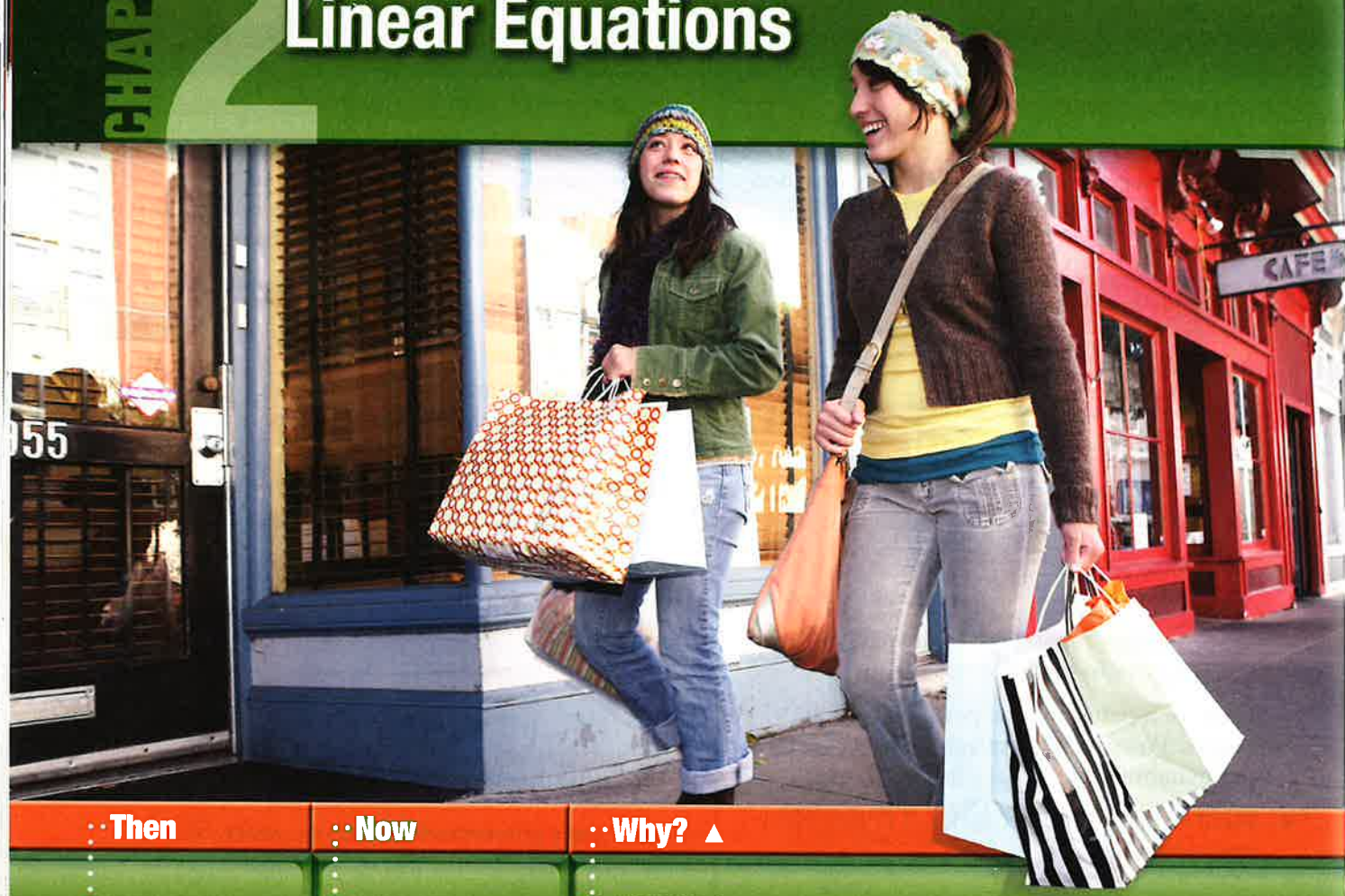
CHAPTER 2

Linear Equations



Then

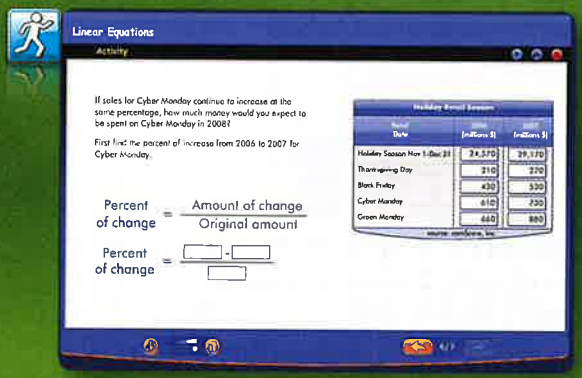
- You learned to simplify algebraic expressions.

Now

- In this chapter, you will:
  - Create equations that describe relationships.
  - Solve linear equations in one variable.
  - Solve proportions.
  - Use formulas to solve real-world problems.

Why? ▲

- **SHOPPING** In recent years, the percent of change in sales per year at shopping malls in the U.S. averaged 5%. A store manager can use this data to set a sales goal for the upcoming year.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Write an algebraic expression for each verbal expression.

1. four less than three times a number n
2. a number d cubed less seven
3. the difference between two times b and eleven

Evaluate each expression.

4. $(9 - 4)^2 \div 3$
5. $\dfrac{3 \cdot 8 - 12 \div 2}{3^2}$
6. $5(8 - 2) \div 3$
7. $\frac{1}{3}(21) + \frac{1}{8}(32)$
8. $72 \div 9 + 3 \cdot 2^3$
9. $\dfrac{11 - 3}{2} + 7$
10. $2[(5 - 3)^2 + 8] + (3 - 1) \div 2$
11. **BAKERY** Sue buys 1 carrot cake for \$14, 6 large chocolate chip cookies for \$1.50 each, and a dozen doughnuts for \$0.45 each. How much money did Sue spend at the bakery?

Find each percent.

12. What percent of 400 is 260?
13. Twelve is what percent of 60?
14. What percent of 25 is 75?
15. **ICE CREAM** What percent of the people surveyed prefer strawberry ice cream?

Favorite Flavor	Number of Responses
vanilla	82
chocolate	76
strawberry	42

QuickReview

Example 1

Write an algebraic expression for the phrase *the product of eight and w increased by nine.*

the product of eight and w increased by nine

$8 \cdot w + 9$

The expression is $8w + 9$.

Example 2

Evaluate $9 - \left[\dfrac{8 + 2^2}{2} - 2(5 \times 2 - 8)\right]$.

$9 - \left[\dfrac{8 + 2^2}{2} - 2(5 \times 2 - 8)\right]$ Original expression

$= 9 - \left[\dfrac{8 + 2^2}{2} - 2(2)\right]$ Evaluate inside the parentheses.

$= 9 - \left(\dfrac{8 + 2^2}{2} - 4\right)$ Multiply.

$= 9 - \left(\dfrac{8 + 4}{2} - 4\right)$ Evaluate the power.

$= 9 - (6 - 4)$ Add and then divide.

$= 7$ Simplify.

Example 3

32 is what percent of 40?

$\dfrac{a}{b} = \dfrac{p}{100}$ Use the percent proportion.

$\dfrac{32}{40} = \dfrac{p}{100}$ Replace a with 32 and b with 40.

$32(100) = 40p$ Find the cross products.

$3200 = 40p$ Multiply.

$80 = p$ Divide each side by 40.

32 is 80% of 40.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 2. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

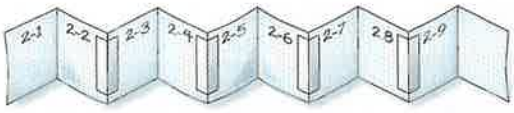
FOLDABLES StudyOrganizer

Linear Equations Make this Foldable to help you organize your Chapter 2 notes about linear equations. Begin with 5 sheets of grid paper.

1 **Fold** each sheet in half along the width.

2 **Unfold** each sheet and tape to form one long piece.

3 **Label** each page with the lesson number as shown. Refold to form a booklet.



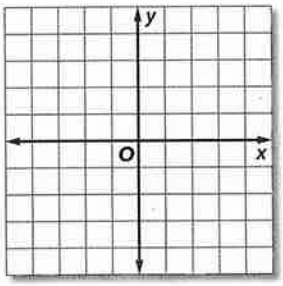
New Vocabulary

English		Español
formula	p. 76	fórmula
solve an equation	p. 83	resolver una ecuación
equivalent equations	p. 83	ecuaciones equivalentes
multi-step equation	p. 91	ecuación de varios pasos
identity	p. 98	identidad
ratio	p. 111	razón
proportion	p. 111	proporción
rate	p. 113	tasa
unit rate	p. 113	tasa unitaria
scale model	p. 114	modelo de escala
percent of change	p. 119	porcentaje de cambio
literal equation	p. 127	ecuación literal
dimensional analysis	p. 128	análisis dimensional
weighted average	p. 132	promedio ponderado

Review Vocabulary

algebraic expression expresion algebraica an expression consisting of one or more numbers and variables along with one or more arithmetic operations

coordinate system sistema de coordenedas the grid formed by the intersection of two number lines, the horizontal axis and the vertical axis



function función a relation in which each element of the domain is paired with exactly one element of the range

LESSON 2-1

Writing Equations

Then

- You evaluated and simplified algebraic expressions.

Now

- 1 Translate sentences into equations.
- 2 Translate equations into sentences.

Why?

- The Daytona 500 is widely considered to be the most important event of the NASCAR circuit. The distance around the track is 2.5 miles, and the race is a total of 500 miles. We can write an equation to determine how many laps it takes to finish the race.

New Vocabulary

formula

Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Write Verbal Expressions To write an equation, identify the unknown for which you are looking and assign a variable to it. Then, write the sentence as an equation. Look for key words such as *is*, *is as much as*, *is the same as*, or *is identical to* that indicate where you should place the equals sign.

Consider the Daytona 500 example above.

Words	The length of each lap	times the	number of laps	is	the length of the race.
Variable	Let ℓ represent the number of laps in the race.				
Equation	2.5	$\times$	ℓ	$=$	500

Example 1 Translate Sentences into Equations

Translate each sentence into an equation.

a. Seven times a number squared is five times the difference of k and m.

Seven	times	n squared	is	five	times	the difference of k and m.
7	$\cdot$	n^2	$=$	5	$\cdot$	$(k - m)$

The equation is $7n^2 = 5(k - m)$.

b. Fifteen times a number subtracted from 80 is 25.

You can rewrite the verbal sentence so it is easier to translate. *Fifteen times a number subtracted from 80 is 25* is the same as *80 minus 15 times a number is 25*. Let n represent the number.

80	minus	15	times	a number	is	25.
80	$-$	15	$\cdot$	n	$=$	25

The equation is $80 - 15n = 25$.

Guided Practice

1A. Two plus the quotient of a number and 8 is the same as 16.

1B. Twenty-seven times k is h squared decreased by 9.

Translating sentences to algebraic expressions and equations is a valuable skill in solving real-world problems.

Real-WorldLink

In 1919, Britain and France offered a flight that carried two passengers at a time. Now there are more than 87,000 flights each day in the U.S.

Source: NATCA

Real-World Example 2 Use the Four-Step Problem-Solving Plan

AIR TRAVEL **Refer to the information at the left. In how many days will 261,000 flights have occurred in the United States?**

Understand The information given in the problem is that there are approximately 87,000 flights per day in the United States. We are asked to find how many days it will take for 261,000 flights to have occurred.

Plan Write an equation. Let d represent the number of days needed.

87,000	times	the number of days	equals	261,000.
87,000	•	d	=	261,000

Solve $87{,}000\,d = 261{,}000$ Find d by asking, "What number times 87,000 is 261,000?"

$d = 3$

Check Check your answer by substituting 3 for d in the equation.

$87{,}000(3) \stackrel{?}{=} 261{,}000$ Substitute 3 for d.

$261{,}000 = 261{,}000$ ✓ Multiply.

The answer makes sense and works for the original problem.

GuidedPractice

2. GOVERNMENT There are 50 members in the North Carolina Senate. This is 70 fewer than the number in the North Carolina House of Representatives. How many members are in the North Carolina House of Representatives?

A rule for the relationship between certain quantities is called a **formula**. These equations use variables to represent numbers and form general rules.

Example 3 Write a Formula

GEOMETRY **Translate the sentence into a formula.**

The area of a triangle equals the product of $\frac{1}{2}$ the length of the base and the height.

Words	The	area of a triangle	equals	the product of $\frac{1}{2}$ the length of the base and the height.
Variables	Let A = area, b = base, and h = height.			
Equation	A	=		$\frac{1}{2}bh$

The formula for the area of a triangle is $A = \frac{1}{2}bh$.

GuidedPractice

3. GEOMETRY Translate the sentence into a formula.
In a right triangle, the square of the measure of the hypotenuse c is equal to the sum of the squares of the measures of the legs, a and b.

Math History Link

Ahmes (about 1680–1620 B.C.) Ahmes was the Egyptian mathematician and scribe who copied the Rhind Mathematical Papyrus. The papyrus contains 87 algebra problems of the same type. The first set of problems asks how to divide n loaves of bread among 10 people.

2 Write Sentences from Equations

If you are given an equation, you can write a sentence or create your own word problem.

Example 4 Translate Equations into Sentences

Translate each equation into a sentence.

a. $6z - 15 = 45$

$6z$	$-$	15	$=$	45
Six times z	minus	fifteen	equals	forty-five.

b. $y^2 + 3x = w$

y^2	$+$	$3x$	$=$	w
The sum of y squared	and	three times x	is	w.

GuidedPractice

4A. $15 = 25u^2 + 2$

4B. $\frac{3}{2}r - t^3 = 132$

When given a set of information, you can create a problem that relates a story.

Example 5 Write a Problem

Write a problem based on the given information.

t = the time that Maxine drove in each turn; $t + 4$ = the time that Tia drove in each turn; $2t + (t + 4) = 28$

Sample problem:

Maxine and Tia went on a trip, and they took turns driving. During her turn, Tia drove 4 hours more than Maxine. Maxine took 2 turns, and Tia took 1 turn. Together they drove for 28 hours. How many hours did Maxine drive?

GuidedPractice

5. p = Beth's salary; $0.1p$ = bonus; $p + 0.1p = 525$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Translate each sentence into an equation.**

1. Three times r less than 15 equals 6.
2. The sum of q and four times t is equal to 29.
3. A number n squared plus 12 is the same as the quotient of p and 4.
4. Half of j minus 5 is the sum of k and 13.
5. The sum of 8 and three times k equals the difference of 5 times k and 3.
6. Three fourths of w plus 5 is one half of w increased by nine.
7. The quotient of 25 and t plus 6 is the same as twice t plus 1.
8. Thirty-two divided by y is equal to the product of three and y minus four.

Example 2

9. **FINANCIAL LITERACY** Samuel has $1900 in the bank. He wishes to increase his account to a total of $2500 by depositing $30 per week from his paycheck. Write and solve an equation to find how many weeks he needs to reach his goal.

10. **MODELING** Miguel is earning extra money by painting houses. He charges a $200 fee plus $12 per can of paint needed to complete the job. Write and use an equation to find how many cans of paint he needs for a $260 job.

Translate each sentence into a formula.

Example 3

11. The perimeter of a regular pentagon is 5 times the length of each side.

12. The area of a circle is the product of π and the radius r squared.

13. Four times π times the radius squared is the surface area of a sphere.

14. One third the product of the length of the side squared and the height is the volume of a pyramid with a square base.

Example 4

Translate each equation into a sentence.

15. $7m - q = 23$

16. $6 + 9k + 5j = 54$

17. $3(g + 8) = 4h - 10$

18. $6d^2 - 7f = 8d + f^2$

Example 5

Write a problem based on the given information.

19. g = gymnasts on a team; $3g = 45$

20. c = cost of a notebook; $0.25c$ = markup; $c + 0.25c = 3.75$

Practice and Problem Solving

Extra Practice is on page R2.

Example 1

Translate each sentence into an equation.

21. The difference of f and five times g is the same as 25 minus f.

22. Three times b less than 100 is equal to the product of 6 and b.

23. Four times the sum of 14 and c is a squared.

Example 2

24. **MUSIC** A piano has 52 white keys. Write and use an equation to find the number of octaves on a piano keyboard.

25. **GARDENING** A flat of plants contains 12 plants. Yoshi wants a garden that has three rows with 10 plants per row. Write and solve an equation for the number of flats Yoshi should buy.

Example 3

Translate each sentence into a formula.

26. The perimeter of a rectangle is equal to 2 times the length plus twice the width.

27. Celsius temperature C is five ninths times the difference of the Fahrenheit temperature F and 32.

28. The density of an object is the quotient of its mass and its volume.

29. Simple interest is computed by finding the product of the principal amount p, the interest rate r, and the time t.

Example 4

Translate each equation into a sentence.

30. $j + 16 = 35$

31. $4m = 52$

32. $7(p + 23) = 102$

33. $r^2 - 15 = t + 19$

34. $\frac{2}{5}v + \frac{3}{4} = \frac{2}{3}x^2$

35. $\frac{1}{3} - \frac{4}{5}z = \frac{4}{3}y^3$

Example 5 **Write a problem based on the given information.**

36. q = quarts of strawberries; $2.50q = 10$

37. p = the principal amount; $0.12p$ = the interest charged; $p + 0.12p = 224$

38. m = number of movies rented; $10 + 1.50m = 14.50$

39. p = the number of players in the game; $5p + 7$ = number of cards in a deck

For Exercises 40–43, match each sentence with an equation.

A. $g^2 = 2(g - 10)$

B. $\frac{1}{2}g + 32 = 15 + 6g$

C. $g^3 = 24g + 4$

D. $3g^2 = 30 + 9g$

40. One half of g plus thirty-two is as much as the sum of fifteen and six times g.

41. A number g to the third power is the same as the product of 24 and g plus 4.

42. The square of g is the same as two times the difference of g and 10.

43. The product of 3 and the square of g equals the sum of thirty and the product of nine and g.

44. FINANCIAL LITERACY Tim's bank contains quarters, dimes, and nickels. He has three more dimes than quarters and 6 fewer nickels than quarters. If he has 63 coins, write and solve an equation to find how many quarters Tim has.

45. SHOPPING Pilar bought 17 items for her camping trip, including tent stakes, packets of drink mix, and bottles of water. She bought 3 times as many packets of drink mix as tent stakes. She also bought 2 more bottles of water than tent stakes. Write and solve an equation to discover how many tent stakes she bought.

46. MULTIPLE REPRESENTATIONS In this problem, you will explore how to translate relations with powers.

x	2	3	4	5	6
y	5	10	17	26	37

a. Verbal Write a sentence to describe the relationship between x and y in the table.

b. Algebraic Write an equation that represents the data in the table.

c. Graphical Graph each ordered pair and draw the function. Describe the graph as discrete or continuous.

H.O.T. Problems Use Higher-Order Thinking Skills

47. OPEN ENDED Write a problem about your favorite television show that uses the equation $x + 8 = 30$.

48. REASONING The surface area of a three-dimensional object is the sum of the areas of the faces. If ℓ represents the length of the side of a cube, write a formula for the surface area of the cube.

49. CHALLENGE Given the perimeter P and width w of a rectangle, write a formula to find the length ℓ.

50. WRITING IN MATH How can you translate a verbal sentence into an algebraic equation?

Standardized Test Practice

51. Which equation *best* represents the relationship between the number of hours an electrician works h and the total charges c?

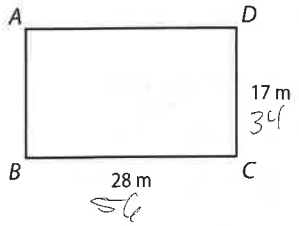
Cost of Electrician	
Emergency House Call	$30 one time fee
Rate	$55/hour

A $c = 30 + 55$
B $c = 30h + 55$
C $c = 30 + 55h$
D $c = 30h + 55h$

52. A car traveled at 55 miles per hour for 2.5 hours and then at 65 miles per hour for 3 hours. How far did the car travel in all?

F 300.5 mi
G 305 mi
H 330 mi
J 332.5 mi

53. SHORT RESPONSE Suppose each dimension of rectangle $ABCD$ is doubled. What is the perimeter of the new $ABCD$?

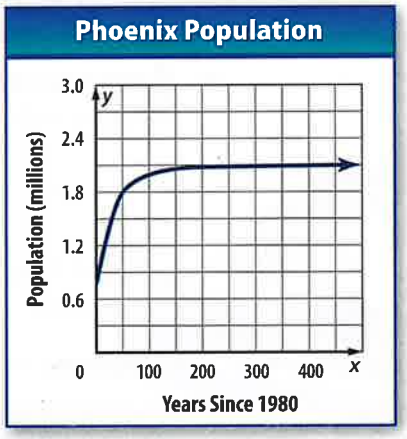


54. STATISTICS Stacy's first five science test scores were 95, 86, 83, 95, and 99. Which of the following is a true statement?

A The mode is the same as the median.
B The median is the same as the mean.
C The range is the same as the mode.
D The mode is the same as the mean.

Spiral Review

55. POPULATION Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph. (Lesson 1-8)



56. SHOPPING Cuties is having a sale on earrings that are regularly $29 for each pair. If you buy 2 pairs, you get 1 pair free. (Lesson 1-7)

a. Make a table that shows the cost of buying 1 to 5 pairs of earrings.

b. Write the data as a set of ordered pairs.

c. Graph the data.

57. GEOMETRY Refer to the table below. (Lesson 1-6)

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Number of Sides	3	4	5	6	7
Interior Angle Sum	180	360	540	720	900

a. Identify the independent and dependent variables.

b. Identify the domain and range for this situation.

c. State whether the function is *discrete* or *continuous*. Explain.

Skills Review

Evaluate each expression.

58. 9^2
59. 10^6
60. 3^5
61. 5^3

EXPLORE

2-2 Algebra Lab
Solving Equations

You can use **algebra tiles** to model solving equations. To **solve an equation** means to find the value of the variable that makes the equation true. An x tile represents the variable x. The 1 tile represents a positive 1. The -1 tile represents a negative 1. And, the $-x$ tile represents the variable negative x. The goal is to get the x-tile by itself on one side of the mat by using the rules stated below.

CCSS Common Core State Standards

Content Standards

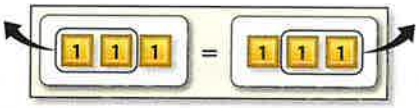
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

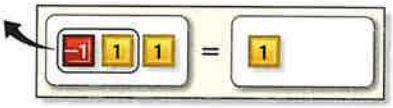
8 Look for and express regularity in repeated reasoning.

Rules for Equation Models When Adding or Subtracting:

- You can remove or add the same number of identical algebra tiles to each side of the mat without changing the equation.



- One positive tile and one negative tile of the same unit are called a zero pair. Since $1 + (-1) = 0$, you can remove or add zero pairs to either side of the equation mat without changing the equation.

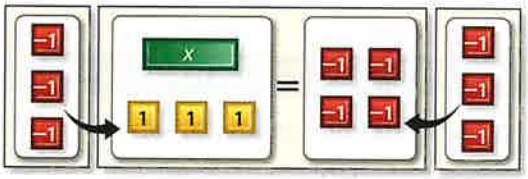


Activity 1 Addition Equation

Use an equation model to solve $x + 3 = -4$.

Step 1 Model the equation. Place 1 x-tile and 3 positive 1-tiles on one side of the mat. Place 4 negative 1-tiles on the other side of the mat.

Step 2 Isolate the x-term. Add 3 negative 1-tiles to each side. The resulting equation is $x = -7$.



$x + 3 = -4$

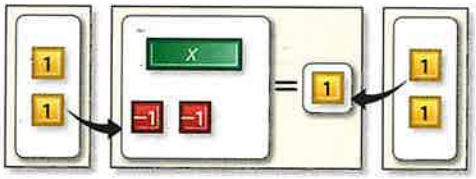
$x + 3 + (-3) = -4 + (-3)$

$x = -7$

Activity 2 Subtraction Equation

Use an equation model to solve $x - 2 = 1$.

Step 1

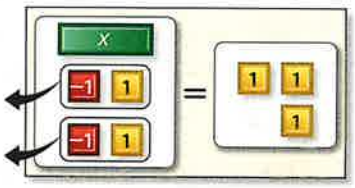


$x - 2 = 1$

$x - 2 + 2 = 1 + 2$

Place 1 x-tile and 2 negative 1-tiles on one side of the mat. Place 1 positive 1-tile on the other side of the mat. Then add 2 positive 1-tiles to each side.

Step 2



$x = 3$

Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is $x = 3$.

(continued on the next page)

Solving Equations *Continued*

Model and Analyze

Use algebra tiles to solve each equation.

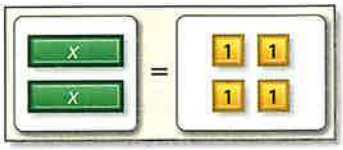
1. $x + 4 = 9$ **2.** $x + (-3) = -4$ **3.** $x + 7 = -2$ **4.** $x + (-2) = 11$

5. WRITING IN MATH If $a = b$, what can you say about $a + c$ and $b + c$? about $a - c$ and $b - c$?

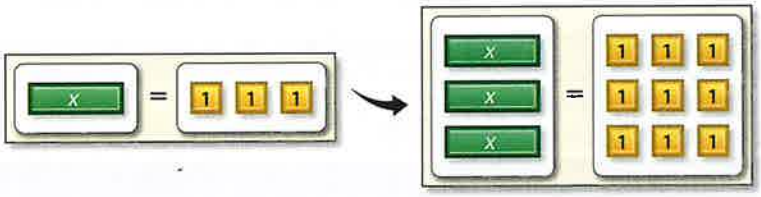
When solving multiplication equations, the goal is still to get the x-tile by itself on one side of the mat by using the rules for dividing.

Rules for the Equation Models When Dividing:

- You can group the tiles on each side of the equation mat into an equal number of groups without changing the equation.



- You can place an equal grouping on each side of the equation mat without changing the equation.

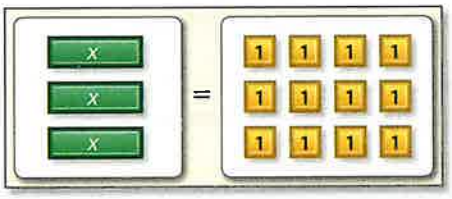


Activity 3 Multiplication Equations

Use an equation model to solve $3x = 12$.

Step 1 Model the equation. Place 3 x-tiles on one side of the mat. Place 12 positive 1-tiles on the other side of the mat.

Step 2 Isolate the x-term. Separate the tiles into 3 equal groups to match the 3 x-tiles. Each x-tile is paired with 4 positive 1-tiles. The resulting equation is $x = 4$.



$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

Model and Analyze

Use algebra tiles to solve each equation.

6. $5x = -15$ **7.** $-3x = -9$ **8.** $4x = 8$ **9.** $-6x = 18$

10. MAKE A CONJECTURE How would you use algebra tiles to solve $\frac{x}{4} = 5$? Discuss the steps you would take to solve this equation algebraically.

LESSON 2-2

Solving One-Step Equations

Then	Now	Why?
• You translated sentences into equations.	• 1 Solve equations by using addition and subtraction. 2 Solve equations by using multiplication and division.	• A record for the most snow angels made at one time was set in Michigan when 3784 people participated. North Dakota had 8910 people register to break the record. To determine how many more people North Dakota had than Michigan, solve the equation $3784 + x = 8910$.

NewVocabulary
solve an equation
equivalent equations

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

6 Attend to precision.

1 Solve Equations Using Addition or Subtraction

In an equation, the variable represents the number that satisfies the equation. To **solve an equation** means to find the value of the variable that makes the equation true.

The process of solving an equation requires assuming that the original equation has a solution and isolating the variable (with a coefficient of 1) on one side of the equation. Each step in this process results in equivalent equations. **Equivalent equations** have the same solution.

KeyConcept Addition Property of Equality

Words	If an equation is true and the same number is added to each side of the equation, the resulting equivalent equation is also true.
Symbols	For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Examples	$14 = 14$ $14 + 3 = 14 + 3$ $17 = 17$ $-3 = -3$ $+ 9 = + 9$ $6 = 6$

Example 1 Solve by Adding

Solve $c - 22 = 54$.

Horizontal Method

$c - 22 = 54$	Original equation
$c - 22 + 22 = 54 + 22$	Add 22 to each side.
$c = 76$	Simplify.

Vertical Method

$$\begin{array}{rl} c - 22 = & 54 \\ + 22 = & + 22 \\ \hline c = & 76 \end{array}$$

To check that 76 is the solution, substitute 76 for c in the original equation.

CHECK

$c - 22 = 54$	Original equation
$76 - 22 \stackrel{?}{=} 54$	Substitute 76 for c.
$54 = 54$ ✓	Subtract.

GuidedPractice

1A. $113 = g - 25$

1B. $j - 87 = -3$

Similar to the Addition Property of Equality, the **Subtraction Property of Equality** can also be used to solve equations.

StudyTip
Subtraction Subtracting a value is equivalent to adding the opposite of the value.

KeyConcept Subtraction Property of Equality

Words	If an equation is true and the same number is subtracted from each side of the equation, the resulting equivalent equation is also true.
Symbols	For any real numbers a, b, and c, if $a = b$, then $a - c = b - c$.
Examples	$87 = 87$ $87 - 17 = 87 - 17$ $70 = 70$
	$13 = 13$ $-28 = -28$ $-15 = -15$

StudyTip
Solving an Equation When solving equations you can use either the horizontal method or the vertical method. Both methods will produce the same result.

Example 2 Solve by Subtracting

Solve $63 + m = 79$.

Horizontal Method

$63 + m = 79$	Original equation
$63 - 63 + m = 79 - 63$	Subtract 63 from each side.
$m = 16$	Simplify.

Vertical Method

$$\begin{array}{rr} 63 + m = & 79 \\ -63 & = -63 \\ \hline m = & 16 \end{array}$$

To check that 16 is the solution, replace m with 16 in the original equation.

CHECK

$63 + m = 79$	Original equation
$63 + 16 \stackrel{?}{=} 79$	Substitution, $m = 16$
$79 = 79$ ✓	Simplify.

GuidedPractice

2A. $27 + k = 30$

2B. $-12 = p + 16$

2 Solve Equations Using Multiplication or Division

In the equation $\frac{x}{3} = 9$, the variable x is divided by 3. To solve for x, undo the division by multiplying each side by 3. This is an example of the **Multiplication Property of Equality**.

KeyConcept Multiplication Property of Equality

Words	If an equation is true and each side is multiplied by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $ac = bc$.
Example	If $x = 5$, then $3x = 15$.

Division Property of Equality

Words	If an equation is true and each side is divided by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
Example	If $x = -20$, then $\frac{x}{5} = \frac{-20}{5}$ or -4.

The reciprocal of a number can be used to solve equations.

ReviewVocabulary
reciprocal the multiplicative inverse of a number

Example 3 Solve by Multiplying or Dividing

Solve each equation.

a. $\frac{2}{3}q = \frac{1}{2}$

$\frac{2}{3}q = \frac{1}{2}$ Original equation

$\frac{3}{2}\left(\frac{2}{3}\right)q = \frac{3}{2}\left(\frac{1}{2}\right)$ Multiply each side by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

$q = \frac{3}{4}$ Check the result.

b. $39 = -3r$

$39 = -3r$ Original equation

$\frac{39}{-3} = \frac{-3r}{-3}$ Divide each side by -3.

$-13 = r$ Check the result.

GuidedPractice

3A. $\frac{3}{5}k = 6$ **3B.** $-\frac{1}{4} = \frac{2}{3}b$

We can also use reciprocals and properties of equality to solve real-world problems.

Ronnie Kaufman/Blend Images/Getty Images

Real-WorldLink
Almost half of 10- to 18-year-olds in the U.S. use a cell phone. Of those, 53% play games on their phones, more than 33% download games, 52% use the calendar/organizer, and nearly all teens with camera phones snap pictures.

Source: Lexdon Business Library

Real-World Example 4 Solve by Multiplying

SURVEYS **Of a group of 13- to 15-year-old girls surveyed, 225, or about $\frac{9}{20}$ said they talk on the telephone while they watch television. About how many girls were surveyed?**

Words	Nine twentieths times those surveyed	is	225.
Variable	Let g = the number of girls surveyed.		
Equation	$\frac{9}{20}g$	=	225

$\frac{9}{20}g = 225$ Original equation

$\left(\frac{20}{9}\right)\frac{9}{20}g = \left(\frac{20}{9}\right)225$ Multiply each side by $\frac{20}{9}$.

$g = \frac{4500}{9}$ $\left(\frac{20}{9}\right)\left(\frac{9}{20}\right) = 1$

$g = 500$ Simplify.

About 500 girls were surveyed.

GuidedPractice

4. STAINED GLASS Allison is making a stained glass window. Her pattern requires that one fifth of the glass should be blue. She has 288 square inches of blue glass. If she intends to use all of her blue glass, how much glass will she need for the entire project?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Solve each equation. Check your solution.

1. $g + 5 = 33$
2. $104 = y - 67$
3. $\frac{2}{3} + w = 1\frac{1}{2}$
4. $-4 + t = -7$
5. $a + 26 = 35$
6. $-6 + c = 32$
7. $1.5 = y - (-5.6)$
8. $3 + g = \frac{1}{4}$
9. $x + 4 = \frac{3}{4}$
10. $\frac{t}{7} = -5$
11. $\frac{a}{36} = \frac{4}{9}$
12. $\frac{2}{3}n = 10$
13. $\frac{8}{9} = \frac{4}{5}k$
14. $12 = \frac{x}{-3}$
15. $-\frac{r}{4} = \frac{1}{7}$

Example 4

16. **FUNDRAISING** The television show "Idol Gives Back" raised money for relief organizations. During this show, viewers could call in and vote for their favorite performer. The parent company contributed $5 million for the 50 million votes cast. What did they pay for each vote?

17. CCSS **REASONING** Hana decides to buy her cat a bed from an online fund that gives $\frac{7}{8}$ of her purchase to shelters that care for animals. How much of Hana's money went to the animal shelter?

Practice and Problem Solving

Extra Practice is on page R2.

Examples 1–3 Solve each equation. Check your solution.

18. $v - 9 = 14$
19. $44 = t - 72$
20. $-61 = d + (-18)$
21. $18 + z = 40$
22. $-4a = 48$
23. $12t = -132$
24. $18 - (-f) = 91$
25. $-16 - (-t) = -45$
26. $\frac{1}{3}v = -5$
27. $\frac{u}{8} = -4$
28. $\frac{a}{6} = -9$
29. $-\frac{k}{5} = \frac{7}{5}$
30. $\frac{3}{4} = w + \frac{2}{5}$
31. $-\frac{1}{2} + a = \frac{5}{8}$
32. $-\frac{t}{7} = \frac{1}{15}$
33. $-\frac{5}{7} = y - 2$
34. $v + 914 = -23$
35. $447 + x = -261$
36. $-\frac{1}{7}c = 21$
37. $-\frac{2}{3}h = -22$
38. $\frac{3}{5}q = -15$
39. $\frac{n}{8} = -\frac{1}{4}$
40. $\frac{c}{4} = -\frac{9}{8}$
41. $\frac{2}{3} + r = -\frac{4}{9}$

Example 4

42. **CATS** A domestic cat can run at speeds of 27.5 miles per hour when chasing prey. A cheetah can run 42.5 miles per hour faster when chasing prey. How fast can the cheetah go?

43. **CARS** The average time t it takes to manufacture a car in the United States is 24.9 hours. This is 8.1 hours longer than the average time it takes to manufacture a car in Japan. Write and solve an equation to find the average time in Japan.

Solve each equation. Check your solution.

44. $\frac{x}{9} = 10$

45. $\frac{b}{7} = -11$

46. $\frac{3}{4} = \frac{c}{24}$

47. $\frac{2}{3} = \frac{1}{8}y$

48. $\frac{2}{3}n = 14$

49. $\frac{3}{5}g = -6$

50. $4\frac{1}{5} = 3p$

51. $-5 = 3\frac{1}{2}x$

52. $6 = -\frac{1}{2}n$

53. $-\frac{2}{5} = -\frac{z}{45}$

54. $-\frac{g}{24} = \frac{5}{12}$

55. $-\frac{v}{5} = -45$

Write an equation for each sentence. Then solve the equation.

56. Six times a number is 132.

57. Two thirds equals negative eight times a number.

58. Five elevenths times a number is 55.

59. Four fifths is equal to ten sixteenths of a number.

60. Three and two thirds times a number equals two ninths.

61 Four and four fifths times a number is one and one fifth.

62. CCSS **PRECISION** Adelina is comparing prices for two brands of health and energy bars at the local grocery store. She wants to get the best price for each bar.

a. Write an equation to find the price for each bar of the Feel Great brand.

b. Write an equation to find the price of each bar for the Super Power brand.

c. Which bar should Adelina buy? Explain.

63. **MEDIA** The world's largest passenger plane, the Airbus A380, was first used by Singapore Airlines in 2005. The following description appeared on a news Web site after the plane was introduced.

"That airline will see the A380 transporting some 555 passengers, 139 more than a similarly set-up 747."

How many passengers will a similarly set-up 747 transport?

64. **FUEL** In 2004, approximately 5 million cars and trucks were classified as flex-fuel, which means they could run on gasoline or ethanol. In 2009, that number increased to about 8 million. How many more cars and trucks were flex-fuel in 2009?

65. **CHEERLEADING** At a certain cheerleading competition, the maximum time per team, including the set up, is 3 minutes. The Ridgeview High School squad's performance time is 2 minutes and 34 seconds. How much time does the squad have left for their set up?

66. **COMIC BOOKS** An X-Men #1 comic book in mint condition recently sold for $45,000. An Action Comics #63 (Mile High), also in mint condition, sold for $15,000. How much more did the X-Men comic book sell for than the Action Comics book?

67. **MOVIES** A certain movie made $1.6 million in ticket sales. Its sequel made $0.8 million in ticket sales. How much more did the first movie make than the sequel?

68. **CAMERAS** An electronics store sells a certain digital camera for $126. This is $\frac{2}{3}$ of the price that a photography store charges. What is the cost of the camera at the photography store?

69 **BLOGS** In 2006, 57 million American adults read online blogs. However, 45 million fewer American adults say that they maintain their own blog. How many American adults maintain a blog?

70. **SCIENCE CAREERS** According to the Bureau of Labor and Statistics, approximately 140,000,000 people were employed in the United States in 2009.

   a. The number of people in production occupations times 20 is the number of working people. Write an equation to represent the number of people employed in production occupations in 2009. Then solve the equation.

   b. The number of people in repair occupations is 2,300,000 less than the number of people in production occupations. How many people are in repair occupations?

71. **DANCES** Student Council has a budget of $1000 for the homecoming dance. So far, they have spent $350 dollars for music.

   a. Write an equation to represent the amount of money left to spend. Then solve the equation.

   b. They then spent $225 on decorations. Write an equation to represent the amount of money left.

   c. If the Student Council spent their entire budget, write an equation to represent how many $6 tickets they must sell to make a profit.

H.O.T. Problems Use Higher-Order Thinking Skills

72. **WHICH ONE DOESN'T BELONG?** Identify the equation that does not belong with the other three. Explain your reasoning.

$n + 14 = 27$	$12 + n = 25$	$n - 16 = 29$	$n - 4 = 9$

73. **OPEN ENDED** Write an equation involving addition and demonstrate two ways to solve it.

74. **REASONING** For which triangle is the height not $4\frac{1}{2}b$, where b is the length of the base?

Triangle	Base (cm)	Height (cm)
$\triangle ABC$	3.8	17.1
$\triangle MQP$	5.4	24.3
$\triangle RST$	6.3	28.5
$\triangle TRW$	1.6	7.2

75. CCSS **STRUCTURE** Determine whether each sentence is *sometimes*, *always*, or *never true*. Explain your reasoning.

   a. $x + x = x$ **b.** $x + 0 = x$

76. **REASONING** Determine the value for each statement below.

   a. If $x - 7 = 14$, what is the value of $x - 2$?

   b. If $t + 8 = -12$, what is the value of $t + 1$?

77. **CHALLENGE** Solve each equation for x. Assume that $a \neq 0$.

   a. $ax = 12$ **b.** $x + a = 15$ **c.** $-5 = x - a$ **d.** $\frac{1}{a}x = 10$

78. **WRITING IN MATH** Consider the Multiplication Property of Equality and the Division Property of Equality. Explain why they can be considered the same property. Which one do you think is easier to use?

Standardized Test Practice

79. Which of the following best represents the equation $w - 15 = 33$?

A Jake added w ounces of water to his bottle, which originally contained 33 ounces of water. How much water did he add?

B Jake added 15 ounces of water to his bottle, for a total of 33 ounces. How much water w was originally in the bottle?

C Jake drank 15 ounces of water from his bottle and 33 ounces were left. How much water w was originally in the bottle?

D Jake drank 15 ounces of water from his water bottle, which originally contained 33 ounces. How much water w was left?

80. SHORT RESPONSE Charlie's company pays him for every mile that he drives on his trip. When he drives 50 miles, he is paid \$30. To the nearest tenth, how many miles did he drive if he was paid \$275?

81. The table shows the results of a survey given to 500 international travelers. Based on the data, which statement is true?

Vacation Plans	
Destination	**Percent**
The Tropics	37
Europe	19
Asia	17
Other	17
No Vacation	10

F Fifty have no vacation plans.

G Fifteen are going to Asia.

H One third are going to the tropics.

J One hundred are going to Europe.

82. GEOMETRY The amount of water needed to fill a pool represents the pool's ____.

A volume

B surface area

C circumference

D perimeter

Spiral Review

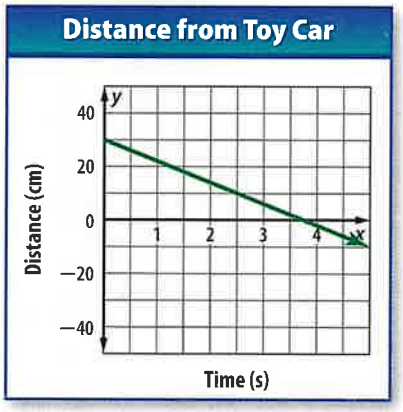
Translate each sentence into an equation. (Lesson 2-1)

83. The sum of twice r and three times k is identical to thirteen.

84. The quotient of t and forty is the same as twelve minus half of u.

85. The square of m minus the cube of p is sixteen.

86. TOYS Identify the function graphed as *linear* or *nonlinear*. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph. (Lesson 1-8)



Skills Review

87. COMMUNICATION Sato communicates with friends for a project. He averages 5 hours using email, 8 hours on the phone, and 2 hours with them in person the first week. If this trend continues, write and evaluate an expression to predict how many hours he will spend communicating with friends over the next 12 weeks.

88. PETS The Poochie Pet supply store has the following items on sale. Write and evaluate an expression to find the total cost of purchasing 1 collar, 2 T-shirts, 3 kerchiefs, 1 leash, and 4 flying disks.

Item	Cost (\$)
studded collar	4.50
kerchief	3.00
doggy T-shirt	6.25
leash	5.50
flying disk	3.25

EXPLORE

2-3 Algebra Lab
Solving Multi-Step Equations

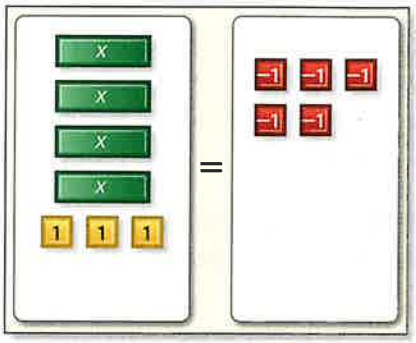
You can use algebra tiles to model solving multi-step equations.

Common Core State Standards
Content Standards
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Activity

Use an equation model to solve $4x + 3 = -5$.

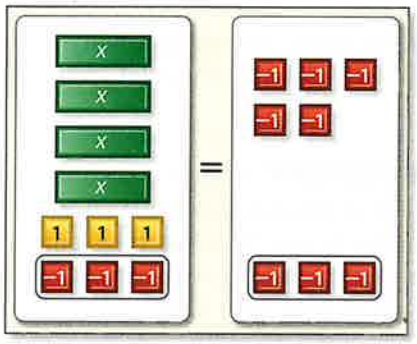
Step 1 Model the equation.



$4x + 3 = -5$

Place 4 x-tiles and 3 positive 1-tiles on one side of the mat. Place 5 negative 1-tiles on the other side.

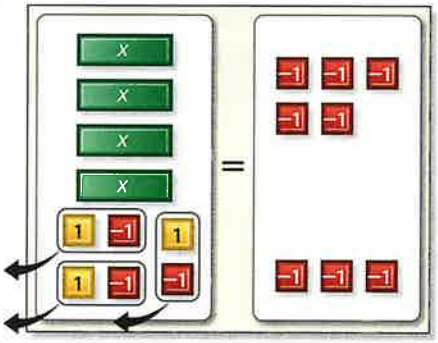
Step 2 Isolate the x-term.



$4x + 3 - 3 = -5 - 3$

Since there are 3 positive 1-tiles with the x-tiles, add 3 negative 1-tiles to each side to form zero pairs.

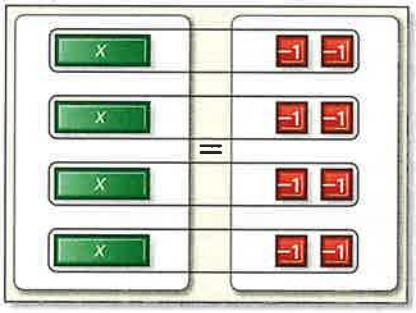
Step 3 Remove zero pairs.



$4x = -8$

Group the tiles to form zero pairs and remove the zero pairs.

Step 4 Group the tiles.



$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$

Separate the remaining tiles into 4 equal groups to match the 4 x-tiles. Each x-tile is paired with 2 negative 1-tiles. The resulting equation is $x = -2$.

Model

Use algebra tiles to solve each equation.

1. $3x - 7 = -10$
2. $2x + 5 = 9$
3. $5x - 7 = 8$
4. $-7 = 3x + 8$
5. $5 + 4x = -11$
6. $3x + 1 = 7$
7. $11 = 2x - 5$
8. $7 + 6x = -11$

9. What would be your first step in solving $8x - 29 = 67$?

10. What steps would you use to solve $9x + 14 = -49$?

LESSON 2-3

Solving Multi-Step Equations

Then
- You solved one-step equations.

Now
1. Solve equations involving more than one operation.
2. Solve equations involving consecutive integers.

Why?
- The Tour de France is the premier cycling event in the world. The map shows the 2007 Tour de France course. If the length of the shortest portion of the race can be represented by k, the expression $4k + 20$ is the length of the longest stage or 236 kilometers. This can be described by the equation $4k + 20 = 236$.

New Vocabulary
- multi-step equation
- consecutive integers
- number theory

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Solve Multi-Step Equations

Since the above equation requires more than one step to solve, it is called a **multi-step equation**. To solve this equation, we must undo each operation by working backward.

Example 1 Solve Multi-Step Equations

Solve each equation. Check your solution.

a. $11x - 4 = 29$

$11x - 4 = 29$	Original equation
$11x - 4 + 4 = 29 + 4$	Add 4 to each side.
$11x = 33$	Simplify.
$\frac{11x}{11} = \frac{33}{11}$	Divide each side by 11.
$x = 3$	Simplify.

b. $\frac{a + 7}{8} = 5$

$\frac{a + 7}{8} = 5$	Original equation
$8\left(\frac{a + 7}{8}\right) = 8(5)$	Multiply each side by 8.
$a + 7 = 40$	Simplify.
$-7 = -7$	Subtract 7 from each side.
$a = 33$	Simplify.

You can check your solutions by substituting the results back into the original equations.

Guided Practice

Solve each equation. Check your solution.

1A. $2a - 6 = 4$

1B. $\frac{n + 1}{-2} = 15$

Real-WorldLink

Shoppers in Shanghai, China, can pay for purchased items at a terminal that can match the buyers' fingerprints with their bank accounts.

Source: *Shanghai Daily*

Real-World Example 2 Write and Solve a Multi-Step Equation

SHOPPING Hiroshi is buying a pair of water skis that are on sale for $\frac{2}{3}$ of the original price. After he uses a $25 gift certificate, the total cost before taxes is $115. What was the original price of the skis? Write an equation for the problem. Then solve the equation.

Words	Two thirds	of	the price	minus	25	is	115.
Variable	Let p = original price of the skis.						
Equation	$\frac{2}{3}$	•	p	−	25	=	115

$$\frac{2}{3}p - 25 = 115 \quad \text{Original equation}$$

$$\frac{2}{3}p - 25 + 25 = 115 + 25 \quad \text{Add 25 to each side.}$$

$$\frac{2}{3}p = 140 \quad \text{Simplify.}$$

$$\frac{3}{2}\left(\frac{2}{3}p\right) = \frac{3}{2}(140) \quad \text{Multiply each side by } \frac{3}{2}.$$

$$p = 210 \quad \text{Simplify.}$$

The original price of the skis was $210.

GuidedPractice

2A. RETAIL A music store has sold $\frac{3}{5}$ of their hip-hop CDs, but 10 were returned. Now the store has 62 hip-hop CDs. How many were there originally?

2B. READING Len read $\frac{3}{4}$ of a graphic novel over the weekend. Monday, he read 22 more pages. If he has read 220 pages, how many pages does the book have?

2 Solve Consecutive Integer Problems

Consecutive integers are integers in counting order, such as 4, 5, and 6 or n, $n + 1$, and $n + 2$. Counting by two will result in *consecutive even integers* if the starting integer n is even and *consecutive odd integers* if the starting integer n is odd.

ConceptSummary Consecutive Integers

Type	Words	Symbols	Example
Consecutive Integers	Integers that come in counting order.	$n, n + 1, n + 2, \ldots$	$\ldots, -2, -1, 0, 1, 2, \ldots$
Consecutive Even Integers	Even integer followed by the next even integer.	$n, n + 2, n + 4, \ldots$	$\ldots, -2, 0, 2, 4, \ldots$
Consecutive Odd Integers	Odd integer followed by the next odd integer.	$n, n + 2, n + 4, \ldots$	$\ldots, -1, 1, -3, 5, \ldots$

Number theory is the study of numbers and the relationships between them.

Example 3 Solve a Consecutive Integer Problem

NUMBER THEORY **Write an equation for the following problem. Then solve the equation and answer the problem.**

Find three consecutive odd integers with a sum of −51.

Let n = the least odd integer.

Then $n + 2$ = the next greater odd integer, and $n + 4$ = the greatest of the three integers.

StudyTip

Regularity You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of n (odd or even) that differs between the two expressions.

Words	The sum of three consecutive odd integers	is	−51.
Equation	$n + (n + 2) + (n + 4)$	$=$	−51.

$n + (n + 2) + (n + 4) = -51$ Original equation

$3n + 6 = -51$ Simplify.

$-6 = -6$ Subtract 6 from each side.

$3n = -57$ Simplify.

$\frac{3n}{3} = \frac{-57}{3}$ Divide each side by 3.

$n = -19$ Simplify.

$n + 2 = -19 + 2$ or -17 $\quad n + 4 = -19 + 4$ or -15

The consecutive odd integers are −19, −17, and −15.

CHECK −19, −17, and −15 are consecutive odd integers.

$-19 + (-17) + (-15) = -51$ ✓

GuidedPractice

3. Write an equation for the following problem. Then solve the equation and answer the problem.

Find three consecutive integers with a sum of 21.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Solve each equation. Check your solution.**

1. $3m + 4 = -11$
2. $12 = -7f - 9$
3. $-3 = 2 + \frac{a}{11}$
4. $\frac{3}{2}a - 8 = 11$
5. $8 = \frac{x - 5}{7}$
6. $\frac{c + 1}{-3} = -21$

Example 2

7. **NUMBER THEORY** Twelve decreased by twice a number equals −34. Write an equation for this situation and then find the number.
8. **BASEBALL** Among the career home run leaders for Major League Baseball, Hank Aaron has 175 fewer than twice the number that Dave Winfield has. Hank Aaron hit 755 home runs. Write an equation for this situation. How many home runs did Dave Winfield hit in his career?

Example 3 **Write an equation and solve each problem.**

9. Find three consecutive odd integers with a sum of 75.
10. Find three consecutive integers with a sum of −36.

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Solve each equation. Check your solution.**

11. $3t + 7 = -8$

12. $8 = 16 + 8n$

13. $-34 = 6m - 4$

14. $9x + 27 = -72$

15. $\frac{y}{5} - 6 = 8$

16. $\frac{f}{-7} - 8 = 2$

17. $1 + \frac{r}{9} = 4$

18. $\frac{k}{3} + 4 = -16$

19. $\frac{n-2}{7} = 2$

20. $14 = \frac{6+z}{-2}$

21. $-11 = \frac{a-5}{6}$

22. $\frac{22-w}{3} = -7$

Example 2

23. **FINANCIAL LITERACY** The Cell+ Cellular Phone store offers the plans shown in the table. Raul chose the business plan and has budgeted $100 per month. Write an equation for this situation, and determine how many minutes per month he can use the phone and stay within budget.

Plan	Flat Monthly Fee	Anytime Minutes	Cost per Minute After Anytime Minutes
personal	$29.99	250	$0.20
business	$49.99	650	$0.15
executive	$59.99	1200	$0.10

Example 3 **Write an equation and solve each problem.**

24. Fourteen less than three fourths of a number is negative eight. Find the number.

25. Seventeen is thirteen subtracted from six times a number. What is the number?

26. Find three consecutive even integers with the sum of -84.

27. Find three consecutive odd integers with the sum of 141.

28. Find four consecutive integers with the sum of 54.

29. Find four consecutive integers with the sum of -142.

Solve each equation. Check your solution.

30. $-6m - 8 = 24$

31. $45 = 7 - 5n$

32. $\frac{2b}{3} + 6 = 24$

33. $\frac{5x}{9} - 11 = -51$

34. $65 = \frac{3}{4}c - 7$

35. $9 + \frac{2}{3}x = 81$

36. $-\frac{5}{2} = \frac{3}{4}z + \frac{1}{2}$

37. $\frac{5}{6}k + \frac{2}{3} = \frac{4}{3}$

38. $-\frac{1}{5} - \frac{4}{9}a = \frac{2}{15}$

39. $-\frac{3}{7} = \frac{3}{4} - \frac{b}{2}$

Write an equation and solve each problem.

40. **REASONING** The ages of three brothers are consecutive integers with the sum of 96. How old are the brothers?

41. **VOLCANOES** Moving lava can build up and form beaches at the coast of an island. The growth of an island in a seaward direction may be modeled as $8y + 2$ centimeters, where y represents the number of years that the lava flows. An island has expanded 60 centimeters seaward. How long has the lava flowed?

Solve each equation. Check your solution.

42. $-5x - 4.8 = 6.7$

43. $3.7q + 26.2 = 111.67$

44. $0.6a + 9 = 14.4$

45. $\frac{c}{2} - 4.3 = 11.5$

46. $9 = \frac{-6p - (-3)}{-8}$

47. $3.6 - 2.4m = 12$

48. If $7m - 3 = 53$, what is the value of $11m + 2$?

49. If $13y + 25 = 64$, what is the value of $4y - 7$?

50. If $-5c + 6 = -69$, what is the value of $6c - 15$?

51. AMUSEMENT PARKS An amusement park offers a yearly membership of \$275 that allows for free parking and admission to the park. Members can also use the water park for an additional \$5 per day. Nonmembers pay \$6 for parking, \$15 for admission, and \$9 for the water park.

a. Write and solve an equation to find the number of visits it would take for the total cost to be the same for a member and a nonmember if they both use the water park at each visit.

b. Make a table for the costs of members and nonmembers after 3, 6, 9, 12, and 15 visits to the park.

c. Plot these points on a coordinate graph and describe what you see.

52. SHOPPING At The Family Farm, you can pick your own fruits and vegetables.

a. The cost of a bag of potatoes is \$1.50 less than $\frac{1}{2}$ of the price of apples. Write and solve an equation to find the cost of potatoes.

b. The price of each zucchini is 3 times the price of winter squash minus \$7. Write and solve an equation to find the cost of zucchini.

c. Write an equation to represent the cost of a pumpkin using the cost of the blueberries.

The Family Farm	
Fruit	**Price (\$)**
Apples	6.99/bag
Pumpkins	5.00 each
Blueberries	2.99/qt
Winter squash	2.99 each

H.O.T. Problems Use Higher-Order Thinking Skills

53. OPEN ENDED Write a problem that can be modeled by the equation $2x + 40 = 60$. Then solve the equation and explain the solution in the context of the problem.

54. CHALLENGE Solve each equation for x. Assume that $a \neq 0$.

a. $ax + 7 = 5$ **b.** $\frac{1}{a}x - 4 = 9$ **c.** $2 - ax = -8$

55. REASONING Determine whether each equation has a solution. Justify your answer.

a. $\frac{a+4}{5+a} = 1$ **b.** $\frac{1+b}{1-b} = 1$ **c.** $\frac{c-5}{5-c} = 1$

56. CCSS REGULARITY Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The sum of three consecutive odd integers equals an even integer.

57. WRITING IN MATH Write a paragraph explaining the order of the steps that you would take to solve a multi-step equation.

Standardized Test Practice

58. Which is the best estimate for the number of minutes on the calling card advertised below?

A 10 min
B 20 min
C 50 min
D 200 min

59. GRIDDED RESPONSE The scale factor for two similar triangles is 2 : 3. The perimeter of the smaller triangle is 56 cm. What is the perimeter of the larger triangle in centimeters?

60. Mr. Morrison is draining his cylindrical pool. The pool has a radius of 10 feet and a standard height of 4.5 feet. If the pool water is pumped out at a constant rate of 5 gallons per minute, about how long will it take to drain the pool? ($1 \text{ ft}^3 = 7.5$ gal)

F 37.8 min
G 7 h
H 25.4 h
J 35.3 h

61. STATISTICS Look at the golf scores for the five players in the table.

Player	1	2	3	4	5
Score	80	91	103	79	78

Which of these is the range of the golf scores?

A 10
B 25
C 35
D 40

Spiral Review

62. GAS MILEAGE A midsize car with a 4-cylinder engine travels 34 miles on a gallon of gas. This is 10 miles more than a luxury car with an 8-cylinder engine travels on a gallon of gas. How many miles does a luxury car travel on a gallon of gas? (Lesson 2-2)

63. DEER In a recent year, 1286 female deer were born in Clark County. That is 93 fewer than the number of male deer born. How many male deer were born that year? (Lesson 2-2)

Translate each equation into a verbal sentence. (Lesson 2-1)

64. $f - 15 = 6$
65. $3h + 7 = 20$
66. $k^2 + 18 = 54 - m$
67. $3p = 8p - r$
68. $\frac{3}{5}t + \frac{1}{3} = t$
69. $\frac{1}{2}v = \frac{2}{3}v + 4$

70. GEOGRAPHY The Pacific Ocean covers about 46% of Earth. If P represents the surface area of the Pacific Ocean and E represents the surface area of Earth, write an equation for this situation. (Lesson 2-1)

Find the value of n in each equation. Then name the property that is used. (Lesson 1-3)

71. $1.5 + n = 1.5$
72. $8n = 1$
73. $4 - n = 0$
74. $1 = 2n$

Skills Review

Evaluate each expression.

75. $5 + 3(4^2)$
76. $\frac{38 - 12}{2 \cdot 13}$
77. $[5(1 + 1)]^3$
78. $[8(2) - 4^2] + 7(4)$

LESSON 2-4

Solving Equations with the Variable on Each Side

Then

- You solved multi-step equations.

Now

1. Solve equations with the variable on each side.
2. Solve equations involving grouping symbols.

Why?

- The equation $y = 1.3x + 19$ represents the number of times Americans eat in their cars each year, where x is the number of years since 1985, and y is the number of times that they eat in their car. The equation $y = -1.3x + 93$ represents the number of times Americans eat in restaurants each year, where x is the number of years since 1985, and y is the number of times that they eat in a restaurant.

The equation $1.3x + 19 = -1.3x + 93$ represents the year when the number of times Americans eat in their cars will equal the number of times Americans eat in restaurants.

Gen Nishino/Taxi/Getty Images

New Vocabulary

identity

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

1 Variables on Each Side

To solve an equation that has variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable terms on one side.

Example 1 Solve an Equation with Variables on Each Side

Solve $2 + 5k = 3k - 6$. Check your solution.

$2 + 5k = 3k - 6$	Original equation
$-3k = -3k$	Subtract $3k$ from each side.
$2 + 2k = -6$	Simplify.
$-2 = -2$	Subtract 2 from each side.
$2k = -8$	Simplify.
$\frac{2k}{2} = \frac{-8}{2}$	Divide each side by 2.
$k = -4$	Simplify.

CHECK

$2 + 5k = 3k - 6$	Original equation
$2 + 5(-4) \stackrel{?}{=} 3(-4) - 6$	Substitution, $k = -4$
$2 + -20 \stackrel{?}{=} -12 - 6$	Multiply.
$-18 = -18$ ✓	Simplify.

Guided Practice

Solve each equation. Check your solution.

1A. $3w + 2 = 7w$

1B. $5a + 2 = 6 - 7a$

1C. $\frac{x}{2} + 1 = \frac{1}{4}x - 6$

1D. $1.3c = 3.3c + 2.8$

2 Grouping Symbols If equations contain grouping symbols such as parentheses or brackets, use the Distributive Property first to remove the grouping symbols.

Example 2 Solve an Equation with Grouping Symbols

Solve $6(5m - 3) = \frac{1}{3}(24m + 12)$.

$6(5m - 3) = \frac{1}{3}(24m + 12)$	Original equation
$30m - 18 = 8m + 4$	Distributive Property
$30m - 18 - 8m = 8m + 4 - 8m$	Subtract $8m$ from each side.
$22m - 18 = 4$	Simplify.
$22m - 18 + 18 = 4 + 18$	Add 18 to each side.
$22m = 22$	Simplify.
$\frac{22m}{22} = \frac{22}{22}$	Divide each side by 22.
$m = 1$	Simplify.

StudyTip
Solving an Equation
You may want to eliminate the terms with a variable from one side before eliminating a constant.

GuidedPractice

Solve each equation. Check your solution.

2A. $8s - 10 = 3(6 - 2s)$ **2B.** $7(n - 1) = -2(3 + n)$

Some equations may have no solution. That is, there is no value of the variable that will result in a true equation. Some equations are true for all values of the variables. These are called **identities**.

Example 3 Find Special Solutions

Solve each equation.

a. $5x + 5 = 3(5x - 4) - 10x$

$5x + 5 = 3(5x - 4) - 10x$	Original equation
$5x + 5 = 15x - 12 - 10x$	Distributive Property
$5x + 5 = 5x - 12$	Simplify.
$-5x \quad = -5x$	Subtract $5x$ from each side.
$5 \neq -12$	

Since $5 \neq -12$, this equation has no solution.

ReadingMath
No Solution The symbol that represents no solution is $\varnothing$.

b. $3(2b - 1) - 7 = 6b - 10$

$3(2b - 1) - 7 = 6b - 10$	Original equation
$6b - 3 - 7 = 6b - 10$	Distributive Property
$6b - 10 = 6b - 10$	Simplify.
$0 = 0$	Subtract $6b - 10$ from each side.

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of b.

GuidedPractice

3A. $7x + 5(x - 1) = -5 + 12x$ **3B.** $6(y - 5) = 2(10 + 3y)$

The steps for solving an equation can be summarized as follows.

ConceptSummary Steps for Solving Equations

Step 1 Simplify the expressions on each side. Use the Distributive Property as needed.

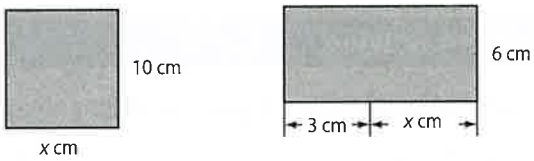
Step 2 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side. Simplify.

Step 3 Use the Multiplication or Division Property of Equality to solve.

There are many situations in which you must simplify expressions with grouping symbols in order to solve an equation.

Standardized Test Example 4 Write an Equation

Find the value of x so that the figures have the same area.



A 3

B 4.5

C 6.5

D 7

Test-TakingTip

CCSS Tools There is often more than one way to solve a problem. In this example, you can write an algebraic equation and solve for x. Or you can substitute each answer choice into the formulas to find the correct answer.

Read the Test Item

The area of the first rectangle is $10x$, and the area of the second is $6(3 + x)$. The equation $10x = 6(3 + x)$ represents this situation.

Solve the Test Item

A $10x = 6(3 + x)$

$10(3) \stackrel{?}{=} 6(3 + 3)$

$30 \stackrel{?}{=} 6(6)$

$30 \neq 36$ ✗

B $10x = 6(3 + x)$

$10(4.5) \stackrel{?}{=} 6(3 + 4.5)$

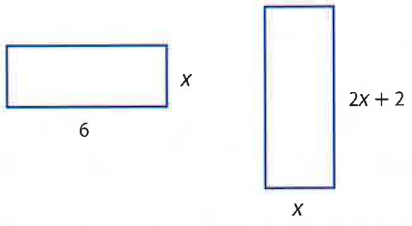
$45 \stackrel{?}{=} 6(7.5)$

$45 = 45$ ✓

Since the value 4.5 results in a true statement, you do not need to check 6.5 and 7. The answer is B.

GuidedPractice

4. Find the value of x so that the figures have the same perimeter.



F 1.5 **G** 2 **H** 3.2 **J** 4

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 Solve each equation. Check your solution.

1. $13x + 2 = 4x + 38$
2. $\frac{2}{3} + \frac{1}{6}q = \frac{5}{6}q + \frac{1}{3}$
3. $6(n + 4) = -18$
4. $7 = -11 + 3(b + 5)$
5. $5 + 2(n + 1) = 2n$
6. $7 - 3r = r - 4(2 + r)$
7. $14v + 6 = 2(5 + 7v) - 4$
8. $5h - 7 = 5(h - 2) + 3$

Example 4

9. **MULTIPLE CHOICE** Find the value of x so that the figures have the same perimeter.

A 4 B 5 C 6 D 7

Practice and Problem Solving

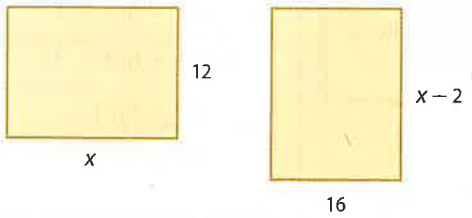
Extra Practice is on page R2.

Examples 1–3 Solve each equation. Check your solution.

10. $7c + 12 = -4c + 78$
11. $2m - 13 = -8m + 27$
12. $9x - 4 = 2x + 3$
13. $6 + 3t = 8t - 14$
14. $\frac{b - 4}{6} = \frac{b}{2}$
15. $\frac{5v - 4}{10} = \frac{4}{5}$
16. $8 = 4(r + 4)$
17. $6(n + 5) = 66$
18. $5(g + 8) - 7 = 103$
19. $12 - \frac{4}{5}(x + 15) = 4$
20. $3(3m - 2) = 2(3m + 3)$
21. $6(3a + 1) - 30 = 3(2a - 4)$

Example 4

22. **GEOMETRY** Find the value of x so the rectangles have the same area.



23. **NUMBER THEORY** Four times the lesser of two consecutive even integers is 12 less than twice the greater number. Find the integers.

24. CCSS **SENSE-MAKING** Two times the least of three consecutive odd integers exceeds three times the greatest by 15. What are the integers?

Solve each equation. Check your solution.

25. $2x = 2(x - 3)$
26. $\frac{2}{5}h - 7 = \frac{12}{5}h - 2h + 3$
27. $-5(3 - q) + 4 = 5q - 11$
28. $2(4r + 6) = \frac{2}{3}(12r + 18)$
29. $\frac{3}{5}f + 24 = 4 - \frac{1}{5}f$
30. $\frac{1}{12} + \frac{3}{8}y = \frac{5}{12} + \frac{5}{8}y$
31. $\frac{2m}{5} = \frac{1}{3}(2m - 12)$
32. $\frac{1}{8}(3d - 2) = \frac{1}{4}(d + 5)$
33. $6.78j - 5.2 = 4.33j + 2.15$
34. $14.2t - 25.2 = 3.8t + 26.8$
35. $3.2k - 4.3 = 12.6k + 14.5$
36. $5[2p - 4(p + 5)] = 25$

37. **NUMBER THEORY** Three times the lesser of two consecutive even integers is 6 less than six times the greater number. Find the integers.

38. **MONEY** Chris has saved twice the number of quarters that Nora saved plus 6. The number of quarters Chris saved is also five times the difference of the number of quarters and 3 that Nora has saved. Write and solve an equation to find the number of quarters they each have saved.

39. **DVD** A company that replicates DVDs spends \$1500 per day in building overhead plus \$0.80 per DVD in supplies and labor. If the DVDs sell for \$1.59 per disk, how many DVDs must the company sell each day before it makes a profit?

40. **MOBILE PHONES** The table shows the number of mobile phone subscribers for two states for a recent year. How long will it take for the numbers of subscribers to be the same?

State	Mobile Phone Subscribers (thousands)	New Subscribers Each Year (thousands)
Alabama	3765	325
Wisconsin	3842	292

41. **MULTIPLE REPRESENTATIONS** In this problem, you will explore $2x + 4 = -x - 2$.

a. **Graphical** Make a table of values with five points for $y = 2x + 4$ and $y = -x - 2$. Graph the points from the tables.

b. **Algebraic** Solve $2x + 4 = -x - 2$.

c. **Verbal** Explain how the solution you found in part **b** is related to the intersection point of the graphs in part **a**.

H.O.T. Problems Use Higher-Order Thinking Skills

42. **REASONING** Solve $5x + 2 = ax - 1$ for x. Assume that $a \neq 0$. Describe each step.

43. **CHALLENGE** Write an equation with the variable on each side of the equals sign, at least one fractional coefficient, and a solution of -6. Discuss the steps you used.

44. **OPEN ENDED** Create an equation with at least two grouping symbols for which there is no solution.

45. **CCSS CRITIQUE** Determine whether each solution is correct. If the solution is not correct, describe the error and give the correct solution.

a.
$$\begin{aligned} 2(g + 5) &= 22 \\ 2g + 5 &= 22 \\ 2g + 5 - 5 &= 22 \\ 2g &= 17 \\ g &= 8.5 \end{aligned}$$

b.
$$\begin{aligned} 5d &= 2d - 18 \\ 5d - 2d &= 2d - 18 - 2d \\ 3d &= -18 \\ d &= -6 \end{aligned}$$

c.
$$\begin{aligned} -6z + 13 &= 7z \\ -6z + 13 - 6z &= 7z - 6z \\ 13 &= z \end{aligned}$$

46. **CHALLENGE** Find the value of k for which each equation is an identity.

a. $k(3x - 2) = 4 - 6x$ **b.** $15y - 10 + k = 2(ky - 1) - y$

47. **WRITING IN MATH** Compare and contrast solving equations with variables on both sides of the equation to solving one-step or multi-step equations with a variable on one side of the equation.

Standardized Test Practice

48. A hang glider, 25 meters above the ground, starts to descend at a constant rate of 2 meters per second. Which equation shows the height h after t seconds of descent?

A $h = 25t + 2t$
B $h = -25t + 2$
C $h = 2t + 25$
D $h = -2t + 25$

49. GEOMETRY Two rectangular walls each with a length of 12 feet and a width of 23 feet need to be painted. It costs \$0.08 per square foot for paint. How much will it cost to paint the walls?

F \$22.08
G \$23.04
H \$34.50
J \$44.16

50. SHORT RESPONSE Maddie works at Game Exchange. They are having a sale as shown.

Item	Price	Special
video games	\$20	Buy 2 get 1 Free
DVDs	\$15	Buy 1 get 1 Free

Her employee discount is 15%. If sales tax is 7.25%, how much does she spend for a total of 4 video games?

51. Solve $\frac{4}{5}x + 7 = \frac{3}{15}x - 3$.

A $-16\frac{2}{3}$
B $-14\frac{4}{9}$
C -10
D $-6\frac{2}{3}$

Spiral Review

Solve each equation. Check your solution. (Lesson 2-3)

52. $5n + 6 = -4$
53. $-1 = 7 + 3c$
54. $\frac{1}{2}z + 7 = 16 - \frac{3}{5}z$
55. $\frac{2}{5}x + 6 = \frac{2}{3}x + 10$
56. $\frac{a}{7} - 3 = -2$
57. $9 + \frac{y}{5} = 6$

58. WORLD RECORDS In 1998, Winchell's House of Donuts in Pasadena, California, made the world's largest donut. It weighed 5000 pounds and had a circumference of 298.3 feet. What was the donut's diameter to the nearest tenth? (*Hint:* $C = \pi d$) (Lesson 2-2)

59. ZOO At a zoo, the cost of admission is posted on the sign. Find the cost of admission for two adults and two children. (Lesson 1-4)

Find the value of n. Then name the property used in each step. (Lesson 1-3)

60. $25n = 25$
61. $n \cdot 1 = 2$
62. $12 \cdot n = 12 \cdot 6$
63. $n + 0 = \frac{2}{3}$
64. $4 \cdot \frac{1}{4} = n$
65. $(10 - 8)(7) = 2(n)$

Skills Review

Translate each sentence into an equation.

66. Twice a number t decreased by eight equals seventy.
67. Five times the sum of m and k is the same as seven times k.
68. Half of p is the same as p minus 3.

Evaluate each expression.

69. $-9 - (-14)$
70. $-10 + (20)$
71. $-15 - 9$
72. $5(14)$
73. $-55 \div (-5)$
74. $-25(-5)$

LESSON 2-5

Solving Equations Involving Absolute Value

Then

- You solved equations with the variable on each side.

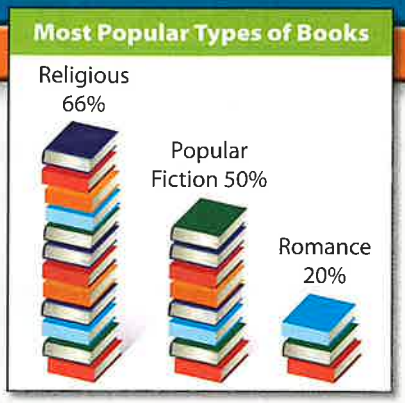
Now

1. Evaluate absolute value expressions.
2. Solve absolute value equations.

Why?

- In 2007, a telephone poll was conducted to determine the reading habits of people in the U.S. People in this survey were allowed to select more than one type of book.

  The survey had a margin of error of ±3%. This means that the results could be three points higher or lower. So, the percent of people who read religious material could be as high as 69% or as low as 63%.



Source: CNN

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

7 Look for and make use of structure.

1 Absolute Value Expressions

Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable.

Example 1 Expressions with Absolute Value

Evaluate $|m + 6| - 14$ if $m = 4$.

$|m + 6| - 14 = |4 + 6| - 14$ Replace m with 4.

$= |10| - 14$ $4 + 6 = 10$

$= 10 - 14$ $|10| = 10$

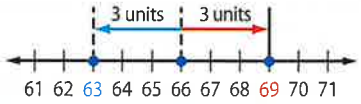
$= -4$ Simplify.

GuidedPractice

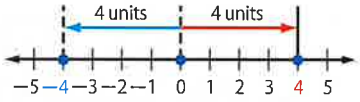
1. Evaluate $23 - |3 - 4x|$ if $x = 2$.

2 Absolute Value Equations

The margin of error in the example at the top of the page is an example of absolute value. The distance between 66 and 69 on a number line is the same as the distance between 63 and 66.



There are three types of open sentences involving absolute value, $|x| = n$, $|x| < n$, and $|x| > n$. In this lesson, we will consider only the first type. Look at the equation $|x| = 4$. This means that the distance between 0 and x is 4.



If $|x| = 4$, then $x = -4$ or $x = 4$. Thus, the solution set is $\{-4, 4\}$.

For each absolute value equation, we must consider both cases. To solve an absolute value equation, first isolate the absolute value on one side of the equals sign if it is not already by itself.

ReadingMath

Absolute Values The expression $|f + 5|$ is read the *absolute value of the quantity f plus 5.*

KeyConcept Absolute Value Equations

Words	When solving equations that involve absolute values, there are two cases to consider.
	Case 1 The expression inside the absolute value symbol is positive or zero.
	Case 2 The expression inside the absolute value symbol is negative.
Symbols	For any real numbers a and b, if $\|a\| = b$ and $b \geq 0$, then $a = b$ or $a = -b$.
Example	$\|d\| = 10$, so $d = 10$ or $d = -10$.

Example 2 Solve Absolute Value Equations

Solve each equation. Then graph the solution set.

a. $|f + 5| = 17$

$|f + 5| = 17$ Original equation

Case 1

$f + 5 = 17$

$f + 5 - 5 = 17 - 5$ Subtract 5 from each side.

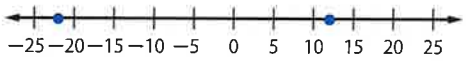
$f = 12$ Simplify.

Case 2

$f + 5 = -17$

$f + 5 - 5 = -17 - 5$

$f = -22$



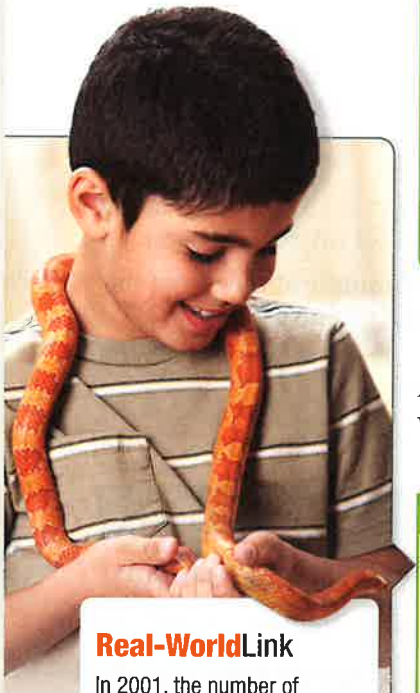
b. $|b - 1| = -3$

$|b - 1| = -3$ means the distance between b and 1 is -3. Since distance cannot be negative, the solution is the empty set $\varnothing$.

GuidedPractice

2A. $|y + 2| = 4$

2B. $|3n - 4| = -1$



Real-WorldLink

In 2001, the number of households in the U.S. that had either a turtle, snake, lizard, or other reptile as a pet was 1,678,000.

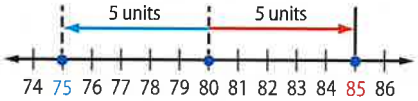
Source: American Veterinary Medical Association

Absolute value equations occur in real-world situations that describe a range within which a value must lie.

Real-World Example 3 Solve an Absolute Value Equation

SNAKES The temperature of an enclosure for a pet snake should be about 80°F, give or take 5°. Find the maximum and minimum temperatures.

You can use a number line to solve.



The distance from 80 to 75 is 5 units.
The distance from 80 to 85 is 5 units.

The solution set is {75, 85}. The maximum and minimum temperatures are 85° and 75°.

Guided Practice

3. ICE CREAM Ice cream should be stored at 5°F with an allowance for 5°. Write and solve an equation to find the maximum and minimum temperatures at which the ice cream should be stored.

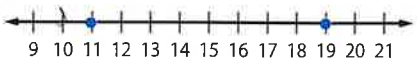
When given two points on a graph, you can write an absolute value equation for the graph.

StudyTip

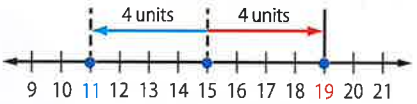
Find the Midpoint To find the point midway between two points, add the values together and divide by 2. For Example 4, $11 + 19 = 30$, $30 \div 2 = 15$. So 15 is the point halfway between 11 and 19.

Example 4 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.



Find the point that is the same distance from 11 and from 19. This is the midpoint between 11 and 19, which is 15.

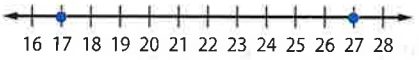


The distance from 15 to 11 is 4 units.
The distance from 15 to 19 is 4 units.

So an equation is $|x - 15| = 4$.

Guided Practice

4. Write an equation involving absolute value for the graph.



Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Evaluate each expression if $f = 3$, $g = -4$, and $h = 5$.**

1. $|3 - h| + 13$ **2.** $16 - |g + 9|$ **3.** $|f + g| - h$

Example 2 **Solve each equation. Then graph the solution set.**

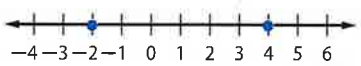
4. $|n + 7| = 5$ **5.** $|3z - 3| = 9$ **6.** $|4n - 1| = -6$

7. $|b + 4| = 2$ **8.** $|2t - 4| = 8$ **9.** $|5h + 2| = -8$

Example 3 **10. FINANCIAL LITERACY** For a company to invest in a product, they must believe they will receive a 12% return on investment (ROI) plus or minus 3%. Write an equation to find the least and the greatest ROI they believe they will receive.

Example 4 **Write an equation involving absolute value for each graph.**

11.



12.

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Evaluate each expression if $a = -2$, $b = -3$, $c = 2$, $x = 2.1$, $y = 3$, and $z = -4.2$.**

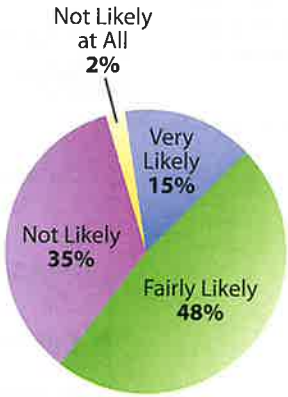
13. $|2x + z| + 2y$
14. $4a - |3b + 2c|$
15. $-|5a + c| + |3y + 2z|$
16. $-a + |2x - a|$
17. $|y - 2z| - 3$
18. $3|3b - 8c| - 3$
19. $|2x - z| + 6b$
20. $-3|z| + 2(a + y)$
21. $-4|c - 3| + 2|z - a|$

Example 2 **Solve each equation. Then graph the solution set.**

22. $|n - 3| = 5$
23. $|f + 10| = 1$
24. $|v - 2| = -5$
25. $|4t - 8| = 20$
26. $|8w + 5| = 21$
27. $|6y - 7| = -1$
28. $\left|\frac{1}{2}x + 5\right| = -3$
29. $|-2y + 6| = 6$
30. $\left|\frac{3}{4}a - 3\right| = 9$

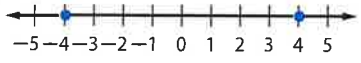
Example 3

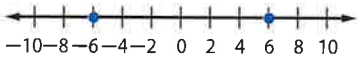
31. **SURVEY** The circle graph at the right shows the results of a survey that asked, "How likely is it that you will be rich some day?" If the margin of error is ±4%, what is the range of the percent of teens who say it is very likely that they will be rich?

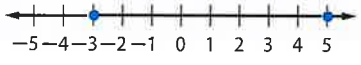


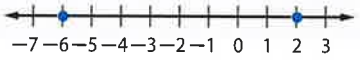
32. **CHEERLEADING** For competition, the cheerleading team is preparing a dance routine that must last 4 minutes, with a variation of ±5 seconds.
    a. Find the least and greatest possible times for the routine in minutes and seconds.
    b. Find the least and greatest possible times in seconds.

Example 4 **Write an equation involving absolute value for each graph.**

33. 

34. 

35. 

36. 

Solve each equation. Then graph the solution set.

37. $\left|-\frac{1}{2}b - 2\right| = 10$
38. $|-4d + 6| = 12$
39. $|5f - 3| = 12$
40. $2|h| - 3 = 8$
41. $4 - 3|q| = 10$
42. $\frac{4}{|p|} + 12 = 14$

43. CCSS **SENSE-MAKING** The 4 × 400 relay is a race where 4 runners take turns running 400 meters, or one lap around the track.
    a. If a runner runs the first leg in 52 seconds plus or minus 2 seconds, write an equation to find the fastest and slowest times.
    b. If the runners of the second and third legs run their laps in 53 seconds plus or minus 1 second, write an equation to find the fastest and slowest times.
    c. Suppose the runner of the fourth leg is the fastest on the team. If he runs an average of 50.5 seconds plus or minus 1.5 seconds, what are the team's fastest and slowest times?

44. FASHION To allow for a model's height, a designer is willing to use models that require him to change hems either up or down 2 inches. The length of the skirts is 20 inches.

a. Write an absolute value equation that represents the length of the skirts.

b. What is the range of the lengths of the skirts?

c. If a 20-inch skirt was fitted for a model that is 5 feet 9 inches tall, will the designer use a 6-foot-tall model?

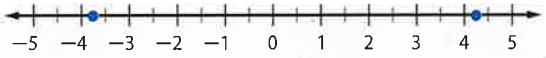
45. CCSS PRECISION Speedometer accuracy can be affected by many details such as tire diameter and axle ratio. For example, there is variation of ±3 miles per hour when calibrated at 50 miles per hour.

a. What is the range of actual speeds of the car if calibrated at 50 miles per hour?

b. A speedometer calibrated at 45 miles per hour has an accepted variation of ±1 mile per hour. What can we conclude from this?

Write an equation involving absolute value for each graph.

46.



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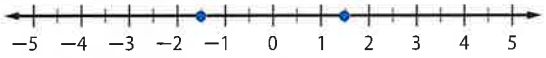

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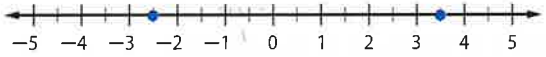

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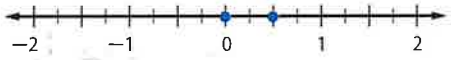

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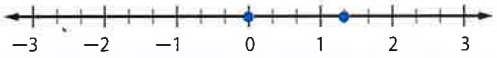

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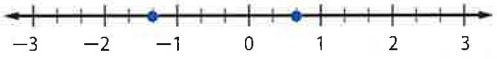


52. MUSIC A CD will record an hour and a half of music plus or minus 3 minutes for time between tracks.

a. Write an absolute value equation that represents the recording time.

b. What is the range of time in minutes that the CD could run?

c. Graph the possible times on a number line.

53. ACOUSTICS The Red Rocks Amphitheater located in the Red Rock Park near Denver, Colorado, is the only naturally occurring amphitheater. The acoustic qualities here are such that a maximum of 20,000 people, plus or minus 1000, can hear natural voices clearly.

a. Write an equation involving an absolute value that represents the number of people that can hear natural voices at Red Rocks Amphitheater.

b. Find the maximum and minimum number of people that can hear natural voices clearly in the amphitheater.

c. What is the range of people in part **b**?

54. **BOOK CLUB** The members of a book club agree to read within ten pages of the last page of the chapter. The chapter ends on page 203.

a. Write an absolute value equation that represents the pages where club members could stop reading.

b. Write the range of the pages where the club members could stop reading.

55. **SCHOOL** Teams from Washington and McKinley High Schools are competing in an academic challenge. A correct response on a question earns 10 points and an incorrect response loses 10 points. A team earns 0 points on an unattempted question. There are 5 questions in the math section.

a. What are the maximum and minimum scores a team can earn on the math section?

b. Suppose the McKinley team has 160 points at the start of the math section. Write and solve an equation that represents the maximum and minimum scores the team could have at the end of the math section.

c. What are all of the possible scores that a school can earn on the math section?

H.O.T. Problems Use Higher-Order Thinking Skills

56. **OPEN ENDED** Describe a real-world situation that could be represented by the absolute value equation $|x - 4| = 10$.

CCSS **STRUCTURE** **Determine whether the following statements are *sometimes*, *always*, or *never* true, if c is an integer. Explain your reasoning.**

57. The value of $|x + 1|$ is greater than zero.

58. The solution of $|x + c| = 0$ is greater than 0.

59. The inequality $|x| + c < 0$ has no solution.

60. The value of $|x + c| + c$ is greater than zero.

61. **REASONING** Explain why an absolute value can never be negative.

62. **CHALLENGE** Use the sentence $x = 7 \pm 4.6$.

a. Describe the values of x that make the sentence true.

b. Translate the sentence into an equation involving absolute value.

63. **ERROR ANALYSIS** Alex and Wesley are solving $|x + 5| = -3$. Is either of them correct? Explain your reasoning.

Alex

$|x + 5| = 3$ or $|x + 5| = -3$

$x + 5 = 3$ $\quad$ $x + 5 = -3$

$-5 \quad -5$ $\quad$ $-5 \quad -5$

$x = -2$ $\quad$ $x = -8$

Wesley

$|x + 5| = -3$

The solution is ∅.

64. **WRITING IN MATH** Explain why there are either two, one, or no solutions for absolute value equations. Demonstrate an example of each possibility.

Standardized Test Practice

65. Which equation represents the second step of the solution process?

Step 1: $4(2x + 7) - 6 = 3x$

Step 2: _____

Step 3: $5x + 28 - 6 = 0$

Step 4: $5x = -22$

Step 5: $x = -4.4$

A $4(2x - 6) + 7 = 3x$
B $4(2x + 1) = 3x$
C $8x + 7 - 6 = 3x$
D $8x + 28 - 6 = 3x$

66. GEOMETRY The area of a circle is 25π square centimeters. What is the circumference?

F 625π cm
G 50π cm
H 25π cm
J 10π cm

67. Tanya makes \$5 an hour and 15% commission of the total dollar value on cosmetics she sells. Suppose Tanya's commission is increased to 17%. How much money will she make if she sells \$300 worth of product and works 30 hours?

A \$201
B \$226
C \$255
D \$283

68. EXTENDED RESPONSE John's mother has agreed to take him driving every day for two weeks. On the first day, John drives for 20 minutes. Each day after that, John drives 5 minutes more than the day before.

a. Write an expression for the minutes John drives on the nth day. Explain.

b. For how many minutes will John drive on the last day? Show your work.

c. John's driver's education teacher requires that each student drive for 30 hours with an adult outside of class. Will John's sessions with his mother fulfill this requirement?

Spiral Review

Write and solve an equation for each sentence. (Lesson 2-4)

69. One half of a number increased by 16 is four less than two thirds of the number.

70. The sum of one half of a number and 6 equals one third of the number.

71. SHOE If ℓ represents the length of a man's foot in inches, the expression $2\ell - 12$ can be used to estimate his shoe size. What is the approximate length of a man's foot if he wears a size 8? (Lesson 2-3)

Skills Review

Write an equation for each problem. Then solve the equation.

72. Seven times a number equals -84. What is the number?

73. Two fifths of a number equals -24. Find the number.

74. Negative 117 is nine times a number. Find the number.

75. Twelve is one fifth of a number. What is the number?

CHAPTER 2

Mid-Chapter Quiz

Lessons 2-1 through 2-5

Translate each sentence into an equation. (Lesson 2-1)

1. The sum of three times a and four is the same as five times a.
2. One fourth of m minus six is equal to two times the sum of m and 9.
3. The product of five and w is the same as w to the third power.

4. **MARBLES** Drew has 50 red, green, and blue marbles. He has six more red marbles than blue marbles and four fewer green marbles than blue marbles. Write and solve an equation to determine how many blue marbles Drew has. (Lesson 2-2)

Solve each equation. Check your solution. (Lesson 2-2)

5. $p + 8 = 13$
6. $-26 = b - 3$
7. $\frac{t}{6} = 3$

8. **MULTIPLE CHOICE** Solve the equation $\frac{3}{5}a = \frac{1}{4}$. (Lesson 2-2)

   A -3

   B $\frac{3}{20}$

   C $\frac{5}{12}$

   D 2

Solve each equation. Check your solution. (Lesson 2-3)

9. $2x + 5 = 13$
10. $-21 = 7 - 4y$
11. $\frac{m}{6} - 3 = 8$
12. $-4 = \frac{d + 3}{5}$

13. **FISH** The average length of a yellow-banded angelfish is 12 inches. This is 4.8 times as long as an average common goldfish. (Lesson 2-3)

    a. Write an equation you could use to find the length of the average common goldfish.

    b. What is the length of an average common goldfish?

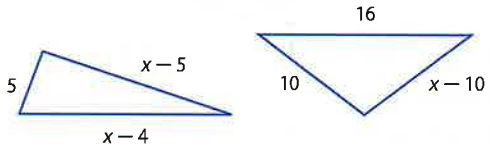
Write an equation and solve each problem. (Lesson 2-3)

14. Three less than three fourths of a number is negative 9. Find the number.
15. Thirty is twelve added to six times a number. What is the number?
16. Find four consecutive integers with a sum of 106.

Solve each equation. Check your solution. (Lesson 2-4)

17. $8p + 3 = 5p + 9$
18. $\frac{3}{4}w + 6 = 9 - \frac{1}{4}w$
19. $\frac{z + 6}{3} = \frac{2z}{4}$

20. **PERIMETER** Find the value of x so that the triangles have the same perimeter. (Lesson 2-4)



21. **PRODUCTION** ABC Sporting Goods Company produces baseball gloves. Their fixed monthly production cost is \$8000 with a per glove cost of \$5. XYZ Sporting Goods Company also produces baseball gloves. Their fixed monthly production cost is \$10,000 with a per glove cost of \$3. Find the value of x, the number of gloves produced monthly, so that the total monthly production cost is the same for both companies. (Lesson 2-4)

Evaluate each expression if $x = -4$, $y = 7$, and $z = -9$. (Lesson 2-5)

22. $|3x - 2| + 2y$
23. $|-4y + 2z| - 7z$

24. **MULTIPLE CHOICE** Solve $|6m - 3| = 9$. (Lesson 2-5)

    F $\{2\}$ **H** $\{-3, 6\}$

    G $\{-1, 2\}$ **J** $\{-3, 3\}$

25. **COFFEE** Some say to brew an excellent cup of coffee, you must have a brewing temperature of 200° F, plus or minus 5 degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

LESSON 2-6

Ratios and Proportions

Then

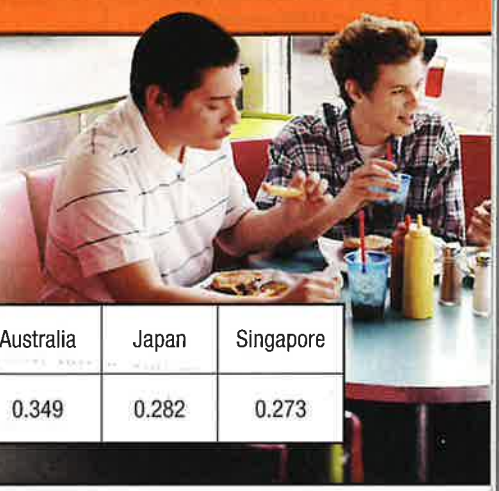
- You evaluated percents by using a proportion.

Now

1. Compare ratios.
2. Solve proportions.

Why?

- Ratios allow us to compare many items by using a common reference. The table below shows the number of restaurants a certain popular fast food chain has per 10,000 people in the United States as well as other countries. This allows us to compare the number of these restaurants using an equal reference.



Countries	United States	New Zealand	Canada	Australia	Japan	Singapore
Number of Restaurants per 10,000 People	0.433	0.369	0.352	0.349	0.282	0.273

NewVocabulary

- ratio
- proportion
- means
- extremes
- rate
- unit rate
- scale
- scale model

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

6 Attend to precision.

1 Ratios and Proportions

The comparison between the number of restaurants and the number of people is a ratio. A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

$$x \text{ to } y \qquad x : y \qquad \frac{x}{y}$$

Suppose you wanted to determine the number of restaurants per 100,000 people in Australia. Notice that this ratio is equal to the original ratio.

$$\frac{0.349 \text{ restaurants}}{10{,}000 \text{ people}} = \frac{3.49 \text{ restaurants}}{100{,}000 \text{ people}}$$

(× 10 above the equation; × 10 below the equation)

An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{0.349}{10{,}000} = \frac{3.49}{100{,}000}$ is a proportion.

Example 1 Determine Whether Ratios Are Equivalent

Determine whether $\frac{2}{3}$ and $\frac{16}{24}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.

$$\frac{2}{3} \overset{\div 1}{=} \frac{2}{3} \qquad \frac{16}{24} \overset{\div 8}{=} \frac{2}{3}$$

When expressed in simplest form, the ratios are equivalent.

GuidedPractice

Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*. Justify your answer.

1A. $\frac{6}{10}, \frac{2}{5}$

1B. $\frac{1}{6}, \frac{5}{30}$

StudyTip

Means and Extremes

To solve a proportion using cross products, write an equation that sets the product of the extremes equal to the product of the means.

There are special names for the terms in a proportion.

1.5 and 1.2 are called the means. They are the middle terms of the proportion.

$0.2 : 1.5 = 1.2 : 9.0$

0.2 and 9.0 are called the extremes. They are the first and last terms of the proportion.

KeyConcept Means-Extremes Property of Proportion

Words	In a proportion, the product of the extremes is equal to the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ and $b, d \neq 0$, then $ad = bc$.
Examples	Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

This is the same as multiplying the means, and multiplying the extremes.

Example 2 Cross Products

Use cross products to determine whether each pair of ratios forms a proportion.

a. $\frac{2}{3.5}, \frac{8}{14}$

$\frac{2}{3.5} \stackrel{?}{=} \frac{8}{14}$ Original proportion

$2(14) \stackrel{?}{=} 3.5(8)$ Cross products

$28 = 28$ ✓ Simplify.

The cross products are equal, so the ratios form a proportion.

b. $\frac{0.3}{1.5}, \frac{0.5}{2.0}$

$\frac{0.3}{1.5} \stackrel{?}{=} \frac{0.5}{2.0}$ Original proportion

$0.3(2.0) \stackrel{?}{=} 1.5(0.5)$ Cross products

$0.6 \neq 0.75$ ✗ Simplify.

The cross products are not equal, so the ratios do not form a proportion.

GuidedPractice

2A. $\frac{0.2}{1.8}, \frac{1}{0.9}$

2B. $\frac{15}{36}, \frac{35}{42}$

StudyTip

Cross Products When you find cross products, you are said to be *cross multiplying*.

2 Solve Proportions

To solve proportions, use cross products.

Example 3 Solve a Proportion

Solve each proportion. If necessary, round to the nearest hundredth.

a. $\frac{x}{10} = \frac{3}{5}$

$\frac{x}{10} = \frac{3}{5}$	Original proportion
$x(5) = 10(3)$	Find the cross products.
$5x = 30$	Simplify.
$\frac{5x}{5} = \frac{30}{5}$	Divide each side by 5.
$x = 6$	Simplify.

b. $\frac{x-2}{14} = \frac{2}{7}$

$\frac{x-2}{14} = \frac{2}{7}$	Original proportion
$(x-2)7 = 14(2)$	Find the cross products.
$7x - 14 = 28$	Simplify.
$7x = 42$	Add 14 to each side.
$x = 6$	Divide each side by 7.

GuidedPractice

3A. $\frac{r}{8} = \frac{25}{40}$ **3B.** $\frac{x+4}{5} = \frac{3}{8}$

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of $9.99 per 10 songs is a rate. A rate that tells how many of one item is being compared to 1 of another item is called a **unit rate**.

Blend Images/SuperStock

Real-WorldCareer

Retail Buyer A retail buyer purchases goods for stores, primarily from wholesalers, for resale to the general public. Buyers use math to determine the amount of each product to order. A bachelor's degree with an emphasis on business studies is usually required.

Real-World Example 4 Rate of Growth

RETAIL In the past two years, a retailer has opened 232 stores. If the rate of growth remains constant, how many stores will the retailer open in the next 3 years?

Understand Let r represent the number of retail stores.

Plan Write a proportion for the problem.

$$\frac{232 \text{ retail stores}}{2 \text{ years}} = \frac{r \text{ retail stores}}{3 \text{ years}}$$

Solve

$\frac{232}{2} = \frac{r}{3}$	Original proportion
$232(3) = 2r$	Find the cross products.
$696 = 2r$	Simplify.
$\frac{696}{2} = \frac{2r}{2}$	Divide each side by 2.
$348 = r$	Simplify.

The retailer will open 348 stores in 3 years.

Check If the clothing retailer continues to open 232 stores every 2 years, then in the next 3 years, it will open 348 stores.

GuidedPractice

4. EXERCISE It takes 7 minutes for Isabella to walk around the gym track twice. At this rate, how many times can she walk around the track in a half hour?

A rate called a **scale** is used to make a **scale model** of something too large or too small to be convenient at actual size.

Real-WorldLink

The Great Smoky Mountains National Park in Tennessee is home to several waterfalls. The Ramsey Cascades is 100 feet tall. It is the tallest in the park.

Source: National Park Service

Real-World Example 5 Scale and Scale Models

MOUNTAIN TRAIL **The Ramsey Cascades Trail is about $1\frac{1}{8}$ inches long on a map with scale 3 inches = 10 miles. What is the actual length of the trail?**

Let ℓ represent the actual length.

scale → actual → $\frac{3}{10} = \frac{1\frac{1}{8}}{\ell}$ ← scale ← actual

$3(\ell) = 1\frac{1}{8}(10)$ Find the cross products.

$3\ell = \frac{45}{4}$ Simplify.

$3\ell \div 3 = \frac{45}{4} \div 3$ Divide each side by 3.

$\ell = \frac{15}{4}$ or $3\frac{3}{4}$ Simplify.

The actual length is about $3\frac{3}{4}$ miles.

GuidedPractice

5. AIRPLANES On a model airplane, the scale is 5 centimeters = 2 meters. If the model's wingspan is 28.5 centimeters, what is the actual wingspan?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 **Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.**

1. $\frac{3}{7}, \frac{9}{14}$
2. $\frac{7}{8}, \frac{42}{48}$
3. $\frac{2.8}{4.4}, \frac{1.4}{2.1}$

Example 3 **Solve each proportion. If necessary, round to the nearest hundredth.**

4. $\frac{n}{9} = \frac{6}{27}$
5. $\frac{4}{u} = \frac{28}{35}$
6. $\frac{3}{8} = \frac{b}{10}$

Example 4

7. **RACE** Jennie ran the first 6 miles of a marathon in 58 minutes. If she is able to maintain the same pace, how long will it take her to finish the 26.2 miles?

Example 5

8. **PRECISION** On a map of North Carolina, Raleigh and Asheville are about 8 inches apart. If the scale is 1 inch = 12 miles, how far apart are the cities?

Practice and Problem Solving

Extra Practice is on page R2.

Examples 1–2 Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.

9. $\frac{9}{11}, \frac{81}{99}$
10. $\frac{3}{7}, \frac{18}{42}$
11. $\frac{8.4}{9.2}, \frac{8.8}{9.6}$
12. $\frac{4}{3}, \frac{6}{8}$
13. $\frac{29.2}{10.4}, \frac{7.3}{2.6}$
14. $\frac{39.68}{60.14}, \frac{6.4}{9.7}$

Example 3 Solve each proportion. If necessary, round to the nearest hundredth.

15. $\frac{3}{8} = \frac{15}{a}$
16. $\frac{t}{2} = \frac{6}{12}$
17. $\frac{4}{9} = \frac{13}{q}$
18. $\frac{15}{35} = \frac{g}{7}$
19. $\frac{7}{10} = \frac{m}{14}$
20. $\frac{8}{13} = \frac{v}{21}$
21. $\frac{w}{2} = \frac{4.5}{6.8}$
22. $\frac{1}{0.19} = \frac{12}{n}$
23. $\frac{2}{0.21} = \frac{8}{n}$
24. $\frac{2.4}{3.6} = \frac{k}{1.8}$
25. $\frac{t}{0.3} = \frac{1.7}{0.9}$
26. $\frac{7}{1.066} = \frac{z}{9.65}$
27. $\frac{x-3}{5} = \frac{6}{10}$
28. $\frac{7}{x+9} = \frac{21}{36}$
29. $\frac{10}{15} = \frac{4}{x-5}$

Example 4

30. **CAR WASH** The B-Clean Car Wash washed 128 cars in 3 hours. At that rate, how many cars can they wash in 8 hours?

Example 5

31. **GEOGRAPHY** On a map of Florida, the distance between Jacksonville and Tallahassee is 2.6 centimeters. If 2 centimeters = 120 miles, what is the distance between the two cities?

32. **CCSS PRECISION** An artist used interlocking building blocks to build a scale model of Kennedy Space Center, Florida. In the model, 1 inch equals 1.67 feet of an actual space shuttle. The model is 110.3 inches tall. How tall is the actual space shuttle? Round to the nearest tenth.

33. **MENU** On Monday, a restaurant made $545 from selling 110 hamburgers. If they sold 53 hamburgers on Tuesday, how much did they make?

Solve each proportion. If necessary, round to the nearest hundredth.

34. $\frac{6}{14} = \frac{7}{x-3}$
35. $\frac{7}{4} = \frac{f-4}{8}$
36. $\frac{3-y}{4} = \frac{1}{9}$
37. $\frac{4v+7}{15} = \frac{6v+2}{10}$
38. $\frac{9b-3}{9} = \frac{5b+5}{3}$
39. $\frac{2n-4}{5} = \frac{3n+3}{10}$

40. **ATHLETES** At Piedmont High School, 3 out of every 8 students are athletes. If there are 1280 students at the school, how many are not athletes?

41. **BRACES** Two out of five students in the ninth grade have braces. If there are 325 students in the ninth grade, how many have braces?

42. **PAINT** Joel used a half gallon of paint to cover 84 square feet of wall. He has 932 square feet of wall to paint. How many gallons of paint should he purchase?

43. **MOVIE THEATERS** Use the table at the right.

Year	Indoor	Drive-In	Total
2003	35,361	634	35,995
2004	36,012	640	36,652
2005	37,092	648	37,740
2006	37,776	649	38,425
2007	38,159	635	38,794
2008	38,201	633	38,834
2009	38,605	628	39,233

Source: North American Theater Owners

a. Write a ratio of the number of indoor theaters to the total number of theaters for each year.

b. Do any two of the ratios you wrote for part **a** form a proportion? If so, explain the real-world meaning of the proportion.

44. **DIARIES** In a survey, 36% of the students said that they kept an electronic diary. There were 900 students who kept an electronic diary. How many students were in the survey?

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how changing the lengths of the sides of a shape by a factor changes the perimeter of that shape.

a. Geometric Draw a square *ABCD*. Draw a square *MNPQ* with sides twice as long as *ABCD*. Draw a square *FGHJ* with sides half as long as *ABCD*.

b. Tabular Complete the table below using the appropriate measures.

ABCD		*MNPQ*		*FGHJ*	
Side length		Side length		Side length	
Perimeter		Perimeter		Perimeter	

c. Verbal Make a conjecture about the change in the perimeter of a square if the side length is increased or decreased by a factor.

H.O.T. Problems Use Higher-Order Thinking Skills

46. **STRUCTURE** In 2007, organic farms occupied 2.6 million acres in the United States and produced goods worth about $1.7 billion. Divide one of these numbers by the other and explain the meaning of the result.

47. **REASONING** Compare and contrast ratios and rates.

48. **CHALLENGE** If $\frac{a+1}{b-1} = \frac{5}{1}$ and $\frac{a-1}{b+1} = \frac{1}{1}$, find the value of $\frac{b}{a}$. (*Hint:* Choose values of a and b for which the proportions are true and evaluate $\frac{b}{a}$.)

49. **WRITING IN MATH** On a road trip, Marcus reads a highway sign and then looks at his gas gauge.

Marcus's gas tank holds 10 gallons and his car gets 32 miles per gallon at his current speed of 65 miles per hour. If he maintains this speed, will he make it to Atlanta without having to stop and get gas? Explain your reasoning.

50. **WRITING IN MATH** Describe how businesses can use ratios. Write about a real-world situation in which a business would use a ratio.

Standardized Test Practice

51. In the figure, $x : y = 2 : 3$ and $y : z = 3 : 5$. If $x = 10$, find the value of z.

A 15
B 20
C 25
D 30

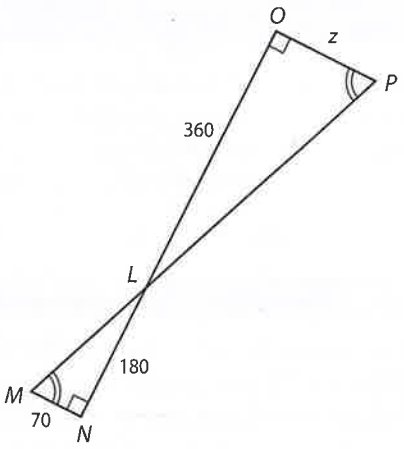
52. GRIDDED RESPONSE A race car driver records the finishing times for recent practice trials.

Trial	Time (seconds)
1	5.09
2	5.10
3	4.95
4	4.91
5	5.05

What is the mean time, in seconds, for the trials?

53. GEOMETRY If $\triangle LMN$ is similar to $\triangle LPO$, what is z?

F 240
G 140
H 120
J 70



54. Which equation below illustrates the Commutative Property?

A $(3x + 4y) + 2z = 3x + (4y + 2z)$
B $7(x + y) = 7x + 7y$
C $xyz = yxz$
D $x + 0 = x$

Spiral Review

Solve each equation. (Lesson 2-5)

55. $|x + 5| = -8$

56. $|b + 9| = 2$

57. $|2p - 3| = 17$

58. $|5c - 8| = 12$

59. HEALTH When exercising, a person's pulse rate should not exceed a certain limit. This maximum rate is represented by the expression $0.8(220 - a)$, where a is age in years. Find the age of a person whose maximum pulse rate is 122 more than their age. (Lesson 2-4)

Solve each equation. Check your solution. (Lesson 2-3)

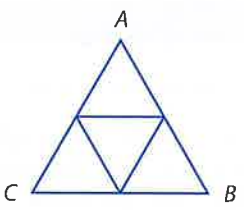
60. $15 = 4a - 5$

61. $7g - 14 = -63$

62. $9 + \frac{y}{5} = 6$

63. $\frac{t}{8} - 6 = -12$

64. GEOMETRY Find the area of $\triangle ABC$ if each small triangle has a base of 5.2 inches and a height of 4.5 inches. (Lesson 1-4)



Evaluate each expression. (Lesson 1-2)

65. $3 + 16 \div 8 \cdot 5$

66. $4^2 \cdot 3 - 5(6 + 3)$

Skills Review

Solve each equation.

67. $4p = 22$

68. $5h = 33$

69. $1.25y = 4.375$

70. $9.8m = 30.87$

EXTEND

2-6 Spreadsheet Lab
Descriptive Modeling

When using numbers to model a real-world situation, it is often helpful to have a metric. A **metric** is a rule for assigning a number to some characteristic or attribute. For example, teachers use metrics to determine grades. Each teacher determines an appropriate metric for assessing a student's performance and assigning a grade.

Common Core State Standards
Content Standards
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

You can use a spreadsheet to calculate different metrics.

Activity

Dorrie wants to buy a house. She has the following expenses: rent of $650, credit card monthly bills of $320, a car payment of $410, and a student loan payment of $115. Dorrie has a yearly salary of $46,500. Use a spreadsheet to find Dorrie's debt-to-income ratio.

Step 1 Enter Dorrie's debts in column B.

Step 2 Add her debts using a function in cell B6. Go to Insert and then Function. Then choose Sum. The sum of 1495 appears in B6.

Step 3 Now insert Dorrie's salary in column C. Remember to find her monthly salary by dividing the yearly salary by 12.

A mortgage company will use the debt-to-income ratio as a metric to determine if Dorrie qualifies for a loan. The **debt-to-income ratio** is calculated as *how much she owes per month* divided by *how much she earns each month.*

Step 4 Enter a formula to find the debt-to-income ratio in cell C6. In the formula bar, enter =B6/C2.

Lab 2-6 B Spreadsheet.xls

	A	B	C
1	Type of Debt	Expenses	Salary
2	Rent	650	3875
3	Credit Cards	320	
4	Car Payment	410	
5	Student Loan	115	
6		1495	0.385806
7			

Sheet 1 | Sheet 2 | Sheet 3

The ratio of about 0.39 appears. An ideal ratio would be 0.36 or less. A ratio higher than 0.36 would cause an increased interest rate or may require a higher down payment.

The spreadsheet shows a debt-to-income ratio of about 0.39. Dorrie should try to eliminate or reduce some debts or try to earn more money in order to lower her debt-to-income ratio.

Exercises

1. How could Dorrie improve her debt-to-income ratio?
2. Another metric mortgage companies use is the ratio of monthly mortgage to total monthly income. An ideal ratio is 0.28. Using this metric, how much could Dorrie afford to pay for a mortgage each month?
3. How effective are each of these metrics as measures of whether Dorrie can afford to buy a house? Explain your reasoning.
4. **MODELING** Metrics are used to compare athletes. For example, ERAs are used to compare pitchers. Find a metric and evaluate its effectiveness for modeling. Compare it to other metrics, and then define your own metric.

LESSON 2-7

Percent of Change

Then	Now	Why?
• You solved proportions.	• 1 Find the percent of change. • 2 Solve problems involving percent of change.	• Every year, millions of people volunteer their time to improve their community. The difference in the number of volunteers from one year to the next can be used to determine a percent to represent the increase or decrease in volunteers.

Jim West/age fotostock

New Vocabulary
- percent of change
- percent of increase
- percent of decrease

Common Core State Standards

Content Standards

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Percent of Change

Percent of change is the ratio of the change in an amount to the original amount expressed as a percent. If the new number is greater than the original number, the percent of change is a **percent of increase.** If the new number is less than the original number, the percent of change is a **percent of decrease.**

Example 1 Percent of Change

Determine whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change.

a. original: 20
final: 23

Subtract the original amount from the final amount to find the amount of change: $23 - 20 = 3$.

Since the new amount is greater than the original, this is a percent of increase.

Use the original number, 20, as the base.

change → $\frac{3}{20} = \frac{r}{100}$ ← r = percent
original amount →

$3(100) = r(20)$

$300 = 20r$

$\frac{300}{20} = \frac{20r}{20}$

$15 = r$

The percent of increase is 15%.

b. original: 25
final: 17

Subtract the original amount from the final amount to find the amount of change: $17 - 25 = -8$.

Since the new amount is less than the original, this is a percent of decrease.

Use the original number, 25, as the base.

change → $\frac{-8}{25} = \frac{r}{100}$ ← r = percent
original amount →

$-8(100) = r(25)$

$-800 = 25r$

$\frac{-800}{25} = \frac{25r}{25}$

$-32 = r$

The percent of decrease is 32%.

Guided Practice

1A. original: 66
new: 30

1B. original: 9.8
new: 12.1

1C. original: 24
new: 40

1D. original: 500
new: 131

Real-WorldLink

In 2009, the total revenue earned by the North American cruise industry was more than $15.16 billion.

Source: Cruise Market Watch

Real-World Example 2 Percent of Change

CRUISE **The total number of passengers on cruise ships increased 10% from 2007 to 2009. If there were 17.22 million passengers in 2009, how many were there in 2007?**

Let f = the number of passengers in 2009. Since 10% is a percent of increase, the number of passengers in 2007 is less than the number of passengers in 2009.

change → $\frac{17.22 - f}{f} = \frac{10}{100}$ Percent proportion

original amount →

$(1722 - f)100 = 10f$ Find the cross products.

$1722 - 100f = 10f$ Distributive Property

$1722 - 100f + 100f = 10f + 100f$ Add $100f$ to each side.

$1722 = 110f$ Simplify.

$\frac{1722}{110} = \frac{110f}{110}$ Divide each side by 110.

$15.65 \approx f$ Simplify.

There were approximately 15.65 million passengers in 2007.

GuidedPractice

2. **TUITION** A recent percent of increase in tuition at Northwestern University, in Evanston, Illinois, was 5.4%. If the new cost is $33,408 per year, find the original cost per year.

2 Solve Problems

Two applications of percent of change are sales tax and discounts. Sales tax is an example of a percent of increase. Discount is an exam of a percent of decrease.

Example 3 Sales Tax

SHOPPING **Marta is purchasing wire and beads to make jewelry. Her merchandi is $28.62 before tax. If the tax is 7.25% of the total sales, what is the final cost?**

Step 1 Find the tax.

The tax is 7.25% of the price of the merchandise.

7.25% of $28.62 = 0.0725 × 28.62 7.25% = 0.0725

= 2.07495 Use a calculator.

Step 2 Find the cost with tax.

Round $2.07495 to $2.07 since tax is always rounded to the nearest cent. Add this amount to the original price: $28.62 + $2.07 = $30.69.

The total cost of Marta's jewelry supplies is $30.69.

GuidedPractice

3. **SHOPPING** A new DVD costs $24.99. If the sales tax is 6.85%, what is the total cost?

To find a discounted amount, you will follow similar steps to those for sales tax.

Example 4 Discounts

DISCOUNT **Since Tyrell has earned good grades in school, he qualifies for the Good Student Discount on his car insurance. His monthly payment without the discount is $85. If the discount is 20%, what will he pay each month?**

Step 1 Find the discount.

The discount is 20% of the original payment.

$20\% \text{ of } \$85 = 0.20 \times 85$ $20\% = 0.20$

$= 17$ Use a calculator.

Step 2 Find the cost after discount.

Subtract $17 from the original payment: $\$85 - \$17 = \$68$.

With the Good Student Discount, Tyrell will pay $68 per month.

StudyTip

CCSS Regularity When translating a problem from word sentences to math sentences, the word "is" translates to =, and the word "of" translates to ×.

Guided Practice

4. **SALES** A picture frame originally priced at $14.89 is on sale for 40% off. What is the discounted price?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.**

1. original: 78
   new: 125
2. original: 41
   new: 24
3. original: 6 candles
   new: 8 candles
4. original: 35 computers
   new: 32 computers

Example 2

5. **GEOGRAPHY** The distance from Phoenix to Tucson is 120 miles. The distance from Phoenix to Flagstaff is about 21.7% longer. To the nearest mile, what is the distance from Phoenix to Flagstaff?

Example 3 **Find the total price of each item.**

6. dress: $22.50
   sales tax: 7.5%
7. video game: $35.99
   sales tax: 6.75%
8. **PROM** A limo costs $85 to rent for 3 hours plus a 7% sales tax. What is the total cost to rent a limo for 6 hours?
9. **GAMES** A computer game costs $49.95 plus a 6.25% sales tax. What is the total cost of the game?

Example 4 **Find the discounted price of each item.**

10. guitar: $95.00
    discount: 15%
11. DVD: $22.95
    discount: 25%
12. **SKATEBOARD** A skateboard costs $99.99. If you have a coupon for 20% off, how much will you save?
13. **CCSS MODELING** Tickets to the county fair are $8 for an adult and $5 for a child. If you have a 15% discount card, how much will 2 adult tickets and 2 child tickets cost?

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.**

14. original: 35
new: 40

15. original: 16
new: 10

16. original: 27
new: 73

17. original: 92
new: 21

18. original: 21.2 grams
new: 10.8 grams

19. original: 11 feet
new: 25 feet

20. original: $68
new: $76

21. original: 21 hours
new: 40 hours

Example 2

22. GASOLINE The average cost of regular gasoline in North Carolina increased by 73% from 2006 to 2007. If the average cost of a gallon of gas in 2006 was $2.069, what was the average cost in 2007? Round to the nearest cent.

23. CARS Beng is shopping for a car. The cost of a new car is $15,500. This is 25% greater than the cost of a used car. What is the cost of the used car?

Example 3 **Find the total price of each item.**

24. messenger bag: $28.00
tax: 7.25%

25. software: $45.00
tax: 5.5%

26. vase: $5.50
tax: 6.25%

27. book: $25.95
tax: 5.25%

28. magazine: $3.50
tax: 5.75%

29. pillow: $9.99
tax: 6.75%

Example 4 **Find the discounted price of each item.**

30. computer: $1099.00
discount: 25%

31. CD player: $89.99
discount: 15%

32. athletic shoes: $59.99
discount: 40%

33. jeans: $24.50
discount: 33%

34. jacket: $125.00
discount: 25%

35. belt: $14.99
discount: 20%

Find the final price of each item.

36. sweater: $14.99
discount: 12%
tax: 6.25%

37. printer: $60.00
discount: 25%
tax: 6.75%

38. board game: $25.00
discount: 15%
tax: 7.5%

39. CONSUMER PRICE INDEX An *index* measures the percent change of a value from a base year. An index of 115 means that there was a 15% increase from the base year. In 2000, the consumer price index of dairy products was 160.7. In 2007, it was 194.0. Determine the percent of change.

40. FINANCIAL LITERACY The current price of each share of a technology company is $135. If this represents a 16.2% increase over the past year, what was the price per share a year ago?

41. CCSS MODELING A group of girls are shopping for dresses to wear to the spring dance. One finds a dress priced $75 with a 20% discount. A second girl finds a dress priced $85 with a 30% discount.

a. Find the amount of discount for each dress.

b. Which girl is getting the better price for the dress?

42. RECREATIONAL SPORTS In 1995, there were 73,567 youth softball teams. By 2007, there were 86,049. Determine the percent of increase.

43. **TOOLS** Which grocery item had the greatest percent increase in cost from 2000 to 2007?

Average Retail Prices of Selected Grocery Items		
Grocery Item	Cost in 2000 ($ per pound)	Cost in 2007 ($ per pound)
milk (gallon)	2.79	3.87
turkey (whole)	0.99	1.01
chicken (whole)	1.08	1.17
ground beef	1.63	2.23
apples	0.82	1.12
iceberg lettuce	0.85	0.95
peanut butter	1.96	1.88

Source: Statistical Abstract of the United States

44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore patterns in percentages.

a. **Tabular** Copy and complete the following table.

1% of	500	is 5.	100% of		is 20.		% of 80 is 20.
2% of		is 5.	50% of		is 20.		% of 40 is 20.
4% of		is 5.	25% of		is 20.		% of 20 is 20.
8% of		is 5.	12.5% of		is 20.		% of 10 is 20.

b. **Verbal** Describe the patterns in the second and fifth columns.

c. **Analytical** Use the patterns to write the fifth row of the table.

H.O.T. Problems Use Higher-Order Thinking Skills

45. **OPEN ENDED** Write a real-world problem to find the total price of an item including sales tax.

46. **REASONING** If you have 75% of a number n, what percent of decrease is it from the number n? If you have 40% of a number a, what percent of decrease do you have from the number a? What pattern do you notice? Is this always true?

47. **ERROR ANALYSIS** Maddie and Xavier are solving for the percent change if the original amount was $25 and the new amount is $28. Is either of them correct? Explain your reasoning.

Maddie

$\frac{3}{28} = \frac{r}{100}$

$3(100) = 28r$

$300 = 28r$

$10.7 = r$

Xavier

$\frac{3}{25} = \frac{r}{100}$

$3(100) = 25r$

$300 = 25r$

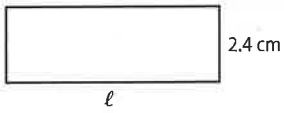
$12 = r$

48. **CHALLENGE** Determine whether the following statement is *sometimes*, *always*, or *never* true. *The percent of change is less than 100%.*

49. **WRITING IN MATH** When is percent of change used in the real world? Explain how to find a percent of change between two values.

Standardized Test Practice

50. GEOMETRY The rectangle has a perimeter of P centimeters. Which equation could be used to find the length ℓ of the rectangle?



A $P = 2.4\ell$
B $P = 4.8 + \ell$
C $P = 2.4 + 2\ell$
D $P = 4.8 + 2\ell$

51. SHORT RESPONSE Henry is painting a room with four walls that are 12 feet by 14 feet. A gallon of paint costs \$18 and covers 350 square feet. If he uses two coats of paint, how much will it cost him to paint the room?

52. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. What was the percent of increase?

F 8.3%
G 14.0%
H 18.5%
J 92.3%

53. PROBABILITY Two dice are rolled. What is the probability that the sum is 10?

A $\frac{1}{3}$ B $\frac{1}{6}$ C $\frac{1}{12}$ D $\frac{1}{36}$

Spiral Review

54. TRAVEL The Chan's minivan requires 5 gallons of gasoline to travel 120 miles. How many gallons of gasoline will they need to travel 360 miles? (Lesson 2-6)

Evaluate each expression if $x = -2$, $y = 6$, and $z = 4$. (Lesson 2-5)

55. $|3 - x| + 7$ **56.** $12 - |z + 9|$ **57.** $|y + x| - z + 4$

Solve each equation. Round to the nearest hundredth. Check your solution. (Lesson 2-4)

58. $1.03p - 4 = -2.15p + 8.72$ **59.** $18 - 3.8t = 7.36 - 1.9t$

60. $5.4w + 8.2 = 9.8w - 2.8$ **61.** $2[d + 3(d - 1)] = 18$

Solve each equation. Check your solution. (Lesson 2-3)

62. $5n + 6 = -4$ **63.** $-11 = 7 + 3c$

64. $15 = 4a - 5$ **65.** $-14 + 7g = -63$

66. RIVERS The Congo River in Africa is 2900 miles long. That is 310 miles longer than the Niger River, which is also in Africa. (Lesson 2-2)

a. Write an equation you could use to find the length of the Niger River.

b. What is the length of the Niger River?

67. FOOD Cameron purchased x pounds of apples for \$0.99 per pound and y pounds of oranges for \$1.29 per pound. Write an algebraic expression that represents the cost of the purchase. (Lesson 1-1)

Skills Review

Translate each equation into a sentence.

68. $d - 14 = 5$ **69.** $2f + 6 = 19$ **70.** $y - 12 = y + 8$

71. $3a + 5 = 27 - 2a$ **72.** $-6c^2 - 4c = 25$ **73.** $d^4 + 64 = 3d^3 + 77$

UNLEASH

2-7 Algebra Lab Percentiles

A **percentile** is a measure that is often used to report test data, such as standardized test scores. It tells us what percent of the total scores were below a given score.

- Percentiles measure rank from the bottom.
- There is no 0 percentile rank. The lowest score is at the 1st percentile.
- There is no 100th percentile rank. The highest score is at the 99th percentile.

Activity

A talent show was held for the twenty finalists in the Teen Idol contest. Each performer received a score from 0 through 30 with 30 being the highest. What is Victor's percentile rank?

Name	Score	Name	Score
Arnold	17	Ishi	27
Benito	9	James	20
Brooke	25	Kat	16
Carmen	21	Malik	10
Daniel	14	Natalie	26
Delia	29	Pearl	4
Fernando	15	Twyla	6
Heather	12	Victor	28
Horatio	5	Warren	22
Ingrid	11	Yolanda	18

Step 1 Write one score on each of 20 slips of paper.

Step 2 Arrange the slips vertically from greatest to least score.

Step 3 Find Victor's percentile rank.

Victor had a score of 28. There are 18 scores below his score. To find his percentile rank, use the following formula.

$$\frac{\text{number of scores below 28}}{\text{total number of scores}} \cdot 100 = \frac{18}{20} \cdot 100 \text{ or } 90$$

Victor scored at the 90th percentile in the contest.

Analyze the Results

1. Find the median, lower quartile, and upper quartile of the scores.
2. Which performer was at the 50th percentile? the 25th percentile? the 75th percentile?
3. Compare and contrast the values for the median, lower quartile, and upper quartile and the scores for the 25th, 50th, and 75th percentiles.
4. While Victor scored at the 90th percentile, what percent of the 30 possible points did he score?
5. CCSS **ARGUMENTS** Compare and contrast the percentile rank and the percent score.
6. Are there any outliers in the data that could alter the results of our computations?
7. **Deciles** are values that divide a set of data into ten equal-sized parts. The 1st decile contains data up to but not including the 10th percentile; the 2nd decile contains data from the 10th percentile up to but not including the 20th percentile, and so on.
   a. Which contestants' scores fall in the 6th decile?
   b. In which decile are Heather and Daniel?

LESSON 2-8

Literal Equations and Dimensional Analysis

Then	Now	Why?
You solved equations with variables on each side.	1 Solve equations for given variables. 2 Use formulas to solve real-world problems.	Each year, more people use credit cards to make everyday purchases. If the entire balance is not paid by the due date, compound interest is applied. The formula for computing the balance of an account with compound interest added annually is $A = P(1 + r)^t$. • A represents the amount of money in the account including the interest, • P is the amount in the account before interest is added, • r is the interest rate written as a decimal, • t is the time in years.

New Vocabulary
literal equation
dimensional analysis
unit analysis

Common Core State Standards

Content Standards
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices
6 Attend to precision.

1 Solve for a Specific Variable

Some equations such as the one above contain more than one variable. At times, you will need to solve these equations for one of the variables.

Example 1 Solve for a Specific Variable

Solve $4m - 3n = 8$ for m.

$4m - 3n = 8$	Original equation
$4m - 3n + 3n = 8 + 3n$	Add $3n$ to each side.
$4m = 8 + 3n$	Simplify.
$\frac{4m}{4} = \frac{8 + 3n}{4}$	Divide each side by 4.
$m = \frac{8}{4} + \frac{3}{4}n$	Simplify.
$m = 2 + \frac{3}{4}n$	Simplify.

Guided Practice

Solve each equation for the variable indicated.

1A. $15 = 3n + 6p$, for n

1B. $\frac{k - 2}{5} = 11j$, for k

1C. $28 = t(r + 4)$, for t

1D. $a(q - 8) = 23$, for q

Sometimes we need to solve equations for a variable that is on both sides of the equation. When this happens, you must get all terms with that variable onto one side of the equation. It is then helpful to use the Distributive Property to isolate the variable for which you are solving.

StudyTip

Solving for a Specific Variable When an equation has more than one variable, it can be helpful to highlight the variable for which you are solving on your paper.

Example 2 Solve for a Specific Variable

Solve $3x - 2y = xz + 5$ for x.

$3x - 2y = xz + 5$	Original equation
$3x - 2y + 2y = xz + 5 + 2y$	Add $2y$ to each side.
$3x - xz = xz - xz + 5 + 2y$	Subtract xz from each side.
$3x - xz = 5 + 2y$	Simplify.
$x(3 - z) = 5 + 2y$	Distributive Property
$\frac{x(3 - z)}{3 - z} = \frac{5 + 2y}{3 - z}$	Divide each side by $3 - z$.
$x = \frac{5 + 2y}{3 - z}$	Simplify.

Since division by 0 is undefined, $3 - z \neq 0$ so $z \neq 3$.

GuidedPractice

Solve each equation for the variable indicated.

2A. $d + 5c = 3d - 1$, for d

2B. $6q - 18 = qr + t$, for q

2 Use Formulas

literal equation. An equation that involves several variables is called a formula or literal equation. To solve a literal equation, apply the process of solving for a specific variable.

Real-WorldLink

The largest yo-yo in the world is 32.7 feet in circumference. It was launched by crane from a height of 189 feet.

Source: *Guinness Book of World Records*

Real-World Example 3 Use Literal Equations

YO-YOS Use the information about the largest yo-yo at the left. The formula for the circumference of a circle is $C = 2\pi r$, where C represents circumference and r represents radius.

a. Solve the formula for r.

$C = 2\pi r$	Formula for circumference
$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$	Divide each side by 2π.
$\frac{C}{2\pi} = r$	Simplify.

b. Find the radius of the yo-yo.

$\frac{C}{2\pi} = r$	Formula for radius
$\frac{32.7}{2\pi} = r$	$C = 32.7$
$5.2 \approx r$	Use a calculator.

The yo-yo has a radius of about 5.2 feet.

GuidedPractice

3. GEOMETRY The formula for the volume of a rectangular prism is $V = \ell wh$, where ℓ is the length, w is the width, and h is the height.

A. Solve the formula for w.

B. Find the width of a rectangular prism that has a volume of 79.04 cubic centimeters, a length of 5.2 centimeters, and a height of 4 centimeters.

When using formulas, you may want to use dimensional analysis. **Dimensional analysis** or **unit analysis** is the process of carrying units throughout a computation.

Example 4 Use Dimensional Analysis

RUNNING **A 10K run is 10 kilometers long. If 1 meter = 1.094 yards, use dimensional analysis to find the length of the race in miles. (*Hint*: 1 mi = 1760 yd)**

Since the given conversion relates meters to yards, first convert 10 kilometers to meters. Then multiply by the conversion factor such that the unit meters are divided out. To convert from yards to miles, multiply by $\frac{1 \text{ mi}}{1760 \text{ yd}}$.

length of run		kilometers to meters		meters to yards		yards to miles
10 km	×	$\frac{1000 \text{ m}}{1 \text{ km}}$	×	$\frac{1.094 \text{ yd}}{1 \text{ m}}$	×	$\frac{1 \text{ mi}}{1760 \text{ yd}}$

Notice how the units cancel, leaving the unit to which you are converting.

$$10 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1.094 \text{ yd}}{1 \text{ m}} \times \frac{1 \text{ mi}}{1760 \text{ yd}} = \frac{10{,}940 \text{ mi}}{1760} \approx 6.2 \text{ mi}$$

A 10K race is approximately 6.2 miles.

StudyTip

CCSS **Precision** As you plan your method of solution, think about what the question is asking and what units of measure will apply to the solution.

GuidedPractice

4. A car travels a distance of 100 feet in about 2.8 seconds. What is the velocity of the car in miles per hour? Round to the nearest whole number.

Check Your Understanding

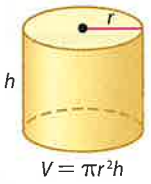
= Step-by-Step Solutions begin on page R13.

Examples 1–2 **Solve each equation or formula for the variable indicated.**

1. $5a + c = -8a$, for a
2. $7h + f = 2h + g$, for g
3. $\frac{k + m}{-7} = n$, for k
4. $q = p(r + s)$, for p

Example 3

5. **PACKAGING** A soap company wants to use a cylindrical container to hold their new liquid soap.



a. Solve the formula for h.

b. What is the height of a container if the volume is 56.52 cubic inches and the radius is 1.5 inches? Round to the nearest tenth.

Example 4

6. **SHOPPING** Scott found a rare video game on an online auction site priced at 35 Australian dollars. If the exchange rate is \$1 U.S. = \$1.24 Australian, find the cost of the game in United States dollars. Round to the nearest cent.

7. CCSS **PRECISION** A fisheye lens has a minimum focus range of 13.5 centimeters. If 1 centimeter is equal in length to about 0.39 inch, what is the minimum focus range of the lens in feet?

Practice and Problem Solving

Extra Practice is on page R2.

Examples 1–2 **Solve each equation or formula for the variable indicated.**

8. $u = vw + z$, for v

9. $x = b - cd$, for c

10. $fg - 9h = 10j$, for g

11. $10m - p = -n$, for m

12. $r = \frac{2}{3}t + v$, for t

13. $\frac{5}{9}v + w = z$, for v

14. $\frac{10ac - x}{11} = -3$, for a

15. $\frac{df + 10}{6} = g$, for f

Example 3

16. FITNESS The formula to compute a person's body mass index is $B = 703 \cdot \frac{w}{h^2}$. B represents the body mass index, w is the person's weight in pounds, and h represents the person's height in inches.

a. Solve the formula for w.

b. What is the weight to the nearest pound of a person who is 64 inches tall and has a body mass index of 21.45?

17. PHYSICS Acceleration is the measure of how fast a velocity is changing. The formula for acceleration is $a = \frac{v_f - v_i}{t}$. a represents the acceleration rate, v_f is the final velocity, v_i is the initial velocity, and t represents the time in seconds.

a. Solve the formula for v_f.

b. What is the final velocity of a runner who is accelerating at 2 feet per second squared for 3 seconds with an initial velocity of 4 feet per second?

Example 4

18. SWIMMING If each lap in a pool is 100 meters long, how many laps equal one mile? Round to the nearest tenth. (*Hint*: 1 foot ≈ 0.3048 meter)

19. CCSS **PRECISION** How many liters of gasoline are needed to fill a 13.2-gallon tank? There are about 1.06 quarts per 1 liter. Round to the nearest tenth.

Solve each equation or formula for the variable indicated.

20. $-14n + q = rt - 4n$, for n

21. $18t + 11v = w - 13t$, for t

22. $ax + z = aw - y$, for a

23. $10c - f = -13 + cd$, for c

Select an appropriate unit from the choices below and convert the rate to that unit.

ft/s | mph | mm/s | km/s

24. a car traveling at 36 ft/s

25. a snail moving at 3.6 m/h

26. a person walking at 3.4 mph

27. a satellite moving at 234,000 m/min

28. DANCING The formula $P = \frac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by a ballroom dancer's heel. In this formula, P is the pressure in pounds per square inch, W is the weight of a person wearing the shoe in pounds, and H is the width of the heel of the shoe in inches.

a. Solve the formula for W.

b. Find the weight of the dancer if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

Write an equation and solve for the variable indicated.

29. Seven less than a number t equals another number r plus 6. Solve for t.

30. Ten plus eight times a number a equals eleven times number d minus six. Solve for a.

31. Nine tenths of a number g is the same as seven plus two thirds of another number k. Solve for k.

32. Three fourths of a number p less two is five sixths of another number r plus five. Solve for r.

33 **GIFTS** Ashley has 214 square inches of paper to wrap a gift box. The surface area S of the box can be found by using the formula $S = 2w(\ell + h) + 2\ell h$, where w is the width of the box, ℓ is the length of the box, and h is the height. If the length of the box is 7 inches and the width is 6 inches, how tall can Ashley's box be?

34. DRIVING A car is driven x miles a year and averages m miles per gallon.

a. Write a formula for g, the number of gallons used in a year.

b. If the average price of gas is p dollars per gallon, write a formula for the total gas cost c in dollars for driving this car each year.

c. Car A averages 15 miles per gallon on the highway, while Car B averages 35 miles per gallon on the highway. If you average 15,000 miles each year, how much money would you save on gas per week by using Car B instead of Car A if the cost of gas averages \$3 per gallon? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

35. CHALLENGE The circumference of an NCAA women's basketball is 29 inches, and the rubber coating is $\frac{3}{16}$ inch thick. Use the formula $v = \frac{4}{3}\pi r^3$, where v represents the volume and r is the radius of the inside of the ball, to determine the volume of the air inside the ball. Round to the nearest whole number.

36. REASONING Select an appropriate unit to describe the highway speed of a car and the speed of a crawling caterpillar. Can the same unit be used for both? Explain.

37. ERROR ANALYSIS Sandrea and Fernando are solving $4a - 5b = 7$ for b. Is either of them correct? Explain.

Sandrea

$4a - 5b = 7$

$-5b = 7 - 4a$

$\frac{-5b}{-5} = \frac{7 - 4a}{-5}$

$b = \frac{7 - 4a}{-5}$

Fernando

$4a - 5b = 7$

$5b = 7 - 4a$

$\frac{5b}{5} = \frac{7 - 4a}{5}$

$b = \frac{7 - 4a}{5}$

38. OPEN ENDED Write a formula for A, the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than A.

39. CCSS **PERSEVERANCE** Solve each equation or formula for the variable indicated.

a. $n = \frac{x + y - 1}{xy}$ for x

b. $\frac{x + y}{x - y} = \frac{1}{2}$ for y

40. **WRITING IN MATH** Why is it helpful to be able to represent a literal equation in different ways?

Standardized Test Practice

41. Eula is investing $6000, part at 4.5% interest and the rest at 6% interest. If d represents the amount invested at 4.5%, which expression represents the amount of interest earned in one year by the amount paying 6%?

A $0.06d$
B $0.06(d - 6000)$
C $0.06(d + 6000)$
D $0.06(6000 - d)$

42. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was Todd's average speed?

F 38.5 mph
G 40 mph
H 44 mph
J 47.5 mph

43. SHORT RESPONSE Brian has 3 more books than Erika. Jasmine has triple the number of books that Brian has. Altogether Brian, Erika, and Jasmine have 22 books. How many books does Jasmine have?

44. GEOMETRY Which of the following best describes a plane?

A a location having neither size nor shape
B a flat surface made up of points having no depth
C made up of points and has no thickness or width
D a boundless, three-dimensional set of all points

Spiral Review

Find the final price of each item. (Lesson 2-7)

45. lamp: $120.00
discount: 20%
tax: 6%

46. dress: $70.00
discount: 30%
tax: 7%

47. camera: $58.00
discount: 25%
tax: 6.5%

48. jacket: $82.00
discount: 15%
tax: 6%

49. comforter: $67.00
discount: 20%
tax: 6.25%

50. lawnmower: $720.00
discount: 35%
tax: 7%

Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

51. $\frac{3}{4.5} = \frac{x}{2.5}$

52. $\frac{2}{0.36} = \frac{7}{p}$

53. $\frac{m}{9} = \frac{2.8}{4.9}$

54. JOBS Laurie mows lawns to earn extra money. She can mow at most 30 lawns in one week. She profits $15 on each lawn she mows. Identify a reasonable domain and range for this situation and draw a graph. (Lesson 1-6)

55. ENTERTAINMENT Each member of the pit orchestra is selling tickets for the school musical. The trombone section sold 50 floor tickets and 90 balcony tickets. Write and evaluate an expression to find how much money the trombone section collected. (Lesson 1-4)

Skills Review

Solve each equation.

56. $8k + 9 = 7k + 6$

57. $3 - 4q = 10q + 10$

58. $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$

59. $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$

60. $4(2a - 1) = -10(a - 5)$

61. $2(w - 3) + 5 = 3(w - 1)$

LESSON 2-9

Weighted Averages

Then

- You translated sentences into equations.

Now

1. Solve mixture problems.
2. Solve uniform motion problems.

Why?

- Baseball players' performance is measured in large part by statistics. Slugging average (SLG) is a weighted average that measures the power of a hitter. The slugging average is calculated by using the following formula.

$$SLG = \frac{1B + (2 \times 2B) + (3 \times 3B) + (4 \times HR)}{\text{at bats}}$$

New Vocabulary

- weighted average
- mixture problem
- uniform motion problem
- rate problem

Common Core State Standards

Content Standards

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

1 Weighted Averages

The batter's slugging percentage is an example of a weighted average. The **weighted average** M of a set of data is found by multiplying each data value by its weight and then finding the mean of the new data set.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages. In a mixture problem, the units are usually the number of gallons or pounds and the value is the cost, value, or concentration per unit.

Real-World Example 1 Mixture Problem

RETAIL **A tea company sells blended tea for $25 per pound. To make blackberry tea, dried blackberries that cost $10.50 per pound are blended with black tea that costs $35 per pound. How many pounds of black tea should be added to 5 pounds of dried blackberries to make blackberry tea?**

Step 1 Let w be the weight of the black tea. Make a table to organize the information.

	Number of Units (lb)	Price per Unit ($)	Total Price (price)(units)
Dried Blackberries	5	10.50	10.50(5)
Black Tea	w	35	$35w$
Blackberry Tea	$5 + w$	25	$25(5 + w)$

Write an equation using the information in the table.

Price of blackberries	plus	price of tea	equals	price of blackberry tea.
$10.50(5)$	$+$	$35w$	$=$	$25(5 + w)$

Step 2 Solve the equation.

$$10.50(5) + 35w = 25(5 + w) \quad \text{Original equation}$$
$$52.5 + 35w = 125 + 25w \quad \text{Distributive Property}$$
$$52.5 + 35w - 25w = 125 + 25w - 25w \quad \text{Subtract } 25w \text{ from each side.}$$
$$52.5 + 10w = 125 \quad \text{Simplify.}$$
$$52.5 - 52.5 + 10w = 125 - 52.5 \quad \text{Subtract 52.5 from each side.}$$
$$10w = 72.5 \quad \text{Simplify.}$$
$$w = 7.25 \quad \text{Divide each side by 10.}$$

To make the blackberry tea, 7.25 pounds of black tea will need to be added to the dried blackberries.

StudyTip

Mixture Problems When you organize the information in mixture problems, remember that the final mixture must contain the sum of the parts in the correct quantities and at the correct percents.

GuidedPractice

1. **COFFEE** How many pounds of Premium coffee beans should be mixed with 2 pounds of Supreme coffee to make the Blend coffee?

Sometimes mixture problems are expressed in terms of percents.

Real-World Example 2 Percent Mixture Problem

FRUIT PUNCH **Mrs. Matthews has 16 cups of punch that is 3% pineapple juice. She also has a punch that is 33% pineapple juice. How many cups of the 33% punch will she need to add to the 3% punch to obtain a punch that is 20% pineapple juice?**

Problem-SolvingTip

Make a Table Using a table is a great way to organize the given information. It also helps you understand how to write an equation to solve for the missing value.

Step 1 Let x = the amount of 33% solution to be added. Make a table.

	Amount of Punch (cups)	Amount of Pineapple Juice
3% Punch	16	0.03(16)
33% Punch	x	$0.33x$
20% Punch	$16 + x$	$0.20(16 + x)$

Write an equation using the information in the table.

Amount of pineapple juice in 3% punch	plus	amount of pineapple juice in 33% punch	equals	amount of pineapple juice in 20% punch.
$0.03(16)$	$+$	$0.33x$	$=$	$0.20(16 + x)$

Step 2 Solve the equation.

$$0.03(16) + 0.33x = 0.20(16 + x) \quad \text{Original equation}$$
$$0.48 + 0.33x = 3.2 + 0.20x \quad \text{Simplify.}$$
$$0.48 + 0.33x - 0.20x = 3.2 + 0.20x - 0.20x \quad \text{Subtract } 0.20x \text{ from each side.}$$
$$0.48 + 0.13x = 3.2 \quad \text{Simplify.}$$
$$0.48 - 0.48 + 0.13x = 3.2 - 0.48 \quad \text{Subtract } 0.48 \text{ from each side.}$$
$$0.13x = 2.72 \quad \text{Simplify.}$$
$$\frac{0.13x}{0.13} = \frac{2.72}{0.13} \quad \text{Divide each side by } 0.13.$$
$$x \approx 20.9 \quad \text{Round to the nearest tenth.}$$

Mrs. Matthews should add about 20.9 cups of the 33% punch to the 16 cups of the 3% punch.

GuidedPractice

2. **ANTIFREEZE** One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

2 Uniform Motion Problems

Uniform motion problems or **rate problems** are problems in which an object moves at a certain speed or rate. The formula $d = rt$ is used to solve these problems. In the formula, d represents distance, r represents rate, and t represents time.

Real-WorldLink

Inline skating is the fourth most popular recreational activity in the U.S.

Source: Statistical Abstract of the United States

Real-World Example 3 Speed of One Vehicle

INLINE SKATING **It took Travis and Tony 40 minutes to skate 5 miles. The return trip took them 30 minutes. What was their average speed for the trip?**

Understand We know that the boys did not travel the same amount of time on each portion of their trip. So, we will need to find the weighted average of their speeds. We are asked to find their average speed for both portions of the trip.

Plan First find the rate of the going portion, and then the return portion of the trip. Because the rate is in miles per hour we convert 40 minutes to about 0.667 hours and 30 minutes to 0.5 hours.

Going

$r = \frac{d}{t}$ Formula for rate

$\approx \frac{5 \text{ miles}}{0.667 \text{ hour}}$ or about 7.5 miles per hour Substitution $d = 5$ mi, $t = 0.667$ h

Return

$r = \frac{d}{t}$ Formula for rate

$= \frac{5 \text{ miles}}{0.5 \text{ hour}}$ or 10 miles per hour Substitution $d = 5$ mi, $t = 0.5$ h

Because we are looking for a weighted average we cannot just average their speeds. We need to find the weighted average for the round trip.

Solve $M = \frac{(\text{rate of going})(\text{time of going}) + (\text{rate of return})(\text{time of return})}{\text{time of going} + \text{time of return}}$

$\approx \frac{(7.5)(0.667) + (10)(0.5)}{0.667 + 0.5}$ Substitution

$\approx \frac{10.0025}{1.167}$ or about 8.6 Simplify.

Their average speed was about 8.6 miles per hour.

Check Our solution of 8.6 miles per hour is between the going portion rate, 7.5 miles per hour, and the return rate, 10 miles per hour. So, we know that our answer is reasonable.

GuidedPractice

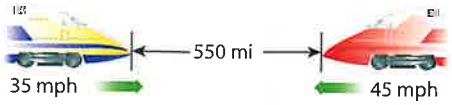
3. EXERCISE Austin jogged 2.5 miles in 16 minutes and then walked 1 mile in 10 minutes. What was his average speed?

The formula $d = rt$ can also be used to solve real-world problems involving two vehicles in motion.

Real-World Example 4 Speeds of Two Vehicles

FREIGHT TRAINS **Two trains are 550 miles apart heading toward each other on parallel tracks. Train A is traveling east at 35 miles per hour, while Train B travels west at 45 miles per hour. When will the trains pass each other?**

Step 1 Draw a diagram.



Step 2 Let t = the number of hours until the trains pass each other. Make a table.

	r	t	$d = rt$
Train A	35	t	$35t$
Train B	45	t	$45t$

Step 3 Write and solve an equation.

Distance traveled by Train A	plus	distance traveled by Train B	equals	550 miles.
$35t$	$+$	$45t$	$=$	550

$35t + 45t = 550$ Original equation

$80t = 550$ Simplify.

$\frac{80t}{80} = \frac{550}{80}$ Divide each side by 80.

$t = 6.875$ Simplify.

The trains will pass each other in about 6.875 hours.

StudyTip

CCSS Sense-Making

Drawing a diagram is not just for geometry problems. You can use diagrams to visualize many problem situations that can be represented by equations.

GuidedPractice

4. **CYCLING** Two cyclists begin traveling in opposite directions on a circular bike trail that is 5 miles long. One cyclist travels 12 miles per hour, and the other travels 18 miles per hour. How long will it be before they meet?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1

1. **FOOD** Tasha ordered soup and salad for lunch. If Tasha ordered 10 ounces of soup for lunch and the total cost was $3.30, how many ounces of salad did Tasha order?

Example 2

2. **CHEMISTRY** Margo has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?

Example 3

3. **TRAVEL** A boat travels 16 miles due north in 2 hours and 24 miles due west in 2 hours. What is the average speed of the boat?

4. **EXERCISE** Felisa jogged 3 miles in 25 minutes and then jogged 3 more miles in 30 minutes. What was her average speed in miles per minute?

Example 4

5. **CYCLING** A cyclist begins traveling 18 miles per hour. At the same time and at the same starting point, an inline skater follows the cyclist's path and begins traveling 6 miles per hour. After how much time will they be 24 miles apart?

Practice and Problem Solving

Extra Practice is on page R2.

Example 1

6. CANDY A candy store wants to create a mix using two hard candies. One is priced at \$5.45 per pound, and the other is priced at \$7.33 per pound. How many pounds of the \$7.33 candy should be mixed with 11 pounds of the \$5.45 candy to sell the mixture for \$6.14 per pound?

7. BUSINESS Party Supplies Inc. sells metallic balloons for \$2 each and helium balloons for \$3.50 per bunch. Yesterday, they sold 36 more metallic balloons than the number of bunches of helium balloons. The total sales for both types of balloons were \$281. Let b represent the number of metallic balloons sold.

a. Copy and complete the table representing the problem.

	Number	Price	Total Price
Metallic Balloons	b		
Bunches of Helium Balloons	$b - 36$		

b. Write an equation to represent the problem.

c. How many metallic balloons were sold?

d. How many bunches of helium balloons were sold?

8. FINANCIAL LITERACY Lakeisha spent \$4.57 on color and black-and-white copies for her project. She made 7 more black-and-white copies than color copies. How many color copies did she make?

Type of Copy	Cost per Page
color	\$0.44
black-and-white	\$0.07

Example 2

9. FISH Rosamaria is setting up a 20-gallon saltwater fish tank that needs to have a salt content of 3.5%. If Rosamaria has water that has 2.5% salt and water that has 3.7% salt, how many gallons of the water with 3.7% salt content should Rosamaria use?

10. CHEMISTRY Hector is performing a chemistry experiment that requires 160 milliliters of 40% sulfuric acid solution. He has a 25% sulfuric acid solution and a 50% sulfuric acid solution. How many milliliters of each solution should he mix to obtain the needed solution?

Example 3

11. TRAVEL A boat travels 36 miles in 1.5 hours and then 14 miles in 0.75 hour. What is the average speed of the boat?

12. CCSS MODELING A person walked 1.5 miles in 28 minutes and then jogged 1.2 more miles in 10 minutes. What was the average speed in miles per minute?

Example 4

13. AIRLINERS Two airliners are 1600 miles apart and heading toward each other at different altitudes. The first plane is traveling north at 620 miles per hour, while the second is traveling south at 780 miles per hour. When will the planes pass each other?

14. SAILING A ship is sailing due east at 20 miles per hour when it passes the lighthouse. At the same time a ship is sailing due west at 15 miles per hour when it passes a point. The point is 175 miles east of the lighthouse. When will these ships pass each other?

15. CHEMISTRY A lab technician has 40 gallons of a 15% iodine solution. How many gallons of a 40% iodine solution must he add to make a 20% iodine solution?

16. **GRADES** At Westbridge High School, a student's grade point average (GPA) is based on the student's grade and the class credit rating. Brittany's grades for this quarter are shown. Find Brittany's GPA if a grade of A equals 4 and a B equals 3.

Class	Credit Rating	Grade
Algebra 1	1	A
Science	1	A
English	1	B
Spanish	1	A
Music	$\frac{1}{2}$	B

17. **SPORTS** In a triathlon, Steve swam 0.5 mile in 15 minutes, biked 20 miles in 90 minutes, and ran 4 miles in 30 minutes. What was Steve's average speed for the triathlon in miles per hour?

18. **MUSIC** Amalia has 10 songs on her digital media player. If 3 songs are 5 minutes long, 3 are 4 minutes long, 2 are 2 minutes long, and 2 are 3.5 minutes long, what is the average length of the songs?

19. **DISTANCE** Garcia is driving to Florida for vacation. The trip is a total of 625 miles.

    a. How far can he drive in 6 hours at 65 miles per hour?

    b. If Garcia maintains a speed of 65 miles per hour, how long will it take him to drive to Florida?

20. **TRAVEL** Two buses leave Smithville at the same time, one traveling north and the other traveling south. The northbound bus travels at 50 miles per hour, and the southbound bus travels at 65 miles per hour. Let t represent the amount of time since their departure.

    a. Copy and complete the table representing the situation.

	r	t	$d = rt$
Northbound bus	?	?	?
Southbound bus	?	?	?

    b. Write an equation to find when the buses will be 345 miles apart.

    c. Solve the equation. Explain how you found your answer.

21. **TRAVEL** A subway travels 60 miles per hour from Glendale to Midtown. Another subway, traveling at 45 miles per hour, takes 11 minutes longer for the same trip. How far apart are Glendale and Midtown?

H.O.T. Problems Use Higher-Order Thinking Skills

22. **OPEN ENDED** Write a problem that depicts motion in opposite directions.

23. **CCSS ARGUMENTS** Describe the conditions so that adding a 50% solution to a 100% solution would produce a 75% solution.

24. **CHALLENGE** Find five consecutive odd integers from least to greatest in which the sum of the first and the fifth is one less than three times the fourth.

25. **CHALLENGE** Describe a situation involving mixtures that could be represented by $1.00x + 0.15(36) = 0.50(x + 36)$.

26. **WRITING IN MATH** Describe how a gallon of 25% solution is added to an unknown amount of 10% solution to get a 15% solution.

Standardized Test Practice

27. If $2x + y = 5$, what is the value of $4x$?

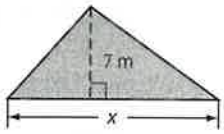
A $10 - y$
B $10 - 2y$
C $\frac{5 - y}{2}$
D $\frac{10 - y}{2}$

28. Which expression is equivalent to $7x^2 3x^{-4}$?

F $21x^{-8}$
G $21x^2$
H $21x^{-6}$
J $21x^{-2}$

29. GEOMETRY What is the base of the triangle if the area is 56 square meters?

A 4 m
B 8 m
C 16 m
D 28 m



30. SHORT RESPONSE Brianne makes blankets for a baby store. She works on the blankets 30 hours per week. The store pays her \$9.50 per hour plus 30% of the profit. If her hourly rate is increased by \$0.75 and her commission is raised to 40%, how much will she earn for a week in which there was a \$300 profit?

Spiral Review

Solve each equation or formula for x. (Lesson 2-8)

31. $2bx - b = -5$

32. $3x - r = r(-3 + x)$

33. $A = 2\pi r^2 + 2\pi rx$

34. SKIING Yuji is registering for ski camp. The cost of the camp is \$1254, but there is a sales tax of 7%. What is the total cost of the camp including tax? (Lesson 2-7)

Translate each equation into a sentence. (Lesson 2-1)

35. $\frac{n}{-6} = 2n + 1$

36. $18 - 5h = 13h$

37. $2x^2 + 3 = 21$

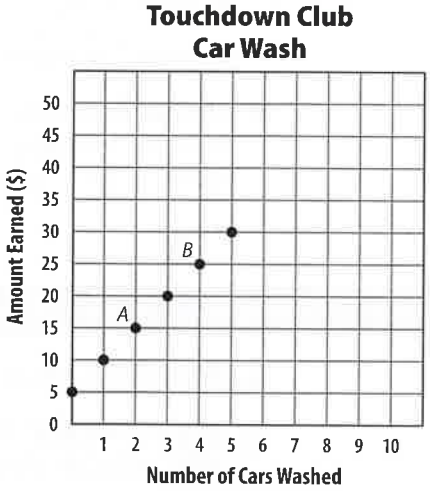
Refer to the graph.

38. Name the ordered pair at point A and explain what it represents. (Lesson 1-6)

39. Name the ordered pair at point B and explain what it represents. (Lesson 1-6)

40. Identify the independent and dependent variables for the function. (Lesson 1-6)

41. BASEBALL Tickets to a baseball game cost \$18.95, \$12.95, or \$9.95. A hot dog and soda combo costs \$5.50. The Madison family is having a reunion. They buy 10 tickets in each price category and plan to buy 30 combos. What is the total cost for the tickets and meals? (Lesson 1-4)



Skills Review

Solve each equation.

42. $a - 8 = 15$

43. $9m - 11 = -29$

44. $18 - 2k = 24$

45. $5 - 8y = 61$

46. $7 = \frac{h}{2} + 3$

47. $\frac{n}{6} + 1 = 5$

CHAPTER 2

Study Guide and Review

Study Guide

KeyConcepts

Writing Equations (Lesson 2-1)

- Identify the unknown you are looking for and assign a variable to it. Then, write the sentence as an equation.

Solving Equations (Lessons 2-2 to 2-4)

- Addition and Subtraction Properties of Equality: If an equation is true and the same number is added to or subtracted from each side, the resulting equation is true.
- Multiplication and Division Properties of Equality: If an equation is true and each side is multiplied or divided by the same nonzero number, the resulting equation is true.
- Steps for Solving Equations:
  - **Step 1** Simplify the expression on each side. Use the Distributive Property as needed.
  - **Step 2** Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side.
  - **Step 3** Use the Multiplication or Division Property of Equality to solve.

Absolute Value Equations (Lesson 2-5)

- For any real numbers a and b, if $|a| = b$ and $b \geq 0$, then $a = b$ or $a = -b$.

Ratios and Proportions (Lesson 2-6)

- The Means-Extremes Property of Proportion states that in a proportion, the product of the extremes is equal to the product of the means.

Percent of Change (Lesson 2-7)

- percent of change $= \frac{\text{the change in an amount}}{\text{the original amount}}$ expressed as a percent

Weighted Averages (Lesson 2-9)

- the weighted average M of a set of data $= \frac{\text{sum of (units} \times \text{the value per unit)}}{\text{the total number of units}}$

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

consecutive integers (p. 92)	percent of decrease (p. 119)
dimensional analysis (p. 128)	percent of increase (p. 119)
equivalent equations (p. 83)	proportion (p. 111)
extremes (p. 112)	rate (p. 113)
formula (p. 76)	ratio (p. 111)
identity (p. 98)	scale (p. 114)
literal equation (p. 127)	scale model (p. 114)
means (p. 112)	solve an equation (p. 83)
multi-step equations (p. 91)	unit analysis (p. 128)
number theory (p. 92)	unit rate (p. 113)
percent of change (p. 119)	weighted average (p. 132)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. In order to write an equation to solve a problem, identify the unknown for which you are looking and assign a(n) number to it.
2. To solve an equation means to find the value of the variable that makes the equation true.
3. The numbers 10, 12, and 14 are an example of consecutive even integers.
4. The absolute value of any number is simply the distance the number is away from zero on a number line.
5. A(n) equation is a comparison of two numbers by division.
6. An equation stating that two ratios are equal is called a(n) proportion.
7. If the new number is less than the original number, the percent of change is a percent of increase.
8. The weighted average of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Lesson-by-Lesson Review

2-1 Writing Equations

Translate each sentence into an equation.

9. The sum of five times a number x and three is the same as fifteen.
10. Four times the difference of b and six is equal to b squared.
11. One half of m cubed is the same as four times m minus nine.

Translate each equation into a sentence.

12. $3p + 8 = 20$
13. $h^2 - 5h + 6 = 0$
14. $\frac{3}{4}w^2 + \frac{2}{3}w - \frac{1}{5} = 2$
15. **FENCING** Adrianne wants to create an outdoor rectangular kennel. The length will be three feet more than twice the width. Write and use an equation to find the length and the width of the kennel if Adrianne has 54 feet of fencing.

Example 1

Translate the following sentence into an equation.

Six times the sum of a number n and four is the same as the difference between two times n to the second power and ten.

$6(n + 4) = 2n^2 - 10$

Example 2

Translate $3d^2 - 9d + 8 = 4(d + 2)$ into a sentence.

Three times a number d squared minus nine times d increased by eight is equal to four times the sum of d and two.

2-2 Solving One-Step Equations

Solve each equation. Check your solution.

16. $x - 9 = 4$
17. $-6 + g = -11$
18. $\frac{5}{9} + w = \frac{7}{9}$
19. $3.8 = m + 1.7$
20. $\frac{a}{12} = 5$
21. $8y = 48$
22. $\frac{2}{5}b = -4$
23. $-\frac{t}{16} = -\frac{7}{8}$
24. **AGE** Max is four years younger than his sister Brenda. Max is 16 years old. Write and solve an equation to find Brenda's age.

Example 3

Solve $x - 13 = 9$. Check your solution.

$x - 13 = 9$	Original equation
$x - 13 + 13 = 9 + 13$	Add 13 to each side.
$x = 22$	$-13 + 13 = 0$ and $9 + 13 = 22$

To check that 22 is the solution, substitute 22 for x in the original equation.

CHECK $x - 13 = 9$	Original equation
$22 - 13 \stackrel{?}{=} 9$	Substitute 22 for x.
$9 = 9$ ✓	Subtract.

2-3 Solving Multi-Step Equations

Solve each equation. Check your solution.

25. $2d - 4 = 8$

26. $-9 = 3t + 6$

27. $14 = -8 - 2k$

28. $\frac{n}{4} - 7 = -2$

29. $\frac{r+4}{3} = 7$

30. $-18 = \frac{9-a}{2}$

31. $6g - 3.5 = 8.5$

32. $0.2c + 4 = 6$

33. $\frac{f}{3} - 9.2 = 3.5$

34. $4 = \frac{-3u-(-7)}{-8}$

35. CONSECUTIVE INTEGERS Find three consecutive odd integers with a sum of 63.

36. CONSECUTIVE INTEGERS Find three consecutive integers with a sum of −39.

Example 4

Solve $7y - 9 = 33$. Check your solution.

$7y - 9 = 33$	Original equation
$7y - 9 + 9 = 33 + 9$	Add 9 to each side.
$7y = 42$	Simplify.
$\frac{7y}{7} = \frac{42}{7}$	Divide each side by 7.
$y = 6$	Simplify.

CHECK

$7y - 9 = 33$	Original equation
$7(6) - 9 \stackrel{?}{=} 33$	Substitute 6 for y.
$42 - 9 \stackrel{?}{=} 33$	Multiply.
$33 = 33$ ✓	Subtract.

2-4 Solving Equations with the Variable on Each Side

Solve each equation. Check your solution.

37. $8m + 7 = 5m + 16$

38. $2h - 14 = -5h$

39. $21 + 3j = 9 - 3j$

40. $\frac{x-3}{4} = \frac{x}{2}$

41. $\frac{6r-7}{10} = \frac{r}{4}$

42. $3(p + 4) = 33$

43. $-2(b - 3) - 4 = 18$

44. $4(3w - 2) = 8(2w + 3)$

Write an equation and solve each problem.

45. Find the sum of three consecutive odd integers if the sum of the first two integers is equal to twenty-four less than four times the third integer.

46. TRAVEL Mr. Jones drove 480 miles to a business meeting. His travel time to the meeting was 8 hours and from the meeting was 7.5 hours. Find his rate of travel for each leg of the trip.

Example 5

Solve $9w - 24 = 6w + 18$.

$9w - 24 = 6w + 18$	Original equation
$9w - 24 - 6w = 6w + 18 - 6w$	Subtract $6w$ from each side.
$3w - 24 = 18$	Simplify.
$3w - 24 + 24 = 18 + 24$	Add 24 to each side.
$3w = 42$	Simplify.
$\frac{3w}{3} = \frac{42}{3}$	Divide each side by 3.
$w = 14$	Simplify.

Example 6

Write an equation to find three consecutive integers such that three times the sum of the first two integers is the same as thirteen more than four times the third integer.

Let x, $x + 1$, and $x + 2$ represent the three consecutive integers.

$3(x + x + 1) = 4(x + 2) + 13$

2-5 Solving Equations Involving Absolute Value

Evaluate each expression if $m = -8$, $n = 4$, and $p = -12$.

47. $|3m - n|$

48. $|-2p + m| - 3n$

49. $-3|6n - 2p|$

50. $4|7m + 3p| + 4n$

Solve each equation. Then graph the solution set.

51. $|x - 6| = 11$

52. $|-4w + 2| = 14$

53. $\left|\frac{1}{3}d - 6\right| = 15$

54. $\left|\frac{2b}{3} + 8\right| = 20$

Example 7

Solve $|y - 9| = 16$. Then graph the solution set.

Case 1

$y - 9 = 16$ Original equation

$y - 9 + 9 = 16 + 9$ Add 9 to each side.

$y = 25$ Simplify.

Case 2

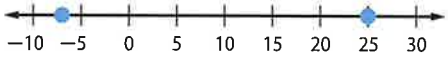
$y - 9 = -16$ Original equation

$y - 9 + 9 = -16 + 9$ Add 9 to each side.

$y = -7$ Simplify.

The solution set is $\{-7, 25\}$.

Graph the points on a number line.



2-6 Ratios and Proportions

Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.

55. $\frac{27}{45}, \frac{3}{5}$

56. $\frac{18}{32}, \frac{3}{4}$

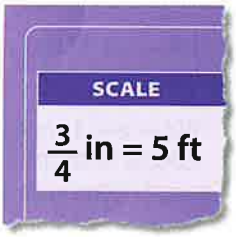
Solve each proportion. If necessary, round to the nearest hundredth.

57. $\frac{4}{9} = \frac{a}{45}$

58. $\frac{3}{8} = \frac{21}{t}$

59. $\frac{9}{12} = \frac{g}{16}$

60. CONSTRUCTION A new gym is being built at Greenfield Middle School. The length of the gym as shown on the builder's blueprints is 12 inches. Find the actual length of the new gym.



Example 8

Determine whether $\frac{7}{9}$ and $\frac{42}{54}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.

First, simplify each ratio. $\frac{7}{9}$ is already in simplest form.

$$\frac{42}{54} = \frac{42 \div 6}{54 \div 6} = \frac{7}{9}$$

When expressed in simplest form, the ratios are equivalent. The answer is yes.

Example 9

Solve $\frac{r}{8} = \frac{3}{4}$. If necessary, round to the nearest hundredth.

$\frac{r}{8} = \frac{3}{4}$ Original equation

$r(4) = 3(8)$ Find the cross products.

$4r = 24$ Simplify.

$\frac{4r}{4} = \frac{24}{4}$ Divide each side by 4.

$r = 6$ Simplify.

2-7 Percent of Change

State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

61. original: 40, new: 50

62. original: 36, new: 24

63. original: $72, new: $60

Find the total price of each item.

64. boots: $64, tax: 7%

65. video game: $49, tax: 6.5%

66. hockey skates: $199, tax: 5.25%

Find the discounted price of each item.

67. digital media player: $69.00, discount: 20%

68. jacket: $129, discount: 15%

69. backpack: $45, discount: 25%

70. ATTENDANCE An amusement park recorded attendance of 825,000 one year. The next year, the attendance increased to 975,000. Determine the percent of increase in attendance.

Example 10

State whether the percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

original: 80
final: 60

Subtract the original amount from the final amount to find the amount of change. $60 - 80 = -20$. Since the new amount is less than the original, this is a percent of decrease.

Use the original number, 80, as the base.

$\frac{\text{change}}{\text{original amount}} \rightarrow \frac{20}{80} = \frac{r}{100}$ Percent proportion

$20(100) = r(80)$ Find cross products.

$2000 = 80r$ Simplify.

$\frac{2000}{80} = \frac{80r}{80}$ Divide each side by 80.

$25 = r$ Simplify.

The percent of decrease is 25%.

2-8 Literal Equations and Dimensional Analysis

Solve each equation or formula for the variable indicated.

71. $3x + 2y = 9$, for y

72. $P = 2\ell + 2w$, for ℓ

73. $-5m + 9n = 15$, for m

74. $14w + 15x = y - 21w$, for w

75. $m = \frac{2}{5}y + n$, for y

76. $7d - 3c = f + 2d$, for d

77. GEOMETRY The formula for the area of a trapezoid is $A = \frac{1}{2}h(a + b)$, where h represents the height and a and b represent the lengths of the bases. Solve for h.

Example 11

Solve $6p - 8n = 12$ for p.

$6p - 8n = 12$ Original equation

$6p - 8n + 8n = 12 + 8n$ Add $8n$ to each side.

$6p = 12 + 8n$ Simplify.

$\frac{6p}{6} = \frac{12 + 8n}{6}$ Divide each side by 6.

$\frac{6p}{6} = \frac{12}{6} + \frac{8}{6}n$ Simplify.

$p = 2 + \frac{4}{3}n$ Simplify.

2-9 Weighted Averages

78. **CANDY** Michael is mixing two types of candy for a party. The chocolate pieces cost $0.40 per ounce, and the hard candy costs $0.20 per ounce. Michael purchases 20 ounces of the chocolate pieces, and the total cost of his candy was $11. How many ounces of hard candy did he purchase?

79. **TRAVEL** A car travels 100 miles east in 2 hours and 30 miles north in half an hour. What is the average speed of the car?

80. **FINANCIAL LITERACY** A candle supply store sells votive wax and low-shrink wax. How many pounds of low-shrink wax should be mixed with 8 pounds of votive wax to obtain a blend that sells for $0.98 a pound?

Example 12

METALS An alloy of metals is 25% copper. Another al is 50% copper. How much of each should be used to make 1000 grams of an alloy that is 45% copper?

Let x = the amount of the 25% copper alloy. Write and solve an equation.

$0.25x + 0.50(1000 - x) = 0.45(1000)$ Original Equati

$0.25x + 500 - 0.50x = 450$ Distributive Pro

$-0.25x + 500 = 450$ Simplify.

$-0.25x + 500 - 500 = 450 - 500$ Subtract 500 fr each side.

$-0.25x = -50$ Simplify.

$\frac{-0.25x}{-0.25} = \frac{-50}{-0.25}$ Divide each side by −0.25.

$x = 200$ Simplify.

200 grams of the 25% alloy and 800 grams of the 50% alloy should be used.

CHAPTER 2 Practice Test

Translate each sentence into an equation.

1. The sum of six and four times d is the same as d minus nine.
2. Three times the difference of two times m and five is equal to eight times m to the second power increased by four.

Solve each equation. Check your solutions.

3. $x - 5 = -11$
4. $\frac{2}{3} = w + \frac{1}{4}$
5. $\frac{t}{6} = -3$

Solve each equation. Check your solution.

6. $2a - 5 = 13$
7. $\frac{p}{4} - 3 = 9$
8. **MULTIPLE CHOICE** At Mama Mia Pizza, the price of a large pizza is determined by $P = 9 + 1.5x$, where x represents the number of toppings added to a cheese pizza. Daniel spent \$13.50 on a large pizza. How many toppings did he get?

   A 0

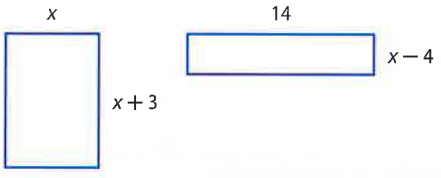
   B 1

   C 3

   D 5

Solve each equation. Check your solution.

9. $5y - 4 = 9y + 8$
10. $3(2k - 2) = -2(4k - 11)$
11. **GEOMETRY** Find the value of x so that the figures have the same perimeter.



12. Evaluate the expression $|3t - 2u| + 5v$ if $t = 2$, $u = -5$, and $v = -3$.

Solve each equation. Then graph the solution set.

13. $|p - 4| = 6$
14. $|2b + 5| = 9$

Solve each proportion. If necessary, round to the nearest hundredth.

15. $\frac{a}{3} = \frac{16}{24}$
16. $\frac{9}{k + 3} = \frac{3}{5}$
17. **MULTIPLE CHOICE** Akiko uses 2 feet of thread for every three squares that she sews for her quilt. How many squares can she sew if she has 38 feet of thread?

   F 19

   G 57

   H 76

   J 228

18. State whether the percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

   original: 54 new: 45

19. Find the total price of a sweatshirt that is priced at \$48 and taxed at 6.5%.
20. **SHOPPING** Kirk wants to purchase a wide-screen TV. He sees an advertisement for a TV that was originally priced at \$3200 and is 20% off. Find the discounted price of the TV.
21. Solve $5x - 3y = 9$ for y.
22. Solve $A = \frac{1}{2}bh$ for h.
23. **CHEMISTRY** Deon has 12 milliliters of a 5% solution. He also has a solution that has a concentration of 30%. How many milliliters of the 30% solution does Deon need to add to the 5% solution to obtain a 20% solution?
24. **BICYCLING** Shanee bikes 5 miles to the park in 30 minutes and 3 miles to the library in 45 minutes. What was her average speed?
25. **MAPS** On a map of North Carolina, the distance between Charlotte and Wilmington is 14.75 inches. If 2 inches equals 24 miles, what is the approximate distance between the two cities?

CHAPTER 2

Preparing for Standardized Tests

Gridded Response Questions

In addition to multiple-choice, short-answer, and extended-response questions, you will likely encounter gridded-response questions on standardized tests. For gridded-response questions, you must print your answer on an answer sheet and mark in the correct circles on the grid to match your answer.

Strategies for Solving Gridded Response Questions

Step 1

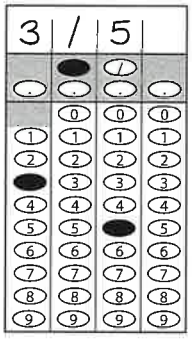
Read the problem carefully.

- **Ask yourself:** "What information is given?" "What do I need to find?" "How do I solve this type of problem?"
- **Solve the Problem:** Use the information given in the problem to solve.
- **Check your answer:** If time permits, check your answer to make sure you have solved the problem correctly.

Step 2

Write your answer in the answer boxes.

- Print only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.



Step 3

Fill in the grid.

- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.
- Fill in each bubble completely and clearly.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

GRIDDED RESPONSE Ashley is 3 years older than her sister, Tina. Combined, the sum of their ages is 27 years. How old is Ashley?

Read the problem carefully. You are told that Ashley is 3 years older than her sister and that their ages combined equal 27 years. You need to find Ashley's age.

Solve the Problem

Words Ashley's age plus Tina's age is equal to 27 years.

Variable Let a represent Ashley's age. Then Tina's age is $a - 3$, since she is 3 years younger than Ashley.

Equation $a + (a - 3) = 27$

Solve the equation for a.

$a + (a - 3) = 27$ Original equation.

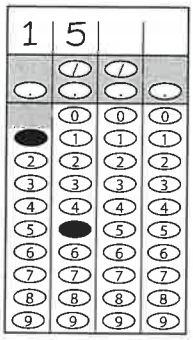
$2a - 3 = 27$ Add like terms.

$2a = 30$ Add 3 to each side.

$a = 15$ Divide each side by 2.

Since we let a represent Ashley's age, we know that she is 15 years old.

Fill in the Grid



Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Copy and complete an answer grid on your paper.

1. Orlando has $1350 in the bank. He wants to increase his balance to a total of $2550 by depositing $40 each week from his paycheck. How many weeks will he need to save in order to reach his goal?

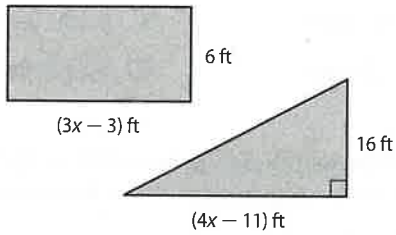
2. Fourteen less than three times a number is equal to 40. Find the number.

3. The table shows the regular prices and sale prices of certain items at a department store this week. What is the percent of discount during the sale?

Item	Regular Price ($)	Sale Price ($)
pillows	25	20
sweaters	30	24
entertainment center	125	100

4. Maureen is driving from Raleigh, North Carolina, to Charlotte, North Carolina, to visit her brother at college. If she averages 65 miles per hour on the trip, then the equation $\frac{d}{2.65} = 65$ can be solved for the distance d. What is the distance to the nearest mile from Raleigh to Charlotte?

5. Find the value of x so that the figures below have the same area.



6. The sum of three consecutive whole numbers is 18. What is the greatest of the numbers?

CHAPTER 2

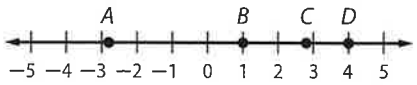
Standardized Test Practice

Cumulative, Chapters 1 and 2

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which point on the number line best represents the position of $\sqrt{8}$?



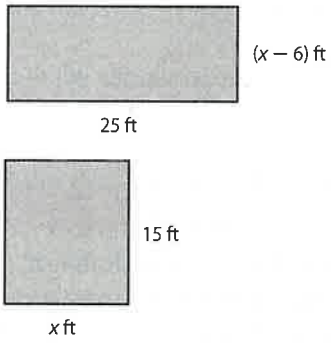
A -2.8

B 1

C 2.8

D 4

2. Find the value of x so that the figures have the same area.



F 10

G 12

H 13

J 15

3. The elevation of Black Mountain is 27 feet more than 16 times the lowest point in the state. If the elevation of the lowest point in the state is 257 feet, what is the elevation of Black Mountain?

A 4,085 feet

B 4,103 feet

C 4,139 feet

D 4,215 feet

4. The expression $(3x^2 + 5x - 12) - 2(x^2 + 4x + 9)$ is equivalent to which of the following?

F $x^2 - 3x - 30$

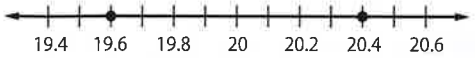
G $x^2 + 13x + 6$

H $5x^2 + x - 18$

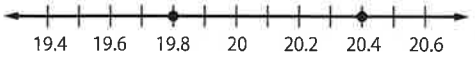
J $x^2 + 3x - 21$

5. The amount of soda, in fluid ounces, dispensed from a machine must satisfy the equation $|a - 0.4| = 20$. Which of the following graphs shows the acceptable minimum and maximum amounts that can be dispensed from the machine?

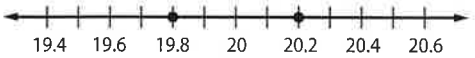
A



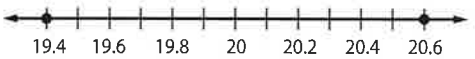
B



C



D



6. If a and b represent integers, $ab = ba$ is an example of which property?

F Associative Property

G Commutative Property

H Distributive Property

J Closure Property

7. The sum of one fifth of a number and three is equal to half of the number. What is the number

A 5

B 10

C 15

D 20

8. Aaron charges \$15 to mow the lawn and \$10 per hour for other gardening work. Which expression represents his earnings?

F $10h$

G $15h$

H $15h + 10$

J $15 + 10h$

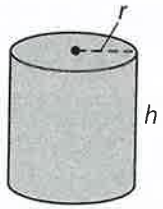
Test-Taking Tip

Question 2 Use the figures and the formula for area to set up an equation. The product of the length and width of each figure should be equal.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The formula for the lateral area of a cylinder is $A = 2\pi rh$, where r is the radius and h is the height. Solve the equation for h.



10. GRIDDED RESPONSE Solve the proportion $\frac{x}{18} = \frac{7}{21}$.

11. GRIDDED RESPONSE The table shows the cost of renting a moving van. If Miguel budgeted \$75, how many miles could he drive the van and maintain his budget?

Moving Van Rentals	
Flat Fee	\$50 for up to 300 miles
Variable Fee	\$0.20 per mile over 300

12. Find the height of a soup can if the area of the label is 302 square centimeters and the radius of the can is 4 centimeters. Round to the nearest whole number.

13. GRIDDED RESPONSE Lara's car needed a particular part that costs \$75. The mechanic charges \$50 per hour to install the part. If the total cost was \$350, how many hours did it take to install the part?

14. Lucinda is buying a set of patio furniture that is on sale for $\frac{4}{5}$ of the original price. After she uses a \$50 gift certificate, the total cost before sales tax is \$222. What was the original price of the patio furniture?

Extended Response

Record your answers on a sheet of paper. Show your work.

15. The city zoo offers a yearly membership that costs \$120. A yearly membership includes free parking. Members can also purchase a ride pass for an additional \$2 per day that allows them unlimited access to the rides in the park. Nonmembers pay \$12 for admission to the park, \$5 for parking, and \$5 for a ride pass.

a. Write an equation that could be solved for the number of visits it would take for the total cost to be the same for a member and a nonmember if they both purchase a ride pass each day. Solve the equation.

b. What would the total cost be for members and nonmembers after this number of visits?

c. Georgena is deciding whether or not to purchase a yearly membership. Explain how she could use the results above to help make her decision.

Need Extra Help?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	0-2	2-4	2-3	1-4	2-5	1-3	2-4	1-1	2-8	2-6	2-3	2-8	2-3	2-3	2-4