

Linear Inequalities



Then

- You solved linear equations.

Now

- In this chapter, you will:
 - Solve one-step and multi-step inequalities.
 - Solve compound inequalities and inequalities involving absolute value.
 - Graph inequalities in two variables.

Why? ▲

- PETS** In the United States, about 75 million dogs are kept as pets. Approximately 16% of these were adopted from animal shelters. About 14% of dog owners own more than 3 dogs.

Linear Inequalities
Activity

The inequality $y < 2x - 3$ represents the number of dog toys you will buy each year depending on the number of dogs you have.

Graph $y < 2x - 3$.

Drag each point to the correct position on the grid.

Click the appropriate line to connect the points.

Select the appropriate choice to shade the area on the grid.

Shade above the line
 Shade below the line

Check Answer

connectED.mcgraw-hill.com Your Digital Math Portal

Animation



Vocabulary



eGlossary



Personal Tutor



Virtual Manipulatives



Graphing Calculator



Audio



Foldables



Self-Check Practice



Worksheets



Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Evaluate each expression for the given values.

- $3x + y$ if $x = -4$ and $y = 2$
- $-2m + 3k$ if $m = -8$ and $k = 3$
- CARS** The expression $\frac{m \text{ mi}}{g \text{ gal}}$ represents the gas mileage of a car. Find the gas mileage of a car that goes 295 miles on 12 gallons of gasoline. Round to the nearest tenth.

Solve each equation.

4. $x - 4 = 9$	5. $x + 8 = -3$
6. $4x = -16$	7. $\frac{x}{3} = 7$
8. $2x + 1 = 9$	9. $4x - 5 = 15$


- $9x + 2 = 3x - 10$
- $3(x - 2) = -2(x + 13)$
- FINANCIAL LITERACY** Claudia opened a savings account with \$325. She saves \$100 per month. Write an equation to determine how much money d , she has after m months.

Solve each equation.

13. $ x + 11 = 18$	14. $ 3x - 2 = 16$
---------------------	---------------------

- SURVEYS** In a survey, 32% of the people chose pizza as their favorite food. The results were reported to within 2% accuracy. What is the maximum and minimum percent of people who chose pizza?

QuickReview



Example 1

Evaluate $-3x^2 + 4x - 6$ if $x = -2$.

$-3x^2 + 4x - 6$	Original expression
$= -3(-2)^2 + 4(-2) - 6$	Replace x with -2 .
$= -3(4) + 4(-2) - 6$	Evaluate the power.
$= -12 + (-8) - 6$	Multiply.
$= -26$	Add and subtract.

Example 2

Solve $-2(x - 4) = 7x - 19$.

$-2(x - 4) = 7x - 19$	Original equation
$-2x + 8 = 7x - 19$	Distributive Property
$-2x + 8 + 2x = 7x - 19 + 2x$	Add $2x$ to each side.
$8 = 9x - 19$	Simplify.
$8 + 19 = 9x - 19 + 19$	Add 19 to each side.
$27 = 9x$	Simplify.
$3 = x$	Divide each side by 3.

Example 3

Solve $|x - 4| = 9$.

If $|x - 4| = 9$, then $x - 4 = 9$ or $x - 4 = -9$.

$x - 4 = 9$	or	$x - 4 = -9$
$x - 4 + 4 = 9 + 4$		$x - 4 + 4 = -9 + 4$
$x = 13$		$x = -5$

So, the solution set is $\{-5, 13\}$.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.



Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

Get Started on the Chapter

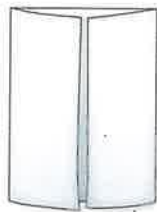
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 5. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer



Linear Inequalities Make this Foldable to help you organize your Chapter 5 notes about linear inequalities. Begin with a sheet of 11" by 17" paper.

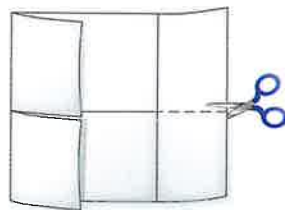
- 1** Fold each side so the edges meet in the center.



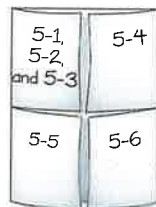
- 2** Fold in half.



- 3** Unfold and cut from each end until you reach the vertical line.



- 4** Label the front of each flap.



New Vocabulary



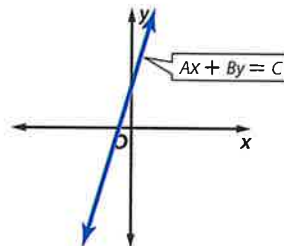
English	Español
inequality p. 285	desigualdad
set-builder notation p. 286	notación de construcción de conjuntos
compound inequality p. 306	desigualdad compuesta
intersection p. 306	intersección
union p. 307	unión
boundary p. 317	frontera
half-plane p. 317	semiplano
closed half-plane p. 317	semiplano cerrada
open half-plane p. 317	semiplano abierto

Review Vocabulary



equivalent equations ecuaciones equivalentes equations that have the same solution

linear equation ecuación lineal an equation in the form $Ax + By = C$, with a graph consisting of points on a straight line



solution set conjunto solución the set of elements from the replacement set that makes an open sentence true

ReadingMath

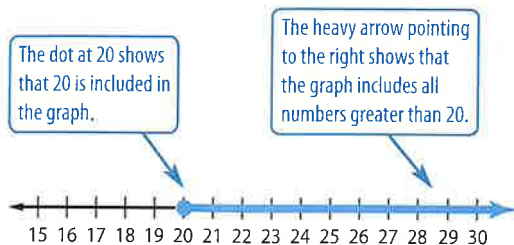
set-builder notation

$\{x \mid x \geq 20\}$ is read *the set of all numbers x such that x is greater than or equal to 20.*

A more concise way of writing a solution set is to use **set-builder notation**.

In set-builder notation, the solution set in Example 1 is $\{x \mid x \geq 20\}$.

This solution set can be graphed on a number line. Be sure to check if the endpoint of the graph of an inequality should be a circle or a dot. If the endpoint is not included in the graph, use a circle, otherwise use a dot.



2 Solve Inequalities by Subtraction

Subtraction can also be used to solve inequalities.

KeyConcept Subtraction Property of Inequalities

Words If the same number is subtracted from each side of a true inequality, the resulting inequality is also true.

Symbols For all numbers a , b , and c , the following are true.

1. If $a > b$, then $a - c > b - c$.
2. If $a < b$, then $a - c < b - c$.

This property is also true for \geq and \leq .

Test-Taking Tip

Isolating the Variable When solving inequalities, the goal is to isolate the variable on one side of the inequality. This is the same as with solving equations.

Standardized Test Example 2 Solve by Subtracting



Solve $m + 19 > 56$.

- A $\{m \mid m < 41\}$ B $\{m \mid m < 37\}$ C $\{m \mid m > 37\}$ D $\{m \mid m > 41\}$

Read the Test Item

You need to find the solution set for the inequality.

Solve the Test Item

Step 1 Solve the inequality.

$$m + 19 > 56 \quad \text{Original inequality}$$

$$m + 19 - 19 > 56 - 19 \quad \text{Subtract 19 from each side.}$$

$$m > 37 \quad \text{Simplify.}$$

Step 2 Write in set-builder notation: $\{m \mid m > 37\}$. The answer is C.

Guided Practice

2. Solve $p + 8 \leq 18$.

- F $\{p \mid p \geq 10\}$ G $\{p \mid p \leq 10\}$ H $\{p \mid p \leq 26\}$ J $\{p \mid p \geq 126\}$



Terms that are constants are not the only terms that can be subtracted. Terms with variables can also be subtracted from each side to solve inequalities.



StudyTip

Writing Inequalities

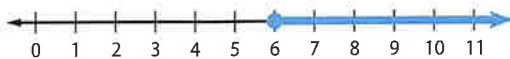
Simplifying the inequality so that the variable is on the left side, as in $a \geq 6$, prepares you to write the solution set in set-builder notation.

Example 3 Variables on Each Side

Solve $3a + 6 \leq 4a$. Then graph the solution set on a number line.

$$\begin{array}{ll} 3a + 6 \leq 4a & \text{Original inequality} \\ 3a - 3a + 6 \leq 4a - 3a & \text{Subtract } 3a \text{ from each side.} \\ 6 \leq a & \text{Simplify.} \end{array}$$

Since $6 \leq a$ is the same as $a \geq 6$, the solution set is $\{a \mid a \geq 6\}$.



GuidedPractice

Solve each inequality. Then graph the solution set on a number line.

3A. $9n - 1 < 10n$

3B. $5h \leq 12 + 4h$

Verbal problems containing phrases like *greater than* or *less than* can be solved by using inequalities. The chart shows some other phrases that indicate inequalities.

ConceptSummary Phrases for Inequalities

$<$	$>$	\leq	\geq
less than fewer than	greater than more than	at most, no more than, less than or equal to	at least, no less than, greater than or equal to



Real-World Example 4 Use an Inequality to Solve a Problem

PETS Felipe needs for the temperature of his leopard gecko's basking spot to be at least 82°F . Currently the basking spot is 62.5°F . How much warmer does the basking spot need to be?

Words The current temperature needs to be at least 82°F .

Variable Let t = the number of degrees that the temperature needs to rise.

Inequality $62.5 + t \geq 82$

$$\begin{array}{ll} 62.5 + t \geq 82 & \text{Original inequality} \\ 62.5 + t - 62.5 \geq 82 - 62.5 & \text{Subtract } 62.5 \text{ from each side.} \\ t \geq 19.5 & \text{Simplify.} \end{array}$$

Felipe needs to raise the temperature of the basking spot 19.5°F or more.

GuidedPractice

4. **SHOPPING** Sanjay has \$65 to spend at the mall. He bought a T-shirt for \$18 and a belt for \$14. If Sanjay wants a pair of jeans, how much can he spend?



Real-WorldLink

Leopard geckos are commonly yellow and white with black spots. They are nocturnal and easy to tame. They do not have toe pads like other geckos, so they do not climb.

Source: Exotic Pets





Examples 1–3 Solve each inequality. Then graph the solution set on a number line.

1. $x - 3 \geq 7$

2. $5 \geq 7 + y$

3. $g + 6 < 2$

4. $11 \leq p + 4$

5. $10 > n - 1$

6. $k + 24 > -5$

7. $8r + 6 < 9r$

8. $8n \geq 7n - 3$

Example 4 Define a variable, write an inequality, and solve each problem. Check your solution.

9. Twice a number increased by 4 is at least 10 more than the number.

10. Three more than a number is less than twice the number.

11. **AMUSEMENT** A thrill ride swings passengers back and forth, a little higher each time up to 137 feet. Suppose the height of the swing after 30 seconds is 45 feet. How much higher will the ride swing?

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–3 Solve each inequality. Then graph the solution set on a number line.

12. $m - 4 < 3$

13. $p - 6 \geq 3$

14. $r - 8 \leq 7$

15. $t - 3 > -8$

16. $b + 2 \geq 4$

17. $13 > 18 + r$

18. $5 + c \leq 1$

19. $-23 \geq q - 30$

20. $11 + m \geq 15$

21. $h - 26 < 4$

22. $8 \leq r - 14$

23. $-7 > 20 + c$

24. $2a \leq -4 + a$

25. $z + 4 \geq 2z$

26. $w - 5 \leq 2w$

27. $3y + 6 \leq 2y$

28. $6x + 5 \geq 7x$

29. $-9 + 2a < 3a$

Example 4 Define a variable, write an inequality, and solve each problem. Check your solution.

30. Twice a number is more than the sum of that number and 9.

31. The sum of twice a number and 5 is at most 3 less than the number.

32. The sum of three times a number and -4 is at least twice the number plus 8.

33. Six times a number decreased by 8 is less than five times the number plus 21.

CCSS MODELING Define a variable, write an inequality, and solve each problem. Then interpret your solution.

34. **FINANCIAL LITERACY** Keisha is babysitting at \$8 per hour to earn money for a car. So far she has saved \$1300. The car that Keisha wants to buy costs at least \$5440. How much money does Keisha still need to earn to buy the car?

35. **TECHNOLOGY** A recent survey found that more than 21 million people between the ages of 12 and 17 use the Internet. Of those, about 16 million said they use the Internet at school. How many teens that are online do not use the Internet at school?

36. **MUSIC** A DJ added 20 more songs to his digital media player, making the total more than 61. How many songs were originally on the player?



37. TEMPERATURE The water temperature in a swimming pool increased 4°F this morning. The temperature is now less than 81°F . What was the water temperature this morning?

38. BASKETBALL A player's goal was to score at least 150 points this season. So far, she has scored 123 points. If there is one game left, how many points must she score to reach her goal?

39. SPAS Samantha received a \$75 gift card for a local day spa for her birthday. She plans to get a haircut and a manicure. How much money will be left on her gift card after her visit?

Service	Cost (\$)
haircut	at least 32
manicure	at least 26

40. VOLUNTEER Kono knows that he can only volunteer up to 25 hours per week. If he has volunteered for the times recorded at the right, how much more time can Kono volunteer this week?

Center	Time (h)
Shelter	3 h 15 min
Kitchen	2 h 20 min

Solve each inequality. Check your solution, and then graph it on a number line.

41. $c + (-1.4) \geq 2.3$

42. $9.1g + 4.5 < 10.1g$

43. $k + \frac{3}{4} > \frac{1}{3}$

44. $\frac{3}{2}p - \frac{2}{3} \leq \frac{4}{9} + \frac{1}{2}p$

45. MULTIPLE REPRESENTATIONS In this problem, you will explore multiplication and division in inequalities.

a. Geometric Suppose a balance has 12 pounds on the left side and 18 pounds on the right side. Draw a picture to represent this situation.



b. Numerical Write an inequality to represent the situation.

c. Tabular Create a table showing the result of doubling, tripling, or quadrupling the weight on each side of the balance. Create a second table showing the result of reducing the weight on each side of the balance by a factor of $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$. Include a column in each table for the inequality representing each situation.

d. Verbal Describe the effect multiplying or dividing each side of an inequality by the same positive value has on the inequality.

CCSS REASONING If $m + 7 \geq 24$, then complete each inequality.

46. $m \geq ?$

47. $m + ? \geq 27$

48. $m - 5 \geq ?$

49. $m - ? \geq 14$

50. $m - 19 \geq ?$

51. $m + ? \geq 43$

H.O.T. Problems Use Higher-Order Thinking Skills

52. REASONING Compare and contrast the graphs of $a < 4$ and $a \leq 4$.

53. CHALLENGE Suppose $b > d + \frac{1}{3}$, $c + 1 < a - 4$, and $d + \frac{5}{8} > a + 2$. Order a , b , c , and d from least to greatest.

54. OPEN ENDED Write three linear inequalities that are equivalent to $y < -3$.

55. WRITING IN MATH Summarize the process of solving and graphing linear inequalities.

56. WRITING IN MATH Explain why $x - 2 > 5$ has the same solution set as $x > 7$.



Standardized Test Practice

57. Which equation represents the relationship shown?

- A $y = 7x - 8$
 B $y = 7x + 8$
 C $y = 8x - 7$
 D $y = 8x + 7$

x	y
1	1
2	9
3	17
4	25
5	33
6	41

58. What is the solution set of the inequality $7 + x < 5$?

- F $\{x \mid x < 2\}$ H $\{x \mid x < -2\}$
 G $\{x \mid x > 2\}$ J $\{x \mid x > -2\}$

59. Francisco has \$3 more than $\frac{1}{4}$ the number of dollars that Kayla has. Which expression represents how much money Francisco has?

- A $3\left(\frac{1}{4}k\right)$ C $3 - \frac{1}{4}k$
 B $\frac{1}{4}k + 3$ D $\frac{1}{4} + 3k$

60. **GRIDDED RESPONSE** The mean score for 10 students on the chemistry final exam was 178. However, the teacher had made a mistake and recorded one student's score as ten points less than the actual score. What should the mean score be?

Spiral Review

Find the inverse of each function. (Lesson 4-7)

61. $f(x) = 7x - 28$

62. $f(x) = \frac{2}{5}x + 12$

63. $f(x) = -\frac{1}{3}x - 8$

64. $f(x) = 12x + 16$

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of each equation. (Lesson 4-4)

65. $(-2, 0)$, $y = x - 6$

66. $(-3, 1)$, $y = -3x + 7$

67. $(1, -3)$, $y = \frac{1}{2}x + 4$

68. $(-2, 7)$, $2x - 5y = 3$

69. **TRAVEL** On an island cruise in Hawaii, each passenger is given a lei. A crew member hands out 3 red, 3 blue, and 3 green leis in that order. If this pattern is repeated, what color lei will the 50th person receive? (Lesson 3-6)

Find the n th term of each arithmetic sequence described. (Lesson 3-5)

70. $a_1 = 52$, $d = 12$, $n = 102$

71. $-9, -7, -5, -3, \dots$ for $n = 18$

72. $0.5, 1, 1.5, 2, \dots$ for $n = 50$

73. **JOBS** Refer to the time card shown. Write a direct variation equation relating your pay to the hours worked and find your pay if you work 30 hours. (Lesson 3-4)

Day	Hours
FRIDAY	2.0
SATURDAY	3.5
SUNDAY	2.0
TOTAL HOURS	7.5
PAY	\$52.50

Skills Review

Solve each equation.

74. $8y = 56$

75. $4p = -120$

76. $-3a = -21$

77. $2c = \frac{1}{5}$

78. $\frac{r}{2} = 21$

79. $-\frac{3}{4}g = -12$

80. $\frac{2}{5}w = -4$

81. $-6x = \frac{2}{3}$



Algebra Lab

Solving Inequalities



You can use algebra tiles to solve inequalities.

CCSS Common Core State Standards
Content Standards

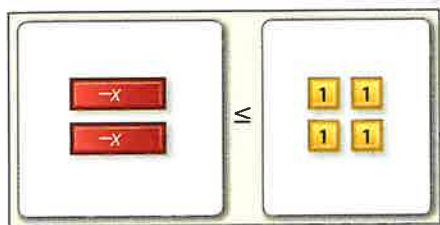
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.



Activity Solve Inequalities

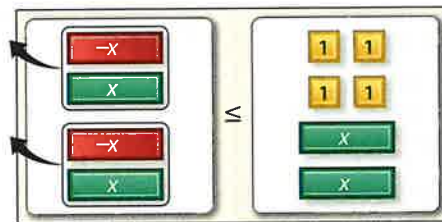
Solve $-2x \leq 4$.

Step 1 Use a self-adhesive note to cover the equals sign on the equation mat. Then write a \leq symbol on the note. Model the inequality.



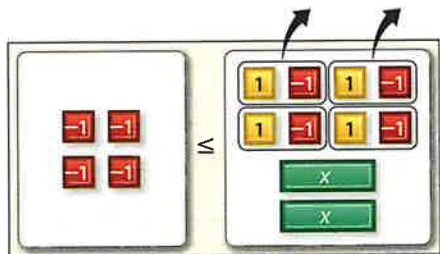
$$-2x \leq 4$$

Step 2 Since you do not want to solve for a negative x -tile, eliminate the negative x -tiles by adding 2 positive x -tiles to each side. Remove the zero pairs.



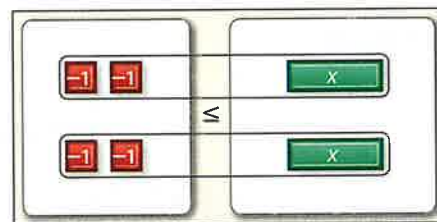
$$-2x + 2x \leq 4 + 2x$$

Step 3 Add 4 negative 1-tiles to each side to isolate the x -tiles. Remove the zero pairs.



$$-4 \leq 2x$$

Step 4 Separate the tiles into 2 groups.



$$-2 \leq x \text{ or } x \geq -2$$

Model and Analyze

Use algebra tiles to solve each inequality.

- $-3x < 9$
 - $-4x > -4$
 - $-5x \geq 15$
 - $-6x \leq -12$
- In Exercises 1–4, is the coefficient of x in each inequality positive or negative?
 - Compare the inequality symbols and locations of the variable in Exercises 1–4 with those in their solutions. What do you find?
 - Model the solution for $3x \leq 12$. How is this different from solving $-3x \leq 12$?
 - Write a rule for solving inequalities involving multiplication and division. (*Hint*: Remember that dividing by a number is the same as multiplying by its reciprocal.)

Solving Inequalities by Multiplication and Division

Then

- You solved equations by using multiplication and division.

Now

- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

Why?

- Terrell received a gift card for \$20 of music downloads. If each download costs \$0.89, the number of downloads he can purchase can be represented by the inequality $0.89d \leq 20$.



Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

- Make sense of problems and persevere in solving them.
- Attend to precision.

1 Solve Inequalities by Multiplication If you multiply each side of an inequality by a positive number, then the inequality remains true.

$$4 > 2 \quad \text{Original inequality}$$

$$4(3) > 2(3) \quad \text{Multiply each side by 3.}$$

$$12 > 6 \quad \text{Simplify.}$$

Notice that the direction of the inequality remains the same.

If you multiply each side of an inequality by a negative number, the inequality symbol changes direction.

$$7 < 9 \quad \text{Original inequality}$$

$$7(-2) < 9(-2) \quad \text{Multiply each side by } -2.$$

$$-14 > -18 \quad \text{Simplify.}$$

These examples demonstrate the **Multiplication Property of Inequalities**.

Key Concept Multiplication Property of Inequalities

Words	Symbols	Examples
If both sides of an inequality that is true are multiplied by a positive number, the resulting inequality is also true.	For any real numbers a and b and any positive real number c , if $a > b$, then $ac > bc$. And, if $a < b$, then $ac < bc$.	$6 > 3.5$ $6(2) > 3.5(2)$ $12 > 7$ and $2.1 < 5$ $2.1(0.5) < 5(0.5)$ $1.05 < 2.5$
If both sides of an inequality that is true are multiplied by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b and any negative real number c , if $a > b$, then $ac < bc$. And, if $a < b$, then $ac > bc$.	$7 > 4.5$ $7(-3) < 4.5(-3)$ $-21 < -13.5$ and $3.1 < 5.2$ $3.1(-4) > 5.2(-4)$ $-12.4 > -20.8$

This property also holds for inequalities involving \leq and \geq .



Real-World Example 1 Write and Solve an Inequality

SURVEYS Of the students surveyed at Madison High School, fewer than eighty-four said they have never purchased an item online. This is about one eighth of those surveyed. How many students were surveyed?

Understand You know the number of students who have never purchased an item online and the portion this is of the number of students surveyed.

Plan Let n = the number of students surveyed. Write an open sentence that represents this situation.

Words	One eighth	times	the number of students surveyed	is less than	84.
Inequality	$\frac{1}{8}$	\cdot	n	$<$	84.

Solve Solve for n .

$$\frac{1}{8}n < 84 \quad \text{Original inequality}$$

$$(8)\frac{1}{8}n < (8)84 \quad \text{Multiply each side by 8.}$$

$$n < 672 \quad \text{Simplify.}$$

Check Check the endpoint with 672 and the direction of the inequality with a value less than 672.

$$\frac{1}{8}(672) \stackrel{?}{=} 84 \quad \text{Check endpoint.} \quad \frac{1}{8}(0) \stackrel{?}{<} 84 \quad \text{Check direction.}$$

$$84 = 84 \checkmark \quad \quad \quad 0 < 84 \checkmark$$

The solution set is $\{n \mid n < 672\}$, so fewer than 672 students were surveyed.

Guided Practice

- BIOLOGY** Mount Kinabalu in Malaysia has the greatest concentration of wild orchids on Earth. It contains more than 750 species, or about one fourth of all orchid species in Malaysia. How many orchid species are there in Malaysia?

You can also use multiplicative inverses with the Multiplication Property of Inequalities to solve an inequality.



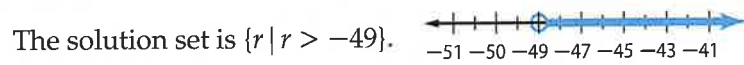
Example 2 Solve by Multiplying

Solve $-\frac{3}{7}r < 21$. Graph the solution on a number line.

$$-\frac{3}{7}r < 21 \quad \text{Original inequality}$$

$$\left(-\frac{7}{3}\right)\left(-\frac{3}{7}r\right) > \left(-\frac{7}{3}\right)21 \quad \text{Multiply each side by } -\frac{7}{3}. \text{ Reverse the inequality symbol.}$$

$$r > -49 \quad \text{Simplify. Check by substituting values.}$$



Guided Practice

Solve each inequality. Check your solution.

- 2A. $-\frac{n}{6} \leq 8$ 2B. $-\frac{4}{3}p > -10$ 2C. $\frac{1}{5}m \geq -3$ 2D. $\frac{3}{8}t < 5$

StudyTip

CCSS Sense-Making In

Example 1, you could also check the solution by substituting a number greater than 672 and verifying that the resulting inequality is false.



Real-WorldLink

More than 30,000 different orchid species flower in the wild on every continent except Antarctica.

Source: Aloha Orchid Nursery



Watch Out!

Negatives A negative sign in an inequality does not necessarily mean that the direction of the inequality should change. For example, when solving $\frac{x}{6} > -3$, do not change the direction of the inequality.

2 Solve Inequalities by Division

If you divide each side of an inequality by a positive number, then the inequality remains true.

Notice that the direction of the inequality remains the same. If you divide each side of an inequality by a negative number, the inequality symbol changes direction.

$$\begin{array}{ll} -10 < -5 & \text{Original inequality} \\ \frac{-10}{5} ? \frac{-5}{5} & \text{Divide each side by } -5. \\ -2 < -1 & \text{Simplify.} \end{array}$$

$$\begin{array}{ll} 15 < 18 & \text{Original inequality} \\ \frac{15}{-3} ? \frac{18}{-3} & \text{Divide each side by } -3. \\ -5 > -6 & \text{Simplify.} \end{array}$$

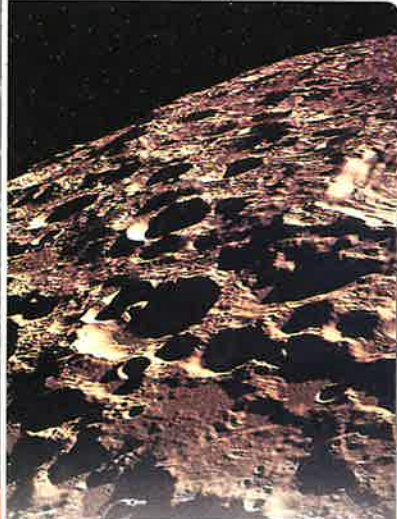
These examples demonstrate the **Division Property of Inequalities**.

Key Concept Division Property of Inequalities		
Words	Symbols	Examples
If both sides of a true inequality are divided by a positive number, the resulting inequality is also true.	For any real numbers a and b and any positive real number c , if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. And, if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$4.5 > 2.1$ $1.5 < 5$ $\frac{4.5}{3} > \frac{2.1}{3}$ and $\frac{1.5}{0.5} < \frac{5}{0.5}$ $1.5 > 0.7$ $3 < 10$
If both sides of a true inequality are divided by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b , and any negative real number c , if $a > b$, then $\frac{a}{c} < \frac{b}{c}$. And, if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.	$6 > 2.4$ $-1.8 < 3.6$ $\frac{6}{-6} < \frac{2.4}{-6}$ and $\frac{-1.8}{-9} < \frac{3.6}{-9}$ $-1 < -0.4$ $0.2 > -0.4$

This property also holds true for inequalities involving \leq and \geq .

Math History Link

Thomas Harriot
(1560–1621) Harriot was a prolific astronomer. He was the first to map the Moon's surface and to see sunspots. Harriot is best known for his work in algebra.



Example 3 Divide to Solve an Inequality

Solve each inequality. Graph the solution on a number line.

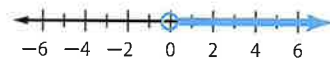
a. $60t > 8$

$$60t > 8 \quad \text{Original inequality}$$

$$\frac{60t}{60} > \frac{8}{60} \quad \text{Divide each side by 60.}$$

$$t > \frac{2}{15} \quad \text{Simplify.}$$

$$\left\{ t \mid t > \frac{2}{15} \right\}$$



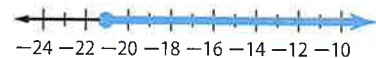
b. $-7d \leq 147$

$$-7d \leq 147 \quad \text{Original inequality}$$

$$\frac{-7d}{-7} \geq \frac{147}{-7} \quad \text{Divide each side by } -7.$$

$$d \geq -21 \quad \text{Simplify.}$$

$$\{ d \mid d \geq -21 \}$$



Guided Practice

3A. $8p \leq 58$

3C. $-12h > 15$

3B. $-42 \geq 6r$

3D. $-\frac{1}{2}n \leq 6$



Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Example 1

1. **FUNDRAISING** The Jefferson Band Boosters raised more than \$5500 from sales of their \$15 band DVD. Define a variable, and write an inequality to represent the number of DVDs they sold. Solve the inequality and interpret your solution.

Examples 2–3 Solve each inequality. Graph the solution on a number line.

2. $30 > \frac{1}{2}n$ 3. $-\frac{3}{4}r \leq -6$ 4. $-\frac{c}{6} \geq 7$ 5. $\frac{h}{2} < -5$
 6. $9t > 108$ 7. $-84 < 7v$ 8. $-28 \leq -6x$ 9. $40 \geq -5z$

Practice and Problem Solving

Extra Practice is on page R5.

Example 1

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

10. **CELL PHONE PLAN** Mario purchases a prepaid phone plan for \$50 at \$0.13 per minute. How many minutes can Mario talk on this plan?
11. **FINANCIAL LITERACY** Rodrigo needs at least \$560 to pay for his spring break expenses, and he is saving \$25 from each of his weekly paychecks. How long will it be before he can pay for his trip?

Examples 2–3 Solve each inequality. Graph the solution on a number line.

12. $\frac{1}{4}m \leq -17$ 13. $\frac{1}{2}a < 20$ 14. $-11 > -\frac{c}{11}$
 15. $-2 \geq -\frac{d}{34}$ 16. $-10 \leq \frac{x}{-2}$ 17. $-72 < \frac{f}{-6}$
 18. $\frac{2}{3}h > 14$ 19. $-\frac{3}{4}j \geq 12$ 20. $-\frac{1}{6}n \leq -18$
 21. $6p \leq 96$ 22. $4r < 64$ 23. $32 > -2y$
 24. $-26 < 26t$ 25. $-6v > -72$ 26. $-33 \geq -3z$
 27. $4b \leq -3$ 28. $-2d < 5$ 29. $-7f > 5$

30. **CHEERLEADING** To remain on the cheerleading squad, Lakita must attend at least $\frac{3}{5}$ of the study table sessions offered. She attends 15 sessions. If Lakita met the requirements, what is the maximum number of study table sessions?

31. **BRACELETS** How many bracelets can Caitlin buy for herself and her friends if she wants to spend no more than \$22?



32. **CCSS PRECISION** The National Honor Society at Pleasantville High School wants to raise at least \$500 for a local charity. Each student earns \$0.50 for every quarter of a mile walked in a walk-a-thon. How many miles will the students need to walk?

33. **MUSEUM** The American history classes are planning a trip to a local museum. Admission is \$8 per person. Determine how many people can go for \$260.

34. **GASOLINE** If gasoline costs \$3.15 per gallon, how many gallons of gasoline, to the nearest tenth, can Jan buy for \$24?



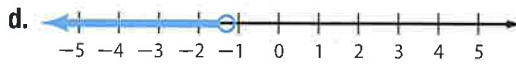
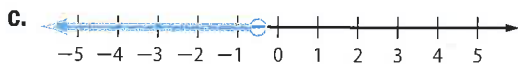
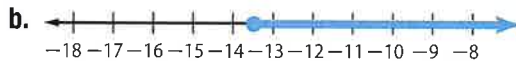
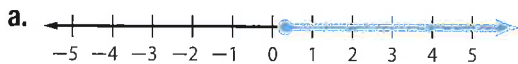
Match each inequality to the graph of its solution.

35. $-\frac{2}{3}h \leq 9$

36. $25j \geq 8$

37. $3.6p < -4.5$

38. $2.3 < -5t$



39. **CANDY** Fewer than 42 employees at a factory stated that they preferred fudge over fruit candy. This is about two thirds of the employees. How many employees are there?

40. **TRAVEL** A certain travel agency employs more than 275 people at all of its branches. Approximately three fifths of all the people are employed at the west branch. How many people work at the west branch?

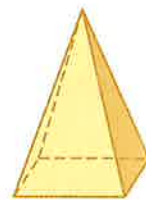
41. **MULTIPLE REPRESENTATIONS** The equation for the volume of a pyramid is $\frac{1}{3}$ the area of the base times the height.

a. **Geometric** Draw a pyramid with a square base b cm long and a height of h cm.

b. **Numerical** Suppose the pyramid has a volume of 72 cm^3 . Write an equation to find the height.

c. **Tabular** Create a table showing the value of h when $b = 1, 3, 6, 9,$ and 12 .

d. **Numerical** Write an inequality for the possible lengths of b such that $b < h$. Write an inequality for the possible lengths of h such that $b > h$.



H.O.T. Problems Use Higher-Order Thinking Skills

42. **ERROR ANALYSIS** Taro and Jamie are solving $6d \geq -84$. Is either of them correct? Explain your reasoning.

Taro

$$\begin{aligned} 6d &\geq -84 \\ \frac{6d}{6} &\geq \frac{-84}{6} \\ d &\geq -14 \end{aligned}$$

Jamie

$$\begin{aligned} 6d &\geq -84 \\ \frac{6d}{6} &\leq \frac{-84}{6} \\ d &\leq -14 \end{aligned}$$

43. **CHALLENGE** Solve each inequality for x . Assume that $a > 0$.

a. $-ax < 5$

b. $\frac{1}{a}x \geq 8$

c. $-6 \geq ax$

44. **CCSS STRUCTURE** Determine whether $x^2 > 1$ and $x > 1$ are equivalent. Explain.

45. **REASONING** Explain whether the statement *If $a > b$, then $\frac{1}{a} > \frac{1}{b}$ is sometimes, always, or never true.*

46. **OPEN ENDED** Create a real-world situation to represent the inequality $-\frac{5}{8} \geq x$.

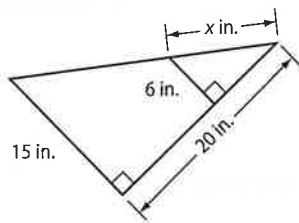
47. **WRITING IN MATH** How are solving linear inequalities and linear equations similar? different?

Standardized Test Practice

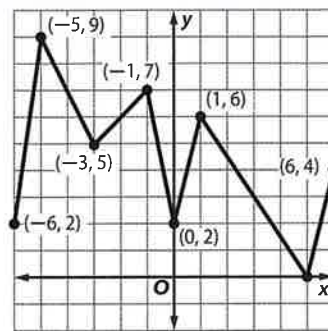
48. Juan's international calling card costs 9¢ for each minute. Which inequality can be used to find how long he can talk to a friend if he does not want to spend more than \$2.50 on the call?

- A $0.09 \geq 2.50m$
- B $0.09 \leq 2.50m$
- C $0.09m \geq 2.50$
- D $0.09m \leq 2.50$

49. **SHORT RESPONSE** Find the value of x .



50. What is the greatest rate of decrease of this function?



- F -5
- G -3
- H -2
- J 1

51. What is the value of x if $4x - 3 = -2x$?

- A -2
- B $-\frac{1}{2}$
- C $\frac{1}{2}$
- D 2

Spiral Review

Solve each inequality. Check your solution, and then graph it on a number line. (Lesson 5-1)

52. $a + 8 > 4$

53. $6y \geq 5y + 21$

54. $b + \frac{1}{2} < 1$

Find the inverse of each function. (Lesson 4-7)

55. $f(x) = -6x + 18$

56. $f(x) = \frac{3}{7}x + 9$

57. $f(x) = 4x - 5$

58. **HOME DECOR** Pam is having blinds installed at her home. The cost c of installation for any number of blinds b can be described by $c = 25 + 6.5b$. Graph the equation and determine how much it would cost if Pam has 8 blinds installed. (Lesson 3-1)

59. **RESCUE** A boater radioed for a helicopter to pick up a sick crew member. At that time, the boat and the helicopter were at the positions shown. How long will it take for the helicopter to reach the boat? (Lesson 2-9)



Solve each equation. (Lesson 2-5)

60. $|x + 3| = 10$

61. $|2x - 8| = 6$

62. $|3x + 1| = -2$

Skills Review

Solve each equation.

63. $4y + 11 = 19$

64. $2x - 7 = 9 + 4x$

65. $\frac{1}{4} + 2x = 4x - 8$

66. $\frac{1}{3}(6w - 3) = 3w + 12$

67. $\frac{7r + 5}{2} = 13$

68. $\frac{1}{2}a = \frac{a - 3}{4}$



Then

You solved multi-step equations.

Now

- 1 Solve linear inequalities involving more than one operation.
- 2 Solve linear inequalities involving the Distributive Property.

Why?

A salesperson may make a base monthly salary and earn a commission on each of her sales. To find the number of sales she needs to make to pay her monthly bills, you can use a multi-step inequality.



Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

7 Look for and make use of structure.

1 Solve Multi-Step Inequalities Multi-step inequalities can be solved by undoing the operations in the same way you would solve a multi-step equation.

Real-World Example 1 Solve a Multi-Step Inequality



SALES Write and solve an inequality to find the sales Mrs. Jones needs if she earns a monthly salary of \$2000 plus a 10% commission on her sales. Her goal is to make at least \$4000 per month. What sales does she need to meet her goal?

base salary + (commission × sales) ≥ income needed

$$\begin{aligned} 2000 + 0.10x &\geq 4000 && \text{Substitution} \\ 0.10x &\geq 2000 && \text{Subtract 2000 from each side.} \\ x &\geq 20,000 && \text{Divide each side by 0.10.} \end{aligned}$$

She must make at least \$20,000 in sales to meet her monthly goal.

Guided Practice

1. **FINANCIAL LITERACY** The Print Shop advertises a special to print 400 flyers for less than the competition. The price includes a \$3.50 set-up fee. If the competition charges \$35.50, what does the Print Shop charge for each flyer?

When multiplying or dividing by a negative number, the direction of the inequality symbol changes. This holds true for multi-step inequalities.

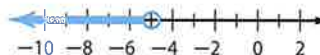
Example 2 Inequality Involving a Negative Coefficient



Solve $-11y - 13 > 42$. Graph the solution on a number line.

$$\begin{aligned} -11y - 13 &> 42 && \text{Original inequality} \\ -11y &> 55 && \text{Add 13 to each side and simplify.} \\ \frac{-11y}{-11} &< \frac{55}{-11} && \text{Divide each side by } -11, \text{ and reverse the inequality.} \\ y &< -5 && \text{Simplify.} \end{aligned}$$

The solution set is $\{y \mid y < -5\}$.



Guided Practice Solve each inequality.

2A. $23 \geq 10 - 2w$

2B. $43 > -4y + 11$



You can translate sentences into multi-step inequalities and then solve them using the Properties of Inequalities.



Example 3 Write and Solve an Inequality

Define a variable, write an inequality, and solve the problem.

Five minus 6 times a number is more than four times the number plus 45.

Let n be the number.

Five	minus	six times a number	is more than	four times a number	plus	forty-five.
5	-	$6n$	$>$	$4n$	+	45

$$5 - 10n > 45 \quad \text{Subtract } 4n \text{ from each side and simplify.}$$

$$-10n > 40 \quad \text{Subtract 5 from each side and simplify.}$$

$$\frac{-10n}{-10} < \frac{40}{-10} \quad \text{Divide each side by } -10, \text{ and reverse the inequality.}$$

$$n < -4 \quad \text{Simplify.}$$

The solution set is $\{n \mid n < -4\}$.

Guided Practice

3. Two more than half of a number is greater than twenty-seven.

2 Solve Inequalities Involving the Distributive Property When solving inequalities that contain grouping symbols, use the Distributive Property to remove the grouping symbols first. Then use the order of operations to simplify the resulting inequality.



Example 4 Distributive Property

Solve $4(3t - 5) + 7 \geq 8t + 3$. Graph the solution on a number line.

$$4(3t - 5) + 7 \geq 8t + 3 \quad \text{Original inequality}$$

$$12t - 20 + 7 \geq 8t + 3 \quad \text{Distributive Property}$$

$$12t - 13 \geq 8t + 3 \quad \text{Combine like terms.}$$

$$4t - 13 \geq 3 \quad \text{Subtract } 8t \text{ from each side and simplify.}$$

$$4t \geq 16 \quad \text{Add 13 to each side.}$$

$$\frac{4t}{4} \geq \frac{16}{4} \quad \text{Divide each side by 4.}$$

$$t \geq 4 \quad \text{Simplify.}$$

The solution set is $\{t \mid t \geq 4\}$.



Guided Practice

Solve each inequality. Graph the solution on a number line.

4A. $6(5z - 3) \leq 36z$

4B. $2(h + 6) > -3(8 - h)$

If solving an inequality results in a statement that is always true, the solution set is the set of all real numbers. This solution set is written as $\{x \mid x \text{ is a real number}\}$. If solving an inequality results in a statement that is never true, the solution set is the empty set, which is written as the symbol \emptyset . The empty set has no members.

Real-World Career

Veterinarian Veterinarians take care of sick and injured animals. Vets can work anywhere from a zoo to a research facility to owning their own practice. Vets need to earn a bachelor's degree, attend vet college for 4 years, and take a test to get licensed.

Watch Out!

Distributive Property

If a negative number is multiplied by a sum or difference, remember to distribute the negative sign along with the number to each term inside the parentheses.



Example 5 Empty Set and All Reals

Solve each inequality. Check your solution.

a. $9t - 5(t - 5) \leq 4(t - 3)$

$$9t - 5(t - 5) \leq 4(t - 3) \quad \text{Original inequality}$$

$$9t - 5t + 25 \leq 4t - 12 \quad \text{Distributive Property}$$

$$4t + 25 \leq 4t - 12 \quad \text{Combine like terms.}$$

$$4t + 25 - 4t \leq 4t - 12 - 4t \quad \text{Subtract } 4t \text{ from each side.}$$

$$25 \leq -12 \quad \text{Simplify.}$$

Since the inequality results in a false statement, the solution set is the empty set, \emptyset .

b. $3(4m + 6) \leq 42 + 6(2m - 4)$

$$3(4m + 6) \leq 42 + 6(2m - 4) \quad \text{Original inequality}$$

$$12m + 18 \leq 42 + 12m - 24 \quad \text{Distributive Property}$$

$$12m + 18 \leq 12m + 18 \quad \text{Combine like terms.}$$

$$12m + 18 - 12m \leq 12m + 18 - 12m \quad \text{Subtract } 12m \text{ from each side.}$$

$$18 \leq 18 \quad \text{Simplify.}$$

All values of m make the inequality true. All real numbers are solutions.

Guided Practice

Solve each inequality. Check your solution.

5A. $18 - 3(8c + 4) \geq -6(4c - 1)$

5B. $46 \leq 8m - 4(2m + 5)$

StudyTip

CCSS Structure Notice that the inequality $4t + 25 < 4t - 12$ means *some number 4t plus 25 is less than or equal to that number minus 12*. No real number makes that inequality true. Observing the meaning of the expressions in each step in this way can lead you to solutions more quickly.

Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Example 1

1. **CANOEING** If four people plan to use the canoe with 60 pounds of supplies, write and solve an inequality to find the allowable average weight per person.



800-lb capacity

2. **SHOPPING** Rita is ordering a movie for \$11.95 and a few CDs. She has \$50 to spend. Shipping and sales tax will be \$10. If each CD costs \$9.99, write and solve an inequality to find the greatest number of CDs that she can buy.

Example 2

CCSS STRUCTURE Solve each inequality. Graph the solution on a number line.

3. $6h - 10 \geq 32$

4. $-3 \leq \frac{2}{3}r + 9$

5. $-3x + 7 > 43$

6. $4m - 17 < 6m + 25$

Example 3

Define a variable, write an inequality, and solve each problem. Then check your solution.

7. Four times a number minus 6 is greater than eight plus two times the number.
8. Negative three times a number plus 4 is less than five times the number plus 8.

Examples 4–5 Solve each inequality. Graph the solution on a number line.

9. $-6 \leq 3(5v - 2)$

10. $-5(g + 4) > 3(g - 4)$

11. $3 - 8x \geq 9 + 2(1 - 4x)$



Examples 1
and 2**CCSS STRUCTURE** Solve each inequality. Graph the solution on a number line.

12. $5b - 1 \geq -11$

14. $-9 \geq \frac{2}{5}m + 7$

16. $-a + 6 \leq 5$

18. $8 - \frac{z}{3} \geq 11$

20. $3b - 6 \geq 15 + 24b$

13. $21 > 15 + 2a$

15. $\frac{w}{8} - 13 > -6$

17. $37 < 7 - 10w$

19. $-\frac{5}{4}p + 6 < 12$

21. $15h + 30 < 10h - 45$

Example 3

Define a variable, write an inequality, and solve each problem. Check your solution.

22. Three fourths of a number decreased by nine is at least forty-two.

23. Two thirds of a number added to six is at least twenty-two.

24. Seven tenths of a number plus 14 is less than forty-nine.

25. Eight times a number minus twenty-seven is no more than the negative of that number plus eighteen.

26. Ten is no more than 4 times the sum of twice a number and three.

27. Three times the sum of a number and seven is greater than five times the number less thirteen.

28. The sum of nine times a number and fifteen is less than or equal to the sum of twenty-four and ten times the number.

Examples 4
and 5**CCSS STRUCTURE** Solve each inequality. Graph the solution on a number line.

29. $-3(7n + 3) < 6n$

31. $2y + 4 > 2(3 + y)$

33. $7 + t \leq 2(t + 3) + 2$

30. $21 \geq 3(a - 7) + 9$

32. $3(2 - b) < 10 - 3(b - 6)$

34. $8a + 2(1 - 5a) \leq 20$

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

35. **CARS** A car salesperson is paid a base salary of \$35,000 a year plus 8% of sales. What are the sales needed to have an annual income greater than \$65,000?36. **ANIMALS** Keith's dog weighs 90 pounds. A healthy weight for his dog would be less than 75 pounds. If Keith's dog can lose an average of 1.25 pounds per week on a certain diet, after how long will the dog reach healthy weight?37. Solve $6(m - 3) > 5(2m + 4)$. Show each step and justify your work.38. Solve $8(a - 2) \leq 10(a + 2)$. Show each step and justify your work.39. **MUSICAL** A high school drama club is performing a musical to benefit a local charity. Tickets are \$5 each. They also received donations of \$565. They want to raise at least \$1500.

a. Write an inequality that describes this situation. Then solve the inequality.

b. Graph the solution.

40. **ICE CREAM** Benito has \$6 to spend. A sundae costs \$3.25 plus \$0.65 per topping. Write and solve an inequality to find how many toppings he can order.



41. SCIENCE The normal body temperature of a camel is 97.7°F in the morning. If it has had no water by noon, its body temperature can be greater than 104°F .

- Write an inequality that represents a camel's body temperature at noon if the camel had no water.
- If C represents degrees Celsius, then $F = \frac{9}{5}C + 32$. Write and solve an inequality to find the camel's body temperature at noon in degrees Celsius.

42. NUMBER THEORY Find all sets of three consecutive positive even integers with a sum no greater than 36.

43. NUMBER THEORY Find all sets of four consecutive positive odd integers with a sum that is less than 42.

Solve each inequality. Check your solution.

44. $2(x - 4) \leq 2 + 3(x - 6)$

45. $\frac{2x - 4}{6} \geq -5x + 2$

46. $5.6z + 1.5 < 2.5z - 4.7$

47. $0.7(2m - 5) \geq 21.7$

GRAPHING CALCULATOR Use a graphing calculator to solve each inequality.

48. $3x + 7 > 4x + 9$

49. $13x - 11 \leq 7x + 37$

50. $2(x - 3) < 3(2x + 2)$

51. $\frac{1}{2}x - 9 < 2x$

52. $2x - \frac{2}{3} \geq x - 22$

53. $\frac{1}{3}(4x + 3) \geq \frac{2}{3}x + 2$

54. MULTIPLE REPRESENTATIONS In this problem, you will solve compound inequalities. A number x is greater than 4, and the same number is less than 9.

- Numerical** Write two separate inequalities for the statement.
- Graphical** Graph the solution set for the first inequality in red. Graph the solution set for the second inequality in blue. Highlight where they overlap.
- Tabular** Make a table using ten points from your number line, including points from each section. Use one column for each inequality and a third column titled "Both are True." Complete the table by writing true or false.
- Verbal** Describe the relationship between the colored regions of the graph and the chart.
- Logical** Make a prediction of what the graph of $4 < x < 9$ looks like.

H.O.T. Problems Use Higher-Order Thinking Skills

55. CCSS REASONING Explain how you could solve $-3p + 7 \geq -2$ without multiplying or dividing each side by a negative number.

56. CHALLENGE If $ax + b < ax + c$ is true for all real values of x , what will be the solution of $ax + b > ax + c$? Explain how you know.

57. CHALLENGE Solve each inequality for x . Assume that $a > 0$.

a. $ax + 4 \geq -ax - 5$

b. $2 - ax < x$

c. $-\frac{2}{a}x + 3 > -9$

58. WHICH ONE DOESN'T BELONG? Name the inequality that does not belong. Explain.

$4y + 9 > -3$

$3y - 4 > 5$

$-2y + 1 < -5$

$-5y + 2 < -13$

59. WRITING IN MATH Explain when the solution set of an inequality will be the empty set or the set of all real numbers. Show an example of each.



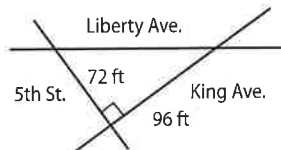
Standardized Test Practice

60. What is the solution set of the inequality $4t + 2 < 8t - (6t - 10)$?

- A $\{t \mid t < -6.5\}$ C $\{t \mid t < 4\}$
 B $\{t \mid t > -6.5\}$ D $\{t \mid t > 4\}$

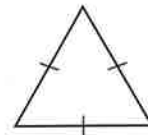
61. **GEOMETRY** The section of Liberty Ave. between 5th St. and King Ave. is temporarily closed. Traffic is being detoured right on 5th St., left on King Ave. and then back on Liberty Ave. How long is the closed section of Liberty Ave.?

- F 100 ft
 G 120 ft
 H 144 ft
 J 180 ft



62. **SHORT RESPONSE** Rhiannon is paid \$52 for working 4 hours. At this rate, how many hours will it take her to earn \$845?

63. **GEOMETRY** Classify the triangle.



- A right
 B parallel
 C obtuse
 D equilateral

Spiral Review

Solve each inequality. Check your solution. (Lesson 5-2)

64. $\frac{y}{2} \leq -5$

65. $12b > -48$

66. $-\frac{2}{3}t \leq -30$

Solve each inequality. Check your solution, and graph it on a number line. (Lesson 5-1)

67. $6 - h > -8$

68. $p - 9 < 2$

69. $3 \geq 4 - m$

Solve each equation by graphing. Verify your answer algebraically. (Lesson 3-2)

70. $2x - 7 = 4x + 9$

71. $5 + 3x = 7x - 11$

72. $2(x - 3) = 5x + 12$

73. **THEME PARKS** In a recent year, 70.9 million people visited the top 5 theme parks in North America. That represents an increase of about 1.14% in the number of visitors from the prior year. About how many people visited the top 5 theme parks in North America in the prior year? (Lesson 2-7)

If $f(x) = 4x - 3$ and $g(x) = 2x^2 + 5$, find each value. (Lesson 1-7)

74. $f(-2)$

75. $g(2) - 5$

76. $f(c + 3)$

77. **COSMETOLOGY** On average, a barber received a tip of \$4 for each of 12 haircuts. Write and evaluate an expression to determine the total amount that she earned. (Lesson 1-4)



Skills Review

Graph each set of numbers on a number line.

78. $\{-4, -2, 2, 4\}$

79. $\{-3, 0, 1, 5\}$

80. $\{\text{integers less than } 3\}$

81. $\{\text{integers greater than or equal to } -2\}$

82. $\{\text{integers between } -3 \text{ and } 4\}$

83. $\{\text{integers less than } -1\}$



Mid-Chapter Quiz

Lessons 5-1 through 5-3

Solve each inequality. Then graph it on a number line.

(Lesson 5-1)

1. $x - 8 > 4$ 2. $m + 2 \geq 6$
 3. $p - 4 < -7$ 4. $12 \leq t - 9$

5. **CONCERTS** Lupe's allowance for the month is \$60. She wants to go to a concert for which a ticket costs \$45.

(Lesson 5-1)

- a. Write and solve an inequality that shows how much money she can spend that month after buying a concert ticket.
 b. She spends \$9.99 on music downloads and \$2 on lunch in the cafeteria. Write and solve an inequality that shows how much she can spend after these purchases and the concert ticket.

Define a variable, write an inequality, and solve each problem.

Check your solution. (Lesson 5-1)

6. The sum of a number and -2 is no more than 6.
 7. A number decreased by 4 is more than -1 .
 8. Twice a number increased by 3 is less than the number decreased by 4.
 9. **MULTIPLE CHOICE** Jane is saving money to buy a new cell phone that costs no more than \$90. So far, she has saved \$52. How much more money does Jane need to save? (Lesson 5-1)

- A \$38
 B more than \$38
 C no more than \$38
 D at least \$38

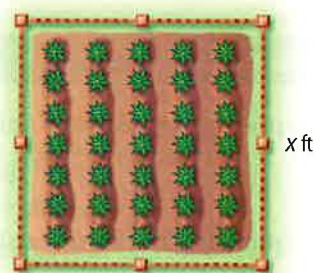
Solve each inequality. Check your solution. (Lesson 5-2)

10. $\frac{1}{3}y \geq 5$ 11. $4 < \frac{c}{5}$
 12. $-8x > 24$ 13. $2m \leq -10$
 14. $\frac{x}{2} < \frac{5}{8}$ 15. $-9a \geq -45$
 16. $\frac{w}{6} > -3$ 17. $\frac{k}{7} < -2$

18. **ANIMALS** The world's heaviest flying bird is the great bustard. A male bustard can be up to 4 feet long and weigh up to 40 pounds. (Lesson 5-2)

- a. Write inequalities to describe the ranges of lengths and weights of male bustards.
 b. Male bustards are usually about four times as heavy as females. Write and solve an inequality that describes the range of weights of female bustards.

19. **GARDENING** Bill is building a fence around a square garden to keep deer out. He has 60 feet of fencing. Find the maximum length of a side of the garden. (Lesson 5-2)



Solve each inequality. Check your solution. (Lesson 5-3)

20. $4a - 2 > 14$
 21. $2x + 11 \leq 5x - 10$
 22. $-p + 4 < -9$
 23. $\frac{d}{4} + 1 \geq -3$
 24. $-2(4b + 1) < -3b + 8$

Define a variable, write an inequality, and solve each problem.

Check your solution. (Lesson 5-3)

25. Three times a number increased by 8 is no more than the number decreased by 4.
 26. Two thirds of a number plus 5 is greater than 17.
 27. **MULTIPLE CHOICE** Shoe rental costs \$2, and each game bowled costs \$3. How many games can Kyle bowl without spending more than \$15? (Lesson 5-3)

- F 2 H 4
 G 3 J 5

5-4 Algebra Lab

Reading Compound Statements



A compound statement is made up of two simple statements connected by the word *and* or *or*. Before you can determine whether a compound statement is true or false, you must understand what the words *and* and *or* mean.

A spider has eight legs, and a dog has five legs.

For a compound statement connected by the word *and* to be true, both simple statements must be true.



A spider has eight legs. → true



A dog has five legs. → false

Since one of the statements is false, the compound statement is false.

A compound statement connected by the word *or* may be *exclusive* or *inclusive*. For example, the statement "With your lunch, you may have milk *or* juice," is exclusive. In everyday language, *or* means one or the other, but not both. However, in mathematics, *or* is inclusive. It means one or the other or both.

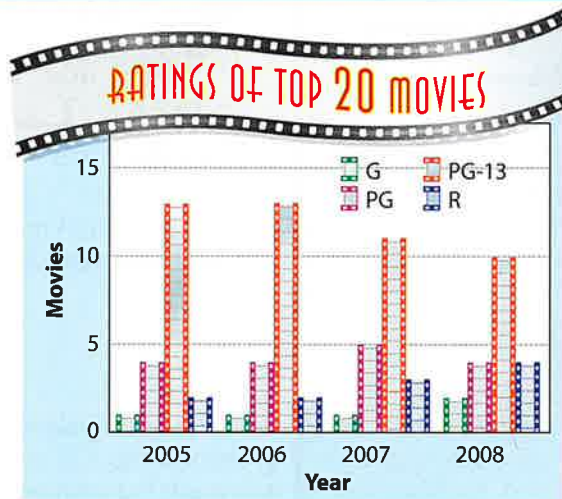
A spider has eight legs, or a dog has five legs.

For a compound statement connected by the word *or* to be true, at least one of the simple statements must be true. Since it is true that a spider has eight legs, the compound statement is true.

Exercises

Is each compound statement *true* or *false*? Explain.

- Most top 20 movies in 2007 were rated PG-13, *or* most top 20 movies in 2005 were rated G.
- In 2008 more top 20 movies were rated PG than were rated G, *and* more were rated PG than rated PG-13.
- For the years shown most top 20 movies are rated PG-13, *and* the least top 20 movies are rated G.
- No top 20 movies in 2008 were rated G, *or* most top 20 movies in 2008 were *not* rated PG.
- $11 < 5$ or $9 < 7$
- $-2 > 0$ and $3 < 7$
- $5 > 0$ and $-3 < 0$
- $-2 > -3$ or $0 = 0$
- $8 \neq 8$ or $-2 > -5$
- $5 > 10$ and $4 > -2$



Source: National Association of Theater Owners

Solving Compound Inequalities

Then

- You solved absolute value equations with two cases.

Now

- Solve compound inequalities containing the word *and*, and graph their solution set.
- Solve compound inequalities containing the word *or*, and graph their solution set.

Why?

- Visitors 4 years of age and older have to purchase a ticket to visit a certain history museum. Visitors less than 18 years of age can purchase a youth ticket. If v represents the age of a visitor, we can write two inequalities to represent when a youth ticket can be purchased.

4 years of age and older less than 18 years of age

$$v \geq 4 \qquad v < 18$$

The inequalities $v \geq 4$ and $v < 18$ can be combined and written without using *and* as $4 \leq v < 18$.



New Vocabulary

compound inequality
intersection
union



Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

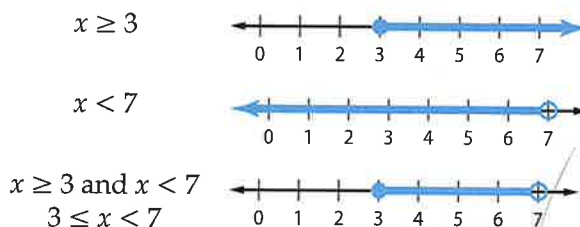
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

- 1 Make sense of problems and persevere in solving them.
- 8 Look for and express regularity in repeated reasoning.

1 Inequalities Containing *and* When considered together, two inequalities such as $h \geq 52$ and $h \leq 72$ form a **compound inequality**. A compound inequality containing *and* is only true if both inequalities are true. Its graph is where the graphs of the two inequalities overlap. This is called the **intersection** of the two graphs.

The intersection can be found by graphing each inequality and then determining where the graphs intersect.



The statement $3 \leq x < 7$ can be read as x is greater than or equal to 3 and less than 7 or x is between 3 and 7 including 3.



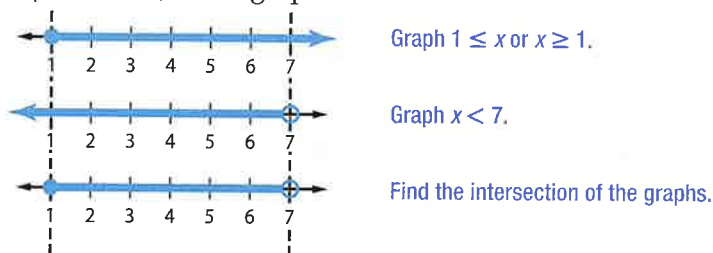
Example 1 Solve and Graph an Intersection

Solve $-2 \leq x - 3 < 4$. Then graph the solution set.

First, express $-2 \leq x - 3 < 4$ using *and*. Then solve each inequality.

$-2 \leq x - 3$	and	$x - 3 < 4$	Write the inequalities.
$-2 + 3 \leq x - 3 + 3$		$x - 3 + 3 < 4 + 3$	Add 3 to each side.
$1 \leq x$		$x < 7$	Simplify.

The solution set is $\{x \mid 1 \leq x < 7\}$. Now graph the solution set.



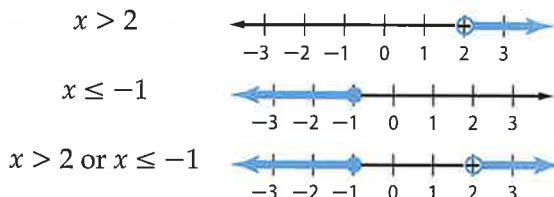
Guided Practice

Solve each compound inequality. Then graph the solution set.

1A. $y - 3 \geq -11$ and $y - 3 \leq -8$

1B. $6 \leq r + 7 < 10$

2 Inequalities Containing or Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if at least one of the inequalities is true. Its graph is the **union** of the graphs of two inequalities.



When solving problems involving inequalities, *within* is meant to be inclusive, so use \geq or \leq . *Between* is meant to be exclusive, so use $<$ or $>$.



Real-World Example 2 Write and Graph a Compound Inequality

SOUND The human ear can only detect sounds between the frequencies 20 Hertz and 20,000 Hertz. Write and graph a compound inequality that describes the frequency of sounds humans cannot hear.

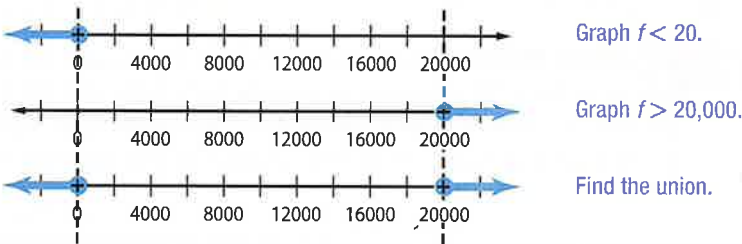
The problem states that humans can hear the frequencies between 20 Hz and 20,000 Hz. We are asked to find the frequencies humans cannot hear.

ReadingMath

At Most The phrase *at most* in Example 2 indicates \leq . It could also have been phrased as *no more than* or *less than or equal to*.

Words	The frequency	is at most	20 Hertz	or	The frequency	is at least	20,000 Hertz.
Variable	Let f be the frequency.						
Inequality	f	\leq	20	or	f	\geq	20,000

Now, graph the solution set.



Notice that the graphs do not intersect. Humans cannot hear sounds at a frequency less than 20 Hertz or greater than 20,000 Hertz. The compound inequality is $\{f | f < 20 \text{ or } f > 20,000\}$.

Guided Practice

2. MANUFACTURING A company is manufacturing an action figure that must be at least 11.2 centimeters and at most 11.4 centimeters tall. Write and graph a compound inequality that describes how tall the action figure can be.

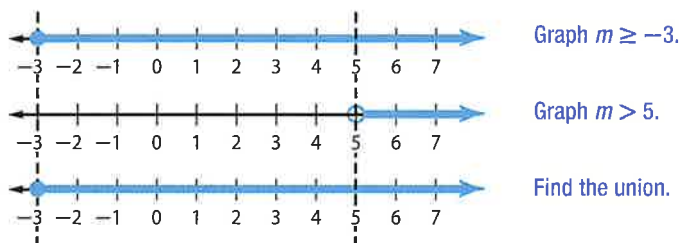




Example 3 Solve and Graph a Union

Solve $-2m + 7 \leq 13$ or $5m + 12 > 37$. Then graph the solution set.

$-2m + 7 \leq 13$	or	$5m + 12 > 37$
$-2m + 7 - 7 \leq 13 - 7$	Subtract.	$5m + 12 - 12 > 37 - 12$
$-2m \leq 6$	Simplify.	$5m > 25$
$\frac{-2m}{-2} \geq \frac{6}{-2}$	Divide.	$\frac{5m}{5} > \frac{25}{5}$
$m \geq -3$	Simplify.	$m > 5$



StudyTip

Intersections and Unions

The graphs of compound inequalities containing *and* will be an intersection. The graphs of compound inequalities containing *or* will be a union.

Notice that the graph of $m \geq -3$ contains every point in the graph of $m > 5$. So, the union is the graph of $m \geq -3$. The solution set is $\{m \mid m \geq -3\}$.

GuidedPractice

Solve each compound inequality. Then graph the solution set.

3A. $a + 1 < 4$ or $a - 1 \geq 3$

3B. $x \leq 9$ or $2 + 4x < 10$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.



Examples 1, 3 Solve each compound inequality. Then graph the solution set.

1. $4 \leq p - 8$ and $p - 14 \leq 2$

2. $r + 6 < -8$ or $r - 3 > -10$

3. $4a + 7 \geq 31$ or $a > 5$

4. $2 \leq g + 4 < 7$

Example 2

5. **CCSS SENSE-MAKING** The recommended air pressure for the tires of a mountain bike is at least 35 pounds per square inch (psi), but no more than 80 pounds per square inch. If a bike's tires have 24 pounds per square inch, what is the recommended range of air that should be put into the tires?

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1, 3 Solve each compound inequality. Then graph the solution set.

6. $f - 6 < 5$ and $f - 4 \geq 2$

7. $n + 2 \leq -5$ and $n + 6 \geq -6$

8. $y - 1 \geq 7$ or $y + 3 < -1$

9. $t + 14 \geq 15$ or $t - 9 < -10$

10. $-5 < 3p + 7 \leq 22$

11. $-3 \leq 7c + 4 < 18$

12. $5h - 4 \geq 6$ and $7h + 11 < 32$

13. $22 \geq 4m - 2$ or $5 - 3m \leq -13$

14. $-4a + 13 \geq 29$ and $10 < 6a - 14$

15. $-y + 5 \geq 9$ or $3y + 4 < -5$



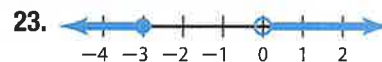
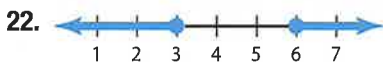
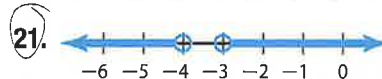
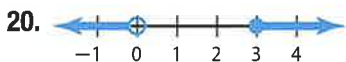
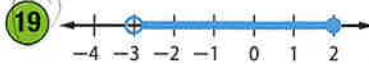
Example 2

16. **SPEED** The posted speed limit on an interstate highway is shown. Write an inequality that represents the sign. Graph the inequality.



17. **NUMBER THEORY** Find all sets of two consecutive positive odd integers with a sum that is at least 8 and less than 24.

Write a compound inequality for each graph.



Solve each compound inequality. Then graph the solution set.

24. $3b + 2 < 5b - 6 \leq 2b + 9$

25. $-2a + 3 \geq 6a - 1 > 3a - 10$

26. $10m - 7 < 17m$ or $-6m > 36$

27. $5n - 1 < -16$ or $-3n - 1 < 8$

28. **COUPON** Juanita has a coupon for 10% off any digital camera at a local electronics store. She is looking at digital cameras that range in price from \$100 to \$250.

- How much are the cameras after the coupon is used?
- If the tax amount is 6.5%, how much should Juanita expect to spend?

Define a variable, write an inequality, and solve each problem. Then check your solution.

29. Eight less than a number is no more than 14 and no less than 5.

30. The sum of 3 times a number and 4 is between -8 and 10 .

31. The product of -5 and a number is greater than 35 or less than 10 .

32. One half a number is greater than 0 and less than or equal to 1 .

33. **SNAKES** Most snakes live where the temperature ranges from 75°F to 90°F , inclusive. Write an inequality for temperatures where snakes will *not* thrive.

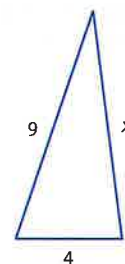
34. **FUNDRAISING** Yumas is selling gift cards to raise money for a class trip. He can earn prizes depending on how many cards he sells. So far, he has sold 34 cards. How many more does he need to sell to earn a prize in category 4?

Cards	Prize
1-15	1
16-30	2
31-45	3
46-60	4
+61	5

35. **TURTLES** Atlantic sea turtle eggs that incubate below 23°C or above 33°C rarely hatch. Write the temperature requirements in two ways: as a pair of simple inequalities, and as a compound inequality.

36. **CCSS STRUCTURE** The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.

- Write and solve three inequalities to express the relationships among the measures of the sides of the triangle shown at the right.
- What are four possible lengths for the third side of the triangle?
- Write a compound inequality for the possible values of x .



- 37 HURRICANES** The Saffir-Simpson Hurricane Scale rates hurricanes on a scale from 1 to 5 based on their wind speed.

Category	Wind Speed (mph)	Example (year)
1	74–95	Gaston (2004)
2	96–110	Frances (2004)
3	111–130	Ivan (2004)
4	131–155	Charley (2004)
5	> 155	Andrew (1992)

- Write a compound inequality for the wind speeds of a category 3 and a category 4 hurricane.
- What is the intersection of the two graphs of the inequalities you found in part a?

- 38. MULTIPLE REPRESENTATIONS** In this problem, you will investigate measurements. The **absolute error** of a measurement is equal to one half the unit of measure. The **relative error** of a measure is the ratio of the absolute error to the expected measure.

- a. **Tabular** Copy and complete the table.

Measure	Absolute Error	Relative Error
14.3 cm	$\frac{1}{2}(0.1) = 0.05$ cm	$\frac{\text{absolute error}}{\text{expected measure}} = \frac{0.05 \text{ cm}}{14.3 \text{ cm}}$ ≈ 0.0035 or 0.4%
1.85 cm		
61.2 cm		
237 cm		

- Analytical** You measured a length of 12.8 centimeters. Compute the absolute error and then write the range of possible measures.
- Logical** To what precision would you have to measure a length in centimeters to have an absolute error of less than 0.05 centimeter?
- Analytical** To find the relative error of an area or volume calculation, add the relative errors of each linear measure. If the measures of the sides of a rectangular box are 6.5 centimeters, 7.2 centimeters, and 10.25 centimeters, what is the relative error of the volume of the box?

H.O.T. Problems Use Higher-Order Thinking Skills

- 39. ERROR ANALYSIS** Chloe and Jonas are solving $3 < 2x - 5 < 7$. Is either of them correct? Explain your reasoning.

Chloe	Jonas
$3 < 2x - 5 < 7$	$3 < 2x - 5 < 7$
$3 < 2x < 12$	$8 < 2x < 7$
$\frac{3}{2} < x < 6$	$4 < x < \frac{7}{2}$

- CCSS PERSEVERANCE** Solve each inequality for x . Assume a is constant and $a > 0$.
 - $-3 < ax + 1 \leq 5$
 - $-\frac{1}{a}x + 6 < 1$ or $2 - ax > 8$
- OPEN ENDED** Create an example of a compound inequality containing *or* that has infinitely many solutions.
- CHALLENGE** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain. *The graph of a compound inequality that involves an or statement is bounded on the left and right by two values of x .*
- WRITING IN MATH** Give an example of a compound inequality you might encounter at an amusement park. Does the example represent an intersection or a union?



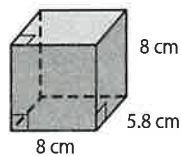
Standardized Test Practice

44. What is the solution set of the inequality $-7 < x + 2 < 4$?

A $\{x \mid -5 < x < 6\}$ C $\{x \mid -9 < x < 2\}$
 B $\{x \mid -5 < x < 2\}$ D $\{x \mid -9 < x < 6\}$

45. **GEOMETRY** What is the surface area of the rectangular solid?

F 249.6 cm^2
 G 278.4 cm^2
 H 313.6 cm^2
 J 371.2 cm^2



46. **GRIDDED RESPONSE** What is the next term in the sequence?

$$\frac{13}{2}, \frac{18}{5}, \frac{23}{8}, \frac{28}{11}, \frac{33}{14}, \dots$$

47. After paying a \$15 membership fee, members of a video club can rent movies for \$2. Nonmembers can rent movies for \$4. What is the least number of movies which must be rented for it to be less expensive for members?

A 9 C 7
 B 8 D 6

Spiral Review

48. **BABYSITTING** Marilyn earns \$150 per month delivering newspapers plus \$7 an hour babysitting. If she wants to earn at least \$300 this month, how many hours will she have to babysit? (Lesson 5-3)
49. **MAGAZINES** Carlos has earned more than \$260 selling magazine subscriptions. Each subscription was sold for \$12. How many did Carlos sell? (Lesson 5-2)
50. **PUNCH** Raquel is mixing lemon-lime soda and a fruit juice blend that is 45% juice. If she uses 3 quarts of soda, how many quarts of fruit juice must be added to produce punch that is 30% juice? (Lesson 2-9)

Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

51. $\frac{14}{x} = \frac{20}{8}$

52. $\frac{0.47}{6} = \frac{1.41}{m}$

53. $\frac{16}{7} = \frac{9}{b}$

54. $\frac{2+y}{5} = \frac{10}{3}$

55. $\frac{8}{9} = \frac{2r-3}{4}$

56. $\frac{6-2y}{8} = \frac{2}{18}$

Determine whether each relation is a function. Explain. (Lesson 1-7)

57.

Domain	2	6	10	7
Range	5	0	5	0

58.

Domain	-5	2	-3	2
Range	-10	-7	-5	-3

59. $\{(-4, 11), (-2, 7), (1, 3), (-4, -1)\}$

60. $\{(2, 7), (5, -3), (7, 6), (10, 7)\}$

Evaluate each expression. (Lesson 1-2)

61. $5 + (4 - 2^2)$

62. $\frac{3}{8}[8 \div (7 - 4)]$

63. $2(4 \cdot 9 - 3) + 5 \cdot \frac{1}{5}$

Skills Review

Solve each equation.

64. $4p - 2 = -6$

65. $18 = 5p + 3$

66. $9 = 1 + \frac{m}{7}$

67. $1.5a - 8 = 11$

68. $20 = -4c - 8$

69. $\frac{b+4}{-2} = -17$

70. $\frac{n-3}{8} = 20$

71. $6y - 16 = 44$

72. $130 = 11k + 9$





Then

You solved equations involving absolute value.

Now

- Solve and graph absolute value inequalities ($<$).
- Solve and graph absolute value inequalities ($>$).

Why?

Some companies use absolute value inequalities to control the quality of their product. To make baby carrots, long carrots are sliced into 3-inch sections and peeled. If the machine is accurate to within $\frac{1}{8}$ of an inch, the length ranges from $2\frac{7}{8}$ inches to $3\frac{1}{8}$ inches.



Common Core State Standards

Content Standards

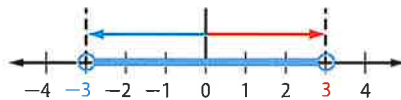
A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Practices

- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.

1 Absolute Value Inequalities ($<$) The inequality $|x| < 3$ means that the distance between x and 0 is less than 3.



So, $x > -3$ and $x < 3$. The solution set is $\{x \mid -3 < x < 3\}$.

When solving absolute value inequalities, there are two cases to consider.

Case 1 The expression inside the absolute value symbols is nonnegative.

Case 2 The expression inside the absolute value symbols is negative.

The solution is the intersection of the solutions of these two cases.



Example 1 Solve Absolute Value Inequalities ($<$)

Solve each inequality. Then graph the solution set.

a. $|m + 2| < 11$

Rewrite $|m + 2| < 11$ for Case 1 and Case 2.

Case 1 $m + 2$ is nonnegative.

and **Case 2** $m + 2$ is negative.

$$m + 2 < 11$$

$$-(m + 2) < 11$$

$$m + 2 - 2 < 11 - 2$$

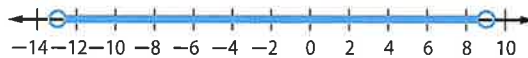
$$m + 2 > -11$$

$$m < 9$$

$$m + 2 - 2 > -11 - 2$$

$$m > -13$$

So, $m < 9$ and $m > -13$. The solution set is $\{m \mid -13 < m < 9\}$.



b. $|y - 1| < -2$

$|y - 1|$ cannot be negative. So it is not possible for $|y - 1|$ to be less than -2 . Therefore, there is no solution, and the solution set is the empty set, \emptyset .

Guided Practice

1A. $|n - 8| \leq 2$

1B. $|2c - 5| < -3$





Real-World Example 2 Apply Absolute Value Inequalities

INTERNET A recent survey showed that 65% of young adults watched online video clips. The margin of error was within 3 percentage points. Find the range of young adults who use video sharing sites.

The difference between the actual number of viewers and the number from the survey is less than or equal to 3. Let x be the actual number of viewers. Then $|x - 65| \leq 3$.

Solve each case of the inequality.

Case 1 $x - 65$ is nonnegative. and **Case 2** $x - 65$ is negative.

$x - 65 \leq 3$	$-(x - 65) \leq 3$
$x - 65 + 65 \leq 3 + 65$	$x - 65 \geq -3$
$x \leq 68$	$x \geq 62$

The range of young adults who use video sharing sites is $\{x \mid 62 \leq x \leq 68\}$.

Guided Practice

2. CHEMISTRY The melting point of ice is 0°C . During a chemistry experiment, Jill observed ice melting within 2°C of this measurement. Write the range of temperatures that Jill observed.



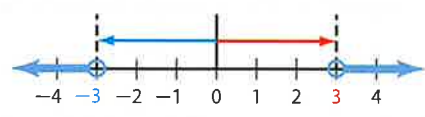
Real-WorldLink

One in five Americans use the Internet to view videos. Young adults tend to watch funny videos, while other age groups tend to watch the news.

Source: Pew Internet and American Life Project

2 Absolute Value Inequalities ($>$)

The inequality $|x| > 3$ means that the distance between x and 0 is greater than 3.



So, $x < -3$ or $x > 3$. The solution set is $\{x \mid x < -3 \text{ or } x > 3\}$.

As in the previous example, we must consider both cases.

Case 1 The expression inside the absolute value symbols is nonnegative.

Case 2 The expression inside the absolute value symbols is negative.



Example 3 Solve Absolute Value Inequalities ($>$)

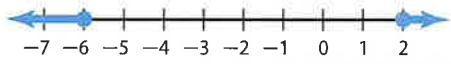
Solve $|3n + 6| \geq 12$. Then graph the solution set.

Rewrite $|3n + 6| \geq 12$ for Case 1 or Case 2.

Case 1 $3n + 6$ is nonnegative. or **Case 2** $3n + 6$ is negative.

$3n + 6 \geq 12$	$-(3n + 6) \geq 12$
$3n + 6 - 6 \geq 12 - 6$	$3n + 6 \leq -12$
$3n \geq 6$	$3n \leq -18$
$n \geq 2$	$n \leq -6$

So, $n \geq 2$ or $n \leq -6$. The solution set is $\{n \mid n \geq 2 \text{ or } n \leq -6\}$.



Guided Practice

Solve each inequality. Then graph the solution set.

3A. $|2k + 1| > 7$

3B. $|r - 6| \geq -5$

StudyTip

CCSS Structure For $|a| \geq b$, where a is any linear expression in one variable and b is a negative number, the solution set will always be the set of all real numbers. Since $|a|$ is always greater than or equal to zero, $|a|$ is always greater than or equal to b .





Examples 1–3 Solve each inequality. Then graph the solution set.


- | | | |
|-------------------|---------------------|----------------------|
| 1. $ a - 5 < 3$ | 2. $ u + 3 < 7$ | 3. $ t + 4 \leq -2$ |
| 4. $ c + 2 > -2$ | 5. $ n + 5 \geq 3$ | 6. $ p - 2 \geq 8$ |

Example 2 7. **FINANCIAL LITERACY** Jerome bought stock in his favorite fast-food restaurant chain at \$70.85. However, it has fluctuated up to \$0.75 in a day. Find the range of prices for which the stock could trade in a day.

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–3 Solve each inequality. Then graph the solution set.

- | | | |
|------------------------|---|-----------------------|
| 8. $ x + 8 < 16$ |  9. $ r + 1 \leq 2$ | 10. $ 2c - 1 \leq 7$ |
| 11. $ 3h - 3 < 12$ | 12. $ m + 4 < -2$ | 13. $ w + 5 < -8$ |
| 14. $ r + 2 > 6$ | 15. $ k - 4 > 3$ | 16. $ 2h - 3 \geq 9$ |
| 17. $ 4p + 2 \geq 10$ | 18. $ 5v + 3 > -9$ | 19. $ -2c - 3 > -4$ |

Example 2 20. **SCUBA DIVING** The pressure of a scuba tank should be within 500 pounds per square inch (psi) of 2500 psi. Write the range of optimum pressures.

Solve each inequality. Then graph the solution set.

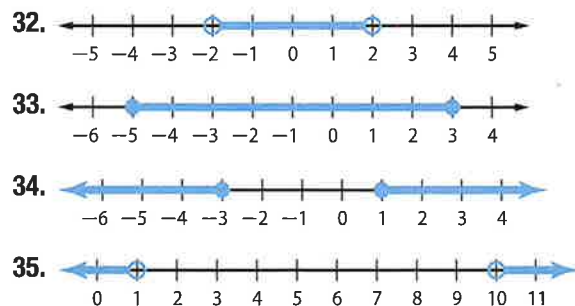
- | | | |
|--|---|--|
| 21. $ 4n + 3 \geq 18$ | 22. $ 5t - 2 \leq 6$ | 23. $\left \frac{3h + 1}{2} \right < 8$ |
| 24. $\left \frac{2p - 8}{4} \right \geq 9$ | 25. $\left \frac{7c + 3}{2} \right \leq -5$ | 26. $\left \frac{2g + 3}{2} \right > -7$ |
| 27. $ -6r - 4 < 8$ | 28. $ -3p - 7 > 5$ | 29. $ -h + 1.5 < 3$ |

30. **MUSIC DOWNLOADS** Kareem is allowed to download \$10 worth of music each month. This month he has spent within \$3 of his allowance.
- What is the range of money he has spent on music downloads this month?
 - Graph the range of the money that he spent.



31. **CHEMISTRY** Water can be present in our atmosphere as a solid, liquid, or gas. Water freezes at 32°F and vaporizes at 212°F.
- Write the range of temperatures in which water is not a liquid.
 - Graph this range.
 - Write the absolute value inequality that describes this situation.

 **REGULARITY** Write an open sentence involving absolute value for each graph.



36. **ANIMALS** A sheep's normal body temperature is 39°C . However, a healthy sheep may have body temperatures 1°C above or below this temperature. What is the range of body temperatures for a sheep?
37. **MINIATURE GOLF** Ginger's score was within 5 strokes of her average score of 52. Determine the range of scores for Ginger's game.

Express each statement using an inequality involving absolute value. Do not solve.

38. The pH of a swimming pool must be within 0.3 of a pH of 7.5.
39. The temperature inside a refrigerator should be within 1.5 degrees of 38°F .
40. Ramona's bowling score was within 6 points of her average score of 98.
41. The cruise control of a car should keep the speed within 3 miles per hour of 55.
42. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the graphs of linear inequalities on a coordinate plane.

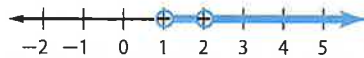
- a. **Tabular** Copy and complete the table. Substitute the x and $f(x)$ values for each point into each inequality. Mark whether the resulting statement is *true* or *false*.

Point	$f(x) \geq x - 1$	true/false	$f(x) \leq x - 1$	true/false
$(-4, 2)$				
$(-2, 2)$				
$(0, 2)$				
$(2, 2)$				
$(4, 2)$				

- b. **Graphical** Graph $f(x) = x - 1$.
- c. **Graphical** Plot each point from the table that made $f(x) \geq x - 1$ a true statement on the graph in red. Plot each point that made $f(x) \leq x - 1$ a true statement in blue.
- d. **Logical** Make a conjecture about what the graphs of $f(x) \geq x - 1$ and $f(x) \leq x - 1$ look like. Complete the table with other points to verify your conjecture.
- e. **Logical** Use what you discovered to describe the graph of a linear inequality.

H.O.T. Problems Use Higher-Order Thinking Skills

43. **ERROR ANALYSIS** Lucita sketched a graph of her solution to $|2a - 3| > 1$. Is she correct? Explain your reasoning.
44. **REASONING** The graph of an absolute value inequality is *sometimes*, *always*, or *never* the union of two graphs. Explain.
45. **CCSS ARGUMENTS** Demonstrate why the solution of $|t| > 0$ is not all real numbers. Explain your reasoning.
46. **WRITING IN MATH** How are symbols used to represent mathematical ideas? Use an example to justify your reasoning.
47. **WRITING IN MATH** Explain how to determine whether an absolute value inequality uses a compound inequality with *and* or a compound inequality with *or*. Then summarize how to solve absolute value inequalities.



Standardized Test Practice

48. The formula for acceleration in a circle is $a = \frac{v^2}{r}$. Which of the following shows the equation solved for r ?

A $r = v$

C $r = av^2$

B $r = \frac{v^2}{a}$

D $r = \frac{\sqrt{a}}{v}$

49. An engraver charges a \$3 set-up fee and \$0.25 per word. Which table shows the total price p for w words?

F

w	p
15	\$3
20	\$4.25
25	\$5.50
30	\$7.75

H

w	p
15	\$3.75
20	\$5
25	\$6.25
30	\$8.50

G

w	p
15	\$6.75
20	\$7
25	\$7.25
30	\$7.50

J

w	p
15	\$6.75
20	\$8
25	\$9.25
30	\$10.50

50. **SHORT RESPONSE** The table shows the items in stock at the school store the first day of class. What is the probability that an item chosen at random was a notebook?

Item	Number Purchased
pencil	57
pen	38
eraser	6
folder	25
notebook	18

51. Solve for n .

$$|2n - 3| = 5$$

A $\{-4, -1\}$

B $\{-1, 4\}$

C $\{1, 1\}$

D $\{4, 4\}$

Spiral Review

Solve each compound inequality. Then graph the solution set. (Lesson 5-4)

52. $b + 3 < 11$ and $b + 2 > -3$

53. $6 \leq 2t - 4 \leq 8$

54. $2c - 3 \geq 5$ or $3c + 7 \leq -5$

55. **FINANCIAL LITERACY** Jackson's bank charges him a monthly service fee of \$6 for his checking account and \$2 for each out-of-network ATM withdrawal. Jackson's account balance is \$87. Write and solve an inequality to find how many out-of-network ATM withdrawals of \$20 Jackson can make without overdrawing his account. (Lesson 5-3)

56. **GEOMETRY** One angle of a triangle measures 10° more than the second. The measure of the third angle is twice the sum of the measure of the first two angles. Find the measure of each angle. (Lesson 2-4)

Solve each equation. Then check your solution. (Lesson 2-2)

57. $c - 7 = 11$

58. $2w = 24$

59. $9 + p = -11$

60. $\frac{t}{5} = 20$

Skills Review

Graph each equation.

61. $y = 4x - 1$

62. $y - x = 3$

63. $2x - y = -4$

64. $3y + 2x = 6$

65. $4y = 4x - 16$

66. $2y - 2x = 8$

67. $-9 = -3x - y$

68. $-10 = 5y - 2x$



5-6 Graphing Inequalities in Two Variables



Then

You graphed linear equations.

Now

- 1 Graph linear inequalities on the coordinate plane.
- 2 Solve inequalities by graphing.

Why?

Hannah has budgeted \$35 every three months for car maintenance. From this she must buy oil costing \$3 and filters that cost \$7 each. How much oil and how many filters can Hannah buy and stay within her budget?

New Vocabulary
 boundary
 half-plane
 closed half-plane
 open half-plane

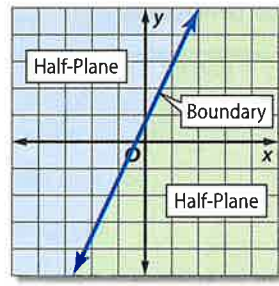
Common Core State Standards

Content Standards
 A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematical Practices
 5 Use appropriate tools strategically.

1 Graph Linear Inequalities The graph of a linear inequality is the set of points that represent all of the possible solutions of that inequality. An equation defines a **boundary**, which divides the coordinate plane into two **half-planes**.



The boundary may or may not be included in the solution. When it is included, the solution is a **closed half-plane**. When not included, the solution is an **open half-plane**.

Key Concept Graphing Linear Inequalities

- Step 1** Graph the boundary. Use a solid line when the inequality contains \leq or \geq . Use a dashed line when the inequality contains $<$ or $>$.
- Step 2** Use a test point to determine which half-plane should be shaded.
- Step 3** Shade the half-plane that contains the solution.



Example 1 Graph an Inequality ($<$ or $>$)

Graph $3x - y < 2$.

Step 1 First, solve for y in terms of x .

$$3x - y < 2$$

$$-y < -3x + 2$$

$$y > 3x - 2$$

Then, graph $y = 3x - 2$. Because the inequality involves $>$, graph the boundary with a dashed line.

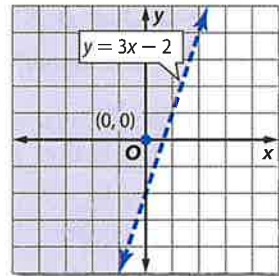
Step 2 Select $(0, 0)$ as a test point.

$$3x - y < 2 \quad \text{Original inequality}$$

$$3(0) - 0 < 2 \quad x = 0 \text{ and } y = 0$$

$$0 < 2 \quad \text{true}$$

Step 3 So, the half-plane containing the origin is the solution. Shade this half-plane.



Guided Practice Graph each inequality.

1A. $y > \frac{1}{2}x + 3$

1B. $x - 1 > y$





Example 2 Graph an Inequality (\leq or \geq)

Graph $x + 5y \leq 10$.

Step 1 Solve for y in terms of x .

$$x + 5y \leq 10 \quad \text{Original inequality}$$

$$5y \leq -x + 10 \quad \text{Subtract } x \text{ from each side and simplify.}$$

$$y \leq -\frac{1}{5}x + 2 \quad \text{Divide each side by 5.}$$

Graph $y = -\frac{1}{5}x + 2$. Because the inequality symbol is \leq , graph the boundary with a solid line.

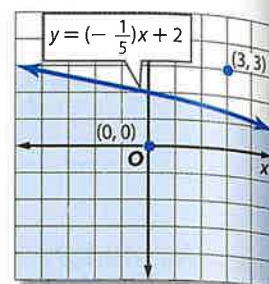
Step 2 Select a test point. Let's use $(3, 3)$. Substitute the values into the original inequality.

$$x + 5y \leq 10 \quad \text{Original inequality}$$

$$3 + 5(3) \leq 10 \quad x = 3 \text{ and } y = 3$$

$$18 \not\leq 10 \quad \text{Simplify.}$$

Step 3 Since this statement is false, shade the other half-plane.



StudyTip

Selecting a Test Point

When selecting a test point, a standard choice is the origin because it offers easy calculations. However, if it lies on the border, you must choose another point that is not on the border.

GuidedPractice

Graph each inequality.

2A. $x - y \leq 3$

2B. $2x + 3y \geq 18$

2 Solve Linear Inequalities

We can use a coordinate plane to solve inequalities with one variable.



Example 3 Solve Inequalities From Graphs

Use a graph to solve $3x + 5 < 14$.

Step 1 First graph the boundary, which is the related equation. Replace the inequality sign with an equals sign, and solve for x .

$$3x + 5 < 14 \quad \text{Original inequality}$$

$$3x + 5 = 14 \quad \text{Change } < \text{ to } =.$$

$$3x = 9 \quad \text{Subtract 5 from each side and simplify.}$$

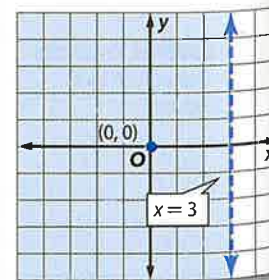
$$x = 3 \quad \text{Divide each side by 3.}$$

Graph $x = 3$ with a dashed line.

Step 2 Choose $(0, 0)$ as a test point. These values in the original inequality give us $5 < 14$.

Step 3 Since this statement is true, shade the half-plane that contains the point $(0, 0)$.

Notice that the x -intercept of the graph is at 3. Since the half-plane to the left of the x -intercept is shaded, the solution is $x < 3$.



GuidedPractice

Use a graph to solve each inequality.

3A. $4x - 3 \geq 17$

3B. $-2x + 6 > 12$



Check Your Understanding

Step-by-Step Solutions begin on page R13.



Examples 1–2 Graph each inequality.

1. $y > x + 3$

2. $y \geq -8$

3. $x + y > 1$

4. $y \leq x - 6$

5. $y < 2x - 4$

6. $x - y \leq 4$

Example 3 Use a graph to solve each inequality.

7. $7x + 1 < 15$

8. $-3x - 2 \geq 11$

9. $3y - 5 \leq 34$

10. $4y - 21 > 1$

Example 4 11. **FINANCIAL LITERACY** The surf shop has a weekly overhead of \$2300.

- Write an inequality to represent the number of skimboards and longboards the shop sells each week to make a profit.
- How many skimboards and longboards must the shop sell each week to make a profit?



Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–2 Graph each inequality.

12. $y < x - 3$

13. $y > x + 12$

14. $y \geq 3x - 1$

15. $y \leq -4x + 12$

16. $6x + 3y > 12$

17. $2x + 2y < 18$

18. $5x + y > 10$

19. $2x + y < -3$

20. $-2x + y \geq -4$

21. $8x + y \leq 6$

22. $10x + 2y \leq 14$

23. $-24x + 8y \geq -48$

Example 3 Use a graph to solve each inequality.

24. $10x - 8 < 22$

25. $20x - 5 > 35$

26. $4y - 77 \geq 23$

27. $5y + 8 \leq 33$

28. $35x + 25 < 6$

29. $14x - 12 > -31$

Example 4 30. **CCSS MODELING** Sybrina is decorating her bedroom. She has \$300 to spend on paint and bed linens. A gallon of paint costs \$14, while a set of bed linens costs \$60.

- Write an inequality for this situation.
- How many gallons of paint and bed linen sets can Sybrina buy and stay within her budget?

Use a graph to solve each inequality.

31. $3x + 2 < 0$

32. $4x - 1 > 3$

33. $-6x - 8 \geq -4$

34. $-5x + 1 < 3$

35. $-7x + 13 < 10$

36. $-4x - 4 \leq -6$

37. **SOCCER** The girls' soccer team wants to raise \$2000 to buy new goals. How many of each item must they sell to buy the goals?

- Write an inequality that represents this situation.
- Graph this inequality.
- Make a table of values that shows at least five possible solutions.
- Plot the solutions from part c.

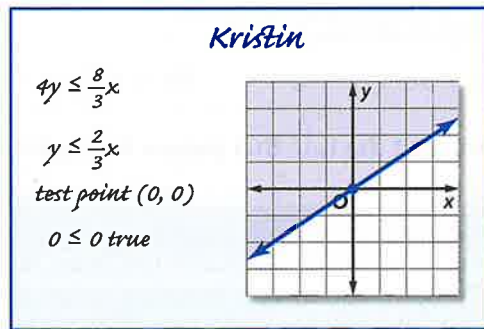
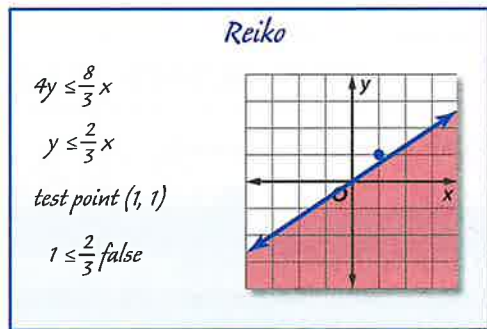


Graph each inequality. Determine which of the ordered pairs are part of the solution set for each inequality.

38. $y \geq 6$; $\{(0, 4), (-2, 7), (4, 8), (-4, -8), (1, 6)\}$
39. $x < -4$; $\{(2, 1), (-3, 0), (0, -3), (-5, -5), (-4, 2)\}$
40. $2x - 3y \leq 1$; $\{(2, 3), (3, 1), (0, 0), (0, -1), (5, 3)\}$
41. $5x + 7y \geq 10$; $\{(-2, -2), (1, -1), (1, 1), (2, 5), (6, 0)\}$
42. $-3x + 5y < 10$; $\{(3, -1), (1, 1), (0, 8), (-2, 0), (0, 2)\}$
43. $2x - 2y \geq 4$; $\{(0, 0), (0, 7), (7, 5), (5, 3), (2, -5)\}$
44. **RECYCLING** Mr. Jones would like to spend no more than \$37.50 per week on recycling. A curbside recycling service will remove up to 50 pounds of plastic bottles and paper products per week. They charge \$0.25 per pound of plastic and \$0.75 per pound of paper products.
- Write an inequality that describes the number of pounds of each product that can be included in the curbside service.
 - Write an inequality that describes Mr. Jones' weekly cost for the service if he stays within his budget.
 - Graph an inequality for the weekly costs for the service.
45. **MULTIPLE REPRESENTATIONS** Use inequalities A and B to investigate graphing compound inequalities on a coordinate plane.
- A. $7(y + 6) \leq 21x + 14$ B. $-3y \leq 3x - 12$
- Numerical** Solve each inequality for y .
 - Graphical** Graph both inequalities on one graph. Shade the half-plane that makes A true in red. Shade the half-plane that makes B true in blue.
 - Verbal** What does the overlapping region represent?

H.O.T. Problems Use Higher-Order Thinking Skills

46. **ERROR ANALYSIS** Reiko and Kristin are solving $4y \leq \frac{8}{3}x$ by graphing. Is either of them correct? Explain your reasoning.



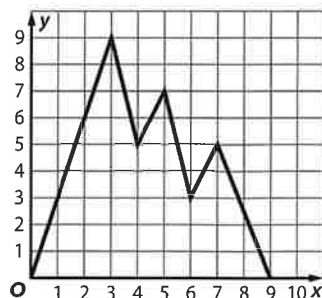
47. **CCSS TOOLS** Write a linear inequality for which $(-1, 2)$, $(0, 1)$, and $(3, -4)$ are solutions but $(1, 1)$ is not.
48. **REASONING** Explain why a point on the boundary should not be used as a test point.
49. **OPEN ENDED** Write a two-variable inequality with a restricted domain and range to represent a real-world situation. Give the domain and range, and explain why they are restricted.
50. **WRITING IN MATH** Summarize the steps to graph an inequality in two variables.



Standardized Test Practice

51. What is the domain of this function?

- A $\{x \mid 0 \leq x \leq 3\}$
- B $\{x \mid 0 \leq x \leq 9\}$
- C $\{y \mid 0 \leq y \leq 9\}$
- D $\{y \mid 0 \leq y \leq 3\}$



52. **EXTENDED RESPONSE** An arboretum will close for the winter when all of the trees have lost their leaves. The table shows the number of trees each day that still have leaves.

Day	5	10	15	20
Trees with Leaves	325	260	195	130

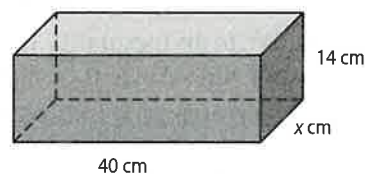
- a. Write an equation that represents the number of trees with leaves y after d days.
- b. Find the y -intercept. What does it mean in the context of this problem?
- c. After how many days will the arboretum close? Explain how you got your answer.

53. Which inequality best represents the statement below?

A jar contains 832 gumballs. Ebony's guess was within 46 pieces.

- F $|g - 832| \leq 46$
- G $|g + 832| \leq 46$
- H $|g - 832| \geq 46$
- J $|g + 832| \geq 46$

54. **GEOMETRY** If the rectangular prism has a volume of $10,080 \text{ cm}^3$, what is the value of x ?



- A 12
- B 14
- C 16
- D 18

Spiral Review

Solve each open sentence. (Lesson 5-5)

55. $|y - 2| > 4$

56. $|t - 6| \leq 5$

57. $|3 + d| < -4$

Solve each compound inequality. (Lesson 5-4)

58. $4c - 4 < 8c - 16 < 6c - 6$

59. $5 < \frac{1}{2}p + 3 < 8$

60. $0.5n \geq -7$ or $2.5n + 2 \leq 9$

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

61. $(1, -3)$ and $(2, 5)$

62. $(-2, -4)$ and $(-7, 3)$

63. $(-6, -8)$ and $(-8, -5)$

64. **FITNESS** The table shows the maximum heart rate to maintain during aerobic activities. Write an equation in function notation for the relation. Determine what would be the maximum heart rate to maintain in aerobic training for an 80-year-old. (Lesson 3-5)

Age (yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

Skills Review

65. **WORK** The formula $s = \frac{w - 10r}{m}$ is used to find keyboarding speeds. In the formula, s represents the speed in words per minute, w the number of words typed, r the number of errors, and m the number of minutes typed. Solve for r .





You can use a graphing calculator to investigate the graphs of inequalities.

CCSS Common Core State Standards
Content Standards

A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematical Practices
5 Use appropriate tools strategically.



Activity 1 Less Than

Graph $y \leq 2x + 5$.

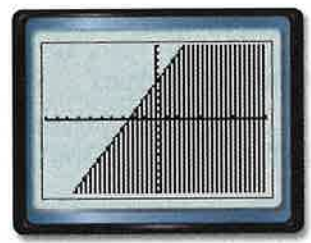
Clear all functions from the Y= list.

KEYSTROKES: **Y=** **CLEAR**

Graph $y \leq 2x + 5$ in a standard viewing window.

KEYSTROKES: 2 **X,T,θ,n** **+** 5 **◀◀◀◀◀◀** **ENTER**
ENTER **ENTER** **ZOOM** 6

All ordered pairs for which y is *less than or equal to* $2x + 5$ lie *below or on* the line and are solutions.



[-10, 10] scl: 1 by [-10, 10] scl: 1



Activity 2 Greater Than

Graph $y - 2x \geq 5$.

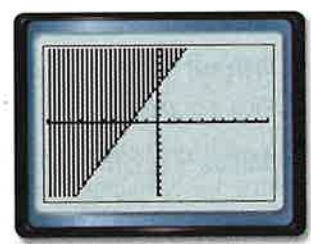
Clear the graph that is currently displayed.

KEYSTROKES: **Y=** **CLEAR**

Rewrite $y - 2x \geq 5$ as $y \geq 2x + 5$ and graph it.

KEYSTROKES: 2 **X,T,θ,n** **+** 5 **◀◀◀◀◀◀** **ENTER**
ENTER **ZOOM** 6

All ordered pairs for which y is *greater than or equal to* $2x + 5$ lie *above or on* the line and are solutions.



[-10, 10] scl: 1 by [-10, 10] scl: 1

Exercises

1. Compare and contrast the two graphs shown above.
2. Graph $y \geq -3x + 1$ in the standard viewing window. Using your graph, name four solutions of the inequality.
3. Suppose student water park tickets cost \$16, and adult water park tickets cost \$20. You would like to buy at least 10 tickets but spend no more than \$200.
 - a. Let x = number of student tickets and y = number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets.
 - b. Graph the inequalities. Use the viewing window $[0, 20]$ scl: 1 by $[0, 20]$ scl: 1.
 - c. Name four possible combinations of student and adult tickets.

CHAPTER 5 Study Guide and Review

Study Guide

Key Concepts

Solving One-Step Inequalities (Lessons 5-1 and 5-2)

For all numbers a , b , and c , the following are true.

- If $a > b$ and c is positive, $ac > bc$.
- If $a > b$ and c is negative, $ac < bc$.

Multi-Step and Compound Inequalities (Lessons 5-3 and 5-4)

- Multi-step inequalities can be solved by undoing the operations in the same way you would solve a multi-step equation.
- A compound inequality containing *and* is only true if both inequalities are true.
- A compound inequality containing *or* is true if at least one of the inequalities is true.

Absolute Value Inequalities (Lesson 5-5)

- The absolute value of any number x is its distance from zero on a number line and is written as $|x|$. If $x \geq 0$, then $|x| = x$. If $x < 0$, then $|x| = -x$.
- If $|x| < n$ and $n > 0$, then $-n < x < n$.
- If $|x| > n$ and $n > 0$, then $x > n$ or $x < -n$.

Inequalities in Two Variables (Lesson 5-6)

To graph an inequality:

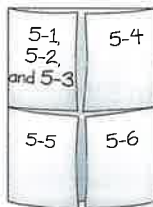
Step 1 Graph the boundary. Use a solid line when the inequality contains \leq or \geq . Use a dashed line when the inequality contains $<$ or $>$.

Step 2 Use a test point to determine which half-plane should be shaded.

Step 3 Shade the half-plane.

FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

- boundary (p. 317)
- closed half-plane (p. 317)
- compound inequality (p. 306)
- half-plane (p. 317)
- inequality (p. 285)
- intersection (p. 306)
- open half-plane (p. 317)
- set-builder notation (p. 286)
- union (p. 307)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. Set-builder notation is a less concise way of writing a solution set.
2. There are two types of compound inequalities.
3. The graph of a compound inequality containing *and* shows the union of the individual graphs.
4. A compound inequality containing *or* is true if one or both of the inequalities is true. Its graph is the union of the graphs of the two inequalities.
5. The graph of an inequality of the form $y < ax + b$ is a region on the coordinate plane called a half-plane.
6. A point defines the boundary of an open half-plane.
7. The boundary is the graph of the equation of the line that defines the edge of each half-plane.
8. The solution set to the inequality $y \geq x$ includes the boundary.
9. When solving an inequality, multiplying each side by a negative number reverses the inequality symbol.
10. The graph of a compound inequality that contains *and* is the intersection of the graphs of the two inequalities.



Lesson-by-Lesson Review

5-1 Solving Inequalities by Addition and Subtraction

Solve each inequality. Then graph it on a number line.

11. $w - 4 > 9$ 12. $x + 8 \leq 3$
 13. $6 + h < 1$ 14. $-5 < a + 2$
 15. $13 - p \geq 15$ 16. $y + 1 \leq 8$

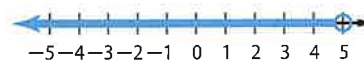
17. **FIELD TRIP** A bus can hold 44 people. If there are 35 students in Samantha's class, how many more people can ride on the bus?

Example 1

Solve $x - 9 < -4$. Then graph it on a number line.

$$\begin{aligned} x - 9 &< -4 && \text{Original inequality} \\ x - 9 + 9 &< -4 + 9 && \text{Add 9 to each side.} \\ x &< 5 && \text{Simplify.} \end{aligned}$$

The solution set is $\{x \mid x < 5\}$.



5-2 Solving Inequalities by Multiplication and Division

Solve each inequality. Graph the solution on a number line.

18. $\frac{1}{3}x > 6$ 19. $\frac{1}{5}g \geq -4$
 20. $4p < 32$ 21. $-55 \leq -5w$
 22. $-2m > 100$ 23. $\frac{2}{3}t < -48$

24. **MOVIE RENTAL** Jack has no more than \$24 to spend on DVDs for a party. Each DVD rents for \$4. Find the maximum number of DVDs Jack can rent for his party.

Example 2

Solve $-14h < 56$. Check your solution.

$$\begin{aligned} -14h &< 56 && \text{Original inequality} \\ \frac{-14h}{-14} &> \frac{56}{-14} && \text{Divide each side by } -14. \\ h &> -4 && \text{Simplify.} \\ \{h \mid h > -4\} &&& \end{aligned}$$

CHECK To check, substitute three different values into the original inequality: -4 , a number less than -4 , and a number greater than -4 .

5-3 Solving Multi-Step Inequalities

Solve each inequality. Graph the solution on a number line.

25. $3h - 7 < 14$ 26. $4 + 5b > 34$
 27. $18 \leq -2x + 8$ 28. $\frac{t}{3} - 6 > -4$
29. Four times a number decreased by 6 is less than -2 . Define a variable, write an inequality, and solve for the number.
30. **TICKET SALES** The drama club collected \$160 from ticket sales for the spring play. They need to collect at least \$400 to pay for new lighting for the stage. If tickets sell for \$3 each, how many more tickets need to be sold?

Example 3

Solve $-6y - 13 > 29$. Check your solution.

$$\begin{aligned} -6y - 13 &> 29 && \text{Original inequality} \\ -6y - 13 + 13 &> 29 + 13 && \text{Add 13 to each side.} \\ -6y &> 42 && \text{Simplify.} \\ \frac{-6y}{-6} &< \frac{42}{-6} && \text{Divide each side by } -6 \\ &&& \text{and change } > \text{ to } < . \\ y &< -7 && \text{Simplify.} \end{aligned}$$

The solution set is $\{y \mid y < -7\}$.

$$\begin{aligned} \text{CHECK } -6y - 13 &> 29 && \text{Original inequality} \\ -6(-10) - 13 &> 29 && \text{Substitute } -10 \text{ for } y. \\ 47 &> 29 && \checkmark \text{ Simplify.} \end{aligned}$$

5-4 Solving Compound Inequalities

Solve each compound inequality. Then graph the solution set.

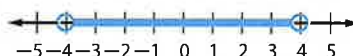
31. $m - 3 < 6$ and $m + 2 > 4$
32. $-4 < 2t - 6 < 8$
33. $3x + 2 \leq 11$ or $5x - 8 > 22$
34. **KITES** A kite can be flown in wind speeds no less than 7 miles per hour and no more than 16 miles per hour. Write an inequality for the wind speeds at which the kite can fly.

Example 4

Solve $-3w + 4 > -8$ and $2w - 11 > -19$. Then graph the solution set.

$$\begin{aligned} -3w + 4 &> -8 & \text{and} & \quad 2w - 11 > -19 \\ w < 4 & & & \quad w > -4 \end{aligned}$$

To graph the solution set, graph $w < 4$ and graph $w > -4$. Then find the intersection.



5-5 Inequalities Involving Absolute Value

Solve each inequality. Then graph the solution set.

35. $|x - 4| < 9$
36. $|p + 2| > 7$
37. $|2c + 3| \leq 11$
38. $|f - 9| \geq 2$
39. $|3d - 1| \leq 8$
40. $\left| \frac{4b - 2}{3} \right| < 12$
41. $\left| \frac{2t + 6}{2} \right| > 10$
42. $|-4y - 3| < 13$
43. $|m + 19| \leq 1$
44. $|-k - 7| \geq 4$

Example 5

Solve $|x - 6| < 9$. Then graph the solution set.

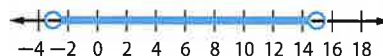
Case 1 $x - 6$ is nonnegative.

$$\begin{aligned} x - 6 &< 9 \\ x &< 15 \end{aligned}$$

Case 2 $x - 6$ is negative.

$$\begin{aligned} -(x - 6) &< 9 \\ x &> -3 \end{aligned}$$

The solution set is $\{x \mid -3 < x < 15\}$.



5-6 Graphing Inequalities in Two Variables

Graph each inequality.

45. $y > x - 3$
46. $y < 2x + 1$
47. $3x - y \leq 4$
48. $y \geq -2x + 6$
49. $5x - 2y < 10$
50. $x + y \geq 1$

Graph each inequality. Determine which of the ordered pairs are part of the solution set for each inequality.

51. $y \leq 4$; $\{(3, 6), (1, 2), (-4, 8), (3, -2), (1, 7)\}$
52. $-2x + 3y \geq 12$; $\{(-2, 2), (-1, 1), (0, 4), (2, 2)\}$

53. **BAKERY** Ben has \$24 to spend on cookies and cupcakes. Write and graph an inequality that represents what Ben can buy.



\$2



\$3

Example 6

Graph $2x - y > 3$.

Solve for y in terms of x .

$$\begin{aligned} 2x - y &> 3 \\ -y &> -2x + 3 \\ y &< 2x - 3 \end{aligned}$$

Original inequality

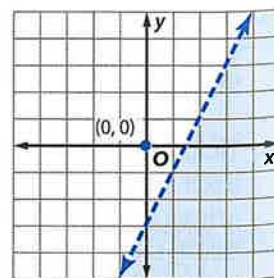
Subtract $2x$ from each side.

Multiply each side by -1 .

Graph the boundary using a dashed line. Choose $(0, 0)$ as a test point.

$$\begin{aligned} 2(0) - 0 &\not> 3 \\ 0 &\not> 3 \end{aligned}$$

Since 0 is not greater than 3, shade the plane that does not contain $(0, 0)$.



CHAPTER 5 Practice Test

Solve each inequality. Then graph it on a number line.

- $x - 9 < -4$
- $6p \geq 5p - 3$
- MULTIPLE CHOICE** Drew currently has 31 comic books in his collection. His friend Connor has 58 comic books. How many more comic books does Drew need to add to his collection in order to have a larger collection than Connor?
 A no more than 21
 B 27
 C at least 28
 D more than 30

Solve each inequality. Graph the solution on a number line.

- $\frac{1}{5}h > 3$
- $7w \leq -42$
- $-\frac{2}{3}t \geq 24$
- $-9m < -36$
- $3c - 7 < 11$
- $\frac{8}{4} + 3 \leq -9$
- $-2(x - 4) > 5x - 13$
- ZOO** The 8th grade science class is going to the zoo. The class can spend up to \$300 on admission.

Zoo Admission	
Visitor	Cost
student	\$8
adult	\$10

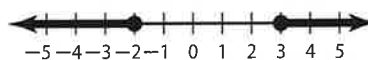
- Write an inequality for this situation.
- If there are 32 students in the class and 1 adult will attend for every 8 students, can the entire class go to the zoo?

Solve each compound inequality. Then graph the solution set.

- $y - 8 < -3$ or $y + 5 > 19$
- $-11 \leq 2h - 5 \leq 13$
- $3z - 2 > -5$ and $7z + 4 < -17$

Define a variable, write an inequality, and solve the problem. Check your solution.

- The difference of a number and 4 is no more than 8.
- Nine times a number decreased by four is at least twenty-three.
- MULTIPLE CHOICE** Write a compound inequality for the graph shown below.



- F $-2 \leq x < 3$ H $x < -2$ or $x \geq 3$
 G $x \leq -2$ or $x \geq 3$ J $-2 < x \leq 3$

Solve each inequality. Then graph the solution set.

- $|p - 5| < 3$
- $|2f + 7| \geq 21$
- $|-4m + 3| \leq 15$
- $\left| \frac{x - 3}{4} \right| > 5$
- RETAIL** A sporting goods store is offering a \$15 coupon on any pair of shoes.
 - The most and least expensive pairs of shoes are \$149.95 and \$24.95. What is the range of costs for customers with coupons?
 - When buying a pair of \$109.95 shoes, you can use a coupon or a 15% discount. Which option is best?

Graph each inequality.

- $y < 4x - 1$
- $2x + 3y \geq 12$
- Graph $y > -2x + 5$. Then determine which of the ordered pairs in $\{(-2, 0), (-1, 5), (2, 3), (7, 3)\}$ are in the solution set.
- PRESCHOOL** Mrs. Jones is buying new books and puzzles for her preschool classroom. Each book costs \$6, and each puzzle costs \$4. Write and graph an inequality to determine how many books and puzzles she can buy for \$96.



Write and Solve an Inequality

Many multiple-choice items will require writing and solving inequalities. Follow the steps below to help you successfully solve these types of problems.

Strategies for Writing and Solving Inequalities

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What are the unknowns for which I need to solve?

Step 2

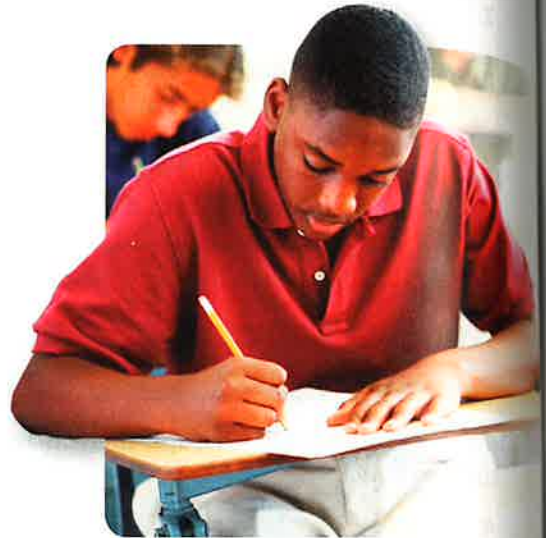
Translate the problem statement into an inequality.

- Assign variables to the unknown(s).
- Write the word sentence as a mathematical number sentence looking for words such as *greater than*, *less than*, *no more than*, *up to*, or *at least* to indicate the type of inequality as well as where to place the inequality sign.

Step 3

Solve the inequality.

- Solve for the unknowns in the inequality.
- Remember that multiplying or dividing each side by a negative number reverses the direction of the inequality.
- Check your answer to make sure it makes sense.



Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

Pedro has earned scores of 89, 74, 79, 85, and 88 on his tests this semester. He needs a test average of at least 85 in order to earn an A for the semester. There will be one more test given this semester.

- Write an inequality to model the situation.
- What score must he have on his final test to earn an A for the semester?

Read the problem carefully. You are given Pedro's first 5 test scores and told that he needs an average of *at least* 85 after his next test to earn an A for the semester.

a. Write the inequality.

Words	Pedro needs a test average of <i>at least</i> 85.
Variable	Let t represent Pedro's score on the final test.
Equation	$\frac{89 + 74 + 79 + 85 + 88 + t}{6} \geq 85$

b. Solve the inequality for t .

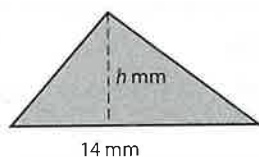
$$\begin{aligned} \frac{89 + 74 + 79 + 85 + 88 + t}{6} &\geq 85 \\ 89 + 74 + 79 + 85 + 88 + t &\geq 85(6) \\ 415 + t &\geq 510 \\ t &\geq 95 \end{aligned}$$

So, Pedro's final test score must be greater than or equal to 95 in order for him to earn an A for the semester.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

- Craig has \$20 to order a pizza. The pizza costs \$12.50 plus \$0.95 per topping. If there is also a \$3 delivery fee, how many toppings can Craig order?
- To join an archery club, Nina had to pay an initiation fee of \$75, plus \$40 per year in membership dues.
 - Write an equation to model the total cost, y , of belonging to the club for x years.
 - How many years will it take her to spend more than \$400 to belong to the club?
- The area of the triangle below is no more than 84 square millimeters. What is the height of the triangle?



- Rosa earns \$200 a month delivering newspapers, plus an average of \$11 per hour babysitting. If her goal is to earn at least \$295 this month, how many hours will she have to babysit?
- To earn money for a new bike, Ethan is selling some of his baseball cards. He has saved \$245. If the bike costs \$1400, and he can sell 154 cards, for how much money will he need to sell each card to reach his goal?
- In a certain lacrosse league, there can be no more than 22 players on each team, and no more than 10 teams per age group. There are 6 age groups.
 - Write an inequality to represent this situation.
 - What is the greatest number of players that can play lacrosse in this league?
- Sarah has \$120 to shop for herself and to buy some gifts for 6 of her friends. She has purchased a shirt for herself for \$32. Assuming that she spends an equal amount on each friend, what is the maximum that she can spend per person?

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Miguel received a \$100 gift certificate for a graduation gift. He wants to buy a CD player that costs \$38 and CDs that cost \$12 each. Which of the following inequalities represents how many CDs Miguel can buy?

A $n \leq 6$
 B $n \geq 5$
 C $n < 5$
 D $n \leq 5$

2. Craig is paid time-and-a-half for any additional hours over 40 that he works.

Time	Pay Rate
Up to 40 hours	\$12.80/hr
Additional hours worked over 40	\$19.20/hr

If Craig's goal is to earn at least \$600 next week, what is the minimum number of hours he needs to work?

F 43 hours H 45 hours
 G 44 hours J 46 hours

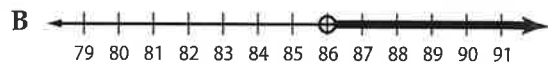
3. Which equation has a slope of $-\frac{2}{3}$ and a y -intercept of 6?

A $y = 6x + \frac{2}{3}$ C $y = -\frac{2}{3}x + 6$
 B $y = -\frac{2}{3}x - 6$ D $y = 6x - \frac{2}{3}$

4. The highest score that is on record on a video game is 10,219 points. The lowest score on record is 257 points. Which of the following inequalities best shows the range of scores recorded on the game?

F $x \leq 10,219$
 G $x \geq 257$
 H $257 < x < 10,219$
 J $257 \leq x \leq 10,219$

5. The current temperature is 82° . If the temperature rises more than 4 degrees, there will be a new record high for the date. Which number line represents the temperatures that would set a new record high?



6. The girls' volleyball team is selling T-shirts and pennants to raise money for new uniforms. The team hopes to raise more than \$250.

Item	Price
T-shirt	\$10
Pennant	\$4

Which of the following combinations of items sold would meet this goal?

F 16 T-shirts and 20 pennants
 G 20 T-shirts and 12 pennants
 H 18 T-shirts and 18 pennants
 J 15 T-shirts and 20 pennants

7. What type of line does not have a defined slope?

A horizontal C perpendicular
 B parallel D vertical

8. Which expression below illustrates the Associative Property?

F $abc = bac$
 G $2(x - 3) = 2x - 6$
 H $(p + 3) - t = p + (3 - t)$
 J $5 + (-5) = 0$

Test-Taking Tip

Question 2 You can check your answer by finding Craig's earnings for the hours worked.

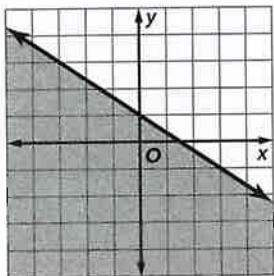
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Solve $-4 < 3x + 8 \leq 23$.

10. **GRIDDED RESPONSE** Tien is saving money for a new television. She needs to save at least \$720 to pay for her expenses. Each week Tien saves \$50 toward her new television. How many weeks will it take so she can pay for the television?

11. Write an inequality that best represents the graph.



12. Solve $|x - 4| < 2$.

13. **GRIDDED RESPONSE** Daniel wants to ship a set of golf clubs and several boxes of golf balls in a box that can hold up to 20 pounds. If the set of clubs weighs 9 pounds and each box of golf balls weighs 12 ounces, how many boxes of golf balls can Daniel ship?

14. Graph the solution set for the inequality $3x - 6 \leq 4x - 4 \leq 3x + 1$.

15. Write an equation that represents the data in the table.

x	y
3	12.5
4	16
5	19.5
6	23
7	26.5

16. A sporting goods company near the beach rents bicycles for \$10 plus \$5 per hour. Write an equation in slope-intercept form that shows the total cost, y , of renting a bicycle for x hours. How much would it cost Emily to rent a bicycle for 6 hours?

Extended Response

Record your answers on a sheet of paper. Show your work.

17. Theresa is saving money for a vacation. She needs to save at least \$640 to pay for her expenses. Each week, she puts \$35 towards her vacation savings.
- Let w represent the number of weeks Theresa saves money. Write an inequality to model the situation.
 - Solve the inequality from part a. What is the minimum number of weeks Theresa must save money in order to reach her goal?
 - If Theresa were to save \$45 each week instead, by how many weeks would the minimum savings time be decreased?

Need Extra Help?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Go to Lesson...	5-3	5-3	4-1	5-4	5-1	5-6	3-3	1-3	5-4	5-2	5-6	5-5	5-3	5-4	3-5	4-2	5-2

