



Then

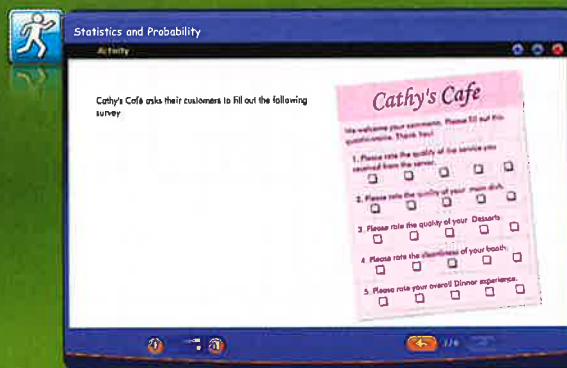
- You calculated simple probability.

Now

- In this chapter, you will:
 - Design surveys and evaluate results.
 - Use permutations and combinations.
 - Find probabilities of compound events.
 - Design and use simulations.

Why? ▲

- RESTAURANTS** A restaurant may ask their customers to complete a survey about their visit. The survey data can be analyzed using statistical methods. The restaurant staff can learn more about their customers and how to improve their experiences in the restaurant.



connectED.mcgraw-hill.com Your Digital Math Portal

Animation



Vocabulary



eGlossary



Personal Tutor



Virtual Manipulatives



Graphing Calculator



Audio



Foldables



Self-Check Practice



Worksheets



Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Determine the probability of each event if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, 3 blue cubes, and 1 green cube.

- $P(\text{red})$
- $P(\text{blue})$
- $P(\text{not red})$
- $P(\text{white})$
- Jim rolls a die with 6 sides. What is the probability of rolling a 5?
- Malika spins a spinner that is divided into 8 equal sections. Each section is a different color, including blue. What is the probability the spinner lands on the blue section?

Find each product.

- $\frac{5}{4} \cdot \frac{2}{3}$
- $\frac{4}{19} \cdot \frac{7}{20}$
- $\frac{4}{32} \cdot \frac{7}{32}$
- $\frac{5}{12} \cdot \frac{6}{11}$
- $\frac{56}{100} \cdot \frac{24}{100}$
- $\frac{9}{34} \cdot \frac{17}{27}$

Write each fraction as a percent. Round to the nearest tenth.

- $\frac{14}{17}$
- $\frac{7}{8}$
- $\frac{107}{125}$
- $\frac{625}{1024}$
- SHOPPERS** At the mall, 700 of the 2000 people shopping were under the age of 21. What percent of the shoppers were under 21?

QuickReview



Example 1

Determine the probability of selecting a green cube if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, and 1 green cube.

There is 1 green cube and a total of 11 cubes in the bag.

$$\frac{1}{11} = \frac{\text{number of green cubes}}{\text{total number of cubes}}$$

The probability of selecting a green cube is $\frac{1}{11}$.

Example 2

Find $\frac{4}{5} \cdot \frac{3}{4}$.

$$\begin{aligned} \frac{4}{5} \cdot \frac{3}{4} &= \frac{4 \cdot 3}{5 \cdot 4} \\ &= \frac{12}{20} \\ &= \frac{3}{5} \end{aligned}$$

Multiply the numerators and the denominators.

Simplify.

Rename in simplest form.

Example 3

Write the fraction $\frac{33}{80}$ as a percent. Round to the nearest tenth.

$$\frac{33}{80} \approx 0.413$$

Simplify and round.

$$0.413 \cdot 100 = 41.3$$

Multiply the decimal by 100.

$\frac{33}{80}$ written as a percent is about 41.3%.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.



Get Started on the Chapter

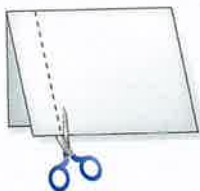
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 12. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer



Statistics and Probability Make this Foldable to help you organize your Chapter 12 notes about Statistics and Probability. Begin with 8 sheets of $8\frac{1}{2}$ " by 11" paper.

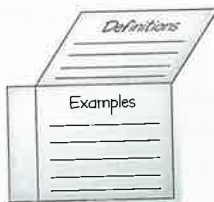
- 1** **Fold** each sheet of paper in half. Cut 1 inch from the end to the fold. Then cut 1 inch along the fold.



- 2** **Label** 7 of the 8 sheets with the lesson number and title.



- 3** **Label** the inside of each sheet with Definitions and Examples.



- 4** **Stack** the sheets. Staple along the left side. Write the title of the chapter on the first page.



NewVocabulary



English		Español
population	p. 747	población
sample	p. 747	muestra
bias	p. 748	tendencia
observational study	p. 749	estudio de observación
experiment	p. 749	experimento
statistic	p. 757	estadística
parameter	p. 757	parámetro
standard deviation	p. 759	desviación estándar
distribution	p. 764	distribución
symmetric distribution	p. 764	distribución simétrica
theoretical probability	p. 780	probabilidad teórica
experimental probability	p. 780	probabilidad experimental
simulation	p. 781	simulación
permutation	p. 786	permutación
combination	p. 787	combinación
compound event	p. 793	evento compuesto
independent events	p. 793	eventos independientes
dependent events	p. 794	eventos dependientes
mutually exclusive	p. 795	mutuamente exclusivos
random variable	p. 803	variable aleatoria
probability distribution	p. 804	distribución de probabilidad
expected value	p. 805	valor esperado

ReviewVocabulary



probability **probilidad** the ratio of favorable outcomes to the total possible outcomes

sample space **espacio muestral** the list of all possible outcomes

Samples and Studies



Then

You displayed results from studies.

Now

- 1 Classify and analyze samples.
- 2 Classify and analyze studies.

Why?



A high school principal is trying to determine whether the school should change its mascot and decides to survey some of the students.



New Vocabulary

- population
- sample
- simple random sample
- systematic sample
- self-selected sample
- convenience sample
- stratified sample
- bias
- survey
- observational study
- experiment

1 Sampling A **population** consists of all of the members of a group of interest. Since it may be impractical to examine every member of a population, a **sample** or subset is sometimes selected to represent the population. The sample can then be analyzed to draw conclusions about the entire population. For instance, in the example above, the population is the entire student body. The sample is the students who participate in the survey.

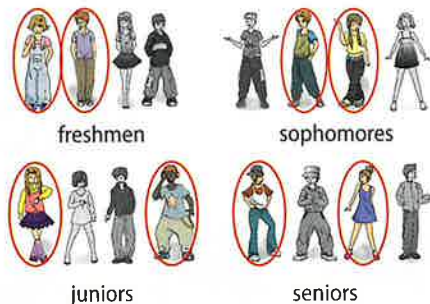
In a **simple random sample**, each member of the population has an equal chance of being selected as part of the sample. The principal can generate a simple random sample by randomly drawing student ID numbers from a hat that contains the ID number of every student.

Some other types of samples are described below.



In a **systematic sample**, members are selected according to a specified interval from a random starting point, such as selecting every third student.

In a **self-selected sample**, members volunteer to be included in the sample.



In a **convenience sample**, members that are readily available or easy to reach are selected, such as the students on a particular bus.

In a **stratified sample**, the population is first divided into similar, nonoverlapping groups. Members are then randomly selected from each group.

Factors that can influence the type of sample used include cost, time, and the availability of willing participants.



Real-World Example 1 Classify a Random Sample

200S Animals in a zoo are divided by species. Then two animals are selected at random from each group to have their blood tested.

a. Identify the sample, and suggest a population from which it was selected.

Sample: the two animals selected from each species

Population: all of the animals in the zoo

b. Classify the sample as *simple*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain your reasoning.

This is a stratified sample. The animals were divided into categories before there was a random selection.

Guided Practice

Identify each sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain your reasoning.

1A. CONTESTS Refer to the information at the left. After the competition, contestants were asked to complete an online survey about possible future categories.

1B. FOOD At a popular diner, the manager checks the quality of the burgers every 20 minutes, starting at a randomly selected time.



Real-WorldLink

The Jamaican Jerk Festival is held annually in Pembroke Pines, Florida. Cooks may enter a competition where each entrant must prepare any three dishes from the following categories: jerked pork, jerked chicken, jerked seafood, or other.

Source: Jamaican Jerk Festival

Sample data are often used to estimate a characteristic of a population. Therefore, a sample should be selected so that it is representative of the entire population. Also, the larger the sample size, or the more samples taken, the more closely it approximates the population.

A **bias** is an error that results in a misrepresentation of a population. If a sample favors one conclusion over another, the sample is biased and the data are invalid.

Example 2 Biased and Unbiased Samples

Identify each sample as *biased* or *unbiased*. Explain your reasoning.

a. **MUSIC** Every fifth person coming into a grocery store is asked to name their favorite radio station.

The sample is unbiased because the participants are randomly selected. The fact that the sample is selected at a grocery store has no bearing on the conclusion because many different kinds of people shop at grocery stores.

b. **MUSIC** Every fifth person at the Country Music Showcase is asked to name their favorite radio station.

The sample is biased because the participants are selected at a country music show, and therefore people may be more likely to select a country music station.

Guided Practice

2A. POLITICS A journalist surveys 15 student members of the Young Republicans to determine the overall political opinions of the student body.

2B. SHOES A shoe store conducts a study of which shoes are most popular. The store surveys every third girl and boy that enters the store.



2 Studies After a sample is selected, information can be collected using one of the following study types.

Study Tip

Census A census is a survey in which every member of the population is questioned. So there is no sample when a census is conducted.

Key Concept Study Types		
Type	Definition	Example
survey	Data are collected from responses given by a sample regarding their characteristics, behaviors, or opinions.	To determine whether the student body is happy with the spring dance theme, the dance committee asks a sample of students for their opinion.
observational study	Members of a sample are measured or observed without being affected by the study.	A gaming company watches a group of teens play a selection of video games and notes the ones they play the most.
experiment	<p>The sample is divided into two groups:</p> <ul style="list-style-type: none"> an <i>experimental group</i> that undergoes a change, and a <i>control group</i> that does not undergo the change. <p>The effect on the experimental group is then compared to the control group.</p>	A teacher administers a paper-and-pencil test to one of two Algebra classes. He administers a computer-based test that covers the same material to the other class. The teacher compares the scores and the completion times of the two classes.

Factors that can influence the type of study conducted are cost, time, and the objective of the study.



Example 3 Classify Study Techniques

Determine whether each situation describes a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

- a. **CHARITY** A local charity is interested in finding out whether residents would use a curbside pick-up service for donations. They distribute 30 questionnaires to people living in the neighborhood.

This is a survey. The data are gathered from responses given by members of the sample.

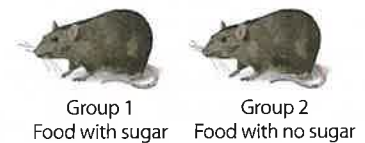
- b. **ADVERTISING** A company shows five different commercials that advertise the same product to a group of students. The company records the students' reactions to each.

This is an observational study. The data are gathered from observing the students.

Guided Practice

Determine whether each situation describes a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

- 3A. **RESEARCH** Scientists study the behavior of two groups of rats to determine their reaction to sugar.



- 3B. **RECREATION** The city council wants to build a community recreational center. They call 1000 random citizens asking if they would consider paying a fee to join if the center is built.



left: Greenberndane/fotostock

The design of a survey can introduce bias. Survey questions should not:

- be confusing,
- encourage the members of the sample to answer a certain way,
- cause a strong reaction, or
- address more than one issue at a time.



Example 4 Biased and Unbiased Survey Questions

Identify each survey question as *biased* or *unbiased*. If biased, explain your reasoning.

- a. Is your favorite ice cream flavor plain vanilla or delicious chocolate?

This question is biased. It limits the responses to vanilla or chocolate and encourages the participant to answer a certain way by using the adjectives *plain* and *delicious*.

- b. What type(s) of reading material(s) do you enjoy?

This question is unbiased. It does not encourage participants to answer a certain way, and it is clearly stated.

Guided Practice

Identify each survey question as *biased* or *unbiased*. If biased, explain your reasoning.

- 4A. Do you like animals, and would you ever consider having a dog, a cat, or a hamster as a pet?
- 4B. How much time do you spend working on school work each night?



Real-World Career

Dog Trainer Dog trainers may seek professional certification by passing an examination after compiling 300 hours experience in dog training, acquiring a high school diploma, and obtaining three letters of reference.

Source: Certification Council for Professional Dog Trainers

Bias can also be introduced in the design of an experiment. To avoid this, members of the control group and the experimental group should be randomly selected. Also, the only difference between the two groups should be the change being observed.



Real-World Example 5 Biased and Unbiased Experimental Designs

TRAINER A dog trainer wants to test training speed with a new method. She selects three terriers as the control group and teaches a trick using her normal method. She selects three greyhounds as the experimental group and teaches the same trick using the new method. She then compares the training times. Identify the experiment as *biased* or *unbiased*. If biased, explain your reasoning.

This experiment is biased. A terrier may be more apt to learn tricks than a greyhound, or vice versa. The bias could be corrected by using a random sample consisting entirely of terriers or greyhounds.

Guided Practice

5. **TRACK** A track coach wants to test whether new shoes that claim to increase speed will produce faster times for his runners. He randomly selects five runners as the control group and records their times throughout the season using their normal shoes. He randomly selects five runners as the experimental group and records their times throughout the season using the new shoes. At the end of the season, he compares any improvements made by each group. Identify the experiment as *biased* or *unbiased*. If biased, explain your reasoning.





Example 1 Identify each sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain your reasoning.

- 1. SHOWER** At a bridal shower, a sticker was placed on the bottom of three random plates. The guests who receive the stickered plates will win a prize.
- 2. BOOK CLUB** Mr. Peterson surveys the students in his English classes to gauge the student body's interest in forming a book club.

Example 2 Identify each sample as *biased* or *unbiased*. Explain your reasoning.

- 3. ELECTION** A group of students stands at the door of the school and asks every tenth student who they would vote for in the upcoming class elections and why.
- 4. SHOPPING** Every fifteenth shopper at a clothing store is asked what they would want most for their birthday.

Example 3 Determine whether each situation describes a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

- 5. TELEVISION** A television network wants to conduct a cartoon marathon. To choose the episodes, they mail a questionnaire to people selected at random throughout the country.
- 6. FOOD** A frozen food company is considering creating frozen meals with tofu instead of meat. At a testing, they randomly give half of a group of 100 people the meals with meat and the other half the same meals with tofu and ask the people how they like the meals.

Example 4 Identify each survey question as *biased* or *unbiased*. If biased, explain your reasoning.

- 7** What are you planning to do over summer vacation?
- Do you think we should serve mouth-watering steak or chicken?
- Don't you think Suzanne should be the class president?
- What type of music do you listen to?

Example 5 Identify the experiment as *biased* or *unbiased*. If biased, explain your reasoning.

- 11. POOLS** A national pool manufacturer wants to determine if a new advertising strategy will increase sales. The company continues to use the normal advertising strategy in its stores located in Indiana, Pennsylvania, and Kentucky. The company uses the new advertising strategy in Florida, Louisiana, and Georgia. The company then compares the sales.
- 12. EDUCATION** A school district wants to determine if having school in session year-round will improve the performance of students. They select one of their schools to be in session year-round and compare the test scores of those students with the test scores of the other students in the district.



Example 1 Identify each sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain your reasoning.

- SPORTS CARDS** Greg divides his baseball cards by teams. Then he randomly selects four cards from each team and records the players' RBIs.
- CARS** The service manager at a car dealership inspects every fifth car to make sure that cars are detailed after being serviced.
- RAFFLE** The students who attended a prom committee meeting were each given a raffle ticket for a drawing of five prizes.
- MUSIC** A music store asks its customers to submit suggestions for local bands that should play on Friday nights.

Example 2 Identify each sample as *biased* or *unbiased*. Explain your reasoning.

- ACTORS** A random sample of ten people is asked to name their favorite actor.
- BASKETBALL** Every fifth athlete at a basketball camp is asked to name their favorite brand of basketball shoe.
- TELEVISION** Every tenth person entering a gas station is asked to name their favorite television program.
- MUSIC** Every fifth person entering a play is asked to name their favorite style of music.

Example 3 Determine whether each situation describes a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

- PARTIES** Federico is throwing a party for one of his friends. He is trying to decide on a theme. He sends a piece of paper in each invitation, asking guests questions to get their opinions.
- VOLUNTEER** Jaime finds 50 students, half of whom volunteer at a homeless shelter, and compares their grade point averages, extracurricular activities, and involvement in school clubs.
- SOCCER** A researcher organizes a soccer game in hot weather. One team wears short-sleeved shirts, while the other team wears long sleeves.
- SALONS** A salon emails customers, asking them to rate their experience during their last appointment.

Example 4 Identify each survey question as *biased* or *unbiased*. If biased, explain your reasoning.

- What outdoor activities do you enjoy?
- Do you think the comedian's stupid antics are funny?
- Do you like to listen to music, read a book, or watch movies?
- What is your favorite Web site?

Example 5 Identify the experiment as *biased* or *unbiased*. If biased, explain your reasoning.

- EMPLOYMENT** The management of a company hopes to increase the morale of their employees. They select some employees at random and move them into two identical office buildings. They build a recreation room in one of the buildings for employees to use. They then compare the morale of the employees in each building.
- CONCERTS** The manager of a band wants to see if a light show will improve their concerts. For the final date of the band's tour in their hometown, they perform while doing the light show. The manager then compares the reviews of this concert with the reviews of the rest of the tour.



Identify the observational study as *biased* or *unbiased*. If biased, explain your reasoning.

31. **GOLF** A golf club manufacturer watches a group of randomly selected golfers test a prototype club and notes their performances and reactions.
32. **MOVIES** A production company sets up a test audience of friends and family of the production crew to view the new movie and notes their reactions.

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

33. **GYMS** A gym owner wants to test whether changing the color of the walls improves member satisfaction.
34. **GAMING** A manufacturer invites twenty randomly selected teens to try out a new gaming system and notes their reactions as they play.
35. **SCHOOLS** A student asks 100 randomly selected neighbors if they think the school should build a new football field.
36. **SHOES** A shoe company's Web site allows customers to design their own shoes. This program keeps a count of styles and colors chosen by customers.
- Identify the sample. From what population was the sample selected?
 - State the method of data collection.
 - Tell whether the sample is *biased* or *unbiased*. Explain.
 - If unbiased, classify the sample as *simple*, *stratified*, *systematic*, *self-selected*, or *convenience*.

H.O.T. Problems Use Higher-Order Thinking Skills

37. **ERROR ANALYSIS** Amy and Esteban are describing one way to increase the accuracy of a survey. Is either of them correct? Explain your reasoning.

Amy
The survey should include as many people in the population as possible.

Esteban
The sample for the survey should be chosen randomly. Several random samples should be taken.

38. **CCSS CRITIQUE** Consider the following survey proposal.
Question: How do students feel about the new dress code?
Method: Take a simple random sample from each of the four classes. Use this sample to conduct the survey.
Discuss the strengths and weaknesses of this survey.
39. **REASONING** Charlie wants to determine who the most popular athlete is. He conducts three different surveys. For the first survey, he asks 20 random students at school. For the second survey, he asks 50 random people at the mall. For the third survey, he asks 150 random people at a concert. The most popular athlete was different for each survey. Which survey do you think would most likely represent the population? Explain your reasoning.
40. **OPEN ENDED** Design and conduct a simple experiment.
41. **E? WRITING IN MATH** Why are accurate studies important to companies?



Standardized Test Practice

42. GRIDDED RESPONSE The first stage of a rocket burns 28 seconds longer than the second stage. If the total burning time for both stages is 152 seconds, how many seconds does the first stage burn?

43. Ms. Brinkman invested \$30,000; part at 5%, and part at 8%. The total interest on the investment was \$2100 after one year. How much did she invest at 8%?

- A \$10,000 C \$20,000
B \$15,000 D \$25,000

44. A pair of \$25 jeans is on sale for 15% off. What is the sale price?

- F \$21.25 H \$23.25
G \$22.25 J \$24.25

45. GEOMETRY A piece of wire 42 centimeters long is bent into the shape of a rectangle with a width that is twice its length. Find the dimensions of the rectangle.

- A 5 cm, 12 cm C 9 cm, 16 cm
B 7 cm, 14 cm D 11 cm, 18 cm

Spiral Review

Solve each equation. State any extraneous solutions. (Lesson 11-8)

46. $\frac{3}{c} = \frac{2}{c+2}$

47. $\frac{4}{f} = \frac{2}{f-3}$

48. $\frac{j}{j+2} = \frac{j-6}{j-2}$

49. $\frac{h-2}{h} = \frac{h-2}{h-5}$

50. $\frac{3m}{4} + \frac{1}{3} = \frac{3m+4}{6}$

51. $\frac{6}{5} + \frac{4p}{3} = \frac{8p}{5}$

52. $\frac{r-2}{r+2} - \frac{3r}{r-2} = -2$

53. $\frac{t-3}{t+3} - \frac{2t}{t-3} = -1$

54. $\frac{4v}{2v+3} - \frac{2v}{2v-3} = 1$

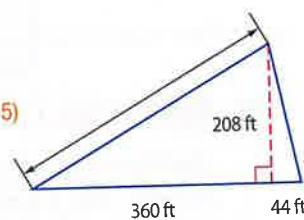
55. SPORTS When air is pumped into a ball, the pressure required can be computed by using the formula $P = \frac{3412.94}{\frac{4\pi r^3}{3}}$, where P represents the pressure in pounds per

square inch (psi), and r is the radius of the ball in inches. (Lesson 11-7)

- Simplify the complex fraction.
- Suppose the air pressure inside the ball is 8 psi. Approximate the radius of the ball to the nearest hundredth.

56. ROLLER COASTERS Suppose a roller coaster climbs 208 feet higher than its starting point, while making a horizontal advance of 360 feet. When it comes down, it makes a horizontal advance of 44 feet. (Lesson 10-5)

- How far will it travel to get to the top of the ride?
- How far will it travel on the downhill track?



Skills Check

57. PHYSICAL SCIENCE Mr. Blackwell's students recorded the height of an object above the ground at several intervals after it was dropped from a height of 5 meters.

Time (s)	0	0.2	0.4	0.6	0.8	1
Height (cm)	500	480	422	324	186	10

Draw a graph showing the relationship between the height of the falling object and time.





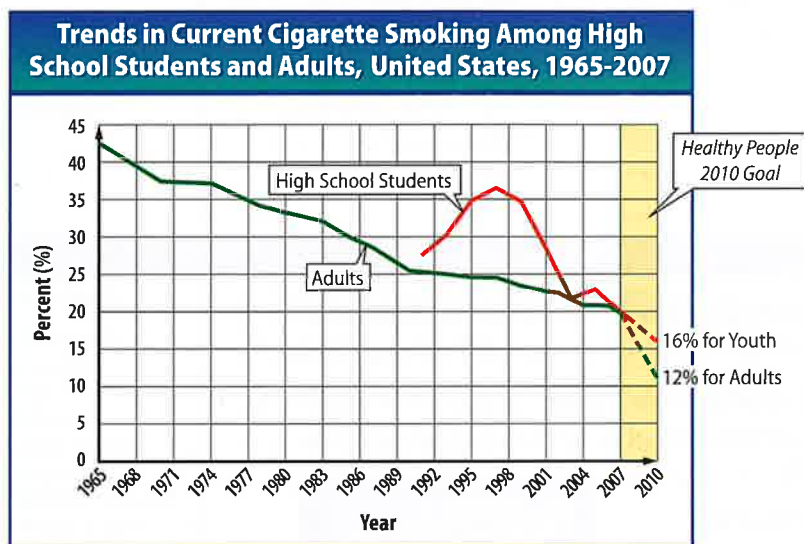
An article in the *International Business Times* asserts that “the rate of decline in teen smoking in the United States has slowed down.” This claim is supported by survey results from a study by the Centers for Disease Control and Prevention (CDC). This survey was conducted over an 18-year period from 1991 to 2009 and included a minimum of 10,904 to a maximum of 16,410 students in 110 to 159 high schools. The study reported cigarette use among high school students as shown in the following table. Current use was defined as smoking a cigarette at least one day in the last 30, while frequent use was defined as smoking at least 20 days in the last 30.

**CCSS Common Core State Standards
Mathematical Practices**

3 Construct viable arguments and critique the reasoning of others.

Year	1991	1993	1995	1997	1999	2001	2003	2005	2007	2009
Current Use (%)	27.5	30.5	34.8	36.4	34.8	28.5	21.9	23.0	20.0	19.5
Frequent Use (%)	12.7	13.8	16.1	16.7	16.8	13.8	9.7	9.4	8.1	7.3

The report contained the following graph comparing the numbers of current high school smokers and adults who consider themselves smokers.



Activity 1 Evaluate the Design of a Study

Evaluate the design of this study and the source of the data.

- What might be a problem that could occur when conducting this survey?
- What biases could occur from their choice of a sample group?
 - A survey about activities that are deemed unacceptable might not get truthful responses from all students who complete the survey. Many students may refuse to complete the survey if they think the data might be used to punish them.
 - The study does not identify what types of schools were used in the survey. The sample may not represent all ethnic and economic groups that are found throughout the United States.

(continued on the next page)

Algebra Lab

Evaluating Published Data *Continued*

Bias may be introduced when data are collected or by how the results are analyzed or reported.

Activity 2 Evaluate an Analysis and Display

For this study, evaluate how the data are analyzed and displayed.

- a. Is there a problem with the conclusion made in the article that the rate of decline in teen smoking is slowing? If so, what is it?
- b. The graph appears to show that percentage of high school smokers is greater than the percentage of adult smokers. Is this accurate? Explain.
 - a. The assertion generalizes all teens, while the survey was taken from high school students. Teens who have graduated or dropped out of school were not included and may have vastly different smoking habits than the sample group.
 - b. The line labeled *Adults* represents the percent of adults who classify themselves as current cigarette smokers. These adults would most likely be those who smoke on a daily basis. The line labeled *High School Students* is for those students who have smoked at least once in the last 30 days. A better comparison with adult smokers would be those high school students who were frequent smokers. These percents would be less than those for adults.

Activity 3 Compare Data

On a graphing calculator, make a double line graph comparing the rates of change of the current use and frequent use of cigarettes by high school students.

Step 1 First enter the data into L1, L2, and L3. Press **STAT** 1 and enter the years from 1991 to 2009 in L1, then the percents of current use in L2, and the percents for frequent use in L3.

Step 2 Use the **Stat Plot** function to produce a double line graph. Use **Plot1** to graph the line for the current use and **Plot2** to graph the line for the frequent use.

Step 3 Next, adjust the window settings and graph the double line graph.



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Analyze the Results

1. Would a graph of frequent smokers imply a conclusion similar to the one implied by the graph of current smokers for all high school students? Explain.
2. What might be a reason that the CDC would use the percents of current use rather than frequent use?
3. List five things to consider whenever you are evaluating published data in the media.
4. Find a report in the media and evaluate the published data using your list from Exercise 3.



Then

You analyzed data collection techniques.

Now

- 1 Identify sample statistics and population parameters.
- 2 Analyze data sets using statistics.

Why?

At the start of every class period for one week, each of Mr. Day's algebra students randomly draws 9 pennies from a jar of 1000 pennies. Each student calculates the mean age of the random sample of pennies drawn and then returns the pennies to the jar.

How does the mean age for 9 pennies compare to the mean age of all 1000 pennies?

New Vocabulary

- statistical inference
- statistic
- parameter
- mean absolute deviation (MAD)
- standard deviation
- variance

Common Core State Standards

Content Standards
S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Mathematical Practices

- 2 Reason abstractly and quantitatively.
- 6 Attend to precision.

1 Statistics and Parameters The statistics of a sample are used to draw conclusions about the entire population. This is called **statistical inference**. In the scenario above, each student takes a random sample of pennies from the jar. The jar of 1000 pennies represents the population.

A **statistic** is a measure that describes a characteristic of a sample. A **parameter** is a measure that describes a characteristic of a population. Parameters are fixed values that can be determined by the entire population, but are typically estimated based on the statistics of a carefully chosen random sample. A statistic can and usually will vary from sample to sample. A parameter will not change, for it represents the entire population.

Example 1 Statistics and Parameters

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

a. At a local university, a random sample of 40 scholarship applicants is selected. The mean grade-point average of the 40 applicants is calculated.

- Sample: the group of 40 scholarship applicants
- Population: all applicants
- Sample statistic: mean grade-point average of the sample
- Population parameter: mean grade-point average of all applicants

b. A stratified random sample of registered nurses is selected from all hospitals in a three-county area, and the median salary is calculated.

- Sample: randomly selected registered nurses from hospitals in the three-county area
- Population: all nurses at the hospitals in the same region
- Sample statistic: median salary of nurses in the sample
- Population parameter: median salary of all nurses in all hospitals in a three-county area.

Guided Practice

1. CEREAL Starting with a randomly selected box of cereal from the manufacturing line, every 50th box of cereal is removed and weighed. The mode weight of a day's sample is calculated.



2 Statistical Analysis As shown in Lesson 0-12, univariate data can be represented by measures of central tendency, such as the mean, median, and mode. Univariate data can also be represented by measures of variation that assess the variability of the data. Some examples are the range, quartiles, interquartile range, mean absolute deviation, and standard deviation.

The **mean absolute deviation (MAD)** is the average of the absolute values of the differences between the mean and each value in the data set. The mean absolute deviation is used to predict errors and judge how well the mean represents the data.

StudyTip

Ellipsis The ellipsis in the formula for MAD denotes “and so on”. All of the terms between the second term and the last term are implied to save space.

KeyConcept Mean Absolute Deviation

Step 1 Find the mean, \bar{x} .

Step 2 Find the absolute value of the difference between each data value x_n and the mean, $|\bar{x} - x_n|$.

Step 3 Find the sum of all of the values in Step 2.

Step 4 Divide the sum by the number of values in the set of data n .

$$\text{Formula MAD} = \frac{|\bar{x} - x_1| + |\bar{x} - x_2| + \dots + |\bar{x} - x_n|}{n}$$



Example 2 Mean Absolute Deviation

MARKETING Each person who visited the Comic Book Shoppe’s Web site was asked to enter the number of comic books they buy each month. They received the following responses in one day: {2, 2, 3, 4, 14}. Find and interpret the mean absolute deviation.

Step 1 Find the mean.

$$\bar{x} = \frac{2 + 2 + 3 + 4 + 14}{5} \text{ or } 5$$

Step 2 Find the absolute values of the differences.

$$x_1 = 2: |\bar{x} - x_1| = |5 - 2| \text{ or } 3$$

$$x_2 = 2: |\bar{x} - x_2| = |5 - 2| \text{ or } 3$$

$$x_3 = 3: |\bar{x} - x_3| = |5 - 3| \text{ or } 2$$

$$x_4 = 4: |\bar{x} - x_4| = |5 - 4| \text{ or } 1$$

$$x_5 = 14: |\bar{x} - x_5| = |5 - 14| \text{ or } 9$$

Step 3 Find the sum.

$$3 + 3 + 2 + 1 + 9 = 18$$

Step 4 Find the mean absolute deviation.

$$\text{MAD} = \frac{|\bar{x} - x_1| + |\bar{x} - x_2| + \dots + |\bar{x} - x_n|}{n}$$

Formula for Mean Absolute Deviation

$$= \frac{18}{5} \text{ or } 3.6$$

The sum is 18 and $n = 5$.

A mean absolute deviation of 3.6 indicates that the data, on average, are 3.6 units away from the mean. This value is significantly influenced by the outlier 14. Without the outlier, the data set would have a mean of 2.75 and a mean absolute deviation of 0.75.

GuidedPractice

2. DANCES The prom committee kept a count of how many tickets it sold each day during lunch: {12, 32, 36, 41, 22, 47, 51, 33, 37, 49}. Find and interpret the mean absolute deviation of these data.



In a set of data, the **standard deviation** shows how the data deviate from the mean. A low standard deviation indicates that the data tend to be very close to the mean, while a high standard deviation indicates that the data are spread out over a larger range of values.

The standard deviation is represented by the lowercase Greek letter sigma, σ . The **variance** σ^2 of the data is the square of the standard deviation.

StudyTip

Symbols The mean of a sample and the mean of a population are calculated the same way. \bar{x} refers to the mean of a sample and μ refers to the mean of a population. In this text, \bar{x} will refer to both.

KeyConcept Standard Deviation

- Step 1** Find the mean, \bar{x} .
- Step 2** Find the square of the difference between each data value x_n and the mean, $(\bar{x} - x_n)^2$.
- Step 3** Find the sum of all of the values in Step 2.
- Step 4** Divide the sum by the number of values in the set of data n . This value is the variance.
- Step 5** Take the square root of the variance.

$$\text{Formula } \sigma = \sqrt{\frac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + \dots + (\bar{x} - x_n)^2}{n}}$$



Example 3 Variance and Standard Deviation

ELECTRONICS Ed surveys his classmates to find out how many electronic gadgets each person has in their home. Find and interpret the standard deviation of the data set.

{9, 10, 11, 6, 9, 11, 9, 8, 11, 8, 7, 9, 11, 11, 5}

- Step 1** Find the mean.

$$\bar{x} = \frac{9 + 10 + 11 + 6 + 9 + 11 + 9 + 8 + 11 + 8 + 7 + 9 + 11 + 11 + 5}{15} \text{ or } 9$$

- Step 2** Find the square of the differences, $(\bar{x} - x_n)^2$.

$$\begin{array}{cccccc} (9 - 9)^2 = 0 & (9 - 10)^2 = 1 & (9 - 11)^2 = 4 & (9 - 6)^2 = 9 & (9 - 9)^2 = 0 \\ (9 - 11)^2 = 4 & (9 - 9)^2 = 0 & (9 - 8)^2 = 1 & (9 - 11)^2 = 4 & (9 - 8)^2 = 1 \\ (9 - 7)^2 = 4 & (9 - 9)^2 = 0 & (9 - 11)^2 = 4 & (9 - 11)^2 = 4 & (9 - 5)^2 = 16 \end{array}$$

- Step 3** Find the sum.

$$0 + 1 + 4 + 9 + 0 + 4 + 0 + 1 + 4 + 1 + 4 + 0 + 4 + 4 + 16 = 52$$

- Step 4** Find the variance.

$$\begin{aligned} \sigma^2 &= \frac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + \dots + (\bar{x} - x_n)^2}{n} && \text{Formula for Variance} \\ &= \frac{52}{15} \text{ or about } 3.47 && \text{The sum is } 52 \text{ and } n = 15. \end{aligned}$$

- Step 5** Find the standard deviation.

$$\begin{aligned} \sigma &= \sqrt{\sigma^2} && \text{Square Root of the Variance} \\ &\approx \sqrt{3.47} \text{ or about } 1.86 \end{aligned}$$

A standard deviation of 1.86 is small compared to the mean of 9. This suggests that most of the data values are relatively close to the mean.

GuidedPractice

- 3. DIET** Caleb tracked his Calorie intake for a week. Find and interpret the standard deviation of his Calorie intake.

1950, 2000, 2100, 2000, 1900, 2100, 2000



The mean and standard deviation can be used to compare two different sets of data.



StudyTip

Symbols The standard deviation of a sample s and the standard deviation of a population σ are calculated in different ways. In this text, you will calculate the standard deviation of a population.

Example 4 Compare Two Sets of Data

Miguel plays golf at Table Rock and Blackhawk golf courses. Compare the means and standard deviations of each set of Miguel's scores.

Table Rock				
81	78	79	82	80
80	79	83	81	80

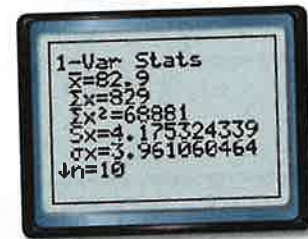
Blackhawk				
84	79	86	78	77
88	85	79	87	86

Use a graphing calculator to find the mean and standard deviation. Clear all lists. Then press **STAT** **ENTER**, and enter each data value into L1. To view the statistics, press **STAT** **▶** 1 **ENTER**.

Table Rock



Blackhawk



Miguel's mean score at Table Rock is 80.3 with a standard deviation of about 1.4. His mean score at Blackhawk is 82.9 with a standard deviation of about 4.0. Therefore, he tends to score lower at Table Rock. The greater standard deviation at Blackhawk indicates that there is greater variability to his scores at that course, but he is more consistent at Table Rock.

Guided Practice

4. **SWIMMING** Anna is considering two different lineups for her 4×100 relay team. Below are the times in minutes recorded for each lineup. Compare the means and standard deviations of each set of data.

Lineup A				
4.25	4.31	4.19	4.40	4.23
4.18	4.71	4.56	4.32	4.39

Lineup B				
4.47	4.68	4.25	4.41	4.49
4.18	4.27	4.69	4.32	4.44

Check Your Understanding

= Step-by-Step Solutions begin on page R13.



- Example 1** 1. **BOOKS** A stratified random sample of 1000 college students in the United States is surveyed about how much money they spend on books per year. Identify the sample and the population. Then describe the sample statistic and the population parameter.
- Example 2** 2. **AMUSEMENT PARKS** An amusement park manager kept track of how many bags of cotton candy they sold each hour on a Saturday: {16, 24, 15, 17, 22, 16, 18, 24, 17, 13, 25, 21}. Find and interpret the mean absolute deviation.
- Example 3** 3. **PART-TIME JOBS** Ms. Johnson asks all of the members of the girls' tennis team to find the number of hours each week they work at part-time jobs: {10, 12, 0, 6, 9, 15, 12, 10, 11, 20}. Find and interpret the standard deviation of the data set.
- Example 4** 4. **CCSS MODELING** Mr. Jones recorded the number of pull-ups done by his students. Compare the means and standard deviations of each group.
Boys: {5, 16, 3, 8, 4, 12, 2, 15, 0, 1, 9, 3} Girls: {2, 4, 0, 3, 5, 4, 6, 1, 3, 8, 3, 4}



Example 1 Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

- POLITICS** A random sample of 1003 Mercy County voters is asked if they would vote for the incumbent for governor. The percent responding *yes* is calculated.
- ACTIVITIES** A stratified random sample of high school students from each school in the county was polled about the time spent each week on extracurricular activities.
- MONEY** A stratified random sample of 2500 high school students across the country was asked how much money they spent each month.

Example 2

- DVDS** A math teacher asked all of his students to count the number of DVDs they owned. Find and interpret the mean absolute deviation.

Number of DVDs					
26	39	5	82	12	14
0	3	15	19	41	6
2	0	11	1	19	29

- SWIMMING** The owner of a public swimming pool tracked the daily attendance. Find and interpret the mean absolute deviation.

Daily Attendance					
86	45	91	104	95	88
111	85	79	102	166	103
89	94	79	103	88	84

Example 3

- CCSS REASONING** Samantha wants to see if she is getting a fair wage for babysitting at \$8.50 per hour. She takes a survey of her friends to see what they charge per hour. The results are {\$8.00, \$8.50, \$9.00, \$7.50, \$15.00, \$8.25, \$8.75}. Find and interpret the standard deviation of the data.
- ARCHERY** Carla participates in competitive archery. Each competition allows a maximum of 90 points. Carla's results for the last 8 competitions are {76, 78, 81, 75, 80, 80, 76, 77}. Find and interpret the standard deviation of the data.

Example 4

- BASKETBALL** The coach of the Wildcats basketball team is comparing the number of fouls called against his team with the number called against their rivals, the Trojans. He records the number of fouls called against each team for each game of the season. Compare the means and standard deviations of each set of data.

Wildcats			
15	12	13	9
11	12	14	12
8	16	9	9
11	13	12	14

Trojans			
9	10	14	13
7	8	10	10
9	7	11	9
12	11	13	8

- MOVIE RATINGS** Two movies were rated by the same group of students. Ratings were from 1 to 10, with 10 being the best.

- Compare the means and standard deviations of each set of data.
- Provide an argument for why Movie A would be preferred. Movie B?

Movie A			
7	8	7	6
8	6	7	8
6	8	8	6
7	7	8	8

Movie B			
9	5	10	6
3	10	9	4
8	3	9	9
2	8	10	3



14. **PENNIES** Mr. Day has another jar of pennies on his desk. There are 30 pennies in this jar. Theo chooses 5 pennies from the jar. Lola chooses 10 pennies, and Peter chooses 20 pennies. Pennies are chosen and replaced.

- Theo's pennies are {1974, 1975, 1981, 1999, 1992}. Find the mean absolute deviation.
- Lola's pennies are {2004, 1999, 2004, 2005, 1991, 2003, 2005, 2000, 2001, 1998}. Find the mean absolute deviation.
- Peter's pennies are {2007, 2005, 1975, 2003, 2005, 1997, 1992, 1994, 1991, 1992, 2000, 1999, 2005, 1982, 2005, 2004, 1998, 2001, 2002, 2006}. Find the mean absolute deviation.
- Find the mean absolute deviation for all of the pennies in the jar. Which sample most accurately reflected the population mean? Explain.

Years of Pennies in Jar					
2001	1990	2000	1982	1991	1975
2007	1981	2005	2007	2003	2005
1997	1974	1992	1994	1991	1992
2000	1995	1999	2005	2006	2005
2004	2004	1998	2001	2002	2006

15. **RUNNING** The results of a 5K race are published in a local paper. Over a thousand people participated, but only the times of the top 15 finishers are listed.

15th Annual 5K Road Race					
Place	Time (min:s)	Place	Time (min:s)	Place	Time (min:s)
1	15:56	6	16:34	11	17:14
2	16:06	7	16:41	12	17:46
3	16:11	8	16:54	13	17:56
4	16:21	9	17:00	14	17:57
5	16:26	10	17:03	15	18:03

- Find the mean and standard deviation of the top 15 running times. (*Hint: Convert each time to seconds.*)
- Identify the sample and population.
- Analyze the sample. Classify the data as *quantitative* or *qualitative*. Can a statistical analysis of the sample be applied to the population? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

16. **CCSS CRITIQUE** Jennifer and Megan are determining one way to decrease the size of the standard deviation of a set of data. Is either of them correct? Explain.

Jennifer
Remove the outliers from the data set.

Megan
Add data values to the data set that are equal to the mean.

- REASONING** Determine whether the statement *Two random samples taken from the same population will have the same mean and standard deviation is sometimes, always, or never true.* Explain.
- OPEN ENDED** Describe a situation in which it would be useful to use a sample mean to help estimate a population mean. How could you collect a random sample?
- CHALLENGE** Write a set of data with a standard deviation that is equal to the mean absolute deviation.

WRITING IN MATH Compare and contrast each of the following.

- statistics and parameters
- standard deviation and mean absolute deviation



Standardized Test Practice

22. Melina bought a shirt that was marked 20% off. She paid \$15.75. What was the original price?

- A \$16.69 C \$18.69
B \$17.69 D \$19.69

23. **SHORT RESPONSE** A group of student ambassadors visited the Capitol building. Twenty students met with the local representative. This was 16% of the students. How many student ambassadors were there altogether?

24. The tallest 7 trees in a park have heights in meters of 19, 24, 17, 26, 24, 20, and 18. Find the mean absolute deviation of their heights.

- F 3.0 H 3.4
G 3.2 J 21

25. It takes 3 hours for a boat to travel 27 miles upstream. The same boat can travel 30 miles downstream in 2 hours. Find the speed of the boat.

- A 3 mph C 12 mph
B 5 mph D 14 mph

Spiral Review

Identify each sample as *biased* or *unbiased*. Explain your reasoning. (Lesson 12-1)

26. **SHOPPING** Every tenth person walking into the mall is asked to name their favorite store.

27. **MUSIC** Every fifth person at a rock concert is asked to name their favorite radio station.

Simplify each expression. (Lesson 11-3)

28. $\frac{x^2 - 8x + 15}{x^2 + 3x - 18}$

29. $\frac{x^2 - x - 12}{x^2 - 6x + 8}$

30. $\frac{x^2 - x - 30}{x^2 - 4x - 12}$

31. **SOCCER** The number of members of the local soccer association has increased by 6% every year. As of the beginning of 2010, there were 880 members. (Lesson 7-6)

- a. Write an equation for the number of members of the association t years after 2010.
b. If this trend continues, predict how many members the association will have in 2020.

32. **GEOMETRY** If the side length of a cube is s , the volume is represented by s^3 , and the surface area is represented by $6s^2$. (Lessons 7-1 and 7-2)



- a. Are the expressions for volume and surface area monomials? Explain.
b. If the side of a cube measures 3 feet, find the volume and surface area.
c. Find a side length s such that the volume and surface area have the same measure.
d. The volume of a cylinder can be found by multiplying the radius squared times the height times π , or $V = \pi r^2 h$. Suppose you have two cylinders. Each measure of the second is twice the measure of the first, so $V = \pi(2r)^2(2h)$. What is the ratio of the volume of the first cylinder to the second cylinder?

Skills Review

Find the range, median, lower quartile, and upper quartile for each set of data.

33. {15, 23, 46, 36, 15, 19}

34. {55, 57, 39, 72, 46, 53, 81}

35. {21, 25, 19, 18, 22, 16, 27}

36. {52, 29, 72, 64, 33, 49, 51, 68}

37. {8, 12, 9, 11, 11, 10, 14, 18}

38. {133, 119, 147, 94, 141, 106, 118, 149}





Then

- You calculated measures of central tendency and variation.

Now

- Describe the shape of a distribution.
- Use the shapes of distributions to select appropriate statistics.

Why?

- While training for the 100-meter dash, Sarah pulled a muscle in her lower back. After being cleared for practice, she continued to train. Sarah's median time was about 12.34 seconds, but her average time dropped to about 12.53 seconds.



New Vocabulary

- distribution
- negatively skewed distribution
- symmetric distribution
- positively skewed distribution



Common Core State Standards

- Content Standards**
 S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
 S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

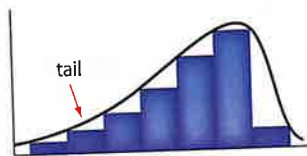
Mathematical Practices

- Use appropriate tools strategically.

1 Describing Distributions A **distribution** of data shows the observed or theoretical frequency of each possible data value. Recall that a histogram is a type of bar graph used to display data that have been organized into equal intervals. A histogram is useful when viewing the overall distribution of the data within a set over its range. You can see the shape of the distribution by drawing a curve over the histogram.

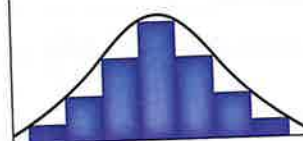
Key Concept Symmetric and Skewed Distributions

Negatively Skewed Distribution



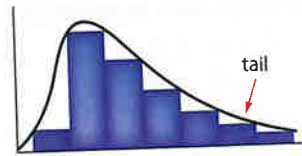
The majority of the data are on the right.

Symmetric Distribution



The data are evenly distributed.

Positively Skewed Distribution



The majority of the data are on the left.

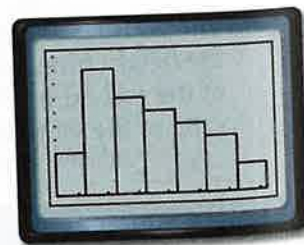
Example 1 Distribution Using a Histogram

Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

- 25, 22, 31, 25, 26, 35, 18, 39, 22, 32, 34, 26, 42, 23, 40, 36, 18, 30
 26, 30, 37, 23, 19, 33, 24, 29, 39, 21, 43, 25, 34, 24, 26, 30, 21, 22

First, press **STAT** **ENTER** and enter each data value. Then, press **2nd** **[STAT PLOT]** **ENTER** **ENTER** and choose **1**. Press **ZOOM** **[ZoomStat]** to adjust the window.

The graph is high on the left and has a tail on the right. Therefore, the distribution is positively skewed.



[17, 45] scl: 4 by [0, 10] scl: 1

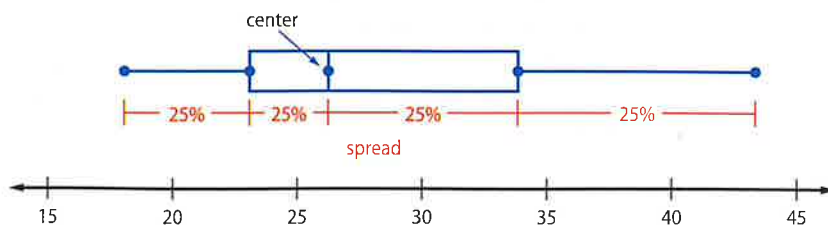
Guided Practice

- Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

- 8, 11, 15, 25, 21, 26, 20, 12, 32, 20, 31, 14, 19, 27, 22, 21, 14, 8
 6, 23, 18, 16, 28, 25, 16, 20, 29, 24, 17, 35, 20, 27, 10, 16, 22, 12



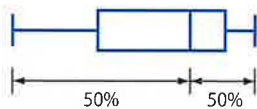
A box-and-whisker plot can also be used to identify the shape of a distribution. Recall from Lesson 0-13 that a box-and-whisker plot displays the spread of a data set by dividing it into four quartiles. The data from Example 1 are displayed below.



Notice that the left whisker is shorter than the right whisker, and that the line representing the median is closer to the left whisker. This represents a peak on the left and a tail to the right.

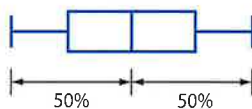
Key Concept Symmetric and Skewed Box-and-Whisker Plots

Negatively Skewed



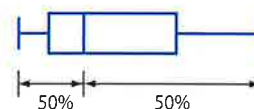
The left whisker is longer than the right. The median is closer to the shorter whisker.

Symmetric



The whiskers are the same length. The median is in the center of the data.

Positively Skewed



The right whisker is longer than the left. The median is closer to the shorter whisker.

Study Tip

Outliers In Example 2, notice that the outlier does not affect the shape of the distribution.

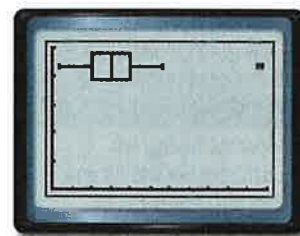
Example 2 Distribution Using a Box-and-Whisker Plot

Use a graphing calculator to construct a box-and-whisker plot for the data, and use it to determine the shape of the distribution.

9, 17, 15, 10, 16, 2, 17, 19, 10, 18, 14, 8, 20, 20, 3, 21, 12, 11
5, 26, 15, 28, 12, 5, 27, 26, 15, 53, 12, 7, 22, 11, 8, 16, 22, 15

Enter the data as L1. Press **2nd** **[STAT PLOT]** **ENTER** **ENTER** and choose **1-1**. Adjust the window to the dimensions shown.

The lengths of the whiskers are approximately equal, and the median is in the middle of the data. This indicates that the data are equally distributed to the left and right of the median. Thus, the distribution is symmetric.



[0, 55] scl: 5 by [0, 5] scl: 1

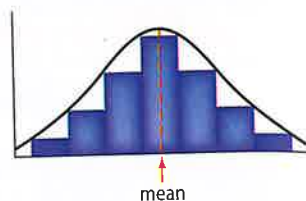
Guided Practice

2. Use a graphing calculator to construct a box-and-whisker plot for the data, and use it to describe the shape of the distribution.

40, 50, 35, 48, 43, 31, 52, 42, 54, 38, 50, 46, 49, 43, 40, 50, 32, 53
51, 43, 47, 41, 49, 50, 34, 54, 51, 44, 54, 39, 47, 35, 51, 44, 48, 37

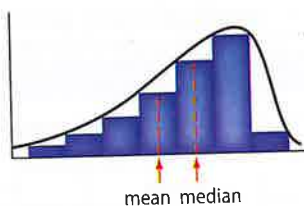
2 Analyzing Distributions You have learned that data can be described using statistics. The mean and median describe the center. The standard deviation and quartiles describe the spread. You can use the shape of the distribution to choose the most appropriate statistics that describe the center and spread of a set of data.

When a distribution is symmetric, the mean accurately reflects the center of the data. However, when a distribution is skewed, this statistic is not as reliable.

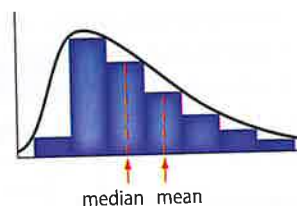


In Lesson 0-12, you discovered that outliers can have a strong effect on the mean of a data set, while the median is less affected. So, when a distribution is skewed, the mean lies away from the majority of the data toward the tail. The median is less affected and stays near the majority of the data.

Negatively Skewed Distribution



Positively Skewed Distribution



When choosing appropriate statistics to represent a set of data, first determine the shape of the distribution.

- If the distribution is relatively symmetric, the mean and standard deviation can be used.
- If the distribution is skewed or has outliers, use the five-number summary.

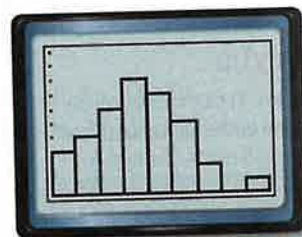


Example 3 Choose Appropriate Statistics

Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a histogram for the data.

21, 28, 16, 30, 25, 34, 21, 47, 18, 36, 24, 28, 30, 15, 33, 24, 32, 22, 27, 38, 23, 29, 15, 27, 33, 19, 34, 29, 23, 26, 19, 30, 25, 13, 20, 25

Use a graphing calculator to create a histogram. The graph is high in the middle and low on the left and right. Therefore, the distribution is symmetric.



[12, 48] scl: 4 by [0, 10] scl: 1

The distribution is symmetric, so use the mean and standard deviation to describe the center and spread.

Press **STAT** **▶** **ENTER** **ENTER**.



The mean \bar{x} is about 26.1 with standard deviation σ of about 7.1.

TechnologyTip

CCSS Tools On a graphing calculator, each bar is called a *bin*. The width of each bin can be adjusted by pressing **WINDOW** and changing Xscl. View the histogram using different bin widths and compare the results to determine the appropriate bin width.

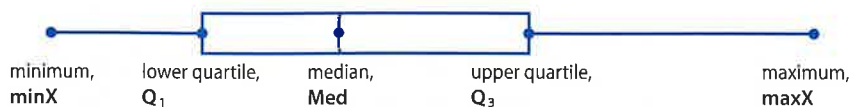


Guided Practice

3. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by creating a histogram for the data.

19, 2, 25, 14, 24, 20, 27, 30, 14, 25, 19, 32, 21, 31, 25, 16, 24, 22
29, 6, 26, 32, 17, 26, 24, 26, 32, 10, 28, 19, 26, 24, 11, 23, 19, 8

A box-and-whisker plot is helpful when viewing a skewed distribution since it is constructed using the five-number summary.



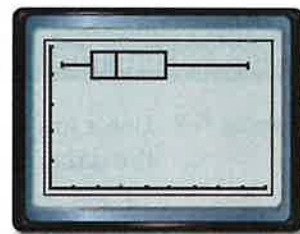
Real-World Example 4 Choose Appropriate Statistics



- COMMUNITY SERVICE** The number of community service hours each of Ms. Tucci's students completed is shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

Community Service Hours												
6	13	8	7	19	12	2	19	11	22	7	33	13
3	8	10	5	25	16	6	14	7	20	10	30	

Use a graphing calculator to create a box-and-whisker plot. The right whisker is longer than the left and the median is closer to the left whisker. Therefore, the distribution is positively skewed.



[0, 36] scl: 4 by [0, 5] scl: 1

The distribution is positively skewed, so use the five-number summary. The range is $33 - 2$ or 31. The median number of hours completed is 11, and half of the students completed between 7 and 19 hours.



Guided Practice

4. **FUNDRAISER** The money raised per student in Mr. Bulanda's 5th period class is shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by creating a box-and-whisker plot for the data.

Money Raised per Student (dollars)									
41	27	52	18	42	32	16	95	27	65
36	45	5	34	50	15	62	38	57	20
38	21	33	58	25	42	31	8	40	28



Real-WorldLink

Volunteers in the Peace Corps must be at least 18 years old, and more than 90% of volunteers have college degrees. Volunteers work in another country for 27 months and are placed in host countries that have the greatest needs for skilled volunteers.

Source: Peace Corps



Check Your Understanding

= Step-by-Step Solutions begin on page R13.



Examples 1–2 Use a graphing calculator to construct a histogram and a box-and-whisker plot for the data. Then describe the shape of the distribution.

- 80, 84, 68, 64, 57, 88, 61, 72, 76, 80, 83, 77, 78, 82, 65, 70, 83, 78, 73, 79, 70, 62, 69, 66, 79, 80, 86, 82, 73, 75, 71, 81, 74, 83, 77, 73
- 30, 24, 35, 84, 60, 42, 29, 16, 68, 47, 22, 74, 34, 21, 48, 91, 66, 51, 33, 29, 18, 31, 54, 75, 23, 45, 25, 32, 57, 40, 23, 32, 47, 67, 62, 23

Example 3 Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a histogram for the data.

- 58, 66, 52, 75, 60, 56, 78, 63, 59, 54, 60, 67, 72, 80, 68, 88, 55, 60, 59, 61, 82, 70, 67, 60, 58, 86, 74, 61, 92, 76, 58, 62, 66, 74, 69, 64

Example 4

- PRESENTATIONS** The length of the students' presentations in Ms. Monroe's 2nd period class are shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.



Practice and Problem Solving

Extra Practice is on page R12.

Examples 1–2 Use a graphing calculator to construct a histogram and a box-and-whisker plot for the data. Then describe the shape of the distribution.

- 55, 65, 70, 73, 25, 36, 33, 47, 52, 54, 55, 60, 45, 39, 48, 55, 46, 38, 50, 54, 63, 31, 49, 54, 68, 35, 27, 45, 53, 62, 47, 41, 50, 76, 67, 49
- 42, 48, 51, 39, 47, 50, 48, 51, 54, 46, 49, 36, 50, 55, 51, 43, 46, 37, 50, 52, 43, 40, 33, 51, 45, 53, 44, 40, 52, 54, 48, 51, 47, 43, 50, 46

Example 3 Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a histogram for the data.

- 32, 44, 50, 49, 21, 12, 27, 41, 48, 30, 50, 23, 37, 16, 49, 53, 33, 25, 35, 40, 48, 39, 50, 24, 15, 29, 37, 50, 36, 43, 49, 44, 46, 27, 42, 47
- 82, 86, 74, 90, 70, 81, 89, 88, 75, 72, 69, 91, 96, 82, 80, 78, 74, 94, 85, 77, 80, 67, 76, 84, 80, 83, 88, 92, 87, 79, 84, 96, 85, 73, 82, 83

Example 4

- WEATHER** The daily low temperatures for New Carlisle over a 30-day period are shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

Temperature (°F)														
48	50	55	53	57	53	44	61	57	49	51	58	46	54	57
50	55	47	57	48	58	53	49	56	59	52	48	55	53	51



10. **TRACK** Refer to the beginning of the lesson. Sarah's 100-meter dash times are shown.

- Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.
- Sarah's slowest time prior to pulling a muscle was 12.50 seconds. Use a graphing calculator to create a box-and-whisker plot that *does not* include the times that she ran after pulling the muscle. Then describe the center and spread of the new data set.
- What effect does removing the times recorded after Sarah pulled a muscle have on the shape of the distribution and on how you should describe the center and spread?

100-meter dash (seconds)				
12.20	12.35	13.60	12.24	12.72
12.18	12.06	12.41	12.28	13.06
12.87	12.04	12.38	12.20	13.12
12.30	13.27	12.93	12.16	12.02
12.50	12.14	11.97	12.24	13.09
12.46	12.33	13.57	11.96	13.34

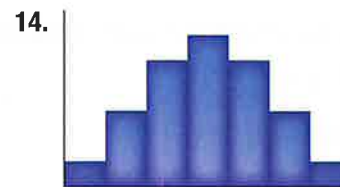
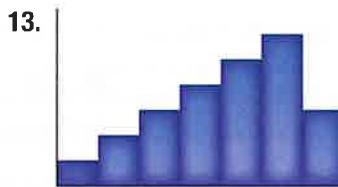
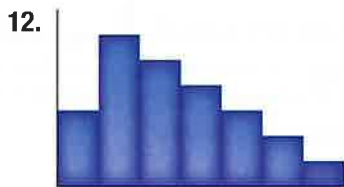
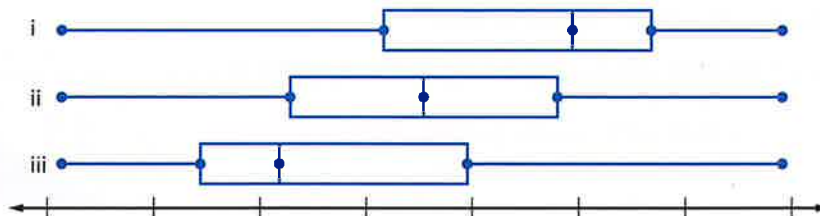
11. **MENU** The prices for entrees at a restaurant are shown.

- Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.
- The owner of the restaurant decides to eliminate all entrees that cost more than \$15. Use a graphing calculator to create a box-and-whisker plot that reflects this change. Then describe the center and spread of the new data set.

Entree Prices (\$)				
9.00	11.25	16.50	9.50	13.00
18.50	7.75	11.50	13.75	9.75
8.00	16.50	12.50	10.25	17.75
13.00	10.75	16.75	8.50	11.50

H.O.T. Problems Use Higher-Order Thinking Skills

CHALLENGE Identify the box-and-whisker plot that corresponds to each of the following histograms.



- CCSS ARGUMENTS** Research and write a definition for a *bimodal distribution*. How can the measures of center and spread of a bimodal distribution be described?
- OPEN ENDED** Give an example of a set of real-world data with a distribution that is symmetric and one with a distribution that is not symmetric.
- WRITING IN MATH** Explain why the mean and standard deviation are used to describe the center and spread of a symmetrical distribution and the five-number summary is used to describe the center and spread of a skewed distribution.



Standardized Test Practice

18. At the county fair, 1000 tickets were sold. Adult tickets cost \$8.50, children's tickets cost \$4.50, and a total of \$7300 was collected. How many children's tickets were sold?
- A 700 C 400
B 600 D 300
19. Edward has 20 dimes and nickels, which together total \$1.40. How many nickels does he have?
- F 12 H 8
G 10 J 6
20. If 4.5 kilometers is about 2.8 miles, about how many miles is 6.1 kilometers?
- A 3.2 miles C 3.8 miles
B 3.6 miles D 4.0 miles
21. **EXTENDED RESPONSE** Three times the width of a certain rectangle exceeds twice its length by three inches, and four times its length is twelve more than its perimeter.
- Translate the sentences into equations.
 - Find the dimensions of the rectangle.
 - What is the area of the rectangle?

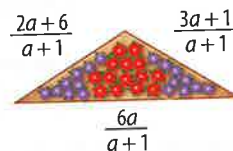
Spiral Review

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter. (Lesson 12-2)

22. **AMUSEMENT PARK** A systematic sample of 250 guests is asked how much money they spent on concessions inside the park. The median amount of money is calculated.
23. **PROM** A random sample of 100 high school seniors at North Boyton High School is surveyed, and the mean amount of money spent on prom by a senior is calculated.

Identify each survey question as *biased* or *unbiased*. If biased, explain your reasoning. (Lesson 12-1)

24. What do you like the most about reality television shows, and which one is your favorite?
25. Are you planning on seeing the school play?
26. Don't you agree that the school should renovate the library?
27. **GARDENING** They planted a triangular garden. Write an expression for the perimeter of the triangle. (Lesson 11-6)



Find the inverse of each function. (Lesson 4-7)

28. $f(x) = 2x - 14$

30. $f(x) = \frac{1}{4}x + 3$

32. $f(x) = \frac{2}{3}x + 6$

29. $f(x) = 17 - 5x$

31. $f(x) = -\frac{1}{7}x - 1$

33. $f(x) = 12 - \frac{3}{5}x$

Skills Review

A bowl contains 3 red chips, 6 green chips, 5 yellow chips, and 8 orange chips. A chip is drawn randomly. Find each probability.

34. red

35. orange

36. yellow or green

37. not orange

38. not green

39. red or orange



Comparing Sets of Data

Then

- You calculated measures of central tendency and variation.

Now

- Determine the effect that transformations of data have on measures of central tendency and variation.
- Compare data using measures of central tendency and variation.

Why?

- Tom gets paid hourly to do landscaping work. Because he is such a good employee, Tom is planning to ask his boss for a bonus. Tom's initial pay for a month is shown. He is trying to decide whether he should ask for an extra \$5 per day or a 10% increase in his daily wages.

Tom's Pay (\$)		
44	52	50
40	48	46
44	52	54
58	42	52
54	50	52
42	52	46
56	48	44
50	42	



New Vocabulary
linear transformation

Common Core State Standards

Content Standards

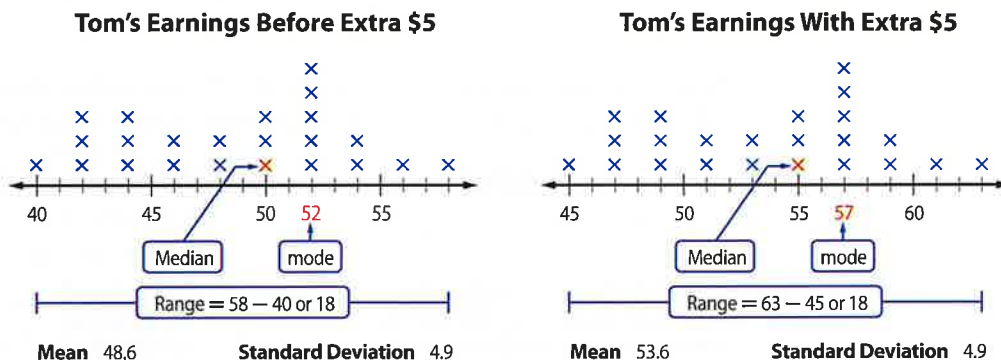
S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Mathematical Practices

- 1 Make sense of problems and persevere in solving them.

1 Transformations of Data To see the effect that an extra \$5 per day would have on Tom's daily pay, we can find the new daily pay values and compare the measures of center and variation for the two sets of data. The new data can be found by performing a **linear transformation**. A **linear transformation** is an operation performed on a data set that can be written as a linear function. Tom's daily pay after the \$5 bonus can be found using $y = 5 + x$, where x represents his original daily pay and y represents his daily pay after the bonus.



Notice that each value was translated 5 units to the right. Thus, the mean, median, and mode increased by 5. Since the new minimum and maximum values also increased by 5, the range remained the same. The standard deviation is unchanged because the amount by which each value deviates from the mean stayed the same.

These results occur when any positive or negative number is added to every value in a set of data.

Key Concept Transformations Using Addition

If a real number k is added to every value in a set of data, then:

- the mean, median, and mode of the new data set can be found by adding k to the mean, median, and mode of the original data set, and
- the range and standard deviation will not change.



Example 1 Transformation Using Addition

Find the mean, median, mode, range, and standard deviation of the data set obtained after adding 7 to each value.

13, 5, 8, 12, 7, 4, 5, 8, 14, 11, 13, 8

Method 1 Find the mean, median, mode, range, and standard deviation of the original data set.

Mean	9	Mode	8	Standard Deviation	3.3
Median	8	Range	10		

Add 7 to the mean, median, and mode. The range and standard deviation are unchanged.

Mean	16	Mode	15	Standard Deviation	3.3
Median	15	Range	10		

Method 2 Add 7 to each data value.

20, 12, 15, 19, 14, 11, 12, 15, 21, 18, 20, 15

Find the mean, median, mode, range, and standard deviation of the new data set.

Mean	16	Mode	15	Standard Deviation	3.3
Median	15	Range	10		

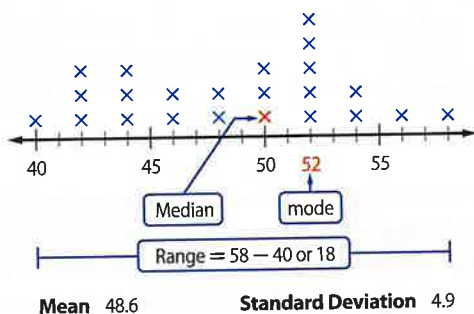
Guided Practice

1. Find the mean, median, mode, range, and standard deviation of the data set obtained after adding -4 to each value.

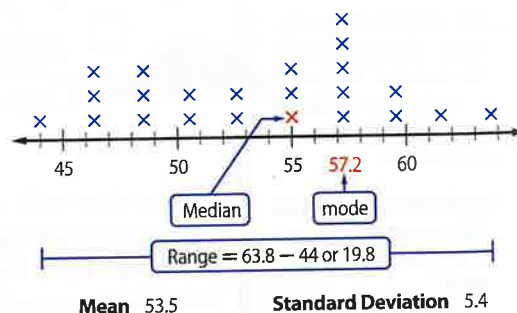
27, 41, 15, 36, 26, 40, 53, 38, 37, 24, 45, 26

To see the effect that a daily increase of 10% has on the data set, we can multiply each value by 1.10 and recalculate the measures of center and variation.

Tom's Earnings Before Extra 10%



Tom's Earnings With Extra 10%



Notice that each value did not increase by the same amount, but did increase by a factor of 1.10. Thus, the mean, median, and mode increased by a factor of 1.10. Since each value was increased by a constant percent and not by a constant amount, the range and standard deviation both changed, also increasing by a factor of 1.10.

KeyConcept Transformations Using Multiplication

If every value in a set of data is multiplied by a constant k , $k > 0$, then the mean, median, mode, range, and standard deviation of the new data set can be found by multiplying each original statistic by k .

TechnologyTip

1-Var Stats To quickly calculate the mean \bar{x} , median Med, standard deviation σ , and range of a data set, enter the data as L1 in a graphing calculator, and then press

STAT **▶** **ENTER**

ENTER. Subtract minX from maxX to find the range.

Since the medians for both bonuses are equal and the means are approximately equal, Tom should ask for the bonus that he thinks he has the best chance of receiving.



Example 2 Transformation Using Multiplication

Find the mean, median, mode, range, and standard deviation of the data set obtained after multiplying each value by 3.

21, 12, 15, 18, 16, 10, 12, 19, 17, 18, 12, 22

Find the mean, median, mode, range, and standard deviation of the original data set.

Mean	16	Mode	12	Standard Deviation	3.7
Median	16.5	Range	12		

Multiply the mean, median, mode, range, and standard deviation by 3.

Mean	48	Mode	36	Standard Deviation	11.1
Median	49.5	Range	36		

Guided Practice

- Find the mean, median, mode, range, and standard deviation of the data set obtained after multiplying each value by 0.8.

63, 47, 54, 60, 55, 46, 51, 60, 58, 50, 56, 60

2 Comparing Distributions Recall that when choosing appropriate statistics to represent data, you should first analyze the shape of the distribution. The same is true when comparing distributions.

- Use the mean and standard deviation to compare two symmetric distributions.
- Use the five-number summaries to compare two skewed distributions or a symmetric distribution and a skewed distribution.



Example 3 Compare Data Using Histograms

QUIZ SCORES Robert and Elaine's quiz scores for the first semester of Algebra 1 are shown below.

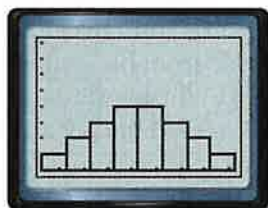
Robert's Quiz Scores
85, 95, 70, 87, 78, 82, 84, 84, 85, 99, 88, 74, 75, 89, 79, 80, 92, 91, 96, 81

Elaine's Quiz Scores
89, 76, 87, 86, 92, 77, 78, 83, 83, 82, 81, 82, 84, 85, 85, 86, 89, 93, 77, 85

- Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.

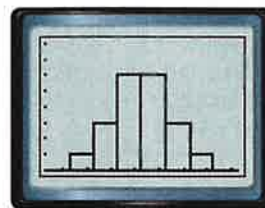
Enter Robert's quiz scores as L1 and Elaine's quiz scores as L2.

Robert's Quiz Scores



[69, 101] scl: 4 by [0, 8] scl: 1

Elaine's Quiz Scores



[69, 101] scl: 4 by [0, 8] scl: 1

Both distributions are high in the middle and low on the left and right. Therefore, both distributions are symmetric.

Technology Tip

Histograms To create a histogram for a set of data in L2, press **2nd** [STAT PLOT] **ENTER** **ENTER**, choose **▭**, and enter L2 for Xlist.



Technology Tip

Multiple Data Sets In order to calculate statistics for a set of data in L2, press

STAT ► ENTER
2nd [L2] ENTER

- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

Both distributions are symmetric, so use the means and standard deviations to describe the centers and spreads.

Robert's Quiz Scores



Elaine's Quiz Scores



The means for the students' quiz scores are approximately equal, but Robert's quiz scores have a much higher standard deviation than Elaine's quiz scores. This means that Elaine's quiz scores are generally closer to her mean than Robert's quiz scores are to his mean.

Guided Practice

COMMUTE The students in two of Mr. Martin's classes found the average number of minutes that they each spent traveling to school each day.

- 3A. Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.
- 3B. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

2nd Period (minutes)
8, 4, 18, 7, 13, 26, 12, 6, 20, 5, 9, 24, 8, 16, 31, 13, 17, 10, 8, 22, 12, 25, 13, 11, 18, 12, 16, 22, 25, 33

7th Period (minutes)
21, 4, 20, 13, 22, 6, 10, 23, 13, 25, 14, 16, 19, 21, 19, 8, 20, 18, 9, 14, 21, 17, 19, 22, 4, 19, 21, 26

Box-and-whisker plots are useful for comparisons of data because they can be displayed on the same screen.



Real-World Link

Only about 2% of all high school student athletes attend a college with the aid of athletic scholarships.

Source: NCAA

Real-World Example 4 Compare Data Using Box-and-Whisker Plots

FOOTBALL Kurt's total rushing yards per game for his junior and senior seasons are shown.

Junior Season (yards)					
16	20	72	4	25	18
34	10	42	17	56	12

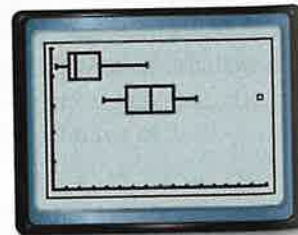
Senior Season (yards)					
77	54	109	60	156	72
39	83	73	101	46	80

- a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

Enter Kurt's rushing yards from his junior season as L1 and his rushing yards from his senior season as L2. Graph both box-and-whisker plots on the same screen by graphing L1 as Plot1 and L2 as Plot2.

For Kurt's junior season, the right whisker is longer than the left, and the median is closer to the left whisker. The distribution is positively skewed.

For Kurt's senior season, the lengths of the whiskers are approximately equal, and the median is in the middle of the data. The distribution is symmetric.



[0, 160] scl: 10 by [0, 5] scl: 1



StudyTip

Box-and-Whisker Plots

Recall that a box-and-whisker plot displays the spread of a data set by dividing it into four quartiles. Each quartile accounts for 25% of the data.

- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

One distribution is symmetric and the other is skewed, so use the five-number summaries to compare the data.

The upper quartile for Kurt's junior season was 38, while the minimum for his senior season was 39. This means that Kurt rushed for more yards in every game during his senior season than 75% of the games during his junior season.

The maximum for Kurt's junior season was 72, while his median for his senior season was 75. This means that in half of his games during his senior year, he rushed for more yards than in any game during his junior season. Overall, we can conclude that Kurt rushed for many more yards during his senior season than during his junior season.

GuidedPractice

BASKETBALL The points Vanessa scored per game during her junior and senior seasons are shown.

- 4A. Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.
- 4B. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

Junior Season (points)
10, 12, 6, 10, 13, 8, 12, 3, 21, 14, 7, 0, 15, 6, 16, 8, 17, 3, 17, 2

Senior Season (points)
10, 32, 3, 22, 20, 30, 26, 24, 5, 22, 28, 32, 26, 21, 6, 20, 24, 18, 12, 25

Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Example 1 Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

1. 10, 13, 9, 8, 15, 8, 13, 12, 7, 8, 11, 12; + (-7) 2. 38, 36, 37, 42, 31, 44, 37, 45, 29, 42, 30, 42; + 23

Example 2 Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

3. 6, 10, 3, 7, 4, 9, 3, 8, 5, 11, 2, 1; $\times 3$ 4. 42, 39, 45, 44, 37, 42, 38, 37, 41, 49, 42, 36; $\times 0.5$

Example 3 5. **TRACK** Mark and Kyle's long jump distances are shown.

Kyle's Distances (ft)
17.2, 18.28, 18.56, 17.28, 17.36, 18.08, 17.43, 17.71, 17.46, 18.26, 17.51, 17.58, 17.41, 18.21, 17.34, 17.63, 17.55, 17.26, 17.18, 17.78, 17.51, 17.83, 17.92, 18.04, 17.91

Mark's Distances (ft)
18.88, 19.24, 17.63, 18.69, 17.74, 19.18, 17.92, 18.96, 18.19, 18.21, 18.46, 17.47, 18.49, 17.86, 18.93, 18.73, 18.34, 18.67, 18.56, 18.79, 18.47, 18.84, 18.87, 17.94, 18.7

- a. Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.
- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.



Example 4

6. **TIPS** Miguel and Stephanie are servers at a restaurant. The tips that they earned to the nearest dollar over the past 15 workdays are shown.

Miguel's Tips (\$)
14, 68, 52, 21, 63, 32, 43, 35, 70, 37, 42, 16, 47, 38, 48

Stephanie's Tips (\$)
34, 52, 43, 39, 41, 50, 46, 36, 37, 47, 39, 49, 44, 36, 50

- a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.
- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

Practice and Problem Solving

Extra Practice is on page R12.

Example 1

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

7. 52, 53, 49, 61, 57, 52, 48, 60, 50, 47; + 8 8. 101, 99, 97, 88, 92, 100, 97, 89, 94, 90; + (-13)
9. 27, 21, 34, 42, 20, 19, 18, 26, 25, 33; + (-4) 10. 72, 56, 71, 63, 68, 59, 77, 74, 76, 66; + 16

Example 2

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

11. 11, 7, 3, 13, 16, 8, 3, 11, 17, 3; $\times 4$ 12. 64, 42, 58, 40, 61, 67, 58, 52, 51, 49; $\times 0.2$
13. 33, 37, 38, 29, 35, 37, 27, 40, 28, 31; $\times 0.8$ 14. 1, 5, 4, 2, 1, 3, 6, 2, 5, 1; $\times 6.5$

Example 3

15. **BOOKS** The page counts for the books that the students chose are shown.

1st Period
388, 439, 206, 438, 413, 253, 311, 427, 258, 511, 283, 578, 291, 358, 297, 303, 325, 506, 331, 482, 343, 372, 456, 267, 484, 227

6th Period
357, 294, 506, 392, 296, 467, 308, 319, 485, 333, 352, 405, 359, 451, 378, 490, 379, 401, 409, 421, 341, 438, 297, 440, 500, 312, 502

- a. Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.
- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.
16. **TELEVISIONS** The prices for a sample of televisions are shown.

The Electronics Superstore
46, 25, 62, 45, 30, 43, 40, 46, 33, 53, 35, 38, 39, 40, 52, 42, 44, 48, 50, 35, 32, 55, 28, 58

Game Central
53, 49, 26, 61, 40, 50, 42, 35, 45, 48, 31, 48, 33, 50, 35, 55, 38, 50, 42, 53, 44, 54, 48, 58

- a. Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.
- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

Example 4

17. **BRAINTEASERS** The time that it took Leon and Cassie to complete puzzles is shown.

Leon's Times (minutes)
4.5, 1.8, 3.2, 5.1, 2.0, 2.6, 4.8, 2.4, 2.2, 2.8, 1.8, 2.2, 3.9, 2.3, 3.3, 2.4

Cassie's Times (minutes)
2.3, 5.8, 4.8, 3.3, 5.2, 4.6, 3.6, 5.7, 3.8, 4.2, 5.0, 4.3, 5.5, 4.9, 2.4, 5.2

- a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.
- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.



18. **DANCE** The total amount of money that a sample of students spent to attend the homecoming dance is shown.

Boys (dollars)
114, 98, 131, 83, 91, 64, 94, 77, 96, 105, 72, 108, 87, 112, 58, 126

Girls (dollars)
124, 74, 105, 133, 85, 162, 90, 109, 94, 102, 98, 171, 138, 89, 154, 76

- a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.
- b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.
19. **LANDSCAPING** Refer to the beginning of the lesson. Rhonda, another employee that works with Tom, earned the following over the past month.

Rhonda's Pay (\$)		
45	55	53
47	53	54
44	56	59
63	47	53
60	57	62
44	50	45
60	53	49
62	47	

- a. Find the mean, median, mode, range, and standard deviation of Rhonda's earnings.
- b. A \$5 bonus had been added to each of Rhonda's daily earnings. Find the mean, median, mode, range, and standard deviation of Rhonda's earnings before the \$5 bonus.

20. **SHOPPING** The items Lorenzo purchased are shown.

- a. Find the mean, median, mode, range, and standard deviation of the prices.
- b. A 7% sales tax was added to the price of each item. Find the mean, median, mode, range, and standard deviation of the items without the sales tax.

<i>Baseball hat</i>	<i>\$14.98</i>
<i>Jeans</i>	<i>\$24.61</i>
<i>T-shirt</i>	<i>\$12.84</i>
<i>T-shirt</i>	<i>\$16.05</i>
<i>Backpack</i>	<i>\$42.80</i>
<i>Folders</i>	<i>\$2.14</i>
<i>Sweatshirt</i>	<i>\$19.26</i>

H.O.T. Problems Use Higher-Order Thinking Skills

21. **CHALLENGE** A salesperson has 15 SUVs priced between \$33,000 and \$37,000 and 5 luxury cars priced between \$44,000 and \$48,000. The average price for all of the vehicles is \$39,250. The salesperson decides to reduce the prices of the SUVs by \$2000 per vehicle. What is the new average price for all of the vehicles?
22. **REASONING** If every value in a set of data is multiplied by a constant k , $k < 0$, then how can the mean, median, mode, range, and standard deviation of the new data set be found?
23. **WRITING IN MATH** Compare and contrast the benefits of displaying data using histograms and box-and-whisker plots.
24. **CCSS REGULARITY** If k is added to every value in a set of data, and then each resulting value is multiplied by a constant m , $m > 0$, how can the mean, median, mode, range, and standard deviation of the new data set be found? Explain your reasoning.
25. **WRITING IN MATH** Explain why the mean and standard deviation are used to compare the center and spread of two symmetrical distributions and the five-number summary is used to compare the center and spread of two skewed distributions or a symmetric distribution and a skewed distribution.

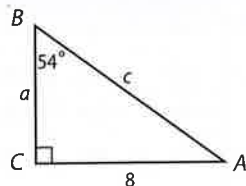


Standardized Test Practice

26. A store manager recorded the number of customers each day for a week: {46, 57, 63, 78, 91, 110, 101}. Find the mean absolute deviation.

A 16.8 C 19.4
B 18.1 D 22.7

27. **SHORT RESPONSE** Solve the right triangle. Round each side length to the nearest tenth.



28. A research company divides a group of volunteers by age, and then randomly selects volunteers from each group to complete a survey. What type of sample is this?

F simple H self-selected
G systematic J stratified

29. Which set of measures can be the measures of the sides of a right triangle?

A 6, 7, 9
B 9, 12, 19
C 12, 15, 17
D 14, 48, 50

Spiral Review

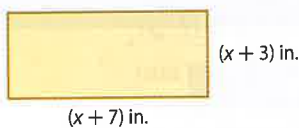
30. Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution. (Lesson 12-3)

23, 45, 50, 22, 37, 24, 36, 46, 24, 52, 25, 42, 25, 26, 54, 47, 27, 55
63, 28, 29, 30, 45, 31, 55, 43, 32, 34, 30, 23, 30, 35, 27, 35, 38, 40

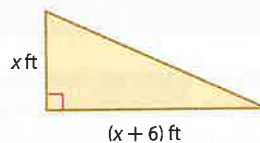
31. **SUBSCRIPTIONS** Ms. Wilson's students are selling magazine subscriptions. Her students recorded the total number of subscriptions they each sold: {8, 12, 10, 7, 4, 3, 0, 4, 9, 0, 5, 3, 23, 6, 2}. Find and interpret the standard deviation of the data set. (Lesson 12-2)

Find the value of x for each figure. Round to the nearest tenth if necessary. (Lesson 9-4)

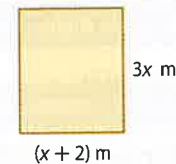
32. $A = 45 \text{ in}^2$



33. $A = 20 \text{ ft}^2$



34. $A = 42 \text{ m}^2$



Factor each polynomial. (Lesson 8-6)

35. $x^2 - 4x - 21$

36. $11x + x^2 + 30$

37. $32 + x^2 - 12x$

38. $-36 - 9x + x^2$

39. $x^2 + 12x + 20$

40. $-x + x^2 - 42$

41. **MANUFACTURING** A company is designing a box for dry pasta in the shape of a rectangular prism. The length is 2 inches more than twice the width, and the height is 3 inches more than the length. Write an expression for the volume of the box. (Lesson 8-3)

Skills Review

Find the degree of each polynomial.

42. $2x^2 + 5y - 21$

43. $16xy^3 - 17x^2y - 16z^3$

44. $3ac^3d + 14a^2$

45. 18

46. $3a^2b^3 + 11ab^2c$

47. $7x + 11$



Mid-Chapter Quiz

Lessons 12-1 through 12-4

Identify each sample, and suggest a population from which it was selected. Then classify the sample as *simple*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain your reasoning. (Lesson 12-1)

- CEREAL** A cereal company invites 100 random children and parents to test a new cereal and records the reactions.
- SCHOOL LUNCH** A school is creating a new lunch menu. They send out a questionnaire to all students with odd homeroom numbers to determine what items should be on the new menu.
- MASCOTS** The cheerleaders send out a flyer with pictures of possible options for the new mascot to all the girls in the school. The girls mark their favorite mascot and send it back. The new mascot is chosen from the survey.

Identify each sample as *biased* or *unbiased*. Explain your reasoning. (Lesson 12-1)

- ART** Every fifth person leaving the art museum is asked to name their favorite piece.
- SHOPPING** Each person leaving the Earring Pagoda is asked to name their favorite store in the mall.
- FOOTBALL** Every 10th student leaving the student union at Ohio State is asked to name their favorite college football team.

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter. (Lesson 12-2)

- DINING** At a restaurant, a random sample of 15 diners is selected. The amount of money spent on each meal is recorded.
- POOLS** A random sample of 25 children at a community pool is asked if they visit the pool at least once each week. The percent responding *yes* is calculated.
- PLAY AREA** Ian listed the ages of the children playing at the play area at the mall. Find and interpret the standard deviation of the data set. (Lesson 12-2)

{2, 3, 2, 2, 4, 2, 3, 2, 8, 3, 4, 2}

- MULTIPLE CHOICE** Several friends are chipping in to buy a gift for their teacher. Indigo is keeping track of how much each friend spends. Find the mean absolute deviation. (Lesson 12-2)

{\\$10, \\$5, \\$3, \\$6, \\$7, \\$8}

- A \$1.83 C \$2.40
B \$2.22 D \$6.50

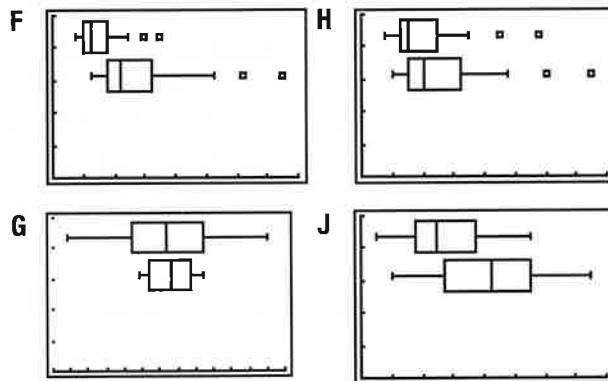
- Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution. (Lesson 12-3)

19, 36, 26, 36, 40, 31, 30, 33, 23, 38, 23, 46

- Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data. (Lesson 12-3)

9, 11, 2, 6, 8, 10, 6, 3, 10, 11, 9, 8, 3,
8, 5, 11, 14, 6, 8, 6, 11, 5, 9, 10, 8

- MULTIPLE CHOICE** Which pair of box-and-whisker plots depicts two positively skewed sets of data in which 75% of one set of data is larger than 75% of the other set of data? (Lesson 12-4)



Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value. (Lesson 12-4)

- 6, 9, 0, 15, 9, 14, 11, 13, 9, 5, 8, 6; + (-3)
- 19, 22, 10, 17, 26, 24, 12, 22, 18, 17; + 8

12-5 Simulations



Then

- You calculated simple probability.

Now

- 1 Calculate experimental probabilities.
- 2 Design simulations and summarize data from simulations.

Why?

- Alex has been practicing his penalty kicks. He expects to be able to make at least 66% of his penalty kicks. To test this, he takes 50 penalty kicks, of which he makes 35.



New Vocabulary

theoretical probability
experimental probability
relative frequency
simulation
probability model



Common Core State Standards

Mathematical Practices
4 Model with mathematics.

1 Experimental Probability A **theoretical probability** is the ratio of the number of favorable outcomes to the total number of outcomes. The theoretical probability that a coin lands heads up is $\frac{1}{2}$ or 50%. The theoretical probability suggests that 5 of 10 tosses will most likely result in the coin landing heads up.

We can toss a coin 10 times and record the outcomes. The **experimental probability**, or **relative frequency**, is determined using data from experiments. It is the ratio of the number of times an outcome occurs to the total number of events or trials. In this case, the experimental probability of tossing the coin and having it land heads up is $\frac{3}{10}$ or 30%.

Outcome	Frequency
heads	3
tails	7



Standardized Test Example 1 Find Experimental Probability

A die is rolled 50 times and the results are recorded. Find the experimental probability of rolling at least a 5.

- A $\frac{6}{50}$ B $\frac{7}{50}$ C $\frac{13}{50}$ D $\frac{37}{50}$

Read the Test Item

We are asked to find the probability of rolling at least a 5. Therefore, we need to consider rolling a 5 and rolling a 6.

Solve the Test Item

$$P(\text{at least a 5}) = \frac{\text{number of 5s and 6s rolled}}{\text{total number of rolls}}$$

$$= \frac{6 + 7}{50} \text{ or } \frac{13}{50}$$

The experimental probability of rolling at least a 5 is $\frac{13}{50}$ or 26%.

The correct answer is C.

Outcome	Frequency
1	6
2	10
3	14
4	7
5	6
6	7

Guided Practice

- Students were asked how they travel to school each morning. Find the experimental probability of randomly selecting a student who does not ride in a car or bus.

- F $\frac{9}{47}$ G $\frac{17}{47}$ H $\frac{21}{47}$ J $\frac{38}{47}$

Mode	Frequency
bike	3
bus	21
car	17
walk	6



Notice that the experimental probability of rolling a 3 was $\frac{7}{25}$ or 28%, but the theoretical probability is $\frac{1}{6}$ or about 16.7%. This is because theoretical probability is what is *expected* to happen and experimental probability is what *actually* happens.

2 Simulations An experiment that would be difficult or impractical to perform can be modeled by a **simulation**. In a simulation, a **probability model** is used to recreate a situation so that the experimental probability of an outcome can be found. A probability model is a mathematical model used to represent the theoretical probability of the outcomes in an experiment. Coins, dice, random number generators, or other objects can be used as probability models.

To design a simulation, use the following steps.

KeyConcept Designing a Simulation

- Step 1** Determine each possible outcome and its theoretical probability.
- Step 2** Describe an appropriate probability model for the situation that accurately represents the theoretical probability of each outcome.
- Step 3** Define what a trial is for the situation, and state the number of trials to be conducted.



Real-World Example 2 Design a Simulation

QUALITY CONTROL Eloy inspects bike frames as they come through the assembly line. From previous observations, he expects to find a weld defect in one out of every ten frames and a design defect in one out of every twenty frames that he inspects. Design a simulation using Eloy's expectation of defects. Assume that a frame can only have one of the defects.

- Step 1** There are three possible outcomes: weld defect, design defect, and no defects. Use Eloy's expectation of defects to calculate the theoretical probability of each outcome.

Possible Outcomes	Theoretical Probability
Weld defect	10%
Design defect	5%
No defects	85%

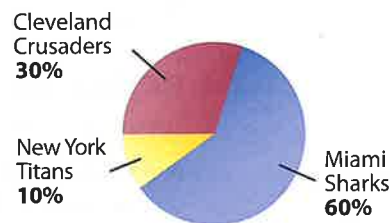
- Step 2** We can use the random number generator on a graphing calculator. Assign the integers 0–19 to accurately represent the probability data.

Outcome	Represented by
Weld defect	0, 1
Design defect	2
No defects	3–19

- Step 3** A trial will represent selecting a frame at random. The simulation can consist of any number of trials. We will use 40.

GuidedPractice

- 2. FREE AGENT** A survey asked team executives for which team they thought free agent Bonny Solomon would choose to play next season. The results are shown. Design a simulation that can be used to estimate the probability that Bonny chooses one of these three teams.



Real-World Career

Assemblers Assemblers may work as part of a team, where all members are capable of performing each task. In the automobile manufacturing industry, the mean wage for a team assembler in 2009 was \$49,360.

Source: Bureau of Labor Statistics

StudyTip

CCSS Modeling Simulations should involve data that are easier to obtain than the actual data you are modeling. Another probability model for Example 2 would be to place 20 marbles in a bag letting 2 red marbles represent a weld defect, 1 orange marble represent a design defect, and 17 yellow marbles represent no defects.



After designing a simulation, conduct it and analyze the results.



Technology Tip

Random Number Generator After running the random number generator, you can store the results as a list by pressing **STO** and entering L1.



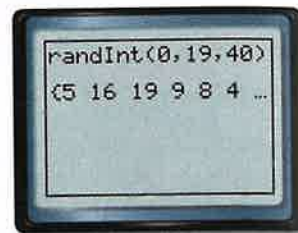
Math History Link

Ada Byron Lovelace (1815–1852) Lovelace is known for generating a sequence of numbers using an early model of a computer. She is credited with being the first computer programmer. In 1980, the U.S. Department of Defense named a programming language after her.

Example 3 Conduct and Evaluate a Simulation

QUALITY CONTROL Refer to the simulation in Example 2. Conduct the simulation and report the results.

Press **MATH** and select **randInt** (. Then press 0 , 19 , 40) **ENTER**. Use the left and right arrow buttons to view the results. Make a frequency table and record the results.



Outcome	Tally	Frequency
Weld defect		3
Design defect		2
No defects		35
Total		40

Calculate the experimental probability of finding each type of defect.

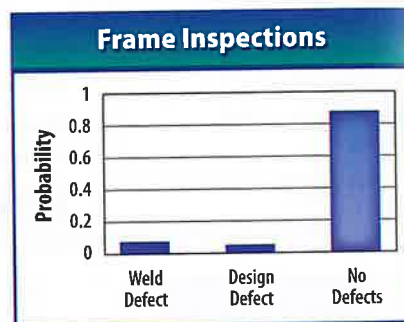
Weld defect $\frac{3}{40}$ or 0.075

Design defect $\frac{2}{40}$ or 0.05

No defects $\frac{35}{40}$ or 0.875

The experimental probabilities that a frame will have a weld defect, a design defect, or no defects in this case are 7.5%, 5%, and 87.5%, respectively. Notice that the probabilities are close to, and in the case of a design defect, the same as, the theoretical probabilities.

Make a bar graph of these results.



Guided Practice

3. FREE AGENT Conduct the simulation in Guided Practice 2. Then report the results.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **1. MULTIPLE CHOICE** A movie theater employee surveyed a sample of customers as they exited the theater. Find the experimental probability of randomly selecting a customer who is older than 12 but younger than 46.

- A $\frac{24}{127}$
- B $\frac{157}{254}$
- C $\frac{99}{127}$
- D $\frac{213}{254}$

Examples 2–3 **2. FOOTBALL** Rico is the kicker on the football team. Last season, he made 94% of his extra points.

- a. Design a simulation that can be used to estimate the probability that Rico will make his next extra point.
- b. Conduct the simulation, and report the results.

Age	Frequency
0–7	13
8–12	28
13–17	48
18–23	42
24–31	27
32–45	40
46–64	33
65+	23



- Example 1** 3 **CARDS** Javier is drawing a card from a standard deck of cards, recording the suit, and then replacing the card in the deck. The table below shows his results.

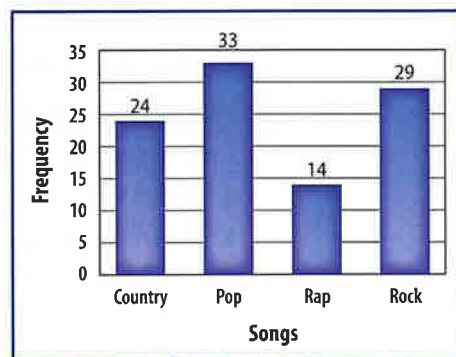
Suit	clubs	diamonds	hearts	spades
Frequency	7	4	5	9

- Find the experimental probability of drawing a heart.
- Find the experimental probability of drawing a black card.
- Javier repeated his test. The results of the second test are shown below. Combine the results from the second test with the results from the first test, and then find the experimental probability of drawing a spade.

Suit	clubs	diamonds	hearts	spades
Frequency	5	8	6	6

4. **MUSIC** Shannon's digital media player has a large collection of songs. She randomly toggles through the songs and then records the genre. The graph shows her results.

- Find the experimental probability of selecting a country song.
- Find the experimental probability of selecting a song that is not rock.



- Examples 2–3** 5. **BATTING AVERAGE** In a computer baseball game, a player has a batting average of .300. That is, he gets a hit 300 out of 1000, or 30%, of the times he is at bat.

- Design a simulation that can be used to estimate the probability that the player will get a hit at his next at bat.
- Conduct the simulation, and report the results.

6. **JEANS** Julie examines the stitching on pairs of jeans that are produced at a manufacturing plant. She expects to find defects in 1 out of every 20 pairs.

- Design a simulation that can be used to estimate the probability that the next pair of jeans that Julie examines has a defect.
- Conduct the simulation, and report the results.

7. **FOOD** For a promotion, the concession stands at a football stadium are giving away free items. For every tenth customer, a wheel is spun to choose the customer's prize. Each prize is equally likely.

- Design a simulation that can be used to estimate the probability that the next spin is one of the five prizes.
- Conduct the simulation, and report the results.



8. **CCSS MODELING** For its twentieth anniversary, a store randomly gives each customer a prize from the following choices: a free music download, a free game download, a free bag of popcorn, or a free DVD. The chances of winning each prize are equal.

- Design a simulation that can be used to estimate the probability that the next prize given is one of the four prizes.
- Conduct the simulation, and report the results.



- 9 GAMES** Games at the fair require the majority of players to lose in order for game owners to make a profit. New games are tested to make sure they have sufficient difficulty. The results of three test groups are listed in the table. The owners want a maximum of 33% of players to win. There were 50 participants in each test group.

- a. What is the experimental probability that the participant was a winner in the second group?
- b. What is the experimental probability of winning for all three groups?

Result	Group 1	Group 2	Group 3
Winners	13	15	19
Losers	37	35	31

- c. **DECISION MAKING** Should this game be used? Explain your reasoning.

- 10. TEST** Jack forgot to study for his multiple-choice science quiz and is going to guess for each question. There are 20 questions, each with 4 possible answers.

- a. Design a simulation that can be used to estimate the number of questions that Jack answers correctly.
- b. Conduct the experiment from part a five times, and complete the table.
- c. How many should Jack expect to answer correctly?

Simulation	Number of Correct Answers
1	
2	
3	
4	
5	

- 11. MULTIPLE REPRESENTATIONS** In this problem, you will explore the effect the number of trials has on the experimental probability of an event.

- a. **Verbal** What is the probability of rolling a 1 on a die?
- b. **Analytical** Design a simulation that can be used to estimate the probability that the next number rolled on a die is a 1.
- c. **Analytical** Conduct the simulation from part b for 10, 20, 50, and 100 trials, and complete the table.
- d. **Analytical** As the number of trials increases, what is happening to the experimental probability?
- e. **Verbal** The *Law of Large Numbers* states that as the number of trials increases, the experimental probability gets closer to the theoretical probability. If you continued the simulation, each time increasing the number of trials, what would you expect the experimental probability for rolling a 1 to approach?

Rolling a 1		
Trials	Frequency	Experimental Probability
10		
20		
50		
100		

H.O.T. Problems Use Higher-Order Thinking Skills

- 12. CCSS ARGUMENTS** The experimental probability of heads when a coin is tossed 15 times is *sometimes, never, or always* equal to the theoretical probability. Explain your reasoning.
- 13. OPEN ENDED** Describe a situation at your school that could be represented by a simulation. Then design the simulation.
- 14. CHALLENGE** *True or false:* If the theoretical probability of an event is 1, the experimental probability of the event cannot be 0. Explain your reasoning.
- 15. REASONING** Jeremy tosses a coin several times and finds that the experimental probability for it landing heads up is 25%. Should Jeremy be concerned about the fairness of the coin? Explain your reasoning.
- 16. WRITING IN MATH** What should you consider when using the results of a simulation to make a prediction?



Standardized Test Practice

17. GEOMETRY Suppose a covered water tank in the shape of a right circular cylinder is thirty feet long and eight feet in diameter. What is the surface area of the cylinder?

- A $272\pi \text{ ft}^2$ C $224\pi \text{ ft}^2$
 B $248\pi \text{ ft}^2$ D $153\pi \text{ ft}^2$

18. SHORT RESPONSE Two consecutive numbers have a sum of 91. What are the numbers?

19. Solve $\frac{2x}{x-2} + \frac{8}{x} = 6$.

- F 1 H 2
 G 1, 4 J 2, 6

20. Mr. Bahn has \$20,000 to invest. He invests part at 6% and the rest at 7%. He earns \$1280 in interest within a year. How much did he invest at 7%?

- A \$12,000 C \$9950
 B \$11,275 D \$8000

Spiral Review

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value. (Lesson 12-4)

- 21.** 12, 16, 4, 8, 7, 11, 9, 4; + 5 **22.** 1, 4, 3, 9, 12, 6, 7, 3; + 12 **23.** 18, 12, 8, 13, 7, 15, 8, 6; + (-3)

24. DANCE RECITAL The number of dance students in each act of a dance recital is shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data. (Lesson 12-3)

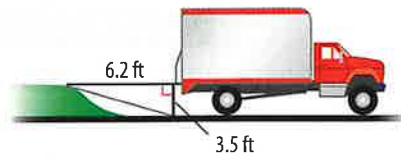
Number of Dance Students														
13	15	1	20	14	4	18	2	17	10	22	1	22	15	17
21	10	18	14	18	2	10	20	15	18	4	19	12	16	5

25. PARTIES Student Council is planning a party for the school volunteers. There are five 66-ounce unopened bottles of soda left from a recent dance. When poured over ice, $5\frac{1}{2}$ ounces of soda fills a cup. How many servings of soda do they have? (Lesson 11-7)

Write an inverse variation equation that relates x and y . Assume that y varies inversely as x . Then solve. (Lesson 11-1)

- 26.** If $y = 8.5$ when $x = -1$, find x when $y = -1$. **27.** If $y = 8$ when $x = 1.55$, find x when $y = -0.62$. **28.** If $y = 6.4$ when $x = 4.4$, find x when $y = 3.2$.

29. DELIVERY Ben and Amado are delivering a freezer. The bank in front of the house is the same height as the back of the truck. They set up their ramp as shown. What is the length of the slanted part of the ramp to the nearest tenth of a foot? (Lesson 10-5)



Skills Review

Write each fraction as a percent rounded to the nearest whole number.

- 30.** $\frac{26}{58}$ **31.** $\frac{55}{125}$ **32.** $\frac{14}{128}$
33. $\frac{82}{110}$ **34.** $\frac{76}{124}$ **35.** $\frac{23}{86}$





Then

- You used the Fundamental Counting Principle.

Now

- 1 Use permutations.
- 2 Use combinations.

Why?

- Angie's coach told her that she would bat sixth in the softball game. When a coach decides on the team's lineup before a game, the order in which she fills in the names is important because it determines the order in which the players will bat.



New Vocabulary

- permutation
- factorial
- combination



Common Core State Standards

Mathematical Practices

- 8 Look for and express regularity in repeated reasoning.

1 Permutations When the objects in a sample space are arranged so that order is important, each possible arrangement is called a **permutation**. Three permutations of batting orders for the first four batters are shown below.



Use the Fundamental Counting Principle to find the total number of permutations.

$$\begin{array}{ccccccccc} & \text{number of} & & \text{choices for} & & \text{choices for} & & \text{choices for} & & \text{choices for} \\ & \text{permutations} & & \text{1st batter} & & \text{2nd batter} & & \text{3rd batter} & & \text{4th batter} \\ \text{Permutations} & = & 4 & \cdot & 3 & \cdot & 2 & \cdot & 1 \\ & = & 24 & & & & & & & \end{array}$$

There are 24 ways to arrange the first four batters. The expression used to find the number of arrangements, $4 \cdot 3 \cdot 2 \cdot 1$, can be written as $4!$, which is read *4 factorial*.

KeyConcept Factorial

- Words** The **factorial** of a positive integer n is the product of the positive integers less than or equal to n . $0!$ is defined to be 1.
- Symbols** $n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 1$, where $n \geq 1$



Example 1 Permutations Using Factorials

TRAVEL A travel agency is planning a European vacation package. How many ways can the agency arrange the 5 cities along the tour?

There are five choices for the first city, four choices for the second city, and so on.

$$\begin{aligned} \text{Number of ways to arrange the cities} &= 5! \\ &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 120 \end{aligned}$$

There are 120 ways to arrange the cities.

Guided Practice

- MOVIES** Lloyd and five friends go to a movie. In how many different ways can they sit together in a row of 6 empty seats?



Suppose Angie's coach had 5 players in mind for the first 3 spots in the lineup. How many different batting orders could she make?

$$\begin{array}{ccccccc} \text{choices for} & & \text{choices for} & & \text{choices for} & & \\ \text{1st batter} & & \text{2nd batter} & & \text{3rd batter} & & \\ 5 & \cdot & 4 & \cdot & 3 & = & 60 \text{ permutations} \end{array}$$

Once the 3 spots are filled, there are $5 - 3$ or 2 players left over. Notice that $5 \cdot 4 \cdot 3$ can also be written as $= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{5!}{2!}$ or $\frac{5!}{(5-3)!}$. This result is generalized below.

ReadingMath

Notation The number of permutations $P(n, r)$ of n objects taken r at a time can also be written as ${}_n P_r$.

KeyConcept Permutation Formula

Words The number of permutations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!$.

Symbols $P(n, r) = \frac{n!}{(n - r)!}$

Example 2 Use the Permutation Formula



LIBRARY The librarian is placing 6 of 10 magazines in a school showcase. How many ways can she arrange the magazines in the case?

$$P(n, r) = \frac{n!}{(n - r)!} \quad \text{Permutation Formula}$$

$$P(10, 6) = \frac{10!}{(10 - 6)!} \quad n = 10 \text{ and } r = 6$$

$$= \frac{10!}{4!} \quad \text{Simplify.}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \quad \text{Divide by common factors.}$$

$$= 151,200 \quad \text{Simplify.}$$

There are 151,200 ways for the librarian to arrange the magazines.

GuidedPractice

- TROPHIES** James wants to place 5 of his 8 trophies on the fireplace. How many ways can he arrange the trophies?

StudyTip

Permutations and Combinations If order matters in a group, the group is a *permutation*. If order does not matter in a group, the group is a *combination*.

2 Combinations Recall that the coach had 60 possible lineups when considering 5 players for 3 spots. What if order didn't matter and he only cares if players are in the lineup? A selection of objects in which order is *not* important is called a **combination**. From how many combinations of 3 players can the coach choose?



This same combination of players can be arranged $3!$ or 6 different ways. So 6 out of the 60 possible lineups include Angie, Beth, and Cora. The same will be true of every other combination. Therefore, out of the 60 possible lineups, there are $\frac{60}{6}$ or 10 different combinations of players.



Notice that we divided the number of permutations by the number of possible arrangements for every combination of elements $r!$ to obtain the number of combinations.

StudyTip

Combination Formula

The number of combinations is the quotient of the number of permutations of n objects taken r at a time and the number of permutations of r objects.

KeyConcept Combination Formula

Words The number of combinations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!r!$.

Symbols
$$C(n, r) = \frac{n!}{(n - r)!r!}$$



Example 3 Use the Combination Formula

RETAIL Marques works part-time at a local department store. His manager asked him to make a display using 5 different styles of shirts from 8 available styles. How many ways can Marques choose the shirts?

$$C(n, r) = \frac{n!}{(n - r)!r!} \quad \text{Combination Formula}$$

$$C(8, 5) = \frac{8!}{(8 - 5)!5!} \quad n = 8 \text{ and } r = 5$$

$$= \frac{8!}{3!5!} \quad \text{Simplify.}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \quad \text{Divide by common factors.}$$

$$= \frac{336}{6} \text{ or } 56 \quad \text{Simplify.}$$

GuidedPractice

3. **SPRING DANCE** A group of four students is selecting corsages and boutonnieres to wear to the spring dance. They can choose from 4 roses, 6 carnations, and 8 tulips. If all of the flowers are different, in how many ways can 4 flowers be chosen?

Part of the difficulty in learning permutations and combinations is distinguishing between them. Remember that order matters with permutations.



Example 4 Identifying Permutations and Combinations

Identify each situation as a *permutation* or a *combination*.

- a. A playlist of songs on a digital media player is being played so that songs do not repeat. In how many ways can the songs be played?

Since the question is really about the different orders of songs, this is a permutation.

- b. Any orders an ice cream cone with three different flavors. There are 26 possible flavors. How many different sets of three flavors can she choose?

Since the order in which the flavors are selected is not important, this is a combination.

GuidedPractice

- 4A. A teacher uses a random number generator to create a seating chart for 20 students.
- 4B. Ten athletes enter a race. The top three finishers move on to the next round.

WatchOut!

Wording Notice that if we were asked for the order that the scoops were placed on the cone, then this would have been a permutation.



We can use permutations or combinations to find the probability of an event.



Example 5 Probability with Permutations and Combinations

SPANISH CLUB The Spanish club is forming two teams for the International Festival. Rebekah and Lydia are among the nine who volunteered to lead a team. If the advisor assigns positions at random, what is the probability that Rebekah is chosen to lead the food team and Lydia is chosen to lead the tickets team?

Step 1 Find the total number of outcomes.

Since we care about specific positions, this is a permutation. Find the number of permutations of 9 people taken 2 at a time.

$$P(n, r) = \frac{n!}{(n-r)!} \quad \text{Permutation Formula}$$

$$P(9, 2) = \frac{9!}{(9-2)!} \text{ or } 72 \quad n = 9 \text{ and } r = 2$$

There are 72 possible outcomes

Step 2 Find the successes.

Of the 72 permutations, only one has Rebekah leading the food team and Lydia leading the tickets team.

Step 3 Find the probability.

$$P = \frac{\text{number of successes}}{\text{number of possible outcomes}} = \frac{1}{72} \quad \text{Probability Formula}$$

The probability of Rebekah leading the food team and Lydia leading the tickets team is $\frac{1}{72}$.

StudyTip

Probability Find the probability of an event by dividing the number of favorable outcomes by the number of possible outcomes.

Guided Practice

5. STUDENT COUNCIL Four of the twelve members of the student council are being randomly selected to hand out flyers at the mall. What is the probability that Hasina, Amy, Bonny, and Tim are all selected?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.



Example 1 **1. CHARITY** A youth charity group is holding a raffle and wants to display a picture of the 6 prizes on a flyer. How many ways can they arrange the prizes in a row?

Examples 2–4 Identify each situation as a *permutation* or a *combination*.

- choosing 3 different pizza toppings from a list of 12
- choosing team captains for a football team
- choosing the first-, second-, and third-place winner of an art competition

Evaluate each expression.

5. $P(7, 2)$ 6. $P(9, 3)$ **7** $C(6, 4)$ 8. $C(5, 2)$

9. MENUS There are 14 toppings listed in a pizza menu. How many different six-topping pizzas are possible?

Example 5 **10. CCSS MODELING** Students are given 5-digit passwords for their accounts on the school's computer system. If no numbers can repeat, what is the probability that a student's password is 93152?



- Example 1** 11. **SCIENCE FAIR** There are 8 finalists in a science fair competition. How many ways can they stand on the stage?

Evaluate each expression.

12. $P(6, 6)$ 13. $P(5, 1)$ 14. $P(4, 1)$ 15. $P(7, 3)$
 16. $C(7, 6)$ 17. $C(5, 3)$ 18. $C(5, 5)$ 19. $C(3, 0)$

Examples
2–4

Identify each situation as a *permutation* or a *combination*.

20. selecting 5 books to read from a list of 8
 21. an arrangement of the letters in the word *probability*
 22. a list of students by class ranking
 23. a playlist of songs on a digital media player
 24. selecting 4 different ingredients out of 8 for a salad
 25. **CCSS TOOLS** Abigail works at the jewelry store in the mall. Her manger asks her to place 3 of the 12 birthstone necklaces in the front display case. How many ways can she arrange the necklaces in the display case?



26. **RECYCLING** Juana is setting two recycling bins at the end of her driveway for pick-up. She has four bins from which to choose. How many ways can she pick the bins to set out?

Example 5

27. **MUSIC** What is the probability that the first 8 songs that are played on Kenneth's playlist are country songs?

28. **AMUSEMENT PARKS** Tino is entering an amusement park with 5 of his friends. At the gate they must go through a turnstile one at a time. How many ways can Tino and his friends go through the turnstile?

29. **PAGEANTS** The Teen Miss USA pageant has 51 delegates. If the judges choose Teen Miss USA and four runners-up, how many ways can they be chosen?

30. **BASKETBALL** A coach has to select 5 of 12 players on the basketball team to start the game. How many different groups of players could be selected as starters?

31. **ICE CREAM** How many ways can a customer choose 3 flavors of ice cream at The Dairy Barn?

32. **GAMES** Tonisha is playing a board game in which you make words to score points. There are 12 letters left in the box, and she must choose 4. She cannot see the letters.

a. Suppose the 12 letters are all different. How many ways can she choose 4 of the 12?

b. She chooses *A*, *T*, *R*, and *E*. How many different arrangements of three letters can she make from these letters?

c. How many of the three-letter arrangements are words? List them.

33. **DANCE** At the spring dance, Christy and 7 of her friends sit on one side of a table. How many ways can Christy and her friends fill the 8 empty seats?

Exercise Playlist

Country	8 songs
Rock	6 songs
Rap	4 songs



THE DAIRY BARN

5 varieties of chocolate
4 varieties of candy-flavor
6 varieties of berry-flavor



34. **HORSEBACK RIDING** Trish and Charliqua entered a horseback riding camp with 22 other people. Six riders are randomly selected to work with the head instructor. How many different groups of people can be placed with the head instructor?
35. **BOWLING** Chris and Kelly entered a bowling tournament.
- What is the probability that Chris and Kelly are selected to bowl against each other in *any* lane?
 - What is the probability that Chris and Kelly are selected to bowl against each other in the *last* lane?
36. **SECURITY** Banks lock an account after three incorrect PIN entries. Suppose a bank that uses four-digit PINs in which the digits cannot repeat allowed unlimited incorrect entries. What is the maximum time it would take a hacker using a computer program that can enter 100 different codes per second to enter the correct PIN?
37. **DECISION MAKING** Westerville High School is putting on a play. In all, 4 freshmen, 5 sophomores, 6 juniors, and 8 seniors tried out for the 12 open spots.
- How many ways can the 12 spots be chosen?
 - If the students are chosen randomly, what is the probability that at least one senior will be chosen?
 - What probability model can we use to randomly select the first spot?
 - How does this model change when selecting the second spot?
38. **COMPOUND PROBABILITY** There are 7 red marbles, 8 purple marbles, and 6 green marbles in a bag. Fifteen marbles are randomly selected. Use the following steps to determine the probability of selecting 5 of each color.
- Find the number of possible outcomes.
 - Find the number of successes for selecting 5 of 7 red marbles, for selecting 5 of 8 purple marbles, and for selecting 5 of 6 green marbles.
 - Use the Fundamental Counting Principle to find the total number of successes.
 - Determine the probability of selecting 5 of each color.



H.O.T. Problems Use Higher-Order Thinking Skills

39. **ERROR ANALYSIS** Sydney and Ming are determining how many 4-person committees are possible if 10 people are available. Is either of them correct? Explain.

<p><i>Sydney</i></p> ${}_{10}P_4 = \frac{10!}{(10-4)!}$ $= 5040$	<p><i>Ming</i></p> ${}_{10}C_4 = \frac{10!}{(10-4)!4!}$ $= 210$
--	---

40. **CCSS PERSEVERANCE** Seven identical mathematics books and 4 identical science books are to be stored on one shelf. How many different ways can the books be arranged?
41. **WHICH ONE DOESN'T BELONG?** Determine which situation does not belong. Explain.

choosing 5 players on a quiz team

choosing 10 colored marbles from a bag

choosing 4 horses from 6 to run a race

ranking students in a senior class

42. **REASONING** Determine whether the statement ${}_nP_r = {}_nC_r$ is *sometimes*, *always*, or *never* true. Explain your reasoning.
43. **WRITING IN MATH** Write a situation in which 3 of 8 objects are selected and order is not important.



Standardized Test Practice

44. In how many ways can 3 of 8 different flowers be planted along one side of a road?
 A 342 B 338 C 336 D 328
45. If Jack can eat 21 hard-boiled eggs in 15 minutes, how many can he eat in 25 minutes if he continues eating at the same pace?
 F 18 G 35 H 36 J 37
46. Shante has 30 coins, quarters and dimes, that total \$5.70. How many quarters does she have?
 A 12 C 18
 B 15 D 20
47. **SHORT RESPONSE** There are 3 red candies in a bag of 20 candies. What is the probability of selecting a red candy?

Spiral Review

Vacationers were asked how many evenings they spent eating out during their trip. Find each experimental probability. (Lesson 12-5)

Number of Evenings	Frequency
0	14
1	39
2	28
3	21
4	10
5+	13

48. a vacationer ate out at least once
49. a vacationer ate out less than three times
50. Find the mean, median, mode, range, and standard deviation of the data set obtained after multiplying each value by 1.3. (Lesson 12-4)

26, 15, 19, 31, 47, 44, 38, 26, 28, 19

51. **PET CARE** Kendra takes care of pets while their owners are out of town. One week she has three dogs that all eat the same kind of dog food. How many bags of food should Kendra buy for one week? (Lesson 11-6)



Max
12 days/bag



Miles
15 days/bag



Stormy
16 days/bag

Find each product. (Lesson 11-4)

52. $\frac{8}{x^2} \cdot \frac{x^4}{4x}$

53. $\frac{10r^3}{6n^3} \cdot \frac{42n^2}{35r^3}$

54. $\frac{10y^3z^2}{6wx^3} \cdot \frac{12w^2x^2}{25y^2z^4}$

55. $\frac{(n-1)(n+1)}{(n+1)} \cdot \frac{(n-4)}{(n-1)(n+4)}$

56. $\frac{(x-8)}{(x+8)(x-3)} \cdot \frac{(x+4)(x-3)}{(x-8)}$

57. $\frac{3a^2b}{2gh} \cdot \frac{24g^2h}{15ab^2}$

58. **COOKING** The formula $t = \frac{40(25 + 1.85a)}{50 - 1.85a}$ relates the time t in minutes that it takes to cook an average-size potato in an oven at an altitude of a thousands of feet. (Lesson 11-3)

- a. What is the value of a for an altitude of 4500 feet?
- b. Calculate the time it takes to cook a potato at an altitude of 3500 feet and at 7000 feet. How do your cooking times compare at these two altitudes?

Skills Review

Ten red tiles, 12 blue tiles, 8 green tiles, 4 yellow tiles, 10 red tiles, and 12 black tiles are placed in a bag and selected at random. Find each probability.

59. $P(\text{blue})$

60. $P(\text{red})$

61. $P(\text{black or yellow})$

62. $P(\text{green or red})$

63. $P(\text{not blue})$

64. $P(\text{not green})$



Then

You calculated simple probability.

Now

1 Find probabilities of independent and dependent events.

2 Find probabilities of mutually exclusive events.

Why?

Evita is flying from Cleveland to Honolulu. The airline reports that the flight from Cleveland to Honolulu has a 40% on-time record. The airline also reported that they lose luggage 5% of the time. What is the probability that both the flight will be on time and Evita's luggage will arrive?



New Vocabulary

- compound event
- joint probability
- independent events
- dependent events
- mutually exclusive events



Common Core State Standards

Mathematical Practices

- 5 Use appropriate tools strategically.

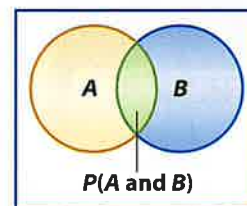
1 Independent and Dependent Events Recall that one event, like flying from Cleveland to Honolulu, is called a *simple event*. A **compound event** is made up of two or more simple events. So, the probability that the flight will be on time and the luggage arrives is an example of a compound event. The probability of compound events is called **joint probability**. The plane being on time may not affect whether luggage is lost. These two events are called **independent events** because the outcome of one event does not affect the outcome of the other.

Key Concept Probability of Independent Events

Words If two events, A and B , are independent, then the probability of both events occurring is the product of the probability of A and the probability of B .

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

Model



Real-World Example 1 Independent Events

MARBLES A bag contains 6 black marbles, 9 blue marbles, 4 yellow marbles, and 2 green marbles. A marble is selected, replaced, and a second marble is selected. Find the probability of selecting a black marble, then a yellow marble.

First marble: $P(\text{black}) = \frac{6}{21}$ ← $\frac{\text{number of black marbles}}{\text{total number of marbles}}$

Second marble: $P(\text{yellow}) = \frac{4}{21}$ ← $\frac{\text{number of yellow marbles}}{\text{total number of marbles}}$

$P(\text{black, yellow}) = P(\text{black}) \cdot P(\text{yellow})$ Probability of independent events
 $= \frac{6}{21} \cdot \frac{4}{21}$ or $\frac{24}{441}$ Substitution

The probability is $\frac{24}{441}$ or about 5.4%.

Guided Practice Find each probability.

1A. $P(\text{blue, green})$

1B. $P(\text{not black, blue})$



When the outcome of one event affects the outcome of another event, they are **dependent events**. In Example 1, if the marble was not placed back in the bag, then drawing the two marbles would have been dependent events. The probability of drawing the second marble depends on what marble was drawn first.

Key Concept Probability of Dependent Events

Words If two events, A and B , are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Recall that the complement of a set is the set of all objects that do *not* belong to the given set. In a standard deck of cards, the complement of drawing a heart is drawing a diamond, club, or spade. So, the probability of drawing a heart is $\frac{13}{52}$, and the probability of not drawing a heart is $\frac{52 - 13}{52}$ or $\frac{39}{52}$.

The sum of the probabilities for any two complementary events is 1.



Real-World Example 2 Dependent Events

CARDS Cynthia randomly draws three cards from a standard deck one at a time without replacement. Find the probability that the cards are drawn in the given order.

a. $P(\text{diamond, spade, diamond})$

$$\text{First card: } P(\text{diamond}) = \frac{13}{52} \text{ or } \frac{1}{4} \quad \begin{array}{l} \leftarrow \text{number of diamonds} \\ \leftarrow \text{total number of cards} \end{array}$$

$$\text{Second card: } P(\text{spade}) = \frac{13}{51} \quad \begin{array}{l} \leftarrow \text{number of spades} \\ \leftarrow \text{number of cards remaining} \end{array}$$

$$\text{Third card: } P(\text{diamond}) = \frac{12}{50} \text{ or } \frac{6}{25} \quad \begin{array}{l} \leftarrow \text{number of diamonds remaining} \\ \leftarrow \text{number of cards remaining} \end{array}$$

$$\begin{aligned} P(\text{diamond, spade, diamond}) &= P(\text{diamond}) \cdot P(\text{spade}) \cdot P(\text{diamond}) \\ &= \frac{1}{4} \cdot \frac{13}{51} \cdot \frac{6}{25} \text{ or } \frac{13}{850} \quad \text{Substitution} \end{aligned}$$

The probability is $\frac{13}{850}$ or about 1.5%.

b. $P(\text{four, four, not a jack})$

After Cynthia draws the first two fours from the deck of 52 cards, there are 50 cards left. Since neither of these cards are jacks, there are still four jacks left in the deck. So, there are $52 - 2 - 4$ or 46 cards that are not jacks.

$$\begin{aligned} P(\text{four, four, not a jack}) &= P(\text{four}) \cdot P(\text{four}) \cdot P(\text{not a jack}) \\ &= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{46}{50} \\ &= \frac{552}{132,600} \text{ or } \frac{23}{5525} \end{aligned}$$

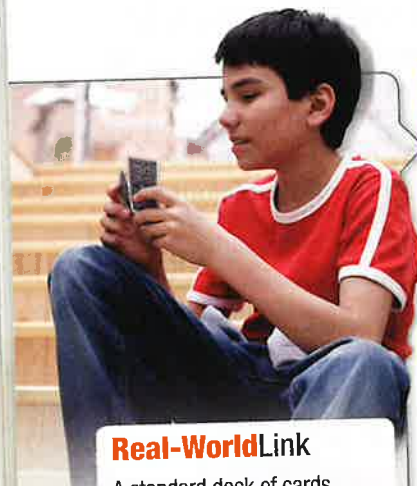
The probability is $\frac{23}{5525}$ or about 0.4%.

Guided Practice

Find each probability.

2A. $P(\text{two, five, not a five})$

2B. $P(\text{heart, not a heart, heart})$



Real-WorldLink

A standard deck of cards consists of 52 cards. There are 4 suits: hearts, diamonds, clubs, and spades. There are 13 cards of each suit. An ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king. The hearts and diamonds are red, and the clubs and spades are black.

Problem-SolvingTip

CCSS Tools Acting out the situation can help you understand what the question is asking. Use a deck of cards to represent the situation described in the problem.



2 Mutually Exclusive Events Events that cannot occur at the same time are called **mutually exclusive events**. Suppose you wanted to find the probability of drawing a heart or a diamond. Since a card cannot be both a heart and a diamond, the events are mutually exclusive.

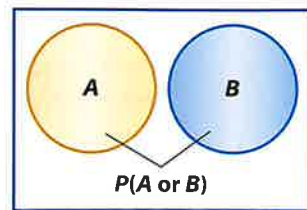
StudyTip

and and *or* While probabilities involving *and* deal with independent and dependent events, probabilities involving *or* deal with mutually exclusive and non-mutually exclusive events.

KeyConcept Probability of Mutually Exclusive Events

Words If two events, A and B , are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

Model



Symbols $P(A \text{ or } B) = P(A) + P(B)$



Real-World Example 3 Mutually Exclusive Events

A die is being rolled. Find each probability.

a. $P(3 \text{ or } 5)$

Since a die cannot show both a 3 and a 5 at the same time, these events are mutually exclusive.

$$P(\text{rolling a } 3) = \frac{1}{6} \quad \leftarrow \begin{array}{l} \text{number of sides with a } 3 \\ \text{total number of sides} \end{array}$$

$$P(\text{rolling a } 5) = \frac{1}{6} \quad \leftarrow \begin{array}{l} \text{number of sides with a } 5 \\ \text{total number of sides} \end{array}$$

$$\begin{aligned} P(3 \text{ or } 5) &= P(\text{rolling a } 3) + P(\text{rolling a } 5) && \text{Probability of mutually exclusive events} \\ &= \frac{1}{6} + \frac{1}{6} && \text{Substitution} \\ &= \frac{2}{6} \text{ or } \frac{1}{3} && \text{Add.} \end{aligned}$$

The probability of rolling a 3 or a 5 is $\frac{1}{3}$ or about 33%.

b. $P(\text{at least } 4)$

Rolling at least a 4 means you can roll either a 4, 5, or a 6. So, you need to find the probability of rolling a 4, 5, or a 6.

$$P(\text{rolling a } 4) = \frac{1}{6} \quad \leftarrow \begin{array}{l} \text{number of sides with a } 4 \\ \text{total number of sides} \end{array}$$

$$P(\text{rolling a } 5) = \frac{1}{6} \quad \leftarrow \begin{array}{l} \text{number of sides with a } 5 \\ \text{total number of sides} \end{array}$$

$$P(\text{rolling a } 6) = \frac{1}{6} \quad \leftarrow \begin{array}{l} \text{number of sides with a } 6 \\ \text{total number of sides} \end{array}$$

$$\begin{aligned} P(\text{at least } 4) &= P(\text{rolling a } 4) + P(\text{rolling a } 5) + P(\text{rolling a } 6) && \text{Mutually exclusive events} \\ &= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} && \text{Substitution} \\ &= \frac{3}{6} \text{ or } \frac{1}{2} && \text{Add.} \end{aligned}$$

The probability of rolling at least a 4 is $\frac{1}{2}$ or about 50%.

StudyTip

Alternative Method In Example 3a, you could have placed the number of possible outcomes over the total number of outcomes. $\frac{1+1}{6} = \frac{2}{6}$ or $\frac{1}{3}$

GuidedPractice

3A. $P(\text{less than } 3)$

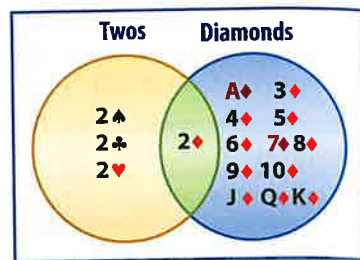
3B. $P(\text{even})$



ReadingMath

A or B Unlike everyday language, the expression *A or B* allows the possibility of both *A* and *B* occurring.

Suppose you want to find the probability of randomly drawing a 2 or a diamond from a standard deck of cards. Since it is possible to draw a card that is both a 2 and a diamond, these events are not mutually exclusive.



$$\begin{array}{ccc} P(2) & P(\text{diamond}) & P(2, \text{diamond}) \\ \frac{4}{52} & \frac{13}{52} & \frac{1}{52} \end{array}$$

In the first two fractions above, the probability of drawing the two of diamonds is counted twice, once for a two and once for a diamond. To find the correct probability, subtract $P(2 \text{ of diamonds})$ from the sum of the first two probabilities.

$$\begin{aligned} P(2 \text{ or a diamond}) &= P(2) + P(\text{diamond}) - P(2 \text{ of diamonds}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} \text{ or } \frac{4}{13} \end{aligned} \quad \text{The probability is } \frac{4}{13} \text{ or about } 31\%.$$

WatchOut!

Intersection of Events

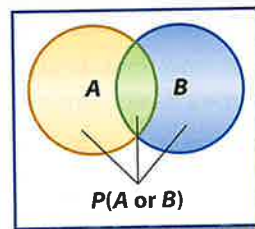
When determining the probability of events that are not mutually exclusive, you may count the intersection of the events twice since it occurs in both events. It only actually occurs once.

KeyConcept Probability of Events that are Not Mutually Exclusive

Words

If two events, *A* and *B*, are not mutually exclusive, then the probability that either *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occurring.

Model



Symbols

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Real-World Example 4 Events that are Not Mutually Exclusive

STUDENT ATHLETES Of 240 girls, 176 are on the Honor Roll, 48 play sports, and 36 are on the Honor Roll and play sports. What is the probability that a randomly selected student plays sports or is on the Honor Roll?

Since some students play sports and are on the Honor Roll, the events are not mutually exclusive.

$$P(\text{sports}) = \frac{48}{240} \quad P(\text{Honor Roll}) = \frac{176}{240} \quad P(\text{sports and Honor Roll}) = \frac{36}{240}$$

$$\begin{aligned} P(\text{sports or Honor Roll}) &= P(\text{sports}) + P(\text{HR}) - P(\text{sports and HR}) \\ &= \frac{48}{240} + \frac{176}{240} - \frac{36}{240} \quad \text{Substitution} \\ &= \frac{188}{240} \text{ or } \frac{47}{60} \quad \text{Simplify.} \end{aligned}$$

The probability is $\frac{47}{60}$ or about 78%.

Guided Practice

4. **PETS** Out of 5200 households surveyed, 2107 had a dog, 807 had a cat, and 303 had both a dog and a cat. What is the probability that a randomly selected household has a dog or a cat?





Examples 1–2 Determine whether the events are *independent* or *dependent*. Then find the probability.

- 1. BABYSITTING** A toy bin contains 12 toys, 8 stuffed animals, and 3 board games. Marsha randomly chooses 2 items for the child she is babysitting. What is the probability that she chose 2 stuffed animals as the first two choices?
- 2. FRUIT** A basket contains 6 apples, 5 bananas, 4 oranges, and 5 peaches. Drew randomly chooses one piece of fruit, eats it, and chooses another. What is the probability that he chose a banana and then an apple?
- 3. MONEY** Nakos has 4 quarters, 3 dimes, and 2 nickels in his pocket. Nakos randomly picks two coins out of his pocket. What is the probability that he did not choose a dime either time, if he replaced the first coin before choosing a second coin?
- 4. BOOKS** Joanna needs a book to prop up a table leg. She randomly selects a book, puts it back on the shelf, and selects another book. What is the probability that Joanna selected two math books?



Examples 3–4 A card is drawn from a standard deck of playing cards. Determine whether the events are *mutually exclusive* or *not mutually exclusive*. Then find the probability.

- 5.** $P(\text{two or queen})$
- 6.** $P(\text{diamond or heart})$
- 7.** $P(\text{seven or club})$
- 8.** $P(\text{spade or ace})$

Practice and Problem Solving

Extra Practice is on page R12.

Examples 1–2 Determine whether the events are *independent* or *dependent*. Then find the probability.

- 9. COINS** If a coin is tossed 4 times, what is the probability of getting tails all 4 times?
- 10. DICE** A die is rolled twice. What is the probability of rolling two different numbers?
- 11. CANDY** A box of chocolates contains 10 milk chocolates, 8 dark chocolates, and 6 white chocolates. Sung randomly chooses a chocolate, eats it, and then randomly chooses another. What is the probability that Sung chose a milk chocolate and then a white chocolate?
- 12. DICE** A die is rolled twice. What is the probability of rolling the same numbers?
- 13. PETS** Chuck and Rashid went to a pet store to buy dog food. They chose from 10 brands of dry food, 6 brands of canned food, and 3 brands of pet snacks. What is the probability that both chose dry food, if Chuck randomly chose first and liked the first brand he picked up?
- 14. CCSS MODELING** A rental agency has 12 white sedans, 8 gray sedans, 6 red sedans, and 3 green sedans for rent. Mr. Escobar rents a sedan, returns it because the radio is broken, and gets another sedan. Assuming the returned sedan remains in circulation, what is the probability that Mr. Escobar was given a green sedan and then a gray sedan?



32. **TILES** Kirsten and José are playing a game. Kirsten places tiles numbered 1 to 50 in a bag. José selects a tile at random. If he selects a prime number or a number greater than 40, then he wins the game. What is the probability that José will win on his first turn?

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore **conditional probability**, which is the probability that event B occurs given that event A has already occurred. It is calculated by dividing the probability of the occurrence of both events by the probability of the occurrence of the first event. The notation for conditional probability is $P(B | A)$, read *the probability of B , given A* .

- a. **Graphical** Draw a Venn diagram to illustrate $P(A \text{ and } B)$.
- b. **Verbal** Tell how to find $P(B | A)$ given the Venn diagram.
- c. **Analytical** A jar contains 12 marbles, of which 8 marbles are red and 4 marbles are green. If marbles are chosen without replacement, find $P(\text{red})$ and $P(\text{red, green})$.
- d. **Analytical** Using the probabilities from part c and the Venn diagram in part a, determine the probability of choosing a green marble on the second selection, given that the first marble selected was red.
- e. **Analytical** Write a formula for finding a conditional probability.
- f. **Analytical** Use the definition from part e to answer the following: At a basketball game, 80% of the fans cheered for the home team. In the same crowd, 20% of the fans were waving banners and cheering for the home team. What is the probability that a fan waved a banner given that the fan cheered for the home team?

H.O.T. Problems Use Higher-Order Thinking Skills

34. **ERROR ANALYSIS** George and Aliyah are determining the probability of randomly choosing a blue or red marble from a bag of 8 blue marbles, 6 red marbles, 8 yellow marbles, and 4 white marbles. Is either of them correct? Explain.

George

$$\begin{aligned} P(\text{blue or red}) &= P(\text{blue}) \cdot P(\text{red}) \\ &= \frac{8}{26} \cdot \frac{6}{26} \\ &= \frac{48}{676} \\ &\text{about } 7\% \end{aligned}$$

Aliyah

$$\begin{aligned} P(\text{blue or red}) &= P(\text{blue}) + P(\text{red}) \\ &= \frac{8}{26} + \frac{6}{26} \\ &= \frac{14}{26} \\ &\text{about } 54\% \end{aligned}$$

35. **CHALLENGE** In some cases, if one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance that the whole string will light?
36. **CCSS REGULARITY** Suppose there are three events A , B , and C that are not mutually exclusive. List all of the probabilities you would need to consider in order to calculate $P(A \text{ or } B \text{ or } C)$. Then write the formula you would use to calculate it.
37. **OPEN ENDED** Describe a situation in your life that involves dependent and independent events. Explain why the events are dependent or independent.
38. **WRITING IN MATH** Explain why the subtraction occurs when finding the probability of two events that are not mutually exclusive.



Standardized Test Practice

39. In how many ways can a committee of 4 be selected from a group of 12 people?

- A 48
- B 483
- C 495
- D 11,880

40. A total of 925 tickets were sold for \$5925. If adult tickets cost \$7.50 and children's tickets cost \$3.00, how many adult tickets were sold?

- F 700
- G 600
- H 325
- J 225

41. **SHORT RESPONSE** A circular swimming pool with a diameter of 28 feet has a deck of uniform width built around it. If the area of the deck is 60π square feet, find its width.

42. The probability of heads landing up when you flip a coin is $\frac{1}{2}$. What is the probability of getting tails if you flip it again?

- A $\frac{1}{4}$
- B $\frac{1}{3}$
- C $\frac{1}{2}$
- D $\frac{3}{4}$

Spiral Review

43. **SHOPPING** The Millers have twelve grandchildren, 5 boys and 7 girls. For their anniversary, the grandchildren decided to pool their money and have three of them shop for the entire group. (Lesson 12-6)

- a. Does this situation represent a *combination* or *permutation*?
- b. How many ways are there to choose the three?
- c. What is the probability that all three will be girls?

44. **HAIR STYLIST** Tia is a hair stylist. Last week, 70% of her clients who called to make appointments made an appointment for a basic haircut. (Lesson 12-5)

- a. Design a simulation that can be used to estimate the probability that the next client that makes an appointment will schedule a basic haircut.
- b. Conduct the simulation, and report the results.

Solve each equation. State any extraneous solutions. (Lesson 11-8)

45. $\frac{4}{a} = \frac{3}{a-2}$

46. $\frac{3}{x} = \frac{1}{x-2}$

47. $\frac{x}{x+1} = \frac{x-6}{x-1}$

48. $\frac{2n}{3} + \frac{1}{2} = \frac{2n-3}{6}$

49. **COOKING** Hannah was making candy using a two-quart pan. As she stirred the mixture, she noticed that the pan was about $\frac{2}{3}$ full. If each piece of candy has a volume of about $\frac{3}{4}$ ounce, approximately how many pieces of candy will Hannah make? (Hint: There are 32 ounces in a quart.) (Lesson 11-3)

50. **GEOMETRY** A rectangle has a width of $3\sqrt{5}$ centimeters and a length of $4\sqrt{10}$ centimeters. Find the area of the rectangle. Write as a simplified radical expression. (Lesson 10-2)

Skills Review

Solve each equation. Check your solution.

51. $\sqrt{-3a} = 6$

52. $\sqrt{a} = 100$

53. $\sqrt{-k} = 4$

54. $5\sqrt{2} = \sqrt{x}$

55. $3\sqrt{7} = \sqrt{-y}$

56. $3\sqrt{4a} - 2 = 10$



12-7 Algebra Lab Two-Way Frequency Tables



Joana sent out a survey to the freshmen and sophomores, asking if they were planning on attending the dance. One way of organizing her responses is to use a two-way frequency table.

A **two-way frequency table** or *contingency table* is used to show the frequencies of data from a survey or experiment classified according to two categories, with the rows indicating one category and the columns indicating the other.



Common Core State Standards Content Standards

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

For Joana's survey, the two categories are *class* and *attendance*. These categories can be split into subcategories: *freshman* and *sophomore* for *class*, and *attending* and *not attending* for *attendance*.

Class	Attending	Not Attending	Totals
Freshman			
Sophomore			
Totals			

subcategories

Activity 1 Two-Way Frequency Table

DANCE Sixty-six freshmen responded to the survey, with 32 saying that they would be attending. Of the 84 sophomores that responded, 46 said they would attend. Organize the data in a two-way table.

Step 1 Find the values for every combination of subcategories. One combination is freshmen/not attending. Since 32 of 66 freshmen are attending, $66 - 32$ or 34 freshmen are *not* attending. These combinations are called **joint frequencies**.

Step 2 Place every combination in the corresponding cell.

Step 3 Find the totals of each subcategory and place them in their corresponding cell. These values are called **marginal frequencies**.

Step 4 Find the sum of each set of marginal frequencies. These two sums should be equal. Place the value in the bottom right corner.

Class	Attending	Not Attending	Totals
Freshman	32	34	66
Sophomore	46	38	84
Totals	78	72	150

joint frequencies

marginal frequencies

marginal frequencies

Analyze the Results

- How many students responded to the survey?
- How many of the students that were surveyed are attending the dance?
- How many of the surveyed sophomores are not attending the dance?
- What does each of the joint frequencies represent?
- What does each of the marginal frequencies represent?
- WORK** Heather sent out a survey asking who was working during the holiday. Of the 50 boys who responded, 34 said *yes*. Of the 45 girls who responded, 21 said *no*. Create a two-way frequency table of the results.
- SOCCER** Pamela asked if anyone would be interested in a co-ed soccer team. Of the 28 boys who responded, 18 said that they would play and 4 were undecided. Of the 22 girls who responded, 6 said they did not want to play and 3 were undecided. Create a two-way frequency table of the results.

Two-Way Frequency Tables *Continued*

A **relative frequency** is the ratio of the number of observations in a category to the total number of observations. Relative frequencies are also probabilities. To create a relative frequency two-way table, divide each of the values by the total number of observations and replace them with their corresponding decimals or percents.

Class	Attending	Not Attending	Totals
Freshman	$\frac{32}{150} \approx 21.3\%$	22.7%	44%
Sophomore	30.7%	25.3%	56%
Totals	52%	48%	100%

A **conditional relative frequency** is the ratio of the joint frequency to the marginal frequency. For example, given that a student is a freshman, what is the conditional relative frequency that he or she is going to the dance? In other words, what is the probability that a freshman is going to the dance?

Activity 2 Two-Way Conditional Relative Frequency Table

DANCE Joana wants to determine the conditional relative frequencies (or probabilities) given the fact that she knows the class of the respondents.

Step 1 Refer to the table in Activity 1. A total of 66 freshmen responded, and 32 said *yes*. Therefore, the conditional relative frequency that a respondent said *yes* given that the respondent is a freshman is $\frac{32}{66}$.

Step 2 Place every conditional relative frequency in the corresponding cell.

Step 3 The conditional relative frequencies for each row should sum to 100%.

Conditional Relative Frequencies by Class			
Class	Attending	Not Attending	Totals
Freshman	$\frac{32}{66} \approx 48\%$	$\frac{34}{66} \approx 52\%$	100%
Sophomore	$\frac{46}{84} \approx 55\%$	$\frac{38}{84} \approx 45\%$	100%

Analyze the Results

- Given that a respondent was a sophomore, what is the probability that he or she said *no*?
- What does each of the conditional relative frequencies represent?
- Why do you think that the columns do not sum to 100%?
- Create a two-way conditional relative frequency table for the category *attendance*.
- Given that a respondent was not attending, what is the probability that he or she is a freshman?
- ACTIVITIES** The managers, staff, and assistants were given three options for the holiday activity: a potluck, a dinner at a restaurant, and a gift exchange. Five of the 11 managers want a dinner, while 3 want a potluck. Eleven of the 45 staff members want a gift exchange, while 18 want a dinner. Ten of the 32 assistants want a dinner, while 8 of them want a gift exchange.
 - Create a two-way frequency table.
 - Convert the two-way frequency table into a relative frequency table.
 - Create two conditional relative frequency tables: one for the activities and one for the employees.

Then

- You found probabilities of events.

Now

- Find probabilities by using random variables.
- Find the expected value of a probability distribution.

Why?

- A gaming software company with five online games on the market is interested in how many games each of their customers play. They surveyed 1000 randomly chosen customers. The results of the survey are shown.

Number of Computer Games	Number of Customers
1	130
2	110
3	150
4	500
5	110

New Vocabulary

random variable
discrete random variable
probability distribution
probability graph
expected value

Common Core State Standards

Mathematical Practices
4 Model with mathematics.

1 Random Variables and Probability A variable with a value that is the numerical outcome of a random event is called a **random variable**. A random variable with a countable number of possibilities is a **discrete random variable**. We can let the random variable G represent the number of different games. So, G can equal 1, 2, 3, 4, or 5.



Example 1 Random Variables

A graduation supply company offers five items that can be purchased for graduation: a diploma frame, graduation picture, cap and gown, senior key ring, and class pin. The school takes a poll of the seniors to see how many of these items each senior is buying. The results are shown.

Number of Items Being Purchased	Number of Seniors
0	12
1	122
2	134
3	115
4	145
5	97

- a. Find the probability that a randomly chosen senior is buying 3 items.

Let X represent the number of items being purchased. There is only one outcome in which 3 items are being purchased, and there is a total of 625 seniors.

$$P(X = 3) = \frac{\text{3 items being purchased}}{\text{seniors surveyed}} \quad P(X = n) \text{ is the probability of } X \text{ occurring } n \text{ times.}$$

$$= \frac{115}{625} \text{ or } \frac{23}{125}$$

The probability is $\frac{23}{125}$ or 18.4%.

- b. Find the probability that a randomly chosen senior buys at least 4 items.

There are $145 + 97$ or 242 seniors who are purchasing at least 4 items.

$$P(X \geq 4) = \frac{242}{625} \quad \text{The probability is } \frac{242}{625} \text{ or about } 38.7\%.$$

Guided Practice

GRADES After an algebra test, there are 7 students with As, 9 with Bs, 11 with Cs, 3 with Ds, and 2 with Fs.

- Find the probability that a randomly chosen student has a C.
- Find the probability that a randomly chosen student has at least a B.



StudyTip

Discrete and Continuous Data Data are *discrete* if the observations can be counted. For example, the number of kittens in a litter is discrete data. Data are *continuous* if the data can take on any value within an interval. For example, the height of each person in a sample is continuous data.

2 Probability Distributions A **probability distribution** is the probability of every possible value of the random variable. A **probability graph** is a bar graph that displays a probability distribution.

KeyConcept Properties of Probability Distributions

- The probability of each value of X is greater than or equal to 0 and is less than or equal to 1.
- The sum of the probabilities of all values of X is 1.



Real-World Example 2 Probability Distribution

PIZZA The table shows the probability distribution of the number of times a customer orders pizza each month.

Pizzas Ordered Per Month	
X = Number of Pizzas	Probability
0	0.10
1	0.12
2	0.44
3	0.24
4+	0.10

a. Show that the distribution is valid.

- For each value of X , the probability is greater than or equal to 0 and less than or equal to 1.
- The sum of the probabilities, $0.10 + 0.12 + 0.44 + 0.24 + 0.10$, is 1.

b. What is the probability that a customer orders pizza fewer than three times per month?

The probability of a compound event is the sum of the probabilities of each individual event. The probability of a customer ordering fewer than 3 times per month is the sum of the probability of ordering 2 times per month plus the probability of ordering one time per month.

$$\begin{aligned} P(X < 3) &= P(X = 2) + P(X = 1) + P(X = 0) \\ &= 0.44 + 0.12 + 0.10 \\ &= 0.66 \end{aligned}$$

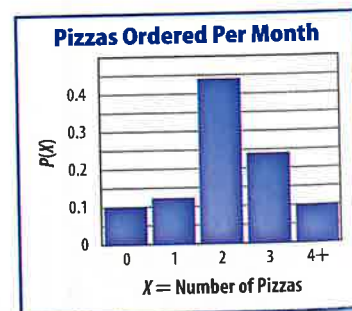
Sum of individual probabilities

$$P(X = 2) = 0.44, P(X = 1) = 0.12, \text{ and } P(X = 0) = 0.10$$

Add.

c. Make a probability graph of the data.

Use the data from the probability distribution table to draw a bar graph. Remember to label each axis and give the graph a title.

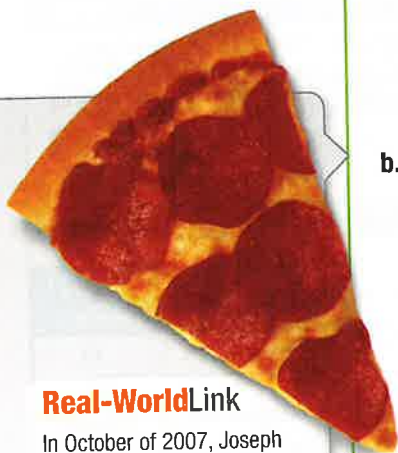


GuidedPractice

The table shows the probability distribution of adults who play golf by age range.

- Show that the distribution is valid.
- What is the probability that an adult golfer is 35 years old or older?
- Make a probability graph of the data.

Golfers By Age	
A = Ages	Probability
18–24	0.13
25–34	0.18
35–44	0.21
45–54	0.19
55–64	0.12
65+	0.17



Real-WorldLink

In October of 2007, Joseph Jones, then a high school senior, ate 83 slices of pepperoni pizza within 10 minutes at an eating competition.

Source: About Pizza



The **expected value** $E(X)$ of a discrete random variable is the weighted average of the variable. The “weight” applied to each value is its theoretical probability. The expected value tells you what average value to expect after many trials.

Key Concept Expected Value of a Discrete Random Variable

Words The expected value of a discrete random variable is the weighted average of the values of the variable. It is calculated by finding the sum of the products of every possible value of X and its associated probability $P(X)$.

Symbols $E(X) = [X_1 \cdot P(X_1)] + [X_2 \cdot P(X_2)] + \dots + [X_n \cdot P(X_n)]$, where n is the total number of values of X



Real-World Example 3 Expected Value

GAMES A candy bar manufacturer is holding a contest. The potential prizes and the probability of winning each prize are shown in the table.

Prize	gift certificate	year supply of candy bars	trip	new car
Prize Value	\$50	\$250	\$4000	\$25,000
Probability	1 in 500	1 in 2000	1 in 2,000,000	1 in 20,000,000

a. Create a probability distribution.

Find the probability associated with each prize. Note that the probability of winning \$0 is found by subtracting the probability of winning something from 1.

X	$P(X)$
\$50	0.002
\$250	0.0005
\$4000	0.0000005
\$25,000	0.00000005
\$0	0.99749945

b. Calculate the expected value.

$$\begin{aligned} E(X) &= [X_1 \cdot P(X_1)] + [X_2 \cdot P(X_2)] + \dots + [X_n \cdot P(X_n)] \\ &= 50(0.002) + 250(0.0005) + 4000(0.0000005) + 25,000(0.00000005) \\ &\quad + 0(0.997\dots) \\ &= 0.1 + 0.125 + 0.002 + 0.00125 + 0 \text{ or } 0.22825 \end{aligned}$$

The expected value is 0.22825 or about \$0.23.

c. Interpret your results.

The expected value of 0.22825 means that one candy bar purchase can be expected to win about \$0.23.

Guided Practice

3. **CONTEST** Wendy entered a contest with the following prize values.

Prize Value	\$100	\$500	\$5000	\$50,000
Probability	1 in 1000	1 in 5000	1 in 100,000	1 in 1,000,000

- Create a probability distribution.
- Calculate the expected value.
- Interpret your results.

Real-WorldLink

Sweepstakes The odds of winning the grand prize in some sweepstakes can be 1 in over 500 million.

StudyTip

Probability of Winning

\$0 The probability of winning \$0 is included in order for the sum of the probabilities in the probability distribution to equal 1.





Example 1

1. **GPS** A car dealership surveys 10,000 of its customers with a GPS system in their vehicles to ask how often they have used the system within the past year. The results are shown in the table.
 - a. Find the probability that a randomly chosen customer will have used the GPS system more than 20 times.
 - b. Find the probability that a randomly chosen customer will have used the GPS system no more than 10 times.

Uses	Customers
0	1382
1–5	2350
6–10	2010
11–15	1863
16–20	1925
21+	470

Example 2

2. **JEANS** A fashion boutique ordered jeans with different numbers of stripes down the outside seams. The table shows the probability distribution of the number of each type of jean that was sold in a particular week.
 - a. Show that the distribution is valid.
 - b. What is the probability that a randomly chosen pair of jeans has fewer than 3 stripes?
 - c. Make a probability graph of the data.

Number of Stripes	Probability
0	0.15
1	0.19
2	0.26
3	0.22
4	0.18

Example 3

3. **CCSS ARGUMENTS** The producers of a game show provided the probability of winning the prizes for one of the games.
 - a. Create a probability distribution.
 - b. Calculate the expected value.
 - c. Interpret your results.

Prize Value	Probability
\$1000	1 in 80
\$5000	1 in 200
\$25,000	1 in 1000
\$100,000	1 in 10,000

Practice and Problem Solving

Extra Practice is on page R12.

Example 1

4. **HOME THEATER** An electronics store sells components and speakers for home theaters individually. The store surveyed its home theater customers to see how many of the 10 components they bought. The results are shown in the table.
 - a. Find the probability that a randomly chosen customer bought 5 or 6 components.
 - b. Find the probability that a randomly chosen customer bought fewer than 5 components.

Components	Customers
0–2	26
3–4	42
5–6	33
7–8	24
9–10	40

5. **FOOD DRIVE** Ms. Valdez’s biology class held a food drive that lasted four days. The class kept track of the number of packages donated each day.
 - a. Find the probability that a randomly chosen product was donated on the fourth day.
 - b. Find the probability that a randomly chosen product was donated on the first or second day.

Day	Packages
1	36
2	22
3	12
4	45



Example 2

6. **MUSIC** A Web site conducted a survey on the number of different formats on which teens have music. The table shows a probability distribution of the results.
- Show that the distribution is valid.
 - What is the probability that a student randomly chosen will have music on 2 or more formats?
 - Make a probability graph of the data.
7. **GRADES** Mr. Rockwell's Algebra class took a chapter test last week. The table shows the probability distribution of the results.
- Show that the distribution is valid.
 - What is the probability that a student chosen at random will have no higher than a 3?
 - Make a probability graph of the data.

Formats	Probability
1	0.35
2	0.31
3	0.19
4	0.11
5	0.02
6+	0.02

Score	Probability
4	0.29
3	0.43
2	0.17
1	0.11
0	0

Example 3

8. **CCSS ARGUMENTS** Kylie entered a drawing at the county fair. The table shows the value and probability of winning each prize.
- Create a probability distribution.
 - Calculate the expected value.
 - Interpret your results.
9. **CONTESTS** Nikia entered a contest where each ticket cost \$1. The table shows the value and probability of each prize.
- Create a probability distribution.
 - Calculate the expected value.
 - Interpret your results.
10. **MARKETING** A retail marketing group conducted a survey on teen shopping habits and asked the teens for the number of stores they visited to complete their holiday shopping. The table shows the probability distribution of the results.

Prize Value	Probability
\$20	1 in 50
\$50	1 in 100
\$100	1 in 250
\$250	1 in 1000

Prize Value	Probability
\$200	1 in 500
\$1000	1 in 5000
\$5000	1 in 25,000
\$25,000	1 in 100,000

Number of Stores	0–2	3–5	6–8	9–11	12+
Probability	0.35	0.32	0.17	0.11	0.05

- Show that the distribution is valid.
 - What is the probability that a shopper chosen at random will shop at more than 5 stores but fewer than 12?
 - Make a probability graph of the data.
11. **DECISION MAKING** An auto insurance company uses many variables to calculate each driver's six-month payment. The table at the right shows the probability of a specific driver getting into an accident that will cost the company \$10,000.
- Taking all of the probabilities into account, what is the probability of the driver having an accident?
 - What is the company's expected payout for this driver?
 - What should the company charge the driver for a six-month policy? Explain your reasoning.

Variable	Probability
driving record	1 in 50
vehicle type	1 in 500
age	1 in 250
gender	1 in 200
residence	1 in 1000



12. **DECISION MAKING** Amber is investing in stocks for her math class. After analyzing five stocks, she has determined the following probabilities.

Stock	Cost Per Share	Probability of \$20 Gain	Probability of No Gain	Probability of \$30 Loss
A	\$100	47%	25%	28%
B	\$200	40%	40%	20%
C	\$200	50%	20%	30%
D	\$200	60%	0%	40%
E	\$300	30%	60%	10%

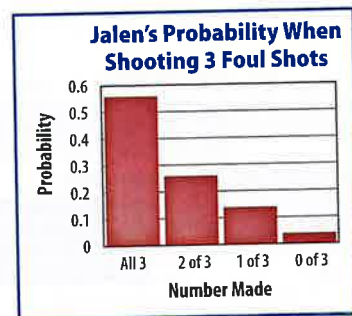
- a. Calculate the expected gain or loss for one share of each stock.
- b. If Amber is allowed to spend \$1000, what combination of stocks should she purchase to ensure the greatest expected value? Explain your reasoning.
13. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate probability distributions and simulations.

- a. **Tabular** Construct a relative-frequency table showing the theoretical probability for the sum obtained from rolling a die and spinning the spinner at the right.
- b. **Graphical** Make a probability graph of the data.
- c. **Analytical** Calculate the expected value of one spin.
- d. **Concrete** Design a simulation for 50 trials. Explain your reasoning. Conduct the simulation and tally your results.
- e. **Graphical** Make a probability graph of the data in the simulation. Compare and contrast the two graphs.



H.O.T. Problems

14. **CHALLENGE** What is wrong with the probability distribution shown? Explain your reasoning.
15. **REASONING** Suppose two dice are rolled twelve times. Which sum is most likely to occur? Make a table to show the probability distribution. Then make a probability graph to confirm your answer.
16. **CCSS STRUCTURE** Explain why the sum of the probabilities in a probability distribution should always be 1. Include an example.



17. **REASONING** Determine whether the following statement is *true* or *false*. Explain your reasoning.

Discrete random variables can take on an infinite number of values.

18. **OPEN ENDED** Write a real-world problem in which you could find a probability distribution. Create a probability graph for your data.

19. **REASONING** Determine whether the following statement is *true* or *false*. Explain your reasoning.

The expected value of a random variable is the value for the random variable most likely to occur.

20. **WRITING IN MATH** Write a real-world story in which you are the owner of a business. Explain how you could use a probability distribution to help you make a business decision.



Standardized Test Practice

21. A coin is flipped and a die is rolled. What is the probability of the coin landing heads up and rolling a 3?

A $\frac{1}{12}$

C $\frac{1}{8}$

B $\frac{1}{9}$

D $\frac{2}{3}$

22. **SHORT RESPONSE** How many different ways can the letters P, Q, R, S be arranged?

23. Suppose there are 10 tickets in a box for a drawing numbered as follows: 1, 2, 2, 3, 4, 4, 6, 6, 9, and 9. A single ticket is randomly chosen from the box. What is the probability of drawing a ticket with a number less than 10?

F $\frac{1}{5}$

G $\frac{3}{10}$

H 1

J 0

24. **GEOMETRY** The height of a triangle is five inches less than the length of its base. If the area of the triangle is 52 square inches, find the base and the height.

A 15 in., 9 in.

C 13 in., 8 in.

B 11 in., 7 in.

D 17 in., 11 in.

Spiral Review

25. **PET TOYS** Johnda is looking for 2 puppy toys at the pet store. They have a box of clearance items that contains 6 balls, 5 tug toys, 8 rawhide chews, and 4 chew toys. If Johnda reaches in the box and pulls out two items, what is the probability that she will pull out two tug toys? (Lesson 12-7)

26. **GAMES** For a certain game, each player rolls four dice at the same time. (Lesson 12-6)

a. Do the outcomes of rolling the four dice represent a permutation or combination? Explain.

b. How many outcomes are possible?

c. What is the probability that four dice show the same number on a single roll?

Find each sum. (Lesson 11-6)

27. $\frac{4}{a^2} + \frac{6}{a}$

28. $\frac{3}{b^3} + \frac{7}{b^2}$

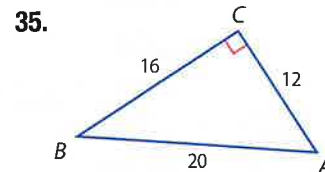
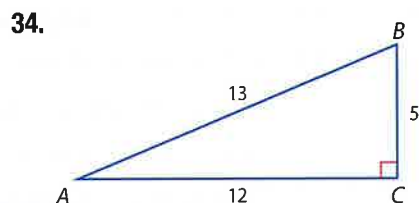
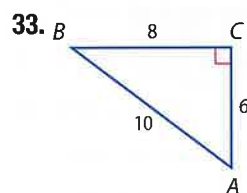
29. $\frac{4}{d+6} + \frac{5}{d-5}$

30. $\frac{f}{f+5} + \frac{4}{f-4}$

31. $\frac{8h}{h+6} + \frac{h}{h-3}$

32. $\frac{7k}{k-3} + \frac{k}{k+2}$

Find the values of the three trigonometric ratios for angle A. (Lesson 10-6)



Simplify each expression. (Lesson 10-2)

36. $\sqrt{\frac{50}{x^4}}$

37. $\frac{\sqrt{t^3}}{\sqrt{18}}$

38. $\sqrt{\frac{15}{14}} \cdot \sqrt{\frac{21}{10}}$

39. $\frac{6}{3-\sqrt{5}}$

40. $\frac{3}{\sqrt{7}+\sqrt{6}}$

41. $\frac{\sqrt{2}}{\sqrt{8}-\sqrt{6}}$



12-8 Graphing Technology Lab The Normal Curve



When there are a large number of values in a data set, the frequency distribution tends to cluster around the mean of the set in a distribution (or shape) called a **normal distribution**. The graph of a normal distribution is called a **normal curve**. Since the shape of the graph resembles a bell, the graph is also called a **bell curve**.

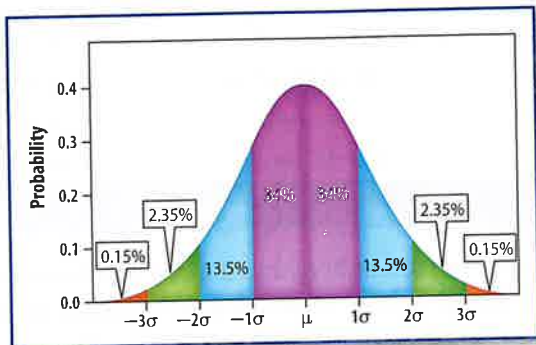
Data sets that have a normal distribution include reaction times of drivers that are the same age, achievement test scores, and the heights of people that are the same age.

CCSS Common Core State Standards Content Standards

S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Mathematical Practices

2 Reason abstractly and quantitatively.



You can use a graphing calculator to graph and analyze a normal distribution if the mean and standard deviation of the data are known.



Activity 1 Graph a Normal Distribution

HEIGHT The mean height of 15-year-old boys in the city where Isaac lives is 67 inches, with a standard deviation of 2.8 inches. Use a normal distribution to represent these data.

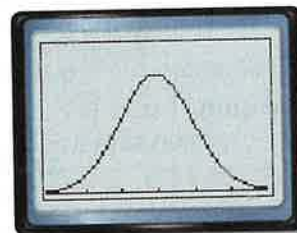
Step 1 Set the viewing window. **WINDOW**

- $X_{min} = 67 - 3 \times 2.8$ **ENTER** 58.6
- $X_{max} = 67 + 3 \times 2.8$ **ENTER** 75.4
- $X_{scl} = 2.8$ **ENTER**
- $Y_{min} = 0$ **ENTER**
- $Y_{max} = 1 \div (2 \times 2.8)$ **ENTER** .17857142...
- $Y_{scale} = 1$ **ENTER**



Step 2 By entering the mean and standard deviation into the calculator, we can graph the corresponding normal curve. Enter the values using the following keystrokes.

KEYSTROKES: **Y=** **2nd** **[DISTR]** **ENTER**
 X, T, θ, n , 67 , 2.8
) **GRAPH**



[58.6, 75.4] scl: 2.8 by [0, 0.17857142] scl: 1

Graphing Technology Lab

The Normal Curve *Continued*

The probability of a range of values is the area under the curve.

Activity 2 Analyze a Normal Distribution

Use the graph to answer questions about the data. What is the probability that Isaac will be at most 67 inches tall when he is 15?

The sum of all the y -values up to $x = 67$ would give us the probability that Isaac's height will be less than or equal to 67 inches. This is also the area under the curve. We will shade the area under the curve from negative infinity to 67 inches and find the area of the shaded portion of the graph.

Step 1 Use the **ShadeNorm** function.

KEYSTROKES: **2nd** **[DISTR]** **▶** **ENTER**

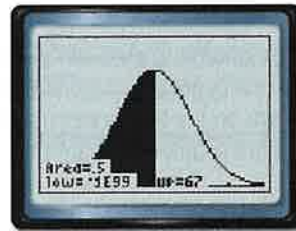


Step 2 Shade the graph.

Next enter the lowest value, highest value, mean, and standard deviation.

On the TI-84 Plus, -1×10^{99} represents negative infinity.

KEYSTROKES: **(←)** **1** **2nd** **[EE]** **99** **,** **67** **,**
67 **,** **2.8** **)** **ENTER**



[58.6, 75.4] scl: 2.8 by [0, 0.17857142] scl: 1

The area is given as 0.5. The probability that Isaac will be 67 inches tall is 0.5 or 50%. Since the mean value is 67, we expect the probability to be 50%.

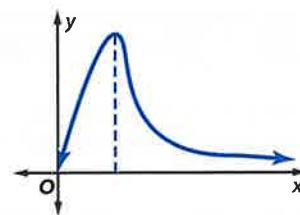
Exercises

1. What is the probability that Isaac will be at least 6 feet tall when he is 15?
2. What is the probability that Isaac will be between 65 and 68 inches?
3. The **z-score** represents the number of standard deviations that a given data value is from the mean. The z -score for a data value X is given by $z = \frac{X - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation. Find and interpret the z -score of a height of 73 inches.
4. Find and interpret the z -score of a height of 61 inches.

Extension

Refer to the curve at the right.

5. Compare this curve to the normal curve in Activity 1.
6. Describe where an outlier of the data set would be graphed on this curve.



Study Guide

Key Concepts

Samples and Studies (Lesson 12-1)

- Some types of samples are simple random, systematic, convenience, self-selected, and stratified.
- Three study types are the survey, observational study, and experiment.

Statistics and Parameters (Lesson 12-2)

- The mean absolute deviation is used to predict errors and judge how well the mean represents the data.
- A low standard deviation indicates that the data tend to be very close to the mean, while a high standard deviation indicates that the data are spread out over a larger range.

Distributions of Data and Comparing Sets of Data

(Lessons 12-3 and 12-4)

- In a negatively skewed distribution, the majority of the data are on the right. In a positively skewed distribution, the majority of the data are on the left. In a symmetric distribution, the data are evenly distributed.

Simulations (Lesson 12-5)

- Simulations are used to perform experiments that would be difficult or impossible to perform in real life.

Permutations and Combinations and Probability of Compound Events (Lessons 12-6 and 12-7)

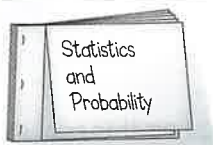
- A selection of objects in which order is important is called a permutation. A selection of objects in which order is not important is called a combination.
- For independent events, the outcome of one does not affect the outcome of the other.

Probability Distributions (Lesson 12-8)

- For each value of X , $0 \leq P(X) \leq 1$. The sum of the probabilities of each value of X is 1.
- The expected value $E(X)$ of a discrete random variable of a probability distribution is its weighted average.

FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary



bias (p. 748)	parameter (p. 757)
combination (p. 787)	permutation (p. 786)
compound event (p. 793)	population (p. 747)
convenience sample (p. 747)	probability distribution (p. 804)
dependent events (p. 794)	probability graph (p. 804)
discrete random variable (p. 803)	random variable (p. 803)
distribution (p. 764)	sample (p. 747)
expected value $E(X)$ (p. 805)	self-selected sample (p. 747)
experiment (p. 749)	simple random sample (p. 747)
experimental probability (p. 780)	simulation (p. 781)
independent events (p. 793)	standard deviation (p. 759)
linear transformation (p. 771)	statistic (p. 757)
mean absolute deviation (MAD) (p. 758)	statistical inference (p. 757)
mutually exclusive events (p. 795)	stratified sample (p. 747)
observational study (p. 749)	survey (p. 749)
	systematic sample (p. 747)
	theoretical probability (p. 780)
	variance (p. 759)

Vocabulary Check

Choose the term that best completes each sentence.

1. An arrangement in which order is important is called a (combination, permutation).
2. A (parameter, statistic) is a measure that describes the characteristic of a sample.
3. A (sample, population) consists of all of the members of a group.
4. (Experimental probability, Theoretical probability) is the ratio of the number of favorable outcomes to the total number of outcomes.
5. A variable with a value that is the numerical outcome of a random event is called a (discrete random variable, random variable).



Lesson-by-Lesson Review

12-1 Samples and Studies

Identify the sample as *biased* or *unbiased*. Explain your reasoning.

6. **GOVERNMENT** To determine whether voters support a new trade agreement, 5 people from the list of registered voters in each state are selected at random.

Determine whether each situation describes a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

7. **SCHOOL DANCE** The homecoming dance committee sends out a questionnaire to all the girls in the school to decide on a theme for the homecoming dance.
8. **MILKSHAKE** Mary wants to test her milkshake recipe using honey instead of sugar. She randomly gives half of her 6 friends milkshakes sweetened with honey and the other half the same milkshakes sweetened with sugar. Then she asks them how they like the milkshakes.

Example 1

MUSIC A randomly selected group of people who listen to a country music radio station is asked to name their favorite type of music. Identify the sample as *biased* or *unbiased*. Explain your reasoning.

The sample is biased. People listening to a country music radio station are likely to vote for country music.

Example 2

BOOK COVER An artist is trying to choose a cover for a children's book. She sends out a flyer with the two covers to all of the students at one school. She asks them to check the favorite cover. Determine whether this situation describes a *survey*, an *observational study*, or an *experiment*. Explain your reasoning.

This is a survey. The data are gathered from the responses given by members in the sample.

12-2 Statistics and Parameters

9. **SHOVELING** Ben shovels driveways to raise money. The number of driveways he shovels each day is {2, 4, 3, 5, 3}. Find and interpret the mean absolute deviation.

10. **CANDY BARS** Luci is keeping track of the number of candy bars each member of the drill team sold. The results are {20, 25, 30, 50, 40, 60, 20, 10, 42}. Find and interpret the mean absolute deviation.

11. **FOOD** A fast food company polls a random sample of its day and night customers to find how many times a month they eat out. Compare the means and standard deviations of each data set.

Day Customers	Night Customers
10, 3, 12, 15, 7, 8, 4, 12, 9, 14, 12, 9	15, 12, 13, 9, 11, 12, 14, 12, 8, 16, 9, 9

Example 3

GIFTS Joshua is collecting money from his family for a Mother's Day gift. He keeps track of how much each person has donated: {10, 5, 20, 15, 10}. Find and interpret the mean absolute deviation.

Step 1 Find the mean: $\bar{x} = \frac{10 + 5 + 20 + 15 + 10}{5}$ or 12.

Step 2 Find the absolute values of the differences.

$$\begin{aligned} x_1 = 10: & |12 - 10| \text{ or } 2 & x_2 = 5: & |12 - 5| \text{ or } 7 \\ x_3 = 20: & |12 - 20| \text{ or } 8 & x_4 = 15: & |12 - 15| \text{ or } 3 \\ x_5 = 10: & |12 - 10| \text{ or } 2 \end{aligned}$$

Step 3 Find the sum: $2 + 7 + 8 + 3 + 2 = 22$.

Step 4 Find the mean absolute deviation.

$$\begin{aligned} \text{MAD} &= \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_n - \bar{x}|}{n} \\ &= \frac{22}{5} \text{ or } 4.4 \end{aligned}$$

A mean absolute deviation of 4.4 indicates that the data, on average, are 4.4 units away from the mean.

12-3 Distributions of Data

Use a graphing calculator to construct a histogram for the data. Then describe the shape of the distribution.

- 55, 62, 32, 56, 31, 59, 19, 61, 8, 48, 41, 69, 32, 63, 48, 60, 43, 66, 71, 70, 49, 56, 21, 67
- 4, 19, 62, 28, 26, 59, 33, 39, 36, 72, 46, 48, 49, 44, 72, 76, 55, 53, 55, 62, 66, 69, 71, 74
- MILK** A grocery store manager tracked the amount of milk in gallons sold each day. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

Gallons of Milk Sold Per Day					
383	296	354	288	195	372
421	367	411	355	296	321
403	357	432	229	180	266

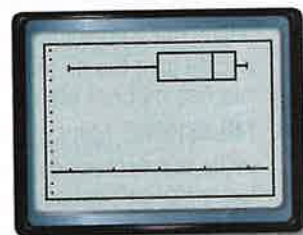
Example 4

DRIVING TESTS Several driving test results are shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

Driving Test Scores					
80	95	100	95	95	100
100	90	75	60	90	80

Use a graphing calculator to create a box-and-whisker plot.

The left whisker is longer than the right and the median is closer to the right whisker. Therefore, the distribution is negatively skewed.



Use the five-number summary. The range is 40.

The median score is 92.5, and half of the drivers scored between 80 and 97.5.

[56, 104] scl: 10 by [-2, 12] scl: 1

12-4 Comparing Sets of Data

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

- 27, 21, 34, 42, 20, 19, 18, 26, 25, 33; $+(-4)$
- 72, 56, 71, 63, 68, 59, 77, 74, 76, 66; $+16$
- SCHOOL** Principal Andrews tracked the number of disciplinary actions given by Ms. Miller and Ms. Anderson to their students each week.

Ms. Miller
9, 16, 12, 11, 12, 9, 10, 14, 13, 10, 9, 10, 11, 9, 12, 10, 11, 12

Ms. Anderson
7, 1, 0, 4, 2, 1, 6, 2, 2, 1, 4, 3, 0, 7, 0, 2, 5, 0

- Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.
- Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

Example 5

Find the mean, median, mode, range, and standard deviation of the data set obtained after adding 6 to each value.

12, 15, 11, 12, 14, 16, 15, 12, 10, 13

Find the mean, median, mode, range, and standard deviation of the original data set.

Mean 13 Mode 12 Standard Deviation 1.8

Median 12.5 Range 6

Add 6 to the mean, median, and mode. The range and standard deviation are unchanged.

Mean 19 Mode 18 Standard Deviation 1.8

Median 18.5 Range 6

12-5 Simulations

- 18. GAMES** While watching a game at a carnival where participants guess which of three shells is covering a ball, Jeremy tallies the following results.

Ball Location	left	middle	right
Frequency	16	18	33

- Find the experimental probability of the ball being under the right shell.
 - Find the experimental probability of the ball not being under the middle shell.
- 19. SCHOOL BUS** Christy has determined that the school bus is late 60% of the time.
- Design a simulation that can be used to estimate the probability that the school bus is late today.
 - Conduct the simulation, and report the results.

Example 6

GROUPS Before the random drawing of groups, Dawn has determined that she is 20% likely to get placed in the same group as Sherry. Design a simulation that can be used to estimate the probability of Dawn and Sherry being in the same group.

Step 1 There are two possible outcomes.

Possible Outcomes	Theoretical Probability
Grouped together	20%
Not grouped together	80%

Step 2 We can use the random number generator on a graphing calculator. Assign the integers 1–5 to accurately represent the probability data.

Outcome	Represented by
Grouped together	1
Not grouped together	2–5

Step 3 A trial will represent one drawing of groups. The simulation can consist of any number of trials. We will use 20.

12-6 Permutations and Combinations

Identify each situation as a *permutation* or a *combination*.

- selecting 3 different toppings for pizza from a list of 15
- an arrangement of textbooks on a bookshelf
- a list of teams participating in a tournament
- a ranking of students by scores
- CLASS PHOTO** The Spanish teacher at South High School wants to arrange 7 students who traveled to Mexico for a yearbook photo. In how many ways can the students be arranged?
- GOLF BALLS** A golf bag contains 5 white golf balls, 6 yellow golf balls, and 4 orange golf balls. Two balls are pulled from the bag at random. What is the probability that both balls are orange golf balls?

Example 7

BOOK COVERS An artist is trying to choose five covers for children's books. There are 10 different covers to choose from. How many ways can the artist choose the covers?

$$C(n, r) = \frac{n!}{(n-r)!r!} \quad \text{Combination Formula}$$

$$C(10, 5) = \frac{10!}{(10-5)!5!} \quad n = 10 \text{ and } r = 5$$

$$= \frac{10!}{5!5!} \quad \text{Simplify.}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{30,240}{120} \text{ or } 252 \quad \text{Simplify.}$$

There are 252 ways for the artist to choose 5 covers.

12-7 Probability of Compound Events

26. **MUSIC** Tracie is playing a mix playlist with 6 classic rock, 8 pop, and 4 dance songs.
- If she selects random play and the songs can repeat, what is the probability that she hears 2 classic rock songs and then a dance song?
 - If the songs cannot repeat, what is the probability that she hears 3 dance songs in a row?

A box contains 8 red chips, 6 blue chips, and 12 white chips. Chips are randomly drawn from the box and are not replaced. Find each probability.

27. $P(\text{red, white, blue})$ 28. $P(\text{red, red, red})$
 29. $P(\text{red, white, white})$ 30. $P(\text{blue, blue})$

One card is randomly drawn from a standard deck of 52 cards. Find each probability.

31. $P(\text{heart or red})$
 32. $P(10 \text{ or spade})$

Example 8

Determine if rolling a die three times is an *independent* or *dependent* event. Then find the probability of rolling a 5 all three times.

These events are independent because the outcome of one does not affect the outcome of the others.

The probability of rolling a 5 is $\frac{1}{6}$.

$$P(5, 5, 5) = P(5) \cdot P(5) \cdot P(5) \\ = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{216}$$

Example 9

A bag of colored paper clips contains 30 red clips, 22 blue clips, and 22 green clips. Three clips are drawn randomly from the bag and are not replaced. Find $P(\text{blue, red, green})$.

$$\text{First clip: } P(\text{blue}) = \frac{22}{74} \quad \text{Second clip: } P(\text{red}) = \frac{30}{73}$$

$$\text{Third clip: } P(\text{green}) = \frac{22}{72}$$

$$P(\text{blue, red, green}) = \frac{22}{74} \cdot \frac{30}{73} \cdot \frac{22}{72} \text{ or } \frac{605}{16,206}$$

12-8 Probability Distributions

A local cable provider asked its subscribers how many television sets they had in their homes. The results of their survey are shown in the probability distribution.

$X = \text{Number of Televisions}$	Probability
1	0.18
2	0.36
3	0.34
4	0.08
5+	0.04

33. Show that the probability distribution is valid.
 34. If a household is selected at random, what is the probability that it has fewer than 4 televisions?

Example 10

The table shows the probability distribution for the number of extracurricular activities in which students at Midpark High School participate.

$X = \text{Number of Activities}$	Probability
0	0.04
1	0.12
2	0.37
3	0.30
4+	0.17

- a. Show that the distribution is valid.

For each X , the probability is greater than or equal to 0 and less than or equal to 1.

$$0.04 + 0.12 + 0.37 + 0.30 + 0.17 = 1$$

- b. What is the probability that a randomly chosen student participates in 1 to 3 activities?

$$P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) \\ = 0.12 + 0.37 + 0.30 = 0.79 \text{ or } 79\%$$

1. **CHOCOLATE** Rico is selling candy. If Marisa randomly selects two candy bars to purchase, what is the probability that she buys a milk chocolate bar followed by a caramel bar?



2. A die is rolled 200 times. What is the experimental probability of rolling less than 3?

Outcome	Frequency
1	30
2	26
3	44
4	38
5	22
6	40

3. **MULTIPLE CHOICE** Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

16, 18, 14, 31, 19, 18, 10, 29,
12, 12, 28, 19, 17, 26, 15, 20

- A positively skewed C symmetric
B negatively skewed D none of the above

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

4. 9, 17, 31, 21, 17, 25, 13, 9, 12, 9; $\times 3$
5. 16, 14, 23, 41, 38, 29, 18, 13, 16; $\times 0.25$

Identify each sample as *biased* or *unbiased*. Explain your reasoning.

6. **NEWSPAPERS** A survey is sent to all people who subscribe to the *Dispatch* to determine what newspaper people prefer to read.
7. **SHOPPING** Each person leaving the Maxtowne Mall is asked to name their favorite clothing store in the mall.

8. **SALES** Nate is keeping track of how much people spent at the school bookstore in one day. Find and interpret the mean absolute deviation for the data: 1, 1, 2, 3, 4, 5, 12.
9. **PIZZA** How many ways can 3 different toppings be chosen from a list of 10 toppings?
10. **EDUCATION** Kristin surveys 200 people in her school to determine how many nights per week students do homework. The results are shown in the table.

Number of Nights	Number of Students
0	10
1	30
2	50
3	90
4	10
5 or more	10

- a. Find the probability that a randomly chosen student will have studied more than 4 nights.
b. Find the probability that a randomly chosen student will have studied no more than 3 nights.
11. **MULTIPLE CHOICE** The second graders are divided into boys and girls. Then 2 girls and 2 boys are chosen at random to represent the class at the Pride Assembly. Which of the following best describes the sample?
- F simple H systematic
G stratified J self-selected
- Identify each situation as a *permutation* or a *combination*.
12. a student's daily class schedule
13. a list of teachers' names at school
14. gold, silver, and bronze medalists
15. **RAFFLE** Carmen is considering paying \$1 for a raffle ticket. What is the expected value of this ticket?

Prize Value	\$10	\$50	\$500
Probability	1 in 100	1 in 500	1 in 5000



Organize Data

Sometimes you may be given a set of data that you need to analyze in order to solve problems on a standardized test. Use this lesson to practice organizing data to help you solve problems.

Strategies for Organizing Data

Step 1

When you are given a problem statement containing data, consider:

- making a list of the data.
- using a table to organize the data.
- using a data display (such as a bar graph, Venn diagram, circle graph, line graph, or box-and-whisker plot) to organize the data.

Step 2

Organize the data.

- Create your table, list, or data display.
- If possible, fill in any missing values that can be found by intermediate computations.

Step 3

Analyze the data to solve the problem.

- Reread the problem statement to determine what you are being asked to solve.
- Use the properties of algebra to work with the organized data and solve the problem.
- If time permits, go back and check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

Of the 24 students in a music class, 10 play the flute, 14 play the piano, and 13 play the guitar. Two students play the flute only, 5 the piano only, and 7 the guitar only. One student plays the flute and the guitar but not the piano. Two students play the piano and guitar but not the flute. Three students play all the instruments. If a student is selected at random, what is the probability that he or she plays the piano and flute, but not the guitar?



Scoring Rubric	
Criteria	Score
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: <ul style="list-style-type: none"> The answer is correct, but the explanation is incomplete. The answer is incorrect, but the explanation is correct. 	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

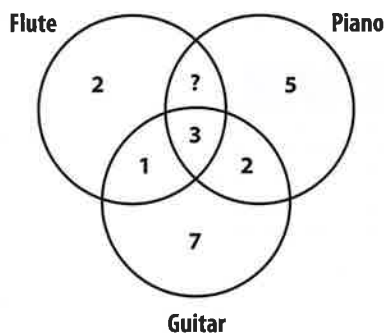
Read the problem carefully. The data is difficult to analyze as it is presented. Use a Venn diagram to organize the data and solve the problem.

Example of a 2-point response:

Use a Venn diagram to organize the data. Fill in all of the information given in the problem statement. There are 14 students who play the piano, so $14 - 5 - 2 - 3$ or 4 students play the piano and the flute, but not the guitar. Find the probability.

$$P(\text{piano and flute}) = \frac{4}{24} \text{ or } \frac{1}{6}$$

So, the probability that a randomly selected student plays the piano and flute but not the guitar is $\frac{1}{6}$.



Exercises

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

- There are 40 students, 9 camp counselors, and 5 teachers at Camp Kern. Each person is assigned to one activity this afternoon. There are 9 students going hiking and 17 students going horseback riding. Of the camp counselors, 2 will supervise the hike and 3 will help with the canoe trip. There are 2 teachers helping with the canoe trip and 2 going horseback riding. Suppose a person is selected at random during the afternoon activities. What is the probability that the one selected is a student on the canoe trip or a camp counselor on a horse? Express your answer as a fraction.
- The table shows the number of coins in a piggy bank.

Coin	Number
Penny	16
Nickel	18
Dime	20
Quarter	10

 - Find the probability that a randomly selected coin will be a dime.
 - Find the probability that a randomly selected coin will be either a nickel or a quarter.
- It takes Craig 40 minutes to mow his family's lawn. His brother Jacob can do the same job in 50 minutes. How long would it take them to mow the lawn together? Round your answer to the nearest tenth of a minute.

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What are the excluded values of the variable in the expression $\frac{x^2 - x - 12}{x^2 - x - 2}$?
- A -1, 2 C -2, 1
B -2, 2 D -3, 4
2. The table shows the number of Calories in twelve different snacks. Find the mean absolute deviation.

Number of Calories in Snacks			
122	91	149	121
64	138	342	72
179	105	99	114

- F 46 H 1.5
G 43 J 0.8
3. Which of the following is *not* a factor of $x^4 - 6x^2 - 27$?
- A $x^2 + 3$ C $x + 3$
B $x - 3$ D $x^2 - 3$
4. Eduardo has 20 CDs. He wants to choose 3 of them at random to take on a road trip. How many different ways can he do this if the order is *not* important?
- F 60 H 1140
G 84 J 6840
5. Which of the following does *not* accurately describe the graph $y = -2x^2 + 4$?
- A The parabola is symmetric about the y -axis.
B The parabola opens downward.
C The parabola has the origin as its vertex.
D The parabola crosses the x -axis in two different places.

6. The highest point in North Carolina is Mt. Mitchell at an elevation of 2,037 meters above sea level. Suppose the position of a hiker is given by the function $p(t) = -2.5t + 2,037$, where t is the number of minutes. Which of the following is the best interpretation of the slope of the function?
- F The hiker's initial position was 2,037 feet below sea level.
G The hiker's initial position was 2,037 feet above sea level.
H The hiker is descending at a rate of 2.5 meters per minute.
J The hiker is ascending at a rate of 2.5 meters per minute
7. Jorge has made 39 out of 52 free throw attempts this season. What is the experimental probability that he makes a free throw?
- A 54% C 75%
B 68% D 79%
8. The graph of which equation passes through the points $(-1, -3)$ and $(-2, 3)$?
- F $y = -6x - 9$
G $y = -\frac{1}{4}x + 3$
H $y = 4x - 5$
J $y = \frac{2}{3}x + 1$
9. At a museum, each child admission costs \$5.75 and each adult costs \$8.25. How much does it cost a family that consists of 2 adults and 4 children?
- A \$34.50 C \$44.50
B \$39.50 D \$49.50

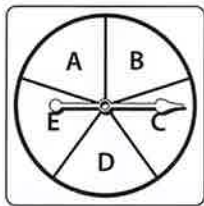
Test-Taking Tip

Question 4 Since order is not important, you are looking for the number of combinations of CDs that can be chosen.

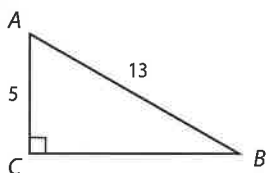
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. **GRIDDED RESPONSE** Suppose Colleen spins the spinner below 80 times and records the results in a frequency table. How many times should she expect to spin a vowel?



11. What is the value of $\sin B$? Express your answer as a fraction.



12. Graph $f(x) \geq |x - 2|$ on a coordinate grid.
13. **GRIDDED RESPONSE** Find the standard deviation of the set of data below to the nearest tenth.

14	11	9	6
10	16	15	13
9	12	19	10

14. Larissa has 5 peanut butter cookies, 7 chocolate chip cookies, 4 sugar cookies, and 9 oatmeal raisin cookies in a jar. If she picks two cookies at random without replacing them, what is the probability that she will choose a peanut butter cookie then a sugar cookie? Express your answer as a fraction.

15. Write an expression that describes the area in square units of a triangle with a height of $4c^3d^2$ and a base of $3cd^4$.

16. Casey made 84 field goals during the basketball season for a total of 183 points. Each field goal was worth either 2 or 3 points. How many 2-point and 3-point field goals did Casey make during the season?

17. **GRIDDED RESPONSE** The booster club pays \$180 to rent a concession stand at a football game. They purchase cans of soda for \$0.25 and sell them at the game for \$1.15. How many cans of soda must they sell to break even?

Extended Response

Record your answers on a sheet of paper. Show your work.

18. To predict whether or not an issue on a ballot will pass or fail, a committee randomly calls 250 houses with area codes that are inside the voting district and asks the opinions of registered voters. Based on these efforts, the committee determines that 71% ($\pm 2.5\%$) of the voting population supports the issue. The committee concludes that the issue will pass.
- Identify the sample.
 - Describe the population.
 - What method of data collection did the committee use: survey, experiment, or observational survey? Explain.
 - Is the sample *biased* or *unbiased*. Explain.
 - If unbiased, classify the sample as *simple*, *systematic*, *self-selected*, *convenience*, or *stratified*. Explain.

Need Extra Help?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Go to Lesson...	11-3	12-2	8-9	12-6	9-3	3-3	12-5	4-2	1-3	12-5	10-6	9-7	12-2	12-7	7-1	6-5	2-4	12-1

