. $(6 - 2y)7$

23. $-5(3m - 2)$

24. **DVD SALES** A video store chain has three locations. Use the information in the table below to write and evaluate an expression to estimate the total number of DVDs sold over a 4-day period. (Lesson 1-4)

Location	Daily Sales Numbers
Location 1	145
Location 2	211
Location 3	184

25. **MULTIPLE CHOICE** Rewrite the expression $(8 - 3p)(-2)$ using the Distributive Property. (Lesson 1-4)

F $16 - 6p$

G $-10p$

H $-16 + 6p$

J $10p$

·· Then	·· Now	·· Why?
● You simplified expressions.	**1** Solve equations with one variable. **2** Solve equations with two variables.	● Mark's baseball team scored 3 runs in the first inning. At the top of the third inning, their score was 4. The open sentence below represents the change in their score. $3 + r = 4$ The solution is 1. The team got 1 run in the second inning.

CCSS Common Core State Standards

Content Standards
A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

1 Solve Equations A mathematical statement that contains algebraic expressions and symbols is an **open sentence**. A sentence that contains an equals sign, $=$, is an **equation**.

$$\boxed{\text{expression}} \longrightarrow 3x + 7 \qquad 3x + 7 = 13 \longleftarrow \boxed{\text{equation}}$$

Finding a value for a variable that makes a sentence true is called **solving** the open sentence. This replacement value is a **solution**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A **set** is a collection of objects or numbers that is often shown using braces. Each object or number in the set is called an **element**, or member. A **solution set** is the set of elements from the replacement set that make an open sentence true.

Example 1 Use a Replacement Set

Find the solution set of the equation $2q + 5 = 13$ if the replacement set is {2, 3, 4, 5, 6}.

Use a table to solve. Replace q in $2q + 5 = 13$ with each value in the replacement set.

Since the equation is true when $q = 4$, the solution of $2q + 5 = 13$ is $q = 4$.

The solution set is {4}.

q	$2q + 5 = 13$	True or False?
2	$2(2) + 5 = 13$	false
3	$2(3) + 5 = 13$	false
4	$2(4) + 5 = 13$	true
5	$2(5) + 5 = 13$	false
6	$2(6) + 5 = 13$	false

▶ **Guided**Practice

Find the solution set for each equation if the replacement set is {0, 1, 2, 3}.

1A. $8m - 7 = 17$ **1B.** $28 = 4(1 + 3d)$

You can often solve an equation by applying the order of operations.

Standardized Test Example 2 Apply the Order of Operations

Solve $6 + (5^2 - 5) \div 2 = p$.

A 3   B 6   C 13   D 16

Read the Test Item

You need to apply the order of operations to the expression in order to solve for p.

Solve the Test Item

$\begin{aligned}
6 + (5^2 - 5) \div 2 &= p & \text{Original equation}\\
6 + (25 - 5) \div 2 &= p & \text{Evaluate powers.}\\
6 + 20 \div 2 &= p & \text{Subtract 5 from 25.}\\
6 + 10 &= p & \text{Divide 20 by 2.}\\
16 &= p & \text{Add.}
\end{aligned}$

The correct answer is D.

▶ **Guided**Practice

2. Solve $t = 9^2 \div (5 - 2)$.

F 3   G 6   H 14.2   J 27

Some equations have a unique solution. Other equations do not have a solution.

Example 3 Solutions of Equations

Solve each equation.

a. $7 - (4^2 - 10) + n = 10$

Simplify the equation first and then look for a solution.

$\begin{aligned}
7 - (4^2 - 10) + n &= 10 & \text{Original equation}\\
7 - (16 - 10) + n &= 10 & \text{Evaluate powers.}\\
7 - 6 + n &= 10 & \text{Subtract 10 from 16.}\\
1 + n &= 10 & \text{Subtract 6 from 7.}
\end{aligned}$

The only value for n that makes the equation true is 9. Therefore, this equation has a unique solution of 9.

b. $n(3 + 2) + 6 = 5n + (10 - 3)$

$\begin{aligned}
n(3 + 2) + 6 &= 5n + (10 - 3) & \text{Original equation}\\
n(5) + 6 &= 5n + (10 - 3) & \text{Add } 3 + 2.\\
n(5) + 6 &= 5n + 7 & \text{Subtract 3 from 10.}\\
5n + 6 &= 5n + 7 & \text{Commutative } (\times)
\end{aligned}$

No matter what real value is substituted for n, the left side of the equation will always be one less than the right side. So, the equation will never be true. Therefore, there is no solution of this equation.

▶ **Guided**Practice

3A. $(18 + 4) + m = (5 - 3)m$

3B. $8 \cdot 4 \cdot k + 9 \cdot 5 = (36 - 4)k - (2 \cdot 5)$

An equation that is true for every value of the variable is called an **identity**.

Example 4 Identities

Solve $(2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]2$.

$(2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]2$	Original Equation
$(10 - 8)(3h + 6) = [(2h + h) + 6]2$	Multiply $2 \cdot 5$.
$2(3h + 6) = [(2h + h) + 6]2$	Subtract 8 from 10.
$6h + 12 = [(2h + h) + 6]2$	Distributive Property
$6h + 12 = [3h + 6]2$	Add $2h + h$.
$6h + 12 = 6h + 12$	Distributive Property

No matter what value is substituted for h, the left side of the equation will always be equal to the right side. So, the equation will always be true. Therefore, the solution of this equation could be any real number.

▶ **Guided**Practice

Solve each equation.

4A. $12(10 - 7) + 9g = g(2^2 + 5) + 36$ **4B.** $2d + (2^3 - 5) = 10(5 - 2) + d(12 \div 6)$

4C. $3(b + 1) - 5 = 3b - 2$ **4D.** $5 - \frac{1}{2}(c - 6) = 4$

2 Solve Equations with Two Variables Some equations contain two variables. It is often useful to make a table of values and use substitution to find the corresponding values of the second variable.

Example 5 Equations Involving Two Variables

MOVIE RENTALS Mr. Hernandez pays $10 each month for movies delivered by mail. He can also rent movies in the store for $1.50 per title. Write and solve an equation to find the total amount Mr. Hernandez spends this month if he rents 3 movies from the store.

The cost of the movie plan is a flat rate. The variable is the number of movies he rents from the store. The total cost is the price of the plan plus $1.50 times the number of movies from the store. Let C be the total cost and m be the number of movies.

$C = 1.50m + 10$	Original equation
$= 1.50(3) + 10$	Substitute 3 for m.
$= 4.50 + 10$	Multiply.
$= 14.50$	

Mr. Hernandez spends $14.50 on movie rentals in one month.

▶ **Guided**Practice

5. TRAVEL Amelia drives an average of 65 miles per hour. Write and solve an equation to find the time it will take her to drive 36 miles.

Example 1   Find the solution set of each equation if the replacement set is {11, 12, 13, 14, 15}.

1. $n + 10 = 23$

2. $7 = \dfrac{c}{2}$

3. $29 = 3x - 7$

4. $(k - 8)12 = 84$

Example 2   **5. MULTIPLE CHOICE** Solve $\dfrac{d + 5}{10} = 2$.

A 10   **B** 15   **C** 20   **D** 25

Examples 3–4 Solve each equation.

6. $x = 4(6) + 3$

7. $14 - 82 = w$

8. $5 + 22a = 2 + 10 \div 2$

9. $(2 \cdot 5) + \dfrac{c^3}{3} = c^3 \div (1^5 + 2) + 10$

Example 5   **10. RECYCLING** San Francisco has a recycling facility that accepts unused paint. Volunteers blend and mix the paint and give it away in 5-gallon buckets. Write and solve an equation to find the number of buckets of paint given away from the 30,000 gallons that are donated.

Practice and Problem Solving   Extra Practice is on page R1.

Example 1   Find the solution set of each equation if the replacement sets are y: {1, 3, 5, 7, 9} and z: {10, 12, 14, 16, 18}.

11. $z + 10 = 22$

12. $52 = 4z$

13. $\dfrac{15}{y} = 3$

14. $17 = 24 - y$

15. $2z - 5 = 27$

16. $4(y + 1) = 40$

17. $22 = \dfrac{60}{y} + 2$

18. $111 = z^2 + 11$

Examples 2–4 Solve each equation.

19. $a = 32 - 9(2)$

20. $w = 56 \div (2^2 + 3)$

21. $\dfrac{27 + 5}{16} = g$

22. $\dfrac{12 \cdot 5}{15 - 3} = y$

23. $r = \dfrac{9(6)}{(8 + 1)3}$

24. $a = \dfrac{4(14 - 1)}{3(6) - 5} + 7$

25. $(4 - 2^2 + 5)w = 25$

26. $7 + x - (3 + 32 \div 8) = 3$

27. $3^2 - 2 \cdot 3 + u = (3^3 - 3 \cdot 8)(2) + u$

28. $(3 \cdot 6 \div 2)v + 10 = 3^2v + 9$

29. $6k + (3 \cdot 10 - 8) = (2 \cdot 3)k + 22$

30. $(3 \cdot 5)t + (21 - 12) = 15t + 3^2$

31 $(2^4 - 3 \cdot 5)q + 13 = (2 \cdot 9 - 4^2)q + \left(\dfrac{3 \cdot 4}{12} - 1\right)$

32. $\dfrac{3 \cdot 22}{18 + 4}r - \left(\dfrac{4^2}{9 + 7} - 1\right) = r + \left(\dfrac{8 \cdot 9}{3} \div 3\right)$

33. SCHOOL A conference room can seat a maximum of 85 people. The principal and two counselors need to meet with the school's juniors to discuss college admissions. If each student must bring a parent with them, how many students can attend each meeting? Assume that each student has a unique set of parents.

34. CCSS MODELING The perimeter of a regular octagon is 128 inches. Find the length of each side.

Example 5

35 **SPORTS** A 200-pound athlete who trains for four hours per day requires 2836 Calories for basic energy requirements. During training, the same athlete requires an additional 3091 Calories for extra energy requirements. Write an equation to find C, the total daily Calorie requirement for this athlete. Then solve the equation.

36. ENERGY An electric generator can power 3550 watts of electricity. Write and solve an equation to find how many 75-watt light bulbs a generator could power.

Make a table of values for each equation if the replacement set is {−2, −1, 0, 1, 2}.

37. $y = 3x - 2$

38. $3.25x + 0.75 = y$

Solve each equation using the given replacement set.

39. $t - 13 = 7$; {10, 13, 17, 20}

40. $14(x + 5) = 126$; {3, 4, 5, 6, 7}

41. $22 = \frac{n}{3}$; {62, 64, 66, 68, 70}

42. $35 = \frac{g - 8}{2}$; {78, 79, 80, 81}

Solve each equation.

43. $\frac{3(9) - 2}{1 + 4} = d$

44. $j = 15 \div 3 \cdot 5 - 4^2$

45. $c + (3^2 - 3) = 21$

46. $(3^3 - 3 \cdot 9) + (7 - 2^2)b = 24b$

47. **CCSS SENSE-MAKING** Blood flow rate can be expressed as $F = \frac{p_1 - p_2}{r}$, where F is the flow rate, p_1 and p_2 are the initial and final pressure exerted against the blood vessel's walls, respectively, and r is the resistance created by the size of the vessel.

a. Write and solve an equation to determine the resistance of the blood vessel for an initial pressure of 100 millimeters of mercury, a final pressure of 0 millimeters of mercury, and a flow rate of 5 liters per minute.

b. Use the equation to complete the table below.

Initial Pressure p_1 (mm Hg)	Final Pressure p_2 (mm Hg)	Resistance r (mm Hg/L/min)	Blood Flow Rate F (L/min)
100	0		5
100	0	30	
	5	40	4
90		10	6

Determine whether the given number is a solution of the equation.

48. $x + 6 = 15$; 9

49. $12 + y = 26$; 14

50. $2t - 10 = 4$; 3

51. $3r + 7 = -5$; 2

52. $6 + 4m = 18$; 3

53. $-5 + 2p = -11$; −3

54. $\frac{q}{2} = 20$; 10

55. $\frac{w - 4}{5} = -3$; −11

56. $\frac{g}{3} - 4 = 12$; 48

Make a table of values for each equation if the replacement set is {−2, −1, 0, 1, 2}.

57. $y = 3x + 5$

58. $-2x - 3 = y$

59. $y = \frac{1}{2}x + 2$

60. $4.2x - 1.6 = y$

61. GEOMETRY The length of a rectangle is 2 inches greater than the width. The length of the base of an isosceles triangle is 12 inches, and the lengths of the other two sides are 1 inch greater than the width of the rectangle.

a. Draw a picture of each figure and label the dimensions.

b. Write two expressions to find the perimeters of the rectangle and triangle.

c. Find the width of the rectangle if the perimeters of the figures are equal.

62. CONSTRUCTION The construction of a building requires 10 tons of steel per story.

  a. Define a variable and write an equation for the number of tons of steel required if the building has 15 stories.

  b. How many tons of steel are needed?

63. **MULTIPLE REPRESENTATIONS** In this problem, you will further explore writing equations.

  a. Concrete Use centimeter cubes to build a tower similar to the one shown at the right.

  b. Tabular Copy and complete the table shown below. Record the number of layers in the tower and the number of cubes used in the table.
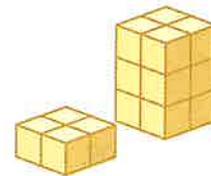
Layers	1	2	3	4	5	6	7
Cubes	?	?	?	?	?	?	?

  c. Analytical As the number of layers in the tower increases, how does the number of cubes in the tower change?

  d. Algebraic Write a rule that gives the number of cubes in terms of the number of layers in the tower.

H.O.T. Problems   Use Higher-Order Thinking Skills

64. REASONING Compare and contrast an expression and an equation.

65. OPEN ENDED Write an equation that is an identity.

66. REASONING Explain why an open sentence always has at least one variable.

67. CCSS CRITIQUE Tom and Li-Cheng are solving the equation $x = 4(3 - 2) + 6 \div 8$. Is either of them correct? Explain your reasoning.

Tom	Li-Cheng
$x = 4(3 - 2) + 6 \div 8$	$x = 4(3 - 2) + 6 \div 8$
$= 4(1) + 6 \div 8$	$= 4(1) + 6 \div 8$
$= 4 + 6 \div 8$	$= 4 + 6 \div 8$
$= 4 + \dfrac{6}{8}$	$= 10 \div 8$
$= 4\dfrac{3}{4}$	$= \dfrac{5}{4}$

68. CHALLENGE Find all of the solutions of $x^2 + 5 = 30$.

69. OPEN ENDED Write an equation that involves two or more operations with a solution of -7.

70. WRITING IN MATH Explain how you can determine that an equation has no real numbers as a solution. How can you determine that an equation has all real numbers as solutions?

71. Which of the following is *not* an equation?

   A $y = 6x - 4$

   B $\dfrac{a+4}{2} = \dfrac{1}{4}$

   C $(4 \cdot 3b) + (8 \div 2c)$

   D $55 = 6 + d^2$

72. SHORT RESPONSE The expected attendance for the Drama Club production is 65% of the student body. If the student body consists of 300 students, how many students are expected to attend?

73. GEOMETRY A speedboat and a sailboat take off from the same port. The diagram shows their travel. What is the distance between the boats?

   F 12 mi

   G 15 mi

   H 18 mi

   J 24 mi
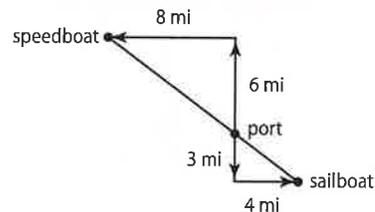
74. Michelle can read 1.5 pages per minute. How many pages can she read in two hours?

   A 90 pages   **C** 150 pages

   B 120 pages   **D** 180 pages

Spiral Review

75. ZOO A zoo has about 500 children and 750 adults visit each day. Write an expression to represent about how many visitors the zoo will have over a month. (Lesson 1-4)

Find the value of p in each equation. Then name the property that is used. (Lesson 1-3)

76. $7.3 + p = 7.3$ **77.** $12p = 1$ **78.** $1p = 4$

79. MOVING BOXES The figure shows the dimensions of the boxes Steve uses to pack. How many cubic inches can each box hold? (Lesson 0-9)
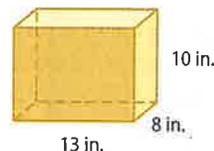
Express each percent as a fraction. (Lesson 0-6)

80. 35%   **81.** 15%   **82.** 28%

For each problem, determine whether you need an estimate or an exact answer. Then solve. (Lessons 0-6 and 0-1)

83. TRAVEL The distance from Raleigh, North Carolina, to Philadelphia, Pennsylvania, is approximately 428 miles. The average gas mileage of José's car is 45 miles per gallon. About how many gallons of gas will be needed to make the trip?

84. PART-TIME JOB An employer pays $8.50 per hour. If 20% of pay is withheld for taxes, what are the take-home earnings from 28 hours of work?

Skills Review

Find each sum or difference.

85. $1.14 + 5.6$ **86.** $4.28 - 2.4$ **87.** $8 - 6.35$

88. $\dfrac{4}{5} + \dfrac{1}{6}$ **89.** $\dfrac{2}{7} + \dfrac{3}{4}$ **90.** $\dfrac{6}{8} - \dfrac{1}{2}$

Relations

- You solved equations with one or two variables.

1 Represent relations.

2 Interpret graphs of relations.

- The deeper in the ocean you are, the greater pressure is on your body. This is because there is more water over you. The force of gravity pulls the water weight down, creating a greater pressure.

The equation that relates the total pressure of the water to the depth is $P = rgh$, where

$P =$ the pressure,
$r =$ the density of water,
$g =$ the acceleration due to gravity, and
$h =$ the height of water above you.

NewVocabulary
coordinate system
coordinate plane
x- and y-axes
origin
ordered pair
x- and y-coordinates
relation
mapping
domain
range
independent variable
dependent variable

Common Core State Standards

Content Standards
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Mathematical Practices
1 Make sense of problems and persevere in solving them.

1 **Represent a Relation** This relationship between the depth and the pressure exerted can be represented by a line on a coordinate grid.

A **coordinate system** is formed by the intersection of two number lines, the *horizontal axis* and the *vertical axis*.

The vertical axis is also called the **y-axis**.

The plane containing the x- and y-axes is the **coordinate plane**.

$(2, 3)$

Each point is named by an *ordered pair*.

The **origin**, at $(0, 0)$, is the point where the axes intersect.

The horizontal axis is also called the **x-axis**.
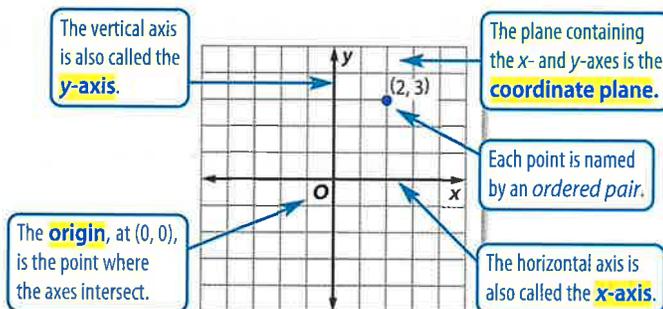
A point is represented on a graph using ordered pairs.

- An **ordered pair** is a set of numbers, or *coordinates*, written in the form (x, y).
- The x-value, called the **x-coordinate**, represents the horizontal placement of the point.
- The y-value, or **y-coordinate**, represents the vertical placement of the point.

A set of ordered pairs is called a **relation**. A relation can be represented in several different ways: as an equation, in a graph, with a table, or with a mapping.

A **mapping** illustrates how each element of the *domain* is paired with an element in the *range*. The set of the first numbers of the ordered pairs is the **domain**. The set of second numbers of the ordered pairs is the **range** of the relation. This mapping represents the ordered pairs $(-2, 4), (-1, 4), (0, 6) (1, 8)$, and $(2, 8)$.
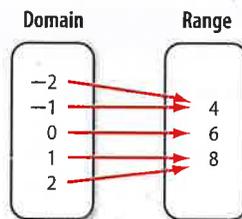
Domain | Range

-2
-1
0
1
2

4
6
8

Study the different representations of the same relation below.

| Ordered Pairs | Table | Graph | Mapping |

Ordered Pairs

(1, 2)
(−2, 4)
(0, −3)

Table

x	y
1	2
−2	4
0	−3

Graph
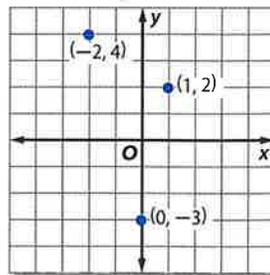
Mapping
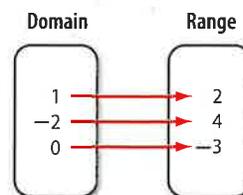
The x-values of a relation are members of the domain and the y-values of a relation are members of the range. In the relation above, the domain is {−2, 1, 0} and the range is {−3, 2, 4}.

Example 1 Representations of a Relation

a. Express {(2, 5), (−2, 3), (5, −2), (−1, −2)} as a table, a graph, and a mapping.

Table
Place the x-coordinates into the first column of the table. Place the corresponding y-coordinates in the second column of the table.

x	y
2	5
−2	3
5	−2
−1	−2

Graph
Graph each ordered pair on a coordinate plane.
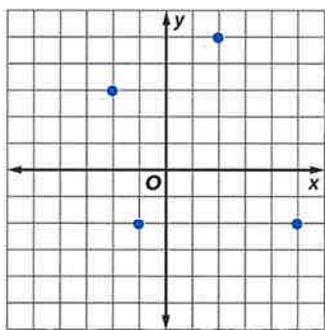
Mapping
List the x-values in the domain and the y-values in the range. Draw arrows from the x-values in the domain to the corresponding y-values in the range.
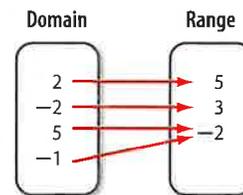
b. Determine the domain and the range of the relation.

The domain of the relation is {2, −2, 5, −1}. The range of the relation is {5, 3, −2}.

▶ **Guided**Practice

1A. Express {(4, −3), (3, 2), (−4, 1), (0, −3)} as a table, graph, and mapping.

1B. Determine the domain and range.

In a relation, the value of the variable that determines the output is called the **independent variable**. The variable with a value that is dependent on the value of the independent variable is called the **dependent variable**. The domain contains values of the independent variable. The range contains the values of the dependent variable.

Real-World Example 2 Independent and Dependent Variables

Identify the independent and dependent variables for each relation.

a. **DANCE** The dance committee is selling tickets to the Fall Ball. The more tickets that they sell, the greater the amount of money they can spend for decorations.

The number of tickets sold is the independent variable because it is unaffected by the money spent on decorations. The money spent on decorations is the dependent variable because it depends on the number of tickets sold.

b. **MOVIES** Generally, the average price of going to the movies has steadily increased over time.

Time is the independent variable because it is unaffected by the cost of attending the movies. The price of going to the movies is the dependent variable because it is affected by time.

GuidedPractice

Identify the independent and dependent variables for each relation.

2A. The air pressure inside a tire increases with the temperature.

2B. As the amount of rain decreases, so does the water level of the river.

Real-WorldLink and photo

Real-World Link

In 1948, a movie ticket cost $0.36. In 2008, the average ticket price in the United States was $7.18.

Source: National Association of Theatre Owners

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2 **Graphs of a Relation** A relation can be graphed without a scale on either axis. These graphs can be interpreted by analyzing their shape.

Example 3 Analyze Graphs

The graph represents the distance Francesca has ridden on her bike. Describe what happens in the graph.

As time increases, the distance increases until the graph becomes a horizontal line.

So, time is increasing but the distance remains constant. At this section Francesca stopped. Then she continued to ride her bike.

Bike Ride
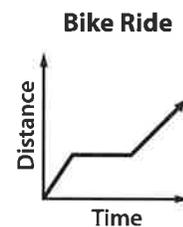
GuidedPractice

Describe what is happening in each graph.

3A. **Driving to School**

3B. **Change in Income**

42 | Lesson 1-6 | Relations

Example 1   Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. {(4, 3), (−2, 2), (5, −6)}

2. {(5, −7), (−1, 4), (0, −5), (−2, 3)}

Example 2   Identify the independent and dependent variables for each relation.

3. Increasing the temperature of a compound inside a sealed container increases the pressure inside a sealed container.

4. Mike's cell phone is part of a family plan. If he uses more minutes than his share, then there are fewer minutes available for the rest of his family.

5. Julian is buying concert tickets for himself and his friends. The more concert tickets he buys the greater the cost.

6. A store is having a sale over Labor Day weekend. The more purchases, the greater the profits.

Example 3   **CCSS** **MODELING** Describe what is happening in each graph.

7. The graph represents the distance the track team runs during a practice.

8. The graph represents revenues generated through an online store.

Example 1   Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

9. {(0, 0), (−3, 2), (6, 4), (−1, 1)}

10. {(5, 2), (5, 6), (3, −2), (0, −2)}

11. {(6, 1), (4, −3), (3, 2), (−1, −3)}

12. {(−1, 3), (3, −6), (−1, −8), (−3, −7)}

13. {(6, 7), (3, −2), (8, 8), (−6, 2), (2, −6)}

14. {(4, −3), (1, 3), (7, −2), (2, −2), (1, 5)}

Example 2   Identify the independent and dependent variables for each relation.

15 The Spanish classes are having a fiesta lunch. Each student that attends is to bring a Spanish side dish or dessert. The more students that attend, the more food there will be.

16. The faster you drive your car, the longer it will take to come to a complete stop.

Example 3   **CCSS** **MODELING** Describe what is happening in each graph.

17. The graph represents the height of a bungee jumper.

18. The graph represents the sales of lawn mowers.

CCSS MODELING Describe what is happening in each graph.

19 The graph represents the value of a rare baseball card.
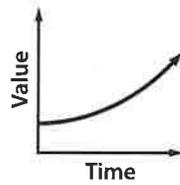
20. The graph represents the distance covered on an extended car ride.
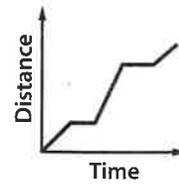
For Exercises 21–23, use the graph at the right.

21. Name the ordered pair at point *A* and explain what it represents.

22. Name the ordered pair at point *B* and explain what it represents.

23. Identify the independent and dependent variables for the relation.
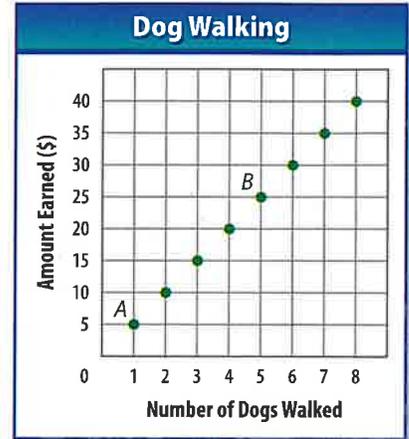
For Exercises 24–26, use the graph at the right.

24. Name the ordered pair at point *C* and explain what it represents.

25. Name the ordered pair at point *D* and explain what it represents.

26. Identify the independent and dependent variables.
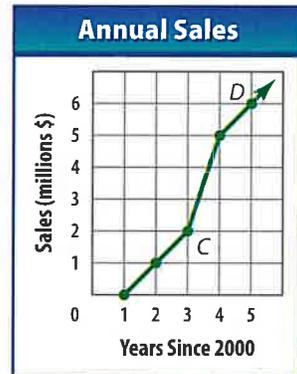
Express each relation as a set of ordered pairs. Describe the domain and range.

27.

Buying Aquarium Fish	
Number of Fish	Total Cost
1	$2.50
2	$4.50
5	$10.50
8	$16.50

28.
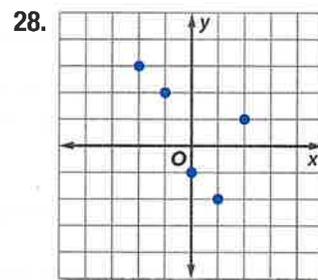
Express the relation in each table, mapping, or graph as a set of ordered pairs.

29.

x	y
4	−1
8	9
−2	−6
7	−3

30.

31.
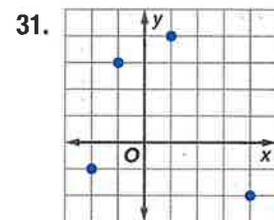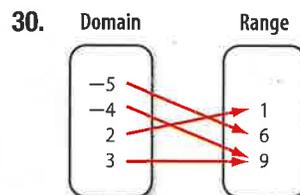
32. SPORTS In a triathlon, athletes swim 2.4 miles, bicycle 112 miles, and run 26.2 miles. Their total time includes transition time from one activity to the next. Which graph best represents a participant in a triathlon? Explain.

Graph A	Graph B	Graph C
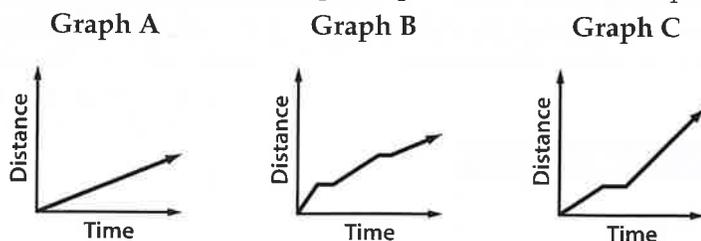
Draw a graph to represent each situation.

33. ANTIQUES A grandfather clock that is over 100 years old has increased in value from when it was first purchased.

34. CAR A car depreciates in value. The value decreases quickly in the first few years.

35. REAL ESTATE A house typically increases in value over time.

36. EXERCISE An athlete alternates between running and walking during a workout.

37 PHYSIOLOGY A typical adult has about 2 pounds of water for every 3 pounds of body weight. This can be represented by the equation $w = 2\left(\frac{b}{3}\right)$, where w is the weight of water in pounds and b is the body weight in pounds.

   a. Make a table to show the relation between body and water weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds. Round to the nearest tenth if necessary.

   b. What are the independent and dependent variables?

   c. State the domain and range, and then graph the relation.

   d. Reverse the independent and dependent variables. Graph this relation. Explain what the graph indicates in this circumstance.

H.O.T. Problems   Use Higher-Order Thinking Skills

38. OPEN ENDED Describe a real-life situation that can be represented using a relation and discuss how one of the quantities in the relation depends on the other. Then represent the relation in three different ways.

39. CHALLENGE Describe a real-world situation where it is reasonable to have a negative number included in the domain or range.

40. CCSS PRECISION Compare and contrast dependent and independent variables.

41. CHALLENGE The table presents a relation. Graph the ordered pairs. Then reverse the y-coordinate and the x-coordinate in each ordered pair. Graph these ordered pairs on the same coordinate plane. Graph the line $y = x$. Describe the relationship between the two sets of ordered pairs.

x	y
0	1
1	3
2	5
3	7

42. WRITING IN MATH Use the data about the pressure of water on page 40 to explain the difference between dependent and independent variables.

43. A school's cafeteria employees surveyed 250 students asking what beverage they drank with lunch. They used the data to create the table below.

Beverage	Number of Students
milk	38
chocolate milk	112
juice	75
water	25

What percent of the students surveyed preferred drinking juice with lunch?

A 25% **C** 35%

B 30% **D** 40%

44. Which of the following is equivalent to $6(3 - g) + 2(11 - g)$?

F $2(20 - g)$ **H** $8(5 - g)$

G $8(14 - g)$ **J** $40 - g$

45. SHORT RESPONSE Grant and Hector want to build a clubhouse at the midpoint between their houses. If Grant's house is at point G and Hector's house is at point H, what will be the coordinates of the clubhouse?
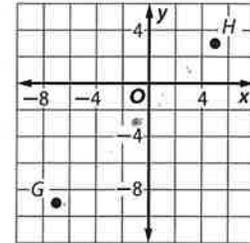
46. If $3b = 2b$, which of the following is true?

A $b = 0$

B $b = \dfrac{2}{3}$

C $b = 1$

D $b = \dfrac{3}{2}$

Solve each equation. (Lesson 1-5)

47. $6(a + 5) = 42$

48. $92 = k + 11$

49. $17 = \dfrac{45}{w} + 2$

50. HOT-AIR BALLOON A hot-air balloon owner charges $150 for a one-hour ride. If he gave 6 rides on Saturday and 5 rides on Sunday, write and evaluate an expression to describe his total income for the weekend. (Lesson 1-4)

51. LOLLIPOPS A bag of lollipops contains 19 cherry, 13 grape, 8 sour apple, 15 strawberry, and 9 orange flavored lollipops. What is the probability of drawing a sour apple flavored lollipop? (Lesson 0-11)

Find the perimeter of each figure. (Lesson 0-7)

52.

53.

54.
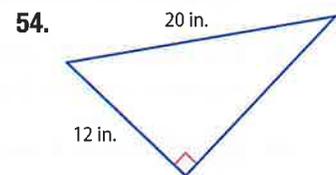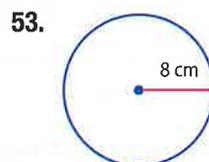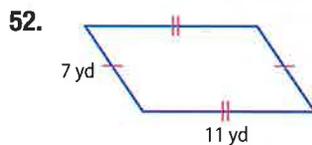
Evaluate each expression.

55. 8^2

56. $(-6)^2$

57. $(2.5)^2$

58. $(-1.8)^2$

59. $(3 + 4)^2$

60. $(1 - 4)^2$

Functions

- You solved equations with elements from a replacement set.

1. Determine whether a relation is a function.
2. Find function values.

- The distance a car travels from when the brakes are applied to the car's complete stop is the stopping distance. This includes time for the driver to react. The faster a car is traveling, the longer the stopping distance. The stopping distance is a function of the speed of the car.

Stopping Distance of a Passenger Car
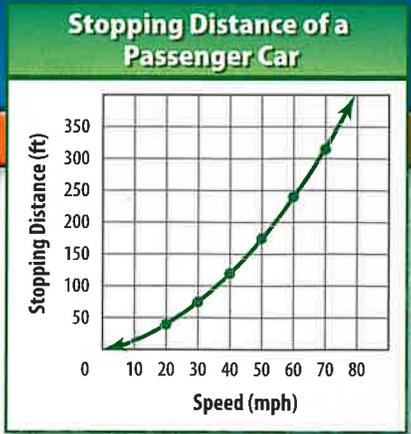
NewVocabulary
function
discrete function
continuous function
vertical line test
function notation
nonlinear function

Common Core State Standards

Content Standards
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

1 Identify Functions

A **function** is a relationship between input and output. In a function, there is exactly one output for each input.

KeyConcept Function

Words A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

Examples

Domain   Range

$-3 \longrightarrow 5$
$0 \longrightarrow 3$
$2 \longrightarrow 2$
$4 \longrightarrow -1$
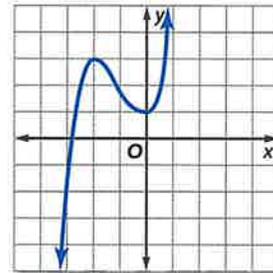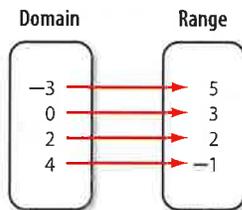
Example 1 Identify Functions

Determine whether each relation is a function. Explain.

a. Domain   Range

$-2 \longrightarrow -3$
$0 \longrightarrow 6$
$3 \longrightarrow 9$
4
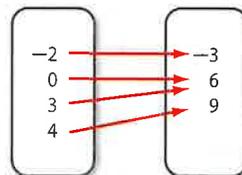
For each member of the domain, there is only one member of the range. So this mapping represents a function. It does not matter if more than one element of the domain is paired with one element of the range.

b.

Domain	1	3	5	1
Range	4	2	4	-4

The element 1 in the domain is paired with both 4 and -4 in the range. So, when x equals 1 there is more than one possible value for y. This relation is not a function.

GuidedPractice

1. $\{(2, 1), (3, -2), (3, 1), (2, -2)\}$

A graph that consists of points that are not connected is a **discrete function**. A function graphed with a line or smooth curve is a **continuous function**.

Real-World Example 2 Draw Graphs

ICE SCULPTING At an ice sculpting competition, each sculpture's height was measured to make sure that it was within the regulated height range of 0 to 6 feet. The measurements were as follows: Team 1, 4 feet; Team 2, 4.5 feet; Team 3, 3.2 feet; Team 4, 5.1 feet; Team 5, 4.8 feet.

a. Make a table of values showing the relation between the ice sculpting team and the height of their sculpture.

Team Number	1	2	3	4	5
Height (ft)	4	4.5	3.2	5.1	4.8

b. Determine the domain and range of the function.

The domain of the function is {1, 2, 3, 4, 5} because this set represents values of the independent variable. It is unaffected by the heights.

The range of the function is {4, 4.5, 3.2, 5.1, 4.8} because this set represents values of the dependent variable. This value depends on the team number.

c. Write the data as a set of ordered pairs. Then graph the data.

Use the table. The team number is the independent variable and the height of the sculpture is the dependent variable. Therefore, the ordered pairs are (1, 4), (2, 4.5), (3, 3.2), (4, 5.1), and (5, 4.8).

Because the team numbers and their corresponding heights cannot be between the points given, the points should not be connected.

Ice Sculpture Competition
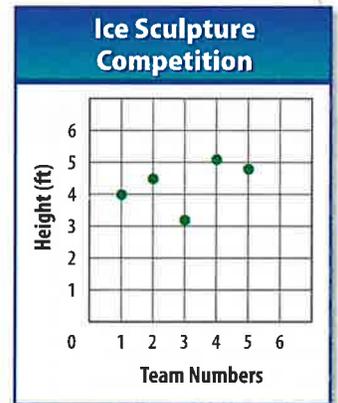
d. State whether the function is *discrete* or *continuous*. Explain your reasoning.

Because the points are not connected, the function is discrete.

GuidedPractice

2. A bird feeder will hold up to 3 quarts of seed. The feeder weighs 2.3 pounds when empty and 13.4 pounds when full.

A. Make a table that shows the bird feeder with 0, 1, 2, and 3 quarts of seed in it weighing 2.3, 6, 9.7, 13.4 pounds respectively.

B. Determine the domain and range of the function.

C. Write the data as a set of ordered pairs. Then graph the data.

D. State whether the function is *discrete* or *continuous*. Explain your reasoning.

You can use the **vertical line test** to see if a graph represents a function. If a vertical line intersects the graph more than once, then the graph is not a function. Otherwise, the relation is a function.

Function	Not a Function	Function
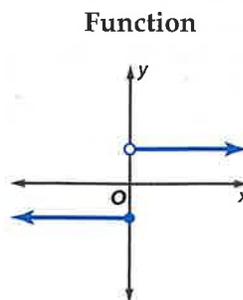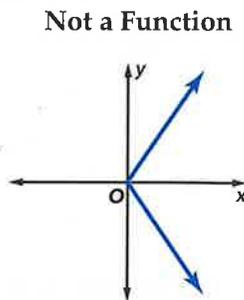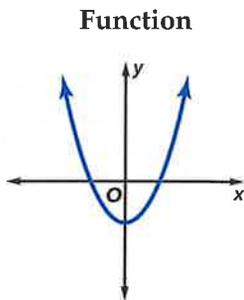
Recall that an equation is a representation of a relation. Equations can also represent functions. Every solution of the equation is represented by a point on a graph. The graph of an equation is the set of all its solutions, which often forms a curve or a line.

Example 3 Equations as Functions

Determine whether $-3x + y = 8$ is a function.

First make a table of values. Then graph the equation.

x	−1	0	1	2
y	5	4.5	11	14
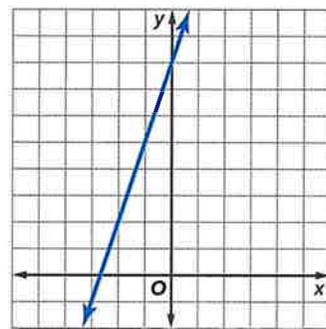
Connect the points with a smooth graph to represent all of the solutions of the equation. The graph is a line. To use the vertical line test, place a pencil at the left of the graph to represent a vertical line. Slowly move the pencil across the graph.

For any value of x, the vertical line passes through no more than one point on the graph. So, the graph and the equation represent a function.

▶ **Guided**Practice **Determine whether each relation is a function.**

3A. $4x = 8$

3B. $4x = y + 8$

A function can be represented in different ways.

Concept Summary Representations of a Function

Table	Mapping	Equation	Graph
	Domain Range		
		$f(x) = \frac{1}{2}x^2 - 1$	

Table:

x	y
−2	1
0	−1
2	1

2 Find Function Values Equations that are functions can be written in a form called **function notation**. For example, consider $y = 3x - 8$.

Equation	Function Notation
$y = 3x - 8$	$f(x) = 3x - 8$

In a function, x represents the elements of the domain, and $f(x)$ represents the elements of the range. The graph of $f(x)$ is the graph of the equation $y = f(x)$. Suppose you want to find the value in the range that corresponds to the element 5 in the domain. This is written $f(5)$ and is read f of 5. The value $f(5)$ is found by substituting 5 for x in the equation.

Example 4 Function Values

For $f(x) = -4x + 7$, find each value.

a. $f(2)$

$$f(2) = -4(2) + 7 \qquad x = 2$$
$$= -8 + 7 \qquad \text{Multiply.}$$
$$= -1 \qquad \text{Add.}$$

b. $f(-3) + 1$

$$f(-3) + 1 = [-4(-3) + 7] + 1 \qquad x = -3$$
$$= 19 + 1 \qquad \text{Simplify.}$$
$$= 20 \qquad \text{Add.}$$

▶ **Guided**Practice

For $f(x) = 2x - 3$, find each value.

4A. $f(1)$ 　　　　　　　　　**4B.** $6 - f(5)$

4C. $f(-2)$ 　　　　　　　　**4D.** $f(-1) + f(2)$

A function with a graph that is not a straight line is a **nonlinear function**.

Example 5 Nonlinear Function Values

If $h(t) = -16t^2 + 68t + 2$, find each value.

a. $h(4)$

$$h(4) = -16(4)^2 + 68(4) + 2 \qquad \text{Replace } t \text{ with 4.}$$
$$= -256 + 272 + 2 \qquad \text{Multiply.}$$
$$= 18 \qquad \text{Add.}$$

b. $2[h(g)]$

$$2[h(g)] = 2[-16(g)^2 + 68(g) + 2] \qquad \text{Replace } t \text{ with } g.$$
$$= 2(-16g^2 + 68g + 2) \qquad \text{Simplify.}$$
$$= -32g^2 + 136g + 4 \qquad \text{Distributive Property}$$

▶ **Guided**Practice

If $f(t) = 2t^3$, find each value.

5A. $f(4)$ 　　　　　　　　　**5B.** $3[f(t)] + 2$

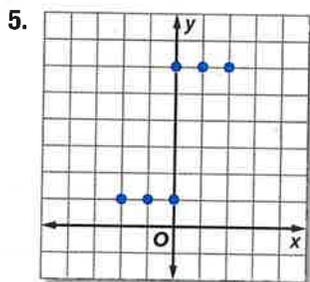**5C.** $f(-5)$ 　　　　　　　　**5D.** $f(-3) - f(1)$

Examples 1, 3 Determine whether each relation is a function. Explain.

1.
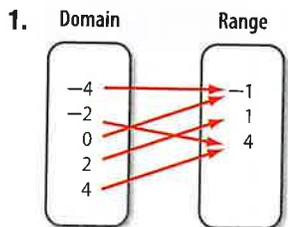
Domain   Range

2.

Domain	Range
2	6
5	7
6	9
6	10

3. $\{(2, 2), (-1, 5), (5, 2), (2, -4)\}$

4. $y = \frac{1}{2}x - 6$

5.

6.

7.

8.
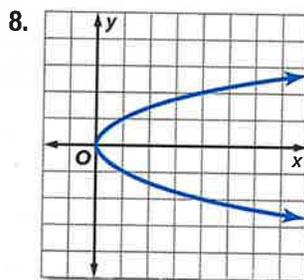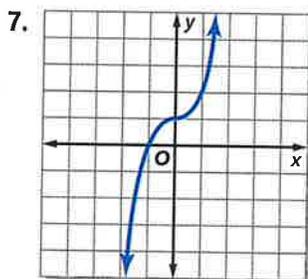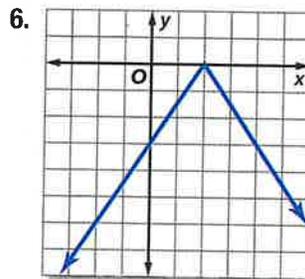
Example 2

9. SCHOOL ENROLLMENT The table shows the total enrollment in U.S. public schools.

School Year	2004–05	2005–06	2006–07	2007–08
Enrollment (in thousands)	48,560	48,710	48,948	49,091

Source: *The World Almanac*

a. Write a set of ordered pairs representing the data in the table if x is the number of school years since 2004–2005.

b. Draw a graph showing the relationship between the year and enrollment.

c. Describe the domain and range of the data.

10. **CCSS REASONING** The cost of sending cell phone pictures is given by $y = 0.25x$, where x is the number of pictures that you send and y is the cost in dollars.

a. Write the equation in function notation. Interpret the function in terms of the context.

b. Find $f(5)$ and $f(12)$. What do these values represent?

c. Determine the domain and range of this function.

Examples 4–5 If $f(x) = 6x + 7$ and $g(x) = x^2 - 4$, find each value.

11 $f(-3)$

12. $f(m)$

13. $f(r - 2)$

14. $g(5)$

15. $g(a) + 9$

16. $g(-4t)$

17. $f(q + 1)$

18. $f(2) + g(2)$

19. $g(-b)$

Example 1   Determine whether each relation is a function. Explain.

20.

Domain	Range
4	5
−6	4
3	3
−2	

21.

Domain	Range
1	5
4	6
−8	7
3	8
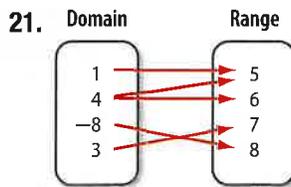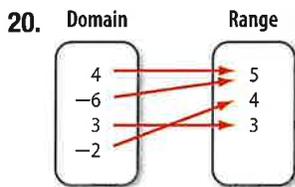
22.

Domain	Range
4	6
−5	3
6	−3
−5	5

23.

Domain	Range
−4	2
3	−5
4	2
9	−7
−3	−5

24.

25.
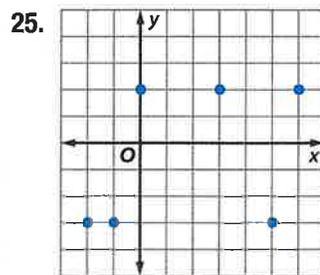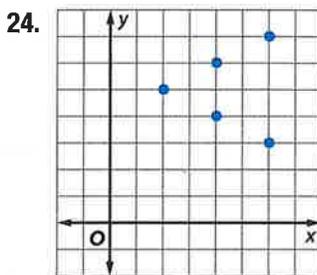
Example 2   **26.** **CCSS** **SENSE-MAKING** The table shows the median home prices in the United States, from 2007 to 2009.

Year	Median Home Price (S)
2007	234,300
2008	213,200
2009	212,200

a. Write a set of ordered pairs representing the data in the table.

b. Draw a graph showing the relationship between the year and price.

c. What is the domain and range for this data?

Example 3   Determine whether each relation is a function.

27. $\{(5, -7), (6, -7), (-8, -1), (0, -1)\}$ **28.** $\{(4, 5), (3, -2), (-2, 5), (4, 7)\}$

29. $y = -8$ **30.** $x = 15$

31. $y = 3x - 2$ **32.** $y = 3x + 2y$

Examples 4–5 If $f(x) = -2x - 3$ and $g(x) = x^2 + 5x$, find each value.

33. $f(-1)$ **34.** $f(6)$ **35.** $g(2)$

36. $g(-3)$ **37.** $g(-2) + 2$ **38.** $f(0) - 7$

39. $f(4y)$ **40.** $g(-6m)$ **41.** $f(c - 5)$

42. $f(r + 2)$ **43.** $5[f(d)]$ **44.** $3[g(n)]$

(45) **EDUCATION** The average national math test scores $f(t)$ for 17-year-olds can be represented as a function of the national science scores t by $f(t) = 0.8t + 72$.

a. Graph this function. Interpret the function in terms of the context.

b. What is the science score that corresponds to a math score of 308?

**c.** What is the domain and range of this function?

Determine whether each relation is a function.

46.

47.
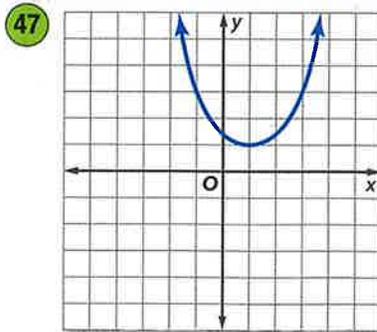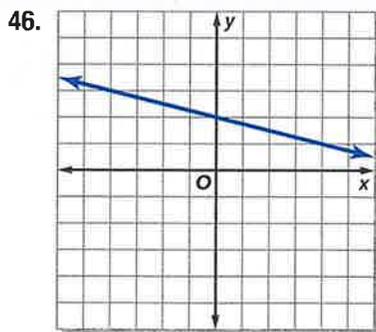
48. **BABYSITTING** Christina earns $7.50 an hour babysitting.

   a. Write an algebraic expression to represent the money Christina will earn if she works h hours.

   b. Choose five values for the number of hours Christina can babysit. Create a table with h and the amount of money she will make during that time.

   c. Use the values in your table to create a graph.

   d. Does it make sense to connect the points in your graph with a line? Why or why not?

H.O.T. Problems   Use Higher-Order Thinking Skills

49. **OPEN ENDED** Write a set of three ordered pairs that represent a function. Choose another display that represents this function.

50. **REASONING** The set of ordered pairs {(0, 1), (3, 2), (3, −5), (5, 4)} represents a relation between x and y. Graph the set of ordered pairs. Determine whether the relation is a function. Explain.

51. **CHALLENGE** Consider $f(x) = -4.3x - 2$. Write $f(g + 3.5)$ and simplify by combining like terms.

52. **WRITE A QUESTION** A classmate graphed a set of ordered pairs and used the vertical line test to determine whether it was a function. Write a question to help her decide if the same strategy can be applied to a mapping.

53. **CCSS PERSEVERANCE** If $f(3b - 1) = 9b - 1$, find one possible expression for $f(x)$.

54. **ERROR ANALYSIS** Corazon thinks $f(x)$ and $g(x)$ are representations of the same function. Maggie disagrees. Who is correct? Explain your reasoning.
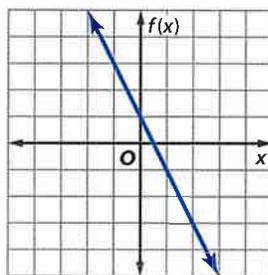
x	$g(x)$
−1	1
0	−1
1	−3
2	−5
3	−7

55. **WRITING IN MATH** How can you determine whether a relation represents a function?

56. Which point on the number line represents a number whose square is less than itself?

A A

B B

C C

D D

57. Determine which of the following relations is a function.

F $\{(-3, 2), (4, 1), (-3, 5)\}$

G $\{(2, -1), (4, -1), (2, 6)\}$

H $\{(-3, -4), (-3, 6), (8, -2)\}$

J $\{(5, -1), (3, -2), (-2, -2)\}$

58. GEOMETRY What is the value of x?

A 3 in.

B 4 in.

C 5 in.

D 6 in.

59. SHORT RESPONSE Camille made 16 out of 19 of her serves during her first volleyball game. She made 13 out of 16 of her serves during her second game. During which game did she make a greater percent of her serves?

Spiral Review

Solve each equation. (Lesson 1-5)

60. $x = \dfrac{27 + 3}{10}$

61. $m = \dfrac{3^2 + 4}{7 - 5}$

62. $z = 32 + 4(-3)$

63. SCHOOL SUPPLIES The table shows the prices of some items Tom needs. If he needs 4 glue sticks, 10 pencils, and 4 notebooks, write and evaluate an expression to determine Tom's cost. (Lesson 1-4)

School Supplies Prices	
glue stick	$1.99
pencil	$0.25
notebook	$1.85

Write a verbal expression for each algebraic expression. (Lesson 1-1)

64. $4y + 2$

65. $\dfrac{2}{3}x$

66. $a^2 b + 5$

Find the volume of each rectangular prism. (Lesson 0-9)

67.

5.4 cm

2.2 cm

3.2 cm

68.

$1\frac{1}{2}$ in.

69.

40 mm

40 mm

180 mm

Skills Review

Evaluate each expression.

70. If $x = 3$, then $6x - 5 = \underline{\ ?\ }$.

71. If $n = -1$, then $2n + 1 = \underline{\ ?\ }$.

72. If $p = 4$, then $3p + 4 = \underline{\ ?\ }$.

73. If $q = 7$, then $7q - 9 = \underline{\ ?\ }$

74. If $k = -11$, then $4k + 6 = \underline{\ ?\ }$

75. If $y = 10$, then $8y - 15 = \underline{\ ?\ }$

You can use TI-Nspire Technology to explore the different ways to represent a function.

Activity

Graph $f(x) = 2x + 3$ on the TI-Nspire graphing calculator.

Step 1 Add a new **Graphs** page.
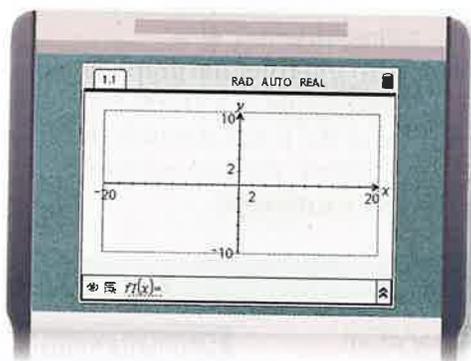
Step 2 Enter $2x + 3$ in the entry line.
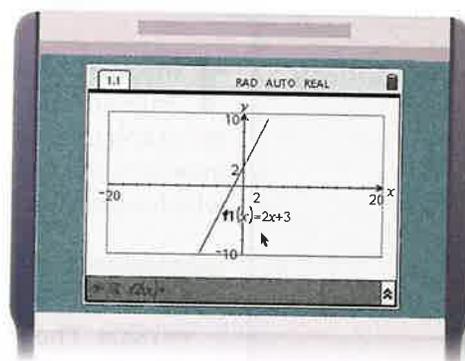
Represent the function as a table.

Step 3 Select the **Show Table** option from the **View** menu to add a table of values on the same display.
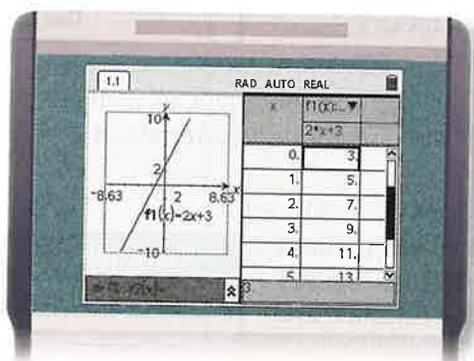
Step 4 Press **ctrl** and **tab** to toggle from the table to the graph. On the graph side, select the line and move it. Notice how the values in the table change.
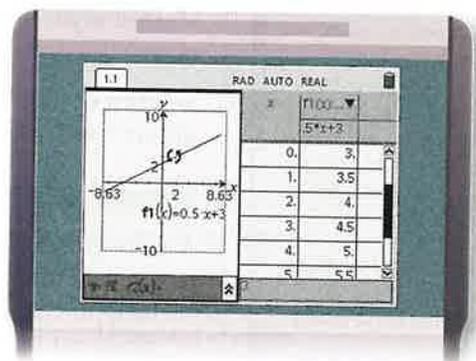
Analyze the Results

CCSS TOOLS Graph each function. Make a table of five ordered pairs that also represents the function.

1. $g(x) = -x - 3$

2. $h(x) = \frac{1}{3}x + 3$

3. $f(x) = -\frac{1}{2}x - 5$

4. $f(x) = 3x - \frac{1}{2}$

5. $g(x) = -2x + 5$

6. $h(x) = \frac{1}{5}x + 4$

Interpreting Graphs of Functions

New Vocabulary
intercept
x-intercept
y-intercept
line symmetry
positive
negative
increasing
decreasing
extrema
relative maximum
relative minimum
end behavior

Common Core State Standards

Content Standards
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Mathematical Practices
1 Make sense of problems and persevere in solving them.

---

1 Interpret Intercepts and Symmetry To interpret the graph of a function, estimate and interpret key features. The **intercepts** of a graph are points where the graph intersects an axis. The *y*-coordinate of the point at which the graph intersects the *y*-axis is called a **y-intercept**. Similarly, the *x*-coordinate of the point at which a graph intersects the *x*-axis is called an **x-intercept**.

🌐 Real-World Example 1 Interpret Intercepts

PHYSICS The graph shows the height *y* of an object as a function of time *x*. Identify the function as *linear* or *nonlinear*. Then estimate and interpret the intercepts.

Linear or Nonlinear: Since the graph is a curve and not a line, the graph is nonlinear.

y-**Intercept:** The graph intersects the *y*-axis at about $(0, 15)$, so the *y*-intercept of the graph is about 15. This means that the object started at an initial height of about 15 meters above the ground.

Height of Launched Object

x-**Intercept(s):** The graph intersects the *x*-axis at about $(7.4, 0)$, so the *x*-intercept is about 7.4. This means that the object struck the ground after about 7.4 seconds.
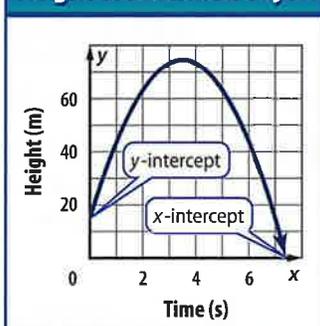
▶ Guided Practice

1. The graph shows the temperature *y* of a medical sample thawed at a controlled rate. Identify the function as *linear* or *nonlinear*. Then estimate and interpret the intercepts.
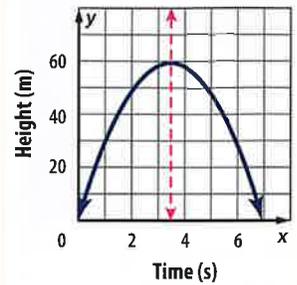
Controlled Thaw of Sample
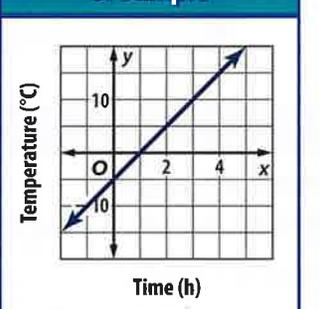
The graphs of some functions exhibit another key feature: symmetry. A graph possesses **line symmetry** in the y-axis or some other vertical line if each half of the graph on either side of the line matches exactly.

Real-World Example 2 Interpret Symmetry

PHYSICS An object is launched. The graph shows the height y of the object as a function of time x. Describe and interpret any symmetry.

The right half of the graph is the mirror image of the left half in approximately the line $x = 3.5$ between approximately $x = 0$ and $x = 7$.

In the context of the situation, the symmetry of the graph tells you that the time it took the object to go up is equal to the time it took to come down.

Height of Launched Object

> **StudyTip**
>
> Symmetry The graphs of most real-world functions do not exhibit symmetry over the entire domain. However, many have symmetry over smaller portions of the domain that are worth analyzing.

▶ **Guided**Practice

2. Describe and interpret any symmetry exhibited by the graph in Guided Practice 1.

---

2 Interpret Extrema and End Behavior Interpreting a graph also involves estimating and interpreting where the function is increasing, decreasing, positive, or negative, and where the function has any extreme values, either high or low.

KeyConcepts Positive, Negative, Increasing, Decreasing, Extrema, and End Behavior

A function is **positive** where its graph lies *above* the x-axis, and **negative** where its graph lies *below* the x-axis.
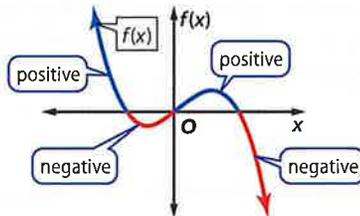
A function is **increasing** where the graph goes *up* and **decreasing** where the graph goes *down* when viewed from left to right.
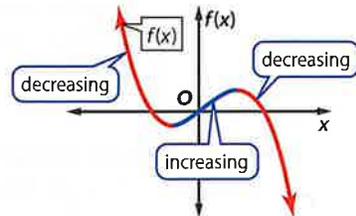

The points shown are the locations of relatively high or low function values called **extrema**. Point A is a **relative minimum**, since no other nearby points have a lesser y-coordinate. Point B is a **relative maximum**, since no other nearby points have a greater y-coordinate.
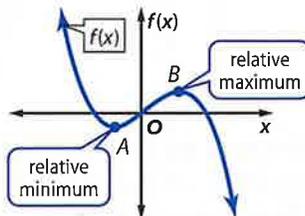
End behavior describes the values of a function at the positive and negative extremes in its domain.

As you move left, the graph goes up. As x decreases, y increases.
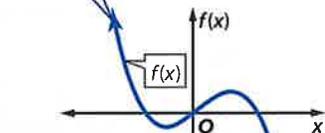
As you move right, the graph goes down. As x increases, y decreases.

> **StudyTip**
>
> End Behavior The end behavior of some graphs can be described as approaching a specific y-value. In this case, a portion of the graph looks like a horizontal line.

VIDEO GAMES U.S. retail sales of video games from 2000 to 2009 can be modeled by the function graphed at the right. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the x-coordinates of any relative extrema, and the end behavior of the graph.

U.S. Video Games Sales

Retail Sales (billions of $)
Years Since 2000

Positive: between about $x = -0.6$ and $x = 10.4$

Negative: for about $x < -0.6$ and $x > 10.4$

This means that there were positive sales between about 2000 and 2010, but the model predicts negative sales after about 2010, indicating the unlikely collapse of the industry.

Increasing: for about $x < 1.5$ and between about $x = 3$ and $x = 8$

Decreasing: between about $x = 2$ and $x = 3$ and for about $x > 8$

This means that sales increased from about 2000 to 2002, decreased from 2002 to 2003, increased from 2003 to 2008, and have been decreasing since 2008.

relative maximum
relative maximum
decreasing
relative minimum
increasing

**Relative Maximums:** at about $x = 1.5$ and $x = 8$

Relative Minimum: at about $x = 3$

The extrema of the graph indicate that the industry experienced two relative peaks in sales during this period: one around 2002 of approximately $10.5 billion and another around 2008 of approximately $22 billion. A relative low of $10 billion in sales came in about 2003.

End Behavior:
As x increases or decreases, the value of y decreases.

The end behavior of the graph indicates negative sales several years prior to 2000 and several years after 2009, which is unlikely. This graph appears to only model sales well between 2000 and 2009 and can only be used to predict sales in 2010.
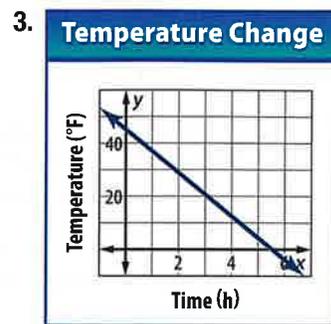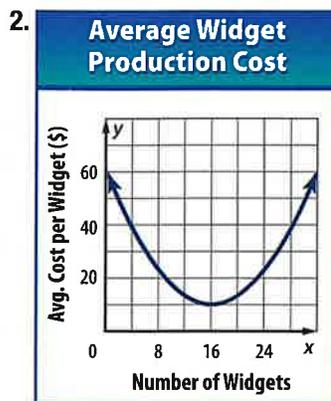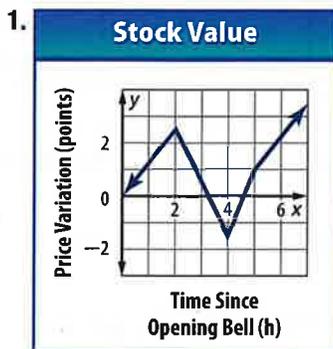

As you move left, the graph goes down.
As you move right, the graph goes down.
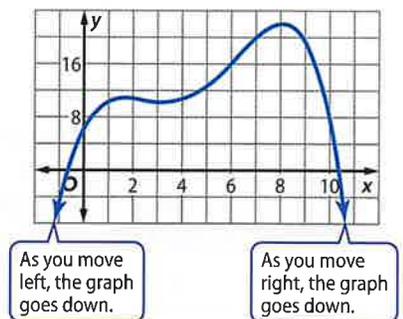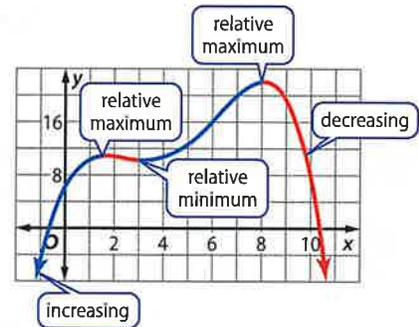
▶ **Guided**Practice

3. Estimate and interpret where the function graphed in Guided Practice 1 is positive, negative, increasing, or decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.
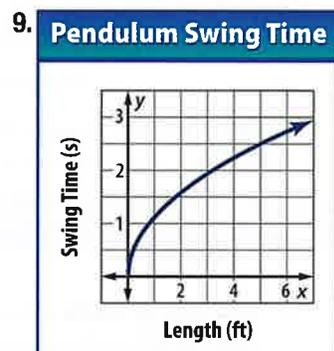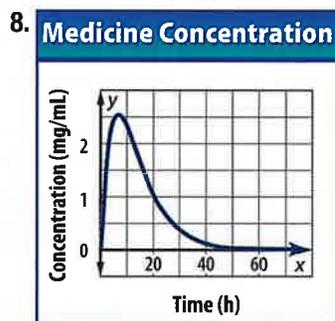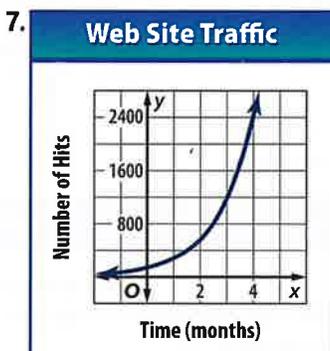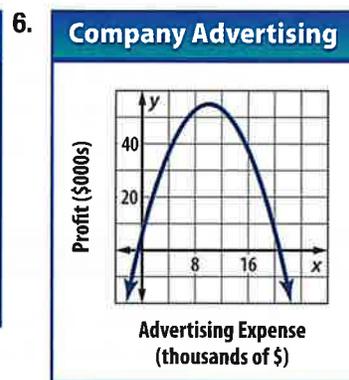
StudyTip
Constant A function is *constant* where the graph does not go up or down as the graph is viewed from left to right.
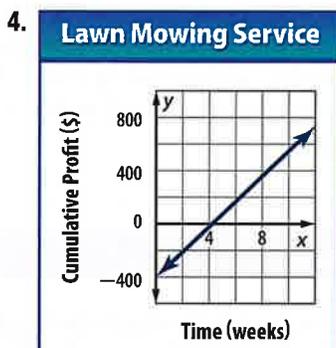
Examples 1–3 CCSS SENSE-MAKING Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the *x*-coordinate of any relative extrema, and the end behavior of the graph.

1.

Stock Value

Price Variation (points)

Time Since Opening Bell (h)

2.

Average Widget Production Cost

Avg. Cost per Widget ($)

Number of Widgets

3.

Temperature Change

Temperature (°F)

Time (h)

Practice and Problem Solving

Extra Practice is on page R1.

Examples 1–3 CCSS SENSE-MAKING Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the *x*-coordinate of any relative extrema, and the end behavior of the graph.

4.

Lawn Mowing Service

Cumulative Profit ($)

Time (weeks)

5

Vehicle Depreciation

Value ($000s)

Years Since Purchase

6.

Company Advertising

Profit ($000s)

Advertising Expense (thousands of $)

7.

Web Site Traffic

Number of Hits

Time (months)

8.

Medicine Concentration

Concentration (mg/mL)

Time (h)

9.

Pendulum Swing Time

Swing Time (s)

Length (ft)

10. FERRIS WHEEL At the beginning of a Ferris wheel ride, a passenger cart is located at the same height as the center of the wheel. The position y in feet of this cart relative to the center t seconds after the ride starts is given by the function graphed at the right. Identify and interpret the key features of the graph. (*Hint:* Look for a pattern in the graph to help you describe its end behavior.)

Ferris Wheel Ride
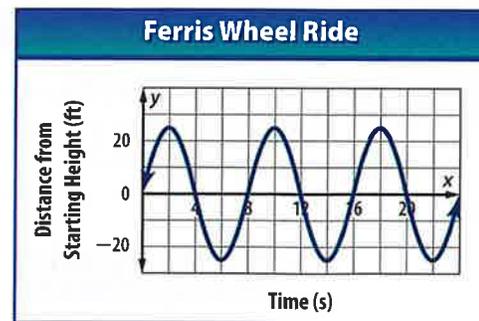
Sketch a graph of a function that could represent each situation. Identify and interpret the intercepts of the graph, where the graph is increasing and decreasing, and any relative extrema.

11. the height of a corn plant from the time the seed is planted until it reaches maturity 120 days later

12. the height of a football from the time it is punted until it reaches the ground 2.8 seconds later

13. the balance due on a car loan from the date the car was purchased until it was sold 4 years later

Sketch graphs of functions with the following characteristics.

14. The graph is linear with an x-intercept at -2. The graph is positive for $x < -2$, and negative for $x > -2$.

15. A nonlinear graph has x-intercepts at -2 and 2 and a y-intercept at -4. The graph has a relative minimum of -4 at $x = 0$. The graph is decreasing for $x < 0$ and increasing for $x > 0$.

16. A nonlinear graph has a y-intercept at 2, but no x-intercepts. The graph is positive and increasing for all values of x.

17. A nonlinear graph has x-intercepts at -8 and -2 and a y-intercept at 3. The graph has relative minimums at $x = -6$ and $x = 6$ and a relative maximum at $x = 2$. The graph is positive for $x < -8$ and $x > -2$ and negative between $x = -8$ and $x = -2$. As x decreases, y increases and as x increases, y increases.

H.O.T. Problems   Use Higher-Order Thinking Skills

18. CCSS CRITIQUE Katara thinks that all linear functions have exactly one x-intercept. Desmond thinks that a linear function can have at most one x-intercept. Is either of them correct? Explain your reasoning.

19. CHALLENGE Describe the end behavior of the graph shown.

20. REASONING Determine whether the following statement is *true* or *false*. Explain.

Functions have at most one y-intercept.

21. OPEN ENDED Sketch the graph of a function with one relative maximum and one relative minimum that could represent a real-world function. Label each axis and include appropriate units. Then identify and interpret the relative extrema of your graph.

22. WRITING IN MATH Describe how you would identify the key features of a graph described in this lesson using a table of values for a function.
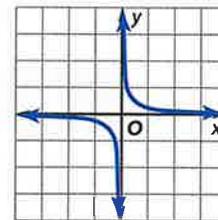
23. Which sentence best describes the end behavior of the function shown?
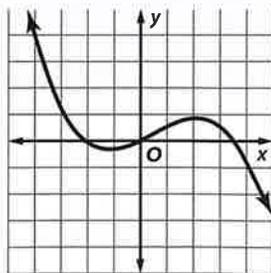
A As x increases, y increases, and as x decreases, y increases.

B As x increases, y increases, and as x decreases, y decreases.

C As x increases, y decreases, and as x decreases, y increases.

D As x increases, y decreases, and as x decreases, y decreases.

24. Which illustrates the Transitive Property of Equality?

F If $c = 1$, then $c \cdot \frac{1}{c} = 1$.

G If $c = d$ and $d = f$, then $c = f$.

H If $c = d$, then $d = c$.

J If $c = d$ and $d = c$, then $c = 1$.

25. Simplify the expression $5d(7 - 3) - 16d + 3 \cdot 2d$.

A $10d$ **C** $21d$
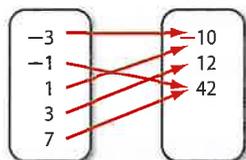
B $14d$ **D** $25d$

26. What is the probability of selecting a red card or an ace from a standard deck of cards?

F $\frac{1}{26}$ **G** $\frac{1}{2}$ **H** $\frac{7}{13}$ **J** $\frac{15}{26}$

Determine whether each relation is a function. (Lesson 1-7)
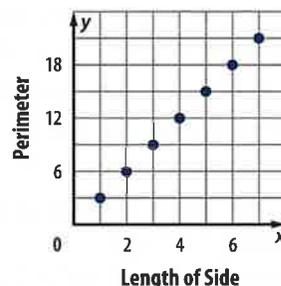
27.
Domain: $-3, -1, 1, 3, 7$ Range: $10, 12, 42$

28. $\{(0, 2), (3, 5), (0, -1), (-2, 4)\}$

29.

x	y
17	6
18	6
19	5
20	4

30. GEOMETRY Express the relation in the graph at the right as a set of ordered pairs. Describe the domain and range. (Lesson 1-6)

Equilateral Triangles

Use the Distributive Property to rewrite each expression. (Lesson 1-4)

31. $\frac{1}{2}d(2d + 6)$ **32.** $-h(6h - 1)$ **33.** $3z - 6x$

34. CLOTHING Robert has 30 socks in his sock drawer. 16 of the socks are white, 6 are black, 2 are red, and 6 are yellow. What is the probability that he randomly pulls out a black sock? (Lesson 0-11)

Evaluate each expression.

35. $(-7)^2$ **36.** 3.2^2 **37.** $(-4.2)^2$ **38.** $\left(\frac{1}{4}\right)^2$

Study Guide

KeyConcepts

Order of Operations (Lesson 1-2)

- Evalute expressions inside grouping symbols.
- Evaluate all powers.
- Multiply and/or divide in order from left to right.
- Add or subtract in order from left to right.

Properties of Equality (Lessons 1-3 and 1-4)

- For any numbers a, b, and c:

Reflexive:	$a = a$
Symmetric:	If $a = b$, then $b = a$.
Transitive:	If $a = b$ and $b = c$, then $a = c$.
Substitution:	If $a = b$, then a may be replaced by b in any expression.
Distributive:	$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
Commutative:	$a + b = b + a$ and $ab = ba$
Associative:	$(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

**Solving Equations** (Lesson 1-5)

- Apply order of operations and the properties of real numbers to solve equations.

Relations, Functions, and Interpreting Graphs of Functions (Lessons 1-6 through 1-8)

- Relations and functions can be represented by ordered pairs, a table, a mapping, or a graph.
- Use the vertical line test to determine if a relation is a function.
- End behavior describes the long-term behavior of a function on either end of its graph.
- Points where the graph of a function crosses an axis are called intercepts.
- A function is positive on a portion of its domain where its graph lies above the x-axis, and negative on a portion where its graph lies below the x-axis.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.
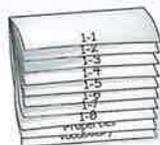
KeyVocabulary

algebraic expression (p. 5)	ordered pair (p. 40)
base (p. 5)	order of operations (p. 10)
coefficient (p. 28)	origin (p. 40)
coordinate system (p. 40)	power (p. 5)
dependent variable (p. 42)	range (p. 40)
domain (p. 40)	reciprocal (p. 17)
end behavior (p. 57)	relation (p. 40)
equation (p. 33)	relative maximum (p. 57)
exponent (p. 5)	relative minimum (p. 57)
function (p. 47)	replacement set (p. 33)
independent variable (p. 42)	simplest form (p. 27)
intercept (p. 56)	solution (p. 33)
like terms (p. 27)	term (p. 5)
line symmetry (p. 57)	variables (p. 5)
mapping (p. 40)	vertical line test (p. 49)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A <u>coordinate system</u> is formed by two intersecting number lines.

2. An <u>exponent</u> indicates the number of times the base is to be used as a factor.

3. An expression is <u>in simplest form</u> when it contains like terms and parentheses.

4. In an expression involving multiplication, the quantities being multiplied are called <u>factors</u>.

5. In a <u>function</u>, there is exactly one output for each input.

6. <u>Order of operations</u> tells us to perform multiplication before subtraction.

7. Since the product of any number and 1 is equal to the number, 1 is called the <u>multiplicative inverse</u>.

Lesson-by-Lesson Review

1-1 Variables and Expressions

Write a verbal expression for each algebraic expression.

8. $h - 7$ 9. $3x^2$ 10. $5 + 6m^3$

Write an algebraic expression for each verbal expression.

11. a number increased by 9

12. two thirds of a number d to the third power

13. 5 less than four times a number

Evaluate each expression.

14. 2^5 15. 6^3 16. 4^4

17. **BOWLING** Fantastic Pins Bowling Alley charges $2.50 for shoe rental plus $3.25 for each game. Write an expression representing the cost to rent shoes and bowl g games.

Example 1

Write a verbal expression for $4x + 9$.

nine more than four times a number x

Example 2

Write an algebraic expression for *the difference of twelve and two times a number cubed*.

Variable   Let x represent the number.

Expression $12 - 2x^3$

Example 3

Evaluate 3^4.

The base is 3 and the exponent is 4.

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ Use 3 as a factor 4 times.

$\quad = 81$ Multiply.

1-2 Order of Operations

Evaluate each expression.

18. $24 - 4 \cdot 5$ 19. $15 + 3^2 - 6$

20. $7 + 2(9 - 3)$ 21. $8 \cdot 4 - 6 \cdot 5$

22. $\left[(2^5 - 5) \div 9\right]11$ 23. $\dfrac{11 + 4^2}{5^2 - 4^2}$

Evaluate each expression if $a = 4$, $b = 3$, and $c = 9$.

24. $c + 3a$

25. $5b^2 \div c$

26. $(a^2 + 2bc) \div 7$

27. **ICE CREAM** The cost of a one-scoop sundae is $2.75, and the cost of a two-scoop sundae is $4.25. Write and evaluate an expression to find the total cost of 3 one-scoop sundaes and 2 two-scoop sundaes.

Example 4

Evaluate the expression $3(9 - 5)^2 \div 8$.

$3(9 - 5)^2 \div 8 = 3(4)^2 \div 8$ Work inside parentheses.

$\qquad\qquad\quad = 3(16) \div 8$ Evaluate 4^2.

$\qquad\qquad\quad = 48 \div 8$ Multiply.

$\qquad\qquad\quad = 6$ Divide.

Example 5

Evaluate the expression $(5m - 2n) \div p^2$ if $m = 8$, $n = 4$, $p = 2$.

$(5m - 2n) \div p^2$

$= (5 \cdot 8 - 2 \cdot 4) \div 2^2$ Replace m with 8, n with 4, and p with 2.

$= (40 - 8) \div 2^2$ Multiply.

$= 32 \div 2^2$ Subtract.

$= 32 \div 4$ Evaluate 2^2.

$= 8$ Divide.

1-3 Properties of Numbers

Evaluate each expression using properties of numbers. Name the property used in each step.

28. $18 \cdot 3(1 \div 3)$

29. $[5 \div (8 - 6)]\frac{2}{5}$

30. $(16 - 4^2) + 9$

31. $2 \cdot \frac{1}{2} + 4(4 \cdot 2 - 7)$

32. $18 + 41 + 32 + 9$

33. $7\frac{2}{5} + 5 + 2\frac{3}{5}$

34. $8 \cdot 0.5 \cdot 5$

35. $5.3 + 2.8 + 3.7 + 6.2$

36. **SCHOOL SUPPLIES** Monica needs to purchase a binder, a textbook, a calculator, and a workbook for her algebra class. The binder costs $9.25, the textbook $32.50, the calculator $18.75, and the workbook $15.00. Find the total cost for Monica's algebra supplies.

Example 6

Evaluate $6(4 \cdot 2 - 7) + 5 \cdot \frac{1}{5}$. Name the property used in each step.

$6(4 \cdot 2 - 7) + 5 \cdot \frac{1}{5}$

$= 6(8 - 7) + 5 \cdot \frac{1}{5}$	Substitution
$= 6(1) + 5 \cdot \frac{1}{5}$	Substitution
$= 6 + 5 \cdot \frac{1}{5}$	Multiplicative Identity
$= 6 + 1$	Multiplicative Inverse
$= 7$	Substitution

1-4 The Distributive Property

Use the Distributive Property to rewrite each expression. Then evaluate.

37. $(2 + 3)6$

38. $5(18 + 12)$

39. $8(6 - 2)$

40. $(11 - 4)3$

41. $-2(5 - 3)$

42. $(8 - 3)4$

Rewrite each expression using the Distributive Property. Then simplify.

43. $3(x + 2)$

44. $(m + 8)4$

45. $6(d - 3)$

46. $-4(5 - 2t)$

47. $(9y - 6)(-3)$

48. $-6(4z + 3)$

49. **TUTORING** Write and evaluate an expression for the number of tutoring lessons Mrs. Green gives in 4 weeks.

Tutoring Schedule	
Day	**Students**
Monday	3
Tuesday	5
Wednesday	4

Example 7

Use the Distributive Property to rewrite the expression $5(3 + 8)$. Then evaluate.

$5(3 + 8) = 5(3) + 5(8)$	Distributive Property
$= 15 + 40$	Multiply.
$= 55$	Simplify.

Example 8

Rewrite the expression $6(x + 4)$ using the Distributive Property. Then simplify.

$6(x + 4) = 6 \cdot x + 6 \cdot 4$	Distributive Property
$= 6x + 24$	Simplify.

Example 9

Rewrite the expression $(3x - 2)(-5)$ using the Distributive Property. Then simplify.

$(3x - 2)(-5)$

$= (3x)(-5) - (2)(-5)$	Distributive Property
$= -15x + 10$	Simplify.

Find the solution set of each equation if the replacement sets are x: $\{1, 3, 5, 7, 9\}$ and y: $\{6, 8, 10, 12, 14\}$.

50. $y - 9 = 3$ **51.** $14 + x = 21$

52. $4y = 32$ **53.** $3x - 11 = 16$

54. $\dfrac{42}{y} = 7$ **55.** $2(x - 1) = 8$

Solve each equation.

56. $a = 24 - 7(3)$

57. $z = 63 \div (3^2 - 2)$

58. **AGE** Shandra's age is four more than three times Sherita's age. Write an equation for Shandra's age. Solve if Sherita is 3 years old

Example 10

Solve the equation $5w - 19 = 11$ if the replacement set is w: $\{2, 4, 6, 8, 10\}$.

Replace w in $5w - 19 = 11$ with each value in the replacement set.

w	$5w - 19 = 11$	True or False?
2	$5(2) - 19 = 11$	false
4	$5(4) - 19 = 11$	false
6	$5(6) - 19 = 11$	true
8	$5(8) - 19 = 11$	false
10	$5(10) - 19 = 11$	false

Since the equation is true when $w = 6$, the solution of $5w - 19 = 11$ is $w = 6$.
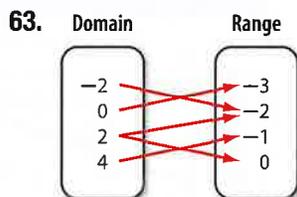
Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

59. $\{(1, 3), (2, 4), (3, 5), (4, 6)\}$

60. $\{(-1, 1), (0, -2), (3, 1), (4, -1)\}$

61. $\{(-2, 4), (-1, 3), (0, 2), (-1, 2)\}$

Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

62.

x	y
5	3
3	-1
1	2
-1	0

63.

Domain: -2, 0, 2, 4 Range: -3, -2, -1, 0

64. **GARDENING** On average, 7 plants grow for every 10 seeds of a certain type planted. Make a table to show the relation between seeds planted and plants growing for 50, 100, 150, and 200 seeds. Then state the domain and range and graph the relation.

Example 11

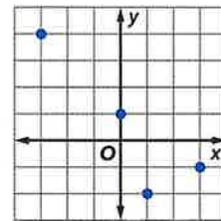Express the relation $\{(-3, 4), (1, -2), (0, 1), (3, -1)\}$ as a table, a graph, and a mapping.

Table

Place the x-coordinates into the first column. Place the corresponding y-coordinates in the second column.

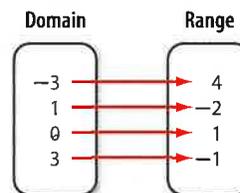| x	y
-3	4
1	-2
0	1
3	-1

Graph

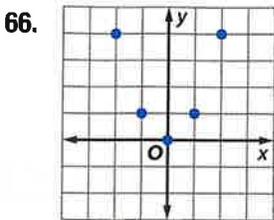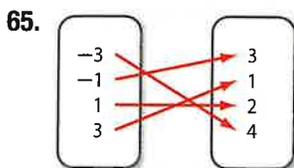Graph each ordered pair on a coordinate plane.

Mapping

List the x-values in the domain and the y-values in the range. Draw arrows from the x-values in set X to the corresponding y-values in set Y.

Domain: -3, 1, 0, 3 Range: 4, -2, 1, -1

1-7 Functions

Determine whether each relation is a function.

65.

66.

67. {(8, 4), (6, 3), (4, 2), (2, 1), (6, 0)}

If $f(x) = 2x + 4$ and $g(x) = x^2 - 3$, find each value.

68. $f(-3)$　　**69.** $g(2)$　　**70.** $f(0)$
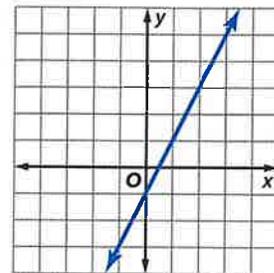
71. $g(-4)$　　**72.** $f(m + 2)$　　**73.** $g(3p)$

74. GRADES A teacher claims that the relationship between number of hours studied for a test and test score can be described by $g(x) = 45 + 9x$, where x represents the number of hours studied. Graph this function.

Example 12

Determine whether $2x - y = 1$ represents a function.

First make a table of values. Then graph the equation.

x	y
−1	−3
0	−1
1	1
2	3
3	5

Using the vertical line test, it can be shown that $2x - y = 1$ does represent a function.

1-8 Interpreting Graphs of Functions

75. Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.
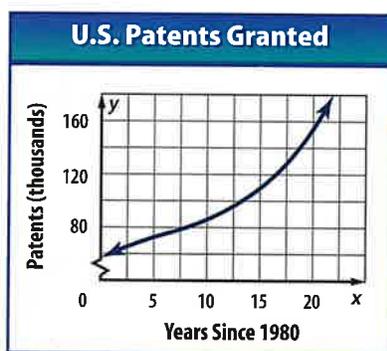
Example 13

POPULATION The population of Haiti from 1994 to 2010 can be modeled by the function graphed below. Estimate and interpret where the function is increasing, and decreasing, the x-coordinates of any relative extrema, and the end behavior of the graph.
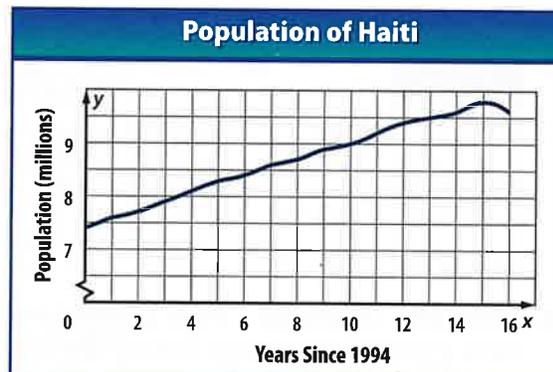
The population increased from 1994 to 2009 and decreased from 2009 to 2010. The relative maximum of the graph indicates that the population peaked in 2009.

As x increases or decreases, y decreases. The end behavior indicates a decline in population from 2009 to 2010.

Write an algebraic expression for each verbal expression.

1. six more than a number

2. twelve less than the product of three and a number

3. four divided by the difference between a number and seven

Evaluate each expression.

4. $32 \div 4 + 2^3 - 3$

5. $\dfrac{(2 \cdot 4)^2}{7 + 3^2}$

6. **MULTIPLE CHOICE** Find the value of the expression $a^2 + 2ab + b^2$ if $a = 6$ and $b = 4$.

    A 68

    B 92

    C 100

    D 121

Evaluate each expression. Name the property used in each step.

7. $13 + (16 - 4^2)$

8. $\frac{2}{9}[9 \div (7 - 5)]$

9. $37 + 29 + 13 + 21$

Rewrite each expression using the Distributive Property. Then simplify.

10. $4(x + 3)$

11. $(5p - 2)(-3)$

12. **MOVIE TICKETS** A company operates three movie theaters. The chart shows the typical number of tickets sold each week at the three locations. Write and evaluate an expression for the total typical number of tickets sold by all three locations in four weeks.

Location	Tickets Sold
A	438
B	374
C	512

Find the solution of each equation if the replacement sets are x: {1, 3, 5, 7, 9} and y: {2, 4, 6, 8, 10}.
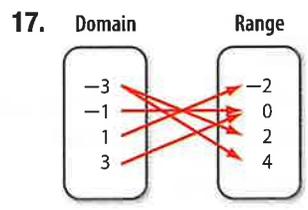
13. $3x - 9 = 12$

14. $y^2 - 5y - 11 = 13$

15. **CELL PHONES** The ABC Cell Phone Company offers a plan that includes a flat fee of $29 per month plus a $0.12 charge per minute. Write an equation to find C, the total monthly cost for m minutes. Then solve the equation for $m = 50$.

Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

16.

x	y
−2	4
1	2
3	0
4	−2

17.

18. **MULTIPLE CHOICE** Determine the domain and range for the relation {(2, 5), (−1, 3), (0, −1), (3, 3), (−4, −2)}.

   F D: {2, −1, 0, 3, −4}, R: {5, 3, −1, 3, −2}

    G D: {5, 3, −1, 3, −2}, R: {2, −1, 0, 3, 4}

    H D: {0, 1, 2, 3, 4}, R: {−4, −3, −2, −1, 0}

    J D: {2, −1, 0, 3, −4}, R: {2, −1, 0, 3, 4}

19. Determine whether the relation {(2, 3), (−1, 3), (0, 4), (3, 2), (−2, 3)} is a function.

If $f(x) = 5 - 2x$ and $g(x) = x^2 + 7x$, find each value.

20. $g(3)$

21. $f(-6y)$

22. Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.
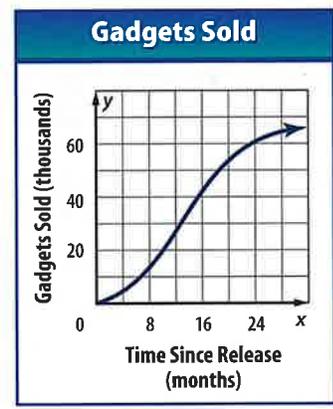
Eliminate Unreasonable Answers

You can eliminate unreasonable answers to help you find the correct one when solving multiple choice test items. Doing so will save you time by narrowing down the list of possible correct answers.

Strategies for Eliminating Unreasonable Answers

Step 1

Read the problem statement carefully to determine exactly what you are being asked to find.

Ask yourself:

- What am I being asked to solve?

- What format (i.e., fraction, number, decimal, percent, type of graph) will the correct answer be?

- What units (if any) will the correct answer have?

Step 2

Carefully look over each possible answer choice and evaluate for reasonableness.

- Identify any answer choices that are clearly incorrect and eliminate them.

- Eliminate any answer choices that are not in the proper format.

- Eliminate any answer choices that do not have the correct units.

Step 3

Solve the problem and choose the correct answer from those remaining. Check your answer.

Standardized Test Example

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

Jason earns 8.5% commission on his weekly sales at an electronics retail store. Last week he had $4200 in sales. What was his commission for the week?

A $332

C $425

B $357

D $441

Using mental math, you know that 10% of $4200 is $420. Since 8.5% is less than 10%, you know that Jason earned less than $420 in commission for his weekly sales. So, answer choices C and D can be eliminated because they are greater than $420. The correct answer is either A or B.

$4200 \times 0.085 = \$357$

So, the correct answer is B.

Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

1. Coach Roberts expects 35% of the student body to turn out for a pep rally. If there are 560 students, how many does Coach Roberts expect to attend the pep rally?

  A 184

  B 196

  C 214

  D 390

2. Jorge and Sally leave school at the same time. Jorge walks 300 yards north and then 400 yards east. Sally rides her bike 600 yards south and then 800 yards west. What is the distance between the two students?
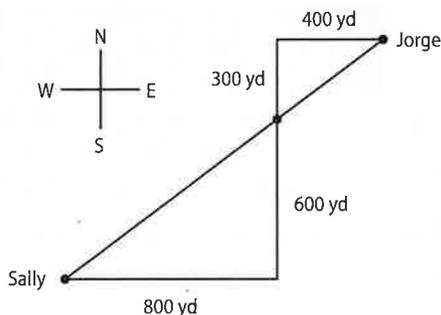
  F 500 yd

  G 750 yd

  H 1,200 yd

  J 1,500 yd

3. What is the range of the relation below?

$\{(1, 2), (3, 4), (5, 6), (7, 8)\}$

  A all real numbers

  B all even numbers

  C $\{2, 4, 6, 8\}$

  D $\{1, 3, 5, 7\}$

4. The expression $3n + 1$ gives the total number of squares needed to make each figure of the pattern where n is the figure number. How many squares will be needed to make Figure 9?
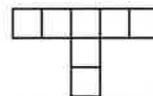

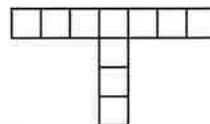Figure 1

Figure 2

Figure 3

  F 28 squares

  G 32.5 squares
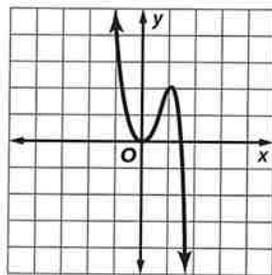
  H 56 squares

  J 88.5 squares

5. The expression $3x - (2x + 4x - 6)$ is equivalent to

  A $-3x - 6$ **C** $3x + 6$

  B $-3x + 6$ **D** $3x - 6$

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Evaluate the expression 2^6.

  A 12

  B 32

  C 64

  D 128

2. Which sentence best describes the end behavior of the function shown?

  F As x increases, y increases, and as x decreases, y increases.

  G As x increases, y increases, and as x decreases, y decreases.

  H As x increases, y decreases, and as x decreases, y increases.

  J As x increases, y decreases, and as x decreases, y decreases.

3. Let y represent the number of yards. Which algebraic expression represents the number of feet in y?

  A $y - 3$

  B $y + 3$

  C $3y$

  D $\frac{3}{y}$

4. What is the domain of the following relation?

$$\{(1, 3), (-6, 4), (8, 5)\}$$

  F $\{3, 4, 5\}$

  G $\{-6, 1, 8\}$

  H $\{-6, 1, 3, 4, 5, 8\}$

  J $\{1, 3, 4, 5, 8\}$

5. The table shows the number of some of the items sold at the concession stand at the first day of a soccer tournament. Estimate how many items were sold from the concession stand throughout the four days of the tournament.

Concession Sales Day 1 Results	
Item	**Number Sold**
Popcorn	78
Hot Dogs	80
Chip	48
Sodas	51
Bottled Water	92

  A 1350 items   **C** 1450 items

  B 1400 items   **D** 1500 items

6. There are 24 more cars than twice the number of trucks for sale at a dealership. If there are 100 cars for sale, how many trucks are there for sale at the dealership?

  F 28   **H** 34

  G 32   **J** 38

7. Refer to the relation in the table below. Which of the following values would result in the relation *not* being a function?

x	-6	-2	0	?	3	5
y	-1	8	3	-3	4	0

  A -1

  B 3

  C 7

  D 8

> **Test-Taking Tip**
>
> **Question 7** A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

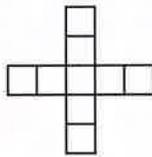**8.** The edge of each box below is 1 unit long.

Figure 1

Figure 2

Figure 3

  a. Make a table showing the perimeters of the first 3 figures in the pattern.

  b. Look for a pattern in the perimeters of the shapes. Write an algebraic expression for the perimeter of Figure n.

  c. What would be the perimeter of Figure 10 in the pattern?

9. The table shows the costs of certain items at a corner hardware store.

Item	Cost
box of nails	$3.80
box of screws	$5.25
claw hammer	$12.95
electric drill	$42.50

  a. Write two expressions to represent the total cost of 3 boxes of nails, 2 boxes of screws, 2 hammers, and 1 electric drill.

  b. What is the total cost of the items purchased?

10. GRIDDED RESPONSE Evaluate the expression below.

$$\frac{5^3 \cdot 4^2 - 5^2 \cdot 4^3}{5 \cdot 4}$$

**11.** Use the equation $y = 2(4 + x)$ to answer each question.

  a. Complete the table for each value of x.

  b. Plot the points from the table on a coordinate grid. What do you notice about the points?

  c. Make a conjecture about the relationship between the change in x and the change in y.

x	y
1	
2	
3	
4	
5	
6	

Record your answers on a sheet of paper. Show your work.

12. The volume of a sphere is four-thirds the product of π and the radius cubed.
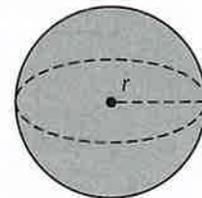
  a. Write an expression for the volume of a sphere with radius r.

  b. Find the volume of a sphere with a radius of 6 centimeters. Describe how you found your answer.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	1-2	1-8	1-1	1-6	1-4	1-5	1-7	1-5	1-3	1-2	1-4	1-1