CHAPTER 4

Equations of Linear Functions

Then

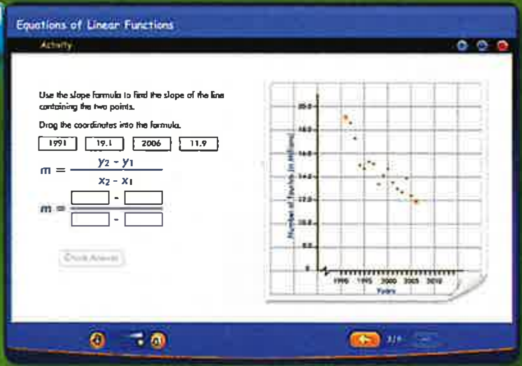
- You graphed linear functions.

Now

- In this chapter, you will:
  - Write and graph linear equations in various forms.
  - Use scatter plots and lines of fit, and write equations of best-fit lines using linear regression.
  - Find inverse linear functions.

Why? ▲

- **OIL** The amount of oil drilled at an oil field changes from year to year. From the yearly data, patterns emerge. Rate of change can be applied to these data to determine a linear model. This can be used to predict the amount of oil drilled in future years.



Use the slope formula to find the slope of the line containing the two points.

Drag the coordinates into the formula.

1991 | 19.1 | 2006 | 11.9

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\square - \square}{\square - \square}$$

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LESSON 4-1

Graphing Equations in Slope-Intercept Form

Then

- You found rates of change and slopes.

Now

1. Write and graph linear equations in slope-intercept from.
2. Model real-world data with equations in slope-intercept form.

Why?

- Jamil has 500 songs on his digital media player. He joins a music club that lets him download 30 songs per month for a monthly fee. The number of songs that Jamil could eventually have in his player if he does not delete any songs is represented by $y = 30x + 500$.

Gregory Costanzo/Lifesize/Getty Images

NewVocabulary

slope-intercept form
constant function

Common Core State Standards

Content Standards

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Mathematical Practices

2 Reason abstractly and quantitatively.

8 Look for and express regularity in repeated reasoning.

1 Slope-Intercept Form

An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is in **slope-intercept form**. The variables m and b are called *parameters* of the equation. Changing either value changes the equation's graph.

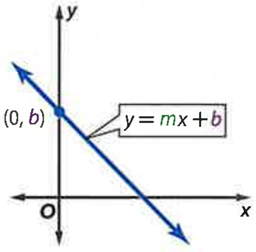
KeyConcept Slope-Intercept Form

Words The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.

Example

$$y = mx + b$$

$$y = \underset{\text{slope}}{2}x + \underset{y\text{-intercept}}{6}$$



Example 1 Write and Graph an Equation

Write an equation in slope-intercept form for the line with a slope of $\frac{3}{4}$ and a y-intercept of -2. Then graph the equation.

$y = mx + b$ Slope-intercept form

$y = \frac{3}{4}x + (-2)$ Replace m with $\frac{3}{4}$ and b with -2.

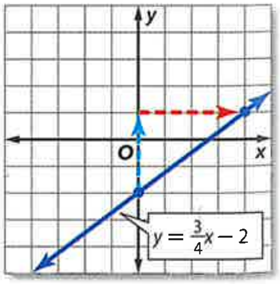
$y = \frac{3}{4}x - 2$ Simplify.

Now graph the equation.

Step 1 Plot the y-intercept $(0, -2)$.

Step 2 The slope is $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$. From $(0, -2)$, move up 3 units and right 4 units. Plot the point.

Step 3 Draw a line through the two points.



GuidedPractice

Write an equation of a line in slope intercept form with the given slope and y-intercept. Then graph the equation.

1A. slope: $-\frac{1}{2}$, y-intercept: 3

1B. slope: -3, y-intercept: -8

When an equation is not written in slope-intercept form, it may be easier to rewrite it before graphing.

Example 2 Graph Linear Equations

Graph $3x + 2y = 6$.

Rewrite the equation in slope-intercept form.

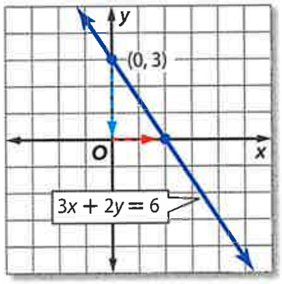
$3x + 2y = 6$	Original equation
$3x + 2y - 3x = 6 - 3x$	Subtract $3x$ from each side.
$2y = 6 - 3x$	Simplify.
$2y = -3x + 6$	$6 - 3x = 6 + (-3x)$ or $-3x + 6$
$\frac{2y}{2} = \frac{-3x + 6}{2}$	Divide each side by 2.
$y = -\frac{3}{2}x + 3$	Slope-intercept form

Now graph the equation. The slope is $-\frac{3}{2}$, and the y-intercept is 3.

Step 1 Plot the y-intercept (0, 3).

Step 2 The slope is $\frac{\text{rise}}{\text{run}} = -\frac{3}{2}$. From (0, 3), move down 3 units and right 2 units. Plot the point.

Step 3 Draw a line through the two points.



Study Tip

Counting and Direction When counting rise and run, a negative sign may be associated with the value in the numerator or denominator. If with the numerator, begin by counting down for the rise. If with the denominator, count left when counting the run. The resulting line will be the same.

Guided Practice

Graph each equation.

2A. $3x - 4y = 12$

2B. $-2x + 5y = 10$

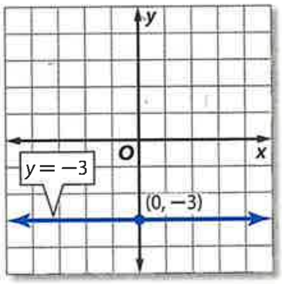
Except for the graph of $y = 0$, which lies on the x-axis, horizontal lines have a slope of 0. They are graphs of **constant functions**, which can be written in slope-intercept form as $y = 0x + b$ or $y = b$, where b is any number. Constant functions do not cross the x-axis. Their domain is all real numbers, and their range is b.

Example 3 Graph Linear Equations

Graph $y = -3$.

Step 1 Plot the y-intercept $(0, -3)$.

Step 2 The slope is 0. Draw a line through the points with y-coordinate -3.



Guided Practice

Graph each equation.

3A. $y = 5$

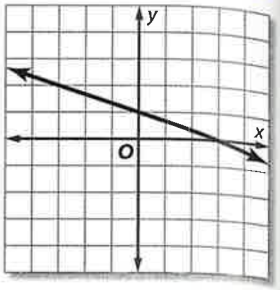
3B. $2y = 1$

Vertical lines have no slope. So, equations of vertical lines cannot be written in slope-intercept form.

There are times when you will need to write an equation when given a graph. To do this, locate the y-intercept and use the rise and run to find another point on the graph. Then write the equation in slope-intercept form.

Standardized Test Example 4 Write an Equation in Slope-Intercept Form

Which of the following is an equation in slope-intercept form for the line shown?



A $y = -3x + 1$

B $y = -3x + 3$

C $y = -\frac{1}{3}x + 1$

D $y = -\frac{1}{3}x + 3$

Test-Taking Tip

Eliminating Choices
Analyze the graph to determine the slope and the y-intercept. Then you can save time by eliminating answer choices that do not match the graph.

Read the Test Item

You need to find the slope and y-intercept of the line to write the equation.

Solve the Test Item

Step 1 The line crosses the y-axis at (0, 1), so the y-intercept is 1. The answer is either A or C.

Step 2 To get from (0, 1) to (3, 0), go down 1 unit and 3 units to the right. The slope is $-\frac{1}{3}$.

Step 3 Write the equation.

$$y = mx + b$$

$$y = -\frac{1}{3}x + 1$$

CHECK The graph also passes through (−3, 2). If the equation is correct, this should be a solution.

$$y = -\frac{1}{3}x + 1$$

$$2 \stackrel{?}{=} -\frac{1}{3}(-3) + 1$$

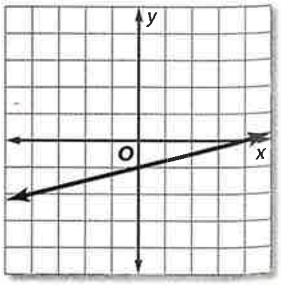
$$2 \stackrel{?}{=} 1 + 1$$

$$2 = 2 \checkmark$$

The answer is C.

GuidedPractice

4. Which of the following is an equation in slope-intercept form for the line shown?



F $y = \frac{1}{4}x - 1$

G $y = \frac{1}{4}x + 4$

H $y = 4x - 1$

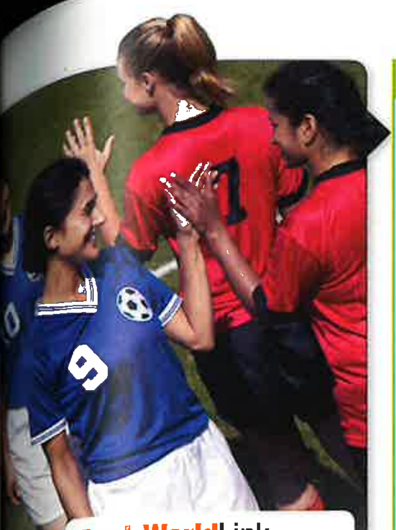
J $y = 4x + 4$

2 Modeling Real-World Data Real-world data can be modeled by a linear equation if there is a constant rate of change. The rate of change represents the slope. The y-intercept is the point where the value of the independent variable is 0.

Rea
In 19
girls
sport
comp
sport
avera
year
Source
School

C
Exar
Exar

Brand X Pictures/PunchStock



Real-WorldLink

In 1997, about 2.6 million girls competed in high school sports. The number of girls competing in high school sports has increased by an average of 0.06 million per year since 1997.

Source: National Federation of High School Associations

Real-World Example 5 Write and Graph a Linear Equation

SPORTS **Use the information at the left about high school sports.**

a. **Write a linear equation to find the number of girls in high school sports after 1997.**

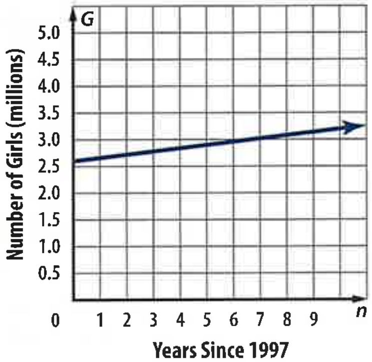
Words	Number of girls competing	equals	rate of change	times	number of years	plus	amount at start.
Variables	Let G = number of girls competing.				Let n = number of years since 1997.		
Equation	G	$=$	0.06	$\times$	n	$+$	2.6

The equation is $G = 0.06n + 2.6$.

b. **Graph the equation.**

The y-intercept is where the data begins. So, the graph passes through (0, 2.6).

The rate of change is the slope, so the slope is 0.06.



c. **Estimate the number of girls competing in 2017.**

The year 2017 is 20 years after 1997.

$G = 0.06n + 2.6$ Write the equation.

$= 0.06(20) + 2.6$ Replace n with 20.

$= 3.8$ Simplify.

There will be about 3.8 million girls competing in high school sports in 2017.

GuidedPractice

5. **FUNDRAISERS** The band boosters are selling sandwiches for \$5 each. They bought \$1160 in ingredients.

A. Write an equation for the profit P made on n sandwiches.

B. Graph the equation.

C. Find the total profit if 1400 sandwiches are sold.

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 **Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.**

① slope: 2, y-intercept: 4

2. slope: -5, y-intercept: 3

3. slope: $\frac{3}{4}$, y-intercept: -1

4. slope: $-\frac{5}{7}$, y-intercept: $-\frac{2}{3}$

Examples 2–3 **Graph each equation.**

5. $-4x + y = 2$

6. $2x + y = -6$

7. $-3x + 7y = 21$

8. $6x - 4y = 16$

9. $y = -1$

10. $15y = 3$

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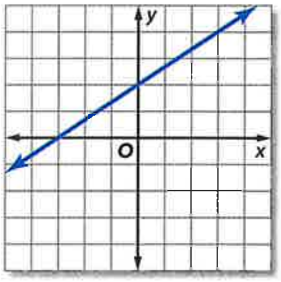
Example 4 **Write an equation in slope-intercept form for each graph shown.**

11.



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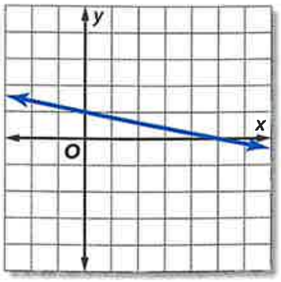

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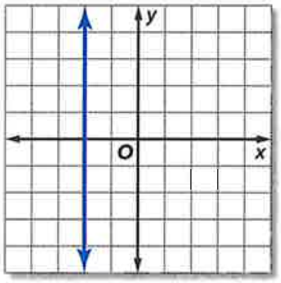

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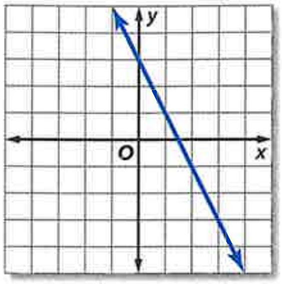


Example 5 **15.** **FINANCIAL LITERACY** Rondell is buying a new stereo system for his car using Jack's Stereo layaway plan.

Jack's Stereo Layaway Plan
\$75 down and
\$10 each week

a. Write an equation for the total amount S that he has paid after w weeks.

b. Graph the equation.

c. Find out how much Rondell will have paid after 8 weeks.

16. **CCSS** **REASONING** Ana is driving from her home in Miami, Florida, to her grandmother's house in New York City. On the first day, she will travel 240 miles to Orlando, Florida, to pick up her cousin. Then they will travel 350 miles each day.

a. Write an equation that models the total number of miles m Ana has traveled, if d represents the number of days after she picks up her cousin.

b. Graph the equation.

c. How long will the drive take if the total length of the trip is 1343 miles?

Practice and Problem Solving

Extra Practice is on page R4.

Example 1 **Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.**

17 slope: 5, y-intercept: 8
18. slope: 3, y-intercept: 10
19. slope: -4, y-intercept: 6
20. slope: -2, y-intercept: 8
21. slope: 3, y-intercept: -4
22. slope: 4, y-intercept: -6

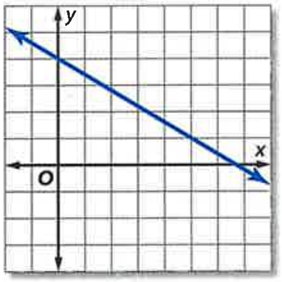
Examples 2–3 **Graph each equation.**

23. $-3x + y = 6$
24. $-5x + y = 1$
25. $-2x + y = -4$
26. $y = 7x - 7$
27. $5x + 2y = 8$
28. $4x + 9y = 27$
29. $y = 7$
30. $y = -\frac{2}{3}$
31. $21 = 7y$
32. $3y - 6 = 2x$

Example 4

Write an equation in slope-intercept form for each graph shown.

33.



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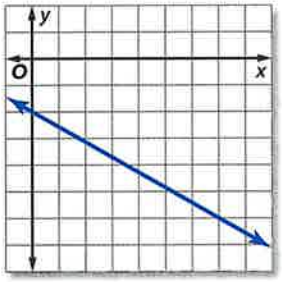

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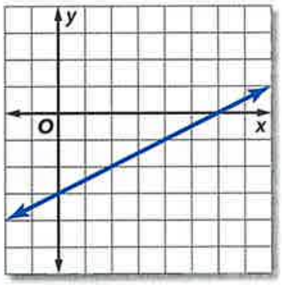

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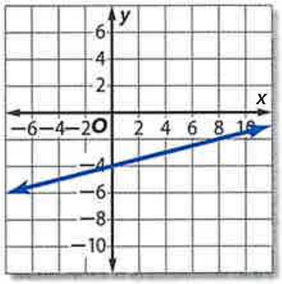


Example 5

37 **MANATEES** In 1991, 1267 manatees inhabited Florida's waters. The manatee population has increased at a rate of 123 manatees per year.

a. Write an equation for the manatee population, P, t years since 1991.

b. Graph this equation.

c. In 2006, the manatee was removed from Florida's endangered species list. What was the manatee population in 2006?

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

38. slope: $\frac{1}{2}$, y-intercept: -3

39. slope: $\frac{2}{3}$, y-intercept: -5

40. slope: $-\frac{5}{6}$, y-intercept: 5

41. slope: $-\frac{3}{7}$, y-intercept: 2

42. slope: 1, y-intercept: 4

43. slope: 0, y-intercept: 5

Graph each equation.

44. $y = \frac{3}{4}x - 2$

45. $y = \frac{5}{3}x + 4$

46. $3x + 8y = 32$

47. $5x - 6y = 36$

48. $-4x + \frac{1}{2}y = -1$

49. $3x - \frac{1}{4}y = 2$

50. **TRAVEL** A rental company charges \$8 per hour for a mountain bike plus a \$5 fee for a helmet.

a. Write an equation in slope-intercept form for the total rental cost C for a helmet and a bicycle for t hours.

b. Graph the equation.

c. What would the cost be for 2 helmets and 2 bicycles for 8 hours?

51. CCSS **REASONING** For Illinois residents, the average tuition at Chicago State University is \$157 per credit hour. Fees cost \$218 per year.

a. Write an equation in slope-intercept form for the tuition T for c credit hours.

b. Find the cost for a student who is taking 32 credit hours.

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

52. slope: -1, y-intercept: 0

53. slope: 0.5, y-intercept: 7.5

54. slope: 0, y-intercept: 7

55. slope: -1.5, y-intercept: -0.25

56. Write an equation of a horizontal line that crosses the y-axis at $(0, -5)$.

57. Write an equation of a line that passes through the origin and has a slope of 3.

58. **TEMPERATURE** The temperature dropped rapidly overnight. Starting at 80°F, the temperature dropped 3° per minute.

a. Draw a graph that represents this drop from 0 to 8 minutes.

b. Write an equation that describes this situation. Describe the meaning of each variable as well as the slope and y-intercept.

59. **FITNESS** Refer to the information at the right.

a. Write an equation that represents the cost C of a membership for m months.

b. What does the slope represent?

c. What does the C-intercept represent?

d. What is the cost of a two-year membership?

GET FIT GYM

startup fee **$145**

$45 monthly fee

60. **MAGAZINES** A teen magazine began with a circulation of 500,000 in its first year. Since then, the circulation has increased an average of 33,388 per year.

a. Write an equation that represents the circulation c after t years.

b. What does the slope represent?

c. What does the c-intercept represent?

d. If the magazine began in 1944, and this trend continues, in what year will the circulation reach 3,000,000?

61. **SMART PHONES** A telecommunications company sold 3305 smart phones in the first year of production. Suppose, on average, they expect to sell 25 phones per day.

a. Write an equation for the number of smart phones P sold t years after the first year of production, assuming 365 days per year.

b. If sales continue at this rate, how many years will it take for the company to sell 100,000 phones?

H.O.T. Problems Use Higher-Order Thinking Skills

62. **OPEN ENDED** Draw a graph representing a real-world linear function and write an equation for the graph. Describe what the graph represents.

63. **REASONING** Determine whether the equation of a vertical line can be written in slope-intercept form. Explain your reasoning.

64. **CHALLENGE** Summarize the characteristics that the graphs $y = 2x + 3$, $y = 4x + 3$, $y = -x + 3$, and $y = -10x + 3$ have in common.

65. CCSS **REGULARITY** If given an equation in standard form, explain how to determine the rate of change.

66. **WRITING IN MATH** Explain how you would use a given y-intercept and the slope to predict a y-value for a given x-value without graphing.

Standardized Test Practice

67. A music store has x CDs in stock. If 350 are sold and $3y$ are added to stock, which expression represents the number of CDs in stock?

A $350 + 3y - x$ **C** $x + 350 + 3y$
B $x - 350 + 3y$ **D** $3y - 350 - x$

68. PROBABILITY The table shows the result of a survey of favorite activities. What is the probability that a student's favorite activity is sports or drama club?

Extracurricular Activity	Students
art club	24
band	134
choir	37
drama club	46
mock trial	19
school paper	26
sports	314

F $\frac{3}{8}$ **G** $\frac{4}{9}$ **H** $\frac{3}{5}$ **J** $\frac{2}{3}$

69. A recipe for fruit punch calls for 2 ounces of orange juice for every 8 ounces of lemonade. If Jennifer uses 64 ounces of lemonade, which proportion can she use to find x, the number of ounces of orange juice needed?

A $\frac{2}{x} = \frac{64}{6}$ **C** $\frac{2}{8} = \frac{x}{64}$
B $\frac{8}{x} = \frac{64}{2}$ **D** $\frac{6}{2} = \frac{x}{64}$

70. EXTENDED RESPONSE The table shows the results of a canned food drive. 1225 cans were collected, and the 12th-grade class collected 55 more cans than the 10th-grade class. How many cans each did the 10th- and 12th-grade classes collect? Show your work.

Grade	Cans
9	340
10	x
11	280
12	y

Spiral Review

For each arithmetic sequence, determine the related function. Then determine if the function is *proportional* or *nonproportional*. (Lesson 3-6)

71. 3, 7, 11, … **72.** 8, 6, 4, … **73.** 0, 3, 6, … **74.** 1, 2, 3, …

75. GAME SHOWS Contestants on a game show win money by answering 10 questions. (Lesson 3-5)

a. Find the value of the 10th question.

b. If all questions are answered correctly, how much are the winnings?

Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve. (Lesson 3-4)

76. If $y = 10$ when $x = 5$, find y when $x = 6$.

77. If $y = -16$ when $x = 4$, find x when $y = 20$.

78. If $y = 6$ when $x = 18$, find y when $x = -12$.

79. If $y = 12$ when $x = 15$, find x when $y = -6$.

Skills Review

Find the slope of the line that passes through each pair of points.

80. (2, 3), (9, 7) **81.** (−3, 6), (2, 4) **82.** (2, 6), (−1, 3) **83.** (−3, 3), (1, 3)

LESSON 4-2 Writing Equations in Slope-Intercept Form

Then

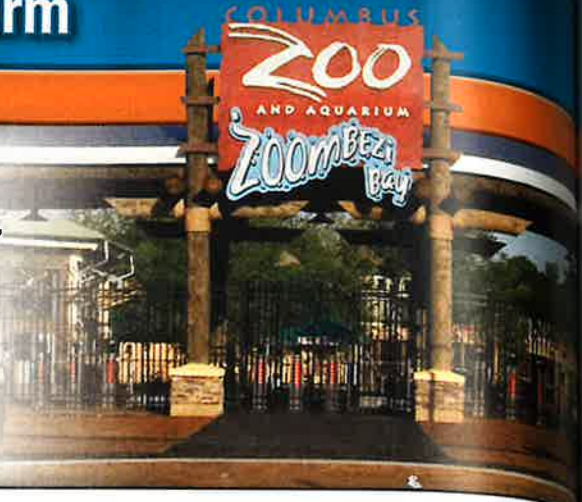
- You graphed lines given the slope and the y-intercept.

Now

1. Write an equation of a line in slope-intercept form given the slope and one point.
2. Write an equation of a line in slope-intercept form given two points.

Why?

- In 2006, the attendance at the Columbus Zoo and Aquarium was about 1.6 million. In 2009, the zoo's attendance was about 2.2 million. You can find the average rate of change for these data. Then you can write an equation that would model the average attendance at the zoo for a given year.



© Columbus Zoo and Aquarium

NewVocabulary

constraint
linear extrapolation

Common Core State Standards

Content Standards

F.BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

1 Write an Equation Given the Slope and a Point

The next example shows how to write an equation of a line if you are given a slope and a point other than the y-intercept.

Example 1 Write an Equation Given the Slope and a Point

PT

Write an equation of the line that passes through (2, 1) with a slope of 3.

You are given the slope but not the y-intercept.

Step 1 Find the y-intercept.

$y = mx + b$	Slope-intercept form
$1 = 3(2) + b$	Replace m with 3, y with 1, and x with 2.
$1 = 6 + b$	Simplify.
$1 - 6 = 6 + b - 6$	Subtract 6 from each side.
$-5 = b$	Simplify.

Step 2 Write the equation in slope-intercept form.

$y = mx + b$	Slope-intercept form
$y = 3x - 5$	Replace m with 3 and b with -5.

Therefore, the equation of the line is $y = 3x - 5$.

GuidedPractice

Write an equation of a line that passes through the given point and has the given slope.

1A. $(-2, 5)$, slope 3

1B. $(4, -7)$, slope -1

2 Write an Equation Given Two Points

If you are given two points through which a line passes, you can use them to find the slope first. Then follow the steps in Example 1 to write the equation.

Study Tip

Choosing a point Given two points on a line, you may select either point to be (x_1, y_1). Be sure to remain consistent throughout the problem.

Example 2 Write an Equation Given Two Points

Write an equation of the line that passes through each pair of points.

a. (3, 1) and (2, 4)

Step 1 Find the slope of the line containing the given points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \frac{4 - 1}{2 - 3}$ $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (2, 4)$

$= \frac{3}{-1}$ or -3 Simplify.

Step 2 Use either point to find the y-intercept.

$y = mx + b$ Slope-intercept form

$4 = (-3)(2) + b$ Replace m with -3, x with 2, and y with 4.

$4 = -6 + b$ Simplify.

$4 - (-6) = -6 + b - (-6)$ Subtract -6 from each side.

$10 = b$ Simplify.

Step 3 Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = -3x + 10$ Replace m with -3 and b with 10.

Therefore, the equation is $y = -3x + 10$.

Study Tip

Slope If the (x_1, y_1) coordinates are negative, be sure to account for both the negative signs and the subtraction symbols in the Slope Formula.

b. (−4, −2) and (−5, −6)

Step 1 Find the slope of the line containing the given points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \frac{-6 - (-2)}{-5 - (-4)}$ $(x_1, y_1) = (-4, -2)$ and $(x_2, y_2) = (-5, -6)$

$= \frac{-4}{-1}$ or 4 Simplify.

Step 2 Use either point to find the y-intercept.

$y = mx + b$ Slope-intercept form

$-2 = 4(-4) + b$ Replace m with 4, x with -4, and y with -2.

$-2 = -16 + b$ Simplify.

$-2 - (-16) = -16 + b - (-16)$ Subtract -16 from each side.

$14 = b$ Simplify.

Step 3 Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = 4x + 14$ Replace m with 4 and b with 14.

Therefore, the equation is $y = 4x + 14$.

Guided Practice

Write an equation of the line that passes through each pair of points.

2A. (−1, 12), (4, −8)

2B. (5, −8), (−7, 0)

© Columbus Zoo and Aquarium

In mathematics, a **constraint** is a condition that a solution must satisfy. Equations can be viewed as constraints in a problem situation. The solutions of the equation meet the constraints of the problem.

Real-WorldCareer

Ground Crew
Airline ground crew responsibilities include checking tickets, helping passengers with luggage, and making sure that baggage is loaded properly and secure. This job usually requires a high school diploma or GED.

Source: Airline Jobs

Real-World Example 3 Use Slope-Intercept Form

FLIGHTS **The table shows the number of domestic flights in the U.S. from 2004 to 2008. Write an equation that could be used to predict the number of flights if it continues to decrease at the same rate.**

Year	Flights (millions)
2004	9.97
2005	10.04
2006	9.71
2007	9.84
2008	9.37

Understand You know the number of flights for 2004–2008.

Plan Let x represent the number of years since 2000, and let y represent the number of flights. Write an equation of the line that passes through (4, 9.97) and (8, 9.37).

Solve Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{9.37 - 9.97}{8 - 4}$	Let $(x_1, y_1) = (4, 9.97)$ and $(x_2, y_2) = (8, 9.37)$.
$= -\frac{0.6}{4}$ or -0.15	Simplify.

Use (8, 9.37) to find the y-intercept of the line.

$y = mx + b$	Slope-intercept form
$9.37 = -0.15(8) + b$	Replace y with 9.37, m with -0.15, and x with 8.
$9.37 = -1.2 + b$	Simplify.
$10.57 = b$	Add 1.2 to each side.

Write the equation using $m = -0.15$ and $b = 10.57$.

$y = mx + b$	Slope-intercept form
$y = -0.15x + 10.57$	Replace m with -0.15 and b with 10.57.

Check Check your result by using the coordinates of the other point.

$y = -0.15x + 10.57$	Original equation
$9.97 \stackrel{?}{=} -0.15(4) + 10.57$	Replace y with 9.97 and x with 4.
$9.97 = 9.97$ ✓	Simplify.

GuidedPractice

3. **FINANCIAL LITERACY** In addition to his weekly salary, Ethan is paid \$16 per delivery. Last week, he made 5 deliveries, and his total pay was \$215. Write a linear equation to find Ethan's total weekly pay T if he makes d deliveries.

You can use a linear equation to make predictions about values that are beyond the range of the data. This process is called **linear extrapolation**.

Problem-SolvingTip

CCSS Precision Deciding whether an answer is reasonable is useful when an exact answer is not necessary.

Real-World Example 4 Predict from Slope-Intercept Form

FLIGHTS **Estimate the number of domestic flights in 2020.**

$y = -0.15x + 10.57$	Original equation
$= -0.15(20) + 10.57$ or 7.57 million	Replace x with 20.

GuidedPractice

4. **MONEY** Use the equation in Guided Practice 3 to predict how much money Ethan will earn in a week if he makes 8 deliveries.

Stephan Goerlich/age fotostock

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 **Write an equation of the line that passes through the given point and has the given slope.**

1. (3, −3), slope 3

2. (2, 4), slope 2

3. (1, 5), slope −1

4. (−4, 6), slope −2

Example 2 **Write an equation of the line that passes through each pair of points.**

5. (4, −3), (2, 3)

6. (−7, −3), (−3, 5)

7. (−1, 3), (0, 8)

8. (−2, 6), (0, 0)

Examples 3, 4 **9.** **WHITEWATER RAFTING** Ten people from a local youth group went to Black Hills Whitewater Rafting Tour Company for a one-day rafting trip. The group paid $425.

a. Write an equation in slope-intercept form to find the total cost C for p people.

b. How much would it cost for 15 people?

Practice and Problem Solving

Extra Practice is on page R4.

Example 1 **Write an equation of the line that passes through the given point and has the given slope.**

10. (3, 1), slope 2

11. (−1, 4), slope −1

12. (1, 0), slope 1

13. (7, 1), slope 8

14. (2, 5), slope −2

15. (2, 6), slope 2

Example 2 **Write an equation of the line that passes through each pair of points.**

16. (9, −2), (4, 3)

17. (−2, 5), (5, −2)

18. (−5, 3), (0, −7)

19. (3, 5), (2, −2)

20. (−1, −3), (−2, 3)

21. (−2, −4), (2, 4)

Examples 3, 4 **22.** **CCSS MODELING** Greg is driving a remote control car at a constant speed. He starts the timer when the car is 5 feet away. After 2 seconds the car is 35 feet away.

a. Write a linear equation to find the distance d of the car from Greg.

b. Estimate the distance the car has traveled after 10 seconds.

23. **ZOOS** Refer to the beginning of the lesson.

a. Write a linear equation to find the attendance (in millions) y after x years. Let x be the number of years since 2000.

b. Estimate the zoo's attendance in 2020.

24. **BOOKS** In 1904, a dictionary cost 30¢. Since then the cost of a dictionary has risen an average of 6¢ per year.

a. Write a linear equation to find the cost C of a dictionary y years after 1904.

b. If this trend continues, what will the cost of a dictionary be in 2020?

Write an equation of the line that passes through the given point and has the given slope.

25. (4, 2), slope $\frac{1}{2}$

26. (3, −2), slope $\frac{1}{3}$

27. (6, 4), slope $-\frac{3}{4}$

28. (2, −3), slope $\frac{2}{3}$

29. (2, −2), slope $\frac{2}{7}$

30. (−4, −2), slope $-\frac{3}{5}$

Stephan Goerlich/age fotostock

31. **DOGS** In 2001, there were about 56.1 thousand golden retrievers registered in the United States. In 2002, the number was 62.5 thousand.

a. Write a linear equation to find the number of thousands of golden retrievers G that will be registered in year t, where $t = 0$ is the year 2000.

b. Graph the equation.

c. Estimate the number of golden retrievers that will be registered in 2017.

32. **GYM MEMBERSHIPS** A local recreation center offers a yearly membership for $265. The center offers aerobics classes for an additional $5 per class.

a. Write an equation that represents the total cost of the membership.

b. Carly spent $500 one year. How many aerobics classes did she take?

33. **SUBSCRIPTION** A magazine offers an online subscription that allows you to view up to 25 archived articles free. To view 30 archived articles, you pay $49.15. To view 33 archived articles, you pay $57.40.

a. What is the cost of each archived article for which you pay a fee?

b. What is the cost of the magazine subscription?

Write an equation of the line that passes through the given points.

34. $(5, -2), (7, 1)$ **35** $(5, -3), (2, 5)$ **36.** $\left(\frac{5}{4}, 1\right), \left(-\frac{1}{4}, \frac{3}{4}\right)$ **37.** $\left(\frac{5}{12}, -1\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$

Determine whether the given point is on the line. Explain why or why not.

38. $(3, -1); y = \frac{1}{3}x + 5$ **39.** $(6, -2); y = \frac{1}{2}x - 5$

For Exercises 40–42, determine which equation best represents each situation. Explain the meaning of each variable.

A $y = -\frac{1}{3}x + 72$

B $y = 2x + 225$

C $y = 8x + 4$

40. **CONCERTS** Tickets to a concert cost $8 each plus a processing fee of $4 per order.

41. **FUNDRAISING** The freshman class has $225. They sell raffle tickets at $2 each to raise money for a field trip.

42. **POOLS** The current water level of a swimming pool in Tucson, Arizona, is 6 feet. The rate of evaporation is $\frac{1}{3}$ inch per day.

43. **CCSS SENSE-MAKING** A manufacturer implemented a program to reduce waste. In 1998 they sent 946 tons of waste to landfills. Each year after that, they reduced their waste by an average 28.4 tons.

a. How many tons were sent to the landfill in 2010?

b. In what year will it become impossible for this trend to continue? Explain.

44. **COMBINING FUNCTIONS** The parents of a college student open an account for her with a deposit of $5000, and they set up automatic deposits of $100 to the account every week.

a. Write a function $d(t)$ to express the amount of money in the account t weeks after the initial deposit.

b. The student plans on spending $600 the first week and $250 in each of the following weeks for room and board and other expenses. Write a function $w(t)$ to express the amount of money taken out of the account each week.

c. Find $B(t) = d(t) - w(t)$. What does this new function represent?

d. Will the student run out of money? If so, when?

㊺ **CONCERT TICKETS** Jackson is ordering tickets for a concert online. There is a processing fee for each order, and the tickets are \$52 each. Jackson ordered 5 tickets and the cost was \$275.

- **a.** Determine the processing fee. Write a linear equation to represent the total cost C for t tickets.
- **b.** Make a table of values for at least three other numbers of tickets.
- **c.** Graph this equation. Predict the cost of 8 tickets.

46. MUSIC A music store is offering a Frequent Buyers Club membership. The membership costs \$22 per year, and then a member can buy CDs at a reduced price. If a member buys 17 CDs in one year, the cost is \$111.25.

- **a.** Determine the cost of each CD for a member.
- **b.** Write a linear equation to represent the total cost y of a one year membership, if x CDs are purchased.
- **c.** Graph this equation.

H.O.T. Problems Use Higher-Order Thinking Skills

47. ERROR ANALYSIS Tess and Jacinta are writing an equation of the line through (3, −2) and (6, 4). Is either of them correct? Explain your reasoning.

Tess

$m = \frac{4-(-2)}{6-3} = \frac{6}{3}$ or 2

$y = mx + b$

$6 = 2(4) + b$

$6 = 8 + b$

$-2 = b$

$y = 2x - 2$

Jacinta

$m = \frac{4-(-2)}{6-3} = \frac{6}{3}$ or 2

$y = mx + b$

$-2 = 2(3) + b$

$-2 = 6 + b$

$-8 = b$

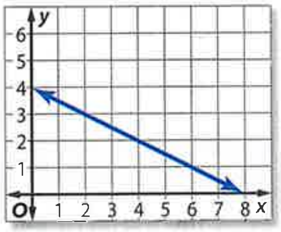
$y = 2x - 8$

48. CHALLENGE Consider three points, (3, 7), (−6, 1) and (9, p), on the same line. Find the value of p and explain your steps.

49. REASONING Consider the standard form of a linear equation, $Ax + By = C$.

- **a.** Rewrite the equation in slope-intercept form.
- **b.** What is the slope?
- **c.** What is the y-intercept?
- **d.** Is this true for all real values of A, B, and C?

50. OPEN ENDED Create a real-world situation that fits the graph at the right. Define the two quantities and describe the functional relationship between them. Write an equation to represent this relationship and describe what the slope and y-intercept mean.



51. WRITING IN MATH Linear equations are useful in predicting future events. Describe some factors in real-world situations that might affect the reliability of the graph in making predictions.

52. CCSS ARGUMENTS What information is needed to write the equation of a line? Explain.

Standardized Test Practice

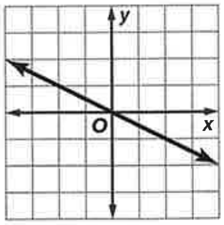
53. Which equation *best* represents the graph?

A $y = 2x$

B $y = -2x$

C $y = \frac{1}{2}x$

D $y = -\frac{1}{2}x$



54. Roberto receives an employee discount of 12%. If he buys a \$355 item at the store, what is his discount to the nearest dollar?

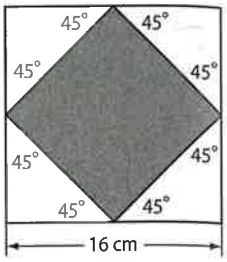
F \$3

G \$4

H \$30

J \$43

55. GEOMETRY The midpoints of the sides of the large square are joined to form a smaller square. What is the area of the smaller square?



A 64 cm^2

B 128 cm^2

C 248 cm^2

D 256 cm^2

56. SHORT RESPONSE If $\frac{5(x + 4)}{2} + 7 = 37$, what is the value of $3x - 9$?

Spiral Review

Graph each equation. (Lesson 4-1)

57. $y = 3x + 2$

58. $y = -4x + 2$

59. $3y = 2x + 6$

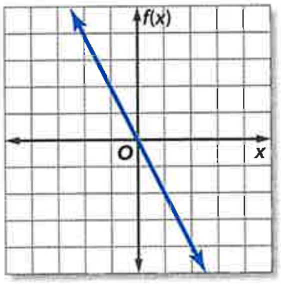
60. $y = \frac{1}{2}x + 6$

61. $3x + y = -1$

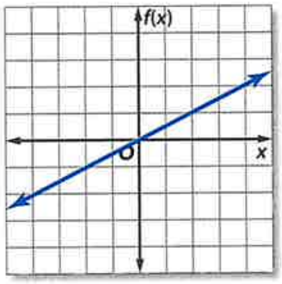
62. $2x + 3y = 6$

Write an equation in function notation for each relation. (Lesson 3-6)

63.



64.



65. METEOROLOGY The distance d in miles that the sound of thunder travels in t seconds is given by the equation $d = 0.21t$. (Lesson 3-4)

a. Graph the equation.

b. Use the graph to estimate how long it will take you to hear thunder from a storm 3 miles away.

Solve each equation. Check your solution. (Lesson 2-3)

66. $-5t - 2.2 = -2.9$

67. $-5.5a - 43.9 = 77.1$

68. $4.2r + 7.14 = 12.6$

69. $-14 - \frac{n}{9} = 9$

70. $\frac{-8b - (-9)}{-10} = 17$

71. $9.5x + 11 - 7.5x = 14$

Skills Review

Find the value of r so the line through each pair of points has the given slope.

72. $(6, -2), (r, -6), m = 4$

73. $(8, 10), (r, 4), m = 6$

74. $(7, -10), (r, 4), m = -3$

75. $(6, 2), (9, r), m = -1$

76. $(9, r), (6, 3), m = -\frac{1}{3}$

77. $(5, r), (2, -3), m = \frac{4}{3}$

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LESSON 4-3 Writing Equations in Point-Slope Form

Then

- You wrote linear equations given either one point and the slope or two points.

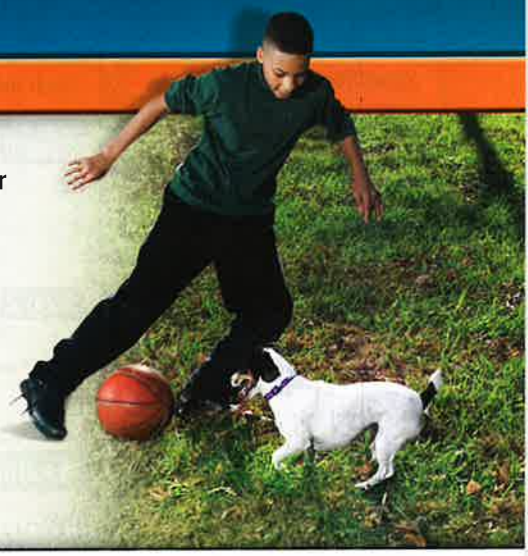
Now

- **1** Write equations of lines in point-slope form.
- **2** Write linear equations in different forms.

Why?

- Most humane societies have foster homes for newborn puppies, kittens, and injured or ill animals. During the spring and summer, a large shelter can place 3000 animals in homes each month.

  If a shelter had 200 animals in foster homes at the beginning of spring, the number of animals in foster homes at the end of the summer could be represented by $y = 3000x + 200$, where x is the number of months and y is the number of animals.



NewVocabulary
point-slope form

Common Core State Standards

Content Standards

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

2 Reason abstractly and quantitatively.

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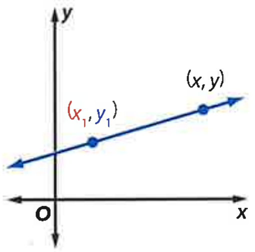
1 Point-Slope Form

Point-Slope Form An equation of a line can be written in **point-slope form** when given the coordinates of one known point on a line and the slope of that line.

KeyConcept Point-Slope Form

Words The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Symbols $y - y_1 = m(x - x_1)$



Example 1 Write and Graph an Equation in Point-Slope Form

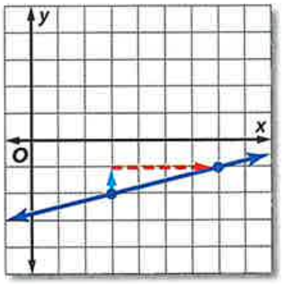
Write an equation in point-slope form for the line that passes through (3, −2) with a slope of $\frac{1}{4}$. Then graph the equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = \frac{1}{4}(x - 3)$ $(x_1, y_1) = (3, -2)$, $m = \frac{1}{4}$

$y + 2 = \frac{1}{4}(x - 3)$ Simplify.

Plot the point at (3, −2) and use the slope to find another point on the line. Draw a line through the two points.



GuidedPractice

1\. Write an equation in point-slope form for the line that passes through (−2, 1) with a slope of −6. Then graph the equation.

2 Forms of Linear Equations

If you are given the slope and the coordinates of one or two points, you can write the linear equation in the following ways.

StudyTip

Slope The slope of the line remains unchanged throughout the line. You can go in either direction along the line using the same rise over run and you will always end at a point on the line.

ConceptSummary Writing Equations

Given the Slope and One Point	Given Two Points
Step 1 Substitute the value of m and let the x and y coordinates be (x_1, y_1). Or, substitute the value of m, x, and y into the slope-intercept form and solve for b.	**Step 1** Find the slope.
Step 2 Rewrite the equation in the needed form.	**Step 2** Choose one of the two points to use.
	Step 3 Follow the steps for writing an equation given the slope and one point.

ReviewVocabulary

Standard form of a linear equation $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1

Example 2 Standard Form

Write $y - 1 = -\frac{2}{3}(x - 5)$ in standard form.

$y - 1 = -\frac{2}{3}(x - 5)$ Original equation

$3(y - 1) = 3\left(-\frac{2}{3}\right)(x - 5)$ Multiply each side by 3 to eliminate the fraction.

$3(y - 1) = -2(x - 5)$ Simplify.

$3y - 3 = -2x + 10$ Distributive Property

$3y = -2x + 13$ Add 3 to each side.

$2x + 3y = 13$ Add $2x$ to each side.

GuidedPractice

2. Write $y - 1 = 7(x + 5)$ in standard form.

To find the y-intercept of an equation, rewrite the equation in slope-intercept form.

Example 3 Slope-Intercept Form

Write $y + 3 = \frac{3}{2}(x + 1)$ in slope-intercept form.

$y + 3 = \frac{3}{2}(x + 1)$ Original equation

$y + 3 = \frac{3}{2}x + \frac{3}{2}$ Distributive Property

$y = \frac{3}{2}x - \frac{3}{2}$ Subtract 3 from each side.

GuidedPractice

3. Write $y + 6 = -3(x - 4)$ in slope-intercept form.

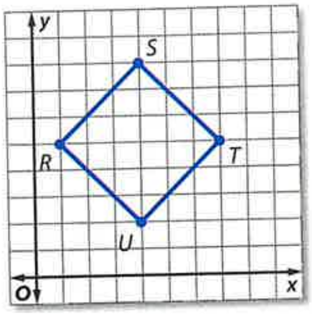
Being able to use a variety of forms of linear equations can be useful in other subjects as well.

StudyTip

Slopes in Squares Nonvertical opposite sides of a square have equal slopes. If the coordinates for one of the vertices are unavailable, use the slope of the opposite side.

Example 4 Point-Slope Form and Standard Form

GEOMETRY The figure shows square $RSTU$.



a. Write an equation in point-slope form for the line containing side $\overline{TU}$.

Step 1 Find the slope of $\overline{TU}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{5 - 2}{7 - 4}$ or 1	$(x_1, y_1) = (4, 2)$ and $(x_2, y_2) = (7, 5)$

Step 2 You can select either point for (x_1, y_1) in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = 1(x - 4)$	$(x_1, y_1) = (4, 2)$
$y - 5 = 1(x - 7)$	$(x_1, y_1) = (7, 5)$

b. Write an equation in standard form for the same line.

$y - 2 = 1(x - 4)$	Original equation	$y - 5 = 1(x - 7)$
$y - 2 = 1x - 4$	Distributive Property	$y - 5 = 1x - 7$
$y = 1x - 2$	Add to each side.	$y = 1x - 2$
$-1x + y = -2$	Subtract $1x$ from each side.	$-1x + y = -2$
$x - y = 2$	Multiply each side by -1.	$x - y = 2$

GuidedPractice

4A. Write an equation in point-slope form of the line containing side $\overline{ST}$.

4B. Write an equation in standard form of the line containing $\overline{ST}$.

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 Write an equation in point-slope form for the line that passes through the given point with the slope provided. Then graph the equation.

① $(-2, 5)$, slope -6

2. $(-2, -8)$, slope $\frac{5}{6}$

3. $(4, 3)$, slope $-\frac{1}{2}$

Example 2 Write each equation in standard form.

4. $y + 2 = \frac{7}{8}(x - 3)$

5. $y + 7 = -5(x + 3)$

6. $y + 2 = \frac{5}{3}(x + 6)$

Example 3 Write each equation in slope-intercept form.

7. $y - 10 = 4(x + 6)$

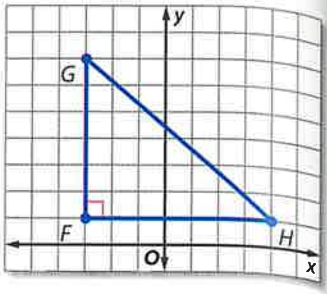
8. $y - 7 = -\frac{3}{4}(x + 5)$

9. $y - 9 = x + 4$

Example 4 **10.** **GEOMETRY** Use right triangle *FGH*.

a. Write an equation in point-slope form for the line containing $\overline{GH}$.

b. Write the standard form of the line containing $\overline{GH}$.



Practice and Problem Solving

Extra Practice is on page R4.

Example 1 **Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.**

11. $(5, 3), m = 7$

12. $(2, -1), m = -3$

13. $(-6, -3), m = -1$

14. $(-7, 6), m = 0$

15. $(-2, 11), m = \frac{4}{3}$

16. $(-6, -8), m = -\frac{5}{8}$

17. $(-2, -9), m = -\frac{7}{5}$

18. $(-6, 0)$, horizontal line

Example 2 **Write each equation in standard form.**

19. $y - 10 = 2(x - 8)$

20. $y - 6 = -3(x + 2)$

21. $y - 9 = -6(x + 9)$

22. $y + 4 = \frac{2}{3}(x + 7)$

23. $y + 7 = \frac{9}{10}(x + 3)$

24. $y + 7 = -\frac{3}{2}(x + 1)$

25. $2y + 3 = -\frac{1}{3}(x - 2)$

26. $4y - 5x = 3(4x - 2y + 1)$

Example 3 **Write each equation in slope-intercept form.**

27. $y - 6 = -2(x - 7)$

28. $y - 11 = 3(x + 4)$

29. $y + 5 = -6(x + 7)$

30. $y - 1 = \frac{4}{5}(x + 5)$

31. $y + 2 = \frac{1}{6}(x - 4)$

32. $y + 6 = -\frac{3}{4}(x + 8)$

33. $y + 3 = -\frac{1}{3}(2x + 6)$

34. $y + 4 = 3(3x + 3)$

Example 4 **(35)** **MOVIE RENTALS** The number of copies of a movie rented at a video kiosk decreased at a constant rate of 5 copies per week. The 6th week after the movie was released, 4 copies were rented. How many copies were rented during the second week?

36. **CCSS REASONING** A company offers premium cable for \$39.95 per month plus a one-time setup fee. The total cost for setup and 6 months of service is \$264.70.

a. Write an equation in point-slope form to find the total price y for any number of months x. (*Hint*: The point (6, 264.70) is a solution to the equation.)

b. Write the equation in slope-intercept form.

c. What is the setup fee?

Write an equation for the line described in standard form.

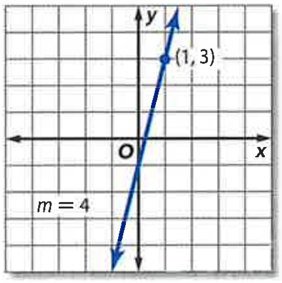
37. through $(-1, 7)$ and $(8, -2)$

38. through $(-4, 3)$ with y-intercept 0

39. with x-intercept 4 and y-intercept 5

Write an equation in point-slope form for each line.

40.



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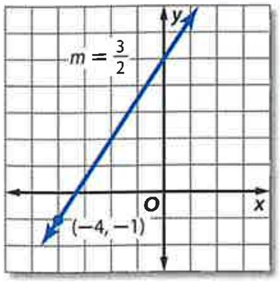

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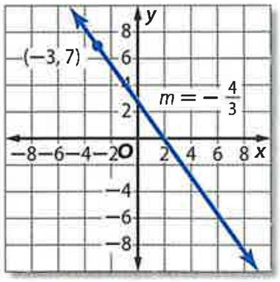


Write each equation in slope-intercept form.

43 $y + \frac{3}{5} = x - \frac{2}{5}$

44. $y - \frac{7}{2} = \frac{1}{2}(x - 4)$

45. $y + \frac{1}{3} = \frac{5}{6}\left(x + \frac{2}{5}\right)$

46. Write an equation in point-slope form, slope-intercept form, and standard form for a line that passes through $(-2, 8)$ with slope $\frac{8}{5}$.

47. Line ℓ passes through $(-9, 4)$ with slope $\frac{4}{7}$. Write an equation in point-slope form, slope-intercept form, and standard form for line ℓ.

48. **WEATHER** The barometric pressure is 598 millimeters of mercury (mmHg) at an altitude of 1.8 kilometers and 577 millimeters of mercury at 2.1 kilometers.

a. Write a formula for the barometric pressure as a function of the altitude.

b. What is the altitude if the pressure is 657 millimeters of mercury?

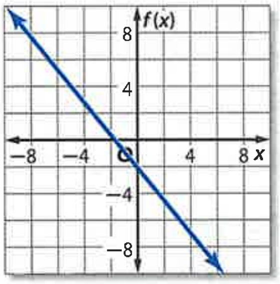
H.O.T. Problems Use Higher-Order Thinking Skills

49. **WHICH ONE DOESN'T BELONG?** Identify the equation that does not belong. Explain your reasoning.

$y - 5 = 3(x - 1)$	$y + 1 = 3(x + 1)$	$y + 4 = 3(x + 1)$	$y - 8 = 3(x - 2)$

50. **CCSS CRITIQUE** Juana thinks that $f(x)$ and $g(x)$ have the same slope but different intercepts. Sabrina thinks that $f(x)$ and $g(x)$ describe the same line. Is either of them correct? Explain your reasoning.

The graph of $g(x)$ is the line that passes through $(3, -7)$ and $(-6, 4)$.



51. **OPEN ENDED** Describe a real-life scenario that has a constant rate of change and a value of y for a particular value of x. Represent this situation using an equation in point-slope form, an equation in standard form, and an equation in slope-intercept form.

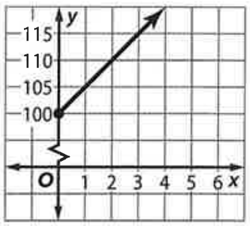
52. **REASONING** Write an equation for the line that passes through $(-4, 8)$ and $(3, -7)$. What is the slope? Where does the line intersect the x-axis? the y-axis?

53. **CHALLENGE** Write an equation in point-slope form for the line that passes through the points (f, g) and (h, j).

54. **WRITING IN MATH** Why do we represent linear equations in more than one form?

Standardized Test Practice

55. Which statement is *most* strongly supported by the graph?



A You have \$100 and spend \$5 weekly.

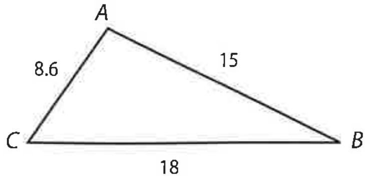
B You have \$100 and save \$5 weekly.

C You need \$100 for a new CD player and save \$5 weekly.

D You need \$100 for a new CD player and spend \$5 weekly.

56. SHORT RESPONSE A store offers customers a \$5 gift certificate for every \$75 they spend. How much would a customer have to spend to earn \$35 worth of gift certificates?

57. GEOMETRY Which triangle is similar to $\triangle ABC$?



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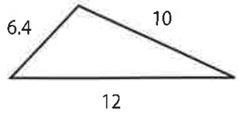

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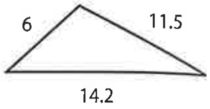

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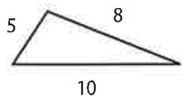

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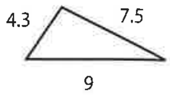


58. In a class of 25 students, 6 have blue eyes, 15 have brown hair, and 3 have blue eyes and brown hair. How many students have neither blue eyes nor brown hair?

A 4

B 7

C 10

D 22

Spiral Review

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

59. $(4, 2), (-2, -4)$

60. $(3, -2), (6, 4)$

61. $(-1, 3), (2, -3)$

62. $(2, -2), (3, 2)$

63. $(7, -2), (-4, -2)$

64. $(0, 5), (-3, 5)$

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept. (Lesson 4-1)

65. slope: -2, y-intercept: 6

66. slope: 3, y-intercept: -5

67. slope: $\frac{1}{2}$, y-intercept: 3

68. slope: $-\frac{3}{5}$, y-intercept: 12

69. slope: 0, y-intercept: 3

70. slope: -1, y-intercept: 0

71. THEATER The Coral Gables Actors' Playhouse has 7 rows of seats in the orchestra section. The number of seats in the rows forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section. Was the section oversold? (Lesson 3-5)

Rows	Number of Seats
7	76
6	68
5	60

Skills Review

Solve each equation or formula for the variable specified.

72. $y = mx + b$, for m

73. $v = r + at$, for a

74. $km + 5x = 6y$, for m

75. $4b - 5 = -t$, for b

LESSON 4-4

Parallel and Perpendicular Lines

:: Then

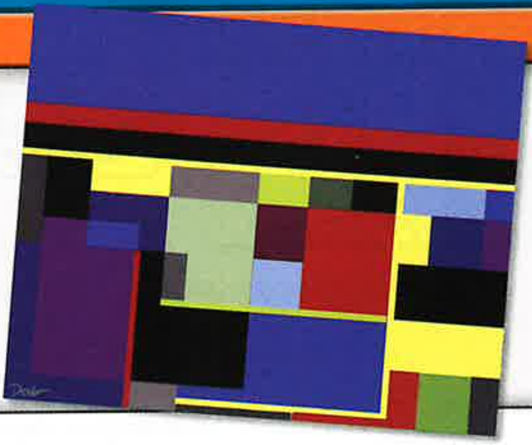
- You wrote equations in point-slope form.

:: Now

- **1** Write an equation of the line that passes through a given point, parallel to a given line.
- **2** Write an equation of the line that passes through a given point, perpendicular to a given line.

:: Why?

- Notice the squares, rectangles and lines in the piece of art shown at the right. Some of the lines intersect forming right angles. Other lines do not intersect at all.



New Vocabulary

parallel lines
perpendicular lines

Common Core State Standards

Content Standards

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

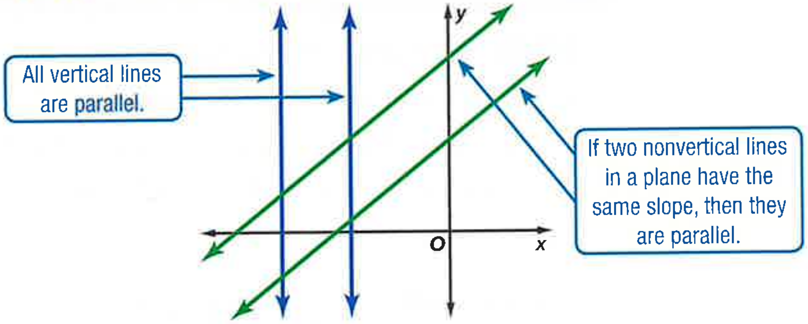
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Mathematical Practices

5 Use appropriate tools strategically.

1 Parallel Lines

Lines in the same plane that do not intersect are called **parallel lines**. Nonvertical parallel lines have the same slope.



You can write an equation of a line parallel to a given line if you know a point on the line and an equation of the given line. First find the slope of the given line. Then, substitute the point provided and the slope from the given line into the point-slope form.

PT

Example 1 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through $(-3, 5)$ and is parallel to the graph of $y = 2x - 4$.

Step 1 The slope of the line with equation $y = 2x - 4$ is 2. The line parallel to $y = 2x - 4$ has the same slope, 2.

Step 2 Find the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 5 = 2[x - (-3)]$ Replace m with 2 and (x_1, y_1) with $(-3, 5)$.

$y - 5 = 2(x + 3)$ Simplify.

$y - 5 = 2x + 6$ Distributive Property

$y - 5 + 5 = 2x + 6 + 5$ Add 5 to each side.

$y = 2x + 11$ Write the equation in slope-intercept form.

Guided Practice

1. Write an equation in point-slope form for the line that passes through $(4, -1)$ and is parallel to the graph of $y = \frac{1}{4}x + 7$.

Purestock/Getty Images

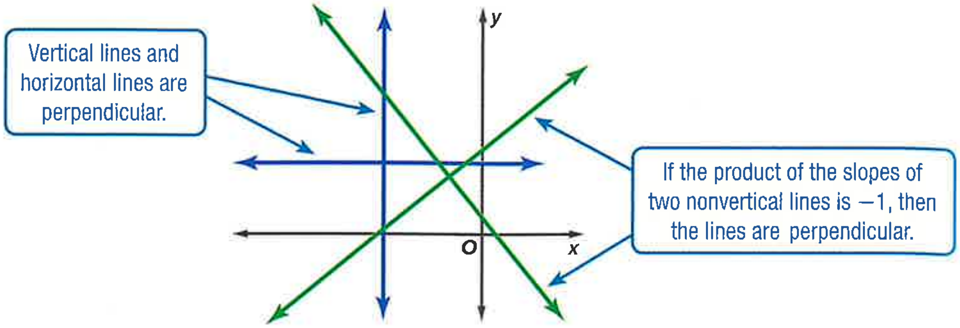
ReviewVocabulary

opposite reciprocals

The opposite reciprocal of $\frac{a}{b}$ is $-\frac{b}{a}$. Their product is -1.

2 Perpendicular Lines

Lines that intersect at right angles are called **perpendicular lines**. The slopes of nonvertical perpendicular lines are opposite reciprocals. That is, if the slope of a line is 4, the slope of the line perpendicular to it is $-\frac{1}{4}$.

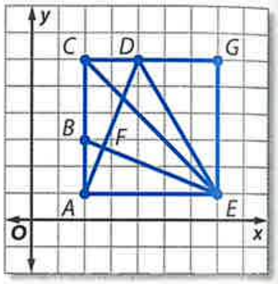

Vertical lines and horizontal lines are perpendicular.

If the product of the slopes of two nonvertical lines is -1, then the lines are perpendicular.

You can use slope to determine whether two lines are perpendicular.

Real-World Example 2 Slopes of Perpendicular Lines

DESIGN **The outline of a company's new logo is shown on a coordinate plane.**



a. Is $\angle DFE$ a right angle in the logo?

If $\overline{BE}$ and $\overline{AD}$ are perpendicular, then $\angle DFE$ is a right angle. Find the slopes of $\overline{BE}$ and $\overline{AD}$.

slope of $\overline{BE}$: $m = \frac{1-3}{7-2}$ or $-\frac{2}{5}$

slope of $\overline{AD}$: $m = \frac{6-1}{4-2}$ or $\frac{5}{2}$

The line segments are perpendicular because $-\frac{2}{5} \times \frac{5}{2} = -1$. Therefore, $\angle DFE$ is a right angle.

b. Is each pair of opposite sides parallel?

If a pair of opposite sides are parallel, then they have the same slope.

slope of $\overline{AC}$: $m = \frac{6-1}{2-2}$ or undefined

Since $\overline{AC}$ and $\overline{GE}$ are both parallel to the y-axis, they are vertical and are therefore parallel.

slope of $\overline{CG}$: $m = \frac{6-6}{7-2}$ or 0

Since $\overline{CG}$ and $\overline{AE}$ are both parallel to the x-axis, they are horizontal and are therefore parallel.

GuidedPractice

2. **CONSTRUCTION** On the plans for a treehouse, a beam represented by $\overline{QR}$ has endpoints $Q(-6, 2)$ and $R(-1, 8)$. A connecting beam represented by $\overline{ST}$ has endpoints $S(-3, 6)$ and $T(-8, 5)$. Are the beams perpendicular? Explain.

Real-WorldLink

Though treehouses are typically built for recreational purposes, they were originally designed as a way to be protected from wild animals, dense population, and other threats.

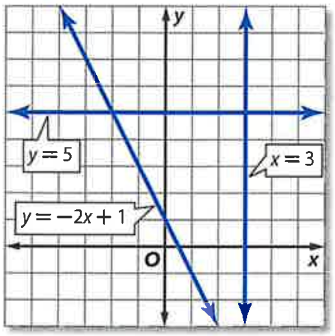
Source: *The Treehouse Book*

Robert Holmgren/Stone+/Getty Images

You can determine whether the graphs of two linear equations are parallel or perpendicular by comparing the slopes of the lines.

Example 3 Parallel or Perpendicular Lines

Determine whether the graphs of $y = 5$, $x = 3$, and $y = -2x + 1$ are *parallel* or *perpendicular*. Explain.



Graph each line on a coordinate plane.

From the graph, you can see that $y = 5$ is parallel to the x-axis and $x = 3$ is parallel to the y-axis. Therefore, they are perpendicular. None of the lines are parallel.

GuidedPractice

3. Determine whether the graphs of $6x - 2y = -2$, $y = 3x - 4$, and $y = 4$ are *parallel* or *perpendicular*. Explain.

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

StudyTip

CCSS Tools Graph the given equation on a coordinate grid and plot the given point. Using a ruler, draw a line perpendicular to the given line that passes through the point.

Example 4 Perpendicular Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through $(-4, 6)$ and is perpendicular to the graph of $2x + 3y = 12$.

Step 1 Find the slope of the given line by solving the equation for y.

$$2x + 3y = 12 \quad \text{Original equation}$$

$$2x - 2x + 3y = -2x + 12 \quad \text{Subtract } 2x \text{ from each side.}$$

$$3y = -2x + 12 \quad \text{Simplify.}$$

$$\frac{3y}{3} = \frac{-2x + 12}{3} \quad \text{Divide each side by 3.}$$

$$y = -\frac{2}{3}x + 4 \quad \text{Simplify.}$$

The slope is $-\frac{2}{3}$.

Step 2 The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{3}$ or $\frac{3}{2}$. Find the equation of the perpendicular line.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 6 = \frac{3}{2}[x - (-4)] \quad (x_1, y_1) = (-4, 6) \text{ and } m = \frac{3}{2}$$

$$y - 6 = \frac{3}{2}(x + 4) \quad \text{Simplify.}$$

$$y - 6 = \frac{3}{2}x + 6 \quad \text{Distributive Property}$$

$$y - 6 + 6 = \frac{3}{2}x + 6 + 6 \quad \text{Add 6 to each side.}$$

$$y = \frac{3}{2}x + 12 \quad \text{Simplify.}$$

GuidedPractice

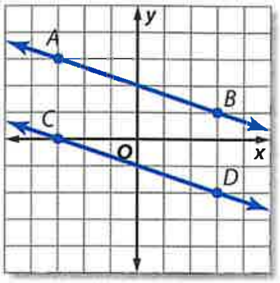
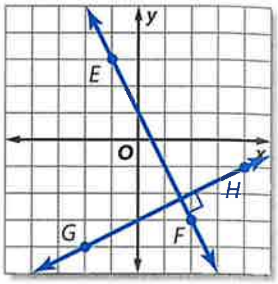
4. Write an equation in slope-intercept form for the line that passes through $(4, 7)$ and is perpendicular to the graph of $y = \frac{2}{3}x - 1$.

Robert Holmgren/Stone+/Getty Images

ReadingMath

Parallel and Perpendicular Lines The symbol for parallel is ||. The symbol for perpendicular is ⊥.

ConceptSummary Parallel and Perpendicular Lines

	Parallel Lines	Perpendicular Lines
Words	Two nonvertical lines are parallel if they have the same slope.	Two nonvertical lines are perpendicular if the product of their slopes is −1.
Symbols	$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	$\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$
Models		

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

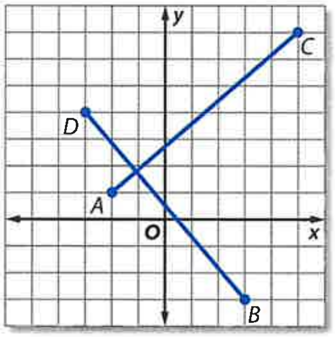
Example 1 **Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.**

1. $(-1, 2), y = \frac{1}{2}x - 3$

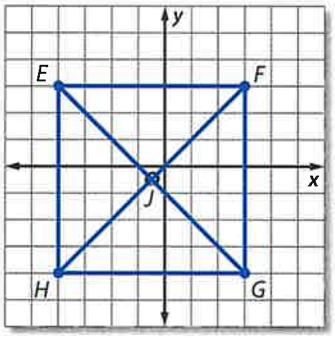
2. $(0, 4), y = -4x + 5$

Example 2

3. **GARDENS** A garden is in the shape of a quadrilateral with vertices $A(-2, 1)$, $B(3, -3)$, $C(5, 7)$, and $D(-3, 4)$. Two paths represented by $\overline{AC}$ and $\overline{BD}$ cut across the garden. Are the paths perpendicular? Explain.



4. CCSS **PRECISION** A square is a quadrilateral that has opposite sides parallel, consecutive sides that are perpendicular, and diagonals that are perpendicular. Determine whether the quadrilateral is a square. Explain.



Example 3 **Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.**

5. $y = -2x, 2y = x, 4y = 2x + 4$

6. $y = \frac{1}{2}x, 3y = x, y = -\frac{1}{2}x$

Example 4 **Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.**

7. $(-2, 3), y = -\frac{1}{2}x - 4$

8. $(-1, 4), y = 3x + 5$

9. $(2, 3), 2x + 3y = 4$

10. $(3, 6), 3x - 4y = -2$

Practice and Problem Solving

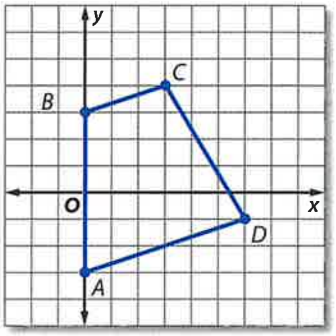
Extra Practice is on page R4.

Example 1 **Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.**

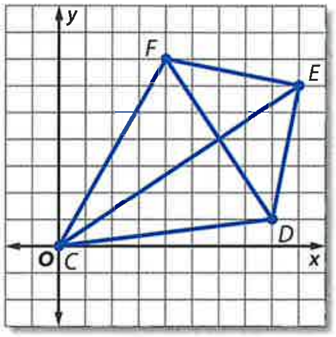
11. $(3, -2), y = x + 4$ **12.** $(4, -3), y = 3x - 5$ **13.** $(0, 2), y = -5x + 8$

14. $(-4, 2), y = -\frac{1}{2}x + 6$ **15.** $(-2, 3), y = -\frac{3}{4}x + 4$ **16.** $(9, 12), y = 13x - 4$

Example 2 **17. GEOMETRY** A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Is *ABCD* a trapezoid? Explain your reasoning.



18. GEOMETRY *CDEF* is a kite. Are the diagonals of the kite perpendicular? Explain your reasoning.



19. Determine whether the graphs of $y = -6x + 4$ and $y = \frac{1}{6}x$ are perpendicular. Explain.

20. MAPS On a map, Elmwood Drive passes through $R(4, -11)$ and $S(0, -9)$, and Taylor Road passes through $J(6, -2)$ and $K(4, -5)$. If they are straight lines, are the two streets perpendicular? Explain.

Example 3 **CCSS PERSEVERANCE** **Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.**

21. $2x - 8y = -24, 4x + y = -2, x - 4y = 4$

22. $3x - 9y = 9, 3y = x + 12, 2x - 6y = 12$

Example 4 **Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.**

23 $(-3, -2), y = -2x + 4$ **24.** $(-5, 2), y = \frac{1}{2}x - 3$ **25.** $(-4, 5), y = \frac{1}{3}x + 6$

26. $(2, 6), y = -\frac{1}{4}x + 3$ **27.** $(3, 8), y = 5x - 3$ **28.** $(4, -2), y = 3x + 5$

Write an equation in slope-intercept form for a line perpendicular to the graph of the equation that passes through the *x*-intercept of that line.

29. $y = -\frac{1}{2}x - 4$ **30.** $y = \frac{2}{3}x - 6$ **31.** $y = 5x + 3$

32. Write an equation in slope-intercept form for the line that is perpendicular to the graph of $3x + 2y = 8$ and passes through the y-intercept of that line.

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

33. $y = 4x + 3$
$4x + y = 3$

34. $y = -2x$
$2x + y = 3$

35. $3x + 5y = 10$
$5x - 3y = -6$

36. $-3x + 4y = 8$
$-4x + 3y = -6$

37. $2x + 5y = 15$
$3x + 5y = 15$

38. $2x + 7y = -35$
$4x + 14y = -42$

39. Write an equation of the line that is parallel to the graph of $y = 7x - 3$ and passes through the origin.

40. EXCAVATION Scientists excavating a dinosaur mapped the site on a coordinate plane. If one bone lies from $(-5, 8)$ to $(10, -1)$ and a second bone lies from $(-10, -3)$ to $(-5, -6)$, are the bones parallel? Explain.

41. ARCHAEOLOGY In the ruins of an ancient civilization, an archaeologist found pottery at $(2, 6)$ and hair accessories at $(4, -1)$. A pole is found with one end at $(7, 10)$ and the other end at $(14, 12)$. Is the pole perpendicular to the line through the pottery and the hair accessories? Explain.

42. GRAPHICS To create a design on a computer, Andeana must enter the coordinates for points on the design. One line segment she drew has endpoints of $(-2, 1)$ and $(4, 3)$. The other coordinates that Andeana entered are $(2, -7)$ and $(8, -3)$. Could these points be the vertices of a rectangle? Explain.

43. MULTIPLE REPRESENTATIONS In this problem, you will explore parallel and perpendicular lines.

a. Graphical Graph the points $A(-3, 3)$, $B(3, 5)$, and $C(-4, 0)$ on a coordinate plane.

b. Analytical Determine the coordinates of a fourth point D that would form a parallelogram. Explain your reasoning.

c. Analytical What is the minimum number of points that could be moved to make the parallelogram a rectangle? Describe which points should be moved, and explain why.

H.O.T. Problems Use Higher-Order Thinking Skills

44. CHALLENGE If the line through $(-2, 4)$ and $(5, d)$ is parallel to the graph of $y = 3x + 4$, what is the value of d?

45. REASONING Which key features of the graphs of two parallel lines are the same, and which are different? Which key features of the graphs of two perpendicular lines are the same, and which are different?

46. OPEN ENDED Graph a line that is parallel and a line that is perpendicular to $y = 2x - 1$.

Example 3

47. CCSS CRITIQUE Carmen and Chase are finding an equation of the line that is perpendicular to the graph of $y = \frac{1}{3}x + 2$ and passes through the point $(-3, 5)$. Is either of them correct? Explain your reasoning.

Carmen

$y - 5 = -3[x - (-3)]$

$y - 5 = -3(x + 3)$

$y = -3x - 9 + 5$

$y = -3x - 4$

Chase

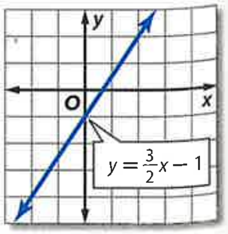
$y - 5 = 3[x - (-3)]$

$y - 5 = 3(x + 3)$

$y = 3x + 9 + 5$

$y = 3x + 14$

48. WRITING IN MATH Illustrate how you can determine whether two lines are parallel or perpendicular. Write an equation for the graph that is parallel and an equation for the graph that is perpendicular to the line shown. Explain your reasoning.



Standardized Test Practice

49. Which of the following is an algebraic translation of the following phrase?

5 less than the quotient of a number and 8

A $5 - \frac{n}{8}$ **C** $5 - \frac{8}{n}$

B $\frac{n}{8} - 5$ **D** $\frac{8}{n} - 5$

50. A line through which two points would be parallel to a line with a slope of $\frac{3}{4}$?

F (0, 5) and (−4, 2) **H** (0, 0) and (0, −2)

G (0, 2) and (−4, 1) **J** (0, −2) and (−4, −2)

51. Which equation best fits the data in the table?

A $y = x + 4$

B $y = 2x + 3$

C $y = 7$

D $y = 4x - 5$

x	y
1	5
2	7
3	9
4	11

52. SHORT RESPONSE Tyler is filling his 6000-gallon pool at a constant rate. After 4 hours, the pool contained 800 gallons. How many total hours will it take to completely fill the pool?

Spiral Review

Write each equation in standard form. (Lesson 4-3)

53. $y - 13 = 4(x - 2)$

54. $y - 5 = -2(x + 2)$

55. $y + 3 = -5(x + 1)$

56. $y + 7 = \frac{1}{2}(x + 2)$

57. $y - 1 = \frac{5}{6}(x - 4)$

58. $y - 2 = -\frac{2}{5}(x - 8)$

59. CANOE RENTAL Latanya and her friends rented a canoe for 3 hours and paid a total of $45. (Lesson 4-2)

a. Write a linear equation to find the total cost C of renting the canoe for h hours.

b. How much would it cost to rent the canoe for 8 hours?

Write an equation of the line that passes through each point with the given slope. (Lesson 4-2)

60. $(5, -2), m = 3$

61. $(-5, 4), m = -5$

62. $(3, 0), m = -2$

63. $(3, 5), m = 2$

64. $(-3, -1), m = -3$

65. $(-2, 4), m = -5$

Simplify each expression. If not possible, write *simplified*. (Lesson 1-4)

66. $13m + m$

67. $14a^2 + 13b^2 + 27$

68. $3(x + 2x)$

69. FINANCIAL LITERACY At a Farmers' Market, merchants can rent a small table for $5.00 and a large table for $8.50. One time, 25 small and 10 large tables were rented. Another time, 35 small and 12 large were rented. (Lesson 1-2)

a. Write an algebraic expression to show the total amount of money collected.

b. Evaluate the expression.

Skills Review

Express each relation as a graph. Then determine the domain and range.

70. $\{(3, 8), (3, 7), (2, -9), (1, -9), (-5, -3)\}$

71. $\{(3, 4), (4, 3), (2, 2), (5, -4), (-4, 5)\}$

72. $\{(0, 2), (-5, 1), (0, 6), (-1, 9), (-4, -5)\}$

73. $\{(-7, 6), (-3, -4), (4, -5), (-2, 6), (-3, 2)\}$

LESSON 4-5

Scatter Plots and Lines of Fit

Then

- You wrote linear equations given a point and the slope.

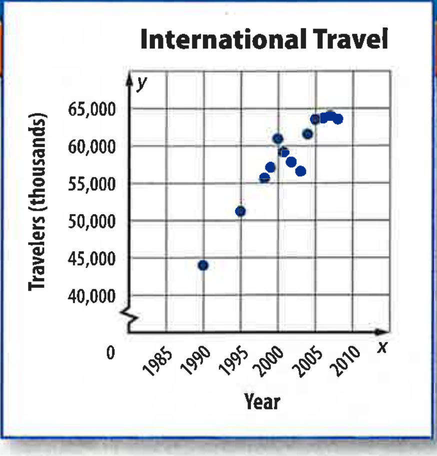
Now

- **1** Investigate relationships between quantities by using points on scatter plots.

  2 Use lines of fit to make and evaluate predictions.

Why?

- The graph shows the number of people from the United States who travel to other countries. The points do not all lie on the same line; however, you may be able to draw a line that is close to all of the points. That line would show a linear relationship between the year *x* and the number of travelers each year *y*. Generally, international travel has increased.



New Vocabulary

bivariate data
scatter plot
line of fit
linear interpolation

Common Core State Standards

Content Standards

S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S.ID.6c Fit a linear function for a scatter plot that suggests a linear association.

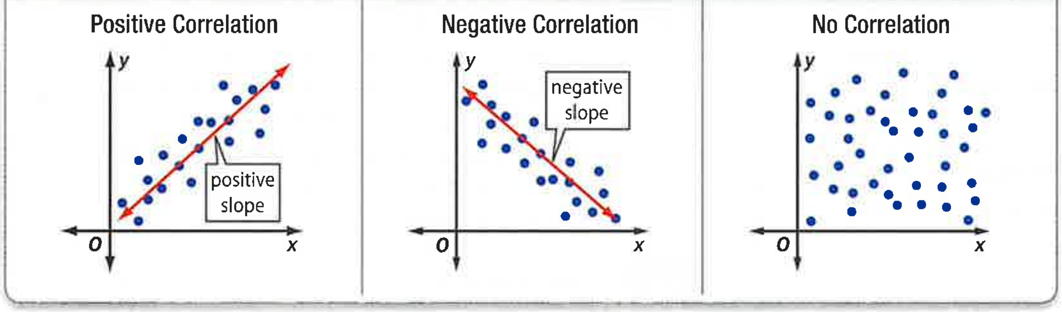
Mathematical Practices

1 Make sense of problems and persevere in solving them.
4 Model with mathematics.

1 Investigate Relationships Using Scatter Plots

Data with two variables are called **bivariate data**. A **scatter plot** shows the relationship between a set of data with two variables, graphed as ordered pairs on a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.

Concept Summary Scatter Plots



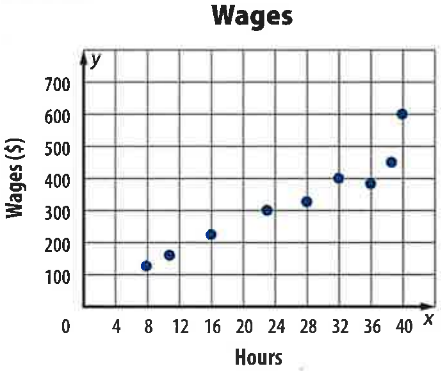
Positive Correlation — As *x* increases, *y* increases

Negative Correlation — As *x* decreases, *y* decreases

No Correlation — *x* and *y* are not related

Real-World Example 1 Evaluate a Correlation

WAGES Determine whether the graph shows a *positive, negative,* or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.



The graph shows a positive correlation. As the number of hours worked increases, the wages usually increase.

Guided Practice

1. Refer to the graph on international travel. Determine whether the graph shows a *positive, negative,* or *no* correlation. If there is a positive or negative correlation, describe its meaning.

2 Use Lines of Fit

Scatter plots can show whether there is a trend in a set of data. When the data points all lie close to a line, a **line of fit** or *trend line* can model the trend.

KeyConcept Using a Linear Function to Model Data

Step 1 Make a scatter plot. Determine whether any relationship exists in the data.

Step 2 Draw a line that seems to pass close to most of the data points.

Step 3 Use two points on the line of fit to write an equation for the line.

Step 4 Use the line of fit to make predictions.

Real-WorldLink

The Kingda Ka roller coaster at Six Flags Great Adventure in Jackson, New Jersey, has broken three records: tallest roller coaster at 456 feet, fastest at 128 miles per hour, and largest vertical drop of 418 feet.

Source: Ultimate Roller Coaster

Real-World Example 2 Write a Line of Fit

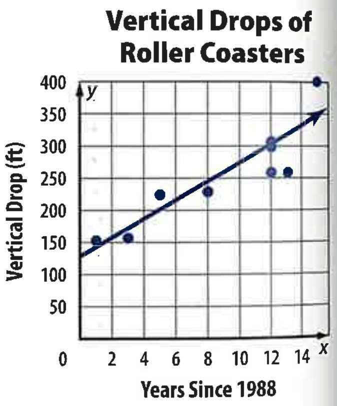
ROLLER COASTERS The table shows the largest vertical drops of nine roller coasters in the United States and the number of years after 1988 that they were opened. Identify the independent and the dependent variables. Is there a relationship in the data? If so, predict the vertical drop in a roller coaster built 30 years after 1988.

Years Since 1988	1	3	5	8	12	12	12	13	15
Vertical Drop (ft)	151	155	225	230	306	300	255	255	400

Source: Ultimate Roller Coaster

Step 1 Make a scatter plot.

The independent variable is the year, and the dependent variable is the vertical drop. As the number of years increases, the vertical drop of roller coasters increases. There is a positive correlation between the two variables.



Vertical Drops of Roller Coasters

Step 2 Draw a line of fit.

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown.

Step 3 Write the slope-intercept form of an equation for the line of fit.

The line of fit passes close to (2, 150) and the data point (12, 300).

Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_1, y_1) = (2, 150),\ (x_2, y_2) = (12, 300)$$

$$= \frac{300 - 150}{12 - 2}$$

$$= \frac{150}{10} \text{ or } 15$$

Use $m = 15$ and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

$$y - y_1 = m(x - x_1)$$

$$y - 150 = 15(x - 2)$$

$$y - 150 = 15x - 30$$

$$y = 15x + 120$$

A slope of 15 means that the vertical drops increased an average of 15 feet per year. To predict the vertical drop of a roller coaster built 30 years after 1988, substitute 30 for x in the equation. The vertical drop is 15(30) + 120 or 570 feet.

GuidedPractice

2. **MUSIC** The table shows the dollar value in millions for the sales of CDs for the year. Make a scatter plot and determine what relationship exists, if any.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
Sales	13,215	12,909	12,044	11,233	11,447	10,520	9373	7452	5471

ReadingMath

Interpolation and Extrapolation The Latin prefix *inter-* means between, and the Latin prefix *extra-* means beyond.

In Lesson 4-2, you learned that linear extrapolation is used to predict values *outside* the range of the data. You can also use a linear equation to predict values *inside* the range of the data. This is called **linear interpolation**.

Real-World Example 3 Use Interpolation or Extrapolation

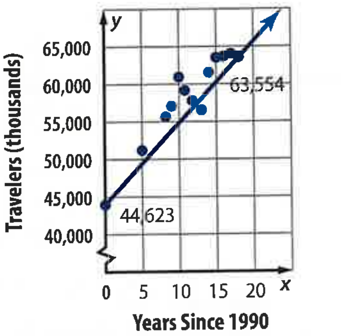
PT

TRAVEL **Use the scatter plot to find the approximate number of United States travelers to international countries in 1996.**

Step 1 Draw a line of fit. The line should be as close to as many points as possible.

Step 2 Write the slope-intercept form of the equation. The line of fit passes through (0, 44,623) and (18, 63,554).

International Travel



Source: *Statistical Abstract of the United States*

Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{63{,}554 - 44{,}623}{18 - 0}$$ $(x_1, y_1) = (0, 44{,}623)$, $(x_2, y_2) = (18, 63{,}554)$

$$= \frac{18{,}931}{18}$$ Simplify.

Use $m = \frac{18{,}931}{18}$ and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

$$y - y_1 = m(x - x_1)$$

$$y - 44{,}623 = \frac{18{,}931}{18}(x - 0)$$

$$y - 44{,}623 = \frac{18{,}931}{18}x$$

$$y = \frac{18{,}931}{18}x + 44{,}623$$

Step 3 Evaluate the function for $x = 1996 - 1990$ or 6.

$$y = \frac{18{,}931}{18}x + 44{,}623$$ Equation of best-fit line

$$= \frac{18{,}931}{18}(6) + 44{,}623$$ $x = 6$

$$= 6310\frac{1}{3} + 44{,}623 \text{ or } 50{,}933\frac{1}{3}$$ Add.

In 1996, there were approximately 50,933 thousand or 50,933,000 people who traveled from the United States to international countries.

GuidedPractice

3. **MUSIC** Use the equation for the line of fit for the data in Guided Practice 2 to estimate CD sales in 2015.

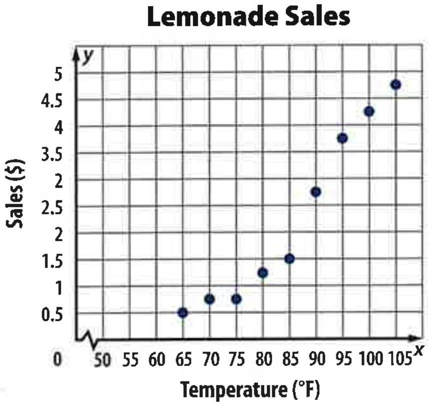
Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 **Determine whether each graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.**

1.

2.



Example 2

3. **SENSE-MAKING** The table shows the median age of females when they were first married.

a. Make a scatter plot and determine what relationship exists, if any, in the data. Identify the independent and the dependent variables.

b. Draw a line of fit for the scatter plot.

c. Write an equation in slope-intercept form for the line of fit.

Example 3

d. Predict what the median age of females when they are first married will be in 2016.

e. Do you think the equation can give a reasonable estimate for the year 2056? Explain.

Year	Age
1996	24.8
1997	25.0
1998	25.0
1999	25.1
2000	25.1
2001	25.1
2002	25.3
2003	25.3
2004	25.3
2005	25.5
2006	25.9

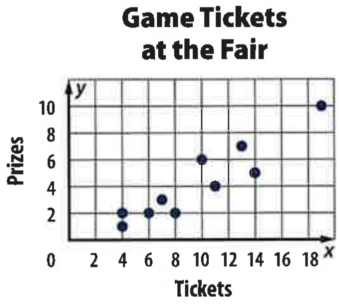
Source: U.S. Bureau of Census

Practice and Problem Solving

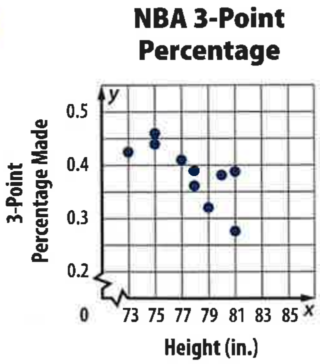
Extra Practice is on page R4.

Example 1 **Determine whether each graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.**

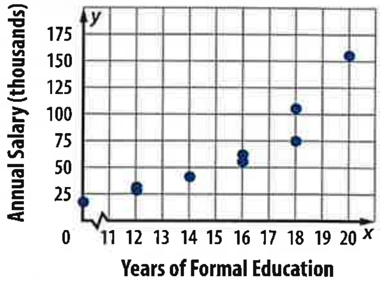
4.



⑤

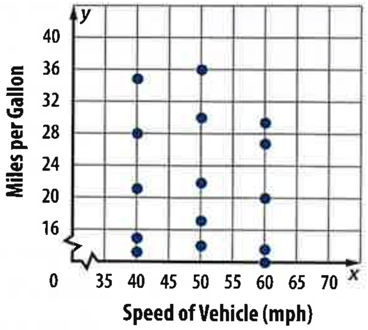


6.



Salaries

7



Gas Mileage of Various Vehicles

Examples 2–3

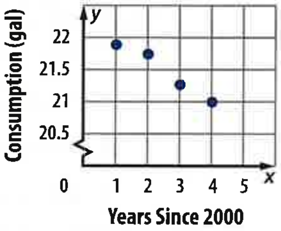
8. MILK Refer to the scatter plot of gallons of milk consumption per person for selected years.

a. Use the points (2, 21.75) and (4, 21) to write the slope-intercept form of an equation for the line of fit.

b. Predict the milk consumption in 2020.

c. Predict in what year milk consumption will be 10 gallons.

d. Is it reasonable to use the equation to estimate the consumption of milk for any year? Explain.



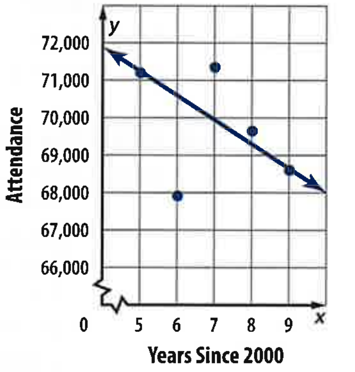
Consumption of Milk in Gallons

9. FOOTBALL Use the scatter plot.

a. Use the points (5, 71,205) and (9, 68,611) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.

b. Predict the average attendance at a game in 2020.

c. Can you use the equation to make a decision about the average attendance in any given year in the future? Explain.



Buffalo Bills Average Game Attendance

10. CCSS **SENSE-MAKING** The Body Mass Index (BMI) is a measure of body fat using height and weight. The heights and weights of twelve men with normal BMI are given in the table at the right.

a. Make a scatter plot comparing the height in inches to the weight in pounds.

b. Draw a line of fit for the data.

c. Write the slope-intercept form of an equation for the line of fit.

d. Predict the normal weight for a man who is 84 inches tall.

e. A man's weight is 188 pounds. Use the equation of the line of fit to predict the height of the man.

Height (in.)	Weight (lb)
62	115
63	124
65	120
67	134
67	140
68	138
68	144
68	152
69	147
72	155
73	168
73	166

⑪ **GEYSERS** The time to the next eruption of Old Faithful can be predicted by using the duration of the current eruption.

Duration (min)	1.5	2	2.5	3	3.5	4	4.5	5
Interval (min)	48	55	70	72	74	82	93	100

a. Identify the independent and the dependent variables. Make a scatter plot and determine what relationship, if any, exists in the data. Draw a line of fit for the scatter plot.

b. Let x represent the duration of the previous interval. Let y represent the time between eruptions. Write the slope-intercept form of the equation for the line of fit. Predict the interval after a 7.5-minute eruption.

c. Make a critical judgment about using the equation to predict the duration of the next eruption. Would the equation be a useful model?

12. **COLLECT DATA** Use a tape measure to measure both the foot size and the height in inches of ten individuals.

a. Record your data in a table.

b. Make a scatter plot and draw a line of fit for the data.

c. Write an equation for the line of fit.

d. Make a conjecture about the relationship between foot size and height.

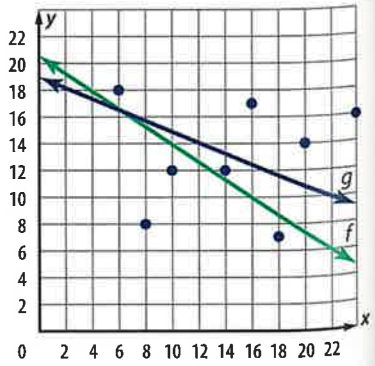
H.O.T. Problems Use Higher-Order Thinking Skills

13. **OPEN ENDED** Describe a real-life situation that can be modeled using a scatter plot. Decide whether there is a *positive*, *negative*, or *no* correlation. Explain what this correlation means.

14. **WHICH ONE DOESN'T BELONG?** Analyze the following situations and determine which one does not belong.

hours worked and amount of money earned	height of an athlete and favorite color
seedlings that grow an average of 2 centimeters each week	number of photos stored on a camera and capacity of camera

15. **ARGUMENTS** Determine which line of fit is better for the scatter plot. Explain your reasoning.



16. **REASONING** What can make a scatter plot and line of fit more useful for accurate predictions? Does an accurate line of fit always predict what will happen in the future? Explain.

17. **WRITING IN MATH** Make a scatter plot that shows the height of a person and age. Explain how you could use the scatter plot to predict the age of a person given his or her height. How can the information from a scatter plot be used to identify trends and make decisions?

Standardized Test Practice

18. Which equation best describes the relationship between the values of x and y in the table?

A $y = x - 5$
B $y = 2x - 5$
C $y = 3x - 7$
D $y = 4x - 7$

x	y
−1	−7
0	−5
2	−1
4	3

19. STATISTICS Mr. Hernandez collected data on the heights and average stride lengths of a random sample of high school students. He then made a scatter plot. What kind of correlation did he most likely see?

F positive
G constant
H negative
J no

20. GEOMETRY Mrs. Aguilar's rectangular bedroom measures 13 feet by 11 feet. She wants to purchase carpet for the bedroom that costs \$2.95 per square foot, including tax. How much will the carpet cost?

A \$70.80
B \$141.60
C \$145.95
D \$421.85

21. SHORT RESPONSE Nikia bought a one-month membership to a fitness center for \$35. Each time she goes, she rents a locker for \$0.25. If she spent \$40.50 at the fitness center last month, how many days did she go?

Spiral Review

Determine whether the graphs of each pair of equations are *parallel, perpendicular,* or *neither.* (Lesson 4-4)

22. $y = -2x + 11$
$y + 2x = 23$

23. $3y = 2x + 14$
$2x + 3y = 2$

24. $y = -5x$
$y = 5x - 18$

25. $y = 3x + 2$
$y = -\frac{1}{3}x - 2$

Write each equation in standard form. (Lesson 4-3)

26. $y - 13 = 4(x - 2)$

27. $y - 5 = -2(x + 2)$

28. $y + 3 = -5(x + 1)$

29. $y + 7 = \frac{1}{2}(x + 2)$

30. $y - 1 = \frac{5}{6}(x - 4)$

31. $y - 2 = -\frac{2}{5}(x - 8)$

Graph each equation. (Lesson 4-1)

32. $y = 2x + 3$

33. $4x + y = -1$

34. $3x + 4y = 7$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

35. (3, 4), (10, 8)

36. (−4, 7), (3, 5)

37. (3, 7), (−2, 4)

38. (−3, 2), (−3, 4)

39. (−2, −6), (−1, 10)

40. (1, −5), (−3, −5)

41. DRIVING Latisha drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles? (Lesson 2-6)

Skills Review

Express each relation as a graph. Then determine the domain and range.

42. {(4, 5), (5, 4), (−2, −2), (4, −5), (−5, 4)}

43. {(7, 6), (3, 4), (4, 5), (−2, 6), (−3, 2)}

LESSON 4-6 Regression and Median-Fit Lines

Then

- You used lines of fit and scatter plots to evaluate trends and make predictions.

Now

- 1 Write equations of best-fit lines using linear regression.

  2 Write equations of median-fit lines.

Why?

- The table shows the total attendance, in millions of people, at the Minnesota State Fair from 2005 to 2009. You can use a graphing calculator to find the equation of a *best-fit line* and use it to make predictions about future attendance at the fair.

Year	Attendance (millions)
2005	1.633
2006	1.681
2007	1.682
2008	1.693
2009	1.790

NewVocabulary

best-fit line
linear regression
correlation coefficient
residual
median-fit line

Common Core State Standards

Content Standards

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Mathematical Practices

5 Use appropriate tools strategically.

1 Best-Fit Lines

You have learned how to find and write equations for lines of fit by hand. Many calculators use complex algorithms that find a more precise line of fit called the **best-fit line**. One algorithm is called **linear regression**.

Your calculator may also compute a number called the **correlation coefficient**. This number will tell you if your correlation is positive or negative and how closely the equation is modeling the data. The closer the correlation coefficient is to 1 or −1, the more closely the equation models the data.

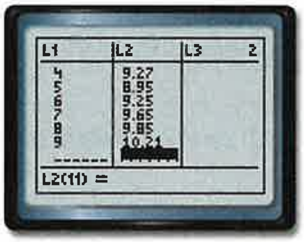
Real-World Example 1 Best-Fit Line

MOVIES The table shows the amount of money made by movies in the United States. Use a graphing calculator to write an equation for the best-fit line for that data.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Income ($ billion)	7.48	8.13	9.19	9.35	9.27	8.95	9.25	9.65	9.85	10.21

Before you begin, make sure that your Diagnostic setting is on. You can find this under the **CATALOG** menu. Press **D** and then scroll down and click **DiagnosticOn**. Then press ENTER.

Step 1 Enter the data by pressing STAT and selecting the **Edit** option. Let the year 2000 be represented by 0. Enter the years since 2000 into List 1 (L1). These will represent the x-values. Enter the income ($ billion) into List 2 (L2). These will represent the y-values.



```
L1      L2      L3      2
 4      9.27
 5      8.95
 6      9.25
 7      9.65
 8      9.85
 9      10.21
------
L2(11) =
```

Step 2 Perform the regression by pressing STAT and selecting the **CALC** option. Scroll down to **LinReg (ax+b)** and press ENTER twice.

```
LinReg
 y=ax+b
 a=.2320606061
 b=8.088727273
 r²=.7665273702
 r=.8755154883
```

John Giustina/Photodisc/Getty Images
Jonathan Larsen/age fotostock

Step 3 Write the equation of the regression line by rounding the a and b values on the screen. The form that we chose for the regression was $ax + b$, so the equation is $y = 0.23x + 8.09$. The correlation coefficient is about 0.8755, which means that the equation models the data fairly well.

Real-WorldLink

In 1994, Minnesota became the first state to sanction girls' ice hockey as a high school varsity sport.

Source: ESPNET SportsZone

GuidedPractice

Write an equation of the best-fit line for the data in each table. Name the correlation coefficient. Round to the nearest ten-thousandth. Let x be the number of years since 2003.

1A. HOCKEY The table shows the number of goals of leading scorers for the Mustang Girls Hockey Team.

Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	30	23	41	35	31	43	33	45

1B. HOCKEY The table gives the number of goals scored by the team each season.

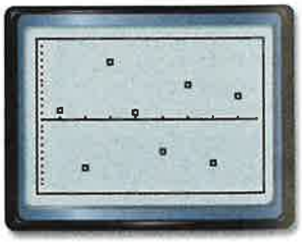
Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	63	44	55	63	81	85	93	84

We know that not all of the points will lie on the best-fit line. The difference between an observed y-value and its predicted y-value (found on the best-fit line) is called a **residual**. Residuals measure how much the data deviate from the regression line. When residuals are plotted on a scatter plot they can help to assess how well the best-fit line describes the data. If the best-fit line is a good fit, there is no pattern in the residual plot.

Real-World Example 2 Graph and Analyze a Residual Plot

HOCKEY Graph and analyze the residual plot for the data for Guided Practice 1A. Determine if the best-fit line models the data well.

After calculating the best-fit line in Guided Practice 1A, you can obtain the residual plot of the data. Turn on **Plot2** under the **STAT PLOT** menu and choose ⋰. Use **L1** for the **Xlist** and **RESID** for the **Ylist**. You can obtain **RESID** by pressing `2nd` **[STAT]** and selecting **RESID** from the list of names. Graph the scatter plot of the residuals by pressing `ZOOM` and choosing **ZoomStat**.



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The residuals appear to be randomly scattered and centered about the line $y = 0$. Thus, the best-fit line seems to model the data well.

GuidedPractice

2. UNEMPLOYMENT Graph and analyze the residual plot for the following data comparing graduation rates and unemployment rates.

Graduation Rate	73	85	64	81	68	82
Unemployment Rate	6.9	4.1	3.2	5.5	4.3	5.1

Purestock/Getty Images

A residual is positive when the observed value is above the line, negative when the observed value is below the line, and zero when it is on the line. One common measure of goodness of fit is the sum of squared vertical distances from the points to the line. The best-fit line, which is also called the *least-squares regression line*, minimizes the sum of the squares of those distances.

We can use points on the best-fit line to estimate values that are not in the data. Recall that when we estimate values that are between known values, this is called *linear interpolation*. When we estimate a number outside of the range of the data, it is called *linear extrapolation*.

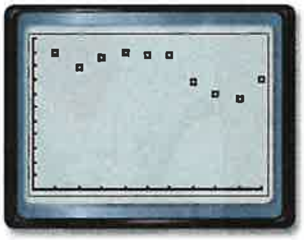
Real-World Example 3 Use Interpolation and Extrapolation

PAINTBALL **The table shows the points received by the top ten paintball teams at a tournament. Estimate how many points the 20th-ranked team received.**

Rank	1	2	3	4	5	6	7	8	9	10
Score	100	89	96	99	97	98	78	70	64	80

Write an equation of the best-fit line for the data. Then extrapolate to find the missing value.

Step 1 Enter the data from the table into the lists. Let the ranks be the x-values and the scores be the y-values. Then graph the scatter plot.



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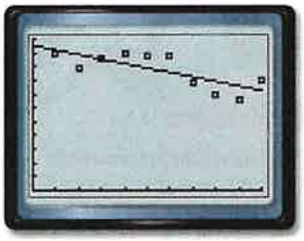
Step 2 Perform the linear regression using the data in the lists. Find the equation of the best-fit line.

The equation is about $y = -3.32x + 105.3$.



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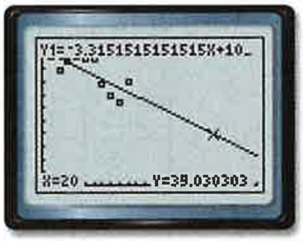
Step 3 Graph the best-fit line. Press Y= VARS and choose **Statistics**. From the **EQ** menu, choose **RegEQ**. Then press GRAPH.

Step 4 Use the graph to predict the points that the 20th-ranked team received. Change the viewing window to include the x-value to be evaluated. Press 2nd **[CALC]** ENTER 20 ENTER to find that when $x = 20$, $y \approx 39$. It is estimated that the 20th ranked team received 39 points.



```
Y1=-3.3151515151515X+10…
X=20          Y=39.030303
```

[0, 25] scl: 1 by [0, 110] scl: 1

GuidedPractice

ONLINE GAMES **Use linear interpolation to estimate the percent of Americans that play online games for the following ages.**

Age	15	20	30	40	50
Percent	81	54	37	29	25

Source: Pew Internet & American Life Survey

3A. 35 years

3B. 18 years

2 Median-Fit Lines

A second type of fit line that can be found using a graphing calculator is a **median-fit line**. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.

Real-World Link

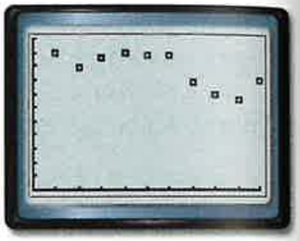
Paintball is more popular with 12- to 17-year-olds than any other age group. In a recent year, 3,649,000 teens participated in paintball while 2,195,000 18- to 24-year-olds participated.

Source: *Statistical Abstract* of the *United States*

Example 4 Median-Fit Line

PAINTBALL **Find and graph the equation of a median-fit line for the data in Example 3. Then predict the score of the 15th ranked team.**

Step 1 Reenter the data if it is not in the lists. Clear the **Y=** list and graph the scatter plot.



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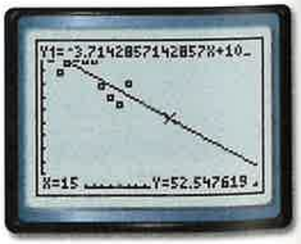
Step 2 To find the median-fit equation, press the STAT key and select the **CALC** option. Scroll down to the **Med-Med** option and press ENTER. The value of a is the slope, and the value of b is the y-intercept.

The equation for the median-fit line is about $y = -3.71x + 108.26$.

Step 3 Copy the equation to the **Y=** list and graph. Use the **value** option to find the value of y when $x = 15$.

The 15th place team scored about 53 points.



```
Y1=-3.7142857142857X+10_
X=15 ..........Y=52.547619
```

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Notice that the equations for the regression line and the median-fit line are very similar.

GuidedPractice

4. Use the data from Guided Practice 3 and a median-fit line to estimate the numbers of 18- and 35-year-olds who play online games. Compare these values with the answers from the regression line.

Alan Thornton/Stone/Getty Images

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1, 2

1. **POTTERY** A local university is keeping track of the number of art students who use the pottery studio each day.

Day	1	2	3	4	5	6	7
Students	10	15	18	15	13	19	20

a. Write an equation of the regression line and find the correlation coefficient.

b. Graph the residual plot and determine if the regression line models the data well.

Example 3

2. **COMPUTERS** The table below shows the percent of Americans with a broadband connection at home in a recent year. Use linear extrapolation and a regression equation to estimate the percentage of 60-year-olds with broadband at home.

Age	25	30	35	40	45	50
Percent	40	42	36	35	36	32

Example 4

3. **VACATION** The Smiths want to rent a house on the lake that sleeps eight people. The cost of the house per night is based on how close it is to the water.

Distance from Lake (mi)	0.0 (houseboat)	0.3	0.5	1.0	1.25	1.5	2.0
Price/Night ($)	785	325	250	200	150	140	100

a. Find and graph an equation for the median-fit line.

b. What would you estimate is the cost of a rental 1.75 miles from the lake?

Practice and Problem Solving

Extra Practice is on page R4.

Example 1

Write an equation of the regression line for the data in each table. Then find the correlation coefficient.

4. **SKYSCRAPERS** The table ranks the ten tallest buildings in the world.

Rank	1	2	3	4	5	6	7	8	9	10
Stories	101	88	110	88	88	80	69	102	78	70

⑤ **MUSIC** The table gives the number of annual violin auditions held by a youth symphony each year since 2004. Let x be the number of years since 2004.

Year	2004	2005	2006	2007	2008	2009	2010
Auditions	22	19	25	37	32	35	42

Example 2

6. **RETAIL** The table gives the sales at a clothing chain since 2004. Let x be the number of years since 2004.

Year	2004	2005	2006	2007	2008	2009	2010
Sales (Millions of Dollars)	6.84	7.6	10.9	15.4	17.6	21.2	26.5

a. Write an equation of the regression line.

b. Graph and analyze the residual plot.

Examples 3, 4 **7** **MARATHON** The number of entrants in the Boston Marathon every five years since 1975 is shown. Let x be the number of years since 1975.

Year	1975	1980	1985	1990	1995	2000	2005	2010
Entrants	2395	5417	5594	9412	9416	17,813	20,453	26,735

a. Find an equation for the median-fit line.

b. According to the equation, how many entrants were there in 2003?

8. **CAMPING** A campground keeps a record of the number of campsites rented the week of July 4 for several years. Let x be the number of years since 2000.

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010
Sites Rented	34	45	42	53	58	47	57	65	59

a. Find an equation for the regression line.

b. Predict the number of campsites that will be rented in 2012.

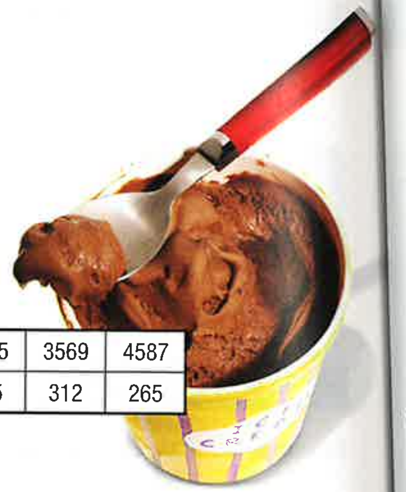
c. Predict the number of campsites that will be rented in 2020.

9. **ICE CREAM** An ice cream company keeps a count of the tubs of chocolate ice cream delivered to each of their stores in a particular area.

a. Find an equation for the median-fit line.

b. Graph the points and the median-fit line.

c. How many tubs would be delivered to a 1500-square-foot store? a 5000-square-foot store?



Store Size (ft^2)	2100	2225	3135	3569	4587
Tubs (hundreds)	110	102	215	312	265

10. CCSS **SENSE-MAKING** The prices of the eight top-selling brands of jeans at Jeanie's Jeans are given in the table below.

Sales Rank	1	2	3	4	5	6	7	8
Price ($)	43	44	50	61	64	135	108	78

a. Find the equation for the regression line.

b. According to the equation, what would be the price of a pair of the 12th best-selling brand?

c. Is this a reasonable prediction? Explain.

11. **STATE FAIRS** Refer to the beginning of the lesson.

a. Graph a scatter plot of the data, where $x = 1$ represents 2005. Then find and graph the equation for the best-fit line.

b. Graph and analyze the residual plot.

c. Predict the total attendance in 2020.

Alex Cao/Photodisc/Getty Images

12. **FIREFIGHTERS** The table shows statistics from the U.S. Fire Administration.

a. Find an equation for the median-fit line.

b. Graph the points and the median-fit line.

c. Does the median-fit line give you an accurate picture of the number of firefighters? Explain.

Age	Number of Firefighters
18	40,919
25	245,516
35	330,516
45	296,665
55	167,087
65	54,559

13. **ATHLETICS** The table shows the number of participants in high school athletics.

Year Since 1970	1	10	20	30	35
Athletes	3,960,932	5,356,913	5,298,671	6,705,223	7,159,904

a. Find an equation for the regression line.

b. According to the equation, how many participated in 1988?

14. **ART** A count was kept on the number of paintings sold at an auction by the year in which they were painted. Let x be the number of years since 1950.

Year Painted	1950	1955	1960	1965	1970	1975
Paintings Solds	8	5	25	21	9	22

a. Find the equation for the linear regression line.

b. How many paintings were sold that were painted in 1961?

c. Is the linear regression equation an accurate model of the data? Explain why or why not.

H.O.T. Problems Use Higher-Order Thinking Skills

15. CCSS **ARGUMENTS** Below are the results of the World Superpipe Championships in 2008.

Men	Score	Rank	Women	Score
Shaun White	93.00	1	Torah Bright	96.67
Mason Aguirre	90.33	2	Kelly Clark	93.00
Janne Korpi	85.33	3	Soko Yamaoka	85.00
Luke Mitrani	85.00	4	Ellery Hollingsworth	79.33
Keir Dillion	81.33	5	Sophie Rodriguez	71.00

Find an equation of the regression line for each, and graph them on the same coordinate plane. Compare and contrast the men's and women's graphs.

16. **REASONING** For a class project, the scores that 10 randomly selected students earned on the first 8 tests of the school year are given. Explain how to find a line of best fit. Could it be used to predict the scores of other students? Explain your reasoning.

17. **OPEN ENDED** For 10 different people, measure their heights and the lengths of their heads from chin to top. Use these data to generate a linear regression equation and a median-fit equation. Make a prediction using both of the equations.

18. **WRITING IN MATH** How are lines of fit and linear regression similar? different?

Alex Cao/Photodisc/Getty Images

Standardized Test Practice

19. GEOMETRY Sam is putting a border around a poster. x represents the poster's width, and y represents the poster's length. Which equation represents how much border Sam will use if he doubles the length and the width?

A $4xy$ **C** $4(x + y)$

B $(x + y)^4$ **D** $16(x + y)$

20. SHORT RESPONSE Tatiana wants to run 5 miles at an average pace of 9 minutes per mile. After 4 miles, her average pace is 9 minutes 10 seconds. In how many minutes must she complete the final mile to reach her goal?

21. What is the slope of the line that passes through (1, 3) and (−3, 1)?

F -2 **H** $\frac{1}{2}$

G $-\frac{1}{2}$ **J** 2

22. What is an equation of the line that passes through (0, 1) and has a slope of 3?

A $y = 3x - 1$

B $y = 3x - 2$

C $y = 3x + 4$

D $y = 3x + 1$

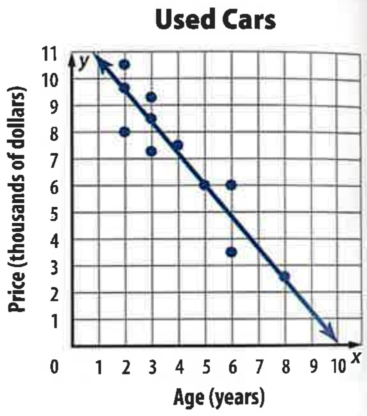
Spiral Review

23. USED CARS Gianna wants to buy a specific make and model of a used car. She researched prices from dealers and private sellers and made the graph shown. (Lesson 4-5)

a. Describe the relationship in the data.

b. Use the line of fit to predict the price of a car that is 7 years old.

c. Is it reasonable to use this line of fit to predict the price of a 10-year-old car? Explain.



24. GEOMETRY A quadrilateral has sides with equations $y = -2x$, $2x + y = 6$, $y = \frac{1}{2}x + 6$, and $x - 2y = 9$. Is the figure a rectangle? Explain your reasoning. (Lesson 4-4)

Write each equation in standard form. (Lesson 4-3)

25. $y - 2 = 3(x - 1)$

26. $y - 5 = 6(x + 1)$

27. $y + 2 = -2(x - 5)$

28. $y + 3 = \frac{1}{2}(x + 4)$

29. $y - 1 = \frac{2}{3}(x + 9)$

30. $y + 3 = -\frac{1}{4}(x + 2)$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

31. (3, 4), (10, 8)

32. (−4, 7), (3, 5)

33. (3, 7), (−2, 4)

34. (−3, 2), (−3, 4)

Skills Review

If $f(x) = x^2 - x + 1$, find each value.

35. $f(-1)$

36. $f(5) - 3$

37. $f(a)$

38. $f(b^2)$

Graph each equation.

39. $y = x + 2$

40. $x + 5y = 4$

41. $2x - 3y = 6$

42. $5x + 2y = 6$

LESSON 4-7

Inverse Linear Functions

Then

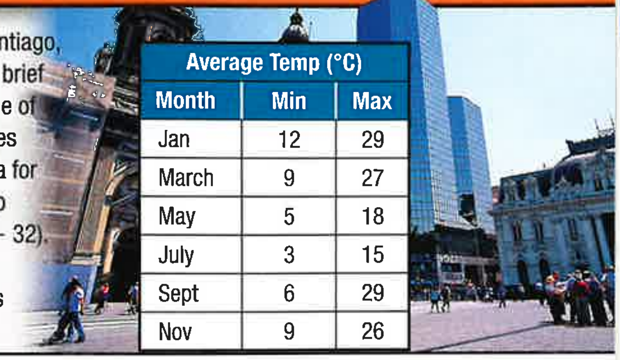
- You represented relations as tables, graphs, and mappings.

Now

1. Find the inverse of a relation.
2. Find the inverse of a linear function.

Why?

- Randall is writing a report on Santiago, Chile, and he wants to include a brief climate analysis. He found a table of temperatures recorded in degrees Celsius. He knows that a formula for converting degrees Fahrenheit to degrees Celsius is $C(x) = \frac{5}{9}(x - 32)$. He will need to find the *inverse* function to convert from degrees Celsius to degrees Fahrenheit.



Average Temp (°C)		
Month	**Min**	**Max**
Jan	12	29
March	9	27
May	5	18
July	3	15
Sept	6	29
Nov	9	26

NewVocabulary

inverse relation
inverse function

Common Core State Standards

Content Standards

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Mathematical Practices

6 Attend to precision.

1 Inverse Relations

An **inverse relation** is the set of ordered pairs obtained by exchanging the x-coordinates with the y-coordinates of each ordered pair in a relation. If (5, 3) is an ordered pair of a relation, then (3, 5) is an ordered pair of the inverse relation.

KeyConcept Inverse Relations

Words If one relation contains the element (a, b), then the inverse relation will contain the element (b, a).

Example A and B are inverse relations.

A		B
$(-3, -16)$	$\rightarrow$	$(-16, -3)$
$(-1, 4)$	$\rightarrow$	$(4, -1)$
$(2, 14)$	$\rightarrow$	$(14, 2)$
$(5, 32)$	$\rightarrow$	$(32, 5)$

Notice that the domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

PT

Example 1 Inverse Relations

Find the inverse of each relation.

a. **{(4, −10), (7, −19), (−5, 17), (−3, 11)}**

To find the inverse, exchange the coordinates of the ordered pairs.

$(4, -10) \rightarrow (-10, 4)$ $\quad$ $(-5, 17) \rightarrow (17, -5)$

$(7, -19) \rightarrow (-19, 7)$ $\quad$ $(-3, 11) \rightarrow (11, -3)$

The inverse is {(−10, 4), (−19, 7), (17, −5), (11, −3)}.

b.

x	−4	−1	5	9
y	−13	−8.5	0.5	6.5

Write the coordinates as ordered pairs. Then exchange the coordinates of each pair.

$(-4, -13) \rightarrow (-13, -4)$ $\quad$ $(5, 0.5) \rightarrow (0.5, 5)$

$(-1, -8.5) \rightarrow (-8.5, -1)$ $\quad$ $(9, 6.5) \rightarrow (6.5, 9)$

The inverse is {(−13, −4), (−8.5, −1), (0.5, 5), (6.5, 9)}.

TongRo Image Stock/Alamy

GuidedPractice

1A. $\{(-6, 8), (-15, 11), (9, 3), (0, 6)\}$

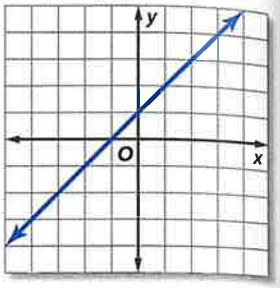
1B.

x	−10	−4	−3	0
y	5	11	12	15

The graphs of relations can be used to find and graph inverse relations.

Example 2 Graph Inverse Relations

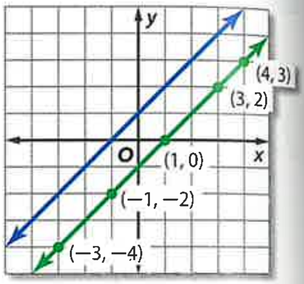
Graph the inverse of the relation.



StudyTip

CCSS Precision Only two points are necessary to graph the inverse of a line, but several should be used to avoid possible error.

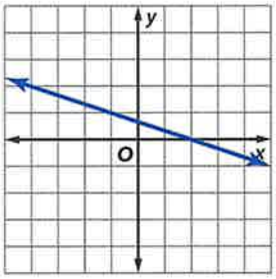
The graph of the relation passes through the points at $(-4, -3)$, $(-2, -1)$, $(0, 1)$, $(2, 3)$, and $(3, 4)$. To find points through which the graph of the inverse passes, exchange the coordinates of the ordered pairs. The graph of the inverse passes through the points at $(-3, -4)$, $(-1, -2)$, $(1, 0)$, $(3, 2)$, and $(4, 3)$. Graph these points and then draw the line that passes through them.



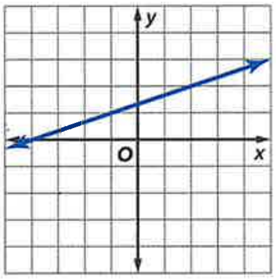
GuidedPractice

Graph the inverse of each relation.

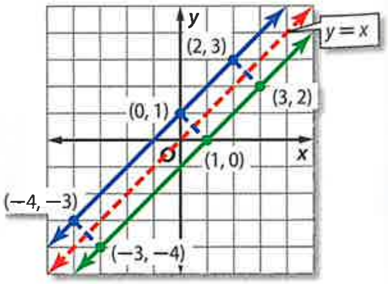
2A.



2B.



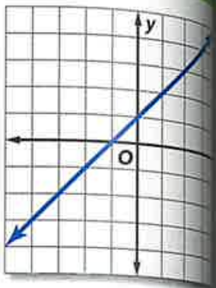
The graphs from Example 2 are graphed on the right with the line $y = x$. Notice that the graph of an inverse is the graph of the original relation reflected in the line $y = x$. For every point (x, y) on the graph of the original relation, the graph of the inverse will include the point (y, x).

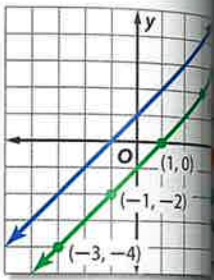


2 **Inverse Functions** A linear relation that is described by a function has an **inverse function** that can generate ordered pairs of the inverse relation. The inverse of the linear function $f(x)$ can be written as $f^{-1}(x)$ and is read *f of x inverse* or *the inverse of f of x*.

−4	−3	0
11	12	15

e relations.





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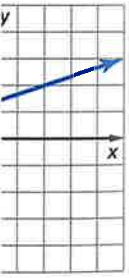
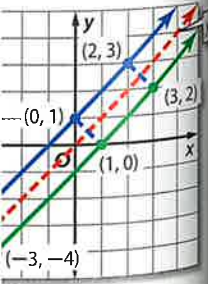
tation The −1 in $f^{-1}(x)$ is ot an exponent.





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verse relation. Th

is read f of x invers

KeyConcept Finding Inverse Functions

To find the inverse function $f^{-1}(x)$ of the linear function $f(x)$, complete the following steps.

Step 1 Replace $f(x)$ with y in the equation for $f(x)$.

Step 2 Interchange y and x in the equation.

Step 3 Solve the equation for y.

Step 4 Replace y with $f^{-1}(x)$ in the new equation.

Example 3 Find Inverse Linear Functions

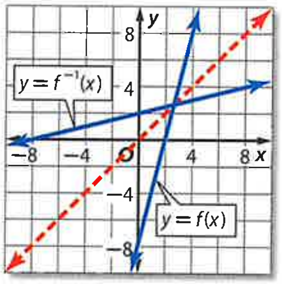
Find the inverse of each function.

a. $f(x) = 4x - 8$

Step 1	$f(x) = 4x - 8$	Original equation
	$y = 4x - 8$	Replace $f(x)$ with y.
Step 2	$x = 4y - 8$	Interchange y and x.
Step 3	$x + 8 = 4y$	Add 8 to each side.
	$\frac{x+8}{4} = y$	Divide each side by 4.
Step 4	$\frac{x+8}{4} = f^{-1}(x)$	Replace y with $f^{-1}(x)$.

The inverse of $f(x) = 4x - 8$ is $f^{-1}(x) = \frac{x+8}{4}$ or $f^{-1}(x) = \frac{1}{4}x + 2$.

CHECK Graph both functions and the line $y = x$ on the same coordinate plane. $f^{-1}(x)$ appears to be the reflection of $f(x)$ in the line $y = x$. ✓



b. $f(x) = -\frac{1}{2}x + 11$

Step 1	$f(x) = -\frac{1}{2}x + 11$	Original equation
	$y = -\frac{1}{2}x + 11$	Replace $f(x)$ with y.
Step 2	$x = -\frac{1}{2}y + 11$	Interchange y and x.
Step 3	$x - 11 = -\frac{1}{2}y$	Subtract 11 from each side.
	$-2(x - 11) = y$	Multiply each side by −2.
	$-2x + 22 = y$	Distributive Property
Step 4	$-2x + 22 = f^{-1}(x)$	Replace y with $f^{-1}(x)$.

The inverse of $f(x) = -\frac{1}{2}x + 11$ is $f^{-1}(x) = -2x + 22$.

GuidedPractice

3A. $f(x) = 4x - 12$

3B. $f(x) = \frac{1}{3}x + 7$

Real-WorldLink

The winter months in Chile occur during the summer months in the U.S. due to Chile's location in the southern hemisphere. The average daily high temperature of Santiago during its winter months is about 60° F.

Source: World Weather Information Service

Real-World Example 4 Use an Inverse Function

TEMPERATURE **Refer to the beginning of the lesson. Randall wants to convert the temperatures from degrees Celsius to degrees Fahrenheit.**

a. Find the inverse function $C^{-1}(x)$.

Step 1 $C(x) = \frac{5}{9}(x - 32)$ Original equation

$y = \frac{5}{9}(x - 32)$ Replace $C(x)$ with y.

Step 2 $x = \frac{5}{9}(y - 32)$ Interchange y and x.

Step 3 $\frac{9}{5}x = y - 32$ Multiply each side by $\frac{9}{5}$.

$\frac{9}{5}x + 32 = y$ Add 32 to each side.

Step 4 $\frac{9}{5}x + 32 = C^{-1}(x)$ Replace y with $C^{-1}(x)$.

The inverse function of $C(x)$ is $C^{-1}(x) = \frac{9}{5}x + 32$.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

x represents the temperature in degrees Celsius. $C^{-1}(x)$ represents the temperature in degrees Fahrenheit.

c. Find the average temperatures for July in degrees Fahrenheit.

The average minimum and maximum temperatures for July are 3° C and 15° C, respectively. To find the average minimum temperature, find $C^{-1}(3)$.

$C^{-1}(x) = \frac{9}{5}x + 32$ Original equation

$C^{-1}(3) = \frac{9}{5}(3) + 32$ Substitute 3 for x.

$= 37.4$ Simplify.

To find the average maximum temperature, find $C^{-1}(15)$.

$C^{-1}(x) = \frac{9}{5}x + 32$ Original equation

$C^{-1}(15) = \frac{9}{5}(15) + 32$ Substitute 15 for x.

$= 59$ Simplify.

The average minimum and maximum temperatures for July are 37.4° F and 59° F, respectively.

GuidedPractice

4. **RENTAL CAR** Peggy rents a car for the day. The total cost $C(x)$ in dollars is given by $C(x) = 19.99 + 0.3x$, where x is the number of miles she drives.

   A. Find the inverse function $C^{-1}(x)$.

   B. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

   C. How many miles did Peggy drive if her total cost was $34.99?

Felipe Dupouy/Lifesize/Getty Images

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 Find the inverse of each relation.

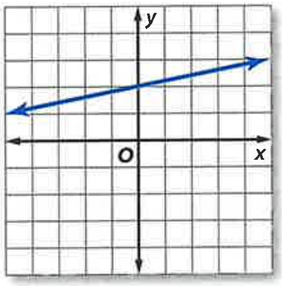
1. $\{(4, -15), (-8, -18), (-2, -16.5), (3, -15.25)\}$

2.

x	−3	0	1	6
y	11.8	3.7	1	−12.5

Example 2 Graph the inverse of each relation.

3.



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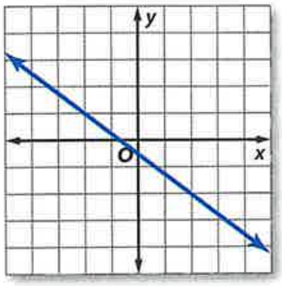


Example 3 Find the inverse of each function.

5. $f(x) = -2x + 7$

6. $f(x) = \frac{2}{3}x + 6$

Example 4 **7.** **CCSS REASONING** Dwayne and his brother purchase season tickets to the Cleveland Crusaders games. The ticket package requires a one-time purchase of a personal seat license costing $1200 for two seats. A ticket to each game costs $70. The cost $C(x)$ in dollars for Dwayne for the first season is $C(x) = 600 + 70x$, where x is the number of games Dwayne attends.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many games did Dwayne attend if his total cost for the season was $950?

Practice and Problem Solving

Extra Practice is on page R4.

Example 1 Find the inverse of each relation.

8. $\{(-5, 13), (6, 10.8), (3, 11.4), (-10, 14)\}$

⑨ $\{(-4, -49), (8, 35), (-1, -28), (4, 7)\}$

10.

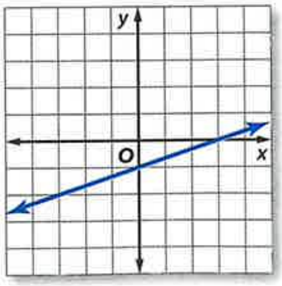
x	y
−8	−36.4
−2	−15.4
1	−4.9
5	9.1
11	30.1

11.

x	y
−3	7.4
−1	4
1	0.6
3	−2.8
5	−6.2

Example 2 Graph the inverse of each relation.

12.



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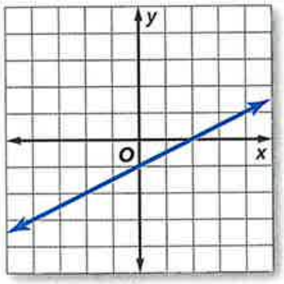


Felipe Dupouy/Lifesize/Getty Images

Example 3 **Find the inverse of each function.**

14. $f(x) = 25 + 4x$

15. $f(x) = 17 - \frac{1}{3}x$

16. $f(x) = 4(x + 17)$

17. $f(x) = 12 - 6x$

18. $f(x) = \frac{2}{5}x + 10$

19. $f(x) = -16 - \frac{4}{3}x$

Example 4 **20. DOWNLOADS** An online music subscription service allows members to download songs for \$0.99 each after paying a monthly service charge of \$3.99. The total monthly cost $C(x)$ of the service in dollars is $C(x) = 3.99 + 0.99x$, where x is the number of songs downloaded.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many songs were downloaded if a member's monthly bill is \$27.75?

21. LANDSCAPING At the start of the mowing season, Chuck collects a one-time maintenance fee of \$10 from his customers. He charges the Fosters \$35 for each cut. The total amount collected from the Fosters in dollars for the season is $C(x) = 10 + 35x$, where x is the number of times Chuck mows the Fosters' lawn.

a. Find the inverse function.

b. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

c. How many times did Chuck mow the Fosters' lawn if he collected a total of \$780 from them?

Write the inverse of each equation in $f^{-1}(x)$ notation.

22. $3y - 12x = -72$

23. $x + 5y = 15$

24. $-42 + 6y = x$

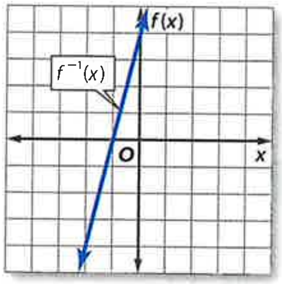
25. $3y + 24 = 2x$

26. $-7y + 2x = -28$

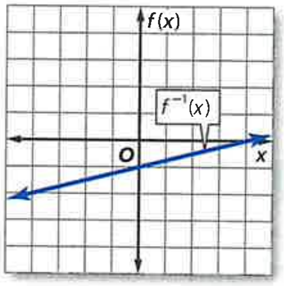
27. $3y - x = 3$

CCSS TOOLS **Match each function with the graph of its inverse.**

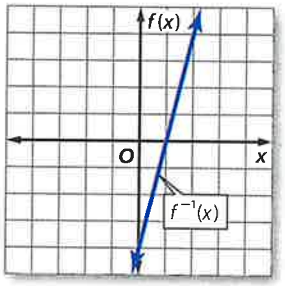
A.



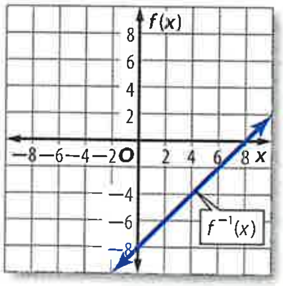
B.



C.



D.



28. $f(x) = x + 4$

29. $f(x) = 4x + 4$

30. $f(x) = \frac{1}{4}x + 1$

31. $f(x) = \frac{1}{4}x - 1$

Write an equation for the inverse function $f^{-1}(x)$ that satisfies the given conditions.

32. slope of $f(x)$ is 7; graph of $f^{-1}(x)$ contains the point (13, 1)

33. graph of $f(x)$ contains the points (−3, 6) and (6, 12)

34. graph of $f(x)$ contains the point (10, 16); graph of $f^{-1}(x)$ contains the point (3, −16)

35. slope of $f(x)$ is 4; $f^{-1}(5) = 2$

36. **CELL PHONES** Mary Ann pays a monthly fee for her cell phone package which includes 700 minutes. She gets billed an additional charge for every minute she uses the phone past the 700 minutes. During her first month, Mary Ann used 26 additional minutes and her bill was $37.79. During her second month, Mary Ann used 38 additional minutes and her bill was $41.39.

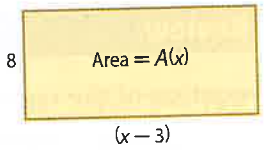
a. Write a function that represents the total monthly cost $C(x)$ of Mary Ann's cell phone package, where x is the number of additional minutes used.

b. Find the inverse function.

c. What do x and $C^{-1}(x)$ represent in the context of the inverse function?

d. How many additional minutes did Mary Ann use if her bill for her third month was $48.89?

37. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the domain and range of inverse functions.



a. Algebraic Write a function for the area $A(x)$ of the rectangle shown.

b. Graphical Graph $A(x)$. Describe the domain and range of $A(x)$ in the context of the situation.

c. Algebraic Write the inverse of $A(x)$. What do x and $A^{-1}(x)$ represent in the context of the situation?

d. Graphical Graph $A^{-1}(x)$. Describe the domain and range of $A^{-1}(x)$ in the context of the situation.

e. Logical Determine the relationship between the domains and ranges of $A(x)$ and $A^{-1}(x)$.

H.O.T. Problems Use Higher-Order Thinking Skills

38. **CHALLENGE** If $f(x) = 5x + a$ and $f^{-1}(10) = -1$, find a.

39. **CHALLENGE** If $f(x) = \frac{1}{a}x + 7$ and $f^{-1}(x) = 2x - b$, find a and b.

CCSS ARGUMENTS **Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain your reasoning.**

40. If $f(x)$ and $g(x)$ are inverse functions, then $f(a) = b$ and $g(b) = a$.

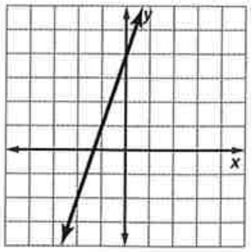
41. If $f(a) = b$ and $g(b) = a$, then $f(x)$ and $g(x)$ are inverse functions.

42. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses by graphing the functions and the line $y = x$ on the same coordinate plane.

43. **WRITING IN MATH** Explain why it may be helpful to find the inverse of a function.

Standardized Test Practice

44. Which equation represents a line that is perpendicular to the graph and passes through the point at (2, 0)?



A $y = 3x - 6$

B $y = -3x + 6$

C $y = -\frac{1}{3}x + \frac{2}{3}$

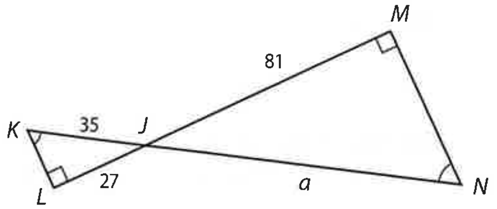
D $y = \frac{1}{3}x - \frac{2}{3}$

45. A giant tortoise travels at a rate of 0.17 mile per hour. Which equation models the time t it would take the giant tortoise to travel 0.8 mile?

F $t = \frac{0.8}{0.17}$ **H** $t = \frac{0.17}{0.8}$

G $t = (0.17)(0.8)$ **J** $0.8 = \frac{0.17}{t}$

46. GEOMETRY If $\triangle JKL$ is similar to $\triangle JNM$ what is the value of a?



A 62.5

B 105

C 125

D 155.5

47. GRIDDED RESPONSE What is the difference in the value of $2.1(x + 3.2)$, when $x = 5$ and when $x = 3$?

Spiral Review

Write an equation of the regression line for the data in each table. (Lesson 4-6)

48.

x	1	3	5	7	9
y	3	8	15	18	21

49.

x	3	5	7	9	11
y	7.2	23.5	41.2	56.4	73.1

50.

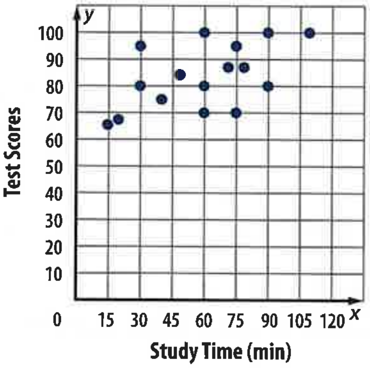
x	1	2	3	4	5
y	21	33	39	54	64

51.

x	2	4	6	8	10
y	1.4	2.4	2.9	3.3	4.2

52. TESTS Determine whether the graph at the right shows a *positive*, *negative*, or *no* correlation. If there is a correlation, describe its meaning. (Lesson 4-5)

Test Scores



Suppose y varies directly as x. (Lesson 3-4)

53. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

54. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

55. If $y = 2.6$ when $x = 0.25$, find y when $x = 1.125$.

56. If $y = 6$ when $x = 0.6$, find x when $y = 12$.

Skills Review

Solve each equation.

57. $104 = k - 67$

58. $-4 + x = -7$

59. $\frac{m}{7} = -11$

60. $\frac{2}{3}p = 14$

61. $-82 = 18 - n$

62. $\frac{9}{t} = -27$

CHAPTER 4

Study Guide and Review

Study Guide

KeyConcepts

Slope-Intercept Form (Lessons 4-1 and 4-2)

- The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.
- If you are given two points through which a line passes, use them to find the slope first.

Point-Slope Form (Lesson 4-3)

- The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Parallel and Perpendicular Lines (Lesson 4-4)

- Nonvertical parallel lines have the same slope.
- Lines that intersect at right angles are called perpendicular lines. The slopes of perpendicular lines are opposite reciprocals.

Scatter Plots and Lines of Fit (Lesson 4-5)

- Data with two variables are called bivariate data.
- A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane.

Regression and Median-Fit Lines (Lesson 4-6)

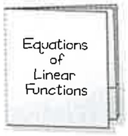
- A graphing calculator can be used to find regression lines and median-fit lines.

Inverse Linear Functions (Lesson 4-7)

- An inverse relation is the set of ordered pairs obtained by exchanging the x-coordinates with the y-coordinates of each ordered pair of a relation.
- A linear function $f(x)$ has an inverse function that can be written as $f^{-1}(x)$ and is read *f of x inverse* or *the inverse of f of x*.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

best-fit line (p. 255)
bivariate data (p. 247)
constant function (p. 217)
constraint (p. 228)
correlation coefficient (p. 255)
identity function (p. 224)
inverse function (p. 264)
inverse relation (p. 263)
linear extrapolation (p. 228)
linear interpolation (p. 249)
linear regression (p. 255)
line of fit (p. 248)
median-fit line (p. 258)
parallel lines (p. 239)
perpendicular lines (p. 240)
point-slope form (p. 233)
scatter plot (p. 247)
slope-intercept form (p. 216)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The y-intercept is the y-coordinate of the point where the graph crosses the y-axis.
2. The process of using a linear equation to make predictions about values that are beyond the range of the data is called linear regression.
3. An inverse relation is the set of ordered pairs obtained by exchanging the x-coordinates with the y-coordinates of each ordered pair of a relation.
4. The correlation coefficient describes whether the correlation between the variables is positive or negative and how closely the regression equation is modeling the data.
5. Lines in the same plane that do not intersect are called parallel lines.
6. Lines that intersect at acute angles are called perpendicular lines.
7. A(n) constant function can generate ordered pairs for an inverse relation.
8. The range of a relation is the range of its inverse function.
9. An equation of the form $y = mx + b$ is in point-slope form.

Lesson-by-Lesson Review

4-1 Graphing Equations in Slope-Intercept Form

Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.

10. slope: 3, y-intercept: 5

11. slope: -2, y-intercept: -9

12. slope: $\frac{2}{3}$, y-intercept: 3

13. slope: $-\frac{5}{8}$, y-intercept: -2

Graph each equation.

14. $y = 4x - 2$

15. $y = -3x + 5$

16. $y = \frac{1}{2}x + 1$

17. $3x + 4y = 8$

18. **SKI RENTAL** Write an equation in slope-intercept form for the total cost of skiing for h hours with one lift ticket.

Example 1

Write an equation of a line in slope-intercept form with slope -5 and y-intercept -3. Then graph the equation.

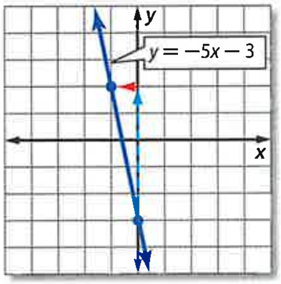
$y = mx + b$ Slope-intercept form

$y = -5x + (-3)$ $m = -5$ and $b = -3$

$y = -5x - 3$ Simplify.

To graph the equation, plot the y-intercept $(0, -3)$.

Then move up 5 units and left 1 unit. Plot the point. Draw a line through the two points.



4-2 Writing Equations in Slope-Intercept Form

Write an equation of the line that passes through the given point and has the given slope.

19. (1, 2), slope 3

20. (2, −6), slope −4

21. (−3, −1), slope $\frac{2}{5}$

22. (5, −2), slope $-\frac{1}{3}$

Write an equation of the line that passes through the given points.

23. (2, −1), (5, 2)

24. (−4, 3), (1, 13)

25. (3, 5), (5, 6)

26. (2, 4), (7, 2)

27. **CAMP** In 2005, a camp had 450 campers. Five years later, the number of campers rose to 750. Write a linear equation that represents the number of campers that attend camp.

Example 2

Write an equation of the line that passes through (3, 2) with a slope of 5.

Step 1 Find the y-intercept.

$y = mx + b$ Slope-intercept form

$2 = 5(3) + b$ $m = 5$, $y = 2$, and $x = 3$

$2 = 15 + b$ Simplify.

$-13 = b$ Subtract 15 from each side.

Step 2 Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = 5x - 13$ $m = 5$ and $b = -13$

CHAPTER 4 Study Guide and Review *Continued*

4-3 Writing Equations in Point-Slope Form

Write an equation in point-slope form for the line that passes through the given point with the slope provided.

28. (6, 3), slope 5

29. (−2, 1), slope −3

30. (−4, 2), slope 0

Write each equation in standard form.

31. $y - 3 = 5(x - 2)$

32. $y - 7 = -3(x + 1)$

33. $y + 4 = \frac{1}{2}(x - 3)$

34. $y - 9 = -\frac{4}{5}(x + 2)$

Write each equation in slope-intercept form.

35. $y - 2 = 3(x - 5)$

36. $y - 12 = -2(x - 3)$

37. $y + 3 = 5(x + 1)$

38. $y - 4 = \frac{1}{2}(x + 2)$

Example 3

Write an equation in point-slope form for the line that passes through (3, 4) with a slope of −2.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = -2(x - 3)$	Replace m with −2 and (x_1, y_1) with (3, 4).

Example 4

Write $y + 6 = -4(x - 3)$ in standard form.

$y + 6 = -4(x - 3)$	Original equation
$y + 6 = -4x + 12$	Distributive Property
$4x + y + 6 = 12$	Add $4x$ to each side.
$4x + y = 6$	Subtract 6 from each side.

4-4 Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

39. (2, 5), $y = x - 3$

40. (0, 3), $y = 3x + 5$

41. (−4, 1), $y = -2x - 6$

42. (−5, −2), $y = -\frac{1}{2}x + 4$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the given equation.

43. (2, 4), $y = 3x + 1$

44. (1, 3), $y = -2x - 4$

45. (−5, 2), $y = \frac{1}{3}x + 4$

46. (3, 0), $y = -\frac{1}{2}x$

Example 5

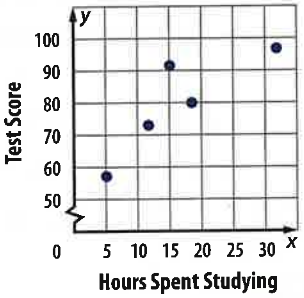
Write an equation in slope-intercept form for the line that passes through (−2, 4) and is parallel to the graph of $y = 6x - 3$.

The slope of the line with equation $y = 6x - 3$ is 6. The line parallel to $y = 6x - 3$ has the same slope, 6.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = 6[x - (-2)]$	Substitute.
$y - 4 = 6(x + 2)$	Simplify.
$y - 4 = 6x + 12$	Distributive Property
$y = 6x + 16$	Add 4 to each side.

4-5 Scatter Plots and Lines of Fit

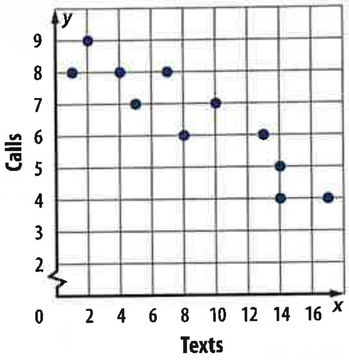
47. Determine whether the graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning.



48. **ATTENDANCE** A scatter plot of data compares the number of years since a business has opened and its annual number of sales. It contains the ordered pairs (2, 650) and (5, 1280). Write an equation in slope-intercept form for the line of fit for this situation.

Example 6

The scatter plot displays the number of texts and the number of calls made daily. Write an equation for the line of fit.



First, find the slope using (2, 9) and (17, 4).

$m = \frac{4-9}{17-2} = \frac{-5}{15}$ or $-\frac{1}{3}$ Substitute and simplify.

Then find the y-intercept.

$9 = -\frac{1}{3}(2) + b$ Substitute.

$9\frac{2}{3} = b$ Add $\frac{2}{3}$ to each side.

Write the equation. $y = -\frac{1}{3}x + 9\frac{2}{3}$

4-6 Regression and Median-Fit Lines

49. **SALE** The table shows the number of purchases made at an outerwear store during a sale. Write an equation of the regression line. Then estimate the daily purchases on day 10 of the sale.

Days Since Sale Began	1	2	3	4	5	6	7
Daily Purchases	15	21	32	30	40	38	51

50. **MOVIES** The table shows ticket sales at a certain theater during the first week after a movie opened. Write an equation of the regression line. Then estimate the daily ticket sales on the 15th day.

Days Since Movie Opened	1	2	3	4	5	6	7
Daily Ticket Sales	85	92	89	78	65	68	55

Example 7

ATTENDANCE **The table shows the annual attendance at an amusement park. Write an equation of the regression line for the data.**

Years Since 2004	0	1	2	3	4	5	6
Attendance (thousands)	75	80	72	68	65	60	53

Step 1 Enter the data by pressing STAT and selecting the **Edit** option.

Step 2 Perform the regression by pressing STAT and selecting the **CALC** option. Scroll down to **LinReg (ax + b)** and press ENTER.

Step 3 Write the equation of the regression line by rounding the a- and b-values on the screen. $y = -4.04x + 79.68$

4-7 Inverse Linear Functions

Find the inverse of each relation.

51. {(7, 3.5), (6.2, 8), (−4, 2.7), (−12, 1.4)}

52. {(1, 9), (13, 26), (−3, 4), (−11, −2)}

53.

X	*Y*
−4	2.7
−1	3.8
0	4.1
3	7.2

54.

X	*Y*
−12	4
−8	0
−4	−4
0	−8

Find the inverse of each function.

55. $f(x) = \frac{5}{11}x + 10$

56. $f(x) = 3x + 8$

57. $f(x) = -4x - 12$

58. $f(x) = \frac{1}{4}x - 7$

59. $f(x) = -\frac{2}{3}x + \frac{1}{4}$

60. $f(x) = -3x + 3$

Example 8

Find the inverse of the relation.

{(5, −3), (11, 2), (−6, 12), (4, −2)}

To find the inverse, exchange the coordinates of the ordered pairs.

$(5, -3) \rightarrow (-3, 5)$ $(-6, 12) \rightarrow (12, -6)$

$(11, 2) \rightarrow (2, 11)$ $(4, -2) \rightarrow (-2, 4)$

The inverse is {(−3, 5), (2, 11), (12, −6), (−2, 4)}.

Example 9

Find the inverse of $f(x) = \frac{1}{4}x + 9$.

$f(x) = \frac{1}{4}x + 9$ Original equation

$y = \frac{1}{4}x + 9$ Replace $f(x)$ with y.

$x = \frac{1}{4}y + 9$ Interchange y and x.

$x - 9 = \frac{1}{4}y$ Subtract 9 from each side.

$4(x - 9) = y$ Multiply each side by 4.

$4x - 36 = y$ Distributive Property

$4x - 36 = f^{-1}(x)$ Replace y with $f^{-1}(x)$.

CHAPTER 4

Practice Test

1. Graph $y = 2x - 3$.

2. **MULTIPLE CHOICE** A popular pizza parlor charges \$12 for a large cheese pizza plus \$1.50 for each additional topping. Write an equation in slope-intercept form for the total cost C of a pizza with t toppings.

A $C = 12t + 1.50$

B $C = 13.50t$

C $C = 12 + 1.50t$

D $C = 1.50t - 12$

Write an equation of a line in slope-intercept form that passes through the given point and has the given slope.

3. $(-4, 2)$; slope -3

4. $(3, -5)$; slope $\frac{2}{3}$

Write an equation of the line in slope-intercept form that passes through the given points.

5. $(1, 4), (3, 10)$

6. $(2, 5), (-2, 8)$

7. $(0, 4), (-3, 0)$

8. $(7, -1), (9, -4)$

9. **PAINTING** The data in the table show the size of a room in square feet and the time it takes to paint the room in minutes.

Room Size	100	150	200	400	500
Painting Time	160	220	270	500	680

a. Use the points (100, 160) and (500, 680) to write an equation in slope-intercept form.

b. Predict the amount of time required to paint a room measuring 750 square feet.

10. **SALARY** The table shows the relationship between years of experience and teacher salary.

Years Experience	1	5	10	15	20
Salary (thousands of dollars)	28	31	42	49	64

a. Write an equation for the best-fit line.

b. Find the correlation coefficient and explain what it tells us about the relationship between experience and salary.

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

11. $(2, -3), y = 4x - 9$

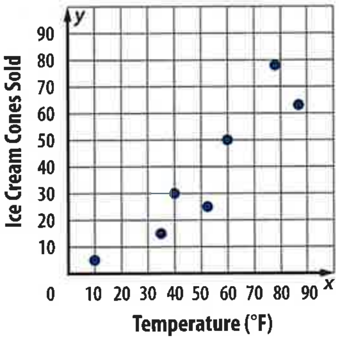
12. $(-5, 1), y = -3x + 2$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

13. $(1, 4), y = -2x + 5$

14. $(-3, 6), y = \frac{1}{4}x + 2$

15. **MULTIPLE CHOICE** The graph shows the relationship between outside temperature and daily ice cream cone sales. What type of correlation is shown?



F positive correlation

G negative correlation

H no correlation

J not enough information

16. **ADOPTION** The table shows the number of children from Ethiopia adopted by U.S. citizens.

Years Since 2000	5	6	7	8	9
Number of Children	442	731	1254	1724	2277

a. Write the slope-intercept form of the equation for the line of fit.

b. Predict the number of children from Ethiopia who will be adopted in 2025.

Find the inverse of each function.

17. $f(x) = -5x - 30$

18. $f(x) = 4x + 10$

19. $f(x) = \frac{1}{6}x - 2$

20. $f(x) = \frac{3}{4}x + 12$