

6 Systems of Linear Equations and Inequalities



Then

- You solved linear equations in one variable.

Now

- In this chapter, you will:
 - Solve systems of linear equations by graphing, substitution, and elimination.
 - Solve systems of linear inequalities by graphing.

Why? ▲

- MUSIC** \$1500 worth of tickets were sold for a marching band competition. Adult tickets were \$12 each, and student tickets were \$8 each. If you knew how many total tickets were sold, you could use a system of equations to determine how many adult tickets and how many student tickets were sold.



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Animation



Vocabulary



eGlossary



Personal Tutor



Virtual Manipulatives



Graphing Calculator



Audio



Foldables



Self-Check Practice



Worksheets



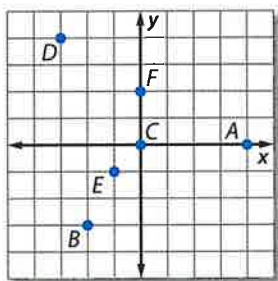
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Name the ordered pair for each point on the coordinate plane.



- | | |
|------|------|
| 1. A | 2. D |
| 3. B | 4. C |
| 5. E | 6. F |

Solve each equation or formula for the variable specified.

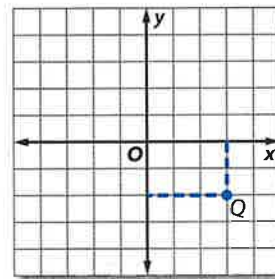
- $2x + 4y = 12$, for x
- $x = 3y - 9$, for y
- $m - 2n = 6$, for m
- $y = mx + b$, for x
- $P = 2\ell + 2w$, for ℓ
- $5x - 10y = 40$, for y
- GEOMETRY** The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A represents the area, b is the base, and h is the height of the triangle. Solve the equation for b .

QuickReview



Example 1

Name the ordered pair for Q on the coordinate plane.



Follow a vertical line from the point to the x -axis. This gives the x -coordinate, 3.

Follow a horizontal line from the point to the y -axis. This gives the y -coordinate, -2 .

The ordered pair is $(3, -2)$.

Example 2

Solve $12x + 3y = 36$ for y .

$$12x + 3y = 36$$

Original equation

$$12x + 3y - 12x = 36 - 12x$$

Subtract $12x$ from each side.

$$3y = 36 - 12x$$

Simplify.

$$\frac{3y}{3} = \frac{36 - 12x}{3}$$

Divide each side by 3.

$$y = 12 - 4x$$

Simplify.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.



Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES Study Organizer



Systems of Linear Equations and Inequalities Make this Foldable to help you organize your Chapter 6 notes about solving systems of equations and inequalities. Begin with a sheet of notebook paper.

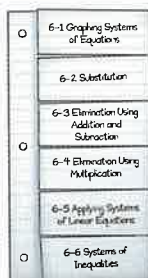
1 Fold lengthwise to the holes.



2 Cut 6 tabs.



3 Label the tabs using the lesson titles.



New Vocabulary



| English | | Español |
|------------------------|--------|--------------------------|
| system of equations | p. 335 | sistema de ecuaciones |
| consistent | p. 335 | consistente |
| independent | p. 335 | independiente |
| dependent | p. 335 | dependiente |
| inconsistent | p. 335 | inconsistente |
| substitution | p. 344 | sustitución |
| elimination | p. 350 | eliminación |
| matrix | p. 370 | matriz |
| element | p. 370 | elemento |
| dimension | p. 370 | dimensión |
| augmented matrix | p. 370 | matriz ampliada |
| row reduction | p. 371 | reducción de fila |
| identity matrix | p. 371 | matriz |
| system of inequalities | p. 372 | sistema de desigualdades |

Review Vocabulary



domain **dominio** the set of the first numbers of the ordered pairs in a relation

intersection **intersección** the graph of a compound inequality containing *and*; the solution is the set of elements common to both graphs

proportion **proporción**
an equation stating that two ratios are equal

Proportion

$$\frac{24}{30} = \frac{4}{5}$$

$\div 6$ (above the fraction bar)
 $\div 6$ (below the fraction bar)

Graphing Systems of Equations

Then

You graphed linear equations.

Now

1 Determine the number of solutions a system of linear equations has.

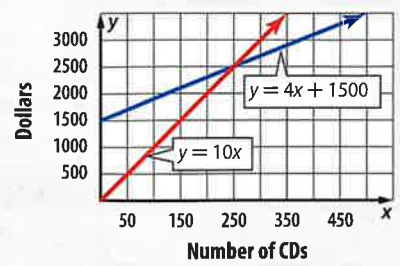
2 Solve systems of linear equations by graphing.

Why?

The cost to begin production on a band's CD is \$1500. Each CD costs \$4 to produce and will sell for \$10. The band wants to know how many CDs they will have to sell to earn a profit.

Graphing a system can show when a company makes a profit. The cost of producing the CD can be modeled by the equation $y = 4x + 1500$, where y represents the cost of production and x is the number of CDs produced.

Band's CD Sales



New Vocabulary
 system of equations
 consistent
 independent
 dependent
 inconsistent

Common Core State Standards
Content Standards
 A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
 A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices
 3 Construct viable arguments and critique the reasoning of others.
 8 Look for and express regularity in repeated reasoning.

1 Possible Number of Solutions The income from the CDs sold can be modeled by the equation $y = 10x$, where y represents the total income of selling the CDs, and x is the number of CDs sold.

If we graph these equations, we can see at which point the band begins making a profit. The point where the two graphs intersect is where the band breaks even. This happens when the band sells 250 CDs. If the band sells more than 250 CDs, they will make a profit.

The two equations, $y = 4x + 1500$ and $y = 10x$, form a **system of equations**. The ordered pair that is a solution of both equations is the solution of the system. A system of two linear equations can have one solution, an infinite number of solutions, or no solution.

- If a system has at least one solution, it is said to be **consistent**. The graphs intersect at one point or are the same line.
- If a consistent system has exactly one solution, it is said to be **independent**. If it has an infinite number of solutions, it is **dependent**. This means that there are unlimited solutions that satisfy both equations.
- If a system has no solution, it is said to be **inconsistent**. The graphs are parallel.

| Concept Summary Possible Solutions | | | |
|---|----------------------------|--------------------------|--------------|
| Number of Solutions | exactly one | infinite | no solution |
| Terminology | consistent and independent | consistent and dependent | inconsistent |
| Graph | | | |

StudyTip

Number of Solutions

When both equations are of the form $y = mx + b$, the values of m and b can determine the number of solutions.

| Compare m and b | Number of Solutions |
|--|---------------------|
| different m values | one |
| same m value, but different b values | none |
| same m value, and same b value | infinite |

Example 1 Number of Solutions

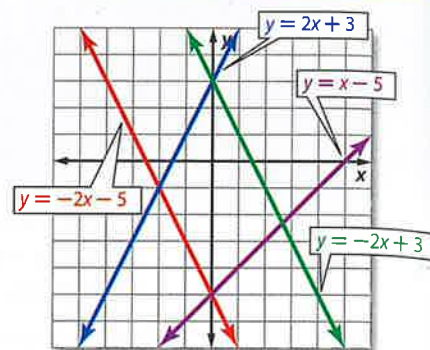
Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.

a. $y = -2x + 3$
 $y = x - 5$

Since the graphs of these two lines intersect at one point, there is exactly one solution. Therefore, the system is consistent and independent.

b. $y = -2x - 5$
 $y = -2x + 3$

Since the graphs of these two lines are parallel, there is no solution of the system. Therefore, the system is inconsistent.



Guided Practice

1A. $y = 2x + 3$
 $y = -2x - 5$

1B. $y = x - 5$
 $y = -2x - 5$

2 Solve by Graphing One method of solving a system of equations is to graph the equations carefully on the same coordinate grid and find their point of intersection. This point is the solution of the system.

Example 2 Solve by Graphing

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

a. $y = -3x + 10$
 $y = x - 2$

The graphs appear to intersect at the point (3, 1). You can check this by substituting 3 for x and 1 for y .

CHECK $y = -3x + 10$ Original equation

$1 \stackrel{?}{=} -3(3) + 10$ Substitution

$1 \stackrel{?}{=} -9 + 10$ Multiply.

$1 = 1$ ✓

$y = x - 2$ Original equation

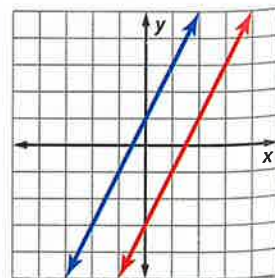
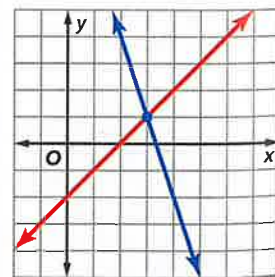
$1 \stackrel{?}{=} 3 - 2$ Substitution

$1 = 1$ ✓ Multiply.

The solution is (3, 1).

b. $2x - y = -1$
 $4x - 2y = 6$

The lines have the same slope but different y -intercepts, so the lines are parallel. Since they do not intersect, there is no solution of this system. The system is inconsistent.



Review Vocabulary

parallel lines never intersect and have the same slope



Guided Practice

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

2A. $x - y = 2$
 $3y + 2x = 9$

2B. $y = -2x - 3$
 $6x + 3y = -9$

We can use what we know about systems of equations to solve many real-world problems involving constraints that are modeled by two or more different functions.

Real-World Example 3 Write and Solve a System of Equations



SPORTS The number of girls participating in high school soccer and track and field has steadily increased over the past few years. Use the information in the table to predict the approximate year when the number of girls participating in these two sports will be the same.

| High School Sport | Number of Girls Participating in 2008 (thousands) | Average rate of increase (thousands per year) |
|-------------------|---|---|
| soccer | 345 | 8 |
| track and field | 458 | 3 |

Source: National Federation of State High School Associations



Real-WorldLink

In 2008, 3.1 million girls participated in high school sports. This was an all-time high for female participation.

Source: National Federation of State High School Associations

| | | | | | | | |
|------------------|--|--------|------------------|-------|----------------------------|------|-------------------------------|
| Words | Number of girls participating | equals | rate of increase | times | number of years after 2008 | plus | number participating in 2008. |
| Variables | Let y = number of girls competing. Let x = number of years after 2008. | | | | | | |
| Equations | Soccer: y | = | 8 | • | x | + | 345 |
| | Track and field: y | = | 3 | • | x | + | 458 |

Graph $y = 8x + 345$ and $y = 3x + 458$. The graphs appear to intersect at approximately (22.5, 525).

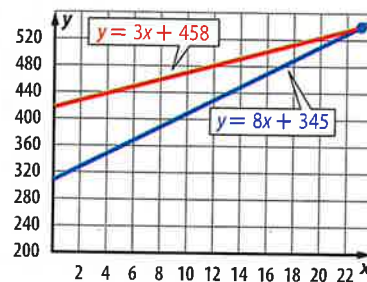
CHECK Use substitution to check this answer.

$$y = 8x + 345 \qquad y = 3x + 458$$

$$525 \stackrel{?}{=} 8(22.5) + 345 \qquad 525 \stackrel{?}{=} 3(22.5) + 458$$

$$525 = 525 \checkmark \qquad 525 \approx 525.5 \checkmark$$

The solution means that approximately 22 years after 2008, or in 2030, the number of girls participating in high school soccer and track and field will be the same, about 525,000.



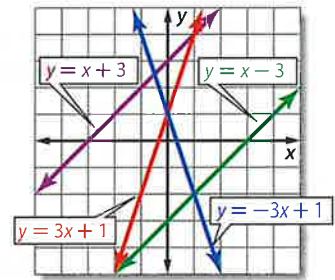
Guided Practice

3. **VIDEO GAMES** Joe and Josh each want to buy a video game. Joe has \$14 and saves \$10 a week. Josh has \$26 and saves \$7 a week. In how many weeks will they have the same amount?





Example 1 Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.



- | | |
|----------------------------------|---------------------------------|
| 1. $y = -3x + 1$ $y = 3x + 1$ | 2. $y = 3x + 1$ $y = x - 3$ |
| 3. $y = x - 3$ $y = x + 3$ | 4. $y = x + 3$ $x - y = -3$ |
| 5. $x - y = -3$ $y = -3x + 1$ | 6. $y = -3x + 1$ $y = x - 3$ |

Example 2 Graph each system and determine the number of solutions that it has. If it has one solution, name it.

- | | |
|--------------------------------|--------------------------------|
| 7. $y = x + 4$ $y = -x - 4$ | 8. $y = x + 3$ $y = 2x + 4$ |
|--------------------------------|--------------------------------|

Example 3 9. **CCSS MODELING** Alberto and Ashanti are reading a graphic novel.

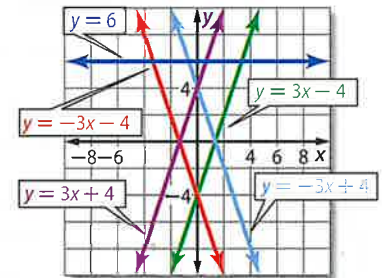


- Write an equation to represent the pages each boy has read.
- Graph each equation.
- How long will it be before Alberto has read more pages than Ashanti? Check and interpret your solution.

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Use the graph at the right to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*.



- | | |
|------------------------------------|-----------------------------------|
| 10. $y = 6$ $y = 3x + 4$ | 11. $y = 3x + 4$ $y = -3x + 4$ |
| 12. $y = -3x + 4$ $y = -3x - 4$ | 13. $y = -3x - 4$ $y = 3x - 4$ |
| 14. $3x - y = -4$ $y = 3x + 4$ | 15. $3x - y = 4$ $3x + y = 4$ |

Example 2 Graph each system and determine the number of solutions that it has. If it has one solution, name it.

- | | | |
|------------------------------------|-----------------------------------|-------------------------------------|
| 16. $y = -3$ $y = x - 3$ | 17. $y = 4x + 2$ $y = -2x - 3$ | 18. $y = x - 6$ $y = x + 2$ |
| 19. $x + y = 4$ $3x + 3y = 12$ | 20. $x - y = -2$ $-x + y = 2$ | 21. $x + 2y = 3$ $x = 5$ |
| 22. $2x + 3y = 12$ $2x - y = 4$ | 23. $2x + y = -4$ $y + 2x = 3$ | 24. $2x + 2y = 6$ $5y + 5x = 15$ |



25. **SCHOOL DANCE** Akira and Jen are competing to see who can sell the most tickets for the Winter Dance. On Monday, Akira sold 22 and then sold 30 per day after that. Jen sold 53 on Monday and then sold 20 per day after that.
- Write equations for the number of tickets each person has sold.
 - Graph each equation.
 - Solve the system of equations. Check and interpret your solution.

26. **CCSS MODELING** If x is the number of years since 2000 and y is the percent of people using travel services, the following equations represent the percent of people using travel agents and the percent of people using the Internet to plan travel.

Travel agents: $y = -2x + 30$

Internet: $y = 6x + 41$

- Graph the system of equations.
- Estimate the year travel agents and the Internet were used equally.

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

27. $y = \frac{1}{2}x$

$y = x + 2$

28. $y = 6x + 6$

$y = 3x + 6$

29. $y = 2x - 17$

$y = x - 10$

30. $8x - 4y = 16$

$-5x - 5y = 5$

31. $3x + 5y = 30$

$3x + y = 18$

32. $-3x + 4y = 24$

$4x - y = 7$

33. $2x - 8y = 6$

$x - 4y = 3$

34. $4x - 6y = 12$

$-2x + 3y = -6$

35. $2x + 3y = 10$

$4x + 6y = 12$

36. $3x + 2y = 10$

$2x + 3y = 10$

37. $3y - x = -2$

$y - \frac{1}{3}x = 2$

38. $\frac{8}{5}y = \frac{2}{5}x + 1$

$\frac{2}{5}y = \frac{1}{10}x + \frac{1}{4}$

39. $\frac{1}{3}x + \frac{1}{3}y = 1$

$x + y = 1$

40. $\frac{3}{4}x + \frac{1}{2}y = \frac{1}{4}$

$\frac{2}{3}x + \frac{1}{6}y = \frac{1}{2}$

41. $\frac{5}{6}x + \frac{2}{3}y = \frac{1}{2}$

$\frac{2}{5}x + \frac{1}{5}y = \frac{3}{5}$

42. **PHOTOGRAPHY** Suppose x represents the number of cameras sold and y represents the number of years since 2000. Then the number of digital cameras sold each year since 2000, in millions, can be modeled by the equation $y = 12.5x + 10.9$. The number of film cameras sold each year since 2000, in millions, can be modeled by the equation $y = -9.1x + 78.8$.

- Graph each equation.
- In which year did digital camera sales surpass film camera sales?
- In what year did film cameras stop selling altogether?
- What are the domain and range of each of the functions in this situation?

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

43. $2y = 1.2x - 10$

$4y = 2.4x$

44. $x = 6 - \frac{3}{8}y$

$4 = \frac{2}{3}x + \frac{1}{4}y$



- 45. WEB SITES** Personal publishing site *Lookatme* had 2.5 million visitors in 2009. Each year after that, the number of visitors rose by 13.1 million. Online auction site *Buyourstuff* had 59 million visitors in 2009, but each year after that the number of visitors fell by 2 million.
- Write an equation for each of the companies.
 - Make a table of values for 5 years for each of the companies.
 - Graph each equation.
 - When will *Lookatme* and *Buyourstuff*'s sites have the same number of visitors?
 - Name the domain and range of these functions in this situation.
- 46. MULTIPLE REPRESENTATIONS** In this problem, you will explore different methods for finding the intersection of the graphs of two linear equations.
- Algebraic** Use algebra to solve the equation $\frac{1}{2}x + 3 = -x + 12$.
 - Graphical** Use a graph to solve $y = \frac{1}{2}x + 3$ and $y = -x + 12$.
 - Analytical** How is the equation in part **a** related to the system in part **b**?
 - Verbal** Explain how to use the graph in part **b** to solve the equation in part **a**.

H.O.T. Problems Use Higher-Order Thinking Skills

- 47. ERROR ANALYSIS** Store A is offering a 10% discount on the purchase of all electronics in their store. Store B is offering \$10 off all the electronics in their store. Francisca and Alan are deciding which offer will save them more money. Is either of them correct? Explain your reasoning.

Francisca

You can't determine which store has the better offer unless you know the price of the items you want to buy.

Alan

store A has the better offer because 10% of the sale price is a greater discount than \$10.

- 48. CHALLENGE** Use graphing to find the solution of the system of equations $2x + 3y = 5$, $3x + 4y = 6$, and $4x + 5y = 7$.
- 49. CCSS ARGUMENTS** Determine whether a system of two linear equations with $(0, 0)$ and $(2, 2)$ as solutions *sometimes*, *always*, or *never* has other solutions. Explain.
- 50. WHICH ONE DOESN'T BELONG?** Which one of the following systems of equations doesn't belong with the other three? Explain your reasoning.
- | | | | |
|--------------------------------|--------------------------------|-----------------------------------|---------------------------------|
| $4x - y = 5$ $-2x + y = -1$ | $-x + 4y = 8$ $3x - 6y = 6$ | $4x + 2y = 14$ $12x + 6y = 18$ | $3x - 2y = 1$ $2x + 3y = 18$ |
|--------------------------------|--------------------------------|-----------------------------------|---------------------------------|
- 51. OPEN ENDED** Write three equations such that they form three systems of equations with $y = 5x - 3$. The three systems should be inconsistent, consistent and independent, and consistent and dependent, respectively.
- 52. WRITING IN MATH** Describe the advantages and disadvantages to solving systems of equations by graphing.



Standardized Test Practice

53. SHORT RESPONSE Certain bacteria can reproduce every 20 minutes, doubling the population. If there are 450,000 bacteria in a population at 9:00 A.M., how many bacteria will be in the population at 2:00 P.M.?

54. GEOMETRY An 84-centimeter piece of wire is cut into equal segments and then attached at the ends to form the edges of a cube. What is the volume of the cube?



- A 294 cm^3 C 1158 cm^3
 B 343 cm^3 D 2744 cm^3

55. What is the solution of the inequality $-9 < 2x + 3 < 15$?

- F $-x \geq 0$ H $-6 < x < 6$
 G $x \leq 0$ J $-5 < x < 5$

56. What is the solution of the system of equations?

$$\begin{aligned} x + 2y &= -1 \\ 2x + 4y &= -2 \end{aligned}$$

- A $(-1, -1)$ C no solution
 B $(2, 1)$ D infinitely many solutions

Spiral Review

Graph each inequality. (Lesson 5-6)

57. $3x + 6y > 0$

58. $4x - 2y < 0$

59. $3y - x \leq 9$

60. $4y - 3x \geq 12$

61. $y < -4x - 8$

62. $3x - 1 > y$

63. LIBRARY To get a grant from the city's historical society, the number of history books must be within 25 of 1500. What is the range of the number of historical books that must be in the library? (Lesson 5-5)

64. SCHOOL Camilla's scores on three math tests are shown in the table. The fourth and final test of the grading period is tomorrow. She needs an average of at least 92 to receive an A for the grading period. (Lesson 5-3)

| Test | Score |
|------|-------|
| 1 | 91 |
| 2 | 95 |
| 3 | 88 |

- a. If m represents her score on the fourth math test, write an inequality to represent this situation.
 b. If Camilla wants an A in math, what must she score on the test?
 c. Is your solution reasonable? Explain.

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of the equation. (Lesson 4-4)

65. $(-3, 1)$, $y = \frac{1}{3}x + 2$

66. $(6, -2)$, $y = \frac{3}{5}x - 4$

67. $(2, -2)$, $2x + y = 5$

68. $(-3, -3)$, $-3x + y = 6$

Skills Review

Find the solution of each equation using the given replacement set.

69. $f - 14 = 8$; $\{12, 15, 19, 22\}$

70. $15(n + 6) = 165$; $\{3, 4, 5, 6, 7\}$

71. $23 = \frac{d}{4}$; $\{91, 92, 93, 94, 95\}$

72. $36 = \frac{t-9}{2}$; $\{78, 79, 80, 81\}$

Evaluate each expression if $a = 2$, $b = -3$, and $c = 11$.

73. $a + 6b$

74. $7 - ab$

75. $(2c + 3a) \div 4$

76. $b^2 + (a^3 - 8)5$





You can use a graphing calculator to graph and solve a system of equations.



Activity 1 Solve a System of Equations

Solve the system of equations. State the decimal solution to the nearest hundredth.

$$5.23x + y = 7.48$$

$$6.42x - y = 2.11$$

Step 1 Solve each equation for y to enter them into the calculator.

$$5.23x + y = 7.48$$

First equation

$$5.23x + y - 5.23x = 7.48 - 5.23x$$

Subtract $5.23x$ from each side.

$$y = 7.48 - 5.23x$$

Simplify.

$$6.42x - y = 2.11$$

Second equation

$$6.42x - y - 6.42x = 2.11 - 6.42x$$

Subtract $6.42x$ from each side.

$$-y = 2.11 - 6.42x$$

Simplify.

$$(-1)(-y) = (-1)(2.11 - 6.42x)$$

Multiply each side by -1 .

$$y = -2.11 + 6.42x$$

Simplify.

Step 2 Enter these equations in the $Y=$ list and graph in the standard viewing window.

KEYSTROKES: $Y=$ 7.48 $-$ 5.23 X,T,θ,n

$ENTER$ $(-)$ 2.11 $+$

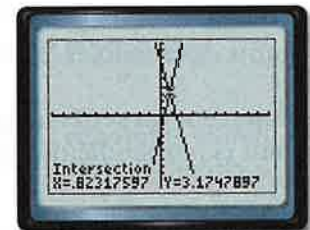
6.42 X,T,θ,n $ZOOM$ 6



Step 3 Use the $CALC$ menu to find the point of intersection.

KEYSTROKES: $2nd$ $[CALC]$ 5 $ENTER$ $ENTER$

$ENTER$



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

The solution is approximately $(0.82, 3.17)$.

When you solve a system of equations with $y = f(x)$ and $y = g(x)$, the solution is an ordered pair that satisfies both equations. The solution always occurs when $f(x) = g(x)$. Thus, the x -coordinate of the solution is the value of x where $f(x) = g(x)$.

One method you can use to solve an equation with one variable is by graphing and solving a system of equations based on the equation. To do this, write a system using both sides of the equation. Then use a graphing calculator to solve the system.

Activity 2 Use a System to Solve a Linear Equation

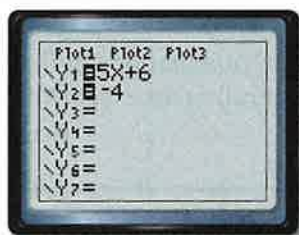
Use a system of equations to solve $5x + 6 = -4$.

Step 1 Write a system of equations. Set each side of the equation equal to y .

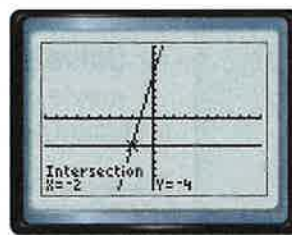
$$y = 5x + 6 \quad \text{First equation}$$

$$y = -4 \quad \text{Second equation}$$

Step 2 Enter these equations in the $Y=$ list and graph.



Step 3 Use the **CALC** menu to find the point of intersection.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

The solution is -2 .

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 2x - 3$
 $y = -0.4x + 5$

3. $x + y = 9.35$
 $5x - y = 8.75$

5. $5.2x - y = 4.1$
 $1.5x + y = 6.7$

7. $7x - 2y = 16$
 $11x + 6y = 32.3$

9. $0.62x + 0.35y = 1.60$
 $-1.38x + y = 8.24$

2. $y = 6x + 1$
 $y = -3.2x - 4$

4. $2.32x - y = 6.12$
 $4.5x + y = -6.05$

6. $1.8 = 5.4x - y$
 $y = -3.8 - 6.2x$

8. $3x + 2y = 16$
 $5x + y = 9$

10. $75x - 100y = 400$
 $33x - 10y = 70$

Use a graphing calculator to solve each equation. Write decimal solutions to the nearest hundredth.

11. $4x - 2 = -6$

12. $3 = 1 + \frac{x}{2}$

13. $\frac{x+4}{-2} = -1$

14. $\frac{3}{2}x + \frac{1}{2} = 2x - 3$

15. $4x - 9 = 7 + 7x$

16. $-2 + 10x = 8x - 1$

17. **WRITING IN MATH** Explain why you can solve an equation like $r = ax + b$ by solving the system of equations $y = r$ and $y = ax + b$.

6-2 Substitution

Then

- You solved systems of equations by graphing.

Now

- Solve systems of equations by using substitution.
- Solve real-world problems involving systems of equations by using substitution.

Why?

- Two movies were released at the same time. Movie A earned \$31 million in its opening week, but fell to \$15 million the following week. Movie B opened earning \$21 million and fell to \$11 million the following week. If the earnings for each movie continue to decrease at the same rate, when will they earn the same amount?



New Vocabulary
substitution



Common Core State Standards

Content Standards

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Solve by Substitution You can use a system of equations to find when the movie earnings are the same. One method of finding an exact solution of a system of equations is called **substitution**.

KeyConcept Solving by Substitution

- Step 1** When necessary, solve at least one equation for one variable.
- Step 2** Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example 1 Solve a System by Substitution

Use substitution to solve the system of equations.

$y = 2x + 1$ ← **Step 1** The first equation is already solved for y .
 $3x + y = -9$

Step 2 Substitute $2x + 1$ for y in the second equation.

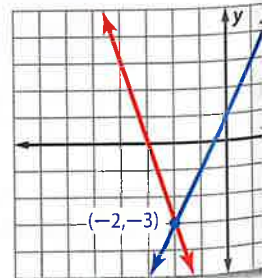
$$\begin{aligned} 3x + y &= -9 && \text{Second equation} \\ 3x + 2x + 1 &= -9 && \text{Substitute } 2x + 1 \text{ for } y. \\ 5x + 1 &= -9 && \text{Combine like terms.} \\ 5x &= -10 && \text{Subtract 1 from each side.} \\ x &= -2 && \text{Divide each side by 5.} \end{aligned}$$

Step 3 Substitute -2 for x in either equation to find y .

$$\begin{aligned} y &= 2x + 1 && \text{First equation} \\ &= 2(-2) + 1 && \text{Substitute } -2 \text{ for } x. \\ &= -3 && \text{Simplify.} \end{aligned}$$

The solution is $(-2, -3)$.

CHECK You can check your solution by graphing.



Guided Practice

1A. $y = 4x - 6$
 $5x + 3y = -1$

1B. $2x + 5y = -1$
 $y = 3x + 10$



StudyTip

Slope-Intercept Form

If both equations are in the form $y = mx + b$, they can simply be set equal to each other and then solved for x . The solution for x can then be used to find the value of y .

If a variable is not isolated in one of the equations in a system, solve an equation for a variable first. Then you can use substitution to solve the system.

PT

Example 2 Solve and then Substitute

Use substitution to solve the system of equations.

$$\begin{aligned}x + 2y &= 6 \\3x - 4y &= 28\end{aligned}$$

Step 1 Solve the first equation for x since the coefficient is 1.

$$\begin{aligned}x + 2y &= 6 && \text{First equation} \\x + 2y - 2y &= 6 - 2y && \text{Subtract } 2y \text{ from each side.} \\x &= 6 - 2y && \text{Simplify.}\end{aligned}$$

Step 2 Substitute $6 - 2y$ for x in the second equation to find the value of y .

$$\begin{aligned}3x - 4y &= 28 && \text{Second equation} \\3(6 - 2y) - 4y &= 28 && \text{Substitute } 6 - 2y \text{ for } x. \\18 - 6y - 4y &= 28 && \text{Distributive Property} \\18 - 10y &= 28 && \text{Combine like terms.} \\18 - 10y - 18 &= 28 - 18 && \text{Subtract 18 from each side.} \\-10y &= 10 && \text{Simplify.} \\y &= -1 && \text{Divide each side by } -10.\end{aligned}$$

Step 3 Find the value of x .

$$\begin{aligned}x + 2y &= 6 && \text{First equation} \\x + 2(-1) &= 6 && \text{Substitute } -1 \text{ for } y. \\x - 2 &= 6 && \text{Simplify.} \\x &= 8 && \text{Add 2 to each side.}\end{aligned}$$

Guided Practice

2A. $4x + 5y = 11$
 $y - 3x = -13$

2B. $x - 3y = -9$
 $5x - 2y = 7$

Generally, if you solve a system of equations and the result is a false statement such as $3 = -2$, there is no solution. If the result is an identity, such as $3 = 3$, then there are an infinite number of solutions.

PT

Example 3 No Solution or Infinitely Many Solutions

Use substitution to solve the system of equations.

$$\begin{aligned}y &= 2x - 4 \\-6x + 3y &= -12\end{aligned}$$

Substitute $2x - 4$ for y in the second equation.

$$\begin{aligned}-6x + 3y &= -12 && \text{Second equation} \\-6x + 3(2x - 4) &= -12 && \text{Substitute } 2x - 4 \text{ for } y. \\-6x + 6x - 12 &= -12 && \text{Distributive Property} \\-12 &= -12 && \text{Combine like terms.}\end{aligned}$$

This statement is an identity. Thus, there are an infinite number of solutions.

StudyTip

Dependent Systems

There are infinitely many solutions of the system in Example 3 because the equations in slope-intercept form are equivalent, and they have the same graph.



Guided Practice Use substitution to solve each system of equations.

3A. $2x - y = 8$
 $y = 2x - 3$

3B. $4x - 3y = 1$
 $6y - 8x = -2$

2 Solve Real-World Problems

You can use substitution to find the solution of a real-world problem involving constraints modeled by a system of equations.



Real-WorldLink

Sound Engineering Technician Sound engineering technicians record, synchronize, mix, and reproduce music, voices, and sound effects in recording studios, sporting arenas, and theater, movie, or video productions. They need to have at least a 2-year associate's degree in electronics.

Real-World Example 4 Write and Solve a System of Equations

MUSIC A store sold a total of 125 car stereo systems and speakers in one week. The stereo systems sold for \$104.95, and the speakers sold for \$18.95. The sales from these two items totaled \$6926.75. How many of each item were sold?

| Number of Units Sold | c | t | 125 |
|----------------------|-----------|----------|---------|
| Sales (\$) | $104.95c$ | $18.95t$ | 6926.75 |

Let c = the number of car stereo systems sold, and let t = the number of speakers sold.

So, the two equations are $c + t = 125$ and $104.95c + 18.95t = 6926.75$.

Notice that $c + t = 125$ represents combinations of car stereo systems and speakers with a sum of 125. The equation $104.95c + 18.95t = 6926.75$ represents the combinations of car stereo systems and speakers with a sales of \$6926.75. The solution of the system of equations represents the option that meets both of the constraints.

Step 1 Solve the first equation for c .

$$c + t = 125 \quad \text{First equation}$$

$$c + t - t = 125 - t \quad \text{Subtract } t \text{ from each side.}$$

$$c = 125 - t \quad \text{Simplify.}$$

Step 2 Substitute $125 - t$ for c in the second equation.

$$104.95c + 18.95t = 6926.75 \quad \text{Second equation}$$

$$104.95(125 - t) + 18.95t = 6926.75 \quad \text{Substitute } 125 - t \text{ for } c.$$

$$13,118.75 - 104.95t + 18.95t = 6926.75 \quad \text{Distributive Property}$$

$$13,118.75 - 86t = 6926.75 \quad \text{Combine like terms.}$$

$$-86t = -6192 \quad \text{Subtract } 13,118.75 \text{ from each side.}$$

$$t = 72 \quad \text{Divide each side by } -86.$$

Step 3 Substitute 72 for t in either equation to find the value of c .

$$c + t = 125 \quad \text{First equation}$$

$$c + 72 = 125 \quad \text{Substitute } 72 \text{ for } t.$$

$$c = 53 \quad \text{Subtract } 72 \text{ from each side.}$$

The store sold 53 car stereo systems and 72 speakers.

Guided Practice

4. **BASEBALL** As of 2009, the New York Yankees and the Cincinnati Reds together had won a total of 32 World Series. The Yankees had won 5.4 times as many as the Reds. How many World Series had each team won?





Examples 1–3 Use substitution to solve each system of equations.

1. $y = x + 5$
 $3x + y = 25$

2. $x = y - 2$
 $4x + y = 2$

3. $3x + y = 6$
 $4x + 2y = 8$

4. $2x + 3y = 4$
 $4x + 6y = 9$

5. $x - y = 1$
 $3x = 3y + 3$

6. $2x - y = 6$
 $-3y = -6x + 18$

Example 4

7. **GEOMETRY** The sum of the measures of angles X and Y is 180° . The measure of angle X is 24° greater than the measure of angle Y.

- Define the variables, and write equations for this situation.
- Find the measure of each angle.

Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–3 Use substitution to solve each system of equations.

8. $y = 5x + 1$
 $4x + y = 10$

9. $y = 4x + 5$
 $2x + y = 17$

10. $y = 3x - 34$
 $y = 2x - 5$

11. $y = 3x - 2$
 $y = 2x - 5$

12. $2x + y = 3$
 $4x + 4y = 8$

13. $3x + 4y = -3$
 $x + 2y = -1$

14. $y = -3x + 4$
 $-6x - 2y = -8$

15. $-1 = 2x - y$
 $8x - 4y = -4$

16. $x = y - 1$
 $-x + y = -1$

17. $y = -4x + 11$
 $3x + y = 9$

18. $y = -3x + 1$
 $2x + y = 1$

19. $3x + y = -5$
 $6x + 2y = 10$

20. $5x - y = 5$
 $-x + 3y = 13$

21. $2x + y = 4$
 $-2x + y = -4$

22. $-5x + 4y = 20$
 $10x - 8y = -40$

Example 4

23. **ECONOMICS** In 2000, the demand for nurses was 2,000,000, while the supply was only 1,890,000. The projected demand for nurses in 2020 is 2,810,414, while the supply is only projected to be 2,001,998.

- Define the variables, and write equations to represent these situations.
- Use substitution to determine during which year the supply of nurses was equal to the demand.

24. **CCSS REASONING** The table shows the approximate number of tourists in two areas of the world during a recent year and the average rates of change in tourism.

| Destination | Number of Tourists | Average Rates of Change in Tourists (millions per year) |
|---------------------------------|--------------------|---|
| South America and the Caribbean | 40.3 million | increase of 0.8 |
| Middle East | 17.0 million | increase of 1.8 |

- Define the variables, and write an equation for each region's tourism rate.
- If the trends continue, in how many years would you expect the number of tourists in the regions to be equal?



Jesse/Alamy, Professor/Alamy, Getty Images, iStockphoto.com



- 25 SPORTS** The table shows the winning times for the Triathlon World Championship.

| Year | Men's | Women's |
|------|---------|---------|
| 2000 | 1:51:39 | 1:54:43 |
| 2009 | 1:44:51 | 1:59:14 |

- The times are in hours, minutes, and seconds. Rewrite the times rounded to the nearest minute.
 - Let the year 2000 be 0. Assume that the rate of change remains the same for years after 2000. Write an equation to represent each of the men's and women's winning times y in any year x .
 - If the trend continues, when would you expect the men's and women's winning times to be the same? Explain your reasoning.
- 26. CONCERT TICKETS** Booker is buying tickets online for a concert. He finds tickets for himself and his friends for \$65 each plus a one-time fee of \$10. Paula is looking for tickets to the same concert. She finds them at another Web site for \$69 and a one-time fee of \$13.60.
- Define the variables, and write equations to represent this situation.
 - Create a table of values for 1 to 5 tickets for each person's purchase.
 - Graph each of these equations.
 - Use the graph to determine who received the better deal? Explain why.

H.O.T. Problems Use Higher-Order Thinking Skills

- 27. ERROR ANALYSIS** In the system $a + b = 7$ and $1.29a + 0.49b = 6.63$, a represents pounds of apples and b represents pounds of bananas. Guillermo and Cara are finding and interpreting the solution. Is either of them correct? Explain.

Guillermo

$$1.29a + 0.49b = 6.63$$

$$1.29a + 0.49(a + 7) = 6.63$$

$$1.29 + 0.49a + 3.43 = 6.63$$

$$0.49a = 3.2$$

$$a = 1.9$$

$a + b = 7$, so $b = 5$. The solution $(2, 5)$ means that 2 pounds of apples and 5 pounds of bananas were bought.

Cara

$$1.29a + 0.49b = 6.63$$

$$1.29(7 - b) + 0.49b = 6.63$$

$$9.03 - 1.29b + 0.49b = 6.63$$

$$-0.8b = -2.4$$

$$b = 3$$

The solution $b = 3$ means that 3 pounds of apples and 3 pounds of bananas were bought.

- 28. CCSS PERSEVERANCE** A local charity has 60 volunteers. The ratio of boys to girls is 7:5. Find the number of boy and the number of girl volunteers.
- 29. REASONING** Compare and contrast the solution of a system found by graphing and the solution of the same system found by substitution.
- 30. OPEN ENDED** Create a system of equations that has one solution. Illustrate how the system could represent a real-world situation and describe the significance of the solution in the context of the situation.
- 31. WRITING IN MATH** Explain how to determine what to substitute when using the substitution method of solving systems of equations.



Standardized Test Practice

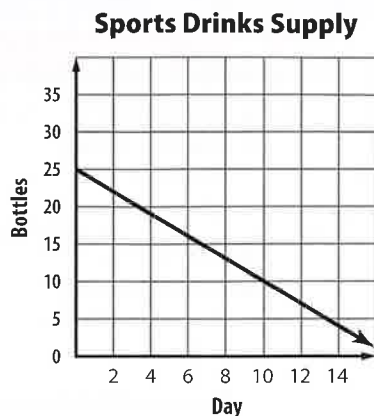
32. The debate team plans to make and sell trail mix. They can spend \$34.

| Item | Cost Per Pound |
|-----------------|----------------|
| sunflower seeds | \$4.00 |
| raisins | \$1.50 |

The pounds of raisins in the mix are to be 3 times the pounds of sunflower seeds. Which system can be used to find r , the pounds of raisins, and p , pounds of sunflower seeds, they should buy?

- A $3p = r$
 $4p + 1.5r = 34$
- B $3p = r$
 $4r + 1.5p = 34$
- C $3r = p$
 $4p + 1.5r = 34$
- D $3r = p$
 $4r + 1.5p = 34$
33. **GRIDDED RESPONSE** The perimeters of two similar polygons are 250 centimeters and 300 centimeters, respectively. What is the scale factor between the first and second polygons?

34. Based on the graph, which statement is true?



- F Mary started with 30 bottles.
 G On day 10, Mary will have 10 bottles left.
 H Mary will be out of sports drinks on day 14.
 J Mary drank 5 bottles the first two days.
35. If p is an integer, which of the following is the solution set for $2|p| = 16$?
- A $\{0, 8\}$
 B $\{-8, 0\}$
 C $\{-8, 8\}$
 D $\{-8, 0, 8\}$

Spiral Review

Graph each system and determine how many solutions it has. If it has one solution, name it. (Lesson 6-1)

36. $y = -5$
 $3x + y = 1$

37. $x = 1$
 $2x - y = 7$

38. $y = x + 5$
 $y = x - 2$

39. $x + y = 1$
 $3y + 3x = 3$

40. **ENTERTAINMENT** Coach Ross wants to take the soccer team out for pizza after their game. Her budget is at most \$70. (Lesson 5-6)

- a. Using the sign, write an inequality that represents this situation.
 b. Are there any restrictions on the variables? Explain.

Solve each inequality. Check your solution. (Lesson 5-3)

41. $6v + 1 \geq -11$
 42. $24 > 18 + 2n$

44. $\frac{a}{8} - 10 > -3$
 45. $-3t + 9 \leq 0$



Welcome to Rini's Pizza

Large Pizza \$12
 Pitcher of Soft Drinks \$2

43. $-11 \geq \frac{2}{5}q + 5$
 46. $54 > -10 - 8n$

Skills Review

Rewrite each product using the Distributive Property. Then simplify.

47. $10b + 5(3 + 9b)$
 48. $5(3t^2 + 4) - 8t$

49. $7h^2 + 4(3h + h^2)$
 50. $-2(7a + 5b) + 5(2a - 7b)$



Elimination Using Addition and Subtraction



Then

- You solved systems of equations by using substitution.

Now

- Solve systems of equations by using elimination with addition.
- Solve systems of equations by using elimination with subtraction.

Why?

- In Chicago, Illinois, there are two more months a when the mean high temperature is below 70°F than there are months b when it is above 70°F . The system of equations, $a + b = 12$ and $a - b = 2$, represents this situation.



New Vocabulary
elimination



Common Core State Standards

Content Standards

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices

7 Look for and make use of structure.

1 Elimination Using Addition If you add these equations, the variable b will be eliminated. Using addition or subtraction to solve a system is called **elimination**.

Key Concept Solving by Elimination

- Step 1** Write the system so like terms with the same or opposite coefficients are aligned.
- Step 2** Add or subtract the equations, eliminating one variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.

Example 1 Elimination Using Addition



Use elimination to solve the system of equations.

$$4x + 6y = 32$$

$$3x - 6y = 3$$

← **Step 1** $6y$ and $-6y$ have opposite coefficients.

Step 2 Add the equations.

$$4x + 6y = 32$$

$$(+)\ 3x - 6y = 3$$

$$7x = 35$$

$$\frac{7x}{7} = \frac{35}{7}$$

$$x = 5$$

The variable y is eliminated.

Divide each side by 7.

Simplify.

Step 3 Substitute 5 for x in either equation to find the value of y .

$$4x + 6y = 32$$

First equation

$$4(5) + 6y = 32$$

Replace x with 5.

$$20 + 6y = 32$$

Multiply.

$$20 + 6y - 20 = 32 - 20$$

Subtract 20 from each side.

$$6y = 12$$

Simplify.

$$\frac{6y}{6} = \frac{12}{6}$$

Divide each side by 6.

$$y = 2$$

Simplify.

The solution is $(5, 2)$.



Guided Practice

1A. $-4x + 3y = -3$
 $4x - 5y = 5$

1B. $4y + 3x = 22$
 $3x - 4y = 14$

We can use elimination to find specific numbers that are described as being related to each other.

Example 2 Write and Solve a System of Equations



Negative three times one number plus five times another number is -11 .
 Three times the first number plus seven times the other number is -1 .
 Find the numbers.

| | | | | |
|------------------------------------|------|---------------------------------|----|---------|
| Negative three times one number | plus | five times another number | is | -11 . |
| $-3x$ | + | $5y$ | = | -11 |
| Three times the first number | plus | seven times the other number | is | -1 . |
| $3x$ | + | $7y$ | = | -1 |

Steps 1 and 2 Write the equations vertically and add.

$$\begin{array}{r}
 -3x + 5y = -11 \\
 (+) 3x + 7y = -1 \\
 \hline
 12y = -12 \\
 \frac{12y}{12} = \frac{-12}{12} \\
 y = -1
 \end{array}$$

The variable x is eliminated.
 Divide each side by 12.
 Simplify.

Step 3 Substitute -1 for y in either equation to find the value of x .

$$\begin{array}{r}
 3x + 7y = -1 \quad \text{Second equation} \\
 3x + 7(-1) = -1 \quad \text{Replace } y \text{ with } -1. \\
 3x + (-7) = -1 \quad \text{Simplify.} \\
 3x + (-7) + 7 = -1 + 7 \quad \text{Add 7 to each side.} \\
 3x = 6 \quad \text{Simplify.} \\
 \frac{3x}{3} = \frac{6}{3} \quad \text{Divide each side by 3.} \\
 x = 2 \quad \text{Simplify.}
 \end{array}$$

The numbers are 2 and -1 .

CHECK

| | |
|-------------------------------------|---|
| $-3x + 5y = -11$ | First equation |
| $-3(2) + 5(-1) \stackrel{?}{=} -11$ | Substitute 2 for x and -1 for y . |
| $-11 = -11 \checkmark$ | Simplify. |
| $3x + 7y = -1$ | Second equation |
| $3(2) + 7(-1) \stackrel{?}{=} -1$ | Substitute 2 for x and -1 for y . |
| $-1 = -1 \checkmark$ | Simplify. |

Study Tip

Coefficients When the coefficients of a variable are the same, subtracting the equations will eliminate the variable. When the coefficients are opposites, adding the equations will eliminate the variable.

Problem-Solving Tip

CCSS Perseverance

Checking your answers in both equations of a system helps ensure there are no calculation errors.

Guided Practice

2. The sum of two numbers is -10 . Negative three times the first number minus the second number equals 2. Find the numbers.



2 Elimination Using Subtraction

Sometimes we can eliminate a variable by subtracting one equation from another.



Standardized Test Example 3

Solve the system of equations.

$$2t + 5r = 6$$

$$9r + 2t = 22$$

A $(-7, 15)$

B $(7, \frac{8}{9})$

C $(4, -7)$

D $(4, -\frac{2}{5})$

Read the Test Item

Since both equations contain $2t$, use elimination by subtraction.

Solve the Test Item

Step 1 Subtract the equations.

$$\begin{array}{r} 5r + 2t = 6 \\ (-) 9r + 2t = 22 \\ \hline -4r \quad = -16 \\ r = 4 \end{array}$$

Write the system so like terms are aligned.

The variable t is eliminated.

Simplify.

Step 2 Substitute 4 for r in either equation to find the value of t .

$$\begin{array}{r} 5r + 2t = 6 \\ 5(4) + 2t = 6 \\ 20 + 2t = 6 \\ 20 + 2t - 20 = 6 - 20 \\ 2t = -14 \\ t = -7 \end{array}$$

First equation

$$r = 4$$

Simplify.

Subtract 20 from each side.

Simplify.

Simplify.

The solution is $(4, -7)$. The correct answer is C.

Guided Practice

3. Solve the system of equations.

$$8b + 3c = 11$$

$$8b + 7c = 7$$

F $(1.5, -1)$

G $(1.75, -1)$

H $(1.75, 1)$

J $(1.5, 1)$



Real-World Example 4 Write and Solve a System of Equations

JOBS Cheryl and Jackie work at an ice cream shop. Cheryl earns \$8.50 per hour and Jackie earns \$7.50 per hour. During a typical week, Cheryl and Jackie earn \$299.50 together. One week, Jackie doubles her work hours, and the girls earn \$412. How many hours does each girl work during a typical week?

Understand You know how much Cheryl and Jackie each earn per hour and how much they earned together.

Plan Let c = Cheryl's hours and j = Jackie's hours.

| | | | | |
|--------------|------|--------------|--------|-----------|
| Cheryl's pay | plus | Jackie's pay | equals | \$299.50. |
| 8.50c | + | 7.50j | = | 299.50 |
| Cheryl's pay | plus | Jackie's pay | equals | \$412. |
| 8.50c | + | 7.50(2j) | = | 412 |



Real-WorldLink

The five most dangerous jobs for teens are: delivery and other driving jobs, working alone in cash-based businesses, traveling youth crews, cooking, and construction.

Source: National Consumers League



Study Tip

Another Method Instead of subtracting the equations, you could also multiply one equation by -1 and then add the equations.

Solve Subtract the equations to eliminate one of the variables. Then solve for the other variable.

$$\begin{array}{r} 8.50c + 7.50j = 299.50 \\ (-) 8.50c + 7.50(2)j = 412 \\ \hline 8.50c + 7.50j = 299.50 \\ (-) 8.50c + 15j = 412 \\ \hline -7.50j = -112.50 \\ \frac{-7.50j}{-7.50} = \frac{-112.50}{-7.50} \\ j = 15 \end{array}$$

Write the equations vertically.

Simplify.

Subtract. The variable c is eliminated.

Divide each side by -7.50 .

Simplify.

Now substitute 15 for j in either equation to find the value of c .

$$\begin{array}{r} 8.50c + 7.50j = 299.50 \\ 8.50c + 7.50(15) = 299.50 \\ 8.50c + 112.50 = 299.50 \\ 8.50c = 187 \\ c = 22 \end{array}$$

First equation

Substitute 15 for j .

Simplify.

Subtract 112.50 from each side.

Divide each side by 8.50.

Check Substitute both values into the other equation to see if the equation holds true. If $c = 22$ and $j = 15$, then $8.50(22) + 15(15)$ or 412 .

Cheryl works 22 hours, while Jackie works 15 hours during a typical week.

Guided Practice

4. **PARTIES** Tamera and Adelina are throwing a birthday party for their friend. Tamera invited 5 fewer friends than Adelina. Together they invited 47 guests. How many guests did each girl invite?

Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Examples 1, 3 Use elimination to solve each system of equations.

- | | |
|-------------------|-------------------|
| 1. $5m - p = 7$ | 2. $8x + 5y = 38$ |
| $7m - p = 11$ | $-8x + 2y = 4$ |
| 3. $7f + 3g = -6$ | 4. $6a - 3b = 27$ |
| $7f - 2g = -31$ | $2a - 3b = 11$ |

Example 2

5. **CCSS REASONING** The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

Example 4

6. **RECYCLING** The recycling and reuse industry employs approximately 1,025,000 more workers than the waste management industry. Together they provide 1,275,000 jobs. How many jobs does each industry provide?



Examples 1, 3 Use elimination to solve each system of equations.

7. $-v + w = 7$

$v + w = 1$

10. $5m - 2p = 24$

$3m + 2p = 24$

13. $6c - 9d = 111$

$5c - 9d = 103$

16. $3j + 4k = 23.5$

$8j - 4k = 4$

8. $y + z = 4$

$y - z = 8$

11. $a + 4b = -4$

$a + 10b = -16$

14. $11f + 14g = 13$

$11f + 10g = 25$

17. $-3x - 8y = -24$

$3x - 5y = 4.5$

9. $-4x + 5y = 17$

$4x + 6y = -6$

12. $6r - 6t = 6$

$3r - 6t = 15$

15. $9x + 6y = 78$

$3x - 6y = -30$

18. $6x - 2y = 1$

$10x - 2y = 5$

Example 2 19. The sum of two numbers is 22, and their difference is 12. What are the numbers?

20. Find the two numbers with a sum of 41 and a difference of 9.

21. Three times a number minus another number is -3 . The sum of the numbers is 11. Find the numbers.

22. A number minus twice another number is 4. Three times the first number plus two times the second number is 12. What are the numbers?

Example 4 23. **TOURS** The Blackwells and Joneses are going to Hershey's Really Big 3D Show in Pennsylvania. Find the adult price and the children's price of the show.

| Family | Number of Adults | Number of Children | Total Cost |
|-----------|------------------|--------------------|------------|
| Blackwell | 2 | 5 | \$31.65 |
| Jones | 2 | 3 | \$23.75 |

Use elimination to solve each system of equations.

24. $4(x + 2y) = 8$

$4x + 4y = 12$

27. $6x - 7y = -26$

$6x + 5y = 10$

25. $3x - 5y = 11$

$5(x + y) = 5$

28. $\frac{1}{2}x + \frac{2}{3}y = 2\frac{3}{4}$

$\frac{1}{4}x - \frac{2}{3}y = 6\frac{1}{4}$

26. $4x + 3y = 6$

$3x + 3y = 7$

29. $\frac{3}{5}x + \frac{1}{2}y = 8\frac{1}{3}$

$-\frac{3}{5}x + \frac{3}{4}y = 8\frac{1}{3}$

30. **CCSS SENSE-MAKING** The total height of an office building b and the granite statue that stands on top of it g is 326.6 feet. The difference in heights between the building and the statue is 295.4 feet.

a. How tall is the statue?

b. How tall is the building?

31. **BIKE RACING** Professional Mountain Bike Racing currently has 66 teams. The number of non-U.S. teams is 30 more than the number of U.S. teams.

a. Let x represent the number of non-U.S. teams and y represent the number of U.S. teams. Write a system of equations that represents the number of U.S. teams and non-U.S. teams.

b. Use elimination to find the solution of the system of equations.

c. Interpret the solution in the context of the situation.

d. Graph the system of equations to check your solution.



32. **SHOPPING** Let x represent the number of years since 2004 and y represent the number of catalogs.

| Catalogs | Number in 2004 | Growth Rate (number per year) |
|----------|----------------|-------------------------------|
| online | 7440 | 1293 |
| print | 3805 | -1364 |

Source: MediaPost Publications

- Write a system of equations to represent this situation.
 - Use elimination to find the solution to the system of equations.
 - Analyze the solution in terms of the situation. Determine the reasonableness of the solution.
33. **MULTIPLE REPRESENTATIONS** Collect 9 pennies and 9 paper clips. For this game, you use objects to score points. Each paper clip is worth 1 point and each penny is worth 3 points. Let p represent the number of pennies and c represent the number of paper clips.

$$9 \text{ points} = \begin{array}{c} \text{penny} \\ \text{penny} \end{array} + \begin{array}{c} \text{paper clip} \\ \text{paper clip} \\ \text{paper clip} \end{array} = 3p + c \\ = 3(2) + 3$$

- Concrete** Choose a combination of 9 objects and find your score.
- Analytical** Write and solve a system of equations to find the number of paper clips and pennies used for 15 points, if 9 total objects are used.
- Tabular** If 9 total objects are used, make a table showing the number of paper clips used and the total number of points when the number of pennies is 0, 1, 2, 3, 4, or 5.
- Verbal** Does the result in the table match the results in part b? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

- REASONING** Describe the solution of a system of equations if after you added two equations the result was $0 = 0$.
- REASONING** What is the solution of a system of equations if the sum of the equations is $0 = 2$?
- OPEN ENDED** Create a system of equations that can be solved by using addition to eliminate one variable. Formulate a general rule for creating such systems.
- CCSS STRUCTURE** The solution of a system of equations is $(-3, 2)$. One equation in the system is $x + 4y = 5$. Find a second equation for the system. Explain how you derived this equation.
- CHALLENGE** The sum of the digits of a two-digit number is 8. The result of subtracting the units digit from the tens digit is -4 . Define the variables and write the system of equations that you would use to find the number. Then solve the system and find the number.
- WRITING IN MATH** Describe when it would be most beneficial to use elimination to solve a system of equations.



Standardized Test Practice

40. SHORT RESPONSE Martina is on a train traveling at a speed of 188 mph between two cities 1128 miles apart. If the train has been traveling for an hour, how many more hours is her train ride?

41. GEOMETRY Ms. Miller wants to tile her rectangular kitchen floor. She knows the dimensions of the floor. Which formula should she use to find the area?

A $A = \ell w$

C $P = 2\ell + 2w$

B $V = Bh$

D $c^2 = a^2 + b^2$

42. If the pattern continues, what is the 8th number in the sequence?

$$2, 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \dots$$

F $\frac{2187}{64}$

G $\frac{2245}{64}$

H $\frac{2281}{64}$

J $\frac{2445}{64}$

43. What is the solution of this system of equations?

$$x + 4y = 1$$

$$2x - 3y = -9$$

A (2, -8)

C no solution

B (-3, 1)

D infinitely many solutions

Spiral Review

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 6-2)

44. $y = 6x$
 $2x + 3y = 40$

45. $x = 3y$
 $2x + 3y = 45$

46. $x = 5y + 6$
 $x = 3y - 2$

47. $y = 3x + 2$
 $y = 4x - 1$

48. $3c = 4d + 2$
 $c = d - 1$

49. $z = v + 4$
 $2z - v = 6$

50. FINANCIAL LITERACY Gregorio and Javier each want to buy a bicycle. Gregorio has already saved \$35 and plans to save \$10 per week. Javier has \$26 and plans to save \$13 per week. (Lesson 6-1)

- In how many weeks will Gregorio and Javier have saved the same amount of money?
- How much will each person have saved at that time?

51. GEOMETRY A *parallelogram* is a quadrilateral in which opposite sides are parallel. Determine whether $ABCD$ is parallelogram. Explain your reasoning. (Lesson 4-4)

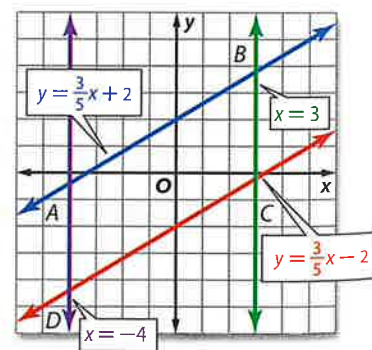
Solve each equation. Check your solution. (Lesson 2-2)

52. $6u = -48$

53. $75 = -15p$

54. $\frac{2}{3}a = 8$

55. $-\frac{3}{4}d = 15$



Skills Review

Simplify each expression. If not possible, write *simplified*.

56. $6q - 3 + 7q + 1$

57. $7w^2 - 9w + 4w^2$

58. $10(2 + r) + 3r$

59. $5y - 7(y + 5)$



Elimination Using Multiplication



Then

You used elimination with addition and subtraction to solve systems of equations.

Now

1 Solve systems of equations by using elimination with multiplication.

2 Solve real-world problems involving systems of equations.

Why?

The table shows the number of cars at Scott's Auto Repair Shop for each type of service.

| Item | Repairs | Maintenance |
|--------|---------|-------------|
| body | 3 | 4 |
| engine | 2 | 2 |

The manager has allotted 1110 minutes for body work and 570 minutes for engine work. The system $3r + 4m = 1110$ and $2r + 2m = 570$ can be used to find the average time for each service.

Common Core State Standards

Content Standards

A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Elimination Using Multiplication

In the system above, neither variable can be eliminated by adding or subtracting. You can use multiplication to solve.

Key Concept Solving by Elimination Using Multiplication

- Step 1** Multiply at least one equation by a constant to get two equations that contain opposite terms.
- Step 2** Add the equations, eliminating one variable. Then solve the equation.
- Step 3** Substitute the value from Step 2 into one of the equations and solve for the other variable. Write the solution as an ordered pair.



Example 1 Multiply One Equation to Eliminate a Variable

Use elimination to solve the system of equations.

$$\begin{aligned} 5x + 6y &= -8 \\ 2x + 3y &= -5 \end{aligned}$$

Steps 1 and 2

$$\begin{array}{r} 5x + 6y = -8 \\ 2x + 3y = -5 \end{array} \xrightarrow{\text{Multiply each term by } -2} \begin{array}{r} 5x + 6y = -8 \\ (+) -4x - 6y = 10 \end{array} \text{ Add.}$$

$$\begin{array}{r} x \\ = \\ 2 \end{array} \text{ } y \text{ is eliminated.}$$

- Step 3** $2x + 3y = -5$ Second equation
- $2(2) + 3y = -5$ Substitution, $x = 2$
- $4 + 3y = -5$ Simplify.
- $3y = -9$ Subtract 4 from each side and simplify.
- $y = -3$ Divide each side by 3 and simplify.

The solution is $(2, -3)$.

Guided Practice

- 1A. $6x - 2y = 10$
 $3x - 7y = -19$
- 1B. $9r + q = 13$
 $3r + 2q = -4$



Sometimes you have to multiply each equation by a different number in order to solve the system.



Example 2 Multiply Both Equations to Eliminate a Variable

Use elimination to solve the system of equations.

$$4x + 2y = 8$$

$$3x + 3y = 9$$

Method 1 Eliminate x .

$$4x + 2y = 8$$

$$3x + 3y = 9$$

Multiply by 3.

Multiply by -4 .

$$12x + 6y = 24$$

$$(+)\ -12x - 12y = -36$$

$$\hline -6y = -12$$

$$\frac{-6y}{-6} = \frac{-12}{-6}$$

$$y = 2$$

Add equations.
 x is eliminated.

Divide each side by -6 .

Simplify.

Now substitute 2 for y in either equation to find the value of x .

$$3x + 3y = 9 \quad \text{Second equation}$$

$$3x + 3(2) = 9 \quad \text{Substitute 2 for } y.$$

$$3x + 6 = 9 \quad \text{Simplify.}$$

$$3x = 3 \quad \text{Subtract 6 from each side and simplify.}$$

$$\frac{3x}{3} = \frac{3}{3} \quad \text{Divide each side by 3.}$$

$$x = 1 \quad \text{The solution is } (1, 2).$$

Method 2 Eliminate y .

$$4x + 2y = 8$$

$$3x + 3y = 9$$

Multiply by 3.

Multiply by -2 .

$$12x + 6y = 24$$

$$(+)\ -6x - 6y = -18$$

$$\hline 6x = 6$$

$$\frac{6x}{6} = \frac{6}{6}$$

$$x = 1$$

Add equations.
 y is eliminated.

Divide each side by 6.

Simplify.

Now substitute 1 for x in either equation to find the value of y .

$$3x + 3y = 9 \quad \text{Second equation}$$

$$3(1) + 3y = 9 \quad \text{Substitute 1 for } x.$$

$$3 + 3y = 9 \quad \text{Simplify.}$$

$$3y = 6 \quad \text{Subtract 3 from each side and simplify.}$$

$$\frac{3y}{3} = \frac{6}{3} \quad \text{Divide each side by 3.}$$

$$y = 2 \quad \text{Simplify.}$$

The solution is $(1, 2)$, which matches the result obtained with Method 1.

CHECK Substitute 1 for x and 2 for y in the first equation.

$$4x + 2y = 8 \quad \text{Original equation}$$

$$4(1) + 2(2) \stackrel{?}{=} 8 \quad \text{Substitute } (1, 2) \text{ for } (x, y).$$

$$4 + 4 \stackrel{?}{=} 8 \quad \text{Multiply.}$$

$$8 = 8 \quad \checkmark \quad \text{Add.}$$

Guided Practice

2A. $5x - 3y = 6$
 $2x + 5y = -10$

2B. $6a + 2b = 2$
 $4a + 3b = 8$

StudyTip

Choosing a Variable to Eliminate Unless the problem is asking for the value of a specific variable, you may use multiplication to eliminate either variable.



Math HistoryLink

Leonardo Pisano (1170–1250) Leonardo Pisano is better known by his nickname *Fibonacci*. His book introduced the Hindu-Arabic place-valued decimal system. Systems of linear equations are studied in this work.



2 Solve Real-World Problems

Sometimes it is necessary to use multiplication before elimination in real-world problem solving too.



Real-World Example 3 Solve a System of Equations

FLIGHT A personal aircraft traveling with the wind flies 520 miles in 4 hours. On the return trip, the airplane takes 5 hours to travel the same distance. Find the speed of the airplane if the air is still.

You are asked to find the speed of the airplane in still air.

Let a = the rate of the airplane if the air is still.

Let w = the rate of the wind.

| | r | t | d | $r \cdot t = d$ |
|------------------|---------|-----|-----|------------------|
| With the Wind | $a + w$ | 4 | 520 | $(a + w)4 = 520$ |
| Against the Wind | $a - w$ | 5 | 520 | $(a - w)5 = 520$ |

So, our two equations are $4a + 4w = 520$ and $5a - 5w = 520$.

$$\begin{array}{r}
 4a + 4w = 520 \\
 5a - 5w = 520
 \end{array}
 \begin{array}{l}
 \text{Multiply by 5.} \\
 \text{Multiply by 4.}
 \end{array}
 \begin{array}{r}
 20a + 20w = 2600 \\
 (+) 20a - 20w = 2080 \\
 \hline
 40a = 4680
 \end{array}
 \begin{array}{l}
 w \text{ is eliminated.} \\
 \\
 \text{Divide each side by 40.} \\
 \\
 \text{Simplify.}
 \end{array}$$

$$\frac{40a}{40} = \frac{4680}{40}$$

$$a = 117$$

The rate of the airplane in still air is 117 miles per hour.

Guided Practice

3. CANOEING A canoeist travels 4 miles downstream in 1 hour. The return trip takes the canoeist 1.5 hours. Find the rate of the boat in still water.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.



Examples 1–2 Use elimination to solve each system of equations.

1. $2x - y = 4$
 $7x + 3y = 27$

2. $2x + 7y = 1$
 $x + 5y = 2$

3. $4x + 2y = -14$
 $5x + 3y = -17$

4. $9a - 2b = -8$
 $-7a + 3b = 12$

Example 3

5. CCSS SENSE-MAKING A kayaking group with a guide travels 16 miles downstream, stops for a meal, and then travels 16 miles upstream. The speed of the current remains constant throughout the trip. Find the speed of the kayak in still water.

| | |
|---------------|------------|
| Leave | 10:00 A.M. |
| Stop for meal | 12:00 noon |
| Return | 1:00 P.M. |
| Finish | 5:00 P.M. |

6. PODCASTS Steve subscribed to 10 podcasts for a total of 340 minutes. He used his two favorite tags, Hobbies and Recreation and Soliloquies. Each of the Hobbies and Recreation episodes lasted about 32 minutes. Each Soliloquies episode lasted 42 minutes. To how many of each tag did Steve subscribe?



Examples 1–2 Use elimination to solve each system of equations.

7. $x + y = 2$
 $-3x + 4y = 15$

9. $x + 5y = 17$
 $-4x + 3y = 24$

11. $2x + 5y = 11$
 $4x + 3y = 1$

13. $3x + 4y = 29$
 $6x + 5y = 43$

15. $8x + 3y = -7$
 $7x + 2y = -3$

17. $12x - 3y = -3$
 $6x + y = 1$

8. $x - y = -8$
 $7x + 5y = 16$

10. $6x + y = -39$
 $3x + 2y = -15$

12. $3x - 3y = -6$
 $-5x + 6y = 12$

14. $8x + 3y = 4$
 $-7x + 5y = -34$

16. $4x + 7y = -80$
 $3x + 5y = -58$

18. $-4x + 2y = 0$
 $10x + 3y = 8$

Example 3

19 NUMBER THEORY Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

20 FOOTBALL A field goal is 3 points and the extra point after a touchdown is 1 point. In a recent post-season, Adam Vinatieri of the Indianapolis Colts made a total of 21 field goals and extra point kicks for 49 points. Find the number of field goals and extra points that he made.

Use elimination to solve each system of equations.

21. $2.2x + 3y = 15.25$
 $4.6x + 2.1y = 18.325$

23. $\frac{1}{4}x + 4y = 2\frac{3}{4}$
 $3x + \frac{1}{2}y = 9\frac{1}{4}$

22. $-0.4x + 0.25y = -2.175$
 $2x + y = 7.5$

24. $\frac{2}{5}x + 6y = 24\frac{1}{5}$
 $3x + \frac{1}{2}y = 3\frac{1}{2}$

25. CCSS MODELING A staffing agency for in-home nurses and support staff places necessary personnel at locations on a daily basis. Each placed nurse works 240 minutes per day at a daily rate of \$90. Each support staff employee works 360 minutes per day at a daily rate of \$120.

- On a given day, 3000 total minutes are worked by the nurses and support staff that were placed. Write an equation that represents this relationship.
- On the same day, earnings for placed nurses and support staff totaled \$1050. Write an equation that represents this relationship.
- Solve the system of equations, and interpret the solution in the context of the situation.

26. GEOMETRY The graphs of $x + 2y = 6$ and $2x + y = 9$ contain two of the sides of a triangle. A vertex of the triangle is at the intersection of the graphs.

- What are the coordinates of the vertex?
- Draw the graph of the two lines. Identify the vertex of the triangle.
- The line that forms the third side of the triangle is the line $x - y = -3$. Draw this line on the previous graph.
- Name the other two vertices of the triangle.



27. **ENTERTAINMENT** At an entertainment center, two groups of people bought batting tokens and miniature golf games, as shown in the table.

| Group | Number of Batting Tokens | Number of Miniature Golf Games | Total Cost |
|-------|--------------------------|--------------------------------|------------|
| A | 16 | 3 | \$30 |
| B | 22 | 5 | \$43 |

- a. Define the variables, and write a system of linear equations from this situation.
 b. Solve the system of equations, and explain what the solution represents.
28. **TESTS** Mrs. Henderson discovered that she had accidentally reversed the digits of a test score and did not give a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score. What was his correct score?

H.O.T. Problems Use Higher-Order Thinking Skills

29. **REASONING** Explain how you could recognize a system of linear equations with infinitely many solutions.
30. **CCSS CRITIQUE** Jason and Daniela are solving a system of equations. Is either of them correct? Explain your reasoning.

Jason

$$\begin{array}{r} 2r + 7t = 11 \\ r - 9t = -7 \\ \hline 2r + 7t = 11 \\ (-) 2r - 18t = -14 \\ \hline 25t = 25 \\ t = 1 \\ 2r + 7t = 11 \\ 2r + 7(1) = 11 \\ 2r + 7 = 11 \\ 2r = 4 \\ \frac{2r}{2} = \frac{4}{2} \\ r = 2 \end{array}$$

The solution is (2, 1).

Daniela

$$\begin{array}{r} 2r + 7t = 11 \\ (-) r - 9t = -7 \\ \hline r = 18 \\ 2r + 7t = 11 \\ 2(18) + 7t = 11 \\ 36 + 7t = 11 \\ 7t = -25 \\ \frac{7t}{7} = \frac{-25}{7} \\ t = -3.6 \end{array}$$

The solution is (18, -3.6).

31. **OPEN ENDED** Write a system of equations that can be solved by multiplying one equation by -3 and then adding the two equations together.
32. **CHALLENGE** The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b . Discuss the steps that you used.
33. **WRITING IN MATH** Why is substitution sometimes more helpful than elimination, and vice versa?



Standardized Test Practice

34. What is the solution of this system of equations?

$$2x - 3y = -9$$

$$-x + 3y = 6$$

- A (3, 3) C (-3, 1)
B (-3, 3) D (1, -3)

35. A buffet has one price for adults and another for children. The Taylor family has two adults and three children, and their bill was \$40.50. The Wong family has three adults and one child. Their bill was \$38. Which system of equations could be used to determine the price for an adult and for a child?

F $x + y = 40.50$ H $2x + 3y = 40.50$

$x + y = 38$ $x + 3y = 38$

G $2x + 3y = 40.50$ J $2x + 2y = 40.50$

$3x + y = 38$ $3x + y = 38$

36. **SHORT RESPONSE** A customer at the paint store has ordered 3 gallons of ivy green paint. Melissa mixes the paint in a ratio of 3 parts blue to one part yellow. How many quarts of blue paint does she use?

37. **PROBABILITY** The table shows the results of a number cube being rolled. What is the experimental probability of rolling a 3?

| Outcome | Frequency |
|---------|-----------|
| 1 | 4 |
| 2 | 8 |
| 3 | 2 |
| 4 | 0 |
| 5 | 5 |
| 6 | 1 |

- A $\frac{2}{3}$ B $\frac{1}{3}$ C 0.2 D 0.1

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-3)

38. $f + g = -3$
 $f - g = 1$

39. $6g + h = -7$
 $6g + 3h = -9$

40. $5j + 3k = -9$
 $3j + 3k = -3$

41. $2x - 4z = 6$
 $x - 4z = -3$

42. $-5c - 3v = 9$
 $5c + 2v = -6$

43. $4b - 6n = -36$
 $3b - 6n = -36$

44. **JOBS** Brandy and Adriana work at an after-school child care center. Together they cared for 32 children this week. Brandy cared for 0.6 times as many children as Adriana. How many children did each girl care for? (Lesson 6-2)

Solve each inequality. Then graph the solution set. (Lesson 5-5)

45. $|m - 5| \leq 8$

46. $|q + 11| < 5$

47. $|2w + 9| > 11$

48. $|2r + 1| \geq 9$

Skills Review

Translate each sentence into a formula.

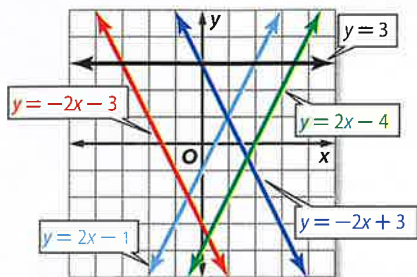
49. The area A of a triangle equals one half times the base b times the height h .
50. The circumference C of a circle equals the product of 2, π , and the radius r .
51. The volume V of a rectangular box is the length ℓ times the width w multiplied by the height h .
52. The volume of a cylinder V is the same as the product of π and the radius r to the second power multiplied by the height h .
53. The area of a circle A equals the product of π and the radius r squared.
54. Acceleration A equals the increase in speed s divided by time t in seconds.



Mid-Chapter Quiz

Lessons 6-1 through 6-4

Use the graph to determine whether each system is *consistent* or *inconsistent* and if it is *independent* or *dependent*. (Lesson 6-1)



1. $y = 2x - 1$
 $y = -2x + 3$
2. $y = -2x + 3$
 $y = -2x - 3$

Graph each system and determine the number of solutions that it has. If it has one solution, name it. (Lesson 6-1)

3. $y = 2x - 3$
 $y = x + 4$
4. $x + y = 6$
 $x - y = 4$
5. $x + y = 8$
 $3x + 3y = 24$
6. $x - 4y = -6$
 $y = -1$
7. $3x + 2y = 12$
 $3x + 2y = 6$
8. $2x + y = -4$
 $5x + 3y = -6$

Use substitution to solve each system of equations. (Lesson 6-2)

9. $y = x + 4$
 $2x + y = 16$
10. $y = -2x - 3$
 $x + y = 9$
11. $x + y = 6$
 $x - y = 8$
12. $y = -4x$
 $6x - y = 30$

13. **FOOD** The cost of two meals at a restaurant is shown in the table below. (Lesson 6-2)

| Meal | Total Cost |
|---------------------|------------|
| 3 tacos, 2 burritos | \$7.40 |
| 4 tacos, 1 burrito | \$6.45 |

- Define variables to represent the cost of a taco and the cost of a burrito.
- Write a system of equations to find the cost of a single taco and a single burrito.
- Solve the systems of equations, and explain what the solution means.
- How much would a customer pay for 2 tacos and 2 burritos?

14. **AMUSEMENT PARKS** The cost of two groups going to an amusement park is shown in the table. (Lesson 6-3)

| Group | Total Cost |
|----------------------|------------|
| 4 adults, 2 children | \$184 |
| 4 adults, 3 children | \$200 |

- Define variables to represent the cost of an adult ticket and the cost of a child ticket.
- Write a system of equations to find the cost of an adult ticket and a child ticket.
- Solve the system of equations, and explain what the solution means.
- How much will a group of 3 adults and 5 children be charged for admission?

15. **MULTIPLE CHOICE** Angelina spent \$16 for 12 pieces of candy to take to a meeting. Each chocolate bar costs \$2, and each lollipop costs \$1. Determine how many of each she bought. (Lesson 6-3)

- 6 chocolate bars, 6 lollipops
- 4 chocolate bars, 8 lollipops
- 7 chocolate bars, 5 lollipops
- 3 chocolate bars, 9 lollipops

Use elimination to solve each system of equations. (Lessons 6-3 and 6-4)

16. $x + y = 9$
 $x - y = -3$
17. $x + 3y = 11$
 $x + 7y = 19$
18. $9x - 24y = -6$
 $3x + 4y = 10$
19. $-5x + 2y = -11$
 $5x - 7y = 1$

20. **MULTIPLE CHOICE** The Blue Mountain High School Drama Club is selling tickets to their spring musical. Adult tickets are \$4 and student tickets are \$1. A total of 285 tickets are sold for \$765. How many of each type of ticket are sold? (Lesson 6-4)

- 145 adult, 140 student
- 120 adult, 165 student
- 180 adult, 105 student
- 160 adult, 125 student

Then

- You solved systems of equations by using substitution and elimination.

Now

- Determine the best method for solving systems of equations.
- Apply systems of equations.

Why?



- In speed skating, competitors race two at a time on a double track. Indoor speed skating rinks have two track sizes for race events: an official track and a short track.

Speed Skating Tracks

| | |
|----------------|-----|
| official track | x |
| short track | y |

The total length of the two tracks is 511 meters. The official track is 44 meters less than four times the short track. The total length is represented by $x + y = 511$. The length of the official track is represented by $x = 4y - 44$.

You can solve the system of equations to find the length of each track.



Common Core State Standards

Content Standards

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical Practices

- Reason abstractly and quantitatively.
- Model with mathematics.

1 Determine the Best Method You have learned five methods for solving systems of linear equations. The table summarizes the methods and the types of systems for which each method works best.

Concept Summary Solving Systems of Equations

| Method | The Best Time to Use |
|----------------------------------|---|
| Graphing | To estimate solutions, since graphing usually does not give an exact solution. |
| Substitution | If one of the variables in either equation has a coefficient of 1 or -1 . |
| Elimination Using Addition | If one of the variables has opposite coefficients in the two equations. |
| Elimination Using Subtraction | If one of the variables has the same coefficient in the two equations. |
| Elimination Using Multiplication | If none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations. |

Substitution and elimination are algebraic methods for solving systems of equations. An algebraic method is best for an exact solution. Graphing, with or without technology, is a good way to estimate a solution.

A system of equations can be solved using each method. To determine the best approach, analyze the coefficients of each term in each equation.



Example 1 Choose the Best Method

Determine the best method to solve the system of equations. Then solve the system.

$$\begin{aligned} 4x - 4y &= 8 \\ -8x + y &= 19 \end{aligned}$$

Understand To determine the best method to solve the system of equations, look closely at the coefficients of each term.

Plan Neither the coefficients of x nor y are the same or additive inverses, so you cannot add or subtract to eliminate a variable. Since the coefficient of y in the second equation is 1, you can use substitution.

Solve First, solve the second equation for y .

$$-8x + y = 19 \quad \text{Second equation}$$

$$-8x + y + 8x = 19 + 8x \quad \text{Add } 8x \text{ to each side.}$$

$$y = 19 + 8x \quad \text{Simplify.}$$

Next, substitute $19 + 8x$ for y in the first equation.

$$4x - 4y = 8 \quad \text{First equation}$$

$$4x - 4(19 + 8x) = 8 \quad \text{Substitution}$$

$$4x - 76 - 32x = 8 \quad \text{Distributive Property}$$

$$-28x - 76 = 8 \quad \text{Simplify.}$$

$$-28x - 76 + 76 = 8 + 76 \quad \text{Add } 76 \text{ to each side.}$$

$$-28x = 84 \quad \text{Simplify.}$$

$$\frac{-28x}{-28} = \frac{84}{-28} \quad \text{Divide each side by } -28.$$

$$x = -3 \quad \text{Simplify.}$$

Last, substitute -3 for x in the second equation.

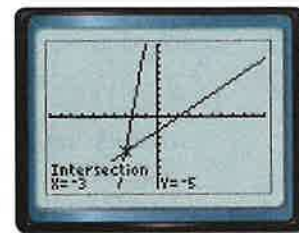
$$-8x + y = 19 \quad \text{Second equation}$$

$$-8(-3) + y = 19 \quad x = -3$$

$$y = -5 \quad \text{Simplify.}$$

The solution of the system of equations is $(-3, -5)$.

Check Use a graphing calculator to check your solution. If your algebraic solution is correct, then the graphs will intersect at $(-3, -5)$.



$[-10, 10]$ scl: 1 $[-10, 10]$ scl: 1

Guided Practice

1A. $5x + 7y = 2$
 $-2x + 7y = 9$

1C. $x - y = 9$
 $7x + y = 7$

1B. $3x - 4y = -10$
 $5x + 8y = -2$

1D. $5x - y = 17$
 $3x + 2y = 5$

StudyTip

CCSS Reasoning The system of equations in Example 1 can also be solved by using elimination with multiplication. You can multiply the first equation by 2 and then add to eliminate the x -term.





Real-WorldLink

Four species of penguins are on the endangered species list. The colonies of penguins that live closest to human inhabitants are the most at risk for extinction.

Source: PBS

2 Apply Systems of Linear Equations

When applying systems of linear equations to problems, it is important to analyze each solution in the context of the situation.



Real-World Example 2 Apply Systems of Linear Equations

PENGUINS Of the 17 species of penguins in the world, the largest species is the emperor penguin. One of the smallest is the Galapagos penguin. The total height of the two penguins is 169 centimeters. The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin. Find the height of each penguin.

The total height of the two species can be represented by $p + g = 169$, where p represents the height of the emperor penguin and g the height of the Galapagos penguin. Next write an equation to represent the height of the emperor penguin.

| | |
|------------------|--|
| Words | The emperor penguin is 22 centimeters more than twice the height of the Galapagos penguin. |
| Variables | Let p = the height of the emperor penguin and g = the height of the Galapagos penguin. |
| Equation | $p = 22 + 2g$ |

First rewrite the second equation.

$$p = 22 + 2g \quad \text{Second equation}$$

$$p - 2g = 22 \quad \text{Subtract } 2g \text{ from each side.}$$

You can use elimination by subtraction to solve this system of equations.

$$p + g = 169 \quad \text{First equation}$$

$$(-) p - 2g = 22 \quad \text{Subtract the second equation.}$$

$$3g = 147 \quad \text{Eliminate } p.$$

$$\frac{3g}{3} = \frac{147}{3} \quad \text{Divide each side by 3.}$$

$$g = 49 \quad \text{Simplify.}$$

Next substitute 49 for g in one of the equations.

$$p = 22 + 2g \quad \text{Second equation}$$

$$= 22 + 2(49) \quad g = 49$$

$$= 120 \quad \text{Simplify.}$$

The height of the emperor penguin is 120 centimeters, and the height of the Galapagos penguin is 49 centimeters.

Does the solution make sense in the context of the problem?

Check by verifying the given information. The penguins' heights added together would be $120 + 49$ or 169 centimeters and $22 + 2(49)$ is 120 centimeters.

Guided Practice

2. **VOLUNTEERING** Jared has volunteered 50 hours and plans to volunteer 3 hours in each coming week. Clementine is a new volunteer who plans to volunteer 5 hours each week. Write and solve a system of equations to find how long it will be before they will have volunteered the same number of hours.





Example 1 Determine the best method to solve each system of equations. Then solve the system.

1. $2x + 3y = -11$ 2. $3x + 4y = 11$ 3. $3x - 4y = -5$ 4. $3x + 7y = 4$
 $-8x - 5y = 9$ $2x + y = -1$ $-3x + 2y = 3$ $5x - 7y = -12$

Example 2 5. **SHOPPING** At a sale, Salazar bought 4 T-shirts and 3 pairs of jeans for \$181. At the same store, Jenna bought 1 T-shirt and 2 pairs of jeans for \$94. The T-shirts were all the same price, and the jeans were all the same price.

- Write a system of equations that can be used to represent this situation.
- Determine the best method to solve the system of equations.
- Solve the system.

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Determine the best method to solve each system of equations. Then solve the system.

6. $-3x + y = -3$ 7. $2x + 6y = -8$ 8. $3x - 4y = -5$
 $4x + 2y = 14$ $x - 3y = 8$ $-3x - 6y = -5$
9. $5x + 8y = 1$ 10. $y + 4x = 3$ 11. $-5x + 4y = 7$
 $-2x + 8y = -6$ $y = -4x - 1$ $-5x - 3y = -14$

Example 2 12. **FINANCIAL LITERACY** For a Future Teachers of America fundraiser, Denzell sold food as shown in the table. He sold 11 more subs than pizzas and earned a total of \$233. Write and solve a system of equations to represent this situation. Then describe what the solution means.

| Item | Selling Price |
|-------|---------------|
| pizza | \$5.00 |
| sub | \$3.00 |

13. **DVDs** Manuela has a total of 40 DVDs of movies and television shows. The number of movies is 4 less than 3 times the number of television shows. Write and solve a system of equations to find the numbers of movies and television shows that she has on DVD.
14. **CAVES** The Caverns of Sonora have two different tours: the Crystal Palace tour and the Horseshoe Lake tour. The total length of both tours is 3.25 miles. The Crystal Palace tour is a half-mile less than twice the distance of the Horseshoe Lake tour. Determine the length of each tour.
15. **CCSS MODELING** The *break-even point* is the point at which income equals expenses. Ridgetmont High School is paying \$13,200 for the writing and research of their yearbook plus a printing fee of \$25 per book. If they sell the books for \$40 each, how many will they have to sell to break even? Explain.
16. **PAINTBALL** Clara and her friends are planning a trip to a paintball park. Find the cost of lunch and the cost of each paintball. What would be the cost for 400 paintballs and lunch?

Paintball in the Park

- \$25 for 500 paintballs
- \$16 for 200 paintballs

Lunch is included



- 17 RECYCLING** Mara and Ling each recycled aluminum cans and newspaper, as shown in the table. Mara earned \$3.77, and Ling earned \$4.65.

| Materials | Pounds Recycled | |
|---------------|-----------------|------|
| | Mara | Ling |
| aluminum cans | 9 | 9 |
| newspaper | 26 | 114 |

- a. Define variables and write a system of linear equations from this situation.
 b. What was the price per pound of aluminum? Determine the reasonableness of your solution.
- 18. BOOKS** The library is having a book sale. Hardcover books sell for \$4 each, and paperback books are \$2 each. If Connie spends \$26 for 8 books, how many hardcover books did she buy?
- 19. MUSIC** An online music club offers individual songs for one price or entire albums for another. Kendrick pays \$14.90 to download 5 individual songs and 1 album. Geoffrey pays \$21.75 to download 3 individual songs and 2 albums.
- a. How much does the music club charge to download a song?
 b. How much does the music club charge to download an entire album?
- 20. CANOEING** Malik canoed against the current for 2 hours and then with the current for 1 hour before resting. Julio traveled against the current for 2.5 hours and then with the current for 1.5 hours before resting. If they traveled a total of 9.5 miles against the current, 20.5 miles with the current, and the current is 3 miles per hour, how fast do Malik and Julio travel in still water?

H.O.T. Problems Use Higher-Order Thinking Skills

- 21. OPEN ENDED** Formulate a system of equations that represents a situation in your school. Describe the method that you would use to solve the system. Then solve the system and explain what the solution means.
- 22. CCSS REASONING** In a system of equations, x represents the time spent riding a bike, and y represents the distance traveled. You determine the solution to be $(-1, 7)$. Use this problem to discuss the importance of analyzing solutions in the context of real-world problems.
- 23. CHALLENGE** Solve the following system of equations by using three different methods. Show your work.

$$4x + y = 13$$

$$6x - y = 7$$

- 24. WRITE A QUESTION** A classmate says that elimination is the best way to solve a system of equations. Write a question to challenge his conjecture.
- 25. WHICH ONE DOESN'T BELONG?** Which system is different? Explain.

$$\begin{aligned} x - y &= 3 \\ x + \frac{1}{2}y &= 1 \end{aligned}$$

$$\begin{aligned} -x + y &= 0 \\ 5x &= 2y \end{aligned}$$

$$\begin{aligned} y &= x - 4 \\ y &= \frac{2}{x} \end{aligned}$$

$$\begin{aligned} y &= x + 1 \\ y &= 3x \end{aligned}$$

- 26. WRITING IN MATH** How do you know what method to use when solving a system of equations?

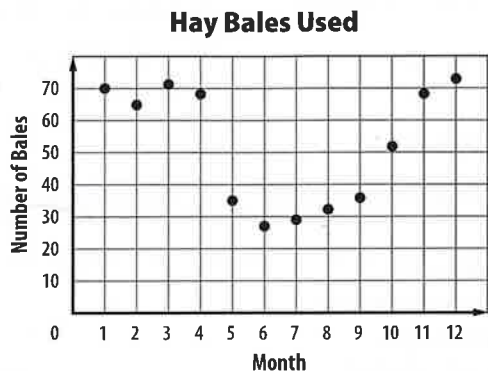


Standardized Test Practice

27. If $5x + 3y = 12$ and $4x - 5y = 17$, what is y ?

- A -1 B 3 C (-1, 3) D (3, -1)

28. **STATISTICS** The scatter plot shows the number of hay bales used on the Bostwick farm during the last year.

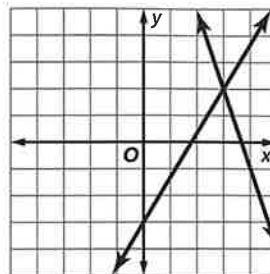


Which is an invalid conclusion?

- F The Bostwicks used less hay in the summer than they did in the winter.
 G The Bostwicks used about 629 bales of hay during the year.
 H On average, the Bostwicks used about 52 bales each month.
 J The Bostwicks used the most hay in February.

29. **SHORT RESPONSE** At noon, Cesar cast a shadow 0.15 foot long. Next to him a streetlight cast a shadow 0.25 foot long. If Cesar is 6 feet tall, how tall is the streetlight?

30. The graph shows the solution to which of the following systems of equations?



- A $y = -3x + 11$
 $3y = 5x - 9$
 B $y = 5x - 15$
 $2y = x + 7$
 C $y = -3x + 11$
 $2y = 4x - 5$
 D $y = 5x - 15$
 $3y = 2x + 18$

Spiral Review

Use elimination to solve each system of equations. (Lesson 6-4)

31. $x + y = 3$

$3x - 4y = -12$

32. $-4x + 2y = 0$

$2x - 3y = 16$

33. $4x + 2y = 10$

$5x - 3y = 7$

34. **TRAVELING** A youth group is traveling in two vans to visit an aquarium. The number of people in each van and the cost of admission for that van are shown. What are the adult and student prices? (Lesson 6-3)

| Van | Number of Adults | Number of Students | Total Cost |
|-----|------------------|--------------------|------------|
| A | 2 | 5 | \$77 |
| B | 2 | 7 | \$95 |

Graph each inequality. (Lesson 5-6)

35. $y < 4$

36. $x \geq 3$

37. $7x + 12y > 0$

38. $y - 3x \leq 4$

Skills Review

Find each sum or difference.

39. $(-3.81) + (-8.5)$

40. $12.625 + (-5.23)$

41. $21.65 + (-15.05)$

42. $(-4.27) + 1.77$

43. $(-78.94) - 14.25$

44. $(-97.623) - (-25.14)$

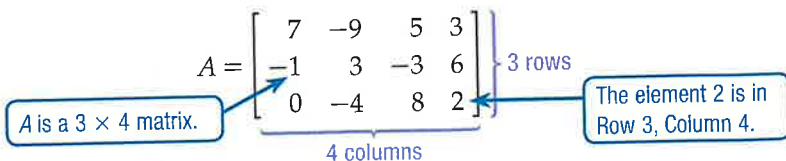




A **matrix** is a rectangular arrangement of numbers, called **elements**, in rows and columns enclosed in brackets. Usually named using an uppercase letter, a matrix can be described by its **dimensions** or by the number of rows and columns in the matrix. A matrix with m rows and n columns is an $m \times n$ matrix (read “ m by n ”).

CCSS Common Core State Standards Content Standards

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.



You can use an augmented matrix to solve a system of equations. An **augmented matrix** consists of the coefficients and the constant terms of a system of equations. Make sure that the coefficients of the x -terms are listed in one column, the coefficients of the y -terms are in another column, and the constant terms are in a third column. The coefficients and constant terms are usually separated by a dashed line.

Linear System

$$\begin{aligned} x - 3y &= 8 \\ -9x + 2y &= -4 \end{aligned}$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 1 & -3 & 8 \\ -9 & 2 & -4 \end{array} \right]$$

Activity 1 Write an Augmented Matrix

Write an augmented matrix for each system of equations.

a. $-2x + 7y = 11$
 $6x - 4y = 2$

Place the coefficients of the equations and the constant terms into a matrix.

$$\begin{aligned} -2x + 7y &= 11 \\ 6x - 4y &= 2 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{cc|c} -2 & 7 & 11 \\ 6 & -4 & 2 \end{array} \right]$$

b. $x - 2y = 5$
 $y = -4$

$$\begin{aligned} x - 2y &= 5 \\ y &= -4 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -4 \end{array} \right]$$

You can solve a system of equations by using an augmented matrix. By performing row operations, you can change the form of the matrix. The operations are the same as the ones used when working with equations.

Key Concept Elementary Row Operations

The following operations can be performed on an augmented matrix.

- Interchange any two rows.
- Multiply all entries in a row by a nonzero constant.
- Replace one row with the sum of that row and a multiple of another row.

Row operations produce a matrix equivalent to the original system. **Row reduction** is the process of performing elementary row operations on an augmented matrix to solve a system.

The goal is to get the coefficients portion of the matrix to have the form $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which is called the **identity matrix**. The first row will give you the solution for x , because the coefficient of y is 0. The second row will give you the solution for y , because the coefficient of x is 0.



Activity 2 Use Row Operations to Solve a System

Use an augmented matrix to solve the system of equations.

$$\begin{aligned} -5x + 3y &= 6 \\ x - y &= 4 \end{aligned}$$

Step 1 Write the augmented matrix: $\left[\begin{array}{cc|c} -5 & 3 & 6 \\ 1 & -1 & 4 \end{array} \right]$.

Step 2 Notice that the first element in the second row is 1. Interchange the rows so 1 can be in the upper left-hand corner.

$$\left[\begin{array}{cc|c} -5 & 3 & 6 \\ 1 & -1 & 4 \end{array} \right] \xrightarrow{\text{Interchange } R_1 \text{ and } R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ -5 & 3 & 6 \end{array} \right]$$

Step 3 To make the first element in the second row a 0, multiply the first row by 5 and add the result to row 2.

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ -5 & 3 & 6 \end{array} \right] \xrightarrow{5R_1 + R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & -2 & 26 \end{array} \right] \quad \begin{array}{l} 1(5) + (-5) = 0; -1(5) + 3 = -2; \\ 4(5) + 6 = 26 \end{array}$$

Step 4 To make the second element in the second row a 1, multiply the second row by $-\frac{1}{2}$.

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & -2 & 26 \end{array} \right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -13 \end{array} \right] \quad \begin{array}{l} 0\left(-\frac{1}{2}\right) = 0; -2\left(-\frac{1}{2}\right) = 1; \\ 26\left(-\frac{1}{2}\right) = -13 \end{array}$$

Step 5 To make the second element in the second row a 0, add the rows together.

$$\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -13 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & -9 \\ 0 & 1 & -13 \end{array} \right] \quad \begin{array}{l} 1 + 0 = 1; -1 + 1 = 0; \\ 4 + (-13) = -9 \end{array}$$

The solution is $(-9, -13)$.

Model and Analyze

Write an augmented matrix for each system of equations. Then solve the system.

1. $x + y = -3$
 $x - y = 1$

2. $x - y = -2$
 $2x + 2y = 12$

3. $3x - 4y = -27$
 $x + 2y = 11$

4. $x + 4y = -6$
 $2x - 5y = 1$

5. $x - 3y = -2$
 $4x + y = 31$

6. $x + 2y = 3$
 $-3x + 3y = 27$

Then

- You graphed and solved linear inequalities.

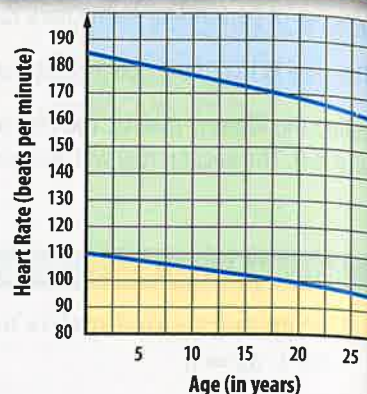
Now

- Solve systems of linear inequalities by graphing.
- Apply systems of linear inequalities.

Why?

- Jacui is beginning an exercise program that involves an intense cardiovascular workout. Her trainer recommends that for a person her age, her heart rate should stay within the following range as she exercises.
 - It should be higher than 102 beats per minute.
 - It should not exceed 174 beats per minute.

The graph shows the maximum and minimum target heart rate for people ages 0 to 30 as they exercise. If the preferred range is in light green, how old do you think Jacui is?



New Vocabulary
system of inequalities



Common Core State Standards

Content Standards

A.REI.12 Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematical Practices

- 1 Make sense of problems and persevere in solving them.
- 6 Attend to precision.

1 Systems of Inequalities The graph above is a graph of two inequalities. A set of two or more inequalities with the same variables is called a **system of inequalities**.

The solution of a system of inequalities with two variables is the set of ordered pairs that satisfy all of the inequalities in the system. The solution set is represented by the overlap, or intersection, of the graphs of the inequalities.

Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

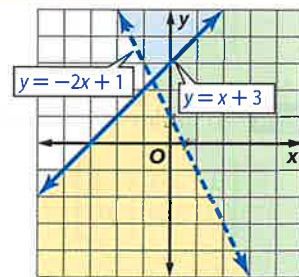
$$y > -2x + 1$$

$$y \leq x + 3$$

The graph of $y = -2x + 1$ is dashed and is not included in the graph of the solution. The graph of $y = x + 3$ is solid and is included in the graph of the solution.

The solution of the system is the set of ordered pairs in the intersection of the graphs of $y > -2x + 1$ and $y \leq x + 3$. This region is shaded in green.

When graphing more than one region, it is helpful to use two different colored pencils or two different patterns for each region. This will make it easier to see where the regions intersect and find possible solutions.



Guided Practice

- | | |
|-----------------|---------------------|
| 1A. $y \leq 3$ | 1B. $2x + y \geq 2$ |
| $x + y \geq 1$ | $2x + y < 4$ |
| 1C. $y \geq -4$ | 1D. $x + y > 2$ |
| $3x + y \leq 2$ | $-4x + 2y < 8$ |

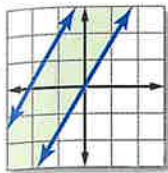
Sometimes the regions never intersect. When this happens, there is no solution because there are no points in common.



Study Tip

Parallel Boundaries

A system of equations represented by parallel lines does not have a solution. However, a system of inequalities with parallel boundaries can have a solution. For example:

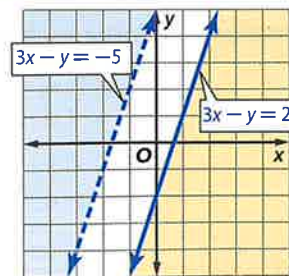


Example 2 No Solution

Solve the system of inequalities by graphing.

$$\begin{aligned} 3x - y &\geq 2 \\ 3x - y &< -5 \end{aligned}$$

The graphs of $3x - y = 2$ and $3x - y = -5$ are parallel lines. The two regions do not intersect at any point, so the system has no solution.



Guided Practice

2A. $y > 3$
 $y < 1$

2B. $x + 6y \leq 2$
 $y \geq -\frac{1}{6}x + 7$

2 Apply Systems of Inequalities When using a system of inequalities to describe constraints on the possible combinations in a real-world problem, sometimes only whole-number solutions will make sense.

Real-World Example 3 Whole-Number Solutions

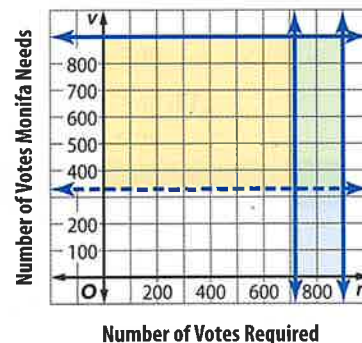
ELECTIONS Monifa is running for student council. The election rules say that for the election to be valid, at least 80% of the 900 students must vote. Monifa knows that she needs more than 330 votes to win.

a. Define the variables, and write a system of inequalities to represent this situation. Then graph the system.

Let r = the number of votes required by the election rules; 80% of 900 students is 720 students. So $r \geq 720$.

Let v = the number of votes that Monifa needs to win. So $v > 330$.

The system of inequalities is $r \geq 720$ and $v > 330$.



b. Name one viable option.

Only whole-number solutions make sense in this problem. One possible solution is $(800, 400)$; 800 students voted and Monifa received 400 votes.

Guided Practice

3. **FUNDRAISING** The Theater Club is selling shirts. They have only enough supplies to print 120 shirts. They will sell sweatshirts for \$22 and T-shirts for \$15, with a goal of at least \$2000 in sales.

A. Define the variables, and write a system of inequalities to represent this situation.

B. Then graph the system.

C. Name one possible solution.

D. Is $(45, 30)$ a solution? Explain.

Real-World Link

Student government might be a good activity for you if you like to bring about change, plan events, and work with others.





Examples 1–2 Solve each system of inequalities by graphing.

- | | |
|---------------------------------------|--|
| 1. $x \geq 4$ $y \leq x - 3$ | 2. $y > -2$ $y \leq x + 9$ |
| 3. $y < 3x + 8$ $y \geq 4x$ | 4. $3x - y \geq -1$ $2x + y \geq 5$ |
| 5. $y \leq 2x - 7$ $y \geq 2x + 7$ | 6. $y > -2x + 5$ $y \geq -2x + 10$ |
| 7. $2x + y \leq 5$ $2x + y \leq 7$ | 8. $5x - y < -2$ $5x - y > 6$ |

Example 3

9. **AUTO RACING** At a racecar driving school there are safety requirements.
- Define the variables, and write a system of inequalities to represent the height and weight requirements in this situation. Then graph the system.
 - Name one possible solution.
 - Is (50, 180) a solution? Explain.



Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–2 Solve each system of inequalities by graphing.

- | | | |
|---------------------------------------|--|--------------------------------------|
| 10. $y < 6$ $y > x + 3$ | 11. $y \geq 0$ $y \leq x - 5$ | 12. $y \leq x + 10$ $y > 6x + 2$ |
| 13. $y < 5x - 2$ $y > -6x + 2$ | 14. $2x - y \leq 6$ $x - y \geq -1$ | 15. $3x - y > -5$ $5x - y < 9$ |
| 16. $y \geq x + 10$ $y \leq x - 3$ | 17. $y < 5x - 5$ $y > 5x + 9$ | 18. $y \geq 3x - 5$ $3x - y > -4$ |
| 19. $4x + y > -1$ $y < -4x + 1$ | 20. $3x - y \geq -2$ $y < 3x + 4$ | 21. $y > 2x - 3$ $2x - y \geq 1$ |
| 22. $5x - y < -6$ $3x - y \geq 4$ | 23. $x - y \leq 8$ $y < 3x$ | 24. $4x + y < -2$ $y > -4x$ |

Example 3

25. **ICE RINKS** Ice resurfacers are used for rinks of at least 1000 square feet and up to 17,000 square feet. The price ranges from as little as \$10,000 to as much as \$150,000.
- Define the variables, and write a system of inequalities to represent this situation. Then graph the system.
 - Name one possible solution.
 - Is (15,000, 30,000) a solution? Explain.
26. **CCSS MODELING** Josefina works between 10 and 30 hours per week at a pizzeria. She earns \$6.50 an hour, but can earn tips when she delivers pizzas.
- Write a system of inequalities to represent the dollars d she could earn for working h hours in a week.
 - Graph this system.
 - If Josefina received \$17.50 in tips and earned a total of \$180 for the week, how many hours did she work?



Solve each system of inequalities by graphing.

27. $x + y \geq 1$
 $x + y \leq 2$

28. $3x - y < -2$
 $3x - y < 1$

29. $2x - y \leq -11$
 $3x - y \geq 12$

30. $y < 4x + 13$
 $4x - y \geq 1$

31. $4x - y < -3$
 $y \geq 4x - 6$

32. $y \leq 2x + 7$
 $y < 2x - 3$

33. $y > -12x + 1$
 $y \leq 9x + 2$

34. $2y \geq x$
 $x - 3y > -6$

35. $x - 5y > -15$
 $5y \geq x - 5$

36. **CLASS PROJECT** An economics class formed a company to sell school supplies. They would like to sell at least 20 notebooks and 50 pens per week, with a goal of earning at least \$60 per week.

- Define the variables, and write a system of inequalities to represent this situation.
- Graph the system.
- Name one possible solution.



37. **FINANCIAL LITERACY** Opal makes \$15 per hour working for a photographer. She also coaches a competitive soccer team for \$10 per hour. Opal needs to earn at least \$90 per week, but she does not want to work more than 20 hours per week.

- Define the variables, and write a system of inequalities to represent this situation.
- Graph this system.
- Give two possible solutions to describe how Opal can meet her goals.
- Is (2, 2) a solution? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

38. **CHALLENGE** Create a system of inequalities equivalent to $|x| \leq 4$.

39. **REASONING** State whether the following statement is *sometimes*, *always*, or *never* true. Explain your answer with an example or counterexample.

Systems of inequalities with parallel boundaries have no solutions.

40. **REASONING** Describe the graph of the solution of this system without graphing.

$6x - 3y \leq -5$

$6x - 3y \geq -5$

41. **OPEN ENDED** One inequality in a system is $3x - y > 4$. Write a second inequality so that the system will have no solution.

42. **CCSS PRECISION** Graph the system of inequalities. Estimate the area of the solution.

$y \geq 1$

$y \leq x + 4$

$y \leq -x + 4$

43. **WRITING IN MATH** Refer to the beginning of the lesson. Explain what each colored region of the graph represents. Explain how shading in various colors can help to clearly show the solution set of a system of inequalities.

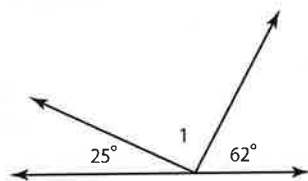


Standardized Test Practice

44. EXTENDED RESPONSE To apply for a scholarship, you must have a minimum of 20 hours of community service and a grade-point average of at least 3.75. Another scholarship requires at least 40 hours of community service and a minimum grade-point average of 3.0.

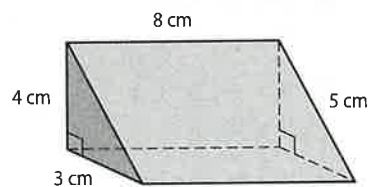
- Write a system of inequalities to represent the credentials you must have to apply for both scholarships.
- Graph the system of inequalities.
- If you are eligible for both scholarships, give one possible solution.

45. GEOMETRY What is the measure of $\angle 1$?



- | | |
|--------------|--------------|
| A 83° | C 90° |
| B 87° | D 93° |

46. GEOMETRY What is the volume of the triangular prism?



- | | |
|----------------------|---------------------|
| F 120 cm^3 | H 48 cm^3 |
| G 96 cm^3 | J 30 cm^3 |

47. Ten pounds of fresh tomatoes make about 15 cups of cooked tomatoes. How many cups of cooked tomatoes does one pound of fresh tomatoes make?

- $1\frac{1}{2}$ cups
- 3 cups
- 4 cups
- 5 cups

Spiral Review

48. CHEMISTRY Orion Labs needs to make 500 gallons of 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. Write and solve a system of equations to find the number of gallons of each solution that should be mixed to make the 34% solution. (Lesson 6-5)

Use elimination to solve each system of equations. (Lesson 6-4)

49. $x + y = 7$
 $2x + y = 11$

50. $a - b = 9$
 $7a + b = 7$

51. $q + 4r = -8$
 $3q + 2r = 6$

52. ENTERTAINMENT A group of 11 adults and children bought tickets for the baseball game. If the total cost was \$156, how many of each type of ticket did they buy? (Lesson 6-4)



Graph each inequality. (Lesson 5-6)

53. $4x - 2 \geq 2y$

54. $9x - 3y < 0$

55. $2y \leq -4x - 6$

Skills Review

Evaluate each expression.

56. 3^3

57. 2^4

58. $(-4)^3$





You can use TI-Nspire technology to explore systems of inequalities. To prepare your calculator, add a new **Graphs** page from the Home screen.

**CCSS Common Core State Standards
Content Standards**

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.



Activity Graph Systems of Inequalities

Mr. Jackson owns a car washing and detailing business. It takes 20 minutes to wash a car and 60 minutes to detail a car. He works at most 8 hours per day and does at most 4 details per day. Write a system of linear inequalities to represent this situation.

First, write a linear inequality that represents the time it takes for car washing and car detailing. Let x represent the number of car washes, and let y represent the number of car details. Then $20x + 60y \leq 480$.

To graph this using a graphing calculator, solve for y .

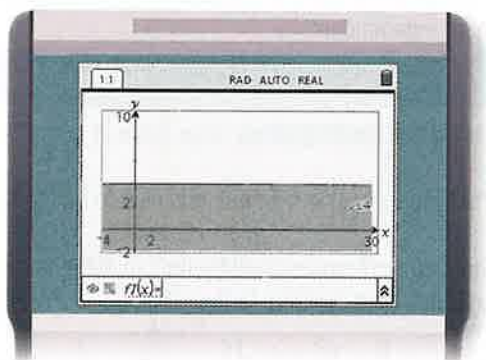
$$20x + 60y \leq 480 \quad \text{Original inequality}$$

$$60y \leq -20x + 480 \quad \text{Subtract } 20x \text{ from each side and simplify.}$$

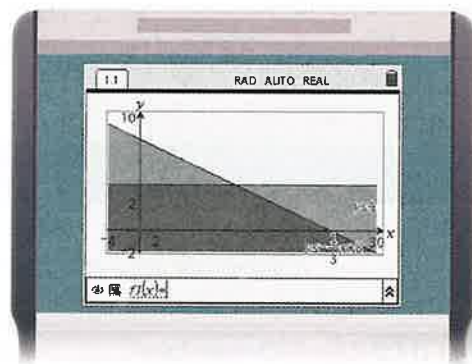
$$y \leq -\frac{1}{3}x + 8 \quad \text{Divide each side by 60 and simplify.}$$

Mr. Jackson does at most 4 details per day. This means that $y \leq 4$.

Step 1 Adjust the viewing window and then graph $y \leq 4$. Use the **Window Settings** option from the **Window/Zoom** menu to adjust the window to -4 to 30 for x and -2 to 10 for y . Keep the scales as **Auto**. Then enter **del ≤ 4 enter**.



Step 2 Graph $y \leq -\frac{1}{3}x + 8$. Press **tab del ≤** and then enter $-\frac{1}{3}x + 8$.



The darkest shaded region of the graph represents the solutions.

Analyze the Results

- If Mr. Jackson charges \$75 for each car he details and \$25 for each car wash, what is the maximum amount of money he could earn in one day?
- What is the greatest number of car washes that Mr. Jackson could do in a day? Explain your reasoning.

Study Guide

Key Concepts

Systems of Equations (Lessons 6-1 through 6-5)

- A system with a graph of two intersecting lines has one solution and is *consistent and independent*.
- Graphing a system of equations can only provide approximate solutions. For exact solutions, you must use algebraic methods.
- In the substitution method, one equation is solved for a variable and the expression substituted into the second equation to find the value of another variable.
- In the elimination method, one variable is eliminated by adding or subtracting the equations.
- Sometimes multiplying one or both equations by a constant makes it easier to use the elimination method.
- The best method for solving a system of equations depends on the coefficients of the variables.

Systems of Inequalities (Lesson 6-6)

- A system of inequalities is a set of two or more inequalities with the same variables.
- The solution of a system of inequalities is the intersection of the graphs.

FOLDABLES Study Organizer

Be sure the Key Concepts are noted in your Foldable.

| | |
|---|--|
| ○ | 6-1 Graphing Systems of Equations |
| | 6-2 Substitution |
| | 6-3 Elimination Using Addition and Subtraction |
| ○ | 6-4 Elimination Using Multiplication |
| | 6-5 Applying Systems of Linear Equations |
| ○ | 6-6 Systems of Inequalities |

Key Vocabulary



- | | |
|---------------------------|---------------------------------|
| augmented matrix (p. 370) | inconsistent (p. 335) |
| consistent (p. 335) | independent (p. 335) |
| dependent (p. 335) | matrix (p. 370) |
| dimension (p. 370) | substitution (p. 344) |
| element (p. 370) | system of equations (p. 335) |
| elimination (p. 350) | system of inequalities (p. 372) |

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. If a system has at least one solution, it is said to be consistent.
2. If a consistent system has exactly two solution(s), it is said to be independent.
3. If a consistent system has an infinite number of solutions, it is said to be inconsistent.
4. If a system has no solution, it is said to be inconsistent.
5. Substitution involves substituting an expression from one equation for a variable in the other.
6. In some cases, dividing two equations in a system together will eliminate one of the variables. This process is called elimination.
7. A set of two or more inequalities with the same variables is called a system of equations.
8. When the graphs of the inequalities in a system of inequalities do not intersect, there are no solutions to the system.



Lesson-by-Lesson Review

6-1 Graphing Systems of Equations

Graph each system and determine the number of solutions that it has. If it has one solution, name it.

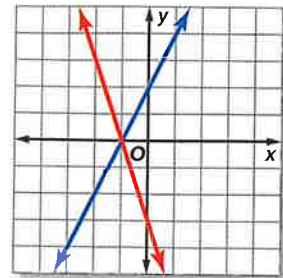
9. $x - y = 1$
 $x + y = 5$
10. $y = 2x - 4$
 $4x + y = 2$
11. $2x - 3y = -6$
 $y = -3x + 2$
12. $-3x + y = -3$
 $y = x - 3$
13. $x + 2y = 6$
 $3x + 6y = 8$
14. $3x + y = 5$
 $6x = 10 - 2y$
15. **MAGIC NUMBERS** Sean is trying to find two numbers with a sum of 14 and a difference of 4. Define two variables, write a system of equations, and solve by graphing.

Example 1

Graph the system and determine the number of solutions it has. If it has one solution, name it.

$$y = 2x + 2$$

$$y = -3x - 3$$



The lines appear to intersect at the point $(-1, 0)$. You can check this by substituting -1 for x and 0 for y .

| | | |
|--------------|--------------------------------|-------------------|
| CHECK | $y = 2x + 2$ | Original equation |
| | $0 \stackrel{?}{=} 2(-1) + 2$ | Substitution |
| | $0 \stackrel{?}{=} -2 + 2$ | Multiply. |
| | $0 = 0$ ✓ | |
| | $y = -3x - 3$ | Original equation |
| | $0 \stackrel{?}{=} -3(-1) - 3$ | Substitution |
| | $0 \stackrel{?}{=} 3 - 3$ | Multiply. |
| | $0 = 0$ ✓ | |

The solution is $(-1, 0)$.

6-2 Substitution

Use substitution to solve each system of equations.

16. $x + y = 3$
 $x = 2y$
17. $x + 3y = -28$
 $y = -5x$
18. $3x + 2y = 16$
 $x = 3y - 2$
19. $x - y = 8$
 $y = -3x$
20. $y = 5x - 3$
 $x + 2y = 27$
21. $x + 3y = 9$
 $x + y = 1$
22. **GEOMETRY** The perimeter of a rectangle is 48 inches. The length is 6 inches greater than the width. Define the variables, and write equations to represent this situation. Solve the system by using substitution.

Example 2

Use substitution to solve the system.

$$3x - y = 18$$

$$y = x - 4$$

| | |
|---------------------|------------------------------|
| $3x - y = 18$ | First equation |
| $3x - (x - 4) = 18$ | Substitute $x - 4$ for y . |
| $2x + 4 = 18$ | Simplify. |
| $2x = 14$ | Subtract 4 from each side. |
| $x = 7$ | Divide each side by 2. |

Use the value of x and either equation to find the value for y .

| | |
|------------------|--------------------------|
| $y = x - 4$ | Second equation |
| $= 7 - 4$ or 3 | Substitute and simplify. |

The solution is $(7, 3)$.

6-3 Elimination Using Addition and Subtraction

Use elimination to solve each system of equations.

23. $x + y = 13$
 $x - y = 5$

25. $x + 4y = -4$
 $x + 10y = -16$

27. $6x + y = 9$
 $-6x + 3y = 15$

29. $2x + 2y = 4$
 $2x - 8y = -46$

24. $-3x + 4y = 21$
 $3x + 3y = 14$

26. $2x + y = -5$
 $x - y = 2$

28. $x - 4y = 2$
 $3x + 4y = 38$

30. $3x + 2y = 8$
 $x + 2y = 2$

31. **BASEBALL CARDS** Cristiano bought 24 baseball cards for \$50. One type cost \$1 per card, and the other cost \$3 per card. Define the variables, and write equations to find the number of each type of card he bought. Solve by using elimination.

Example 3

Use elimination to solve the system of equations.

$$\begin{array}{r} 3x - 5y = 11 \\ x + 5y = -3 \\ \hline 3x - 5y = 11 \\ (+) \quad x + 5y = -3 \\ \hline 4x \qquad = 8 \\ x = 2 \end{array}$$

The variable y is eliminated.
Divide each side by 4.

Now, substitute 2 for x in either equation to find the value of y .

$$\begin{array}{r} 3x - 5y = 11 \\ 3(2) - 5y = 11 \\ 6 - 5y = 11 \\ -5y = 5 \\ y = -1 \end{array}$$

First equation
Substitute.
Multiply.
Subtract 6 from each side.
Divide each side by -5 .

The solution is $(2, -1)$.

6-4 Elimination Using Multiplication

Use elimination to solve each system of equations.

32. $x + y = 4$
 $-2x + 3y = 7$

34. $3x + 4y = 1$
 $5x + 2y = 11$

36. $8x - 3y = -35$
 $3x + 4y = 33$

38. $-7x + 3y = 12$
 $2x - 8y = -32$

33. $x - y = -2$
 $2x + 4y = 38$

35. $-9x + 3y = -3$
 $3x - 2y = -4$

37. $2x + 9y = 3$
 $5x + 4y = 26$

39. $8x - 5y = 18$
 $6x + 6y = -6$

40. **BAKE SALE** On the first day, a total of 40 items were sold for \$356. Define the variables, and write a system of equations to find the number of cakes and pies sold. Solve by using elimination.



Example 4

Use elimination to solve the system of equations.

$$\begin{array}{r} 3x + 6y = 6 \\ 2x + 3y = 5 \end{array}$$

Notice that if you multiply the second equation by -2 , the coefficients of the y -terms are additive inverses.

$$\begin{array}{r} 3x + 6y = 6 \\ 2x + 3y = 5 \quad \xrightarrow{\text{Multiply by } -2} \quad (+) \quad -4x - 6y = -10 \\ \hline -x \qquad = -4 \\ x = 4 \end{array}$$

Now, substitute 4 for x in either equation to find the value of y .

$$\begin{array}{r} 2x + 3y = 5 \\ 2(4) + 3y = 5 \\ 8 + 3y = 5 \\ 3y = -3 \\ y = -1 \end{array}$$

Second equation
Substitution
Multiply.
Subtract 8 from both sides.
Divide each side by 3.

The solution is $(4, -1)$.

6-5 Applying Systems of Linear Equations

Determine the best method to solve each system of equations. Then solve the system.

41. $y = x - 8$
 $y = -3x$

42. $y = -x$
 $y = 2x$

43. $x + 3y = 12$
 $x = -6y$

44. $x + y = 10$
 $x - y = 18$

45. $3x + 2y = -4$
 $5x + 2y = -8$

46. $6x + 5y = 9$
 $-2x + 4y = 14$

47. $3x + 4y = 26$
 $2x + 3y = 19$

48. $11x - 6y = 3$
 $5x - 8y = -25$

49. **COINS** Tionna has saved dimes and quarters in her piggy bank. Define the variables, and write a system of equations to determine the number of dimes and quarters. Then solve the system using the best method for the situation.



50. **FAIR** At a county fair, the cost for 4 slices of pizza and 2 orders of French fries is \$21.00. The cost of 2 slices of pizza and 3 orders of French fries is \$16.50. To find out how much a single slice of pizza and an order of French fries costs, define the variables and write a system of equations to represent the situation. Determine the best method to solve the system of equations. Then solve the system. (Lesson 6-5)

Example 5

Determine the best method to solve the system of equations. Then solve the system.

$$\begin{aligned} 3x + 5y &= 4 \\ 4x + y &= -6 \end{aligned}$$

The coefficient of y is 1 in the second equation. So solving by substitution is a good method. Solve the second equation for y .

$$\begin{aligned} 4x + y &= -6 && \text{Second equation} \\ y &= -6 - 4x && \text{Subtract } 4x \text{ from each side.} \end{aligned}$$

Substitute $-6 - 4x$ for y in the first equation.

$$\begin{aligned} 3x + 5(-6 - 4x) &= 4 && \text{Substitute.} \\ 3x - 30 - 20x &= 4 && \text{Distributive Property} \\ -17x - 30 &= 4 && \text{Simplify.} \\ -17x &= 34 && \text{Add 30 to each side.} \\ x &= -2 && \text{Divide by } -17. \end{aligned}$$

Last, substitute -2 for x in either equation to find y .

$$\begin{aligned} 4x + y &= -6 && \text{Second equation} \\ 4(-2) + y &= -6 && \text{Substitute.} \\ -8 + y &= -6 && \text{Multiply.} \\ y &= 2 && \text{Add 8 to each side.} \end{aligned}$$

The solution is $(-2, 2)$.

6-6 Systems of Inequalities

Solve each system of inequalities by graphing.

51. $x > 3$
 $y < x + 2$

52. $y \leq 5$
 $y > x - 4$

53. $y < 3x - 1$
 $y \geq -2x + 4$

54. $y \leq -x - 3$
 $y \geq 3x - 2$

55. **JOBS** Kishi makes \$7 an hour working at the grocery store and \$10 an hour delivering newspapers. She cannot work more than 20 hours per week. Graph two inequalities that Kishi can use to determine how many hours she needs to work at each job if she wants to earn at least \$90 per week.

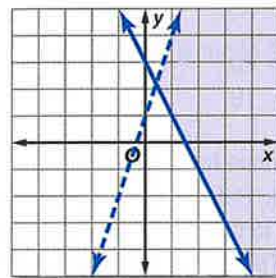
Example 6

Solve the system of inequalities by graphing.

$$y < 3x + 1$$

$$y \geq -2x + 3$$

The solution set of the system is the set of ordered pairs in the intersection of the two graphs. This portion is shaded in the graph below.



Practice Test

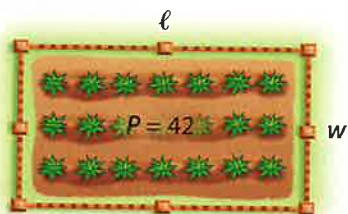
Graph each system and determine the number of solutions that it has. If it has one solution, name it.

- $y = 2x$
 $y = 6 - x$
- $y = x - 3$
 $y = -2x + 9$
- $x - y = 4$
 $x + y = 10$
- $2x + 3y = 4$
 $2x + 3y = -1$

Use substitution to solve each system of equations.

- $y = x + 8$
 $2x + y = -10$
- $x = -4y - 3$
 $3x - 2y = 5$

- GARDENING** Corey has 42 feet of fencing around his garden. The garden is rectangular in shape, and its length is equal to twice the width minus 3 feet. Define the variables, and write a system of equations to find the length and width of the garden. Solve the system by using substitution.



- MULTIPLE CHOICE** Use elimination to solve the system.

$$\begin{aligned} 6x - 4y &= 6 \\ -6x + 3y &= 0 \end{aligned}$$

- (5, 6)
 - (-3, -6)
 - (1, 0)
 - (4, -8)
- SHOPPING** Shelly has \$175 to shop for jeans and sweaters. Each pair of jeans costs \$25, each sweater costs \$20, and she buys 8 items. Determine the number of pairs of jeans and sweaters Shelly bought.

Use elimination to solve each system of equations.

- $x + y = 13$
 $x - y = 5$
- $3x + 7y = 2$
 $3x - 4y = 13$
- $x + y = 8$
 $x - 3y = -4$
- $2x + 6y = 18$
 $3x + 2y = 13$

- MAGAZINES** Julie subscribes to a sports magazine and a fashion magazine. She received 24 issues this year. The number of fashion issues is 6 less than twice the number of sports issues. Define the variables, and write a system of equations to find the number of issues of each magazine.

Determine the best method to solve each system of equations. Then solve the system.

- $y = 3x$
 $x + 2y = 21$
- $x + y = 12$
 $y = x - 4$
- $x + y = 15$
 $x - y = 9$
- $3x + 5y = 7$
 $2x - 3y = 11$

- OFFICE SUPPLIES** At a sale, Ricardo bought 24 reams of paper and 4 inkjet cartridges for \$320. Britney bought 2 reams of paper and 1 inkjet cartridge for \$50. The reams of paper were all the same price and the inkjet cartridges were all the same price. Write a system of equations to represent this situation. Determine the best method to solve the system of equations. Then solve the system.

Solve each system of inequalities by graphing.

- $x > 2$
 $y < 4$
- $x + y \leq 5$
 $y \geq x + 2$
- $3x - y > 9$
 $y > -2x$
- $y \geq 2x + 3$
 $-4x - 3y > 12$



Guess and Check

It is very important to pace yourself and keep track of how much time you have when taking a standardized test. If time is running short, or if you are unsure how to solve a problem, the guess and check strategy may help you determine the correct answer quickly.



Strategies for Guessing and Checking

Step 1

Carefully look over each possible answer choice, and evaluate for reasonableness. Eliminate unreasonable answers.

Ask yourself:

- Are there any answer choices that are clearly incorrect?
- Are there any answer choices that are not in the proper format?
- Are there any answer choices that do not have the proper units for the correct answer?

Step 2

For the remaining answer choices, use the guess and check method.

- **Equations:** If you are solving an equation, substitute the answer choice for the variable and see if this results in a true number sentence.
- **Inequalities:** Likewise, you can substitute the answer choice for the variable and see if it satisfies the inequality.
- **System of Equations:** Find the answer choice that satisfies both equations of the system.

Step 3

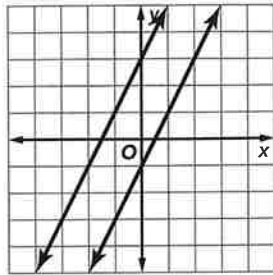
Choose an answer choice and see if it satisfies the constraints of the problem statement. Identify the correct answer.

- If the answer choice you are testing does not satisfy the problem, move on to the next reasonable guess and check it.
- When you find the correct answer choice, stop. You do not have to check the other answer choices.

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following terms *best* describes the system of equations shown in the graph?



- A consistent
 B consistent and dependent
 C consistent and independent
 D inconsistent
2. Use substitution to solve the system of equations below.

$$\begin{cases} y = 4x - 7 \\ 3x - 2y = -1 \end{cases}$$

- F (3, 5) H (5, -2)
 G (4, -1) J (-6, 2)
3. Which ordered pair is the solution of the system of linear equations shown below?

$$\begin{cases} 3x - 8y = -50 \\ 3x - 5y = -38 \end{cases}$$

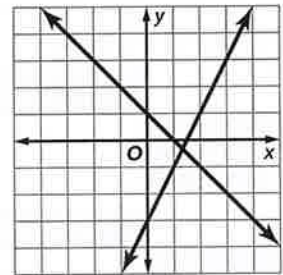
- A $(\frac{5}{8}, \frac{3}{2})$ C $(-\frac{2}{7}, \frac{4}{9})$
 B (4, -9) D (-6, 4)
4. A home goods store received \$881 from the sale of 4 table saws and 9 electric drills. If the receipts from the saws exceeded the receipts from the drills by \$71, what is the price of an electric drill?
- F \$45 H \$108
 G \$59 J \$119

5. A region is defined by this system.

$$\begin{cases} y > -\frac{1}{2}x - 1 \\ y > -x + 3 \end{cases}$$

In which quadrant(s) of the coordinate plane is the region located?

- A I and IV only C I, II, and IV only
 B III only D II and III only
6. Which of the following terms *best* describes the system of equations shown in the graph?



- F consistent
 G consistent and independent
 H consistent and dependent
 J inconsistent
7. Use elimination to solve the system of equations below.

$$\begin{cases} 3x + 2y = -2 \\ 2x - 2y = -18 \end{cases}$$

- A (1, 3) C (-2, -3)
 B (7, -4) D (-4, 5)
8. What is the solution of the following system of equations?

$$\begin{cases} y = 6x - 1 \\ y = 6x + 1 \end{cases}$$

- F (2, 11) H (7, 5)
 G (-3, -14) J no solution

Test-Taking Tip

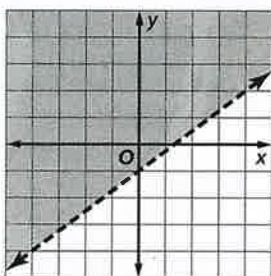
Question 8 You can subtract the second equation from the first equation to eliminate the x -variable. Then solve for y .

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. **GRIDDED RESPONSE** Angie and her sister have \$15 to spend on pizza. A medium pizza costs \$11.50 plus \$0.75 per topping. What is the maximum number of toppings Angie and her sister can get on their pizza?

10. Write an inequality for the graph below.



11. **GRIDDED RESPONSE** Christy is taking a road trip. After she drives 12 more miles, she will have driven at least half of the 108-mile trip. What is the least number of miles she has driven so far?

12. Write an equation in slope-intercept form with a slope of $-\frac{2}{3}$ and a y -intercept of 6.

13. A rental company charges \$9.50 per hour for a scooter plus a \$15 fee. Write an equation in slope-intercept form for the total rental cost C of renting a scooter for h hours.

14. **GRIDDED RESPONSE** A computer supplies store is having a storewide sale this weekend. An inkjet printer that normally sells for \$179.00 is on sale for \$143.20. What is the percent discount of the sale price?

15. In 1980, the population of Kentucky was about 3.66 million people. By 2000, this number had grown to about 4.04 million people. What was the annual rate of change in population from 1980 to 2000?

16. Joseph's cell phone service charges him \$0.15 per text. Write an equation that represents the cost C of his cell phone service for t texts sent each month.

17. A store is offering a \$15 mail-in-rebate on all printers. If Mark is looking at printers that range from \$45 to \$89, how much can he expect to pay?

Extended Response

Record your answers on a sheet of paper. Show your work.

18. The table shows how many canned goods were collected during the first day of a charity food drive.

| Food Drive Day 1 Results | |
|--------------------------|------------------|
| Class | Number Collected |
| 10 th graders | 78 |
| 11 th graders | 80 |
| 12 th graders | 92 |

- Estimate how many canned goods will be collected during the 5-day food drive. Explain your answer.
- Is this estimate a reasonable expectation? Explain.

Need ExtraHelp?

| | | | | | | | | | | | | | | | | | | |
|---------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| If you missed Question... | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Go to Lesson... | 6-1 | 6-2 | 6-3 | 6-3 | 6-6 | 6-1 | 6-3 | 6-3 | 5-3 | 5-6 | 5-3 | 4-2 | 4-2 | 2-7 | 2-7 | 3-4 | 5-4 | 1-4 |

