CHAPTER 3

Linear Functions

Then

You solved linear equations algebraically.

Now

In this chapter you will:

- Identify linear equations, intercepts, and zeros.
- Graph and write linear equations.
- Use rate of change to solve problems.

Why?

AMUSEMENT PARKS The Magic Kingdom in Orlando, Florida, is one of the most popular amusement parks in the world. Yearly attendance figures increase steadily each year. Quantities like populations that change with respect to time can be described using rate of change. Often you can represent these situations with linear functions.

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Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

Flame/Alamy

Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option
Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Graph each ordered pair on a coordinate grid.

1. $(-3, 3)$ **2.** $(-2, 1)$ **3.** $(3, 0)$

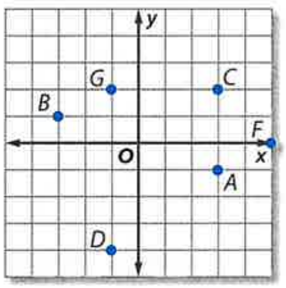
4. $(-5, 5)$ **5.** $(0, 6)$ **6.** $(2, -1)$

Write the ordered pair for each point.

7. *A* **8.** *B*

9. *C* **10.** *D*

11. *F* **12.** *G*



Solve each equation for *y*.

13. $3x + y = 1$

14. $8 - y = x$

15. $5x - 2y = 12$

16. $3x + 4y = 10$

17. $3 - \frac{1}{2}y = 5x$

18. $\frac{y+1}{3} = x + 2$

Evaluate $\frac{a-b}{c-d}$ for each set of values.

19. $a = 7, b = 6, c = 9, d = 5$

20. $a = -3, b = 0, c = 3, d = -1$

21. $a = -5, b = -5, c = 5, d = 8$

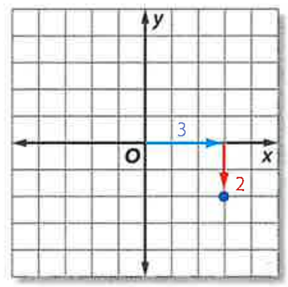
22. $a = -6, b = 3, c = 8, d = 2$

23. **MOVIES** A movie made $297.2 million in 22 weeks. How much did the movie make on average each week?

QuickReview

Example 1

Graph $(3, -2)$ on a coordinate grid.



Example 2

Solve $x - 2y = 8$ for y.

$x - 2y = 8$	Original equation
$x - x - 2y = 8 - x$	Subtract x from each side.
$-2y = 8 - x$	Simplify.
$\frac{-2y}{-2} = \frac{8-x}{-2}$	Divide each side by -2.
$y = \frac{1}{2}x - 4$	Simplify.

Example 3

Evaluate $\frac{a-b}{c-d}$ for $a = 3$, $b = 5$, $c = -2$, and $d = -6$.

$\frac{a-b}{c-d}$	Original expression
$= \frac{3-5}{-2-(-6)}$	Substitute 3 for a, 5 for b, -2 for c, and -6 for d.
$= \frac{-2}{4}$	Simplify.
$= \frac{-2 \div 2}{4 \div 2}$	Divide -2 and 4 by their GCF, 2.
$= \frac{-1}{2}$ or $-\frac{1}{2}$	Simplify. The signs are different so the quotient is negative.

2 Online Option
Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

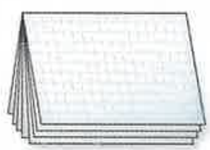
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 3. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

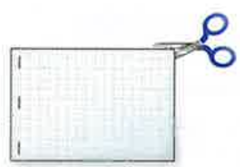
FOLDABLES StudyOrganizer

Linear Functions Make this Foldable to help you organize your Chapter 3 notes about graphing relations and functions. Begin with four sheets of grid paper.

1 **Fold** each sheet of grid paper in half from top to bottom.

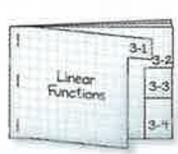


2 **Cut** along fold. Staple the eight half-sheets together to form a booklet.



3 **Cut** tabs into margin. The top tab is 4 lines wide, the next tab is 8 lines wide, and so on. When you reach the bottom of a sheet, start the next tab at the top of the page.

4 **Label** each of the tabs with a lesson number. Use the extra pages for vocabulary.



NewVocabulary

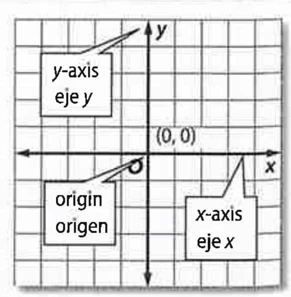
English		Español
linear equation	p. 155	ecuación lineal
standard form	p. 155	forma estándar
constant	p. 155	constante
x-intercept	p. 156	intersección *x*
y-intercept	p. 156	intersección *y*
linear function	p. 163	función lineal
parent function	p. 163	críe la funcitón
family of graphs	p. 163	la familia de gráficas
root	p. 163	raiz
rate of change	p. 172	tasa de cambio
slope	p. 174	pendiente
direct variation	p. 182	variación directa
constant of variation	p. 182	constante de variación
arithmetic sequence	p. 189	sucesión arithmética
inductive reasoning	p. 196	razonamiento inductivo
deductive reasoning	p. 196	razonamiento deductivo

ReviewVocabulary

origin origen
the point where the two axes in a coordinate plane intersect with coordinates (0, 0)

***x*-axis** eje *x* the horizontal number line on a coordinate plane

***y*-axis** eje *y* the vertical number line on a coordinate plane



EXPLORE 3-1

Algebra Lab
Analyzing Linear Graphs

Analyzing a graph can help you learn about the relationship between two quantities. A **linear function** is a function for which the graph is a line. There are four types of linear graphs. Let's analyze each type.

CCSS Common Core State Standards
Content Standards
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Activity 1 Line that Slants Up

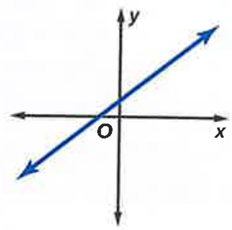
Analyze the function graphed at the right.

a. Describe the domain, range, and end behavior.

b. Describe the intercepts and any maximum or minimum points.

c. Identify where the function is positive, negative, increasing, and decreasing.

d. Describe any symmetry.

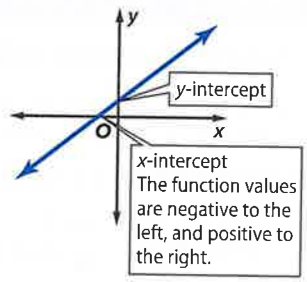


a. The domain and range are all real numbers. As you move left, the graph goes down. So as x decreases, y decreases. As you move right, the graph goes up. So as x increases, y increases.

b. There is one x-intercept and one y-intercept. There are no maximum or minimum points.

c. The function value is 0 at the x-intercept. The function values are negative to the left of the x-intercept and positive to the right. The function goes up from left to right, so it is increasing on the entire domain.

d. The graph has no symmetry.



Lines that slant down from left to right have some different key features.

Activity 2 Line that Slants Down

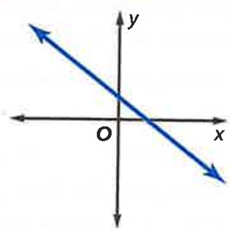
Analyze the function graphed at the right.

a. Describe the domain, range, and end behavior.

b. Describe the intercepts and any maximum or minimum points.

c. Identify where the function is positive, negative, increasing, and decreasing.

d. Describe any symmetry.



a. The domain and range are all real numbers. As you move left, the graph goes up. So as x decreases, y increases. As you move right, the graph goes down. So as x increases, y decreases.

b. There is one x-intercept and one y-intercept. There are no maximum or minimum points.

c. The function values are positive to the left of the x-intercept and negative to the right.
The function goes down from left to right, so it is decreasing on the entire domain.

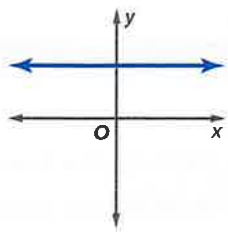
d. The graph has no symmetry.

Analyzing Linear Graphs *Continued*

Horizontal lines represent special functions called **constant functions**.

Activity 3 Horizontal Line

Analyze the function graphed at the right.



a. The domain is all real numbers, and the range is one value. As you move left or right, the graph stays constant. So as x decreases or increases, y is constant.

b. The graph does not intersect the x-axis, so there is no x-intercept. The graph has one y-intercept. There are no maximum or minimum points.

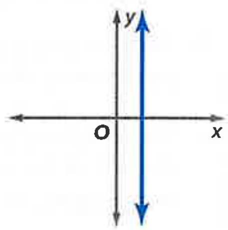
c. The function values are all positive. The function is constant on the entire domain.

d. The graph is symmetric about any vertical line.

Vertical lines represent linear relations that are *not* functions.

Activity 4 Vertical Line

Analyze the relation graphed at the right.



a. The domain is one value, and the range is all real numbers. This relation is not a function. Because you cannot move left or right on the graph, there is no end behavior.

b. There is one x-intercept and no y-intercept. There are no maximum or minimum points.

c. The y-values are positive above the x-axis and negative below. Because you cannot move left or right on the graph, the relation is neither increasing nor decreasing.

d. The graph is symmetric about itself.

Analyze the Results

1. Compare and contrast the key features of lines that slant up and lines that slant down.

2. How would the key features of a horizontal line below the x-axis differ from the features of a line above the x-axis?

3. Consider lines that pass through the origin.

- **a.** How do the key features of a line that slants up and passes through the origin compare to the key features of the line in Activity 1?
- **b.** Compare the key features of a line that slants down and passes through the origin to the key features of the line in Activity 2.
- **c.** Describe a horizontal line that passes through the origin and a vertical line that passes through the origin. Compare their key features to those of the lines in Activities 3 and 4.

4. **CCSS TOOLS** Place a pencil on a coordinate plane to represent a line. Move the pencil to represent different lines and evaluate each conjecture.

- **a.** *True* or *false*: A line can have more than one x-intercept.
- **b.** *True* or *false*: If the end behavior of a line is that as x increases, y increases, then the function values are increasing over the entire domain.
- **c.** *True* or *false*: Two different lines can have the same x- and y-intercepts.

Sketch a linear graph that fits each description.

5. as x increases, y decreases

6. one x-intercept and one y-intercept

7. has symmetry

8. is not a function

LESSON 3-1 Graphing Linear Equations

:: Then	:: Now	:: Why?
• You represented relationships among quantities using equations.	• **1** Identify linear equations, intercepts, and zeros. **2** Graph linear equations.	• Recycling one ton of waste paper saves an average of 17 trees, 7000 gallons of water, 3 barrels of oil, and about 3.3 cubic yards of landfill space. The relationship between the amount of paper recycled and the number of trees saved can be expressed with the equation $y = 17x$, where y represents the number of trees and x represents the tons of paper recycled.

New Vocabulary
- linear equation
- standard form
- constant
- x-intercept
- y-intercept

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Linear Equations and Intercepts

A **linear equation** is an equation that forms a line when it is graphed. Linear equations are often written in the form $Ax + By = C$. This is called the **standard form** of a linear equation. In this equation, C is called a **constant**, or a number. Ax and By are variable terms.

KeyConcept Standard Form of a Linear Equation

Words	The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1.
Examples	In $3x + 2y = 5$, $A = 3$, $B = 2$, and $C = 5$. In $x = -7$, $A = 1$, $B = 0$, and $C = -7$.

Example 1 Identify Linear Equations

Determine whether each equation is a linear equation. Write the equation in standard form.

a. $y = 4 - 3x$

Rewrite the equation so that it appears in standard form.

$y = 4 - 3x$ — Original equation

$y + 3x = 4 - 3x + 3x$ — Add $3x$ to each side.

$3x + y = 4$ — Simplify.

The equation is now in standard form where $A = 3$, $B = 1$, and $C = 4$. This is a linear equation.

b. $6x - xy = 4$

Since the term xy has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

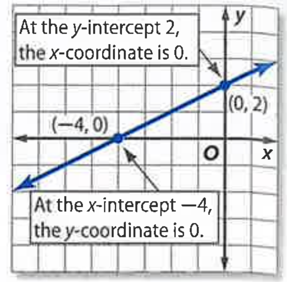
GuidedPractice

1A. $\frac{1}{3}y = -1$

1B. $y = x^2 - 4$

Harald Sund/The Image Bank/Getty Images

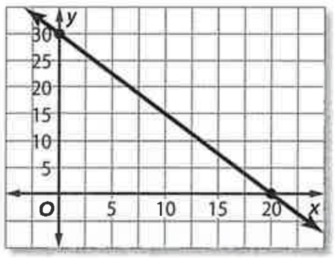
A linear equation can be represented on a coordinate graph. The x-coordinate of the point at which the graph of an equation crosses the x-axis is an **x-intercept**. The y-coordinate of the point at which the graph crosses the y-axis is called a **y-intercept**.



The graph of a linear equation has at most one x-intercept and one y-intercept, unless it is the equation $x = 0$ or $y = 0$, in which case every number is a y-intercept or an x-intercept, respectively.

Standardized Test Example 2 Find Intercepts from a Graph

Find the x- and y-intercepts of the line graphed at the right.



A x-intercept is 0; y-intercept is 30.

B x-intercept is 20; y-intercept is 30.

C x-intercept is 20; y-intercept is 0.

D x-intercept is 30; y-intercept is 20.

Read the Test Item

We need to determine the x- and y-intercepts of the line in the graph.

ReadingMath

Intercepts Usually, the individual coordinates are called the x-intercept and the y-intercept. The x-intercept 20 is located at (20, 0).The y-intercept 30 is located at (0, 30).

Solve the Test Item

Step 1 Find the x-intercept. Look for the point where the line crosses the x-axis.

The line crosses at (20, 0). The x-intercept is 20 because it is the x-coordinate of the point where the line crosses the x-axis.

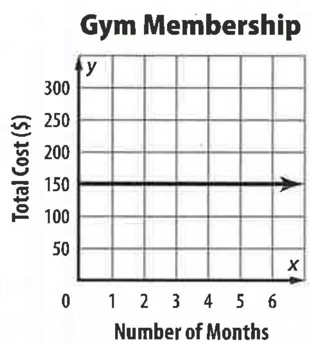
Step 2 Find the y-intercept. Look for the point where the line crosses the y-axis.

The line crosses at (0, 30). The y-intercept is 30 because it is the y-coordinate of the point where the line crosses the y-axis.

Thus, the answer is B.

GuidedPractice

2. HEALTH Find the x- and y-intercepts of the graph.



F x-intercept is 0; y-intercept is 150.

G x-intercept is 150; y-intercept is 0.

H x-intercept is 150; no y-intercept.

J No x-intercept; y-intercept is 150.

Study Tip

Defining Variables In Example 3, time is the *independent* variable, and volume of water is the *dependent* variable.

Real-World Example 3 Find Intercepts from a Table

SWIMMING POOL **A swimming pool is being drained at a rate of 720 gallons per hour. The table shows the function relating the volume of water in a pool and the time in hours that the pool has been draining.**

Draining a Pool	
Time (h)	Volume (gal)
x	y
0	10,080
2	8640
6	5760
10	2880
12	1440
14	0

a. Find the x- and y-intercepts of the graph of the function.

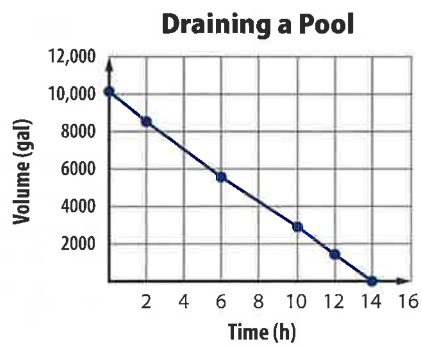
x-intercept = 14 — 14 is the value of x when $y = 0$.

y-intercept = 10,080 — 10,080 is the value of y when $x = 0$.

b. Describe what the intercepts mean in this situation.

The x-intercept 14 means that after 14 hours, the water has a volume of 0 gallons, or the pool is completely drained.

The y-intercept 10,080 means that the pool contained 10,080 gallons of water at time 0, or before it started to drain. This is shown in the graph.



Guided Practice

3. **DRIVING** The table shows the function relating the distance to an amusement park in miles and the time in hours the Torres family has driven. Find the x- and y-intercepts. Describe what the intercepts mean in this situation.

Time (h)	Distance (mi)
0	248
1	186
2	124
3	62
4	0

2 Graph Linear Equations

By first finding the x- and y-intercepts, you have the ordered pairs of two points through which the graph of the linear equation passes. This information can be used to graph the line because only two points are needed to graph a line.

Study Tip

Intercepts The x-intercept is where the graph crosses the x-axis. So the y-value is always 0. The y-intercept is where the graph crosses the y-axis. So, the x-value is always 0.

Example 4 Graph by Using Intercepts

Graph $2x + 4y = 16$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

$2x + 4y = 16$ — Original equation

$2x + 4(0) = 16$ — Replace y with 0.

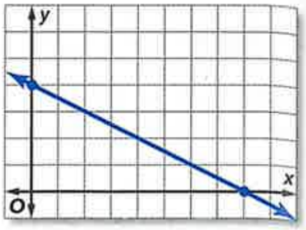
$2x = 16$ — Simplify.

$x = 8$ — Divide each side by 2.

The x-intercept is 8. This means that the graph intersects the x-axis at (8, 0).

To find the y-intercept, let $x = 0$.

$2x + 4y = 16$	Original equation
$2(0) + 4y = 16$	Replace x with 0.
$4y = 16$	Simplify.
$y = 4$	Divide each side by 4.



The y-intercept is 4. This means the graph intersects the y-axis at (0, 4).

Plot these two points and then draw a line through them.

StudyTip

Equivalent Equations Rewriting equations by solving for y may make it easier to find values for y.

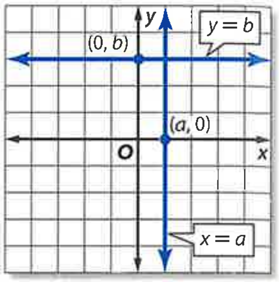
$-x + 2y = 3 \rightarrow y = \frac{x + 3}{2}$

GuidedPractice

Graph each equation by using the x- and y-intercepts.

4A. $-x + 2y = 3$ **4B.** $y = -x - 5$

Note that the graph in Example 4 has both an x- and a y-intercept. Some lines have an x-intercept and no y-intercept or vice versa. The graph of $y = b$ is a horizontal line that only has a y-intercept (unless $b = 0$). The intercept occurs at $(0, b)$. The graph of $x = a$ is a vertical line that only has an x-intercept (unless $a = 0$). The intercept occurs at $(a, 0)$.



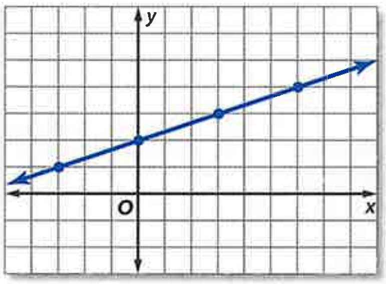
Every ordered pair that makes an equation true represents a point on the graph. So, the graph of an equation represents all of its solutions. Any ordered pair that does not make the equation true represents a point that is not on the line.

Example 5 Graph by Making a Table

Graph $y = \frac{1}{3}x + 2$.

The domain is all real numbers. Select values from the domain and make a table. When the x-coefficient is a fraction, select a number from the domain that is a multiple of the denominator. Create ordered pairs and graph them.

x	$\frac{1}{3}x + 2$	y	(x, y)
−3	$\frac{1}{3}(-3) + 2$	1	(−3, 1)
0	$\frac{1}{3}(0) + 2$	2	(0, 2)
3	$\frac{1}{3}(3) + 2$	3	(3, 3)
6	$\frac{1}{3}(6) + 2$	4	(6, 4)



GuidedPractice

Graph each equation by making a table.

5A. $2x - y = 2$ **5B.** $x = 3$ **5C.** $y = -2$

Check Your Understanding

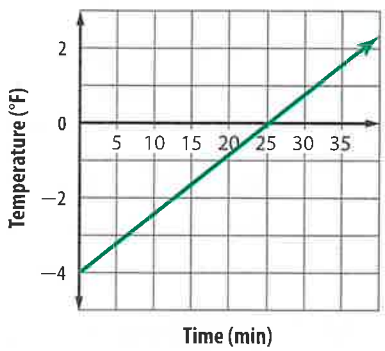
= Step-by-Step Solutions begin on page R13.

Example 1 Determine whether each equation is a linear equation. Write *yes* or *no*. If yes, write the equation in standard form.

1. $x = y - 5$ **2.** $-2x - 3 = y$ **3.** $-4y + 6 = 2$ **4.** $\frac{2}{3}x - \frac{1}{3}y = 2$

Examples 2–3 Find the x- and y-intercepts of the graph of each linear function. Describe what the intercepts mean.

5. **Increasing Temperature**



6.

Position of Scuba Diver	
Time (s)	Depth (m)
x	y
0	–24
3	–18
6	–12
9	–6
12	0

Example 4 Graph each equation by using the x- and y-intercepts.

7. $y = 4 + x$ **8.** $2x - 5y = 1$

Example 5 Graph each equation by making a table.

9. $x + 2y = 4$ **10.** $-3 + 2y = -5$ **11.** $y = 3$

12. CCSS **REASONING** The equation $5x + 10y = 60$ represents the number of children x and adults y who can attend the rodeo for \$60.

a. Use the x- and y-intercepts to graph the equation.

b. Describe what these values mean.

Practice and Problem Solving

Extra Practice is on page R3.

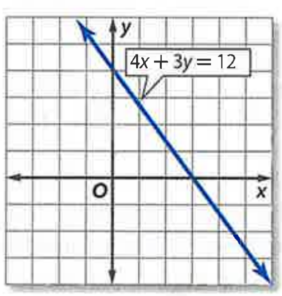
Example 1 Determine whether each equation is a linear equation. Write *yes* or *no*. If yes, write the equation in standard form.

13. $5x + y^2 = 25$ **14.** $8 + y = 4x$ **15.** $9xy - 6x = 7$

16. $4y^2 + 9 = -4$ **17.** $12x = 7y - 10y$ **18.** $y = 4x + x$

Example 2 Find the x- and y-intercepts of the graph of each linear function.

19.

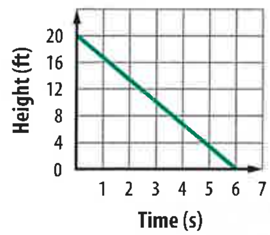


20.

x	y
–3	–1
–2	0
–1	1
0	2
1	3

Example 3 **Find the x- and y-intercepts of each linear function. Describe what the intercepts mean.**

21. **Descent of Eagle**



22.

Eva's Distance from Home	
Time (min)	Distance (mi)
x	y
0	4
2	3
4	2
6	1
8	0

Example 4 **Graph each equation by using the x- and y-intercepts.**

23. $y = 4 + 2x$ **24.** $5 - y = -3x$ **25.** $x = 5y + 5$

26. $x + y = 4$ **27.** $x - y = -3$ **28.** $y = 8 - 6x$

Example 5 **Graph each equation by making a table.**

29. $x = -2$ **30.** $y = -4$ **31.** $y = -8x$

32. $3x = y$ **33.** $y - 8 = -x$ **34.** $x = 10 - y$

35. **TV RATINGS** The number of people who watch a singing competition can be given by $p = 0.15v$, where p represents the number of people in millions who saw the show and v is the number of potential viewers in millions.

a. Make a table of values for the points (v, p).

b. Graph the equation.

c. Use the graph to estimate the number of people who saw the show if there are 14 million potential viewers.

d. Explain why it would not make sense for v to be a negative number.

Determine whether each equation is a linear equation. Write *yes* or *no*. If yes, write the equation in standard form.

36. $x + \frac{1}{y} = 7$ **37.** $\frac{x}{2} = 10 + \frac{2y}{3}$

38. $7n - 8m = 4 - 2m$ **39.** $3a + b - 2 = b$

40. $2r - 3rt + 5t = 1$ **41.** $\frac{3m}{4} = \frac{2n}{3} - 5$

42. **FINANCIAL LITERACY** James earns a monthly salary of \$1200 and a commission of \$125 for each car he sells.

a. Graph an equation that represents how much James earns in a month in which he sells x cars.

b. Use the graph to estimate the number of cars James needs to sell in order to earn \$5000.

Graph each equation.

43. $2.5x - 4 = y$ **44.** $1.25x + 7.5 = y$ **45.** $y + \frac{1}{5}x = 3$

46. $\frac{2}{3}x + y = -7$ **47.** $2x - 3 = 4y + 6$ **48.** $3y - 7 = 4x + 1$

49. CCSS **REASONING** Mrs. Johnson is renting a car for vacation and plans to drive a total of 800 miles. A rental car company charges \$153 for the week including 700 miles and \$0.23 for each additional mile. If Mrs. Johnson has only \$160 to spend on the rental car, can she afford to rent a car? Explain your reasoning.

50. AMUSEMENT PARKS An amusement park charges \$50 for admission before 6 P.M. and \$20 for admission after 6 P.M. On Saturday, the park took in a total of \$20,000.

a. Write an equation that represents the number of admissions that may have been sold. Let x represent the admissions sold before 6 P.M., and let y represent the admissions sold after 6 P.M.

b. Graph the equation.

c. Find the x- and y-intercepts of the graph. What does each intercept represent?

Find the x-intercept and y-intercept of the graph of each equation.

51. $5x + 3y = 15$ **52.** $2x - 7y = 14$ **53.** $2x - 3y = 5$

54. $6x + 2y = 8$ **55.** $y = \frac{1}{4}x - 3$ **56.** $y = \frac{2}{3}x + 1$

57. ONLINE GAMES The percent of teens who play online games can be modeled by $p = \frac{15}{4}t + 66$. p is the percent of students, and t represents time in years since 2000.

a. Graph the equation.

b. Use the graph to estimate the percent of students playing the games in 2008.

58. MULTIPLE REPRESENTATIONS In this problem, you will explore x- and y-intercepts of graphs of linear equations.

a. Graphical If possible, use a straightedge to draw a line on a coordinate plane with each of the following characteristics.

x- and y-intercept	x-intercept, no y-intercept	exactly 2 x-intercepts	no x-intercept, y-intercept	exactly 2 y-intercepts

b. Analytical For which characteristics were you able to create a line and for which characteristics were you unable to create a line? Explain.

c. Verbal What must be true of the x- and y-intercepts of a line?

H.O.T. Problems Use Higher-Order Thinking Skills

59. CCSS REGULARITY Copy and complete each table. State whether any of the tables show a linear relationship. Explain.

Perimeter of a Square	
Side Length	Perimeter
1	
2	
3	
4	

Area of a Square	
Side Length	Area
1	
2	
3	
4	

Volume of a Cube	
Side Length	Volume
1	
2	
3	
4	

60. REASONING Compare and contrast the graphs of $y = 2x + 1$ with the domain $\{1, 2, 3, 4\}$ and $y = 2x + 1$ with the domain of all real numbers.

OPEN ENDED Give an example of a linear equation of the form $Ax + By = C$ for each condition. Then describe the graph of the equation.

61. $A = 0$ **62.** $B = 0$ **63.** $C = 0$

64. WRITING IN MATH Explain how to find the x-intercept and y-intercept of a graph and summarize how to graph a linear equation.

Standardized Test Practice

65. Sancho can ride 8 miles on his bicycle in 30 minutes. At this rate, about how long would it take him to ride 30 miles?

A 8 hours

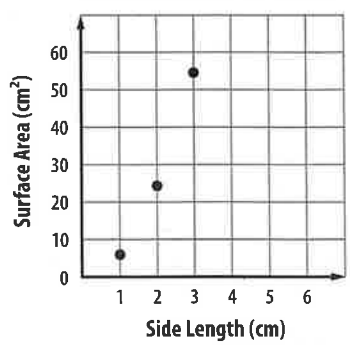
B 6 hours 32 minutes

C 2 hours

D 1 hour 53 minutes

66. GEOMETRY Which is a true statement about the relation graphed?

Surface Area of Cube



F The relation is not a function.

G Surface area is the independent quantity.

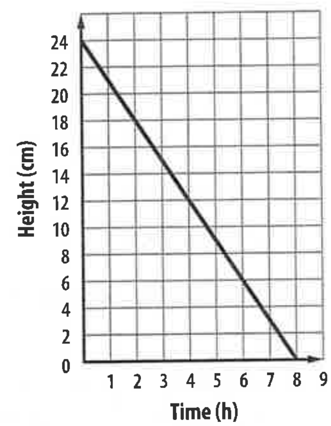
H The surface area of a cube is a function of the side length.

J As the side length of a cube increases, the surface area decreases.

67. SHORT RESPONSE Selena deposited \$2000 into a savings account that pays 1.5% interest compounded annually. If she does not deposit any more money into her account, how much will she earn in interest at the end of one year?

68. A candle burns as shown in the graph.

Candle Height



If the height of the candle is 8 centimeters, approximately how long has the candle been burning?

A 0 hours

B 24 minutes

C 64 minutes

D $5\frac{1}{2}$ hours

Spiral Review

69. FUNDRAISING The Madison High School Marching Band sold solid-color gift wrap for \$4 per roll and print gift wrap for \$6 per roll. The total number of rolls sold was 480, and the total amount of money collected was \$2340. How many rolls of each kind of gift wrap were sold? (Lesson 2-9)

Solve each equation or formula for the variable specified. (Lesson 2-8)

70. $S = \frac{n}{2}(A + t)$, for A

71. $2g - m = 5 - gh$, for g

72. $\frac{y + a}{3} = c$, for y

73. $4z + b = 2z + c$, for z

Skills Review

Evaluate each expression if $x = 2$, $y = 5$, and $z = 7$.

74. $3x^2 - 4y$

75. $\frac{x - y^2}{2z}$

76. $\left(\frac{y}{z}\right)^2 + \frac{xy}{2}$

77. $z^2 - y^3 + 5x^2$

LESSON 3-2

Solving Linear Equations by Graphing

Then

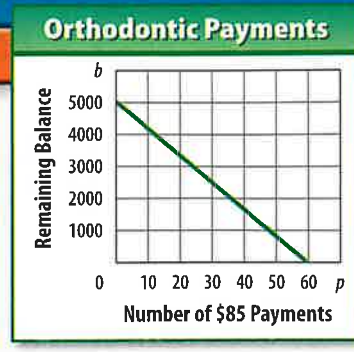
- You graphed linear equations by using tables and finding roots, zeros, and intercepts.

Now

1. Solve linear equations by graphing.
2. Estimate solutions to a linear equation by graphing.

Why?

- The cost of braces can vary widely. The graph shows the balance of the cost of treatments as payments are made. This is modeled by the function $b = -85p + 5100$, where p represents the number of \$85 payments made, and b is the remaining balance.



NewVocabulary

linear function
parent function
family of graphs
root
zeros

Common Core State Standards

Content Standards

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices

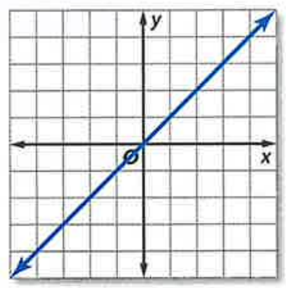
4 Model with mathematics.

1 Solve by Graphing

A **linear function** is a function for which the graph is a line. The simplest linear function is $f(x) = x$ and is called the **parent function** of the family of linear functions. A **family of graphs** is a group of graphs with one or more similar characteristics.

KeyConcept Linear Function

Parent function:	$f(x) = x$
Type of graph:	line
Domain:	all real numbers
Range:	all real numbers

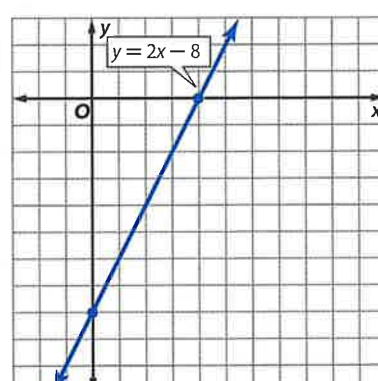


The solution or **root** of an equation is any value that makes the equation true. A linear equation has at most one root. You can find the root of an equation by graphing its related function. To write the related function for an equation, replace 0 with $f(x)$.

Linear Equation	Related Function
$2x - 8 = 0$	$f(x) = 2x - 8$ or $y = 2x - 8$

Values of x for which $f(x) = 0$ are called **zeros** of the function f. The zero of a function is located at the x-intercept of the function. The root of an equation is the value of the x-intercept. So:

- 4 is the x-intercept of $2x - 8 = 0$.
- 4 is the solution of $2x - 8 = 0$.
- 4 is the root of $2x - 8 = 0$.
- 4 is the zero of $f(x) = 2x - 8$.



Example 1 Solve an Equation with One Root

Solve each equation.

a. $0 = \frac{1}{3}x - 2$

Method 1 Solve algebraically.

$0 = \frac{1}{3}x - 2$	Original equation
$0 + 2 = \frac{1}{3}x - 2 + 2$	Add 2 to each side.
$3(2) = 3\left(\frac{1}{3}x\right)$	Multiply each side by 3.
$6 = x$	Solve.

The solution is 6.

b. $3x + 1 = -2$

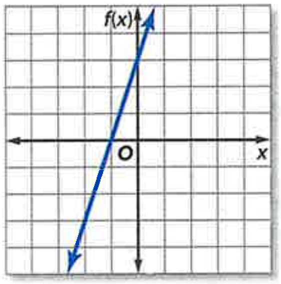
Method 2 Solve by graphing.

Find the related function. Rewrite the equation with 0 on the right side.

$3x + 1 = -2$	Original equation
$3x + 1 + 2 = -2 + 2$	Add 2 to each side.
$3x + 3 = 0$	Simplify.

The related function is $f(x) = 3x + 3$. To graph the function, make a table.

x	$f(x) = 3x + 3$	$f(x)$	$(x, f(x))$
−2	$f(-2) = 3(-2) + 3$	−3	(−2, −3)
−1	$f(-1) = 3(-1) + 3$	0	(−1, 0)



The graph intersects the x-axis at −1. So, the solution is −1.

StudyTip

Zeros from tables The zero is located at the x-intercept, so the value of y will equal 0. When looking at a table, the zero is the x-value when $y = 0$.

Guided Practice

1A. $0 = \frac{2}{5}x + 6$

1B. $-1.25x + 3 = 0$

For equations with the same variable on each side of the equation, use addition or subtraction to get the terms with variables on one side. Then solve.

Example 2 Solve an Equation with No Solution

Solve each equation.

a. $3x + 7 = 3x + 1$

Method 1 Solve algebraically.

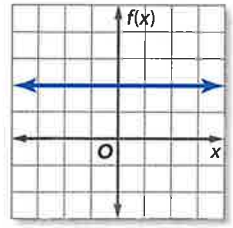
$3x + 7 = 3x + 1$	Original equation
$3x + 7 - 1 = 3x + 1 - 1$	Subtract 1 from each side.
$3x + 6 = 3x$	Simplify.
$3x - 3x + 6 = 3x - 3x$	Subtract $3x$ from each side.
$6 = 0$	Simplify.

The related function is $f(x) = 6$. The root of a linear equation is the value of x when $f(x) = 0$. Since $f(x)$ is always equal to 6, this equation has no solution.

b. $2x - 4 = 2x - 6$

Method 2 Solve by graphing.

$2x - 4 = 2x - 6$	Original equation
$2x - 4 + 6 = 2x - 6 + 6$	Add 6 to each side.
$2x + 2 = 2x$	Simplify.
$2x - 2x + 2 = 2x - 2x$	Subtract $2x$ from each side.
$2 = 0$	Simplify.



Graph the related function, which is $f(x) = 2$. The graph does not intersect the x-axis. Thus, there is no solution.

Guided Practice

2A. $4x + 3 = 4x - 5$

2B. $2 - 3x = 6 - 3x$

2 Estimate Solutions by Graphing

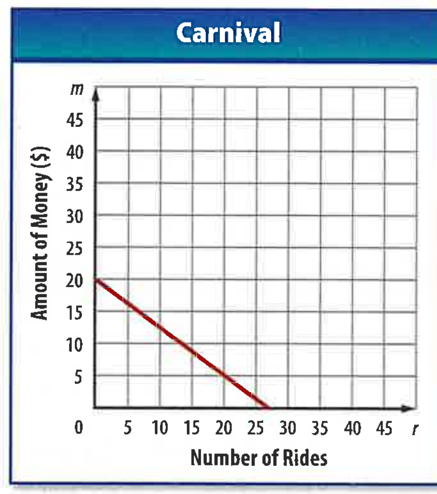
Estimate Solutions by Graphing Graphing may provide only an estimate. In these cases, solve algebraically to find the exact solution.

Real-World Example 3 Estimate by Graphing

CARNIVAL RIDES Emily is going to a local carnival. The function $m = 20 - 0.75r$ represents the amount of money m she has left after r rides. Find the zero of this function. Describe what this value means in this context.

Make a table of values.

r	$m = 20 - 0.75r$	m	(r, m)
0	$m = 20 - 0.75(0)$	20	(0, 20)
5	$m = 20 - 0.75(5)$	16.25	(5, 16.25)



The graph appears to intersect the r-axis at 27.

Next, solve algebraically to check.

$m = 20 - 0.75r$	Original equation
$0 = 20 - 0.75r$	Replace m with 0.
$0 + 0.75r = 20 - 0.75r + 0.75r$	Add $0.75r$ to each side.
$0.75r = 20$	Simplify.
$\frac{0.75r}{0.75} = \frac{20}{0.75}$	Divide each side by 0.75.
$r \approx 26.67$	Simplify and round to the nearest hundredth.

The zero of this function is about 26.67. Since Emily cannot ride part of a ride, she can ride 26 rides before she will run out of money.

Real-World Career

Entertainment Manager An entertainment manager supervises tech tests, calls show cues, schedules performances and performers, coaches employees and guest talent, and manages expenses. Entertainment managers need a college degree in a field such as communication or theater.

Guided Practice

3. FINANCIAL LITERACY Antoine's class is selling candy to raise money for a class trip. They paid \$45 for the candy, and they are selling each candy bar for \$1.50. The function $y = 1.50x - 45$ represents their profit y when they sell x candy bars. Find the zero and describe what it means in the context of this situation.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 Solve each equation by graphing. Verify your answer algebraically.

1. $-2x + 6 = 0$
2. $-x - 3 = 0$
3. $4x - 2 = 0$
4. $9x + 3 = 0$
5. $2x - 5 = 2x + 8$
6. $4x + 11 = 4x - 24$
7. $3x - 5 = 3x - 10$
8. $-6x + 3 = -6x + 5$

Example 3

9. **NEWSPAPERS** The function $w = 30 - \frac{3}{4}n$ represents the weight w in pounds of the papers in Tyrone's newspaper delivery bag after he delivers n newspapers. Find the zero and explain what it means in the context of this situation.

Practice and Problem Solving

Extra Practice is on page R3.

Solve each equation by graphing. Verify your answer algebraically.

10. $0 = x - 5$
11. $0 = x + 3$
12. $5 - 8x = 16 - 8x$
13. $3x - 10 = 21 + 3x$
14. $4x - 36 = 0$
15. $0 = 7x + 10$
16. $2x + 22 = 0$
17. $5x - 5 = 5x + 2$
18. $-7x + 35 = 20 - 7x$
19. $-4x - 28 = 3 - 4x$
20. $0 = 6x - 8$
21. $12x + 132 = 12x - 100$

Example 3

22. **TEXTING** Sean is sending texts to his friends. The function $y = 160 - x$ represents the number of characters y the message can hold after he has typed x characters. Find the zero and explain what it means in the context of this situation.

23. **GIFT CARDS** For her birthday Kwan receives a $50 gift card to download songs. The function $m = -0.50d + 50$ represents the amount of money m that remains on the card after a number of songs d are downloaded. Find the zero and explain what it means in the context of this situation.

Solve each equation by graphing. Verify your answer algebraically.

24. $-7 = 4x + 1$
25. $4 - 2x = 20$
26. $2 - 5x = -23$
27. $10 - 3x = 0$
28. $15 + 6x = 0$
29. $0 = 13x + 34$
30. $0 = 22x - 10$
31. $25x - 17 = 0$
32. $0 = \frac{1}{2} + \frac{2}{3}x$
33. $0 = \frac{3}{4} - \frac{2}{5}x$
34. $13x + 117 = 0$
35. $24x - 72 = 0$

36. **SEA LEVEL** Parts of New Orleans lie 0.5 meter below sea level. After d days of rain the equation $w = 0.3d - 0.5$ represents the water level w in meters. Find the zero, and explain what it means in the context of this situation.

37. **CCSS MODELING** An artist completed an ice sculpture when the temperature was −10°C. The equation $t = 1.25h - 10$ shows the temperature h hours after the sculpture's completion. If the artist completed the sculpture at 8:00 A.M., at what time will it begin to melt?

Solve each equation by graphing. Verify your answer algebraically.

38. $7 - 3x = 8 - 4x$
39. $19 + 3x = 13 + x$
40. $16x + 6 = 14x + 10$
41. $15x - 30 = 5x - 50$
42. $\frac{1}{2}x - 5 = 3x - 10$
43. $3x - 11 = \frac{1}{3}x - 8$

44. **HAIR PRODUCTS** Chemical hair straightening makes curly hair straight and smooth. The percent of the process left to complete is modeled by $p = -12.5t + 100$, where t is the time in minutes that the solution is left on the hair, and p represents the percent of the process left to complete.

a. Find the zero of this function.

b. Make a graph of this situation.

c. Explain what the zero represents in this context.

d. State the possible domain and range of this function.

45. **MUSIC DOWNLOADS** In this problem, you will investigate the change between two quantities.

a. Copy and complete the table.

Number of Songs Downloaded	Total Cost ($)	$\frac{\text{Total Cost}}{\text{Number of Songs Downloaded}}$
2	4	
4	8	
6	12	

b. As the number of songs downloaded increases, how does the total cost change?

c. Interpret the value of the total cost divided by the number of songs downloaded.

H.O.T. Problems Use Higher-Order Thinking Skills

46. **ERROR ANALYSIS** Clarissa and Koko solve $3x + 5 = 2x + 4$ by graphing the related function. Is either of them correct? Explain your reasoning.

Clarissa

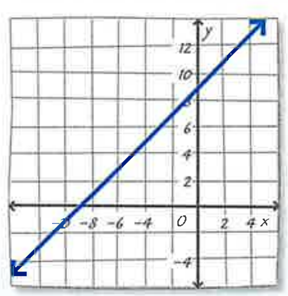
$3x + 5 = 2x + 4$

$x + 5 = 4$

$y = x + 5 + 4$

$y = x + 9$

$x = -9$



Koko

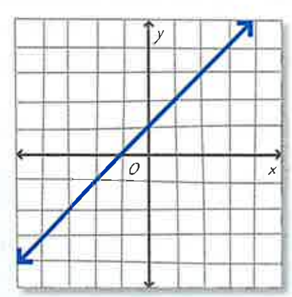
$3x + 5 = 2x + 4$

$x + 5 = 4$

$x + 1 = 0$

$y = x + 1$

$x = -1$



47. **CHALLENGE** Find the solution of $\frac{2}{3}(x + 3) = \frac{1}{2}(x + 5)$ by graphing. Verify your solution algebraically.

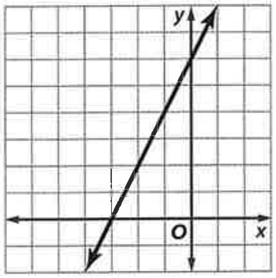
48. **CCSS TOOLS** Explain when it is better to solve an equation using algebraic methods and when it is better to solve by graphing.

49. **OPEN ENDED** Write a linear equation that has a root of $-\frac{3}{4}$. Write its related function.

50. **WRITING IN MATH** Summarize how to solve a linear equation algebraically and graphically.

Standardized Test Practice

51. What are the x- and y-intercepts of the graph of the function?



A $-3, 6$

B $6, -3$

C $3, -6$

D $-6, 3$

52. The table shows the cost C of renting a pontoon boat for h hours.

Hours	1	2	3
Cost ($)	7.25	14.5	21.75

Which equation best represents the data?

F $C = 7.25h$

G $C = h + 7.25$

H $C = 21.75 - 7.25h$

J $C = 7.25h + 21.75$

53. Which is the best estimate for the x-intercept of the graph of the linear function represented in the table?

x	y
0	5
1	3
2	1
3	−1
4	−3

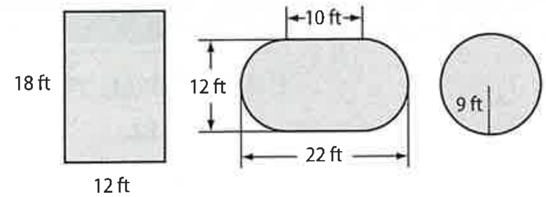
A between 0 and 1

B between 2 and 3

C between 1 and 2

D between 3 and 4

54. EXTENDED RESPONSE Mr. Kauffmann has the following options for a backyard pool.



If each pool has the same depth, which pool would give the greatest area to swim? Explain your reasoning.

Spiral Review

Find the x- and y-intercepts of the graph of each linear equation. (Lesson 3-1)

55. $y = 2x + 10$

56. $3y = 6x - 9$

57. $4x - 14y = 28$

58. FOOD If 2% milk contains 2% butterfat and whipping cream contains 9% butterfat, how much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat? (Lesson 2-9)

Translate each sentence into an equation. (Lesson 2-1)

59. The product of 3 and m plus 2 times n is the same as the quotient of 4 and p.

60. The sum of x and five times y equals twice z minus 7.

Skills Review

Simplify.

61. $\frac{25}{10}$

62. $\frac{-4}{-12}$

63. $\frac{6}{-12}$

64. $\frac{-36}{8}$

Evaluate $\frac{a-b}{c-d}$ for the given values.

65. $a = 6, b = 2, c = 9, d = 3$

66. $a = -8, b = 4, c = 5, d = -3$

67. $a = 4, b = -7, c = -1, d = -2$

EXTEND

3-2 Graphing Technology Lab
Graphing Linear Functions

The power of a graphing calculator is the ability to graph different types of equations accurately and quickly. By entering one or more equations in the calculator you can view features of a graph, such as the *x*-intercept, *y*-intercept, the origin, intersections, and the coordinates of specific points.

Often linear equations are graphed in the **standard viewing window**, which is [−10, 10] by [−10, 10] with a scale of 1 on each axis. To quickly choose the standard viewing window on a TI-83/84 Plus, press [ZOOM] 6.

CCSS Common Core State Standards
Content Standards
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
Mathematical Practices
5 Use appropriate tools strategically.

Activity 1 Graph a Linear Equation

Graph $3x - y = 4$.

Step 1 Enter the equation in the Y= list.

- The Y= list shows the equation or equations that you will graph.
- Equations must be entered with the *y* isolated on one side of the equation. Solve the equation for *y*, then enter it into the calculator.

$3x - y = 4$	Original equation
$3x - y - 3x = 4 - 3x$	Subtract $3x$ from each side.
$-y = -3x + 4$	Simplify.
$y = 3x - 4$	Multiply each side by −1.

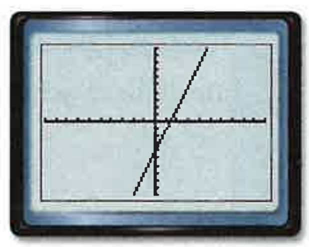
KEYSTROKES: [Y=] 3 [X,T,θ,n] [−] 4

The equals sign appears shaded for graphs that are selected to be displayed.

Step 2 Graph the equation in the standard viewing window.

- Graph the selected equation.

KEYSTROKES: [ZOOM] 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Sometimes a complete graph is not displayed using the standard viewing window. A **complete graph** includes all of the important characteristics of the graph on the screen including the origin and the *x*- and *y*-intercepts. Note that the graph above is a complete graph because all of these points are visible.

When a complete graph is not displayed using the standard viewing window, you will need to change the viewing window to accommodate these important features. Use what you have learned about intercepts to help you choose an appropriate viewing window.

(continued on the next page)

Graphing Technology Lab
Graphing Linear Functions *Continued*

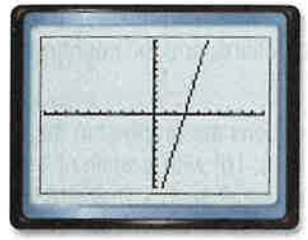
Activity 2 Graph a Complete Graph

Graph $y = 5x - 14$.

Step 1 Enter the equation in the Y= list and graph in the standard viewing window.

- Clear the previous equation from the Y= list. Then enter the new equation and graph.

KEYSTROKES: [Y=] [CLEAR] 5 [X,T,θ,n] [−] 14 [ZOOM] 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 2 Modify the viewing window and graph again.

- The origin and the x-intercept are displayed in the standard viewing window. But notice that the y-intercept is outside of the viewing window.

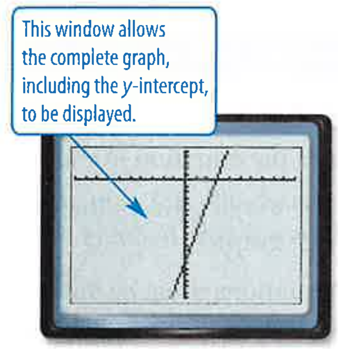
Find the y-intercept.

$y = 5x - 14$ Original equation

$= 5(0) - 14$ Replace x with 0.

$= -14$ Simplify.

Since the y-intercept is -14, choose a viewing window that includes a number less than -14. The window $[-10, 10]$ by $[-20, 5]$ with a scale of 1 on each axis is a good choice.



[−10, 10] scl: 1 by [−20, 5] scl: 1

KEYSTROKES: [WINDOW] −10 [ENTER] 10 [ENTER] 1 [ENTER] −20 [ENTER] 5 [ENTER] 1 [GRAPH]

Exercises

Use a graphing calculator to graph each equation in the standard viewing window. Sketch the result.

1. $y = x + 5$ **2.** $y = 5x + 6$ **3.** $y = 9 - 4x$

4. $3x + y = 5$ **5.** $x + y = -4$ **6.** $x - 3y = 6$

CCSS SENSE-MAKING **Graph each equation in the standard viewing window. Determine whether the graph is complete. If the graph is not complete, adjust the viewing window and graph the equation again.**

7. $y = 4x + 7$ **8.** $y = 9x - 5$ **9.** $y = 2x - 11$

10. $4x - y = 16$ **11.** $6x + 2y = 23$ **12.** $x + 4y = -36$

Consider the linear equation $y = 3x + b$.

13. Choose several different positive and negative values for b. Graph each equation in the standard viewing window.

14. For which values of b is the complete graph in the standard viewing window?

15. How is the value of b related to the y-intercept of the graph of $y = 3x + b$?

EXPLORE

3-3 Algebra Lab
Rate of Change of a Linear Function

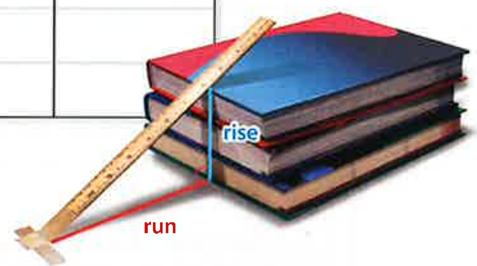
In mathematics, you can measure the steepness of a line using a ratio.

CCSS Common Core State Standards
Content Standards
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F.LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Set Up the Lab

- Stack three books on your desk.
- Lean a ruler on the books to create a ramp.
- Tape the ruler to the desk.
- Measure the rise and the run. Record your data in a table like the one at the right.
- Calculate and record the ratio $\frac{\text{rise}}{\text{run}}$.

rise	run	$\frac{\text{rise}}{\text{run}}$



Activity

Step 1

Move the books to make the ramp steeper. Measure and record the rise and the run. Calculate and record $\frac{\text{rise}}{\text{run}}$.

Step 2

Add books to the stack to make the ramp even steeper. Measure, calculate, and record your data in the table.

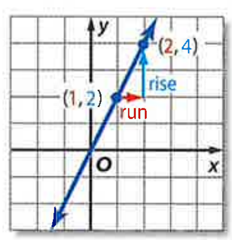
Analyze the Results

1. Examine the ratios you recorded. How did they change as the ramp became steeper?

2. **MAKE A PREDICTION** Suppose you want to construct a skateboard ramp that is not as steep as the one shown at the right. List three different sets of $\frac{\text{rise}}{\text{run}}$ measurements that will result in a less steep ramp. Verify your predictions by calculating the ratio $\frac{\text{rise}}{\text{run}}$ for each ramp.

$m = \frac{18}{24} = \frac{3}{4}$

3. Copy the coordinate graph shown and draw a line through the origin with a $\frac{\text{rise}}{\text{run}}$ ratio greater than the original line. Then draw a line through the origin with a ratio less than that of the original line. Explain using the words *rise* and *run* why the lines you drew have a ratio greater or less than the original line.



4. We have seen what happens on the graph as the $\frac{\text{rise}}{\text{run}}$ ratio gets closer to zero. What would you predict will happen when the ratio is zero? Explain your reasoning. Give an example to support your prediction.

LESSON 3-3

Rate of Change and Slope

::Then	::Now	::Why?
• You graphed ordered pairs in the coordinate plane.	• **1** Use rate of change to solve problems. **2** Find the slope of a line.	• The Daredevil Drop at Wet 'n Wild Emerald Pointe in Greensboro, North Carolina, is a thrilling ride that drops you 76 feet down a steep water chute. A *rate of change* of the ride might describe the distance a rider has fallen over a length of time.

NewVocabulary
rate of change
slope

Common Core State Standards

Content Standards

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Rate of Change

Rate of change is a ratio that describes, on average, how much one quantity changes with respect to a change in another quantity.

KeyConcept Rate of Change

If x is the independent variable and y is the dependent variable, then

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}.$$

Real-World Example 1 Find Rate of Change

PT

ENTERTAINMENT **Use the table to find the rate of change. Then explain its meaning.**

Number of Computer Games	Total Cost ($)
x	y
2	78
4	156
6	234

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} \begin{matrix} \leftarrow \text{dollars} \\ \leftarrow \text{games} \end{matrix}$$

$$= \frac{\text{change in cost}}{\text{change in number of games}}$$

$$= \frac{156 - 78}{4 - 2}$$

$$= \frac{78}{2} \text{ or } \frac{39}{1}$$

The rate of change is $\frac{39}{1}$ or 39. This means that the cost per game is $39.

GuidedPractice

1. REMODELING The table shows how the tiled surface area changes with the number of floor tiles.

Number of Floor Tiles	Area of Tiled Surface (in^2)
x	y
3	48
6	96
9	144

A. Find the rate of change.

B. Explain the meaning of the rate of change.

Nova Stock/SuperStock

So far, you have seen rates of change that are *constant*. Many real-world situations involve rates of change that are not constant.

Real-World Example 2 Compare Rates of Change

AMUSEMENT PARKS **The graph shows the number of people who visited U.S. theme parks in recent years.**

a. Find the rates of change for 2000–2002 and 2002–2004.

Source: *International Association of Amusement Parks and Attractions*

2000–2002:

$$\frac{\text{change in attendance}}{\text{change in time}} = \frac{324 - 317}{2002 - 2000} \begin{matrix}\leftarrow \text{people} \\ \leftarrow \text{years}\end{matrix} \quad \text{Substitute.}$$

$$= \frac{7}{2} \text{ or } 3.5 \quad \text{Simplify.}$$

Over this 2-year period, attendance increased by 7 million, for a rate of change of 3.5 million per year.

2002–2004:

$$\frac{\text{change in attendance}}{\text{change in time}} = \frac{325 - 324}{2004 - 2002} \quad \text{Substitute.}$$

$$= \frac{1}{2} \text{ or } 0.5 \quad \text{Simplify.}$$

Over this 2-year period, attendance increased by 1 million, for a rate of change of 0.5 million per year.

b. Explain the meaning of the rate of change in each case.

For 2000–2002, on average, 3.5 million more people went to a theme park each year than the last.

For 2002–2004, on average, 0.5 million more people attended theme parks each year than the last.

c. How are the different rates of change shown on the graph?

There is a greater vertical change for 2000–2002 than for 2002–2004. Therefore, the section of the graph for 2000–2002 is steeper.

Guided Practice

2. Refer to the graph above. Without calculating, find the 2-year period that has the least rate of change. Then calculate to verify your answer.

Study Tip

CCSS Reasoning A positive rate of change indicates an increase over time. A negative rate of change indicates that a quantity is decreasing.

A rate of change is constant for a function when the rate of change is the same between any pair of points on the graph of the function. Linear functions have a constant rate of change.

Example 3 Constant Rates of Change

Determine whether each function is linear. Explain.

a.

x	y
1	−6
4	−8
7	−10
10	−12
13	−14

x	y	rate of change
1	−6	
4	−8	$\frac{-8-(-6)}{4-1}$ or $-\frac{2}{3}$
7	−10	$\frac{-10-(-8)}{7-4}$ or $-\frac{2}{3}$
10	−12	$\frac{-12-(-10)}{10-7}$ or $-\frac{2}{3}$
13	−14	$\frac{-14-(-12)}{13-10}$ or $-\frac{2}{3}$

The rate of change is constant. Thus, the function is linear.

b.

x	y
−3	10
−1	12
1	16
3	18
5	22

x	y	rate of change
−3	10	
−1	12	$\frac{12-10}{-1-(-3)}$ or 1
1	16	$\frac{16-12}{1-(-1)}$ or 2
3	18	$\frac{18-16}{3-1}$ or 1
5	22	$\frac{22-18}{5-3}$ or 2

This rate of change is not constant. Thus, the function is not linear.

StudyTip

Linear or Nonlinear Function? Notice that the changes in x and y are not the same. For the rate of change to be linear, the change in x-values must be constant and the change in y-values must be constant.

GuidedPractice

3A.

x	y
−3	11
−2	15
−1	19
1	23
2	27

3B.

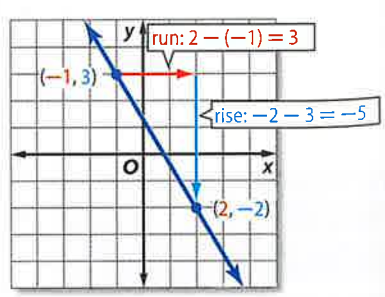
x	y
12	−4
9	1
6	6
3	11
0	16

2 Find Slope

The **slope** of a nonvertical line is the ratio of the change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move from one point to another.

It can be used to describe a rate of change. Slope describes how steep a line is. The greater the absolute value of the slope, the steeper the line.

The graph shows a line that passes through (−1, 3) and (2, −2).



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$= \frac{-2-3}{2-(-1)} \text{ or } -\frac{5}{3}$$

So, the slope of the line is $-\frac{5}{3}$.

Because a linear function has a constant rate of change, any two points on a nonvertical line can be used to determine its slope.

ReadingMath

Subscripts y_1 is read as *y sub one* and x_2 is read as *x sub two*. The 1 and 2 are subscripts and refer to the first and second point to which the *x*- and *y*-values correspond.

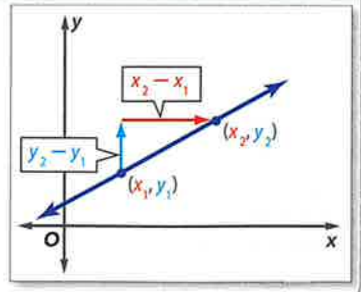
KeyConcept Slope

Words The slope of a nonvertical line is the ratio of the rise to the run.

Symbols The slope *m* of a nonvertical line through any two points, (x_1, y_1) and (x_2, y_2), can be found as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \begin{matrix} \leftarrow \text{change in } y \\ \leftarrow \text{change in } x \end{matrix}$$

Graph



The slope of a line can be positive, negative, zero, or undefined. If the line is not horizontal or vertical, then the slope is either positive or negative.

WatchOut!

Order Be careful not to transpose the order of the *x*-values or the *y*-values.

Example 4 Positive, Negative and Zero Slope

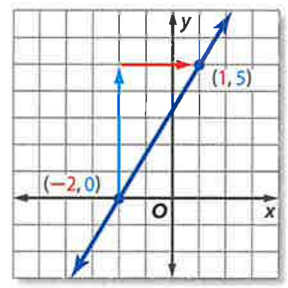
Find the slope of a line that passes through each pair of points.

a. (−2, 0) and (1, 5)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — $\frac{\text{rise}}{\text{run}}$

$= \frac{5 - 0}{1 - (-2)}$ — $(-2, 0) = (x_1, y_1)$ and $(1, 5) = (x_2, y_2)$

$= \frac{5}{3}$ — Simplify.

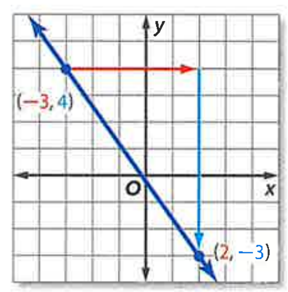


b. (−3, 4) and (2, −3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — $\frac{\text{rise}}{\text{run}}$

$= \frac{-3 - 4}{2 - (-3)}$ — $(-3, 4) = (x_1, y_1)$ and $(2, -3) = (x_2, y_2)$

$= \frac{-7}{5}$ or $-\frac{7}{5}$ — Simplify.

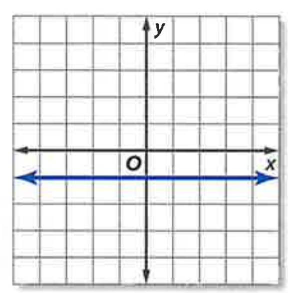


c. (−3, −1) and (2, −1)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — $\frac{\text{rise}}{\text{run}}$

$= \frac{-1 - (-1)}{2 - (-3)}$ — Substitute.

$= \frac{0}{5}$ or 0 — Simplify.



GuidedPractice

Find the slope of the line that passes through each pair of points.

4A. (3, 6), (4, 8) **4B.** (−4, −2), (0, −2) **4C.** (−4, 2), (−2, 10)

4D. (6, 7), (−2, 7) **4E.** (−2, 2), (−6, 4) **4F.** (4, 3), (−1, 11)

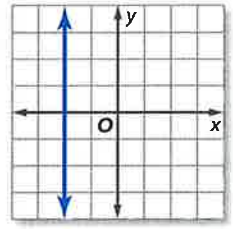
Example 5 Undefined Slope

Find the slope of the line that passes through (−2, 4) and (−2, −3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{\text{rise}}{\text{run}}$$

$$= \frac{-3 - 4}{-2 - (-2)} \qquad \text{Substitute.}$$

$$= \frac{-7}{0} \text{ or undefined} \qquad \text{Simplify.}$$



StudyTip

Zero and Undefined Slopes If the change in *y*-values is 0, then the graph of the line is horizontal. If the change in *x*-values is 0, then the slope is undefined. This graph is a vertical line.

GuidedPractice

Find the slope of the line that passes through each pair of points.

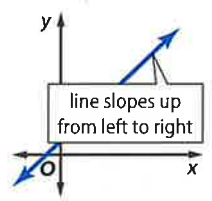
5A. (6, 3), (6, 7)

5B. (−3, 2), (−3, −1)

The graphs of lines with different slopes are summarized below.

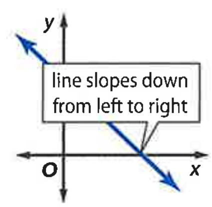
ConceptSummary Slope

positive slope



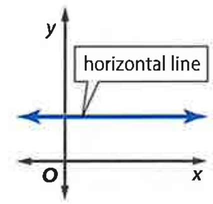
The function values are increasing over the entire domain.

negative slope



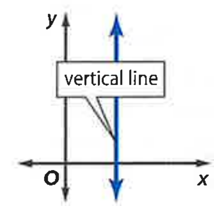
The function values are decreasing over the entire domain.

slope of 0



The function values are constant over the entire domain.

undefined slope



The relation is not a function.

Example 6 Find Coordinates Given the Slope

Find the value of r so that the line through (1, 4) and (−5, r) has a slope of $\frac{1}{3}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$\frac{1}{3} = \frac{r - 4}{-5 - 1} \qquad \text{Let } (1, 4) = (x_1, y_1) \text{ and } (-5, r) = (x_2, y_2).$$

$$\frac{1}{3} = \frac{r - 4}{-6} \qquad \text{Subtract.}$$

$$3(r - 4) = 1(-6) \qquad \text{Find the cross products.}$$

$$3r - 12 = -6 \qquad \text{Distributive Property}$$

$$3r = 6 \qquad \text{Add 12 to each side and simplify.}$$

$$r = 2 \qquad \text{Divide each side by 3 and simplify.}$$

So, the line goes through (−5, 2).

GuidedPractice

Find the value of r so the line that passes through each pair of points has the given slope.

6A. (−2, 6), (r, −4); $m = -5$

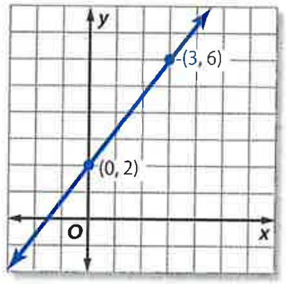
6B. (r, −6), (5, −8); $m = -8$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 Find the rate of change represented in each table or graph.

1.



2.

x	y
3	−6
5	2
7	10
9	18
11	26

Example 2

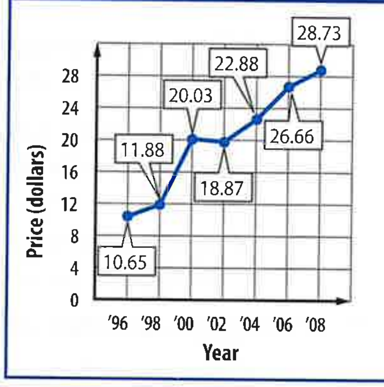
3. CCSS **SENSE-MAKING** Refer to the graph at the right.

a. Find the rate of change of prices from 2006 to 2008. Explain the meaning of the rate of change.

b. Without calculating, find a two-year period that had a greater rate of change than 2006–2008. Explain.

c. Between which years would you guess the new stadium was built? Explain your reasoning.

Houston Astros Tickets Average Price



Source: *Team Marketing Report*

Example 3 Determine whether each function is linear. Write *yes* or *no*. Explain.

4.

x	−7	−4	−1	2	5
y	5	4	3	2	1

5.

x	8	12	16	20	24
y	7	5	3	0	−2

Examples 4–5 Find the slope of the line that passes through each pair of points.

6. (5, 3), (6, 9)

7. (−4, 3), (−2, 1)

8. (6, −2), (8, 3)

9. (1, 10), (−8, 3)

10. (−3, 7), (−3, 4)

11. (5, 2), (−6, 2)

Example 6 Find the value of r so the line that passes through each pair of points has the given slope.

12. $(-4, r), (-8, 3), m = -5$

13. $(5, 2), (-7, r), m = \frac{5}{6}$

Practice and Problem Solving

Extra Practice is on page R3.

Example 1 Find the rate of change represented in each table or graph.

14.

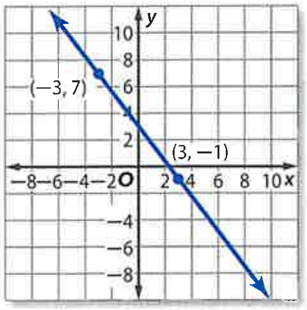
x	y
5	2
10	3
15	4
20	5

15.

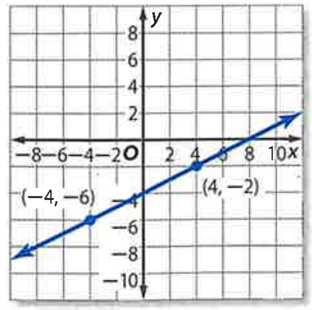
x	y
1	15
2	9
3	3
4	−3

Example 1 **Find the rate of change represented in each table or graph.**

16.



17.



Example 2 **18. SPORTS** What was the annual rate of change from 2004 to 2008 for women participating in collegiate lacrosse? Explain the meaning of the rate of change.

Year	Number of Women
2004	5545
2008	6830

19. RETAIL The average retail price in the spring of 2009 for a used car is shown in the table at the right.

Age (years)	Value ($)
2	17,378
3	16,157

a. Write a linear function to model the price of the car with respect to age.

b. Interpret the meaning of the slope of the line.

c. Assuming a constant rate of change predict the average retail price for a 7-year-old car.

Example 3 **Determine whether each function is linear. Write *yes* or *no*. Explain.**

20.

x	4	2	0	−2	−4
y	−1	1	3	5	7

21.

x	−7	−5	−3	−1	0
y	11	14	17	20	23

22.

x	−0.2	0	0.2	0.4	0.6
y	0.7	0.4	0.1	0.3	0.6

23.

x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

Examples 4–5 **Find the slope of the line that passes through each pair of points.**

24. (4, 3), (−1, 6)
25. (8, −2), (1, 1)
26. (2, 2), (−2, −2)
27. (6, −10), (6, 14)
28. (5, −4), (9, −4)
29. (11, 7), (−6, 2)
30. (−3, 5), (3, 6)
31. (−3, 2), (7, 2)
32. (8, 10), (−4, −6)
33. (−8, 6), (−8, 4)
34. (−12, 15), (18, −13)
35. (−8, −15), (−2, 5)

Example 6 **Find the value of r so the line that passes through each pair of points has the given slope.**

36. (12, 10), (−2, r), $m = -4$
37. (r, −5), (3, 13), $m = 8$
38. (3, 5), (−3, r), $m = \frac{3}{4}$
39. (−2, 8), (r, 4), $m = -\frac{1}{2}$

CCSS **TOOLS** **Use a ruler to estimate the slope of each object.**

40.

41.

42. DRIVING When driving up a certain hill, you rise 15 feet for every 1000 feet you drive forward. What is the slope of the road?

Find the slope of the line that passes through each pair of points.

43.

x	y
4.5	−1
5.3	2

44.

x	y
0.75	1
0.75	−1

45.

x	y
$2\frac{1}{2}$	$-1\frac{1}{2}$
$-\frac{1}{2}$	$\frac{1}{2}$

46. MULTIPLE REPRESENTATIONS In this problem, you will investigate why the slope of a line through any two points on that line is constant.

a. Visual Sketch a line ℓ that contains points A, B, A' and B' on a coordinate plane.

b. Geometric Add segments to form right triangles ABC and $A'B'C'$ with right angles at C and C'. Describe $\overline{AC}$ and $\overline{A'C'}$, and $\overline{BC}$ and $\overline{B'C'}$.

c. Verbal How are triangles ABC and $A'B'C'$ related? What does that imply for the slope between any two distinct points on line ℓ?

47. BASKETBALL The table shown below shows the average points per game (PPG) Michael Redd has scored in each of his first 9 seasons with the NBA's Milwaukee Bucks.

Season	1	2	3	4	5	6	7	8	9
PPG	2.2	11.4	15.1	21.7	23.0	25.4	26.7	22.7	21.2

a. Make a graph of the data. Connect each pair of adjacent points with a line.

b. Use the graph to determine in which period Michael Redd's PPG increased the fastest. Explain your reasoning.

c. Discuss the difference in the rate of change from season 1 through season 4, from season 4 through season 7, from season 7 through season 9.

H.O.T. Problems Use Higher-Order Thinking Skills

48. REASONING Why does the Slope Formula not work for vertical lines? Explain.

49. OPEN ENDED Use what you know about rate of change to describe the function represented by the table.

Time (wk)	Height of Plant (in.)
4	9.0
6	13.5
8	18.0

50. CHALLENGE Find the value of d so the line that passes through (a, b) and (c, d) has a slope of $\frac{1}{2}$.

51. WRITING IN MATH Explain how the rate of change and slope are related and how to find the slope of a line.

52. CCSS ARGUMENTS Kyle and Luna are finding the value of a so the line that passes through $(10, a)$ and $(-2, 8)$ has a slope of $\frac{1}{4}$. Is either of them correct? Explain.

Kyle

$$\frac{-2 - 10}{8 - a} = \frac{1}{4}$$

$$1(8 - a) = 4(-12)$$

$$8 - a = -48$$

$$a = 56$$

Luna

$$\frac{8 - a}{-2 - 10} = \frac{1}{4}$$

$$4(8 - a) = 1(-12)$$

$$32 - 4a = -12$$

$$a = 11$$

Standardized Test Practice

53. The cost of prints from an online photo processor is given by $C(p) = 29.99 + 0.13p$. \$29.99 is the cost of the membership, and p is the number of 4-inch by 6-inch prints. What does the slope represent?

A cost per print

B cost of the membership

C cost of the membership and 1 print

D number of prints

54. Danita bought a computer for \$1200 and its value depreciated linearly. After 2 years, the value was \$250. What was the amount of yearly depreciation?

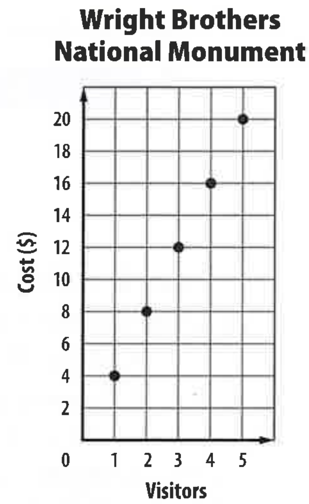
F \$950

G \$475

H \$250

J \$225

55. SHORT RESPONSE The graph represents how much the Wright Brothers National Monument charges visitors. How much does the park charge each visitor?



56. PROBABILITY At a gymnastics camp, 1 gymnast is chosen at random from each team. The Flipstars Gymnastics Team consists of 5 eleven-year-olds, 7 twelve-year-olds, 10 thirteen-year-olds, and 8 fourteen-year-olds. What is the probability that the age of the gymnast chosen is an odd number?

A $\frac{1}{30}$ **B** $\frac{1}{15}$ **C** $\frac{1}{2}$ **D** $\frac{3}{5}$

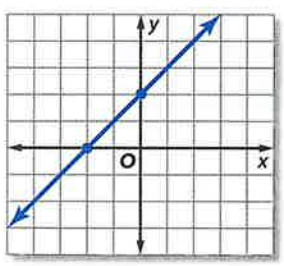
Spiral Review

Solve each equation by graphing. (Lesson 3-2)

57. $3x + 18 = 0$ **58.** $8x - 32 = 0$ **59.** $0 = 12x - 48$

Find the x- and y-intercepts of the graph of each linear function. (Lesson 3-1)

60.



61.

x	y
−3	−4
−2	−2
−1	0
0	2
1	4

62. HOMECOMING Dance tickets are \$9 for one person and \$15 for two people. If a group of seven students wishes to go to the dance, write and solve an equation that would represent the least expensive price p of their tickets. (Lesson 1-3)

Skills Review

Find each quotient.

63. $8 \div \frac{2}{3}$ **64.** $\frac{3}{8} \div \frac{1}{4}$ **65.** $\frac{5}{8} \div 2$

66. $\frac{12 \cdot 6}{9}$ **67.** $\frac{2 \cdot 15}{6}$ **68.** $\frac{18 \cdot 5}{15}$

CHAPTER 3

Mid-Chapter Quiz

Lessons 3-1 through 3-3

Determine whether each equation is a linear equation. Write *yes* or *no*. If yes, write the equation in standard form. (Lesson 3-1)

1. $y = -4x + 3$
2. $x^2 + 3y = 8$
3. $\frac{1}{4}x - \frac{3}{4}y = -1$

Graph each equation using the *x*- and *y*-intercepts. (Lesson 3-1)

4. $y = 3x - 6$
5. $2x + 5y = 10$

Graph each equation by making a table. (Lesson 3-1)

6. $y = -2x$
7. $x = 8 - y$

8. **BOOK SALES** The equation $5x + 12y = 240$ describes the total amount of money collected when selling x paperback books at $5 per book and y hardback books at $12 per book. Graph the equation using the *x*- and *y*-intercepts. (Lesson 3-1)

Find the root of each equation. (Lesson 3-2)

9. $x + 8 = 0$
10. $4x - 24 = 0$
11. $18 + 8x = 0$
12. $\frac{3}{5}x - \frac{1}{2} = 0$

Solve each equation by graphing. (Lesson 3-2)

13. $-5x + 35 = 0$
14. $14x - 84 = 0$
15. $118 + 11x = -3$

16. **MULTIPLE CHOICE** The function $y = -15 + 3x$ represents the outside temperature, in degrees Fahrenheit, in a small Alaskan town where x represents the number of hours after midnight. The function is accurate for x values representing midnight through 4:00 P.M. Find the zero of this function. (Lesson 3-2)

A 0
B 3
C 5
D −15

17. Find the rate of change represented in the table. (Lesson 3-3)

x	y
1	2
4	6
7	10
10	14

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

18. (2, 6), (4, 12)
19. (1, 5), (3, 8)
20. (−3, 4), (2, −6)
21. $\left(\frac{1}{3}, \frac{3}{4}\right), \left(\frac{2}{3}, \frac{1}{4}\right)$

22. **MULTIPLE CHOICE** Find the value of *r* so the line that passes through the pair of points has the given slope. (Lesson 3-3)

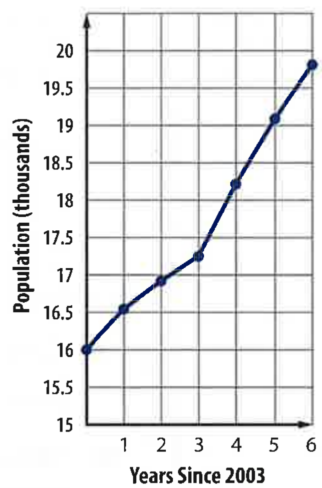
$$(-4, 8), (r, 12), m = \frac{4}{3}$$

F −4
G −1
H 0
J 3

23. Find the slope of the line that passes through the pair of points. (Lesson 3-3)

x	y
2.6	−2
3.1	4

24. **POPULATION GROWTH** The graph shows the population growth in Heckertsville since 2003. (Lesson 3-3)



a. For which time period is the rate of change the greatest?

b. Explain the meaning of the slope from 2003 to 2009.

LESSON 3-4 Direct Variation

:: Then

- You found rates of change of linear functions.

:: Now

1. Write and graph direct variation equations.
2. Solve problems involving direct variation.

:: Why?

- Bianca is saving her money to buy a designer purse that costs $295. To help raise the money, she charges $8 per hour to babysit her neighbors' child. The slope of the line that represents the amount of money Bianca earns is 8, and the rate of change is constant.

NewVocabulary

direct variation
constant of variation
constant of proportionality

Common Core State Standards

Content Standards

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices

1 Make sense of problems and persevere in solving them.
6 Attend to precision.

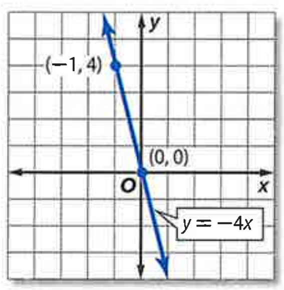
1 Direct Variation Equations

A **direct variation** is described by an equation of the form $y = kx$, where $k \neq 0$. The equation $y = kx$ illustrates a constant rate of change, and k is the **constant of variation**, also called the **constant of proportionality**.

Example 1 Slope and Constant of Variation

Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

a.



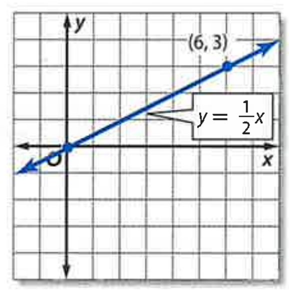
The constant of variation is −4.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{4 - 0}{-1 - 0}$$ $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (-1, 4)$

$$= -4$$ The slope is −4.

b.



The constant of variation is $\frac{1}{2}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{3 - 0}{6 - 0}$$ $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (6, 3)$

$$= \frac{1}{2}$$ The slope is $\frac{1}{2}$.

GuidedPractice

1A. Name the constant of variation for $y = \frac{1}{4}x$. Then find the slope of the line that passes through (0, 0) and (4, 1), two points on the line.

1B. Name the constant of variation for $y = -2x$. Then find the slope of the line that passes through (0, 0) and (1, −2), two points on the line.

The slope of the graph of $y = kx$ is k. Since $0 = k(0)$, the graph of $y = kx$ always passes through the origin. Therefore the x- and y-intercepts are zero.

Example 2 Graph a Direct Variation

Graph $y = -6x$.

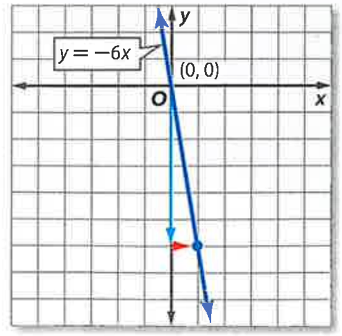
Step 1 Write the slope as a ratio.

$-6 = \frac{-6}{1}$ $\frac{\text{rise}}{\text{run}}$

Step 2 Graph (0, 0).

Step 3 From the point (0, 0), move down 6 units and right 1 unit. Draw a dot.

Step 4 Draw a line containing the points.



> **StudyTip**
> **Constant of Variation** A line with a positive constant of variation will go up from left to right and a line with a negative constant of variation will go down from left to right.

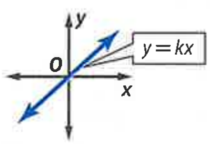
GuidedPractice

2A. $y = 6x$ **2B.** $y = \frac{2}{3}x$ **2C.** $y = -5x$ **2D.** $y = -\frac{3}{4}x$

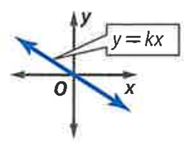
The graphs of all direct variation equations share some common characteristics.

ConceptSummary Direct Variation Graphs

- Direct variation equations are of the form $y = kx$, where $k \neq 0$.
- The graph of $y = kx$ always passes through the origin.
- The slope is positive if $k > 0$.



- The slope is negative if $k < 0$.



If the relationship between the values of y and x can be described by a direct variation equation, then we say that y varies directly as x.

Example 3 Write and Solve a Direct Variation Equation

Suppose y varies directly as x, and $y = 72$ when $x = 8$.

a. Write a direct variation equation that relates x and y.

$y = kx$ Direct variation formula

$72 = k(8)$ Replace y with 72 and x with 8.

$9 = k$ Divide each side by 8.

Therefore, the direct variation equation is $y = 9x$.

b. Use the direct variation equation to find x when $y = 63$.

$y = 9x$ Direct variation formula

$63 = 9x$ Replace y with 63.

$7 = x$ Divide each side by 9.

Therefore, $x = 7$ when $y = 63$.

GuidedPractice

3. Suppose y varies directly as x, and $y = 98$ when $x = 14$. Write a direct variation equation that relates x and y. Then find y when $x = -4$.

2 Direct Variation Problems

One of the most common applications of direct variation is the formula $d = rt$. Distance d varies directly as time t, and the rate r is the constant of variation.

Real-WorldLink

In 2006, domestic airlines transported over 660 million passengers an average distance of 724 miles per flight.

Source: Bureau of Transportation Statistics

Real-World Example 4 Estimate Using Direct Variation

TRAVEL **The distance a jet travels varies directly as the number of hours it flies. A jet traveled 3420 miles in 6 hours.**

a. Write a direct variation equation for the distance d flown in time t.

Words	Distance	equals	rate	times	time.
Variable	Let r = rate.				
Equation	3420	=	r	×	6

Solve for the rate.

$3420 = r(6)$ Original equation

$\frac{3420}{6} = \frac{r(6)}{6}$ Divide each side by 6.

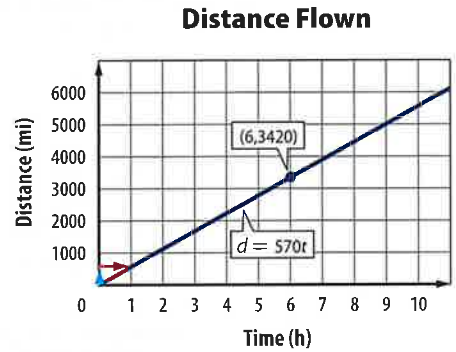
$570 = r$ Simplify.

Therefore, the direct variation equation is $d = 570t$. The airliner flew at a rate of 570 miles per hour.

b. Graph the equation.

The graph of $d = 570t$ passes through the origin with slope 570.

$m = \frac{570}{1}$ $\frac{\text{rise}}{\text{run}}$



Problem-SolvingTip

CCSS Precision Notice that the question asks for an estimate, not an exact answer.

c. Estimate how many hours it will take for an airliner to fly 6500 miles.

$d = 570t$ Original equation

$6500 = 570t$ Replace d with 6500.

$\frac{6500}{570} = \frac{570t}{570}$ Divide each side by 570.

$t \approx 11.4$ Simplify.

It would take the airliner approximately 11.4 hours to fly 6500 miles.

GuidedPractice

4. HOT-AIR BALLOONS A hot-air balloon's height varies directly as the balloon's ascent time in minutes.

A. Write a direct variation for the distance d ascended in time t.

B. Graph the equation.

C. Estimate how many minutes it would take to ascend 2100 feet.

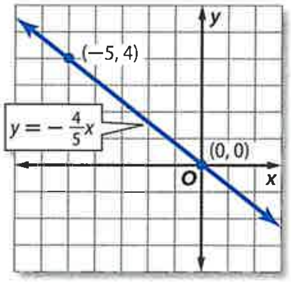
D. About how many minutes would it take to ascend 3500 feet?

Check Your Understanding

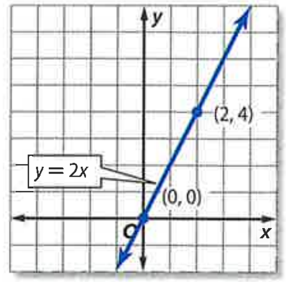
● = Step-by-Step Solutions begin on page R13.

Example 1 **Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.**

1.



2.



Example 2 **Graph each equation.**

3. $y = -x$

4. $y = \frac{3}{4}x$

5. $y = -8x$

6. $y = -\frac{8}{5}$

Example 3 **Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.**

7. If $y = 15$ when $x = 12$, find y when $x = 32$.

8. If $y = -11$ when $x = 6$, find x when $y = 44$.

Example 4

9. CCSS **REASONING** You find that the number of messages you receive on your message board varies directly as the number of messages you post. When you post 5 messages, you receive 12 messages in return.

a. Write a direct variation equation relating your posts to the messages received. Then graph the equation.

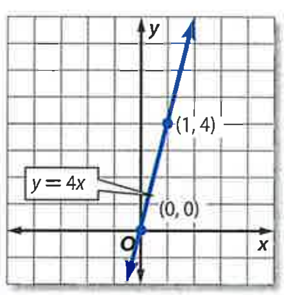
b. Find the number of messages you need to post to receive 96 messages.

Practice and Problem Solving

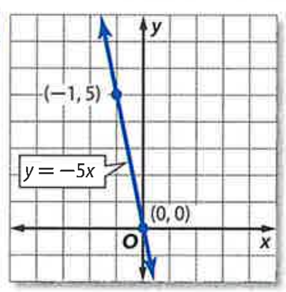
Extra Practice is on page R3.

Example 1 **Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.**

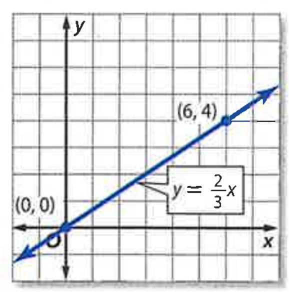
10.



11.



12.



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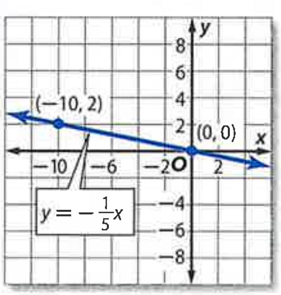

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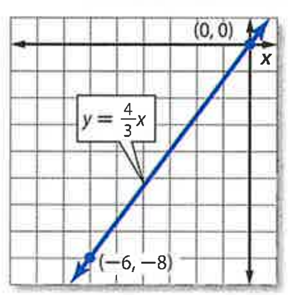

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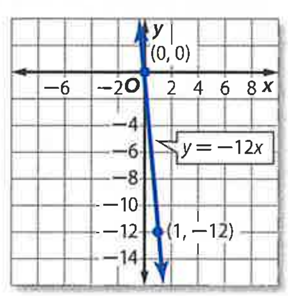


Example 2 **Graph each equation.**

16. $y = 10x$ **17.** $y = -7x$ **18.** $y = x$ **19.** $y = \frac{7}{6}x$

20. $y = \frac{1}{6}x$ **21.** $y = \frac{2}{9}x$ **22.** $y = \frac{6}{5}x$ **23.** $y = -\frac{5}{4}x$

Example 3 **Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.**

24. If $y = 6$ when $x = 10$, find x when $y = 18$.

25. If $y = 22$ when $x = 8$, find y when $x = -16$.

26. If $y = 4\frac{1}{4}$ when $x = \frac{3}{4}$, find y when $x = 4\frac{1}{2}$.

27. If $y = 12$ when $x = \frac{6}{7}$, find x when $y = 16$.

Example 4 **28.** **SPORTS** The distance a golf ball travels at an altitude of 7000 feet varies directly with the distance the ball travels at sea level, as shown.

Hitting a Golf Ball		
Altitude (ft)	0 (sea level)	7000
Distance (yd)	200	210

a. Write and graph an equation that relates the distance a golf ball travels at an altitude of 7000 feet y with the distance at sea level x.

b. What would be a person's average driving distance at 7000 feet if his average driving distance at sea level is 180 yards?

29. **FINANCIAL LITERACY** Depreciation is the decline in a car's value over the course of time. The table below shows the values of a car with an average depreciation.

Age of Car (years)	1	2	3	4	5
Value (dollars)	12,000	10,200	8400	6600	4800

a. Write an equation that relates the age x of the car to the value y that it lost after each year.

b. Find the age of the car if the value is $300.

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.

30. If $y = 3.2$ when $x = 1.6$, find y when $x = 19$.

31. If $y = 15$ when $x = \frac{3}{4}$, find x when $y = 25$.

32. If $y = 4.5$ when $x = 2.5$, find y when $x = 12$.

33. If $y = -6$ when $x = 1.6$, find y when $x = 8$.

CCSS **SENSE-MAKING** **Certain endangered species experience cycles in their populations as shown in the graph at the right. Match each animal below to one of the colored lines in the graph.**

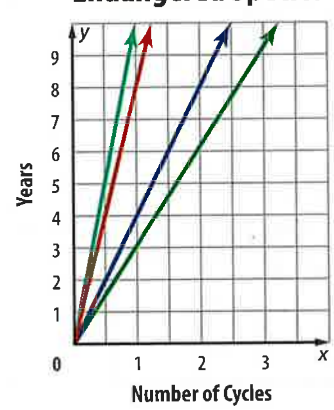
34. red grouse, 8 years per cycle

35. voles, 3 years per cycle

36. lemmings, 4 years per cycle

37. lynx, 10 years per cycle

Population Cycles of Endangered Species



In Exercises 38–40, write and graph a direct variation equation that relates the variables.

38. PHYSICAL SCIENCE The weight W of an object is 9.8 m/s^2 times the mass of the object m.

39. MUSIC Music downloads are \$0.99 per song. The total cost of d songs is T.

40. GEOMETRY The circumference of a circle C is approximately 3.14 times the diameter d.

41. MULTIPLE REPRESENTATIONS In this problem, you will investigate the family of direct variation functions.

a. **Graphical** Graph $y = x$, $y = 3x$, and $y = 5x$ on the same coordinate plane.

b. **Algebraic** Describe the relationship among the constant of variation, the slope of the line, and the rate of change of the graph.

c. **Verbal** Make a conjecture about how you can determine without graphing which of two direct variation equations has the steeper graph.

42. TRAVEL A map of North Carolina is scaled so that 3 inches represents 93 miles. How far apart are Raleigh and Charlotte if they are 1.8 inches apart on the map?

43. INTERNET A company will design and maintain a Web site for your company for \$9.95 per month. Write a direct variation equation to find the total cost C for having a Web page for n months.

44. BASEBALL Before their first game, high school student Todd McCormick warmed all 5200 seats in a new minor league stadium, by sitting in every seat. He started at 11:50 A.M. and finished around 3 P.M.

a. Write a direct variation equation relating the number of seats to time. What is the meaning of the constant of variation in this situation?

b. About how many seats had Todd sat in by 1:00 P.M.?

c. How long would you expect it to take Todd to sit in all of the seats at a major league stadium with more than 40,000 seats?

H.O.T. Problems Use Higher-Order Thinking Skills

45. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong. Explain.

$9 = rt$	$9a = 0$	$z = \frac{1}{9}x$	$w = \frac{9}{t}$

46. REASONING How are the constant of variation and the slope related in a direct variation equation? Explain your reasoning.

47. OPEN ENDED Model a real-world situation using a direct variation equation. Graph the equation and describe the rate of change.

48. CCSS STRUCTURE Suppose y varies directly as x. If the value of x is doubled, then the value of y is also *always*, *sometimes* or *never* doubled. Explain your reasoning.

49. ERROR ANALYSIS Eddy says the slope between any two points on the graph of a direct variation equation $y = kx$ is $\frac{1}{k}$. Adelle says the slope depends on the points chosen. Is either of them correct? Explain.

50. WRITING IN MATH How can you identify the graph of a direct variation equation?

Standardized Test Practice

51. Patricia pays $1.19 each to download songs to her digital media player. If n is the number of downloaded songs, which equation represents the cost C in dollars?

A $C = 1.19n$

B $n = 1.19C$

C $C = 1.19 \div n$

D $C = n + 1.19$

52. Suppose that y varies directly as x, and $y = 8$ when $x = 6$. What is the value of y when $x = 8$?

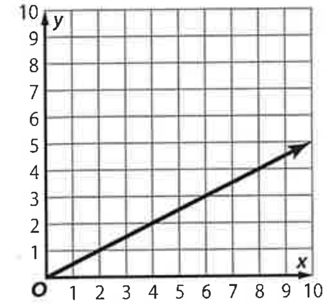
F 6

G 12

H $10\frac{2}{3}$

J 16

53. What is the relationship between the input (x) and output (y)?



A The output is two more than the input.

B The output is two less than the input.

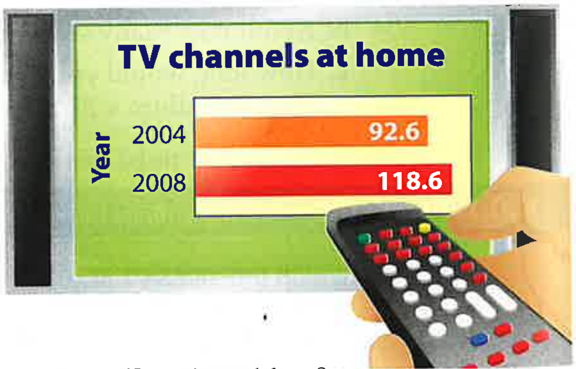
C The output is twice the input.

D The output is half the input.

54. **SHORT RESPONSE** A telephone company charges $40 per month plus $0.07 per minute. How much would a month of service cost a customer if the customer talked for 200 minutes?

Spiral Review

55. **TELEVISION** The graph shows the average number of television channels American households receive. What was the annual rate of change from 2004 to 2008? Explain the meaning of the rate of change. (Lesson 3-3)



Solve each equation by graphing. (Lesson 3-2)

56. $0 = 18 - 9x$

57. $2x + 14 = 0$

58. $-4x + 16 = 0$

59. $-5x - 20 = 0$

60. $8x - 24 = 0$

61. $12x - 144 = 0$

Evaluate each expression if $a = 4$, $b = -2$, and $c = -4$. (Lesson 2-5)

62. $|2a + c| + 1$

63. $4a - |3b + 2|$

64. $-|a + 1| + |3c|$

65. $-a + |2 - a|$

66. $|c - 2b| - 3$

67. $-2|3b - 8|$

Skills Review

Find each difference.

68. $13 - (-1)$

69. $4 - 16$

70. $-3 - 3$

71. $-8 - (-2)$

72. $16 - (-10)$

73. $-8 - 4$

LESSON 3-5 Arithmetic Sequences as Linear Functions

Then

- You identified linear functions.

Now

- **1** Recognize arithmetic sequences.
- **2** Relate arithmetic sequences to linear functions.

Why?

- During a 2000-meter race, the coach of a women's crew team recorded the team's times at several intervals.
  - At 400 meters, the time was 1 minute 32 seconds.
  - At 800 meters, it was 3 minutes 4 seconds.
  - At 1200 meters, it was 4 minutes 36 seconds.
  - At 1600 meters, it was 6 minutes 8 seconds.

  They completed the race with a time of 7 minutes 40 seconds.

New Vocabulary

sequence
terms of the sequence
arithmetic sequence
common difference

Common Core State Standards

Content Standards

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Recognize Arithmetic Sequences

You can relate the pattern of team times to linear functions. A **sequence** is a set of numbers, called the **terms of the sequence**, in a specific order. Look for a pattern in the information given for the women's crew team. Make a table to analyze the data.

Distance (m)	400	800	1200	1600	2000
Time (min : sec)	1:32	3:04	4:36	6:08	7:40

+ 1:32 + 1:32 + 1:32 + 1:32

As the distance increases in regular intervals, the time increases by 1 minute 32 seconds. Since the difference between successive terms is constant, this is an **arithmetic sequence**. The difference between the terms is called the **common difference** d.

KeyConcept Arithmetic Sequence

Words An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the *common difference*.

Examples

$3, 5, 7, 9, 11, \ldots$ with $+2$ $+2$ $+2$ $+2$; $d = 2$

$33, 29, 25, 21, 17, \ldots$ with -4 -4 -4 -4; $d = -4$

The three dots used with sequences are called an *ellipsis*. The ellipsis indicates that there are more terms in the sequence that are not listed.

Math HistoryLink

Mina Rees (1902–1997) Rees received the first award for Distinguished Service to Mathematics from the Mathematical Association of America. She was the first president of the Graduate Center at The City University of New York. Her work in analyzing patterns is still inspiring young women to study mathematics today.

Example 1 Identify Arithmetic Sequences

Determine whether each sequence is an arithmetic sequence. Explain.

a. $-4, -2, 0, 2, \ldots$

$-4 \quad -2 \quad 0 \quad 2$

$+2 \quad +2 \quad +2$

The difference between terms in the sequence is constant. Therefore, this sequence is arithmetic.

b. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, \ldots$

$\frac{1}{2} \quad \frac{5}{8} \quad \frac{3}{4} \quad \frac{13}{16}$

$+\frac{1}{8} \quad +\frac{1}{8} \quad +\frac{1}{16}$

This is not an arithmetic sequence. The difference between terms is not constant.

GuidedPractice

1A. $-26, -22, -18, -14, \ldots$

1B. $1, 4, 9, 25, \ldots$

You can use the common difference of an arithmetic sequence to find the next term.

Example 2 Find the Next Term

Find the next three terms of the arithmetic sequence 15, 9, 3, −3, … .

Step 1 Find the common difference by subtracting successive terms.

$15 \quad 9 \quad 3 \quad -3$

$-6 \quad -6 \quad -6$

The common difference is −6.

Step 2 Add −6 to the last term of the sequence to get the next term.

$-3 \quad -9 \quad -15 \quad -21$

$-6 \quad -6 \quad -6$

The next three terms in the sequence are −9, −15, and −21.

GuidedPractice

2. Find the next four terms of the arithmetic sequence 9.5, 11.0, 12.5, 14.0, ….

StudyTip

CCSS Regularity Notice the regularity in the way expressions in terms of a_1 and d change with each term of the sequence.

Each term in an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Term	Symbol	In Terms of a_1 and d	Numbers
first term	a_1	a_1	8
second term	a_2	$a_1 + d$	$8 + 1(3) = 11$
third term	a_3	$a_1 + 2d$	$8 + 2(3) = 14$
fourth term	a_4	$a_1 + 3d$	$8 + 3(3) = 17$
⋮	⋮	⋮	⋮
nth term	a_n	$a_1 + (n - 1)d$	$8 + (n - 1)(3)$

KeyConcept *n*th Term of an Arithmetic Sequence

The nth term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Example 3 Find the *n*th Term

a. Write an equation for the *n*th term of the arithmetic sequence −12, −8, −4, 0, … .

Step 1 Find the common difference.

$$-12 \quad -8 \quad -4 \quad 0$$
$$+4 \quad +4 \quad +4$$

The common difference is 4.

Step 2 Write an equation.

$a_n = a_1 + (n-1)d$	Formula for the *n*th term
$= -12 + (n-1)4$	$a_1 = -12$ and $d = 4$
$= -12 + 4n - 4$	Distributive Property
$= 4n - 16$	Simplify.

StudyTip

nth Terms Since n represents the number of the term, the inputs for n are the counting numbers.

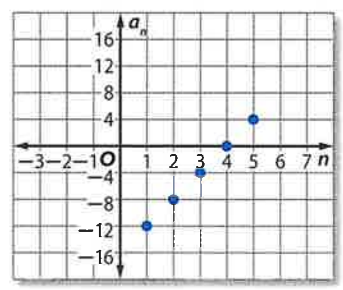
b. Find the 9th term of the sequence.

Substitute 9 for n in the formula for the nth term.

$a_n = 4n - 16$	Formula for the *n*th term
$a_9 = 4(9) - 16$	$n = 9$
$a_9 = 36 - 16$	Multiply.
$a_9 = 20$	Simplify.

c. Graph the first five terms of the sequence.

n	$4n - 16$	a_n	(n, a_n)
1	4(1) − 16	−12	(1, −12)
2	4(2) − 16	−8	(2, −8)
3	4(3) − 16	−4	(3, −4)
4	4(4) − 16	0	(4, 0)
5	4(5) − 16	4	(5, 4)



d. Which term of the sequence is 32?

In the formula for the nth term, substitute 32 for a_n.

$a_n = 4n - 16$	Formula for the *n*th term
$32 = 4n - 16$	$a_n = 32$
$32 + 16 = 4n - 16 + 16$	Add 16 to each side.
$48 = 4n$	Simplify.
$12 = n$	Divide each side by 4.

GuidedPractice

Consider the arithmetic sequence 3, −10, −23, −36, … .

3A. Write an equation for the nth term of the sequence.

3B. Find the 15th term in the sequence.

3C. Graph the first five terms of the sequence.

3D. Which term of the sequence is −114?

2 Arithmetic Sequences and Functions

Arithmetic Sequences and Functions As you can see from Example 3, the graph of the first five terms of the arithmetic sequence lie on a line. An arithmetic sequence is a linear function in which n is the independent variable, a_n is the dependent variable, and d is the slope. The formula can be rewritten as the function $f(n) = (n - 1)d + a_1$, where n is a counting number.

While the domain of most linear functions are all real numbers, in Example 3 the domain of the function is the set of counting numbers and the range of the function is the set of integers on the line.

Real-WorldLink

When a Latina turns 15, her family may host a quinceañera for her birthday. The quinceañera is a traditional Hispanic ceremony and reception that signifies the transition from childhood to adulthood.

Source: Quince Girl

Real-World Example 4 Arithmetic Sequences as Functions

INVITATIONS Marisol is mailing invitations to her quinceañera. The arithmetic sequence \$0.42, \$0.84, \$1.26, \$1.68, … represents the cost of postage.

a. Write a function to represent this sequence.

The first term, a_1, is 0.42. Find the common difference.

0.42 0.84 1.26 1.68

+0.42 +0.42 +0.42

The common difference is 0.42.

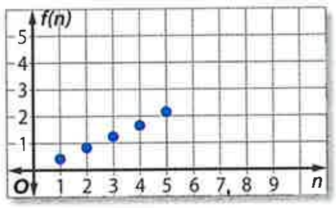
$a_n = a_1 + (n - 1)d$	Formula for the nth term
$= 0.42 + (n - 1)0.42$	$a_1 = 0.42$ and $d = 0.42$
$= 0.42 + 0.42n - 0.42$	Distributive Property
$= 0.42n$	Simplify.

The function is $f(n) = 0.42n$.

b. Graph the function and determine the domain.

The rate of change of the function is 0.42. Make a table and plot points.

n	$f(n)$
1	0.42
2	0.84
3	1.26
4	1.68
5	2.10



The domain of a function is the number of invitations Marisol mails. So, the domain is {1, 2, 3, 4, …}.

GuidedPractice

4. TRACK The chart below shows the length of Martin's long jumps.

Jump	1	2	3	4
Length (ft)	8	9.5	11	12.5

A. Write a function to represent this arithmetic sequence.

B. Then graph the function.

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

1. 18, 16, 15, 13, …
2. 4, 9, 14, 19, …

Example 2 Find the next three terms of each arithmetic sequence.

3. 12, 9, 6, 3, …
4. −2, 2, 6, 10, …

Example 3 Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

5. 15, 13, 11, 9, …
6. −1, −0.5, 0, 0.5, …

Example 4

7. **SAVINGS** Kaia has $525 in a savings account. After one month she has $580 in the account. The next month the balance is $635. The balance after the third month is $690. Write a function to represent the arithmetic sequence. Then graph the function.

Practice and Problem Solving

Extra Practice is on page R3.

Example 1 Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

8. −3, 1, 5, 9, …
9. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots$
10. −10, −7, −4, 1, …
11. −12.3, −9.7, −7.1, −4.5, …

Example 2 Find the next three terms of each arithmetic sequence.

12. 0.02, 1.08, 2.14, 3.2, …
13. 6, 12, 18, 24, …
14. 21, 19, 17, 15, …
15. $-\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots$
16. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$
17. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \ldots$

Example 3 Write an equation for the *n*th term of the arithmetic sequence. Then graph the first five terms in the sequence.

18. −3, −8, −13, −18, …
19. −2, 3, 8, 13, …
20. −11, −15, −19, −23, …
21. −0.75, −0.5, −0.25, 0, …

Example 4

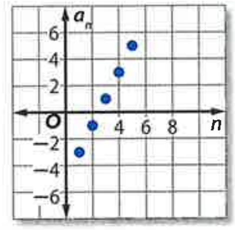
22. **AMUSEMENT PARKS** Shiloh and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours, they had ridden 4 rides. They had ridden 6 rides after 3 hours.
    a. Write a function to represent the arithmetic sequence.
    b. Graph the function and determine the domain.

23. **CCSS MODELING** The table shows how Ryan is paid at his lumber yard job.

Number of 10-ft 2×4 Planks Cut	1	2	3	4	5	6	7
Amount Paid in Commission ($)	8	16	24	32	40	48	56

    a. Write a function to represent Ryan's commission.
    b. Graph the function and determine the domain.

24. The graph is a representation of an arithmetic sequence.



a. List the first five terms.

b. Write the formula for the nth term.

c. Write the function.

25. **NEWSPAPERS** A local newspaper charges by the number of words for advertising. Write a function to represent the advertising costs.

DAILY NEWS ADVERTISING	
10 words \$7.50	20 words \$10.00
15 words \$8.75	25 words \$11.25

26. The fourth term of an arithmetic sequence is 8. If the common difference is 2, what is the first term?

27. The common difference of an arithmetic sequence is −5. If a_{12} is 22, what is a_1?

28. The first four terms of an arithmetic sequence are 28, 20, 12, and 4. Which term of the sequence is −36?

29. **CARS** Jamal's odometer of his car reads 24,521. If Jamal drives 45 miles every day, what will the odometer reading be after 25 days?

30. **YEARBOOKS** The yearbook staff is unpacking a box of school yearbooks. The arithmetic sequence 281, 270, 259, 248 … represents the total number of ounces that the box weighs as each yearbook is taken out of the box.

a. Write a function to represent this sequence.

b. Determine the weight of each yearbook.

c. If the box weighs at least 17 ounces empty and 292 ounces when it is full, how many yearbooks were in the box?

31. **SPORTS** To train for an upcoming marathon, Olivia plans to run 3 miles per day for the first week and then increase the daily distance by a half mile each of the following weeks.

a. Write an equation to represent the nth term of the sequence.

b. If the pattern continues, during which week will she run 10 miles per day?

c. Is it reasonable to think that this pattern will continue indefinitely? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

32. **OPEN ENDED** Create an arithmetic sequence with a common difference of −10.

33. **PERSEVERANCE** Find the value of x that makes $x + 8$, $4x + 6$, and $3x$ the first three terms of an arithmetic sequence.

34. **REASONING** Compare and contrast the domain and range of the linear functions described by $Ax + By = C$ and $a_n = a_1 + (n - 1)d$.

35. **CHALLENGE** Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain. If yes, find the common difference and the next three terms.

a. $2x + 1, 3x + 1, 4x + 1 \ldots$

b. $2x, 4x, 8x, \ldots$

36. **WRITING IN MATH** How are graphs of arithmetic sequences and linear functions similar? different?

Standardized Test Practice

37. GRIDDED RESPONSE The population of Westerville is about 35,000. Each year the population increases by about 400. This can be represented by the following equation, where n represents the number of years from now and p represents the population.

$$p = 35{,}000 + 400n$$

In how many years will the Westerville population be about 38,200?

38. Which relation is a function?

A $\{(-5, 6), (4, -3), (2, -1), (4, 2)\}$

B $\{(3, -1), (3, -5), (3, 4), (3, 6)\}$

C $\{(-2, 3), (0, 3), (-2, -1), (-1, 2)\}$

D $\{(-5, 6), (4, -3), (2, -1), (0, 2)\}$

39. Find the formula for the nth term of the arithmetic sequence.

$$-7, -4, -1, 2, \ldots$$

F $a_n = 3n - 4$

G $a_n = -7n + 10$

H $a_n = 3n - 10$

J $a_n = -7n + 4$

40. STATISTICS A class received the following scores on the ACT. What is the difference between the median and the mode in the scores?

18, 26, 20, 30, 25, 21, 32, 19, 22, 29, 29, 27, 24

A 1

B 2

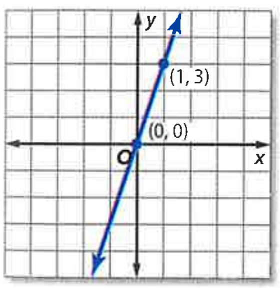
C 3

D 4

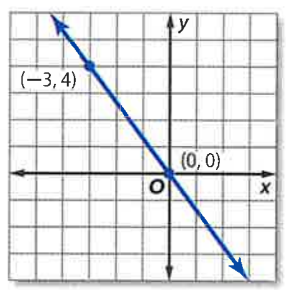
Spiral Review

Name the constant of variation for each direct variation. Then find the slope of the line that passes through each pair of points. (Lesson 3-4)

41.



42.



Find the slope of the line that passes through each pair of points. (Lesson 3-3)

43. $(5, 3), (-2, 6)$

44. $(9, 2), (-3, -1)$

45. $(2, 8), (-2, -4)$

Solve each equation. Check your solution. (Lesson 2-3)

46. $5x + 7 = -8$

47. $8 = 2 + 3n$

48. $12 = \frac{c - 6}{2}$

49. SPORTS The most popular sports for high school girls are basketball and softball. Write and use an equation to find how many more girls play on basketball teams than on softball teams. (Lesson 2-1)

Basketball
453,000 girls

Softball
369,000 girls

Skills Review

Graph each point on the same coordinate plane.

50. $A(2, 5)$

51. $B(-2, 1)$

52. $C(-3, -1)$

53. $D(0, 4)$

54. $F(5, -3)$

55. $G(-5, 0)$

EXTEND 3-5

Algebra Lab
Inductive and Deductive Reasoning

CCSS Common Core State Standards
Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

If Jolene is not feeling well, she may go to a doctor. The doctor will ask her questions about how she is feeling and possibly run other tests. Based on her symptoms, the doctor can diagnose Jolene's illness. This is an example of inductive reasoning. **Inductive reasoning** is used to derive a general rule after observing many events.

To use inductive reasoning:

Step 1 Observe many examples.

Step 2 Look for a pattern.

Step 3 Make a conjecture.

Step 4 Check the conjecture.

Step 5 Discover a likely conclusion.

With **deductive reasoning**, you come to a conclusion by accepting facts. The results of the tests ordered by the doctor may support the original diagnosis or lead to a different conclusion. This is an example of deductive reasoning. There is no conjecturing involved. Consider the two statements below.

1) If the strep test is positive, then the patient has strep throat.

2) Jolene tested positive for strep.

If these two statements are accepted as facts, then the obvious conclusion is that Jolene has strep throat. This is an example of deductive reasoning.

Exercises

1. Explain the difference between *inductive* and *deductive* reasoning. Then give an example of each.

2. When a detective reaches a conclusion about the height of a suspect from the distance between footprints, what kind of reasoning is being used? Explain.

3. When you examine a finite number of terms in a sequence of numbers and decide that it is an arithmetic sequence, what kind of reasoning are you using? Explain.

4. Suppose you have found the common difference for an arithmetic sequence based on analyzing a finite number of terms, what kind of reasoning do you use to find the 100th term in the sequence?

5. **CCSS PERSEVERANCE**
a. Copy and complete the table.

3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9
3	9	27						

b. Write the sequence of numbers representing the numbers in the ones place.

c. Find the number in the ones place for the value of 3^{100}. Explain your reasoning. State the type of reasoning that you used.

LESSON 3-6

Proportional and Nonproportional Relationships

Then

- You recognized arithmetic sequences and related them to linear functions.

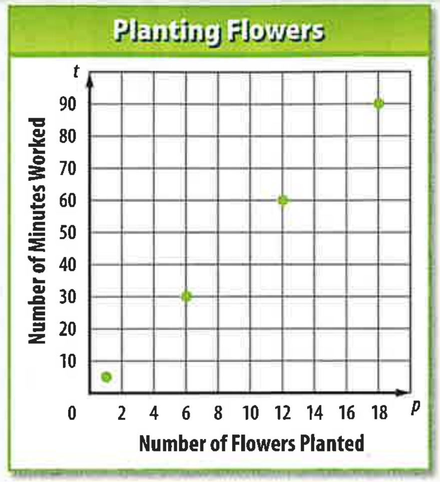
Now

1. Write an equation for a proportional relationship.
2. Write an equation for a nonproportional relationship.

Why?

- Heather is planting flats of flowers. The table shows the number of flowers that she has planted and the amount of time that she has been working in the garden.

Number of flowers planted (p)	1	6	12	18
Number of minutes working (f)	5	30	60	90



The relationship between the flowers planted and the time that Heather worked in minutes can be graphed. Let p represent the number of flowers planted. Let t represent the number of minutes that Heather has worked.

When the ordered pairs are graphed, they form a linear pattern. This pattern can be described by an equation.

CCSS Common Core State Standards

Content Standards

F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

1 Proportional Relationships

If the relationship between the domain and range of a relation is linear, the relationship can be described by a linear equation. If the equation is of the form $y = kx$, then the relationship is proportional. In a proportional relationship, the graph will pass through (0, 0). So, direct variations are proportional relationships.

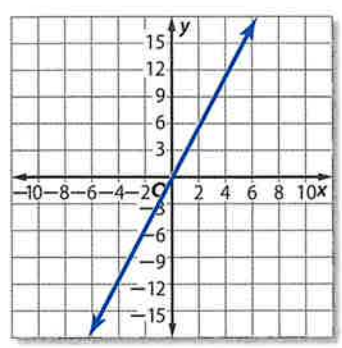
KeyConcept Proportional Relationship

Words A relationship is proportional if its equation is of the form $y = kx$, $k \neq 0$. The graph passes through (0, 0).

Example $y = 3x$

x	0	1	2	3	4
y	0	3	6	9	12

The ratio of the value of x to the value of y is constant when $x \neq 0$.



Proportional relationships are useful when analyzing real-world data. The pattern can be described using a table, a graph, and an equation.

Real-WorldLink

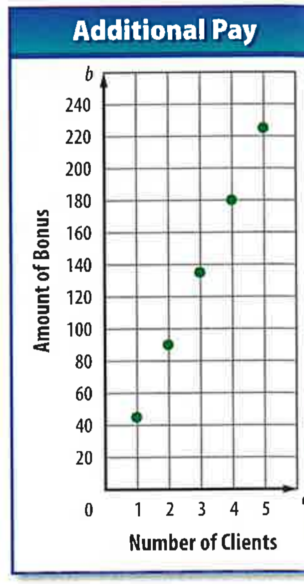
Attendance at fitness clubs has steadily grown over the past fifteen years. Members' ages are expanding to a range of 15–34 on average.

Source: International Health, Racquet, and Sportsclub Association

Real-World Example 1 Proportional Relationships

BONUS PAY **Marcos is a personal trainer at a gym. In addition to his salary, he receives a bonus for each client he sees.**

Number of Clients	1	2	3	4	5
Bonus Pay ($)	45	90	135	180	225



a. Graph the data. What can you deduce from the pattern about the relationship between the number of clients and the bonus pay?

The graph demonstrates a linear relationship between the number of clients and the bonus pay.

The graph also passes through the point (0, 0) because when Marcos sees 0 clients, he does not receive any bonus money. Therefore, the relationship is proportional.

b. Write an equation to describe this relationship.

Look for a pattern that can be described in an equation.

+1 +1 +1 +1

Number of Clients	1	2	3	4	5
Bonus Pay ($)	45	90	135	180	225

+45 +45 +45 +45

StudyTip

CCSS Structure Look for a pattern that shows a constant rate of change between the terms.

The difference between the values for the number of clients c is 1. The difference in the values for the bonus pay b is 45. This suggests that the k-value is $\frac{45}{1}$ or 45. So the equation is $b = 45c$. You can check this equation by substituting values for c into the equation.

CHECK If $c = 1$, then $b = 45(1)$ or 45. ✓
If $c = 5$, then $b = 45(5)$ or 225. ✓

c. Use this equation to predict the amount of Marcos's bonus if he sees 8 clients.

$b = 45c$ Original equation
$= 45(8)$ or 360 $c = 8$

Marcos will receive a bonus of $360 if he sees 8 clients.

GuidedPractice

1. **CHARITY** A professional soccer team is donating money to a local charity for each goal they score.

Number of Goals	1	2	3	4	5
Donation ($)	75	150	225	300	375

A. Graph the data. What can you deduce from the pattern about the relationship between the number of goals and the money donated?

B. Write an equation to describe this relationship.

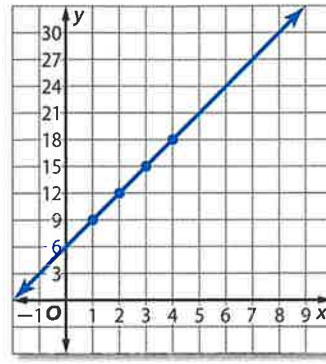
C. Use this equation to predict how much money will be donated for 12 goals.

2 Nonproportional Relationships

Nonproportional Relationships Some linear equations can represent a nonproportional relationship. If the ratio of the value of x to the value of y is different for select ordered pairs that are on the line, the equation is nonproportional and the graph will not pass through (0, 0).

Example 2 Nonproportional Relationships

Write an equation in function notation for the graph.



Understand You are asked to write an equation of the relation that is graphed in function notation.

Plan Find the difference between the x-values and the difference between the y-values.

Solve Select points from the graph and place them in a table.

+1 +1 +1

x	1	2	3	4
y	9	12	15	18

+3 +3 +3

Notice that $\frac{1}{9} \neq \frac{2}{12} \neq \frac{3}{15} \neq \frac{4}{18}$.

The difference between the x-values is 1, while the difference between the y-values is 3. This suggests that $y = 3x$ or $f(x) = 3x$.

If $x = 1$, then $y = 3(1)$ or 3. But the y-value for $x = 1$ is 9. Let's try some other values and see if we can detect a pattern.

x	1	2	3	4
$3x$	3	6	9	12
y	9	12	15	18

y is always 6 more than $3x$.

This pattern shows that 6 should be added to one side of the equation. Thus, the equation is $y = 3x + 6$ or $f(x) = 3x + 6$.

Check Compare the ordered pairs from the table to the graph. The points correspond. ✓

StudyTip

Graphs of Lines A value added to or subtracted from one side of the equation $y = ax$ will cause a shift along the y-axis for the graph of the line.

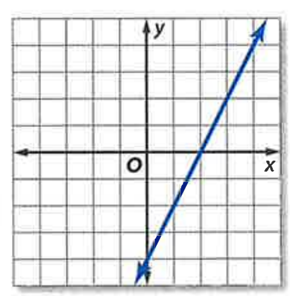
GuidedPractice

2. Write an equation in function notation for the relation shown in the table.

A.

x	1	2	3	4
y	3	2	1	0

B. Write an equation in function notation for the graph.



Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1

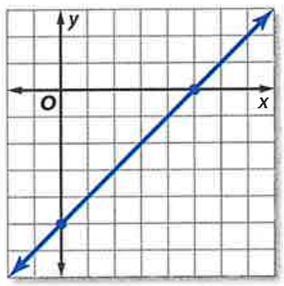
1. **GEOMETRY** The table shows the perimeter of a square with sides of a given length.

Side Length (in.)	1	2	3	4	5
Perimeter (in.)	4	8	12	16	20

   a. Graph the data.
   b. Write an equation to describe the relationship.
   c. What conclusion can you make regarding the relationship between the side and the perimeter?

Example 2

Write an equation in function notation for each relation.

2.


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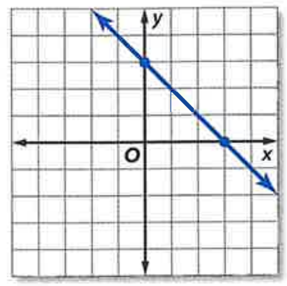


Practice and Problem Solving

Extra Practice is on page R3.

Example 1

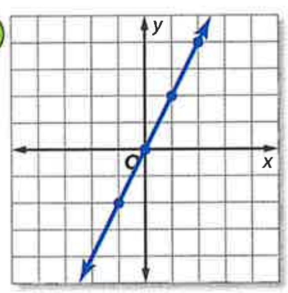
4. CCSS **STRUCTURE** The table shows the pages of comic books read.

Books Read	1	2	3	4	5
Pages Read	35	70	105	140	175

   a. Graph the data.
   b. Write an equation to describe the relationship.
   c. Find the number of pages read if 8 comic books were read.

Example 2

Write an equation in function notation for each relation.

5.


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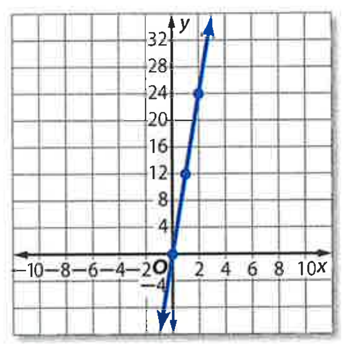

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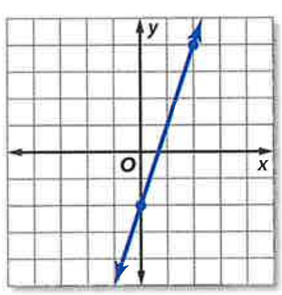

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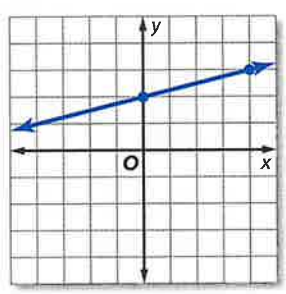


For each arithmetic sequence, determine the related function. Then determine if the function is *proportional* or *nonproportional*. Explain.

9. 0, 3, 6, …

10. −4, 0, 4, …

11. BOWLING Marielle is bowling with her friends. The table shows prices for renting a pair of shoes and bowling. Write an equation to represent the total price y if Marielle buys x games.

Games Bowled	Total Price ($)
2	7.00
4	11.50
6	16.00
8	20.50

12. SNOWFALL The total snowfall each hour of a winter snowstorm is shown in the table below.

Hour	1	2	3	4
Inches of Snowfall	1.65	3.30	4.95	6.60

a. Write an equation to fit the data in the table.

b. Describe the relationship between the hour and inches of snowfall.

13. FUNDRAISER The Cougar Pep Squad wants to sell T-shirts in the bookstore for the spring dance. The cost in dollars to order T-shirts in their school colors is represented by the equation $C = 2t + 3$.

a. Make a table of values that represents this relationship.

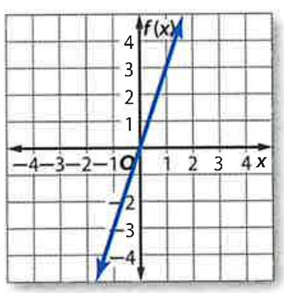
b. Rewrite the equation in function notation.

c. Graph the function.

d. Describe the relationship between the number of T-shirts and the cost.

H.O.T. Problems Use Higher-Order Thinking Skills

14. CCSS CRITIQUE Quentin thinks that $f(x)$ and $g(x)$ are both proportional. Claudia thinks they are not proportional. Is either of them correct? Explain your reasoning.



x	$g(x)$
−2	−7
−1	−4
0	−1
1	2
2	5

15. OPEN ENDED Create an arithmetic sequence in which the first term is 4. Explain the pattern that you used. Write an equation that represents your sequence.

16. CHALLENGE Describe how inductive reasoning can be used to write an equation from a pattern.

17. REASONING A **counterexample** is a specific case that shows that a statement is false. Provide a counterexample to the following statement. *The related function of an arithmetic sequence is always proportional.* Explain your reasoning.

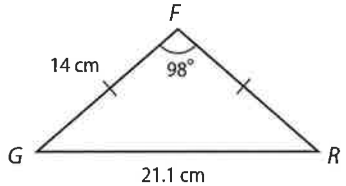
18. WRITING IN MATH Compare and contrast proportional relationships with nonproportional relationships.

Standardized Test Practice

19. What is the slope of a line that contains the point (1, −5) and has the same y-intercept as $2x - y = 9$?

A −9
B −7
C 2
D 4

20. SHORT RESPONSE $\triangle FGR$ is an isosceles triangle. What is the measure of $\angle G$?



21. Luis deposits $25 each week into a savings account from his part-time job. If he has $350 in savings now, how much will he have in 12 weeks?

F $600
G $625
H $650
J $675

22. GEOMETRY Omar and Mackenzie want to build a pulley system by attaching one end of a rope to their 8-foot-tall tree house and anchoring the other end to the ground 28 feet away from the base of the tree house. How long, to the nearest foot, does the piece of rope need to be?

A 26 ft
B 27 ft
C 28 ft
D 29 ft

Spiral Review

Find the next three terms in each sequence. (Lesson 3-5)

23. 3, 13, 23, 33, …

24. −2, −1.4, −0.8, −0.2, …

25. $\frac{3}{4}, \frac{7}{8}, 1, \frac{9}{8}, \ldots$

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve. (Lesson 3-4)

26. If $y = 45$ when $x = 9$, find y when $x = 7$.

27. If $y = -7$ when $x = -1$, find x when $y = -84$.

28. GENETICS About $\frac{2}{25}$ of the male population in the world cannot distinguish red from green. If there are 14 boys in the ninth grade who cannot distinguish red from green, about how many ninth-grade boys are there in all? Write and solve an equation to find the answer. (Lesson 2-2)

29. GEOMETRY The volume V of a cone equals one third times the product of π, the square of the radius r of the base, and the height h. (Lesson 2-1)

a. Write the formula for the volume of a cone.

b. Find the volume of a cone if r is 10 centimeters and h is 30 centimeters.

Skills Review

Solve each equation for y.

30. $3x = y + 7$

31. $2y = 6x - 10$

32. $9y + 2x = 12$

Graph each equation.

33. $y = x - 8$

34. $x - y = -4$

35. $2x + 4y = 8$

CHAPTER 3

Study Guide and Review

Study Guide

Key Concepts

Graphing Linear Equations (Lesson 3-1)

- The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers whose greatest common factor is 1.

Solving Linear Equations by Graphing (Lesson 3-2)

- Values of x for which $f(x) = 0$ are called zeros of the function f. A zero of a function is located at an x-intercept of the graph of the function.

Rate of Change and Slope (Lesson 3-3)

- If x is the independent variable and y is the dependent variable, then rate of change equals

$$\frac{\text{change in } y}{\text{change in } x}.$$

- The slope of a line is the ratio of the rise to the run.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Direct Variation (Lesson 3-4)

- A direct variation is described by an equation of the form $y = kx$, where $k \neq 0$.

Arithmetic Sequences (Lesson 3-5)

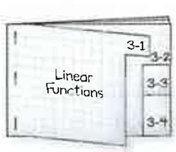
- The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Proportional and Nonproportional Relationships (Lesson 3-6)

- In a proportional relationship, the graph will pass through (0, 0).
- In a nonproportional relationship, the graph will not pass through (0, 0)

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The x-coordinate of the point at which the graph of an equation crosses the x-axis is an x-intercept.
2. A linear equation is an equation of a line.
3. The difference between successive terms of an arithmetic sequence is the constant of variation.
4. The regular form of a linear equation is $Ax + By = C$.
5. Values of x for which $f(x) = 0$ are called zeros of the function f.
6. Any two points on a nonvertical line can be used to determine the slope.
7. The slope of the line $y = 5$ is 5.
8. The graph of any direct variation equation passes through (0, 1).
9. A ratio that describes, on average, how much one quantity changes with respect to a change in another quantity is a rate of change.
10. In the linear equation $4x + 3y = 12$, the constant term is 12.

Lesson-by-Lesson Review

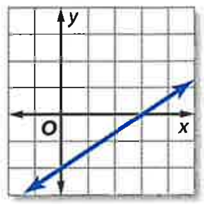
3-1 Graphing Linear Equations

Find the *x*-intercept and *y*-intercept of the graph of each linear function.

11.

x	y
−8	0
−4	3
0	6
4	9
8	12

12.



Graph each equation.

13. $y = -x + 2$

14. $x + 5y = 4$

15. $2x - 3y = 6$

16. $5x + 2y = 10$

17. **SOUND** The distance d in kilometers that sound waves travel through water is given by $d = 1.6t$, where t is the time in seconds.

a. Make a table of values and graph the equation.

b. Use the graph to estimate how far sound can travel through water in 7 seconds.

Example 1

Graph $3x - y = 4$ by using the *x*- and *y*-intercepts.

Find the *x*-intercept.

$3x - y = 4$

$3x - 0 = 4$ Let $y = 0$.

$3x = 4$

$x = \frac{4}{3}$

Find the *y*-intercept.

$3x - y = 4$

$3(0) - y = 4$ Let $x = 0$.

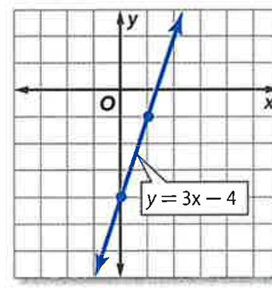
$-y = 4$

$y = -4$

x-intercept: $\frac{4}{3}$

y-intercept: -4

The graph intersects the *x*-axis at $\left(\frac{4}{3}, 0\right)$ and the *y*-axis at $(0, -4)$. Plot these points. Then draw the line through them.



3-2 Solving Linear Equations by Graphing

Find the root of each equation.

18. $0 = 2x + 8$

19. $0 = 4x - 24$

20. $3x - 5 = 0$

21. $6x + 3 = 0$

Solve each equation by graphing.

22. $0 = 16 - 8x$

23. $0 = 21 + 3x$

24. $-4x - 28 = 0$

25. $25x - 225 = 0$

26. **FUNDRAISING** Sean's class is selling boxes of popcorn to raise money for a class trip. Sean's class paid $85 for the popcorn, and they are selling each box for $1. The function $y = x - 85$ represents their profit y for each box of popcorn sold x. Find the zero and describe what it means in this situation.

Example 2

Solve $3x + 1 = -2$ by graphing.

The first step is to find the related function.

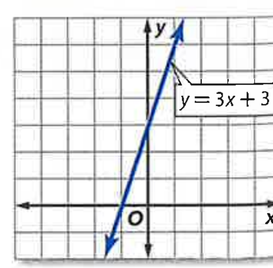
$3x + 1 = -2$ Original equation

$3x + 1 + 2 = -2 + 2$ Add 2 to each side.

$3x + 3 = 0$ Simplify.

The related function is $y = 3x + 3$.

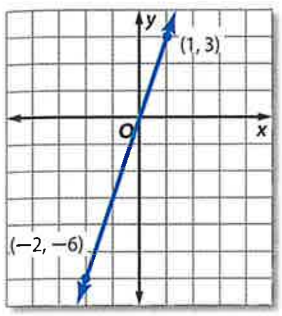
The graph intersects the *x*-axis at −1. So, the solution is −1.



3-3 Rate of Change and Slope

Find the rate of change represented in each table or graph.

27.



28.

x	y
−2	−3
0	−3
4	−3
12	−3

Find the slope of the line that passes through each pair of points.

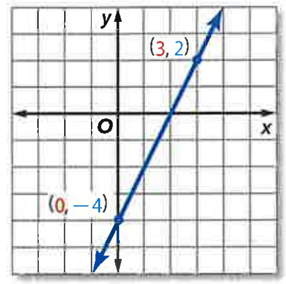
29. (0, 5), (6, 2)

30. (−6, 4), (−6, −2)

31. **PHOTOS** The average cost of online photos decreased from \$0.50 per print to \$0.15 per print between 2002 and 2009. Find the average rate of change in the cost. Explain what it means.

Example 3

Find the slope of the line that passes through (0, −4) and (3, 2).



Let (0, −4) = (x_1, y_1) and (3, 2) = (x_2, y_2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — Slope formula

$= \frac{2 - (-4)}{3 - 0}$ — $x_1 = 0, x_2 = 3, y_1 = -4, y_2 = 2$

$= \frac{6}{3}$ or 2 — Simplify.

3-4 Direct Variation

Graph each equation.

32. $y = x$

33. $y = \frac{4}{3}x$

34. $y = -2x$

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.

35. If $y = 15$ when $x = 2$, find y when $x = 8$.

36. If $y = -6$ when $x = 9$, find x when $y = -3$.

37. If $y = 4$ when $x = -4$, find y when $x = 7$.

38. **JOBS** Suppose you earn \$127 for working 20 hours.

a. Write a direct variation equation relating your earnings to the number of hours worked.

b. How much would you earn for working 35 hours?

Example 4

Suppose y varies directly as x, and $y = -24$ when $x = 8$.

a. Write a direct variation equation that relates x and y.

$y = kx$ — Direct variation equation

$-24 = k(8)$ — Substitute −24 for y and 8 for x.

$\frac{-24}{8} = \frac{k(8)}{8}$ — Divide each side by 8.

$-3 = k$ — Simplify.

So, the direct variation equation is $y = -3x$.

b. Use the direct variation equation to find x when $y = -18$.

$y = -3x$ — Direct variation equation

$-18 = -3x$ — Replace y with −18.

$\frac{-18}{-3} = \frac{-3x}{-3}$ — Divide each side by −3.

$6 = x$ — Simplify.

Therefore, $x = 6$ when $y = -18$.

3-5 Arithmetic Sequences as Linear Functions

Find the next three terms of each arithmetic sequence.

39. 6, 11, 16, 21, ...

40. 1.4, 1.2, 1.0, ...

Write an equation for the nth term of each arithmetic sequence.

41. $a_1 = 6, d = 5$

42. 28, 25, 22, 19, ...

43. **SCIENCE** The table shows the distance traveled by sound in water. Write an equation for this sequence. Then find the time for sound to travel 72,300 feet.

Time (s)	1	2	3	4
Distance ft)	4820	9640	14,460	19,280

Example 5

Find the next three terms of the arithmetic sequence 10, 23, 36, 49, ...

Find the common difference.

10 23 36 49
+13 +13 +13

So, $d = 13$.

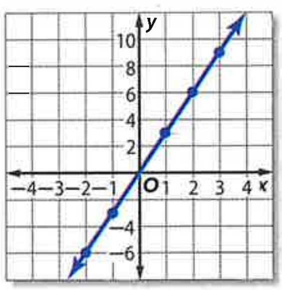
Add 13 to the last term of the sequence. Continue adding 13 until the next three terms are found.

49 62 75 88
+13 +13 +13

The next three terms are 62, 75, and 88.

3-6 Proportional and Nonproportional Relationships

44. Write an equation in function notation for this relation.



45. **ANALYZE TABLES** The table shows the cost of picking your own strawberries at a farm.

Number of Pounds	1	2	3	4
Total Cost ($)	1.25	2.50	3.75	5.00

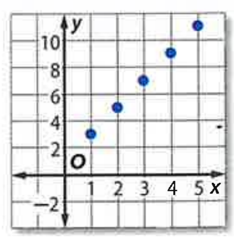
a. Graph the data.

b. Write an equation in function notation to describe this relationship.

c. How much would it cost to pick 6 pounds of strawberries?

Example 6

Write an equation in function notation for this relation.



Make a table of ordered pairs for several points on the graph.

x	1	2	3	4	5
y	3	5	7	9	11

The difference in y-values is twice the difference of x values. This suggests that $y = 2x$. However, $3 \neq 2(1)$. Compare the values of y to the values of $2x$.

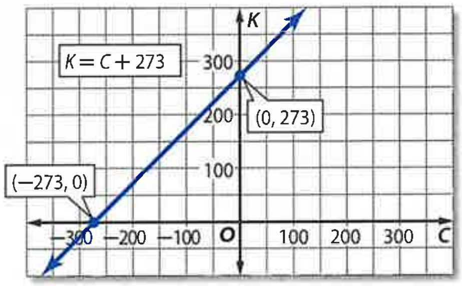
x	1	2	3	4	5
$2x$	2	4	6	8	10
y	3	5	7	9	11

The difference between y and $2x$ is always 1. So the equation is $y = 2x + 1$. Since this relation is also a function, it can be written as $f(x) = 2x + 1$.

CHAPTER 3

Practice Test

1. **TEMPERATURE** The equation to convert Celsius temperature C to Kelvin temperature K is shown.



   a. State the independent and dependent variables. Explain.

   b. Determine the C- and K-intercepts and describe what the intercepts mean in this situation.

Graph each equation.

2. $y = x + 2$
3. $y = 4x$
4. $x + 2y = -1$
5. $-3x = 5 - y$

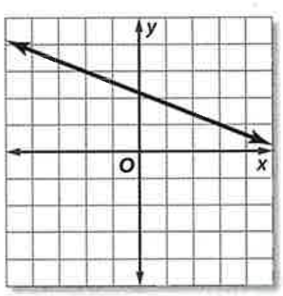
Solve each equation by graphing.

6. $4x + 2 = 0$
7. $0 = 6 - 3x$
8. $5x + 2 = -3$
9. $12x = 4x + 16$

Find the slope of the line that passes through each pair of points.

10. $(5, 8), (-3, 7)$
11. $(5, -2), (3, -2)$
12. $(-4, 7), (8, -1)$
13. $(6, -3), (6, 4)$

14. **MULTIPLE CHOICE** Which is the slope of the linear function shown in the graph?



A $-\frac{5}{2}$

B $-\frac{2}{5}$

C $\frac{2}{5}$

D $\frac{5}{2}$

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.

15. If $y = 6$ when $x = 9$, find x when $y = 12$.
16. When $y = -8$, $x = 8$. What is x when $y = -6$?
17. If $y = -5$ when $x = -2$, what is y when $x = 14$?
18. If $y = 2$ when $x = -12$, find y when $x = -4$.

19. **BIOLOGY** The number of pints of blood in a human body varies directly with the person's weight. A person who weighs 120 pounds has about 8.4 pints of blood in his or her body.

   a. Write and graph an equation relating weight and amount of blood in a person's body.

   b. Predict the weight of a person whose body holds 12 pints of blood.

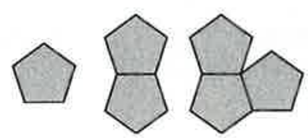
Find the next three terms of each arithmetic sequence.

20. 0, −15, −30, −45, −60, ...
21. 5, 8, 11, 14, ...

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

22. −40, −32, −24, −16, ...
23. 0.75, 1.5, 3, 6, 12, ...
24. 5, 17, 29, 41, ...

25. **MULTIPLE CHOICE** In each figure, only one side of each regular pentagon is shared with another pentagon. The length of each side is 1 centimeter. If the pattern continues, what is the perimeter of a figure that has 6 pentagons?



F 30 cm

G 25 cm

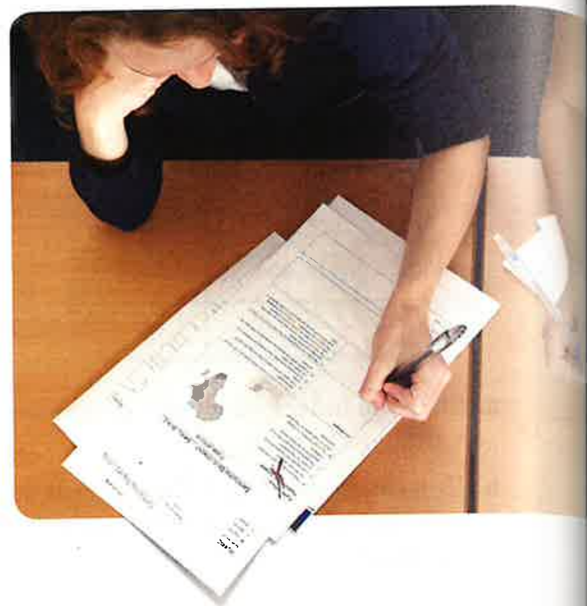
H 20 cm

J 15 cm

CHAPTER 3

Preparing for Standardized Tests

Reading Math Problems



The first step to solving any math problem is to read the problem. When reading a math problem to get the information you need to solve, it is helpful to use special reading strategies.

Strategies for Reading Math Problems

Step 1

Read the problem quickly to gain a general understanding of it.

- **Ask yourself:** "What do I know?" "What do I need to find out?"
- **Think:** "Is there enough information to solve the problem? Is there extra information?"
- **Highlight:** If you are allowed to write in your test booklet, underline or highlight important information. Cross out any information you don't need.

Step 2

Reread the problem to identify relevant facts.

- **Analyze:** Determine how the facts are related.
- **Key Words:** Look for keywords to solve the problem.
- **Vocabulary:** Identify mathematical terms. Think about the concepts and how they are related.
- **Plan:** Make a plan to solve the problem.
- **Estimate:** Quickly estimate the answer.

Step 3

Identify any obvious wrong answers.

- **Eliminate:** Eliminate any choices that are very different from your estimate.
- **Units of Measure:** Identify choices that are possible answers based on the units of measure in the question. For example, if the question asks for area, only answers in square units will work.

Step 4

Look back after solving the problem.

Check: Make sure you have answered the question.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Jamal, Gina, Lisa, and Renaldo are renting a car for a road trip. The cost of renting the car is given by the function $C = 12.5 + 21d$, where C is the total cost for renting the car for d days. What does the slope of the function represent?

A number of people
B cost per day
C number of days
D miles per gallon

Read the problem carefully. The number of people going on the trip is not needed information. You need to know what the slope of the function represents.

Slope is a ratio. The word "per" in answers B and D imply that they are both ratios. Since choices A and C are not ratios, eliminate them.

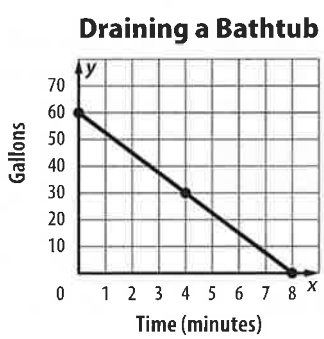
The problem says that C represents the cost of renting the car. So the slope cannot represent the miles per gallon of the car. The slope must represent the cost per day.

The correct answer is B.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. What does the x-intercept mean in the context of the situation given below?



A amount of time needed to drain the bathtub
B number of gallons in the tub when the drain plug is pulled
C number of gallons in the tub after x minutes
D amount of water drained each minute

2. The amount of money raised by a charity carwash varies directly as the number of cars washed. When 11 cars are washed, $79.75 is raised. How many cars must be washed to raise $174.00?

F 10 cars
G 16 cars
H 22 cars
J 24 cars

3. The function $C = 25 + 0.45(x - 450)$ represents the cost of a monthly cell phone bill, when x minutes are used. Which statement best represents the formula for the cost of the bill?

A The cost consists of a flat fee of $0.45 and $25 for each minute used over 450.

B The cost consists of a flat fee of $450 and $0.45 for each minute used over 25.

C The cost consists of a flat fee of $25 and $0.45 for each minute used over 450.

D The cost consists of a flat fee of $25 and $0.45 for each minute used.

CHAPTER 3

Standardized Test Practice

Cumulative, Chapters 1 through 3

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Horatio is purchasing a computer cable for \$15.49. If the sales tax rate in his state is 5.25%, what is the total cost of the purchase?

A \$16.42 **C** \$15.73

B \$16.30 **D** \$15.62

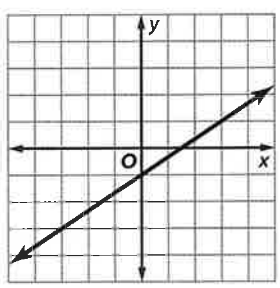
2. What is the value of the expression below?

$$3^2 + 5^3 - 2^5$$

F 14 **H** 102

G 34 **J** 166

3. What is the slope of the linear function graphed below?



A $-\frac{1}{3}$ **C** $\frac{2}{3}$

B $\frac{1}{2}$ **D** $\frac{3}{2}$

4. Find the rate of change for the linear function represented in the table.

Hours Worked	1	2	3	4
Money Earned (\$)	5.50	11.00	16.50	22.00

F increase \$6.50/h

G increase \$5.50/h

H decrease \$5.50/h

J decrease \$6.50/h

5. Suppose that y varies directly as x, and $y = 14$ when $x = 4$. What is the value of y when $x = 9$?

A 25.5 **C** 29.5

B 27.5 **D** 31.5

6. Write an equation for the nth term of the arithmetic sequence shown below.

$$-2, 1, 4, 7, 10, 13, \ldots$$

F $a_n = 2n - 1$ **H** $a_n = 3n + 2$

G $a_n = 2n + 4$ **J** $a_n = 3n - 5$

7. The table shows the labor charges of an electrician for jobs of different lengths.

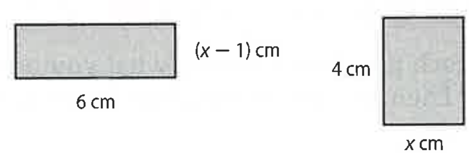
Number of Hours (n)	Labor Charges (c)
1	\$60
2	\$85
3	\$110
4	\$135

Which function represents the situation?

A $C(n) = 25n + 35$ **C** $C(n) = 35n + 25$

B $C(n) = 25n + 30$ **D** $C(n) = 35n + 40$

8. Find the value of x so that the figures have the same area.



F 3 **H** 5

G 4 **J** 6

9. The table shows the total amount of rain during a storm. Write a formula to find out how much rain will fall after a given hour.

Hours (h)	1	2	3	4
Inches (n)	0.45	9.9	1.35	1.8

A $h = 0.45n$ **C** $h = 0.9n$

B $n = 0.45h$ **D** $h = 1.8n$

Test-Taking Tip

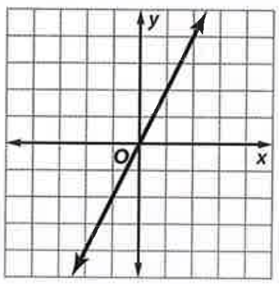
Question 3 You can *eliminate unreasonable answers* to multiple choice items. The line slopes up from left to right, so the slope is positive. Answer choice A can be eliminated.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

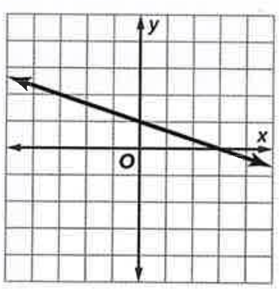
10. The scale on a map is 1.5 inches = 6 miles. If two cities are 4 inches apart on the map, what is the actual distance between the cities?

11. Write a direct variation equation to represent the graph below.

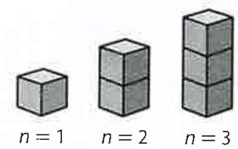


12. Justine bought a car for $18,500 and its value depreciated linearly. After 3 years, the value was $14,150. What is the amount of yearly depreciation?

13. **GRIDDED RESPONSE** Use the graph to determine the solution to the equation $-\frac{1}{3}x + 1 = 0$?



14. Write an expression that represents the total surface area (including the top and bottom) of a tower of n cubes each having a side length of s. (Do not include faces that cover each other.)



15. **GRIDDED RESPONSE** There are 120 members in the North Carolina House of Representatives. This is 70 more than the number of members in the North Carolina Senate. How many members are in the North Carolina Senate?

Extended Response

Record your answers on a sheet of paper. Show your work.

16. A hot air balloon was at a height of 60 feet above the ground when it began to ascend. The balloon climbed at a rate of 15 feet per minute.

a. Make a table that shows the height of the hot air balloon after climbing for 1, 2, 3, and 4 minutes.

b. Let t represent the time in minutes since the balloon began climbing. Write an algebraic equation for a sequence that can be used to find the height, h, of the balloon after t minutes.

c. Use your equation from part b to find the height, in feet, of the hot air balloon after climbing for 8 minutes.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Go to Lesson...	2-7	1-2	3-3	3-3	3-4	3-5	3-6	2-4	3-4	2-6	3-4	3-3	3-2	0-10	2-1	3-5