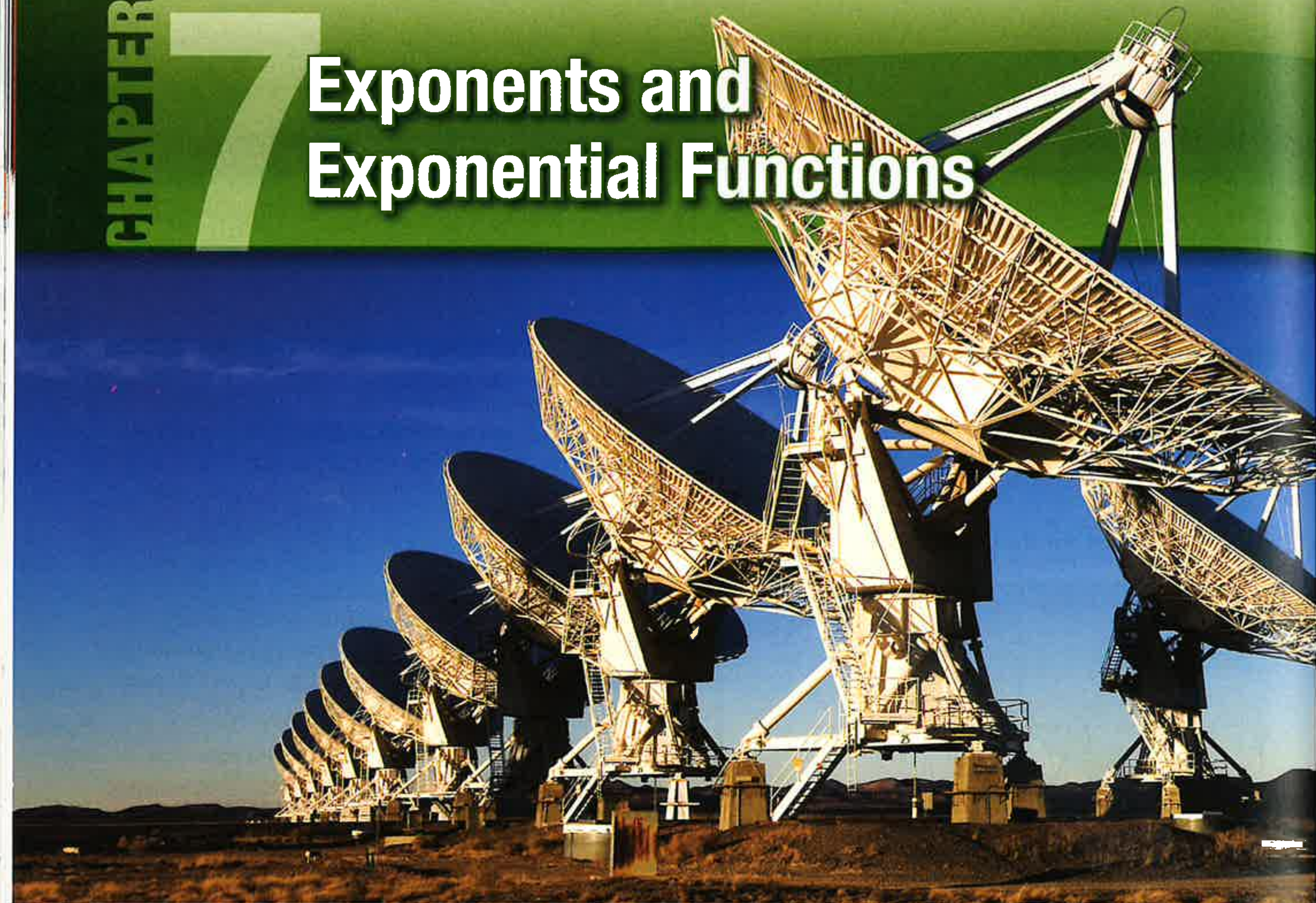
CHAPTER 7

Exponents and Exponential Functions



Then

You evaluated expressions involving exponents.

Now

In this chapter, you will:

- Simplify and perform operations on expressions involving exponents.
- Extend the properties of integer exponents to rational exponents.
- Use scientific notation.
- Graph and use exponential functions.

Why?

SPACE The Very Large Array is an arrangement of 27 radio antennas in a Y pattern. The data the antennas collect is used by astronomers around the world to study the planets and stars. Astrophysicists use and apply properties of exponents to model the distance and orbit of celestial bodies.

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Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

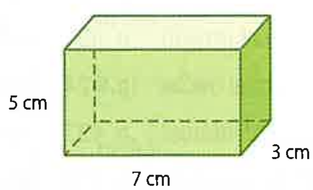
Write each expression using exponents.

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
2. $y \cdot y \cdot y$
3. $6 \cdot 6$
4. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
5. $b \cdot b \cdot b \cdot b \cdot b \cdot b$
6. $m \cdot m \cdot m \cdot p \cdot p \cdot p \cdot p \cdot p \cdot p$
7. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$
8. $\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{w}{z} \cdot \frac{w}{z}$

Find the area or volume of each figure.

9.

10.



11. **PHOTOGRAPHY** A photo is 4 inches by 6 inches. What is the area of the photo?

Evaluate each expression.

12. 2^3
13. $(-5)^2$
14. 3^3
15. $(-4)^3$
16. $\left(\frac{2}{3}\right)^2$
17. $\left(\frac{1}{2}\right)^4$
18. **SCHOOL** The probability of guessing correctly on 5 true-false questions is $\left(\frac{1}{2}\right)^5$. Express this probability as a fraction without exponents.

QuickReview

Example 1

Write $5 \cdot 5 \cdot 5 \cdot 5 + x \cdot x \cdot x$ using exponents.

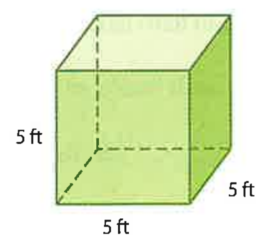
4 factors of 5 is 5^4.

3 factors of x is x^3.

So, $5 \cdot 5 \cdot 5 \cdot 5 + x \cdot x \cdot x = 5^4 + x^3$.

Example 2

Find the volume of the figure.



$V = \ell wh$ Volume of a rectangular prism

$= 5 \cdot 5 \cdot 5$ or 125 $\quad \ell = 5$, $w = 5$, and $h = 5$

The volume is 125 cubic feet.

Example 3

Evaluate $\left(\frac{5}{7}\right)^2$.

$\left(\frac{5}{7}\right)^2 = \frac{5^2}{7^2}$ Power of a Quotient

$= \frac{25}{49}$ Simplify.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

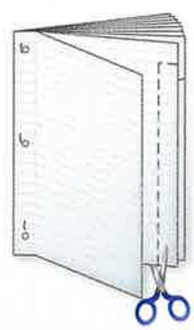
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 7. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES **Study**Organizer

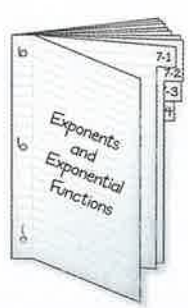
Exponents and Exponential Functions Make this Foldable to help you organize your Chapter 7 notes about exponents and exponential functions. Begin with nine sheets of notebook paper.

1 **Arrange** the paper into a stack.

2 **Staple** along the left side. Starting with the second sheet of paper, cut along the right side to form tabs.



3 **Label** the cover sheet "Exponents and Exponential Functions" and label each tab with a lesson number.



NewVocabulary

English		Español
monomial	p. 391	monomio
constant	p. 391	constante
zero exponent	p. 399	cero exponente
negative exponent	p. 400	exponente negativo
order of magnitude	p. 401	orden de magnitud
rational exponent	p. 406	exponent racional
cube root	p. 407	raíz cúbica
*n*th root	p. 407	raíz enésima
exponential equation	p. 409	ecuación exponencial
scientific notation	p. 414	notación científica
exponential function	p. 424	función exponencial
exponential growth	p. 424	crecimiento exponencial
exponential decay	p. 424	desintegración exponencial
compound interest	p. 433	interés es compuesta
geometric sequence	p. 438	secuencia geométrica
common ratio	p. 438	proporción común
recursive formula	p. 445	fórmula recursiva

ReviewVocabulary

base base In an expression of the form x^n, the base is x.

Distributive Property Propiedad distributiva For any numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

exponent exponente In an expression of the form x^n, the exponent is n. It indicates the number of times x is used as a factor.

exponent

$$x^n = \underbrace{x \cdot x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ times}}$$

base

LESSON 7-1

Multiplication Properties of Exponents

Then

- You evaluated expressions with exponents.

Now

1. Multiply monomials using the properties of exponents.
2. Simplify expressions using the multiplication properties of exponents.

Why?

- Many formulas contain *monomials*. For example, the formula for the horsepower of a car is $H = w\left(\frac{v}{234}\right)^3$. *H* represents the horsepower produced by the engine, *w* equals the weight of the car with passengers, and *v* is the velocity of the car at the end of a quarter of a mile. As the velocity increases, the horsepower increases.

Andreas Stirnberg/Photographer's Choice/Getty Images

NewVocabulary

monomial
constant

Common Core State Standards

Content Standards

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Multiply Monomials

A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. It has only one term. In the formula to calculate the horsepower of a car, the term $w\left(\frac{v}{234}\right)^3$ is a monomial.

An expression that involves division by a variable, like $\frac{ab}{c}$, is not a monomial.

A **constant** is a monomial that is a real number. The monomial $3x$ is an example of a *linear expression* since the exponent of x is 1. The monomial $2x^2$ is a *nonlinear expression* since the exponent is a positive number other than 1.

PT

Example 1 Identify Monomials

Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.

a. 10 — Yes; this is a constant, so it is a monomial.

b. $f + 24$ — No; this expression has addition, so it has more than one term.

c. h^2 — Yes; this expression is a product of variables.

d. j — Yes; single variables are monomials.

GuidedPractice

1A. $-x + 5$

1B. $23abcd^2$

1C. $\frac{xyz^2}{2}$

1D. $\frac{mp}{n}$

Recall that an expression of the form x^n is called a *power* and represents the result of multiplying x by itself n times. x is the *base*, and n is the *exponent*. The word *power* is also used sometimes to refer to the exponent.

exponent

$$3^4 = \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{\text{4 factors}} = 81$$

base

By applying the definition of a power, you can find the product of powers. Look for a pattern in the exponents.

$$2^2 \cdot 2^4 = \underbrace{\overbrace{2 \cdot 2}^{2\text{ factors}} \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2}^{4\text{ factors}}}_{2+4=6\text{ factors}}$$

$$4^3 \cdot 4^2 = \underbrace{\overbrace{4 \cdot 4 \cdot 4}^{3\text{ factors}} \cdot \overbrace{4 \cdot 4}^{2\text{ factors}}}_{3+2=5\text{ factors}}$$

These examples demonstrate the property for the product of powers.

KeyConcept Product of Powers

Words To multiply two powers that have the same base, add their exponents.

Symbols For any real number a and any integers m and p, $a^m \cdot a^p = a^{m+p}$.

Examples $b^3 \cdot b^5 = b^{3+5}$ or b^8 $\quad g^4 \cdot g^6 = g^{4+6}$ or g^{10}

PT

Example 2 Product of Powers

Simplify each expression.

a. $(6n^3)(2n^7)$

$(6n^3)(2n^7) = (6 \cdot 2)(n^3 \cdot n^7)$ Group the coefficients and the variables.

$= (6 \cdot 2)(n^{3+7})$ Product of Powers

$= 12n^{10}$ Simplify.

b. $(3pt^3)(p^3t^4)$

$(3pt^3)(p^3t^4) = (3 \cdot 1)(p \cdot p^3)(t^3 \cdot t^4)$ Group the coefficients and the variables.

$= (3 \cdot 1)(p^{1+3})(t^{3+4})$ Product of Powers

$= 3p^4t^7$ Simplify.

StudyTip

Coefficients and Powers of 1 A variable with no exponent or coefficient shown can be assumed to have an exponent and coefficient of 1. For example, $x = 1x^1$.

GuidedPractice

2A. $(3y^4)(7y^5)$

2B. $(-4rx^2t^3)(-6r^5x^2t)$

We can use the Product of Powers Property to find the power of a power. In the following examples, look for a pattern in the exponents.

$$(3^2)^4 = \overbrace{(3^2)(3^2)(3^2)(3^2)}^{4\text{ factors}}$$
$$= 3^{2+2+2+2}$$
$$= 3^8$$

$$(r^4)^3 = \overbrace{(r^4)(r^4)(r^4)}^{3\text{ factors}}$$
$$= r^{4+4+4}$$
$$= r^{12}$$

These examples demonstrate the property for the power of a power.

KeyConcept Power of a Power

Words To find the power of a power, multiply the exponents.

Symbols For any real number a and any integers m and p, $(a^m)^p = a^{m \cdot p}$.

Examples $(b^3)^5 = b^{3 \cdot 5}$ or b^{15} $\quad (g^6)^7 = g^{6 \cdot 7}$ or g^{42}

StudyTip

CCSS Regularity The power rules are general methods. If you are unsure about when to multiply the exponents and when to add the exponents, write the expression in expanded form.

Standardized Test Example 3 Power of a Power

Simplify $[(2^3)^2]^4$.

A 2^{24} **B** 2^{12} **C** 2^{10} **D** 2^9

Read the Test Item

You need to apply the power of a power rule.

Solve the Test Item

$[(2^3)^2]^4 = (2^{3 \cdot 2})^4$	Power of a Power
$= (2^6)^4$	Simplify.
$= 2^{6 \cdot 4}$ or 2^{24}	Power of a Power

The correct choice is A.

GuidedPractice

3. Simplify $[(2^2)^2]^4$.

F 2^8 **G** 2^{10} **H** 2^{16} **J** 2^{24}

We can use the Product of Powers Property and the Power of a Power Property to find the power of a product. Look for a pattern in the exponents below.

$$(tw)^3 = \overbrace{(tw)(tw)(tw)}^{\text{3 factors}}$$
$$= (t \cdot t \cdot t)(w \cdot w \cdot w)$$
$$= t^3w^3$$

$$(2yz^2)^3 = \overbrace{(2yz^2)(2yz^2)(2yz^2)}^{\text{3 factors}}$$
$$= (2 \cdot 2 \cdot 2)(y \cdot y \cdot y)(z^2 \cdot z^2 \cdot z^2)$$
$$= 2^3y^3z^6 \text{ or } 8y^3z^6$$

These examples demonstrate the property for the power of a product.

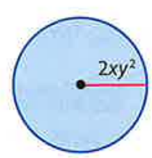
KeyConcept Power of a Product

Words	To find the power of a product, find the power of each factor and multiply.
Symbols	For any real numbers a and b and any integer m, $(ab)^m = a^mb^m$.
Example	$(-2xy^3)^5 = (-2)^5x^5y^{15}$ or $-32x^5y^{15}$

Example 4 Power of a Product

GEOMETRY **Express the area of the circle as a monomial.**

Area $= \pi r^2$	Formula for the area of a circle
$= \pi(2xy^2)^2$	Replace r with $2xy^2$.
$= \pi(2^2x^2y^4)$	Power of a Product
$= 4x^2y^4\pi$	Simplify.

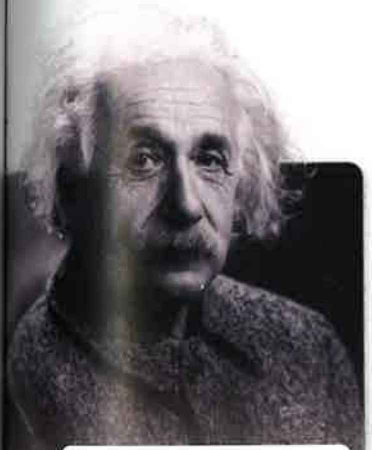


The area of the circle is $4x^2y^4\pi$ square units.

GuidedPractice

4A. Express the area of a square with sides of length $3xy^2$ as a monomial.

4B. Express the area of a triangle with height $4a$ and base $5ab^2$ as a monomial.



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Math-HistoryLink

Albert Einstein

(1879–1955) Albert Einstein is perhaps the most well-known scientist of the 20th century. His formula $E = mc^2$, where E represents the energy, m is the mass of the material, and c is the speed of light, shows that if mass is accelerated enough, it could be converted into usable energy.

2 Simplify Expressions

We can combine and use these properties to simplify expressions involving monomials.

KeyConcept Simplify Monomial Expressions

To simplify a monomial expression, write an equivalent expression in which:

- each variable base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

StudyTip

Simplify When simplifying expressions with multiple grouping symbols, begin at the innermost expression and work outward.

Example 5 Simplify Expressions

Simplify $(3xy^4)^2[(-2y)^2]^3$.

$(3xy^4)^2[(-2y)^2]^3 = (3xy^4)^2(-2y)^6$	Power of a Power
$=(3)^2x^2(y^4)^2(-2)^6y^6$	Power of a Product
$= 9x^2y^8(64)y^6$	Power of a Power
$= 9(64)x^2 \cdot y^8 \cdot y^6$	Commutative
$= 576x^2y^{14}$	Product of Powers

GuidedPractice

5. Simplify $\left(\frac{1}{2}a^2b^2\right)^3[(-4b)^2]^2$.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.**

1. 15 **2.** $2 - 3a$ **3.** $\frac{5c}{d}$

4. $-15g^2$ **5.** $\frac{r}{2}$ **6.** $7b + 9$

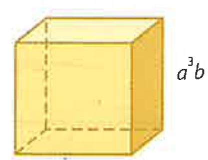
Examples 2–3 **Simplify each expression.**

7. $k(k^3)$ **8.** $m^4(m^2)$ **9.** $2q^2(9q^4)$

10. $(5u^4v)(7u^4v^3)$ **11.** $[(3^2)^2]^2$ **12.** $(xy^4)^6$

13. $(4a^4b^9c)^2$ **14.** $(-2f^2g^3h^2)^3$ **15.** $(-3p^5t^6)^4$

Example 4 **16. GEOMETRY** The formula for the surface area of a cube is $SA = 6s^2$, where SA is the surface area and s is the length of any side.



a. Express the surface area of the cube as a monomial.

b. What is the surface area of the cube if $a = 3$ and $b = 4$?

Example 5 **Simplify each expression.**

17. $(5x^2y)^2(2xy^3z)^3(4xyz)$ **18.** $(-3d^2f^3g)^2[(-3d^2f)^3]^2$

19. $(-2g^3h)(-3gj^4)^2(-ghj)^2$ **20.** $(-7ab^4c)^3[(2a^2c)^2]^3$

Practice and Problem Solving

Extra Practice is on page R7.

Example 1 Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning.

21. 122 **22.** $3a^4$ **23.** $2c + 2$

24. $\frac{-2g}{4h}$ **25.** $\frac{5k}{10}$ **26.** $6m + 3n$

Examples 2–3 Simplify each expression.

27. $(q^2)(2q^4)$ **28.** $(-2u^2)(6u^6)$ **29.** $(9w^2x^8)(w^6x^4)$

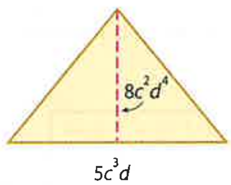
30. $(y^6z^9)(6y^4z^2)$ **31.** $(b^8c^6d^5)(7b^6c^2d)$ **32.** $(14fg^2h^2)(-3f^4g^2h^2)$

33. $(j^5k^7)^4$ **34.** $(n^3p)^4$ **35.** $[(2^2)^2]^2$

36. $[(3^2)^2]^4$ **37.** $[(4r^2t)^3]^2$ **38.** $[(-2xy^2)^3]^2$

Example 4 **GEOMETRY** Express the area of each triangle as a monomial.

39.



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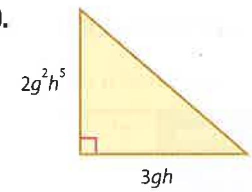


Example 5 Simplify each expression.

41. $(2a^3)^4(a^3)^3$ **42.** $(c^3)^2(-3c^5)^2$

43. $(2gh^4)^3[(-2g^4h)^3]^2$ **44.** $(5k^2m)^3[(4km^4)^2]^2$

45. $(p^5r^2)^4(-7p^3r^4)^2(6pr^3)$ **46.** $(5x^2y)^2(2xy^3z)^3(4xyz)$

47. $(5a^2b^3c^4)(6a^3b^4c^2)$ **48.** $(10xy^5z^3)(3x^4y^6z^3)$

49. $(0.5x^3)^2$ **50.** $(0.4h^5)^3$

51. $\left(-\frac{3}{4}c\right)^3$ **52.** $\left(\frac{4}{5}a^2\right)^2$

53. $(8y^3)(-3x^2y^2)\left(\frac{3}{8}xy^4\right)$ **54.** $\left(\frac{4}{7}m\right)^2(49m)(17p)\left(\frac{1}{34}p^5\right)$

55. $(-3r^3w^4)^3(2rw)^2(-3r^2)^3(4rw^2)^3(2r^2w^3)^4$

56. $(3ab^2c)^2(-2a^2b^4)^2(a^4c^2)^3(a^2b^4c^5)^2(2a^3b^2c^4)^3$

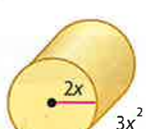
57. **FINANCIAL LITERACY** Cleavon has money in an account that earns 3% simple interest. The formula for computing simple interest is $I = Prt$, where I is the interest earned, P represents the principal that he put into the account, r is the interest rate (in decimal form), and t represents time in years.

a. Cleavon makes a deposit of \$2c and leaves it for 2 years. Write a monomial that represents the interest earned.

b. If c represents a birthday gift of \$250, how much will Cleavon have in this account after 2 years?

CCSS TOOLS Express the volume of each solid as a monomial.

58.



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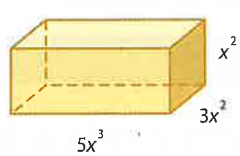
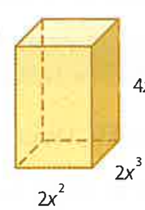


60.



61 **PACKAGING** For a commercial art class, Aiko must design a new container for individually wrapped pieces of candy. The shape that she chose is a cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$.

a. The radius that Aiko would like to use is $2p^3$, and the height is $4p^3$. Write a monomial that represents the volume of her container.

b. Make a table for five possible measures for the radius and height of a cylinder having the same volume.

c. What is the volume of Aiko's container if the height is doubled?

62. **ENERGY** Albert Einstein's formula $E = mc^2$ shows that if mass is accelerated enough, it could be converted into usable energy. Energy E is measured in joules, mass m in kilograms, and the speed c of light is about 300 million meters per second.

a. Complete the calculations to convert 3 kilograms of gasoline completely into energy.

b. What happens to the energy if the amount of gasoline is doubled?

63. **MULTIPLE REPRESENTATIONS** In this problem, you will explore exponents.

a. Tabular Copy and use a calculator to complete the table.

Power	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
Value						$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$

b. Analytical What do you think the values of 5^0 and 5^{-1} are? Verify your conjecture using a calculator.

c. Analytical Complete: For any nonzero number a and any integer n, $a^{-n} = $ _______.

d. Verbal Describe the value of a nonzero number raised to the zero power.

H.O.T. Problems Use Higher-Order Thinking Skills

64. **CCSS PERSEVERANCE** For any nonzero real numbers a and b and any integers m and t, simplify the expression $\left(-\frac{a^m}{b^t}\right)^{2t}$ and describe each step.

65. **REASONING** Copy the table below.

Equation	Related Expression	Power of x	Linear or Nonlinear
$y = x$			
$y = x^2$			
$y = x^3$			

a. For each equation, write the related expression and record the power of x.

b. Graph each equation using a graphing calculator.

c. Classify each graph as *linear* or *nonlinear*.

d. Explain how to determine whether an equation, or its related expression, is linear or nonlinear without graphing.

66. **OPEN ENDED** Write three different expressions that can be simplified to x^6.

67. **WRITING IN MATH** Write two formulas that have monomial expressions in them. Explain how each is used in a real-world situation.

Standardized Test Practice

68. Which of the following is not a monomial?

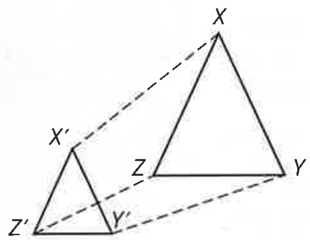
A $-6xy$

B $\frac{1}{2}a^2$

C $-\frac{1}{2b^3}$

D $5gh^4$

69. GEOMETRY The accompanying diagram shows the transformation of $\triangle XYZ$ to $\triangle X'Y'Z'$.



This transformation is an example of a

F dilation

G line reflection

H rotation

J translation

70. CARS In 2002, the average price of a new domestic car was \$19,126. In 2008, the average price was \$28,715. Based on a linear model, what is the predicted average price for 2014?

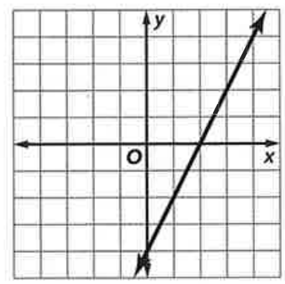
A \$45,495

B \$38,304

C \$35,906

D \$26,317

71. SHORT RESPONSE If a line has a positive slope and a negative y-intercept, what happens to the x-intercept if the slope and the y-intercept are both doubled?



Spiral Review

Solve each system of inequalities by graphing. (Lesson 6-6)

72. $y < 4x$
$2x + 3y \geq -21$

73. $y \geq 2$
$2y + 2x \leq 4$

74. $y > -2x - 1$
$2y \leq 3x + 2$

75. $3x + 2y < 10$
$2x + 12y < -6$

76. SPORTS In the 2006 Winter Olympic Games, the total number of gold and silver medals won by the U.S. was 18. The total points scored for gold and silver medals was 45. Write and solve a system of equations to find how many gold and silver medals were won by the U.S. (Lesson 6-5)

77. DRIVING Tires should be kept within 2 pounds per square inch (psi) of the manufacturer's recommended tire pressure. If the recommendation for a tire is 30 psi, what is the range of acceptable pressures? (Lesson 5-5)

78. BABYSITTING Alexis charges \$10 plus \$4 per hour to babysit. Alexis needs at least \$40 more to buy a television for which she is saving. Write an inequality for this situation. Will she be able to get her television if she babysits for 5 hours? (Lesson 5-6)

Skills Review

Find each quotient.

79. $-64 \div (-8)$

80. $-78 \div 1.3$

81. $42.3 \div (-6)$

82. $-23.94 \div 10.5$

83. $-32.5 \div (-2.5)$

84. $-98.44 \div 4.6$

LESSON 7-2

Division Properties of Exponents

:: Then

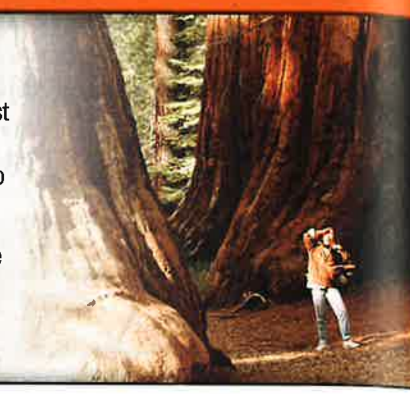
- You multiplied monomials using the properties of exponents.

:: Now

- **1** Divide monomials using the properties of exponents.
- **2** Simplify expressions containing negative and zero exponents.

:: Why?

- The tallest redwood tree is 112 meters or about 10^2 meters tall. The average height of a redwood tree is 15 meters. The closest power of ten to 15 is 10^1, so an average redwood is about 10^1 meters tall. The ratio of the tallest tree's height to the average tree's height is $\frac{10^2}{10^1}$ or 10^1. This means the tallest redwood tree is approximately 10 times as tall as the average redwood tree.



NewVocabulary

zero exponent
negative exponent
order of magnitude

Common Core State Standards

Content Standards

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Divide Monomials

We can use the principles for reducing fractions to find quotients of monomials like $\frac{10^2}{10^1}$. In the following examples, look for a pattern in the exponents.

$$\frac{2^7}{2^4} = \frac{\overbrace{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot 2\cdot 2\cdot 2}^{7\text{ factors}}}{\underbrace{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}}_{4\text{ factors}}} = 2\cdot 2\cdot 2 \text{ or } 2^3 \qquad \frac{t^4}{t^3} = \frac{\overbrace{\overset{1}{\cancel{t}}\cdot\overset{1}{\cancel{t}}\cdot\overset{1}{\cancel{t}}\cdot t}^{4\text{ factors}}}{\underbrace{\underset{1}{\cancel{t}}\cdot\underset{1}{\cancel{t}}\cdot\underset{1}{\cancel{t}}}_{3\text{ factors}}} = t$$

These examples demonstrate the Quotient of Powers Rule.

KeyConcept Quotient of Powers

Words	To divide two powers with the same base, subtract the exponents.
Symbols	For any nonzero number a, and any integers m and p, $\frac{a^m}{a^p} = a^{m-p}$.
Examples	$\frac{c^{11}}{c^8} = c^{11-8}$ or c^3 $\qquad \frac{r^5}{r^2} = r^{5-2} = r^3$

Example 1 Quotient of Powers

Simplify $\frac{g^3h^5}{gh^2}$. Assume that no denominator equals zero.

$$\frac{g^3h^5}{gh^2} = \left(\frac{g^3}{g}\right)\left(\frac{h^5}{h^2}\right) \qquad \text{Group powers with the same base.}$$

$$= \left(g^{3-1}\right)\left(h^{5-2}\right) \qquad \text{Quotient of Powers}$$

$$= g^2h^3 \qquad \text{Simplify.}$$

GuidedPractice

Simplify each expression. Assume that no denominator equals zero.

1A. $\frac{x^3y^4}{x^2y}$

1B. $\frac{k^7m^{10}p}{k^5m^3p}$

We can use the Product of Powers Rule to find the powers of quotients for monomials. In the following example, look for a pattern in the exponents.

$$\left(\frac{3}{4}\right)^3 = \overbrace{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}^{3\text{ factors}} = \frac{\overbrace{3\cdot3\cdot3}^{3\text{ factors}}}{\underbrace{4\cdot4\cdot4}_{3\text{ factors}}} = \frac{3^3}{4^3}$$

$$\left(\frac{c}{d}\right)^2 = \overbrace{\left(\frac{c}{d}\right)\left(\frac{c}{d}\right)}^{2\text{ factors}} = \frac{\overbrace{c\cdot c}^{2\text{ factors}}}{\underbrace{d\cdot d}_{2\text{ factors}}} = \frac{c^2}{d^2}$$

StudyTip

Power Rules with Variables The power rules apply to variables as well as numbers. For example, $\left(\frac{3a}{4b}\right)^3 = \frac{(3a)^3}{(4b)^3}$ or $\frac{27a^3}{64b^3}$.

KeyConcept Power of a Quotient

Words To find the power of a quotient, find the power of the numerator and the power of the denominator.

Symbols For any real numbers a and $b \neq 0$, and any integer m, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Examples $\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4}$ $\quad \left(\frac{r}{t}\right)^5 = \frac{r^5}{t^5}$

Example 2 Power of a Quotient

PT

Simplify $\left(\frac{3p^3}{7}\right)^2$.

$$\left(\frac{3p^3}{7}\right)^2 = \frac{(3p^3)^2}{7^2} \qquad \text{Power of a Quotient}$$

$$= \frac{3^2(p^3)^2}{7^2} \qquad \text{Power of a Product}$$

$$= \frac{9p^6}{49} \qquad \text{Power of a Power}$$

GuidedPractice

Simplify each expression.

2A. $\left(\frac{3x^4}{4}\right)^3$ **2B.** $\left(\frac{5x^5y}{6}\right)^2$ **2C.** $\left(\frac{2y^2}{3z^3}\right)^2$ **2D.** $\left(\frac{4x^3}{5y^4}\right)^3$

Real-WorldCareer

Astronomer An astronomer studies the universe and analyzes space travel and satellite communications. To be a technician or research assistant, a bachelor's degree is required.

A calculator can be used to explore expressions with 0 as the exponent. There are two methods to explain why a calculator gives a value of 1 for 3^0.

Method 1

$\frac{3^5}{3^5} = 3^{5-5}$ Quotient of Powers

$= 3^0$ Simplify.

Method 2

$\frac{3^5}{3^5} = \frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{3}}$ Definition of powers

$= 1$ Simplify.

Since $\frac{3^5}{3^5}$ can only have one value, we can conclude that $3^0 = 1$. A **zero exponent** is any nonzero number raised to the zero power.

KeyConcept Zero Exponent Property

Words	Any nonzero number raised to the zero power is equal to 1.
Symbols	For any nonzero number a, $a^0 = 1$.
Examples	$15^0 = 1$ $\left(\frac{b}{c}\right)^0 = 1$ $\left(\frac{2}{7}\right)^0 = 1$

PT

Example 3 Zero Exponent

Simplify each expression. Assume that no denominator equals zero.

a. $\left(-\frac{4n^2q^5r^2}{9n^3q^2r}\right)^0$

$\left(-\frac{4n^2q^5r^2}{9n^3q^2r}\right)^0 = 1$ $a^0 = 1$

b. $\frac{x^5y^0}{x^3}$

$\frac{x^5y^0}{x^3} = \frac{x^5(1)}{x^3}$ $a^0 = 1$

$= x^2$ Quotient of Powers

GuidedPractice

3A. $\frac{b^4c^2d^0}{b^2c}$

3B. $\left(\frac{2f^4g^7h^3}{15f^3g^9h^6}\right)^0$

StudyTip

Zero Exponent Be careful of parentheses. The expression $(5x)^0$ is 1 but $5x^0 = 5$.

2 Negative Exponents

Any nonzero real number raised to a negative power is a **negative exponent**. To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{c^2}{c^5}$ using two methods.

Method 1

$\frac{c^2}{c^5} = c^{2-5}$ Quotient of Powers

$= c^{-3}$ Simplify.

Method 2

$\frac{c^2}{c^5} = \frac{\cancel{c} \cdot \cancel{c}}{\cancel{c} \cdot \cancel{c} \cdot c \cdot c \cdot c}$ Definition of powers

$= \frac{1}{c^3}$ Simplify.

Since $\frac{c^2}{c^5}$ can only have one value, we can conclude that $c^{-3} = \frac{1}{c^3}$.

KeyConcept Negative Exponent Property

Words	For any nonzero number a and any integer n, a^{-n} is the reciprocal of a^n. Also, the reciprocal of a^{-n} is a^n.
Symbols	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.
Examples	$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\frac{1}{j^{-4}} = j^4$

An expression is considered simplified when it contains only positive exponents, each base appears exactly once, there are no powers of powers, and all fractions are in simplest form.

PT

Example 4 Negative Exponents

Simplify each expression. Assume that no denominator equals zero.

a. $\dfrac{n^{-5}p^4}{r^{-2}}$

$$\frac{n^{-5}p^4}{r^{-2}} = \left(\frac{n^{-5}}{1}\right)\left(\frac{p^4}{1}\right)\left(\frac{1}{r^{-2}}\right)$$ Write as a product of fractions.

$$= \left(\frac{1}{n^5}\right)\left(\frac{p^4}{1}\right)\left(\frac{r^2}{1}\right)$$ $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

$$= \frac{p^4r^2}{n^5}$$ Multiply.

b. $\dfrac{5r^{-3}t^4}{-20r^2t^7u^{-5}}$

$$\frac{5r^{-3}t^4}{-20r^2t^7u^{-5}} = \left(\frac{5}{-20}\right)\left(\frac{r^{-3}}{r^2}\right)\left(\frac{t^4}{t^7}\right)\left(\frac{1}{u^{-5}}\right)$$ Group powers with the same base.

$$= \left(-\frac{1}{4}\right)\left(r^{-3-2}\right)\left(t^{4-7}\right)\left(u^5\right)$$ Quotient of Powers and Negative Exponents Property

$$= -\frac{1}{4}r^{-5}t^{-3}u^5$$ Simplify.

$$= -\frac{1}{4}\left(\frac{1}{r^5}\right)\left(\frac{1}{t^3}\right)\left(u^5\right)$$ Negative Exponent Property

$$= -\frac{u^5}{4r^5t^3}$$ Multiply.

c. $\dfrac{2a^2b^3c^{-5}}{10a^{-3}b^{-1}c^{-4}}$

$$\frac{2a^2b^3c^{-5}}{10a^{-3}b^{-1}c^{-4}} = \left(\frac{2}{10}\right)\left(\frac{a^2}{a^{-3}}\right)\left(\frac{b^3}{b^{-1}}\right)\left(\frac{c^{-5}}{c^{-4}}\right)$$ Group powers with the same base.

$$= \left(\frac{1}{5}\right)\left(a^{2-(-3)}\right)\left(b^{3-(-1)}\right)\left(c^{-5-(-4)}\right)$$ Quotient of Powers and Negative Exponents Property

$$= \frac{1}{5}a^5b^4c^{-1}$$ Simplify.

$$= \frac{1}{5}\left(a^5\right)\left(b^4\right)\left(\frac{1}{c}\right)$$ Negative Exponent Property

$$= \frac{a^5b^4}{5c}$$ Multiply.

GuidedPractice

Simplify each expression. Assume that no denominator equals zero.

4A. $\dfrac{v^{-3}wx^2}{wy^{-6}}$ **4B.** $\dfrac{32a^{-8}b^3c^{-4}}{4a^3b^5c^{-2}}$ **4C.** $\dfrac{5j^{-3}k^2m^{-6}}{25k^{-4}m^{-2}}$

StudyTip

Negative Signs Be aware of where a negative sign is placed. $5^{-1} = \frac{1}{5}$, while $-5^1 \neq \frac{1}{5}$.

Blend Images/SuperStock

Real-WorldLink

An adult human weighs about 70 kilograms and an adult dairy cow weighs about 700 kilograms. Their weights differ by 1 order of magnitude.

Order of magnitude is used to compare measures and to estimate and perform rough calculations. The **order of magnitude** of a quantity is the number rounded to the nearest power of 10. For example, the power of 10 closest to 95,000,000,000 is 10^{11}, or 100,000,000,000. So the order of magnitude of 95,000,000,000 is 10^{11}.

Real-WorldLink

There are over 14,000 species of ants living all over the world. Some ants can carry objects that are 50 times their own weight.

Source: Maine Animal Coalition

Real-World Example 5 Apply Properties of Exponents

HEIGHT **Suppose the average height of a man is about 1.7 meters, and the average height of an ant is 0.0008 meter. How many orders of magnitude as tall as an ant is a man?**

Understand We must find the order of magnitude of the heights of the man and ant. Then find the ratio of the orders of magnitude of the man's heig to that of the ant's height.

Plan Round each height to the nearest power of ten. Then find the ratio o the height of the man to the height of the ant.

Solve The average height of a man is close to 1 meter. So, the order of magnitude is 10^0 meter. The average height of an ant is about 0.001 meter. So, the order of magnitude is 10^{-3} meters.

The ratio of the height of a man to the height of an ant is about $\frac{10^0}{10^{-3}}$.

$$\begin{aligned} \frac{10^0}{10^{-3}} &= 10^{0-(-3)} && \text{Quotient of Powers} \\ &= 10^3 && 0-(-3) = 0+3 \text{ or } 3 \\ &= 1000 && \text{Simplify.} \end{aligned}$$

So, a man is approximately 1000 times as tall as an ant, or a man is 3 orders of magnitude as tall as an ant.

Check The ratio of the man's height to the ant's height is $\frac{1.7}{0.0008} = 2125$. The order of magnitude of 2125 is 10^3. ✓

GuidedPractice

5. **ASTRONOMY** The order of magnitude of the mass of Earth is about 10^{27}. The order of magnitude of the Milky Way galaxy is about 10^{44}. How many orders of magnitude as big is the Milky Way galaxy as Earth?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–4 **Simplify each expression. Assume that no denominator equals zero.**

1. $\frac{t^5u^4}{t^2u}$ **2.** $\frac{a^6b^4c^{10}}{a^3b^2c}$ **3.** $\frac{m^6r^5p^3}{m^5r^2p^3}$ **4.** $\frac{b^4c^6f^8}{b^4c^3f^5}$

5. $\frac{g^8h^2m}{hg^7}$ **6.** $\frac{r^4t^7v^2}{t^7v^2}$ **7.** $\frac{x^3y^2z^6}{z^5x^2y}$ **8.** $\frac{n^4q^4w^6}{q^2n^3w}$

9. $\left(\frac{2a^3b^5}{3}\right)^2$ **10.** $\frac{r^3v^{-2}}{t^{-7}}$ **11.** $\left(\frac{2c^3d^5}{5g^2}\right)^5$

12. $\left(-\frac{3xy^4z^2}{x^3yz^4}\right)^0$ **13.** $\left(\frac{3f^4gh^4}{32f^3g^4h}\right)^0$ **14.** $\frac{4r^2v^0t^5}{2rt^3}$

15. $\frac{f^{-3}g^2}{h^{-4}}$ **16.** $\frac{-8x^2y^8z^{-5}}{12x^4y^{-7}z^7}$ **17.** $\frac{2a^2b^{-7}c^{10}}{6a^{-3}b^2c^{-3}}$

Example 5

18. **FINANCIAL LITERACY** The gross domestic product (GDP) for the United States in 2008 was $14.204 trillion, and the GDP per person was $47,580. Use order of magnitude to approximate the population of the United States in 2008.

Practice and Problem Solving

Extra Practice is on page R7.

Examples 1–4 **Simplify each expression. Assume that no denominator equals zero.**

19. $\dfrac{m^4p^2}{m^2p}$

20. $\dfrac{p^{12}t^3r}{p^2tr}$

21. $\dfrac{3m^{-3}r^4p^2}{12t^4}$

22. $\dfrac{c^4d^4f^3}{c^2d^4f^3}$

23. $\left(\dfrac{3xy^4}{5z^2}\right)^2$

24. $\left(\dfrac{3t^6u^2v^5}{9tuv^{21}}\right)^0$

25. $\left(\dfrac{p^2t^7}{10}\right)^3$

26. $\dfrac{x^{-4}y^9}{z^{-2}}$

27. $\dfrac{a^7b^8c^8}{a^5bc^7}$

28. $\left(\dfrac{3np^3}{7q^2}\right)^2$

29. $\left(\dfrac{2r^3t^6}{5u^9}\right)^4$

30. $\left(\dfrac{3m^5r^3}{4p^8}\right)^4$

31. $\left(-\dfrac{5f^9g^4h^2}{fg^2h^3}\right)^0$

32. $\dfrac{p^{12}t^7r^2}{p^2t^7r}$

33. $\dfrac{p^4t^{-3}}{r^{-2}}$

34. $-\dfrac{5c^2d^5}{8cd^5f^0}$

35. $\dfrac{-2f^3g^2h^0}{8f^2g^2}$

36. $\dfrac{12m^{-4}p^2}{-15m^3p^{-9}}$

37. $\dfrac{k^4m^3p^2}{k^2m^2}$

38. $\dfrac{14f^{-3}g^2h^{-7}}{21k^3}$

39. $\dfrac{39t^4uv^{-2}}{13t^{-3}u^7}$

40. $\left(\dfrac{a^{-2}b^4c^5}{a^{-4}b^{-4}c^3}\right)^2$

41. $\dfrac{r^3t^{-1}x^{-5}}{tx^5}$

42. $\dfrac{g^0h^7j^{-2}}{g^{-5}h^0j^{-2}}$

Example 5

43. **INTERNET** In a recent year, there were approximately 3.95 million Internet hosts. Suppose there were 208 million Internet users. Determine the order of magnitude for the Internet hosts and Internet users. Using the orders of magnitude, how many Internet users were there compared to Internet hosts?

44. **PROBABILITY** The probability of rolling a die and getting an even number is $\frac{1}{2}$. If you roll the die twice, the probability of getting an even number both times is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ or $\left(\frac{1}{2}\right)^2$.
    a. What does $\left(\frac{1}{2}\right)^4$ represent?
    b. Write an expression to represent the probability of rolling a die d times and getting an even number every time. Write the expression as a power of 2.

Simplify each expression. Assume that no denominator equals zero.

45. $\dfrac{-4w^{12}}{12w^3}$

46. $\dfrac{13r^7}{39r^4}$

47. $\dfrac{(4k^3m^2)^3}{(5k^2m^{-3})^{-2}}$

48. $\dfrac{3wy^{-2}}{(w^{-1}y)^3}$

49. $\dfrac{20qr^{-2}t^{-5}}{4q^0r^4t^{-2}}$

50. $\dfrac{-12c^3d^0f^{-2}}{6c^5d^{-3}f^4}$

51. $\dfrac{(2g^3h^{-2})^2}{(g^2h^0)^{-3}}$

52. $\dfrac{(5pr^{-2})^{-2}}{(3p^{-1}r)^3}$

53. $\left(\dfrac{-3x^{-6}y^{-1}z^{-2}}{6x^{-2}yz^{-5}}\right)^{-2}$

54. $\left(\dfrac{2a^{-2}b^4c^2}{-4a^{-2}b^{-5}c^{-7}}\right)^{-1}$

55. $\dfrac{(16x^2y^{-1})^0}{(4x^0y^{-4}z)^{-2}}$

56. $\left(\dfrac{4^0c^2d^3f}{2c^{-4}d^{-5}}\right)^{-3}$

57. **CCSS SENSE-MAKING** The processing speed of an older desktop computer is about 10^8 instructions per second. A new computer can process about 10^{10} instructions per second. The newer computer is how many times as fast as the older one?

Photo by Scott Bauer/USDA

58. **ASTRONOMY** The brightness of a star is measured in magnitudes. The lower the magnitude, the brighter the star. A magnitude 9 star is 2.51 times as bright as a magnitude 10 star. A magnitude 8 star is $2.51 \cdot 2.51$ or 2.51^2 times as bright as a magnitude 10 star.

a. How many times as bright is a magnitude 3 star as a magnitude 10 star?

b. Write an expression to compare a magnitude m star to a magnitude 10 star.

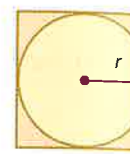
c. A full moon is considered to be magnitude -13, approximately. Does your expression make sense for this magnitude? Explain.

59. **PROBABILITY** The probability of rolling a die and getting a 3 is $\frac{1}{6}$. If you roll the die twice, the probability of getting a 3 both times is $\frac{1}{6} \cdot \frac{1}{6}$ or $\left(\frac{1}{6}\right)^2$.

a. Write an expression to represent the probability of rolling a die d times and getting a 3 each time.

b. Write the expression as a power of 6.

60. **MULTIPLE REPRESENTATIONS** To find the area of a circle, use $A = \pi r^2$. The formula for the area of a square is $A = s^2$.



a. **Algebraic** Find the ratio of the area of the circle to the area of the square.

b. **Algebraic** If the radius of the circle and the length of each side of the square are doubled, find the ratio of the area of the circle to the square.

c. **Tabular** Copy and complete the table.

Radius	Area of Circle	Area of Square	Ratio
r			
$2r$			
$3r$			
$4r$			
$5r$			
$6r$			

d. **Analytical** What conclusion can be drawn from this?

H.O.T. Problems Use Higher-Order Thinking Skills

61. **REASONING** Is $x^y \cdot x^z = x^{yz}$ *sometimes*, *always*, or *never* true? Explain.

62. **OPEN ENDED** Name two monomials with a quotient of $24a^2b^3$.

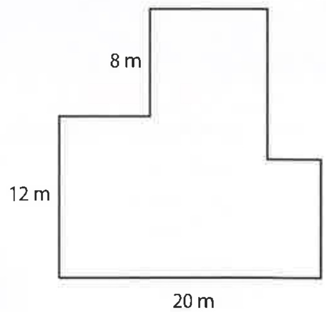
63. **CHALLENGE** Use the Quotient of Powers Property to explain why $x^{-n} = \frac{1}{x^n}$.

64. CCSS **REGULARITY** Write a convincing argument to show why $3^0 = 1$.

65. **WRITING IN MATH** Explain how to use the Quotient of Powers property and the Power of a Quotient property.

Standardized Test Practice

66. What is the perimeter of the figure in meters?

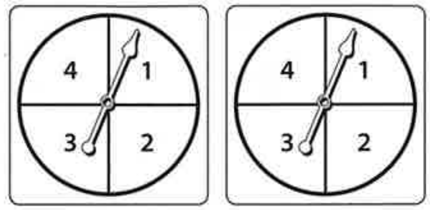


A 40 meters
B 80 meters
C 160 meters
D 400 meters

67. In researching her science project, Leigh learned that light travels at a constant rate and that it takes 500 seconds for light to travel the 93 million miles from the Sun to Earth. Mars is 142 million miles from the Sun. About how many seconds will it take for light to travel from the Sun to Mars?

F 235 seconds
G 327 seconds
H 642 seconds
J 763 seconds

68. EXTENDED RESPONSE Jessie and Jonas are playing a game using the spinners below. Each spinner is equally likely to stop on any of the four numbers. In the game, a player spins both spinners and calculates the product of the two numbers on which the spinners have stopped.



a. What product has the greatest probability of occurring?

b. What is the probability of that product occurring?

69. Simplify $(4^{-2} \cdot 5^0 \cdot 64)^3$.

A $\frac{1}{64}$
B 64
C 320
D 1024

Spiral Review

70. GEOMETRY A rectangular prism has a width of $7x^3$ units, a length of $4x^2$ units, and a height of $3x$ units. What is the volume of the prism? (Lesson 7-1)

Solve each system of inequalities by graphing. (Lesson 6-6)

71. $y \geq 1$
$x < -1$

72. $y \geq -3$
$y - x < 1$

73. $y < 3x + 2$
$y \geq -2x + 4$

74. $y - 2x < 2$
$y - 2x > 4$

Solve each inequality. Check your solution. (Lesson 5-3)

75. $5(2h - 6) > 4h$

76. $22 \geq 4(b - 8) + 10$

77. $5(u - 8) \leq 3(u + 10)$

78. $8 + t \leq 3(t + 4) + 2$

79. $9n + 3(1 - 6n) \leq 21$

80. $-6(b + 5) > 3(b - 5)$

81. GRADES In a high school science class, a test is worth three times as much as a quiz. What is the student's average grade? (Lesson 2-9)

Science Grades

Tests	Quizzes
85	82
92	75
	95

Skills Review

Evaluate each expression.

82. 9^2

83. 11^2

84. 10^6

85. 10^4

86. 3^5

87. 5^3

88. 12^3

89. 4^6

LESSON 7-3 Rational Exponents

:: Then	:: Now	:: Why?
● You used the laws of exponents to find products and quotients of monomials.	● **1** Evaluate and rewrite expressions involving rational exponents. **2** Solve equations involving expressions with rational exponents.	● It's important to protect your skin with sunscreen to prevent damage. The sun protection factor (SPF) of a sunscreen indicates how well it protects you. Sunscreen with an SPF of f absorbs about p percent of the UV-B rays, where $p = 50f^{0.2}$.

NewVocabulary
rational exponent
cube root
nth root
exponential equation

Common Core State Standards

Content Standards

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Mathematical Practices

5 Use appropriate tools strategically.

1 Rational Exponents

You know that an exponent represents the number of times that the base is used as a factor. But how do you evaluate an expression with an exponent that is not an integer like the one above? Let's investigate **rational exponents** by assuming that they behave like integer exponents.

$$\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \quad \text{Write as a multiplication expression.}$$
$$= b^{\frac{1}{2}+\frac{1}{2}} \quad \text{Product of Powers}$$
$$= b^1 \text{ or } b \quad \text{Simplify.}$$

Thus, $b^{\frac{1}{2}}$ is a number with a square equal to b. So $b^{\frac{1}{2}} = \sqrt{b}$.

KeyConcept $b^{\frac{1}{2}}$

Words	For any nonnegative real number b, $b^{\frac{1}{2}} = \sqrt{b}$.	
Examples	$16^{\frac{1}{2}} = \sqrt{16}$ or 4	$38^{\frac{1}{2}} = \sqrt{38}$

Example 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

a. $25^{\frac{1}{2}}$

$25^{\frac{1}{2}} = \sqrt{25}$ Definition of $b^{\frac{1}{2}}$

$= 5$ Simplify.

b. $\sqrt{18}$

$\sqrt{18} = 18^{\frac{1}{2}}$ Definition of $b^{\frac{1}{2}}$

c. $5x^{\frac{1}{2}}$

$5x^{\frac{1}{2}} = 5\sqrt{x}$ Definition of $b^{\frac{1}{2}}$

d. $\sqrt{8p}$

$\sqrt{8p} = (8p)^{\frac{1}{2}}$ Definition of $b^{\frac{1}{2}}$

GuidedPractice

1A. $a^{\frac{1}{2}}$ **1B.** $\sqrt{22}$ **1C.** $(7w)^{\frac{1}{2}}$ **1D.** $2\sqrt{x}$

You know that to find the square root of a number a you find a number with a square of a. In the same way, you can find other roots of numbers. If $a^3 = b$, then a is the **cube root** of b, and if $a^n = b$ for a positive integer n, then a is an **nth root** of b.

KeyConcept nth Root

Words	For any real numbers a and b and any positive integer n, if $a^n = b$, then a is an nth root of b.
Symbols	If $a^n = b$, then $\sqrt[n]{b} = a$.
Example	Because $2^4 = 16$, 2 is a fourth root of 16; $\sqrt[4]{16} = 2$.

StudyTip

CCSS Tools You can use a graphing calculator to find nth roots. Enter n, then press MATH and choose $\sqrt[x]{\ }$.

Since $3^2 = 9$ and $(-3)^2 = 9$, both 3 and -3 are square roots of 9. Similarly, since $2^4 = 16$ and $(-2)^4 = 16$, both 2 and -2 are fourth roots of 16. The positive roots are called *principal roots*. Radical symbols indicate principal roots, so $\sqrt[4]{16} = 2$.

Example 2 nth roots

PT

Simplify.

a. $\sqrt[3]{27}$

$$\sqrt[3]{27} = \sqrt[3]{3 \cdot 3 \cdot 3}$$
$$= 3$$

b. $\sqrt[5]{32}$

$$\sqrt[5]{32} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$
$$= 2$$

GuidedPractice

2A. $\sqrt[3]{64}$ $= 4$

2B. $\sqrt[4]{10{,}000}$ $= 10$

Like square roots, nth roots can be represented by rational exponents.

$$\left(b^{\frac{1}{n}}\right)^n = \underbrace{b^{\frac{1}{n}} \cdot b^{\frac{1}{n}} \cdot \ldots \cdot b^{\frac{1}{n}}}_{n \text{ factors}} \qquad \text{Write as a multiplication expression.}$$

$$= b^{\frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}} \qquad \text{Product of Powers}$$

$$= b^1 \text{ or } b \qquad \text{Simplify.}$$

Thus, $b^{\frac{1}{n}}$ is a number with an nth power equal to b. So $b^{\frac{1}{n}} = \sqrt[n]{b}$.

KeyConcept $b^{\frac{1}{n}}$

Words	For any positive real number b and any integer $n > 1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$.
Example	$8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2}$ or 2

Ariel Skelley/Blend Images/Getty Images

Example 3 Evaluate $b^{\frac{1}{n}}$ Expressions

StudyTip

Rational Exponents on a Calculator Use parentheses to evaluate expressions involving rational exponents on a graphing calculator. For example to find $125^{\frac{1}{3}}$, press 125 [^] [(] 1 [÷] 3 [)] [ENTER].

Simplify.

a. $125^{\frac{1}{3}}$

$$125^{\frac{1}{3}} = \sqrt[3]{125} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$= \sqrt[3]{5 \cdot 5 \cdot 5} \qquad 125 = 5^3$$

$$= 5 \qquad \text{Simplify.}$$

b. $1296^{\frac{1}{4}}$

$$1296^{\frac{1}{4}} = \sqrt[4]{1296} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$= \sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6} \qquad 1296 = 6^4$$

$$= 6 \qquad \text{Simplify.}$$

GuidedPractice

3A. $27^{\frac{1}{3}}$

3B. $256^{\frac{1}{4}}$

The Power of a Power property allows us to extend the definition of $b^{\frac{1}{n}}$ to $b^{\frac{m}{n}}$.

$$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m \qquad \text{Power of a Power}$$

$$= \left(\sqrt[n]{b}\right)^m \text{ or } \sqrt[n]{b^m} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$

KeyConcept $b^{\frac{m}{n}}$

Words For any positive real number b and any integers m and $n > 1$,

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m \text{ or } \sqrt[n]{b^m}.$$

Example $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2$ or 4

Example 4 Evaluate $b^{\frac{m}{n}}$ Expressions

Simplify.

a. $64^{\frac{2}{3}}$

$$64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 \qquad b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$$

$$= \left(\sqrt[3]{4 \cdot 4 \cdot 4}\right)^2 \qquad 64 = 4^3$$

$$= 4^2 \text{ or } 16 \qquad \text{Simplify.}$$

b. $36^{\frac{3}{2}}$

$$36^{\frac{3}{2}} = \left(\sqrt[2]{36}\right)^3 \qquad b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$$

$$= 6^3 \qquad \sqrt{36} = 6$$

$$= 216 \qquad \text{Simplify.}$$

GuidedPractice

4A. $27^{\frac{2}{3}}$

4B. $256^{\frac{5}{4}}$

2 Solve Exponential Equations

In an **exponential equation**, variables occur as exponents. The Power Property of Equality and the other properties of exponents can be used to solve exponential equations.

KeyConcept Power Property of Equality

Words For any real number $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

Examples If $5^x = 5^3$, then $x = 3$. If $n = \frac{1}{2}$, then $4^n = 4^{\frac{1}{2}}$.

Example 5 Solve Exponential Equations

Solve each equation.

a. $\mathbf{6^x = 216}$

$6^x = 216$	Original equation
$6^x = 6^3$	Rewrite 216 as 6^3.
$x = 3$	Property of Equality

CHECK $6^x = 216$

$6^3 \stackrel{?}{=} 216$

$216 = 216$ ✓

b. $\mathbf{25^{x-1} = 5}$

$25^{x-1} = 5$	Original equation
$(5^2)^{x-1} = 5$	Rewrite 25 as 5^2.
$5^{2x-2} = 5^1$	Power of a Power, Distributive Property
$2x - 2 = 1$	Power Property of Equality
$2x = 3$	Add 2 to each side.
$x = \frac{3}{2}$	Divide each side by 2.

CHECK $25^{x-1} = 5$

$25^{\frac{3}{2}-1} \stackrel{?}{=} 5$

$25^{\frac{1}{2}} = 5$ ✓

GuidedPractice

5A. $5^x = 125$

5B. $12^{2x+3} = 144$

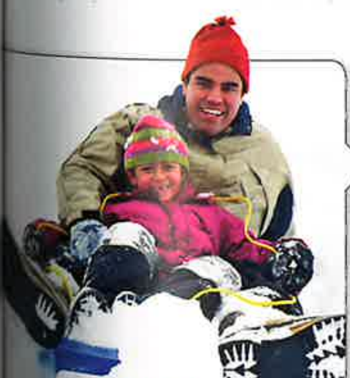
Real-World Example 6 Solve Exponential Equations

SUNSCREEN **Refer to the beginning of the lesson. Find the SPF that absorbs 100% of UV-B rays.**

$p = 50f^{0.2}$	Original equation
$100 = 50f^{0.2}$	$p = 100$
$2 = f^{0.2}$	Divide each side by 50.
$2 = f^{\frac{1}{5}}$	$0.2 = \frac{1}{5}$
$(2^5)^{\frac{1}{5}} = f^{\frac{1}{5}}$	$2 = 2^1 = (2^5)^{\frac{1}{5}}$
$2^5 = f$	Power Property of Equality
$32 = f$	Simplify.

GuidedPractice

6. CHEMISTRY The radius r of the nucleus of an atom of mass number A is $r = 1.2A^{\frac{1}{3}}$ femtometers. Find A if $r = 3.6$ femtometers.



Ryan McVay/Digital Vision/Getty Images

Real-WorldLink

Use extra caution near snow, water, and sand because they reflect the damaging rays of the Sun, which can increase your chance of sunburn.

Source: American Academy of Dermatology

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 Write each expression in radical form, or write each radical in exponential form.

1. $12^{\frac{1}{2}}$
2. $3x^{\frac{1}{2}}$
3. $\sqrt{33}$
4. $\sqrt{8n}$

Examples 2–4 Simplify.

5. $\sqrt[3]{512}$
6. $\sqrt[5]{243}$
7. $343^{\frac{1}{3}}$
8. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$
9. $343^{\frac{2}{3}}$
10. $81^{\frac{3}{4}}$
11. $216^{\frac{4}{3}}$
12. $\left(\frac{1}{49}\right)^{\frac{3}{2}}$

Example 5 Solve each equation.

13. $8^x = 4096$
14. $3^{3x+1} = 81$
15. $4^{x-3} = 32$

Example 6

16. **CCSS TOOLS** A weir is used to measure water flow in a channel. For a rectangular broad crested weir, the flow Q in cubic feet per second is related to the weir length L in feet and height H of the water by $Q = 1.6LH^{\frac{3}{2}}$. Find the water height for a weir that is 3 feet long and has flow of 38.4 cubic feet per second.

Practice and Problem Solving

Extra Practice is on page R7

Example 1 Write each expression in radical form, or write each radical in exponential form.

17. $15^{\frac{1}{2}}$
18. $24^{\frac{1}{2}}$
19. $4k^{\frac{1}{2}}$
20. $(12y)^{\frac{1}{2}}$
21. $\sqrt{26}$
22. $\sqrt{44}$
23. $2\sqrt{ab}$
24. $\sqrt{3xyz}$

Examples 2–4 Simplify.

25. $\sqrt[3]{8}$
26. $\sqrt[5]{1024}$
27. $\sqrt[3]{216}$
28. $\sqrt[4]{10{,}000}$
29. $\sqrt[3]{0.001}$
30. $\sqrt[4]{\frac{16}{81}}$
31. $1331^{\frac{1}{3}}$
32. $64^{\frac{1}{6}}$
33. $3375^{\frac{1}{3}}$
34. $512^{\frac{1}{9}}$
35. $\left(\frac{1}{81}\right)^{\frac{1}{4}}$
36. $\left(\frac{3125}{32}\right)^{\frac{1}{5}}$
37. $8^{\frac{2}{3}}$
38. $625^{\frac{3}{4}}$
39. $729^{\frac{5}{6}}$
40. $256^{\frac{3}{8}}$
41. $125^{\frac{4}{3}}$
42. $49^{\frac{5}{2}}$
43. $\left(\frac{9}{100}\right)^{\frac{3}{2}}$
44. $\left(\frac{8}{125}\right)^{\frac{4}{3}}$

Example 5 **Solve each equation.**

45. $3^x = 243$ **46.** $12^x = 144$ **47.** $16^x = 4$

48. $27^x = 3$ **49.** $9^x = 27$ **50.** $32^x = 4$

51. $2^{x-1} = 128$ **52.** $4^{2x+1} = 1024$ **53.** $6^{x-4} = 1296$

54. $9^{2x+3} = 2187$ **55.** $4^{3x} = 512$ **56.** $128^{3x} = 8$

Example 6

57. CONSERVATION Water collected in a rain barrel can be used to water plants and reduce city water use. Water flowing from an open rain barrel has velocity $v = 8h^{\frac{1}{2}}$, where v is in feet per second and h is the height of the water in feet. Find the height of the water if it is flowing at 16 feet per second.

58. ELECTRICITY The radius r in millimeters of a platinum wire L centimeters long with resistance 0.1 ohm is $r = 0.059L^{\frac{1}{2}}$. How long is a wire with radius 0.236 millimeter?

Write each expression in radical form, or write each radical in exponential form.

59. $17^{\frac{1}{3}}$ **60.** $q^{\frac{1}{4}}$ **61.** $7b^{\frac{1}{3}}$ **62.** $m^{\frac{2}{3}}$

63. $\sqrt[3]{29}$ **64.** $\sqrt[5]{h}$ **65.** $2\sqrt[3]{a}$ **66.** $\sqrt[3]{xy^2}$

Simplify.

67. $\sqrt[3]{0.027}$ **68.** $\sqrt[4]{\frac{n^4}{16}}$ **69.** $a^{\frac{1}{3}} \cdot a^{\frac{2}{3}}$ **70.** $c^{\frac{1}{2}} \cdot c^{\frac{3}{2}}$

71. $(8^2)^{\frac{2}{3}}$ **72.** $\left(y^{\frac{3}{4}}\right)^{\frac{1}{2}}$ **73.** $9^{-\frac{1}{2}}$ **74.** $16^{-\frac{3}{2}}$

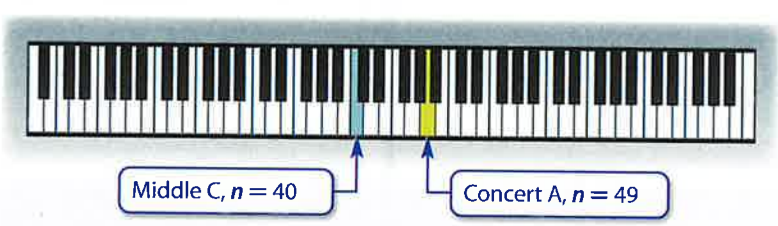
75. $(3^2)^{-\frac{3}{2}}$ **76.** $\left(81^{\frac{1}{4}}\right)^{-2}$ **77.** $k^{-\frac{1}{2}}$ **78.** $\left(d^{\frac{4}{3}}\right)^0$

Solve each equation.

79. $2^{5x} = 8^{2x-4}$ **80.** $81^{2x-3} = 9^{x+3}$ **81.** $2^{4x} = 32^{x+1}$

82. $16^x = \frac{1}{2}$ **83.** $25^x = \frac{1}{125}$ **84.** $6^{8-x} = \frac{1}{216}$

85. CCSS MODELING The frequency f in hertz of the nth key on a piano is $f = 440\left(2^{\frac{1}{12}}\right)^{n-49}$.



a. What is the frequency of Concert A?

b. Which note has a frequency of 220 Hz?

86. RANDOM WALKS Suppose you go on a walk where you choose the direction of each step at random. The path of a molecule in a liquid or a gas, the path of a foraging animal, and a fluctuating stock price are all modeled as random walks. The number of possible random walks w of n steps where you choose one of d directions at each step is $w = d^n$.

a. How many steps have been taken in a 2-direction random walk if there are 4096 possible walks?

b. How many steps have been taken in a 4-direction random walk if there are 65,536 possible walks?

c. If a walk of 7 steps has 2187 possible walks, how many directions could be taken at each step?

87. SOCCER The radius r of a ball that holds V cubic units of air is modeled by $r = 0.62V^{\frac{1}{3}}$. What are the possible volumes of each size soccer ball?

Soccer Ball Dimensions

Size	Diameter (in.)
3	7.3–7.6
4	8.0–8.3
5	8.6–9.0

88. MULTIPLE REPRESENTATIONS In this problem, you will explore the graph of an exponential function.

a. **TABULAR** Copy and complete the table below.

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x) = 4^x$									

b. **GRAPHICAL** Graph $f(x)$ by plotting the points and connecting them with a smooth curve.

c. **VERBAL** Describe the shape of the graph of $f(x)$. What are its key features? Is it linear?

H.O.T. Problems Use Higher-Order Thinking Skills

89. OPEN ENDED Write two different expressions with rational exponents equal to $\sqrt{2}$.

90. CCSS ARGUMENTS Determine whether each statement is *always*, *sometimes*, or *never* true. Assume that x is a nonnegative real number. Explain your reasoning.

a. $x^2 = x^{\frac{1}{2}}$

b. $x^{-2} = x^{\frac{1}{2}}$

c. $x^{\frac{1}{3}} = x^{\frac{1}{2}}$

d. $\sqrt{x} = x^{\frac{1}{2}}$

e. $\left(x^{\frac{1}{2}}\right)^2 = x$

f. $x^{\frac{1}{2}} \cdot x^2 = x$

91. CHALLENGE For what values of x is $x = x^{\frac{1}{3}}$?

92. ERROR ANALYSIS Anna and Jamal are solving $128^x = 4$. Is either of them correct? Explain your reasoning.

Anna	Jamal
$128^x = 4$	$128^x = 4$
$(2^7)^x = 2^2$	$(2^7)^x = 4$
$2^{7x} = 2^2$	$2^{7x} = 4^1$
$7x = 2$	$7x = 1$
$x = \frac{2}{7}$	$x = \frac{1}{7}$

93. WRITING IN MATH Explain why 2 is the principal fourth root of 16.

Standardized Test Practice

94. What is the value of $16^{\frac{3}{4}} + 9^{\frac{3}{2}}$?

A 5 **C** 25
B 11 **D** 35

95. At a movie theater, the costs for various numbers of popcorn and hot dogs are shown.

Hot Dogs	Boxes of Popcorn	Total Cost
1	1	$8.50
2	4	$21.60

Which pair of equations can be used to find p, the cost of a box of popcorn, and h, the cost of a hot dog?

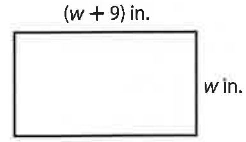
F $p + h = 8.5$
$p + 2h = 10.8$

G $p + h = 8.5$
$2h + 4p = 21.6$

H $p + h = 8.5$
$2p + 4h = 21.6$

J $p + h = 8.5$
$2p + 2h = 21.6$

96. SHORT RESPONSE Find the dimensions of the rectangle if its perimeter is 52 inches.



97. If $3^4 = 9^x$, then $x =$

A 1
B 2
C 4
D 5

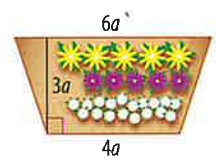
Spiral Review

Simplify each expression. Assume that no denominator equals zero. (Lesson 7-2)

98. $\frac{a^3b^5}{ab^3}$ **99.** $\frac{c^8d^{11}}{c^4d^5}$ **100.** $\frac{4x^3y^3z^6}{xyz^5}$

101. $\frac{a^5b^3c}{a^5bc}$ **102.** $\left(\frac{3m^4}{4p^2}\right)^2$ **103.** $\left(\frac{3df^2}{9d^2f}\right)^0$

104. GARDENING Felipe is planting a flower garden that is shaped like a trapezoid as shown at the right. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$ to find the area of the garden. (Lesson 7-1)



Write each equation in slope-intercept form. (Lesson 4-2)

105. $y - 2 = 3(x - 1)$ **106.** $y - 5 = 6(x + 1)$ **107.** $y + 2 = -2(x + 5)$

108. $y + 3 = \frac{1}{2}(x + 4)$ **109.** $y - 1 = \frac{2}{3}(x + 9)$ **110.** $y + 3 = -\frac{1}{4}(x + 2)$

Skills Check

Find each power.

111. 10^3 **112.** 10^5 **113.** 10^{-1} **114.** 10^{-4}

LESSON 7-4 Scientific Notation

:: Then

- You used the laws of exponents to find products and quotients of monomials.

:: Now

1. Express numbers in scientific notation.
2. Find products and quotients of numbers expressed in scientific notation.

:: Why?

- Space tourism is a multibillion dollar industry. For a price of $20 million, a civilian can travel on a rocket or shuttle and visit the International Space Station (ISS) for a week.

NewVocabulary
scientific notation

Common Core State Standards

Content Standards
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
6 Attend to precision.

1 Scientific Notation

Very large and very small numbers such as $20 million can be cumbersome to use in calculations. For this reason, numbers are often expressed in scientific notation. A number written in **scientific notation** is of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

KeyConcept Standard Form to Scientific Notation

Step 1 Move the decimal point until it is to the right of the first nonzero digit. The result is a real number a.

Step 2 Note the number of places n and the direction that you moved the decimal point.

Step 3 If the decimal point is moved left, write the number as $a \times 10^n$.
If the decimal point is moved right, write the number as $a \times 10^{-n}$.

Step 4 Remove the unnecessary zeros.

Example 1 Standard Form to Scientific Notation

Express each number in scientific notation.

a. 201,000,000

Step 1 201,000,000 ⟶ 2.01000000 $a = 2.01000000$

Step 2 The decimal point moved 8 places to the left, so $n = 8$.

Step 3 $201{,}000{,}000 = 2.01000000 \times 10^8$

Step 4 2.01×10^8

b. 0.000051

Step 1 0.000051 ⟶ 00005.1 $a = 00005.1$

Step 2 The decimal point moved 5 places to the right, so $n = 5$.

Step 3 $0.000051 = 00005.1 \times 10^{-5}$

Step 4 5.1×10^{-5}

GuidedPractice

1A. 68,700,000,000 **1B.** 0.0000725

You can also rewrite numbers in scientific notation in standard form.

WatchOut!

Negative Signs Be careful about the placement of negative signs. A negative sign in the exponent means that the number is between 0 and 1. A negative sign before the number means that it is less than 0.

KeyConcept Scientific Notation to Standard Form

Step 1 In $a \times 10^n$, note whether $n > 0$ or $n < 0$.

Step 2 If $n > 0$, move the decimal point n places right.
If $n < 0$, move the decimal point $-n$ places left.

Step 3 Insert zeros, decimal point, and commas as needed for place value.

Example 2 Scientific Notation to Standard Form

Express each number in standard form.

a. 6.32×10^9

Step 1 The exponent is 9, so $n = 9$.

Step 2 Since $n > 0$, move the decimal point 9 places to the right.
$6.32 \times 10^9 \longrightarrow 6320000000$

Step 3 $6.32 \times 10^9 = 6{,}320{,}000{,}000$ — Rewrite; insert commas.

b. 4×10^{-7}

Step 1 The exponent is -7, so $n = -7$.

Step 2 Since $n < 0$, move the decimal point 7 places to the left.
$4 \times 10^{-7} \longrightarrow 0000004$

Step 3 $4 \times 10^{-7} = 0.0000004$ — Rewrite; insert a 0 before the decimal point.

GuidedPractice

2A. 3.201×10^6

2B. 9.03×10^{-5}

2 Product and Quotients in Scientific Notation

You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

Problem-SolvingTip

CCSS Tools Estimating an answer before computing the solution can help you determine if your answer is reasonable.

Example 3 Multiply with Scientific Notation

Evaluate $(3.5 \times 10^{-3})(7 \times 10^5)$. Express the result in both scientific notation and standard form.

$(3.5 \times 10^{-3})(7 \times 10^5)$	Original expression
$= (3.5 \times 7)(10^{-3} \times 10^5)$	Commutative and Associative Properties
$= 24.5 \times 10^2$	Product of Powers
$= (2.45 \times 10^1) \times 10^2$	$24.5 = 2.45 \times 10$
$= 2.45 \times 10^3$ or 2450	Product of Powers

GuidedPractice

Evaluate each product. Express the results in both scientific notation and standard form.

3A. $(6.5 \times 10^{12})(8.7 \times 10^{-15})$

3B. $(7.8 \times 10^{-4})^2$

Example 4 Divide with Scientific Notation

Evaluate $\frac{3.066 \times 10^8}{7.3 \times 10^3}$. Express the result in both scientific notation and standard form.

$$\frac{3.066 \times 10^8}{7.3 \times 10^3} = \left(\frac{3.066}{7.3}\right)\left(\frac{10^8}{10^3}\right) \quad \text{Product rule for fractions}$$
$$= 0.42 \times 10^5 \quad \text{Quotient of Powers}$$
$$= 4.2 \times 10^{-1} \times 10^5 \quad 0.42 = 4.2 \times 10^{-1}$$
$$= 4.2 \times 10^4 \quad \text{Product of Powers}$$
$$= 42{,}000 \quad \text{Standard form}$$

StudyTip

Quotient of Powers Recall that the Quotient of Powers Property is only valid for powers that have the same base. Since 10^8 and 10^3 have the same base, the property applies.

GuidedPractice

Evaluate each quotient. Express the results in both scientific notation and standard form.

4A. $\frac{2.3958 \times 10^3}{1.98 \times 10^8}$

4B. $\frac{1.305 \times 10^3}{1.45 \times 10^{-4}}$

Real-World Example 5 Use Scientific Notation

MUSIC In the United States, a CD reaches gold status once 500 thousand cop are sold. A CD reaches platinum status once 1 million or more copies are sol

a. Express the number of copies of CDs that need to be sold to reach each st in standard notation.

gold status: 500 thousand = 500,000; platinum status: 1 million = 1,000,000

b. Write each number in scientific notation.

gold status: $500{,}000 = 5 \times 10^5$; platinum status: $1{,}000{,}000 = 1 \times 10^6$

c. How many copies of a CD have sold if it has gone platinum 13 times? Wri your answer in scientific notation and standard form.

A CD reaches platinum status once it sells 1 million records. Since the CD ha gone platinum 13 times, we need to multiply by 13.

$$(13)(1 \times 10^6) \quad \text{Original expression}$$
$$= (13 \times 1)(10^6) \quad \text{Associative Property}$$
$$= 13 \times 10^6 \quad 13 \times 1 = 13$$
$$= (1.3 \times 10^1) \times 10^6 \quad 13 = 1.3 \times 10$$
$$= 1.3 \times 10^7 \quad \text{Product of Powers}$$
$$= 13{,}000{,}000 \quad \text{Standard form}$$

Real-WorldLink

The platinum award was created in 1976. In 2004, the criteria for the award was extended to digital sales. The top-selling artist of all time is the Beatles with 170 million units sold.

Source: Recording Industry Association of America

GuidedPractice

5. SATELLITE RADIO Suppose a satellite radio company earned \$125.4 million in one year.

A. Write this number in standard form.

B. Write this number in scientific notation.

C. If the following year the company earned 2.5 times the amount earned the previous year, determine the amount earned. Write your answer in scientific notation and standard form.

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 **Express each number in scientific notation.**

1. 185,000,000
2. 1,902,500,000
3. 0.000564
4. 0.00000804

MONEY **Express each number in scientific notation.**

5. Teens spend \$13 billion annually on clothing.
6. Teens have an influence on their families' spending habit. They control about \$1.5 billion of discretionary income.

Example 2 **Express each number in standard form.**

7. 1.98×10^7
8. 4.052×10^6
9. 3.405×10^{-8}
10. 6.8×10^{-5}

Example 3 **Evaluate each product. Express the results in both scientific notation and standard form.**

11. $(1.2 \times 10^3)(1.45 \times 10^{12})$
12. $(7.08 \times 10^{14})(5 \times 10^{-9})$
13. $(5.18 \times 10^2)(9.1 \times 10^{-5})$
14. $(2.18 \times 10^{-2})^2$

Example 4 **Evaluate each quotient. Express the results in both scientific notation and standard form.**

15. $\dfrac{1.035 \times 10^8}{2.3 \times 10^4}$
16. $\dfrac{2.542 \times 10^5}{4.1 \times 10^{-10}}$
17. $\dfrac{1.445 \times 10^{-7}}{1.7 \times 10^5}$
18. $\dfrac{2.05 \times 10^{-8}}{4 \times 10^{-2}}$

Example 5

19. CCSS **PRECISION** Salvador bought an air purifier to help him deal with his allergies. The filter in the purifier will stop particles as small as one hundredth of a micron. A micron is one millionth of a millimeter.
 a. Write one hundredth and one micron in standard form.
 b. Write one hundredth and one micron in scientific notation.
 c. What is the smallest size particle in meters that the filter will stop? Write the result in both standard form and scientific notation.

Practice and Problem Solving

Extra Practice is on page R7.

Example 1 **Express each number in scientific notation.**

20. 1,220,000
21. 58,600,000
22. 1,405,000,000,000
23. 0.0000013
24. 0.000056
25. 0.000000000709

EMAIL **Express each number in scientific notation.**

26. Approximately 100 million emails sent to the President are put into the National Archives.
27. By 2015, the email security market will generate \$6.5 billion.

Example 2 **Express each number in standard form.**

28. 1×10^{12}
29. 9.4×10^7
30. 8.1×10^{-3}
31. 5×10^{-4}
32. 8.73×10^{11}
33. 6.22×10^{-6}

Example 2 **INTERNET** **Express each number in standard form.**

34. About 2.1×10^7 people aged 12 to 17 use the Internet.
35. Approximately 1.1×10^7 teens go online daily.

Examples 3–4 **Evaluate each product or quotient. Express the results in both scientific notation and standard form.**

36. $(3.807 \times 10^3)(5 \times 10^2)$
37. $\frac{9.6 \times 10^3}{1.2 \times 10^{-4}}$
38. $\frac{2.88 \times 10^3}{1.2 \times 10^{-5}}$
39. $(6.5 \times 10^7)(7.2 \times 10^{-2})$
40. $(9.5 \times 10^{-18})(9 \times 10^9)$
41. $\frac{8.8 \times 10^3}{4 \times 10^{-4}}$
42. $\frac{9.15 \times 10^{-3}}{6.1 \times 10}$
43. $(1.4 \times 10^6)^2$
44. $(2.58 \times 10^2)(3.6 \times 10^6)$
45. $\frac{5.6498 \times 10^{10}}{8.2 \times 10^4}$
46. $\frac{1.363 \times 10^{16}}{2.9 \times 10^6}$
47. $(5 \times 10^3)(1.8 \times 10^{-7})$
48. $(2.3 \times 10^{-3})^2$
49. $\frac{6.25 \times 10^{-4}}{1.25 \times 10^2}$
50. $\frac{3.75 \times 10^{-9}}{1.5 \times 10^{-4}}$
51. $(7.2 \times 10^7)^2$
52. $\frac{8.6 \times 10^4}{2 \times 10^{-6}}$
53. $(6.3 \times 10^{-5})^2$

Example 5

54. **ASTRONOMY** The distance between Earth and the Sun varies throughout the year. Earth is closest to the Sun in January when the distance is 91.4 million miles. In July, the distance is greatest at 94.4 million miles.
    a. Write 91.4 million in both standard form and in scientific notation.
    b. Write 94.4 million in both standard form and in scientific notation.
    c. What is the percent increase in distance from January to July? Round to the nearest tenth of a percent.

Evaluate each product or quotient. Express the results in both scientific notation and standard form.

55. $(4.65 \times 10^{-2})(5.91 \times 10^6)$
56. $\frac{2.548 \times 10^5}{2.8 \times 10^{-2}}$
57. $\frac{2.135 \times 10^5}{3.5 \times 10^{12}}$
58. $(3.16 \times 10^{-2})^2$
59. $(2.01 \times 10^{-4})(8.9 \times 10^{-3})$
60. $\frac{5.184 \times 10^{-5}}{7.2 \times 10^3}$
61. $(9.04 \times 10^6)(5.2 \times 10^{-4})$
62. $\frac{1.032 \times 10^{-4}}{8.6 \times 10^{-5}}$

LIGHT **The speed of light is approximately 3×10^8 meters per second.**

63. Write an expression to represent the speed of light in kilometers per second.
64. Write an expression to represent the speed of light in kilometers per hour.
65. Make a table to show how many kilometers light travels in a day, a week, a 30-day month, and a 365-day year. Express your results in scientific notation.
66. **CCSS MODELING** A recent cell phone study showed that company A's phone processes up to 7.95×10^5 bits of data every second. Company B's phone processes up to 1.41×10^6 bits of data every second. Evaluate and interpret $\frac{1.41 \times 10^6}{7.95 \times 10^5}$.

67. **EARTH** The population of Earth is about 6.623×10^9. The land surface of Earth is 1.483×10^8 square kilometers. What is the population density for the land surface area of Earth?

68. **RIVERS** A drainage basin separated from adjacent basins by a ridge, hill, or mountain is known as a watershed. The watershed of the Amazon River is 2,300,000 square miles. The watershed of the Mississippi River is 1,200,000 square miles.

a. Write each of these numbers in scientific notation.

b. How many times as large is the Amazon River watershed as the Mississippi River watershed?

69. **AGRICULTURE** In a recent year, farmers planted approximately 92.9 million acres of corn. They also planted 64.1 million acres of soybeans and 11.1 million acres of cotton.

a. Write each of these numbers in scientific notation and in standard form.

b. How many times as much corn was planted as soybeans? Write your results in standard form and in scientific notation. Round your answer to four decimal places.

c. How many times as much corn was planted as cotton? Write your results in standard form and in scientific notation. Round your answer to four decimal places.

H.O.T. Problems Use Higher-Order Thinking Skills

70. **REASONING** Which is greater, 100^{10} or 10^{100}? Explain your reasoning.

71. **ERROR ANALYSIS** Syreeta and Pete are solving a division problem with scientific notation. Is either of them correct? Explain your reasoning.

Syreeta	Pete
$\frac{3.65 \times 10^{-12}}{5 \times 10^{5}} = 0.73 \times 10^{-17}$ $= 7.3 \times 10^{-16}$	$\frac{3.65 \times 10^{-12}}{5 \times 10^{5}} = 0.73 \times 10^{-17}$ $= 7.3 \times 10^{-18}$

72. **CHALLENGE** Order these numbers from least to greatest without converting them to standard form.

$$5.46 \times 10^{-3}, 6.54 \times 10^{3}, 4.56 \times 10^{-4}, -5.64 \times 10^{4}, -4.65 \times 10^{5}$$

73. CCSS **ARGUMENTS** Determine whether the statement is *always*, *sometimes*, or *never* true. Give examples or a counterexample to verify your reasoning.

When multiplying two numbers written in scientific notation, the resulting number can have no more than two digits to the left of the decimal point.

74. **OPEN ENDED** Write two numbers in scientific notation with a product of 1.3×10^{-3}. Then name two numbers in scientific notation with a quotient of 1.3×10^{-3}.

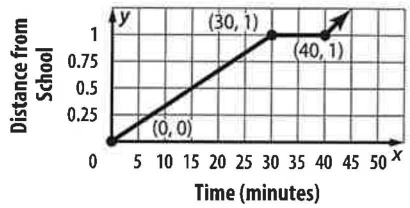
75. **WRITING IN MATH** Write the steps that you would use to divide two numbers written in scientific notation. Then describe how you would write the results in standard form. Demonstrate by finding $\frac{a}{b}$ for $a = 2 \times 10^3$ and $b = 4 \times 10^5$.

Standardized Test Practice

76. Which number represents 0.05604×10^8 written in standard form?

A 0.0000000005604 **C** 5,604,000
B 560,400 **D** 50,604,000

77. Toni left school and rode her bike home. The graph below shows the relationship between her distance from the school and time.



Which explanation could account for the section of the graph from $x = 30$ to $x = 40$?

F Toni rode her bike down a hill.
G Toni ran all the way home.
H Toni stopped at a friend's house on her way home.
J Toni returned to school to get her mathematics book.

78. SHORT RESPONSE In his first four years of coaching football, Coach Delgato's team won 5 games the first year, 10 games the second year, 8 games the third year, and 7 games the fourth year. How many games does the team need to win during the fifth year to have an average of 8 wins per year?

79. The table shows the relationship between Calories and grams of fat contained in an order of fried chicken from various restaurants.

Calories	305	410	320	500	510	440
Fat (g)	28	34	28	41	42	38

Assuming that the data can best be described by a linear model, about how many grams of fat would you expect to be in a 275-Calorie order of fried chicken?

A 22
B 25
C 28
D 30

Spiral Review

80. HEALTH A ponderal index p is a measure of a person's body based on height h in centimeters and mass m in kilograms. One such formula is $p = 100m^{\frac{1}{3}}h^{-1}$. If a person who is 182 centimeters tall has a ponderal index of about 2.2, how much does the person weigh in kilograms? (Lesson 7-3)

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

81. $\frac{8^9}{8^6}$ **82.** $\frac{6^5}{6^3}$ **83.** $\frac{r^8t^{12}}{r^2t^7}$

84. $\left(\frac{3a^4b^4}{8c^2}\right)^4$ **85.** $\left(\frac{5d^3g^2}{3h^4}\right)^2$ **86.** $\left(\frac{4n^2p^4}{8p^3}\right)^3$

87. CHEMISTRY Lemon juice is 10^2 times as acidic as tomato juice. Tomato juice is 10^3 times as acidic as egg whites. How many times as acidic is lemon juice as egg whites? (Lesson 7-2)

Skills Review

Evaluate $a(b^x)$ for each of the given values.

88. $a = 1, b = 2, x = 4$ **89.** $a = 4, b = 1, x = 7$ **90.** $a = 5, b = 3, x = 0$

91. $a = 0, b = 6, x = 8$ **92.** $a = -2, b = 3, x = 1$ **93.** $a = -3, b = 5, x = 2$

CHAPTER 7

Mid-Chapter Quiz

Lessons 7-1 through 7-4

Simplify each expression. (Lesson 7-1)

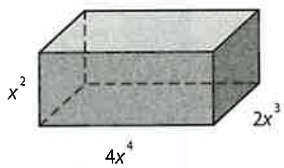
1. $(x^3)(4x^5)$

2. $(m^2p^5)^3$

3. $[(2xy^3)^2]^3$

4. $(6ab^3c^4)(-3a^2b^3c)$

5. MULTIPLE CHOICE Express the volume of the solid as a monomial. (Lesson 7-1)



A $6x^9$

B $8x^9$

C $8x^{24}$

D $7x^{24}$

Simplify each expression. Assume that no denominator equals 0. (Lesson 7-2)

6. $\left(\dfrac{2a^4b^3}{c^6}\right)^3$

7. $\dfrac{2xy^0}{6x}$

8. $\dfrac{m^7n^4p}{m^3n^3p}$

9. $\dfrac{p^4t^{-2}}{r^{-5}}$

10. ASTRONOMY Physicists estimate that the number of stars in the universe has an order of magnitude of 10^{21}. The number of stars in the Milky Way galaxy is around 100 billion. Using orders of magnitude, how many times as many stars are there in the universe as the Milky Way? (Lesson 7-2)

Write each expression in radical form, or write each radical in exponential form. (Lesson 7-3)

11. $42^{\frac{1}{2}}$

12. $11x^{\frac{1}{2}}$

13. $(11g)^{\frac{1}{2}}$

14. $\sqrt{55}$

15. $\sqrt{5k}$

16. $4\sqrt{p}$

Simplify. (Lesson 7-3)

17. $\sqrt[3]{729}$

18. $\sqrt[4]{625}$

19. $1331^{\frac{1}{3}}$

20. $\left(\dfrac{16}{81}\right)^{\frac{1}{4}}$

21. $8^{\frac{2}{3}}$

22. $625^{\frac{3}{4}}$

23. $216^{\frac{5}{3}}$

24. $\left(\dfrac{1}{4}\right)^{\frac{3}{2}}$

Solve each equation. (Lesson 7-3)

25. $4^x = 4096$

26. $5^{2x+1} = 125$

27. $4^{x-3} = 128$

Express each number in scientific notation. (Lesson 7-4)

28. 0.00000054

29. 0.0042

30. 234,000

31. 418,000,000

Express each number in standard form. (Lesson 7-4)

32. 4.1×10^{-3}

33. 2.74×10^5

34. 3×10^9

35. 9.1×10^{-5}

Evaluate each product or quotient. Express the results in scientific notation. (Lesson 7-4)

36. $(2.13 \times 10^2)(3 \times 10^5)$

37. $(7.5 \times 10^6)(2.5 \times 10^{-2})$

38. $\dfrac{7.5 \times 10^8}{2.5 \times 10^4}$

39. $\dfrac{6.6 \times 10^5}{2 \times 10^{-3}}$

40. MAMMALS A blue whale has been caught that was 4.2×10^5 pounds. The smallest mammal is a bumblebee bat, which is about 0.0044 pound. (Lesson 7-4)

a. Write the whale's weight in standard form.

b. Write the bat's weight in scientific notation.

c. How many orders of magnitude as big as a blue whale is a bumblebee bat?

EXPLORE 7-5 Graphing Technology Lab
Family of Exponential Functions

An **exponential function** is a function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. You have studied the effects of changing parameters in linear functions. You can use a graphing calculator to analyze how changing the parameters a and b affects the graphs in the family of exponential functions.

CCSS Common Core State Standards
Content Standards
F.IF.7e Graph exponential and logarithmic functions, showing intercepts and e
behavior, and trigonometric functions, showing period, midline, and amplitude.
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$,
$f(x + k)$ for specific values of k (both positive and negative); find the value of k g
the graphs. Experiment with cases and illustrate an explanation of the effects on
graph using technology.

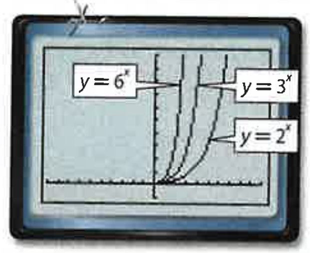
PT

Activity 1 b in $y = b^x$, $b > 1$

Graph the set of equations on the same screen.
Describe any similarities and differences among the graphs.

$y = 2^x$, $y = 3^x$, $y = 6^x$

Enter the equations in the [Y=] list and graph.

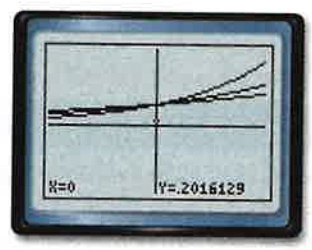


[−10, 10] scl: 1 by [−10, 100] scl: 10

There are many similarities in the graphs. The domain for each function is all real numbers, and the range is all positive real numbers. The functions are increasing over the entire domain. The graphs do not display any line symmetry.

Use the [ZOOM] feature to investigate the key features of the graphs.

Zooming in twice on a point near the origin allows closer inspection of the graphs. The y-intercept is 1 for all three graphs.



[−0.625, 0.625] scl: 1 by [−3.25..., 3.63...] scl: 10

Tracing along the graphs reveals that there are no x-intercepts, no maxima and no minima.

The graphs are different in that the graphs for the equations in which b is greater are steeper.

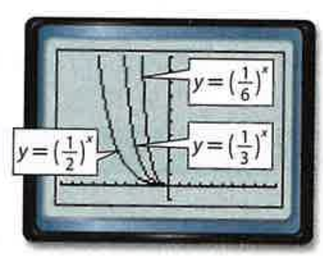
The effect of b on the graph is different when $0 < b < 1$.

Activity 2 b in $y = b^x$, $0 < b < 1$

Graph the set of equations on the same screen.
Describe any similarities and differences among the graphs.

$y = \left(\frac{1}{2}\right)^x$, $y = \left(\frac{1}{3}\right)^x$, $y = \left(\frac{1}{6}\right)^x$

The domain for each function is all real numbers, and the range is all positive real numbers. The function values are all positive and the functions are decreasing over the entire domain. The graphs display no line symmetry. There are no x-intercepts, and the y-intercept is 1 for all three graphs. There are no maxima or minima.



[−10, 10] scl: 1 by [−10, 100] scl: 10

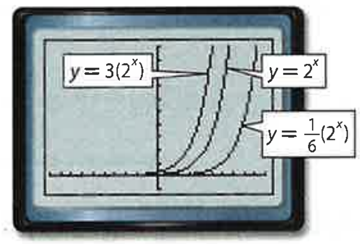
However, the graphs in which b is lesser are steeper.

Activity 3 a in $y = ab^x$, $a > 0$

Graph each set of equations on the same screen. Describe any similarities and differences among the graphs.

$y = 2^x$, $y = 3(2^x)$, $y = \frac{1}{6}(2^x)$

The domain for each function is all real numbers, and the range is all positive real numbers. The functions are increasing over the entire domain. The graphs do not display any line symmetry.



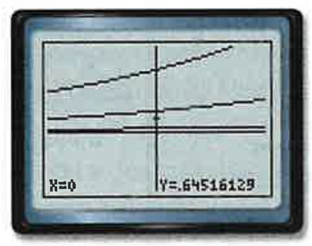
[−10, 10] scl: 1 by [−10, 100] scl: 10

Use the ZOOM feature to investigate the key features of the graphs.

Zooming in twice on a point near the origin allows closer inspection of the graphs.

Tracing along the graphs reveals that there are no x-intercepts, no maxima and no minima.

However, the graphs in which a is greater are steeper. The y-intercept is 1 in the graph of $y = 2^x$, 3 in $y = 3(2^x)$, and $\frac{1}{6}$ in $y = \frac{1}{6}(2^x)$.



[−0.625, 0.625] scl: 1 by [−2.79..., 4.08...] scl: 10

Activity 4 a in $y = ab^x$, $a < 0$

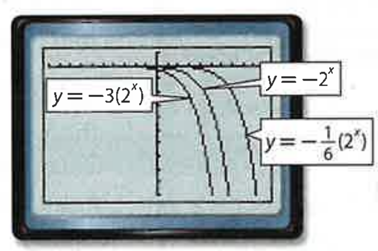
Graph each set of equations on the same screen. Describe any similarities and differences among the graphs.

$y = -2^x$, $y = -3(2^x)$, $y = -\frac{1}{6}(2^x)$

The domain for each function is all real numbers, and the range is all negative real numbers. The functions are decreasing over the entire domain. The graphs do not display any line symmetry.

There are no x-intercepts, no maxima and no minima.

However, the graphs in which the absolute value of a is greater are steeper. The y-intercept is -1 in the graph of $y = -2^x$, -3 in $y = -3(2^x)$, and $-\frac{1}{6}$ in $y = -\frac{1}{6}(2^x)$.



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Model and Analyze

1. How does b affect the graph of $y = ab^x$ when $b > 1$ and when $0 < b < 1$? Give examples.

2. How does a affect the graph of $y = ab^x$ when $a > 0$ and when $a < 0$? Give examples.

3. **CCSS REGULARITY** Make a conjecture about the relationship of the graphs of $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$. Verify your conjecture by graphing both functions.

LESSON 7-5

Exponential Functions

:: Then

- You evaluated numerical expressions involving exponents.

:: Now

1. Graph exponential functions.
2. Identify data that display exponential behavior.

:: Why?

- Tarantulas can appear scary with their large hairy bodies and legs, but they are harmless to humans. The graph shows a tarantula spider population that increases over time. Notice that the graph is not linear.

The graph represents the function $y = 3(2)^x$. This is an example of an *exponential* function.

NewVocabulary

exponential function
exponential growth function
exponential decay function

Common Core State Standards

Content Standards

F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Graph Exponential Functions

An **exponential function** is a function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. Notice that the base is a constant and the exponent is a variable. Exponential functions are nonlinear.

KeyConcept Exponential Function

Words An exponential function is a function that can be described by an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

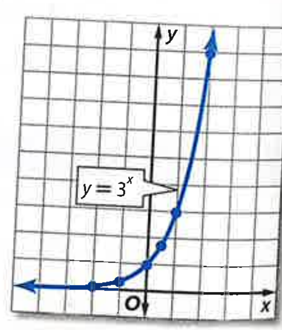
Examples $y = 2(3)^x$ $\quad y = 4^x$ $\quad y = \left(\frac{1}{2}\right)^x$

Example 1 Graph with $a > 0$ and $b > 1$

Graph $y = 3^x$. Find the y-intercept, and state the domain and range.

The graph crosses the y-axis at 1, so the y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers.

x	3^x	y
-2	3^{-2}	$\frac{1}{9}$
-1	3^{-1}	$\frac{1}{3}$
0	3^0	1
$\frac{1}{2}$	$3^{\frac{1}{2}}$	≈ 1.73
1	3^1	3
2	3^2	9



Notice that the graph approaches the x-axis but there is no x-intercept. The graph is increasing on the entire domain.

GuidedPractice

1. Graph $y = 7^x$. Find the y-intercept, and state the domain and range.

Functions of the form $y = ab^x$, where $a > 0$ and $b > 1$, are called **exponential growth functions** and all have the same shape as the graph in Example 1. Functions of the form $y = ab^x$, where $a > 0$ and $0 < b < 1$ are called **exponential decay functions** and also have the same general shape.

StudyTip

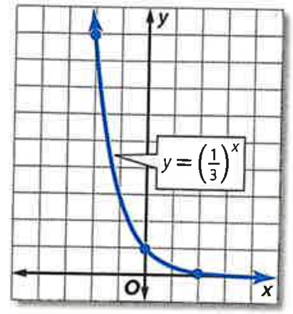
$a < 0$ If the value of a is less than 0, the graph will be reflected across the x-axis.

Example 2 Graph with $a > 0$ and $0 < b < 1$

Graph $y = \left(\frac{1}{3}\right)^x$. Find the y-intercept, and state the domain and range.

The y-intercept is 1. The domain is all real numbers, and the range is all positive real numbers. Notice that as x increases, the y-values decrease less rapidly.

x	$\left(\frac{1}{3}\right)^x$	y
−2	$\left(\frac{1}{3}\right)^{-2}$	9
0	$\left(\frac{1}{3}\right)^0$	1
2	$\left(\frac{1}{3}\right)^2$	$\frac{1}{9}$



GuidedPractice

2. Graph $y = \left(\frac{1}{2}\right)^x - 1$. Find the y-intercept, and state the domain and range.

The key features of the graphs of exponential functions can be summarized as follows.

KeyConcept Graphs of Exponential Functions

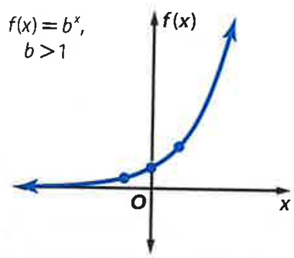
Exponential Growth Functions

Equation: $f(x) = ab^x$, $a > 0$, $b > 1$

Domain, Range: all reals; all positive reals

Intercepts: one y-intercept, no x-intercepts

End behavior: as x increases, $f(x)$ increases; as x decreases, $f(x)$ approaches 0



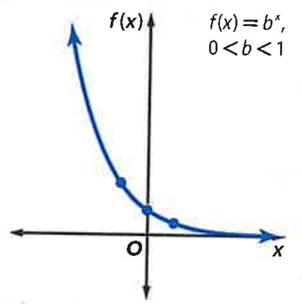
Exponential Decay Functions

Equation: $f(x) = ab^x$, $a > 0$, $0 < b < 1$

Domain, Range: all reals; all positive reals

Intercepts: one y-intercept, no x-intercepts

End behavior: as x increases, $f(x)$ approaches 0; as x decreases, $f(x)$ increases



Exponential functions occur in many real world situations.

BananaStock/PunchStock

Real-WorldLink

The United States is the largest soda consumer in the world. In a recent year, the United States accounted for one third of the world's total soda consumption.

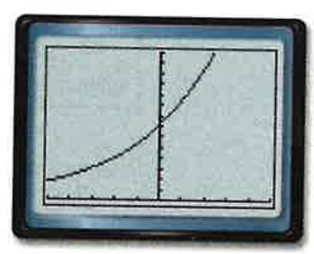
Source: Worldwatch Institute

Real-World Example 3 Use Exponential Functions to Solve Problems

SODA The function $C = 179(1.029)^t$ models the amount of soda consumed in the world, where C is the amount consumed in billions of liters and t is the number of years since 2000.

a. Graph the function. What values of C and t are meaningful in the context of the problem?

Since t represents time, $t > 0$. At $t = 0$, the consumption is 179 billion liters. Therefore, in the context of this problem, $C > 179$ is meaningful.



[−50, 50] scl: 10 by [0, 350] scl: 25

b. How much soda was consumed in 2005?

$C = 179(1.029)^t$ Original equation

$= 179(1.029)^5$ $t = 5$

≈ 206.5 Use a calculator.

The world soda consumption in 2005 was approximately 206.5 billion liters.

GuidedPractice

3. BIOLOGY A certain bacteria population doubles every 20 minutes. Beginning with 10 cells in a culture, the population can be represented by the function $B = 10(2)^t$, where B is the number of bacteria cells and t is the time in 20 minute increments. How many will there be after 2 hours?

2 Identify Exponential Behavior

Identify Exponential Behavior Recall from Lesson 3-3 that linear functions have a constant rate of change. Exponential functions do not have constant rates of change, but they do have constant ratios.

Problem-SolvingTip

Make an Organized List Making an organized list of x-values and corresponding y-values is helpful in graphing the function. It can also help you identify patterns in the data.

Example 4 Identify Exponential Behavior

Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	5	10	15	20	25
y	64	32	16	8	4	2

Method 1 Look for a pattern.

The domain values are at regular intervals of 5. Look for a common factor among the range values.

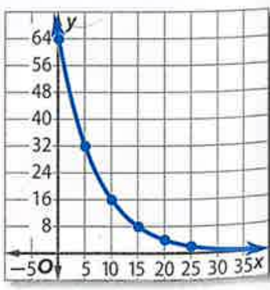
$$64 \xrightarrow{\times\frac{1}{2}} 32 \xrightarrow{\times\frac{1}{2}} 16 \xrightarrow{\times\frac{1}{2}} 8 \xrightarrow{\times\frac{1}{2}} 4 \xrightarrow{\times\frac{1}{2}} 2$$

The range values differ by the common factor of $\frac{1}{2}$.

Since the domain values are at regular intervals and the range values differ by a positive common factor, the data are probably exponential. Its equation may involve $\left(\frac{1}{2}\right)^x$.

Method 2 Graph the data.

Plot the points and connect them with a smooth curve. The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior in which the base is between 0 and 1.



GuidedPractice

4. Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.

x	0	3	6	9	12	15
y	12	16	20	24	28	32

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 **Graph each function. Find the y-intercept and state the domain and range.**

1. $y = 2^x$
2. $y = -5^x$
3. $y = -\left(\frac{1}{5}\right)^x$
4. $y = 3\left(\frac{1}{4}\right)^x$
5. $f(x) = 6^x + 3$
6. $f(x) = 2 - 2^x$

Example 3

7. **BIOLOGY** The function $f(t) = 100(1.05)^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.
   a. What values for the domain and range are reasonable in the context of this situation? Explain.
   b. After two weeks, approximately how many flies are in this population?

Example 4 **Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.**

8.

x	1	2	3	4	5	6
y	−4	−2	0	2	4	6

9.

x	2	4	6	8	10	12
y	1	4	16	64	256	1024

Practice and Problem Solving

Extra Practice is on page R7.

Examples 1–2 **Graph each function. Find the y-intercept and state the domain and range.**

10. $y = 2 \cdot 8^x$
11. $y = 2 \cdot \left(\frac{1}{6}\right)^x$
12. $y = \left(\frac{1}{12}\right)^x$
13. $y = -3 \cdot 9^x$
14. $y = -4 \cdot 10^x$
15. $y = 3 \cdot 11^x$
16. $y = 4^x + 3$
17. $y = \frac{1}{2}(2^x - 8)$
18. $y = 5(3^x) + 1$
19. $y = -2(3^x) + 5$

Example 3

20. **CCSS MODELING** A population of bacteria in a culture increases according to the model $p = 300(2.7)^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.
   a. Use this model to estimate the number of bacteria at 11 A.M.
   b. Graph the function and name the p-intercept. Describe what the p-intercept represents, and describe a reasonable domain and range for this situation.

Example 4 **Determine whether the set of data shown below displays exponential behavior. Write *yes* or *no*. Explain why or why not.**

21.

x	−4	0	4	8	12
y	2	−4	8	−16	32

22.

x	−6	−3	0	3
y	5	10	15	20

23.

x	−8	−6	−4	−2
y	0.25	0.5	1	2

24.

x	20	30	40	50	60
y	1	0.4	0.16	0.064	0.0256

25. **PHOTOGRAPHY** Jameka is enlarging a photograph to make a poster for school. She will enlarge the picture repeatedly at 150%. The function $P = 1.5^x$ models the new size of the picture being enlarged, where x is the number of enlargements. How many times as big is the picture after 4 enlargements?

26. **FINANCIAL LITERACY** Daniel deposited \$500 into a savings account and after 8 years, his investment is worth \$807.07. The equation $A = d(1.005)^{12t}$ models the value of Daniel's investment A after t years with an initial deposit d.

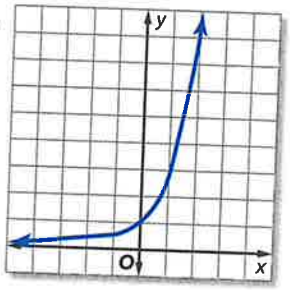
a. What would the value of Daniel's investment be if he had deposited \$1000?

b. What would the value of Daniel's investment be if he had deposited \$250?

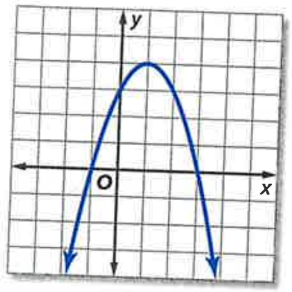
c. Interpret $d(1.005)^{12t}$ to explain how the amount of the original deposit affects the value of Daniel's investment.

Identify each function as *linear*, *exponential*, or *neither*.

27.



28.



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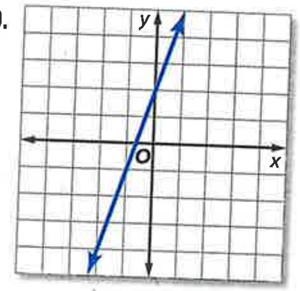


30. $y = 4^x$

31. $y = 2x(x - 1)$

32. $5x + y = 8$

33. **GRADUATION** The number of graduates at a high school has increased by a factor of 1.055 every year since 2001. In 2001, 110 students graduated. The function $N = 110(1.055)^t$ models the number of students N expected to graduate t years after 2001. How many students will graduate in 2012?

Describe the graph of each equation as a transformation of the graph of $y = 2^x$.

34. $y = 2^x + 6$

35. $y = 3(2)^x$

36. $y = -\frac{1}{4}(2)^x$

37. $y = -3 + 2^x$

38. $y = \left(\frac{1}{2}\right)^x$

39. $y = -5(2)^x$

40. **DEER** The deer population at a national park doubles every year. In 2000, there were 25 deer in the park. The function $N = 25(2)^t$ models the number of deer N in the park t years after 2000. What will the deer population be in 2015?

H.O.T. Problems Use Higher-Order Thinking Skills

41. CCSS **PERSEVERANCE** Write an exponential function for which the graph passes through the points at (0, 3) and (1, 6).

42. **REASONING** Determine whether the graph of $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, *sometimes*, *always*, or *never* has an x-intercept. Explain your reasoning.

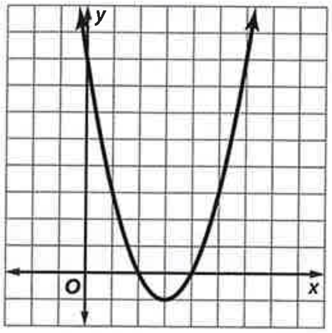
43. **OPEN ENDED** Find an exponential function that represents a real-world situation, and graph the function. Analyze the graph, and explain why the situation is modeled by an exponential function rather than a linear function.

44. **REASONING** Use tables and graphs to compare and contrast an exponential function $f(x) = ab^x + c$, where $a \neq 0$, $b > 0$, and $b \neq 1$, and a linear function $g(x) = ax + c$. Include intercepts, intervals where the functions are increasing, decreasing, positive, or negative, relative maxima and minima, symmetry, and end behavior.

45. **WRITING IN MATH** Explain how to determine whether a set of data displays exponential behavior.

Standardized Test Practice

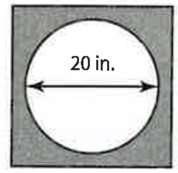
46. SHORT RESPONSE What are the x-intercepts of the function graphed below?



47. Hinto invested \$300 into a savings account. The equation $A = 300(1.005)^{12t}$ models the amount in Hinto's account A after t years. How much will be in Hinto's account after 7 years?

A \$25,326
B \$456.11
C \$385.01
D \$301.52

48. GEOMETRY Ayana placed a circular piece of paper on a square picture as shown below. If the picture extends 4 inches beyond the circle on each side, what is the perimeter of the square picture?



F 64 in.
G 80 in.
H 94 in.
J 112 in.

49. The points with coordinates $(0, -3)$ and $(2, 7)$ are on line ℓ. Line p contains $(3, -1)$ and is perpendicular to line ℓ. What is the x-coordinate of the point where ℓ and p intersect?

A $\frac{1}{2}$
B $-\frac{2}{5}$
C $-\frac{1}{2}$
D -3

Spiral Review

Evaluate each product. Express the results in both scientific notation and standard form. (Lesson 7-4)

50. $(1.9 \times 10^2)(4.7 \times 10^6)$

51. $(4.5 \times 10^{-3})(5.6 \times 10^4)$

52. $(3.8 \times 10^{-4})(6.4 \times 10^{-8})$

Simplify. (Lesson 7-3)

53. $\sqrt[3]{343}$

54. $\sqrt[6]{729}$

55. $\left(\frac{1}{32}\right)^{\frac{1}{5}}$

56. $729^{\frac{5}{6}}$

57. $216^{\frac{5}{3}}$

58. $\left(\frac{1}{81}\right)^{\frac{3}{2}}$

59. DEMOLITION DERBY When a car hits an object, the damage is measured by the collision impact. For a certain car the collision impact I is given by $I = 2v^2$, where v represents the speed in kilometers per minute. What is the collision impact if the speed of the car is 4 kilometers per minute? (Lesson 7-1)

Use elimination to solve each system of equations. (Lesson 6-3)

60. $x + y = -3$
$x - y = 1$

61. $3a + b = 5$
$2a + b = 10$

62. $3x - 5y = 16$
$-3x + 2y = -10$

Skills Review

Find the next three terms of each arithmetic sequence.

63. $1, 3, 5, 7, \ldots$

64. $-6, -4, -2, 0, \ldots$

65. $6.5, 9, 11.5, 14, \ldots$

66. $10, 3, -4, -11, \ldots$

67. $\frac{1}{2}, \frac{5}{4}, 2, \frac{11}{4}, \ldots$

68. $1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \ldots$

EXTEND 7-5

Graphing Technology Lab: Solving Exponential Equations and Inequalities

You can use TI-Nspire Technology to solve exponential equations and inequalities by graphing and by using tables.

CCSS Common Core State Standards
Content Standards
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Mathematical Practices
5 Use appropriate tools strategically.

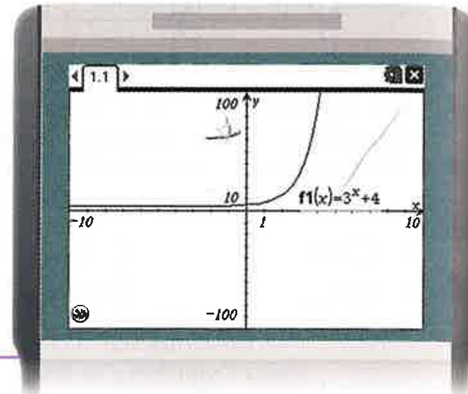
Activity 1 Graph an Exponential Equation

Graph $y = 3^x + 4$ using a graphing calculator.

Step 1 Add a new **Graphs** page.

Step 2 Enter $3^x + 4$ as **f1(x)**.

Step 3 Use the **Window Settings** option from the **Window/Zoom** menu to adjust the window so that x is from -10 to 10 and y is from -100 to 100. Keep the scales as **Auto**.



To solve an equation by graphing, graph both sides of the equation and locate the point(s) of intersection.

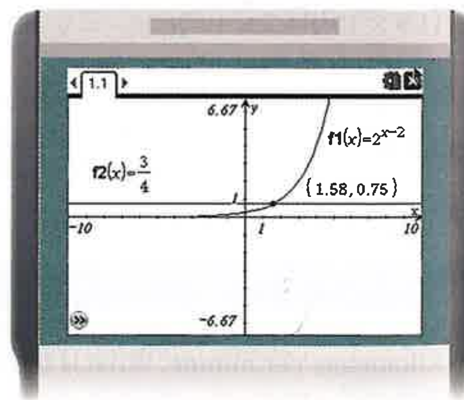
Activity 2 Solve an Exponential Equation by Graphing

Solve $2^{x-2} = \frac{3}{4}$.

Step 1 Add a new **Graphs** page.

Step 2 Enter 2^{x-2} as **f1(x)** and $\frac{3}{4}$ as **f2(x)**.

Step 3 Use the **Intersection Point(s)** tool from the **Points & Lines** menu to find the intersection of the two graphs. Select the graph of **f1(x) enter** and then the graph of **f2(x) enter**.



The graphs intersect at about (1.58, 0.75). Therefore, the solution of $2^{x-2} = \frac{3}{4}$ is 1.58.

Exercises

CCSS TOOLS Use a graphing calculator to solve each equation.

1. $\left(\frac{1}{3}\right)^{x-1} = \frac{3}{4}$

2. $2^{2x-1} = 2x$

3. $\left(\frac{1}{2}\right)^{2x} = 2^{2x}$

4. $5^{\frac{1}{3}x+2} = -x$

5. $\left(\frac{1}{8}\right)^{2x} = -2x + 1$

6. $2^{\frac{1}{4}x-1} = 3^{x+1}$

7. $2^{3x-1} = 4^x$

8. $4^{2x-3} = 5^{-x+1}$

9. $3^{2x-4} = 2^x + 1$

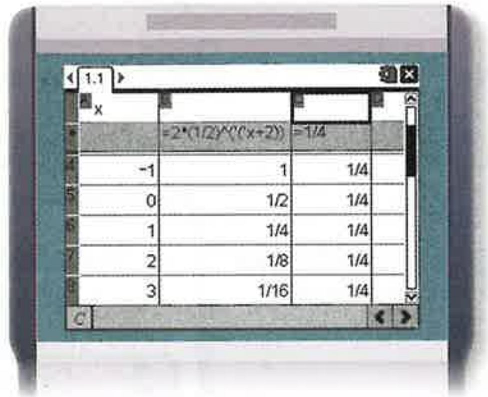
Activity 3 Solve an Exponential Equation by Using a Table

Solve $2\left(\frac{1}{2}\right)^{x+2} = \frac{1}{4}$ using a table.

Step 1 Add a new **Lists & Spreadsheet** page.

Step 2 Label column A as x. Enter values from -4 to 4 in cells A1 to A9.

Step 3 In column B in the formula row, enter the left side of the rational equation. In column C of the formula row, enter $=\frac{1}{4}$. Specify **Variable Reference** when prompted.



Scroll until you see where the values in Columns B and C are equal. This occurs at $x = 1$. Therefore, the solution of $2\left(\frac{1}{2}\right)^{x+2} = \frac{1}{4}$ is 1.

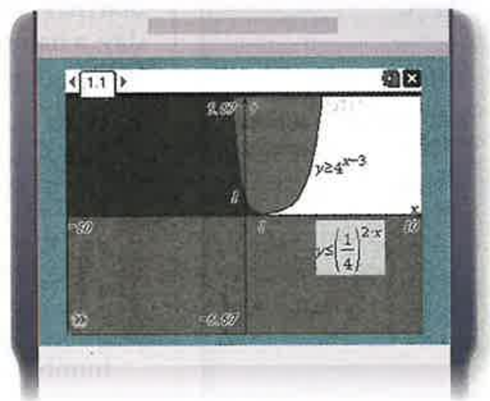
You can also use a graphing calculator to solve exponential inequalities.

Activity 4 Solve an Exponential Inequality

Solve $4^{x-3} \leq \left(\frac{1}{4}\right)^{2x}$.

Step 1 Add a new **Graphs** page.

Step 2 Enter the left side of the inequality into **f1(*x*)**. Press **del**, select $\geq$, and enter 4^{x-3}. Enter the right side of the inequality into **f2(*x*)**. Press **tab del** $\leq$, and enter $\left(\frac{1}{4}\right)^{2x}$.



The x-values of the points in the region where the shading overlap is the solution set of the original inequality. Therefore the solution of $4^{x-3} \leq \left(\frac{1}{4}\right)^{2x}$ is $x \leq 1$.

Exercises

CCSS TOOLS Use a graphing calculator to solve each equation or inequality.

10. $\left(\frac{1}{3}\right)^{3x} = 3^x$

11. $\left(\frac{1}{6}\right)^{2x} = 4^x$

12. $3^{1-x} \leq 4^x$

13. $4^{3x} \leq 2x + 1$

14. $\left(\frac{1}{4}\right)^{x} > 2^{x+4}$

15. $\left(\frac{1}{3}\right)^{x-1} \geq 2^x$

LESSON 7-6

Growth and Decay

Then

- You analyzed exponential functions.

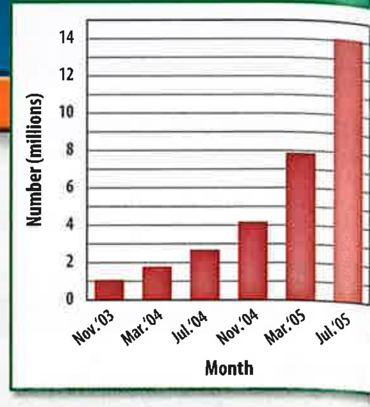
Now

1. Solve problems involving exponential growth.
2. Solve problems involving exponential decay.

Why?

- The number of Weblogs or blogs increased at a monthly rate of about 13.7% over 21 months. The average number of blogs per month can be modeled by $y = 1.1(1 + 0.137)^t$ or $y = 1.1(1.137)^t$, where y represents the total number of blogs in millions and t is the number of months since November 2003.

Growth of Blogs



NewVocabulary

compound interest

Common Core State Standards

Content Standards

F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Mathematical Practices

4 Model with mathematics.

1 Exponential Growth

The equation for the number of blogs is in the form $y = a(1 + r)^t$. This is the general equation for exponential growth.

KeyConcept Equation for Exponential Growth

a is the initial amount. t is time.

$$y = a(1 + r)^t$$

y is the final amount. r is the rate of change expressed as a decimal, $r > 0$.

Real-World Example 1 Exponential Growth

CONTEST **The prize for a radio station contest begins with a \$100 gift card. Once a day, a name is announced. The person has 15 minutes to call or the prize increases by 2.5% for the next day.**

a. Write an equation to represent the amount of the gift card in dollars after t days with no winners.

$y = a(1 + r)^t$	Equation for exponential growth
$y = 100(1 + 0.025)^t$	$a = 100$ and $r = 2.5\%$ or 0.025
$y = 100(1.025)^t$	Simplify.

In the equation $y = 100(1.025)^t$, y is the amount of the gift card and t is the number of days since the contest began.

b. How much will the gift card be worth if no one wins after 10 days?

$y = 100(1.025)^t$	Equation for amount of gift card
$= 100(1.025)^{10}$	$t = 10$
≈ 128.01	Use a calculator.

In 10 days, the gift card will be worth \$128.01.

GuidedPractice

1. **TUITION** A college's tuition has risen 5% each year since 2000. If the tuition in 2000 was \$10,850, write an equation for the amount of the tuition t years after 2000. Predict the cost of tuition for this college in 2015.

Compound interest is interest earned or paid on both the initial investment and previously earned interest. It is an application of exponential growth.

KeyConcept Equation for Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A is the current amount.

n is the number of times the interest is compounded each year, and *t* is time in years.

P is the principal or initial amount.

r is the annual interest rate expressed as a decimal, $r > 0$.

Real-WorldCareer

Financial Advisor Financial advisors help people plan their financial futures. A good financial advisor has mathematical, problem-solving, and communication skills. A bachelor's degree is strongly preferred but not required.

Real-World Example 2 Compound Interest

FINANCE **Maria's parents invested \$14,000 at 6% per year compounded monthly. How much money will there be in the account after 10 years?**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compound interest equation
$= 14{,}000\left(1 + \frac{0.06}{12}\right)^{12(10)}$	$P = 14{,}000$, $r = 6\%$ or 0.06, $n = 12$, and $t = 10$
$= 14{,}000(1.005)^{120}$	Simplify.
$\approx 25{,}471.55$	Use a calculator.

There will be about \$25,471.55 in 10 years.

GuidedPractice

2. **FINANCE** Determine the amount of an investment if \$300 is invested at an interest rate of 3.5% compounded monthly for 22 years.

2 Exponential Decay

In exponential decay, the original amount decreases by the same percent over a period of time. A variation of the growth equation can be used as the general equation for exponential decay.

StudyTip

Growth and Decay Since *r* is added to 1, the value inside the parentheses will be greater than 1 for exponential growth functions. For exponential decay functions, this value will be less than 1 since *r* is subtracted from 1.

KeyConcept Equation for Exponential Decay

$$y = a(1 - r)^t$$

a is the initial amount.

t is time.

y is the final amount.

r is the rate of decay expressed as a decimal, $0 < r < 1$.

Real-World Example 3 Exponential Decay

SWIMMING **A fully inflated child's raft for a pool is losing 6.6% of its air every day. The raft originally contained 4500 cubic inches of air.**

a. Write an equation to represent the loss of air.

$y = a(1 - r)^t$	Equation for exponential decay
$= 4500(1 - 0.066)^t$	$a = 4500$ and $r = 6.6\%$ or 0.066
$= 4500(0.934)^t$	Simplify.

$y = 4500(0.934)^t$, where y is the air in the raft in cubic inches after t days.

b. Estimate the amount of air in the raft after 7 days.

$y = 4500(0.934)^t$ — Equation for air loss

$= 4500(0.934)^7$ — $t = 7$

≈ 2790 — Use a calculator.

The amount of air in the raft after 7 days will be about 2790 cubic inches.

GuidedPractice

3. POPULATION The population of Campbell County, Kentucky, has been decreasing at an average rate of about 0.3% per year. In 2000, its population wa 88,647. Write an equation to represent the population since 2000. If the trend continues, predict the population in 2010.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1

1. SALARY Ms. Acosta received a job as a teacher with a starting salary of $34,000. According to her contract, she will receive a 1.5% increase in her salary every year. How much will Ms. Acosta earn in 7 years?

Example 2

2. MONEY Paul invested $400 into an account with a 5.5% interest rate compounded monthly. How much will Paul's investment be worth in 8 years?

Example 3

3. ENROLLMENT In 2000, 2200 students attended Polaris High School. The enrollment has been declining 2% annually.

a. Write an equation for the enrollment of Polaris High School t years after 2000.

b. If this trend continues, how many students will be enrolled in 2015?

Practice and Problem Solving

Extra Practice is on page R7

Example 1

4. MEMBERSHIPS The Work-Out Gym sold 550 memberships in 2001. Since then the number of memberships sold has increased 3% annually.

a. Write an equation for the number of memberships sold at Work-Out Gym t years after 2001.

b. If this trend continues, predict how many memberships the gym will sell in 2020.

5. COMPUTERS The number of people who own computers has increased 23.2% annually since 1990. If half a million people owned a computer in 1990, predict how many people will own a computer in 2015.

6. COINS Camilo purchased a rare coin from a dealer for $300. The value of the coin increases 5% each year. Determine the value of the coin in 5 years.

Example 2

7. INVESTMENTS Theo invested $6600 at an interest rate of 4.5% compounded monthly. Determine the value of his investment in 4 years.

8. COMPOUND INTEREST Paige invested $1200 at an interest rate of 5.75% compounded quarterly. Determine the value of her investment in 7 years.

9. CCSS **PRECISION** Brooke is saving money for a trip to the Bahamas that costs $295.99. She puts $150 into a savings account that pays 7.25% interest compounded quarterly. Will she have enough money in the account after 4 years? Explain.

Example 3

10. INVESTMENTS Jin's investment of $4500 has been losing its value at a rate of 2.5% each year. What will his investment be worth in 5 years?

11. **POPULATION** In the years from 2010 to 2015, the population of the District of Columbia is expected decrease about 0.9% annually. In 2010, the population was about 530,000. What is the population of the District of Columbia expected to be in 2015?

12. **CARS** Leonardo purchases a car for \$18,995. The car depreciates at a rate of 18% annually. After 6 years, Manuel offers to buy the car for \$4500. Should Leonardo sell the car? Explain.

13. **HOUSING** The median house price in the United States increased an average of 1.4% each year between 2005 and 2007. Assume that this pattern continues.

a. Write an equation for the median house price for t years after 2007.

b. Predict the median house price in 2018.

Median House Price	
2005	\$240,900
2006	\$246,500
2007	\$247,900

Source: *Real Estate Journal*

14. **ELEMENTS** A radioactive element's half-life is the time it takes for one half of the element's quantity to decay. The half-life of Plutonium-241 is 14.4 years. The number of grams A of Plutonium-241 left after t years can be modeled by $A = p(0.5)^{\frac{t}{14.4}}$, where p is the original amount of the element.

a. How much of a 0.2-gram sample remains after 72 years?

b. How much of a 5.4-gram sample remains after 1095 days?

15. **COMBINING FUNCTIONS** A swimming pool holds a maximum of 20,500 gallons of water. It evaporates at a rate of 0.5% per hour. The pool currently contains 19,000 gallons of water.

a. Write an exponential function $w(t)$ to express the amount of water remaining in the pool after time t where t is the number of hours after the pool has reached 19,000 gallons.

b. At this same time, a hose is turned on to refill the pool at a net rate of 300 gallons per hour. Write a function $p(t)$ where t is time in hours the hose is running to express the amount of water that is pumped into the pool.

c. Find $C(t) = p(t) + w(t)$. What does this new function represent?

d. Use the graph of $C(t)$ to determine how long the hose must run to fill the pool to its maximun capacity.

H.O.T. Problems Use Higher-Order Thinking Skills

16. **REASONING** Determine the growth rate (as a percent) of a population that quadruples every year. Explain.

17. CCSS **PRECISION** Santos invested \$1200 into an account with an interest rate of 8% compounded monthly. Use a calculator to approximate how long it will take for Santos's investment to reach \$2500.

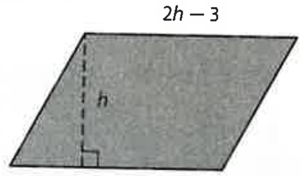
18. **REASONING** The amount of water in a container doubles every minute. After 8 minutes, the container is full. After how many minutes was the container half full? Explain.

19. **WRITING IN MATH** What should you consider when using exponential models to make decisions?

20. **WRITING IN MATH** Compare and contrast the exponential growth formula and the exponential decay formula.

Standardized Test Practice

21. GEOMETRY The parallelogram has an area of 35 square inches. Find the height h of the parallelogram.



A 3.5 inches
B 4 inches
C 5 inches
D 7 inches

22. Which is greater than $64^{\frac{1}{3}}$?

F 2^2
G $64^{\frac{1}{6}}$
H $64^{\frac{1}{2}}$
J 64^{-3}

23. Thi purchased a car for $22,900. The car depreciated at an annual rate of 16%. Which of the following equations models the value of Thi's car after 5 years?

A $A = 22{,}900(1.16)^5$
B $A = 22{,}900(0.16)^5$
C $A = 16(22{,}900)^5$
D $A = 22{,}900(0.84)^5$

24. GRIDDED RESPONSE A deck measures 12 feet by 18 feet. If a painter charges $2.65 per square foot, including tax, how much will it cost in dollars to have the deck painted?

Spiral Review

Graph each function. Find the y-intercept and state the domain and range. (Lesson 7-5)

25. $y = 3^x$

26. $y = \left(\frac{1}{2}\right)^x$

27. $y = 6^x$

Evaluate each product. Express the results in both scientific notation and standard form. (Lesson 7-4)

28. $(4.2 \times 10^3)(3.1 \times 10^{10})$

29. $(6.02 \times 10^{23})(5 \times 10^{-14})$

30. $(7 \times 10^5)^2$

31. $(1.1 \times 10^{-2})^2$

32. $(9.1 \times 10^{-2})(4.2 \times 10^{-7})$

33. $(3.14 \times 10^2)(6.1 \times 10^{-3})$

34. EVENT PLANNING A hall does not charge a rental fee as long as at least $4000 is spent on food. For the prom, the hall charges $28.95 per person for a buffet. How many people must attend the prom to avoid a rental fee for the hall? (Lesson 5-2)

Determine whether the graphs of each pair of equations are *parallel, perpendicular,* or *neither*. (Lesson 4-4)

35. $y = -2x + 11$
$y + 2x = 23$

36. $3y = 2x + 14$
$-3x - 2y = 2$

37. $y = -5x$
$y = 5x - 18$

38. AGES The table shows equivalent ages for horses and humans. Write an equation that relates human age to horse age and find the equivalent horse age for a human who is 16 years old. (Lesson 3-4)

Horse age (x)	0	1	2	3	4	5
Human age (y)	0	3	6	9	12	15

Find the total price of each item. (Lesson 2-7)

39. umbrella: $14.00
tax: 5.5%

40. sandals: $29.99
tax: 5.75%

41. backpack: $35.00
tax: 7%

Skills Review

Graph each set of ordered pairs.

42. (3, 0), (0, 1), (−4, −6)

43. (0, −2), (−1, −6), (3, 4)

44. (2, 2), (−2, −3), (−3, −6)

7-6 Algebra Lab
Transforming Exponential Expressions

You can use the properties of rational exponents to transform exponential functions into other forms in order to solve real-world problems.

CCSS Common Core State Standards
Content Standards
A.SSE.3c Use the properties of exponents to transform expressions for exponential functions.
F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.

Activity Write Equivalent Exponential Expressions

Monique is trying to decide between two savings account plans. Plan A offers a monthly compounding interest rate of 0.25%, while Plan B offers 2.5% interest compounded annually. Which is the better plan? Explain.

In order to compare the plans, we must compare rates with the same compounding frequency. One way to do this is to compare the approximate monthly interest rates of each plan, also called the *effective* monthly interest rate. While you can use the compound interest formula to find this rate, you can also use the properties of exponents.

Write a function to represent the amount A Monique would earn after t years with Plan B. For convenience, let the initial amount of Monique's investment be \$1.

$y = a(1 + r)^t$	Equation for exponential growth
$A(t) = 1(1 + 0.025)^t$	$y = A(t)$, $a = 1$, $r = 2.5\%$ or 0.025
$= 1.025^t$	Simplify.

Now write a function equivalent to $A(t)$ that represents 12 compoundings per year, with a power of $12t$, instead of 1 per year, with a power of $1t$.

$A(t) = 1.025^{1t}$	Original function
$= 1.025^{\left(\frac{1}{12} \cdot 12\right)t}$	$1 = \frac{1}{12} \cdot 12$
$= \left(1.025^{\frac{1}{12}}\right)^{12t}$	Power of a Power
$\approx 1.0021^{12t}$	$(1.025)^{\frac{1}{12}} = \sqrt[12]{1.025}$ or about 1.0021

From this equivalent function, we can determine that the effective monthly interest by Plan B is about 0.0021 or about 0.21% per month. This rate is less than the monthly interest rate of 0.25% per month offered by Plan A, so Plan A is the better plan.

Model and Analyze

1. Use the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to determine the effective monthly interest rate for Plan B. How does this rate compare to the rate calculated using the method in the Activity above?
2. Write a function to represent the amount A Monique would earn after t months by Plan A. Then use the properties of exponents to write a function equivalent to $A(t)$ that represents the amount earned after t years.
3. From the expression you wrote in Exercise 2, identify the effective annual interest rate by Plan A. Use this rate to explain why Plan A is the better plan.
4. Suppose Plan A offered a quarterly compounded interest rate of 1.5%. Use the properties of exponents to explain which is the better plan.

LESSON

7-7 Geometric Sequences as Exponential Functions

:: Then	:: Now	:: Why?
You related arithmetic sequences to linear functions.	**1** Identify and generate geometric sequences. **2** Relate geometric sequences to exponential functions.	You send a chain email to a friend who forwards the email to five more people. Each of these five people forwards the email to five more people. The number of new email generated forms a geometric sequence.

NewVocabulary

geometric sequence
common ratio

Common Core State Standards

Content Standards

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Mathematical Practices

7 Look for and make use of structure.

1 Recognize Geometric Sequences

The first person generates 5 emails. If ea of these people sends the email to 5 more people, 25 emails are generated. If eac of the 25 people sends 5 emails, 125 emails are generated. The sequence of emails generated, 1, 5, 25, 125, … is an example of a **geometric sequence**.

In a geometric sequence, the first term is nonzero and each term after the first is found by multiplying the previous term by a nonzero constant r called the **commor ratio**. The common ratio can be found by dividing any term by its previous term.

PT

Example 1 Identify Geometric Sequences

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.

a. 256, 128, 64, 32, …

Find the ratios of consecutive terms.

256 128 64 32

$\frac{128}{256} = \frac{1}{2}$ $\frac{64}{128} = \frac{1}{2}$ $\frac{32}{64} = \frac{1}{2}$

Since the ratios are constant, the sequence is geometric. The common ratio is $\frac{1}{2}$

b. 4, 9, 12, 18, …

Find the ratios of consecutive terms.

4 9 12 18

$\frac{9}{4} = 2\frac{1}{4}$ $\frac{12}{9} = 1\frac{1}{3}$ $\frac{18}{12} = 1\frac{1}{2}$

The ratios are not constant, so the sequence is not geometric.

Find the differences of consecutive terms.

4 9 12 18

$9 - 4 = 5$ $12 - 9 = 3$ $18 - 12 = 6$

There is no common difference, so the sequence is not arithmetic.
Thus, the sequence is neither geometric nor arithmetic.

GuidedPractice

1A. 1, 3, 9, 27, … **1B.** −20, −15, −10, −5, … **1C.** 2, 8, 14, 22, …

Once the common ratio is known, more terms of a sequence can be generated. The formula can be rewritten as $a_n = a_1 r^{n-1}$, where n is a counting number and r is the common ratio.

Example 2 Find Terms of Geometric Sequences

Find the next three terms in each geometric sequence.

a. **1, −4, 16, −64, ...**

Step 1 Find the common ratio.

$$1 \qquad -4 \qquad 16 \qquad -64$$

$$\frac{-4}{1} = -4 \qquad \frac{16}{-4} = -4 \qquad \frac{-64}{16} = -4$$

Step 2 Multiply each term by the common ratio to find the next three terms.

$$-64 \qquad 256 \qquad -1024 \qquad 4096$$

$$\times(-4) \qquad \times(-4) \qquad \times(-4)$$

The next three terms are 256, −1024, and 4096.

b. **$9, 3, 1, \frac{1}{3} \ldots$**

Step 1 Find the common ratio.

$$9 \qquad 3 \qquad 1 \qquad \frac{1}{3}$$

$$\frac{3}{9} = \frac{1}{3} \qquad \frac{1}{3} = \frac{1}{3} \qquad \frac{\frac{1}{3}}{3} = \frac{1}{3}$$

The value of r is $\frac{1}{3}$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$$\frac{1}{3} \qquad \frac{1}{9} \qquad \frac{1}{27} \qquad \frac{1}{81}$$

$$\times\frac{1}{3} \qquad \times\frac{1}{3} \qquad \times\frac{1}{3}$$

The next three terms are $\frac{1}{9}$, $\frac{1}{27}$, and $\frac{1}{81}$.

Guided Practice

2A. −3, 15, −75, 375, ...

2B. 24, 36, 54, 81, ...

Study Tip

CCSS Structure If the terms of a geometric sequence alternate between positive and negative terms or vice versa, the common ratio is negative.

Math HistoryLink

Thomas Robert Malthus (1766–1834) Malthus studied populations and had pessimistic views about the future population of the world. In his work, he stated: "Population increases in a geometric ratio, while the means of subsistence increases in an arithmetic ratio."

2 Geometric Sequences and Functions

Finding the nth term of a geometric sequence would be tedious if we used the above method. The table below shows a rule for finding the nth term of a geometric sequence.

Position, n	1	2	3	4	...	n
Term, a_n	a_1	$a_1 r$	$a_1 r^2$	$a_1 r^3$	...	$a_1 r^{n-1}$

Notice that the common ratio between the terms is r. The table shows that to get the nth term, you multiply the first term by the common ratio r raised to the power $n - 1$. A geometric sequence can be defined by an exponential function in which n is the independent variable, a_n is the dependent variable, and r is the base. The domain is the counting numbers.

KeyConcept *n*th term of a Geometric Sequence

The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by the following formula, where n is any positive integer and $a_1, r \neq 0$.

$$a_n = a_1 r^{n-1}$$

WatchOut!

Negative Common Ratio If the common ratio is negative, as in Example 3, make sure to enclose the common ratio in parentheses. $(-2)^8 \neq -2^8$

Example 3 Find the *n*th Term of a Geometric Sequence

a. Write an equation for the nth term of the sequence $-6, 12, -24, 48, \ldots$.

The first term of the sequence is -6. So, $a_1 = -6$. Now find the common ratio.

$-6 \qquad 12 \qquad -24 \qquad 48$

$\frac{12}{-6} = -2 \qquad \frac{-24}{12} = -2 \qquad \frac{48}{-24} = -2$

The common ratio is -2.

$a_n = a_1 r^{n-1}$ Formula for nth term

$a_n = -6(-2)^{n-1}$ $a_1 = -6$ and $r = 2$

b. Find the ninth term of this sequence.

$a_n = a_1 r^{n-1}$ Formula for nth term

$a_9 = -6(-2)^{9-1}$ For the nth term, $n = 9$.

$= -6(-2)^8$ Simplify.

$= -6(256)$ $(-2)^8 = 256$

$= -1536$

GuidedPractice

3. Write an equation for the nth term of the geometric sequence 96, 48, 24, 12, … . Then find the tenth term of the sequence.

Real-WorldLink

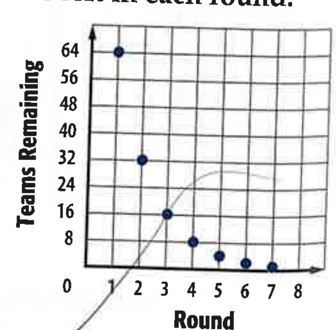
The first NCAA Division I women's basketball tournament was held in 1982. The University of Tennessee has won the most national titles with 8 titles as of 2012.

Source: NCAA Sports

Real-World Example 4 Graph a Geometric Sequence

BASKETBALL The NCAA women's basketball tournament begins with 64 teams. In each round, one half of the teams are left to compete, until only one team remains. Draw a graph to represent how many teams are left in each round.

Compared to the previous rounds, one half of the teams remain. So, $r = \frac{1}{2}$. Therefore, the geometric sequence that models this situation is 64, 32, 16, 8, 4, 2, 1. So in round two, 32 teams compete, in round three 16 teams compete and so forth. Use this information to draw a graph.



GuidedPractice

4. TENNIS A tennis ball is dropped from a height of 12 feet. Each time the ball bounces back to 80% of the height from which it fell. Draw a graph to represent the height of the ball after each bounce.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

1. 200, 40, 8, … **2.** 2, 4, 16, … **3.** −6, −3, 0, 3, … **4.** 1, −1, 1, −1, …

Example 2 **Find the next three terms in each geometric sequence.**

5. 10, 20, 40, 80, … **6.** 100, 50, 25, … **7.** $4, -1, \frac{1}{4}, \ldots$ **8.** −7, 21, −63, …

Example 3 **Write an equation for the *n*th term of each geometric sequence, and find the indicated term.**

9. the fifth term of −6, −24, −96, …

10. the seventh term of −1, 5, −25, …

11. the tenth term of 72, 48, 32, …

12. the ninth term of 112, 84, 63, …

Example 4 **13.** **EXPERIMENT** In a physics class experiment, Diana drops a ball from a height of 16 feet. Each bounce has 70% the height of the previous bounce. Draw a graph to represent the height of the ball after each bounce.

Practice and Problem Solving

Extra Practice is on R7.

Example 1 **Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain.**

14. 4, 1, 2, … **15.** 10, 20, 30, 40, … **16.** 4, 20, 100, …

17. 212, 106, 53, … **18.** −10, −8, −6, −4, … **19.** 5, −10, 20, 40, …

Example 2 **Find the next three terms in each geometric sequence.**

20. 2, −10, 50, … **21.** 36, 12, 4, … **22.** 4, 12, 36, …

23. 400, 100, 25, … **24.** −6, −42, −294, … **25.** 1024, −128, 16, …

Example 3 **26.** The first term of a geometric series is 1 and the common ratio is 9. What is the 8th term of the sequence?

27. The first term of a geometric series is 2 and the common ratio is 4. What is the 14th term of the sequence?

28. What is the 15th term of the geometric sequence −9, 27, −81, …?

29. What is the 10th term of the geometric sequence 6, −24, 96, …?

Example 4 **30.** **PENDULUM** The first swing of a pendulum is shown. On each swing after that, the arc length is 60% of the length of the previous swing. Draw a graph that represents the arc length after each swing.

31. Find the eighth term of a geometric sequence for which $a_3 = 81$ and $r = 3$.

32. CCSS **REASONING** At an online mapping site, Mr. Mosley notices that when he clicks a spot on the map, the map zooms in on that spot. The magnification increases by 20% each time.

a. Write a formula for the *n*th term of the geometric sequence that represents the magnification of each zoom level. (*Hint:* The common ratio is not just 0.2.)

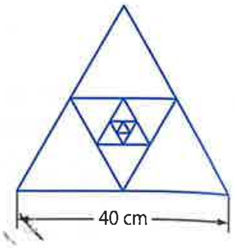
b. What is the fourth term of this sequence? What does it represent?

33. **ALLOWANCE** Danielle's parents have offered her two different options to earn her allowance for a 9-week period over the summer. She can either get paid \$30 each week or \$1 the first week, \$2 for the second week, \$4 for the third week, and so on.

a. Does the second option form a geometric sequence? Explain.

b. Which option should Danielle choose? Explain.

34. **SIERPINSKI'S TRIANGLE** Consider the inscribed equilateral triangles at the right. The perimeter of each triangle is one half of the perimeter of the next larger triangle. What is the perimeter of the smallest triangle?



35. If the second term of a geometric sequence is 3 and the third term is 1, find the first and fourth terms of the sequence.

36. If the third term of a geometric sequence is -12 and the fourth term is 24, find the first and fifth terms of the sequence.

37. **EARTHQUAKES** The Richter scale is used to measure the force of an earthquake. The table shows the increase in magnitude for the values on the Richter scale.

Richter Number (x)	Increase in Magnitude (y)	Rate of Change (slope)
1	1	–
2	10	9
3	100	
4	1000	
5	10,000	

a. Copy and complete the table. Remember that the rate of change is the change in y divided by the change in x.

b. Plot the ordered pairs (Richter number, increase in magnitude).

c. Describe the graph that you made of the Richter scale data. Is the rate of change between any two points the same?

d. Write an exponential equation that represents the Richter scale.

H.O.T. Problems Use Higher-Order Thinking Skills

38. **CHALLENGE** Write a sequence that is both geometric and arithmetic. Explain your answer.

39. CCSS **CRITIQUE** Haro and Matthew are finding the ninth term of the geometric sequence $-5, 10, -20, \ldots$. Is either of them correct? Explain your reasoning.

Haro	Matthew
$r = \frac{10}{-5}$ or -2	$r = \frac{10}{-5}$ or -2
$a_9 = -5(-2)^{9-1}$	$a_9 = -5 \cdot (-2)^{9-1}$
$= -5(512)$	$= -5 \cdot -256$
$= -2560$	$= 1280$

40. **REASONING** Write a sequence of numbers that form a pattern but are neither arithmetic nor geometric. Explain the pattern.

41. **WRITING IN MATH** How are graphs of geometric sequences and exponential functions similar? different?

42. **WRITING IN MATH** Summarize how to find a specific term of a geometric sequence.

Standardized Test Practice

43. Find the eleventh term of the sequence 3, −6, 12, −24, … .

A 6144 C 33
B 3072 D −6144

44. What is the total amount of the investment shown in the table below if interest is compounded monthly?

Principal	\$500
Length of Investment	4 years
Annual Interest Rate	5.25%

F \$613.56 H \$616.56
G \$616.00 J \$718.75

45. SHORT RESPONSE Gloria has \$6.50 in quarters and dimes. If she has 35 coins in total, how many of each coin does she have?

46. What are the domain and range of the function $y = 4(3^x) - 2$?

A D = {all real numbers}, R = $\{y \mid y > -2\}$
B D = {all real numbers}, R = $\{y \mid y > 0\}$
C D = {all integers}, R = $\{y \mid y > -2\}$
D D = {all integers}, R = $\{y \mid y > 0\}$

Spiral Review

Find the next three terms in each geometric sequence. (Lesson 7-6)

47. 2, 6, 18, 54, …
48. −5, −10, −20, −40, …
49. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$
50. −3, 1.5, −0.75, 0.375, …
51. 1, 0.6, 0.36, 0.216, …
52. 4, 6, 9, 13.5, …

Graph each function. Find the *y*-intercept and state the domain and range. (Lesson 7-5)

53. $y = \left(\frac{1}{4}\right)^x - 5$
54. $y = 2(4)^x$
55. $y = \frac{1}{2}(3^x)$

56. LANDSCAPING A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, write a system of equations to represent their growth. Find and interpret the solution in the context of the situation. (Lesson 6-2)

57. MONEY City Bank requires a minimum balance of \$1500 to maintain free checking services. If Mr. Hayashi is going to write checks for the amounts listed in the table, how much money should he start with in order to have free checking? (Lesson 5-1)

Check	Amount
750	\$1300
751	\$947

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept. (Lesson 4-2)

58. slope: 4, y-intercept: 2
59. slope: −3, y-intercept: $-\frac{2}{3}$
60. slope: $-\frac{1}{4}$, y-intercept: −5
61. slope: $\frac{1}{2}$, y-intercept: −9
62. slope: $-\frac{2}{5}$, y-intercept: $\frac{3}{4}$
63. slope: −6, y-intercept: −7

Skills Review

Simplify each expression. If not possible, write *simplified*.

64. $3u + 10u$
65. $5a - 2 + 6a$
66. $6m^2 - 8m$
67. $4w^2 + w + 15w^2$
68. $13(5 + 4a)$
69. $(4t - 6)16$

EXTEND 7-7

Algebra Lab
Average Rate of Change of Exponential Functions

You know that the rate of change of a linear function is the same for any two points on the graph. The rate of change of an exponential function is not constant.

CCSS Common Core State Standards
Content Standards
F.IF.6 Calculate and interpret the average rate of change of a functi[on] (presented symbolically or as a table) over a specified interval. Estim[ate] the rate of change from a graph.

Activity Evaluating Investment Plans

John has \$2000 to invest in one of two plans. Plan 1 offers to increase his principal by \$75 each year, while Plan 2 offers to pay 3.6% interest compounded monthly. The dollar value of each investment after t years is given by $A_1 = 2000 + 75t$ and $A_2 = 2000(1.003)^{12t}$, respectively. Use the function values, the average rate of change, and the graphs of the equations to interpret and compare the plans.

Step 1 Copy and complete the table below by finding the missing values for A_1 and A_2.

t	0	1	2	3	4	5
A_1						
A_2						

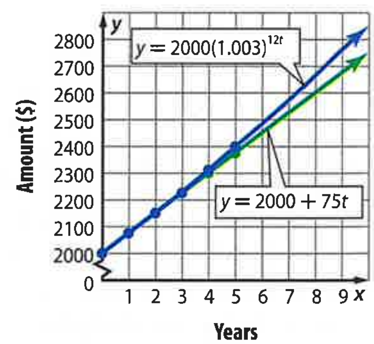
Step 2 Find the average rate of change for each plan from $t = 0$ to 1, $t = 3$ to 4, and $t = 0$ to 5.

Plan 1: $\frac{2075 - 2000}{1 - 0}$ or 75 $\quad \frac{2300 - 2225}{4 - 3}$ or 75 $\quad \frac{2375 - 2000}{5 - 0}$ or 75

Plan 2: $\frac{2073.2 - 2000}{1 - 0}$ or 73.2 $\quad \frac{2309.27 - 2227.74}{4 - 3}$ or about 82 $\quad \frac{2393.79 - 2000}{5 - 0}$ or about 79

Step 3 Graph the ordered pairs for each function. Connect each set of points with a smooth curve.

Step 4 Use the graph and the rates of change to compare the plans. Both graphs have a rate of change for the first year of about \$75 per year. From year 3 to 4, Plan 1 continues to increase at \$75 per year, but Plan 2 grows at a rate of more than \$81 per year. The average rate of change over the first five years for Plan 1 is \$75 per year and for Plan 2 is over \$78 per year. This indicates that as the number of years increases, the investment in Plan 2 grows at an increasingly faster pace. This is supported by the widening gap between their graphs.



Exercises

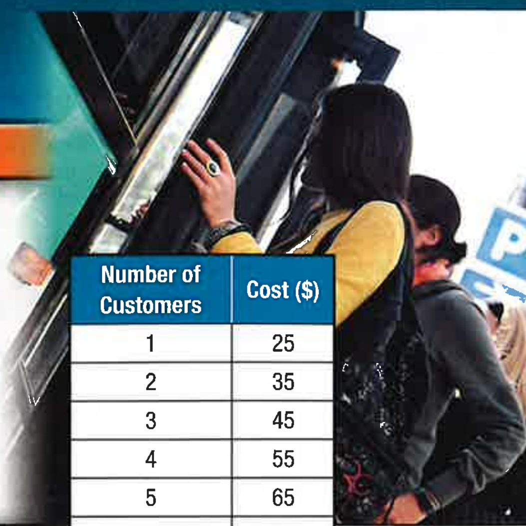
The value of a company's piece of equipment decreases over time due to depreciation. The function $y = 16{,}000(0.985)^{2t}$ represents the value after t years.

1. What is the average rate of change over the first five years?
2. What is the average rate of change of the value from year 5 to year 10?
3. What conclusion about the value can we make based on these average rates of change?
4. **CCSS REGULARITY** Copy and complete the table for $y = x^4$.

x	-3	-2	-1	0	1	2	3
y							

Compare and interpret the average rate of change for $x = -3$ to 0 and for $x = 0$ to 3.

LESSON 7-8 Recursive Formulas



Then

- You wrote explicit formulas to represent arithmetic and geometric sequences.

Now

1. Use a recursive formula to list terms in a sequence.
2. Write recursive formulas for arithmetic and geometric sequences.

Why?

- Clients of a shuttle service get picked up from their homes and driven to premium outlet stores for shopping. The total cost of the service depends on the total number of customers. The costs for the first six customers are shown.

Number of Customers	Cost ($)
1	25
2	35
3	45
4	55
5	65
6	75

NewVocabulary
recursive formula

Common Core State Standards

Content Standards
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.

1 Using Recursive Formulas

An explicit formula allows you to find any term a_n of a sequence by using a formula written in terms of n. For example, $a_n = 2n$ can be used to find the fifth term of the sequence 2, 4, 6, 8, …: $a_5 = 2(5)$ or 10.

A **recursive formula** allows you to find the nth term of a sequence by performing operations to one or more of the preceding terms. Since each term in the sequence above is 2 greater than the term that preceded it, we can add 2 to the fourth term to find that the fifth term is 8 + 2 or 10. We can then write a recursive formula for a_n.

$$\begin{aligned}
a_1 &= & &= 2\\
a_2 &= a_1 + 2 \text{ or } 2 + 2 & &= 4\\
a_3 &= a_2 + 2 \text{ or } 4 + 2 & &= 6\\
a_4 &= a_3 + 2 \text{ or } 6 + 2 & &= 8\\
&\vdots & &\vdots\\
a_n &= a_{n-1} + 2
\end{aligned}$$

A recursive formula for the sequence above is $a_1 = 2$, $a_n = a_{n-1} + 2$, for $n \geq 2$ where n is an integer. The term denoted a_{n-1} represents the term immediately before a_n. Notice that the first term a_1 is given, along with the domain for n.

Example 1 Use a Recursive Formula

Find the first five terms of the sequence in which $a_1 = 7$ and $a_n = 3a_{n-1} - 12$, if $n \geq 2$.

Use $a_1 = 7$ and the recursive formula to find the next four terms.

$$\begin{aligned}
a_2 &= 3a_{2-1} - 12 && n = 2\\
&= 3a_1 - 12 && \text{Simplify.}\\
&= 3(7) - 12 \text{ or } 9 && a_1 = 7
\end{aligned}$$

$$\begin{aligned}
a_3 &= 3a_{3-1} - 12 && n = 3\\
&= 3a_2 - 12 && \text{Simplify.}\\
&= 3(9) - 12 \text{ or } 15 && a_2 = 9
\end{aligned}$$

$$\begin{aligned}
a_4 &= 3a_{4-1} - 12 && n = 4\\
&= 3a_3 - 12 && \text{Simplify.}\\
&= 3(15) - 12 \text{ or } 33 && a_3 = 15
\end{aligned}$$

$$\begin{aligned}
a_5 &= 3a_{5-1} - 12 && n = 5\\
&= 3a_4 - 12 && \text{Simplify.}\\
&= 3(33) - 12 \text{ or } 87 && a_4 = 33
\end{aligned}$$

The first five terms are 7, 9, 15, 33, and 87.

GuidedPractice

1. Find the first five terms of the sequence in which $a_1 = -2$ and $a_n = (-3)a_{n-1} + 4$, if $n \geq 2$.

PhotoAlto

2 Writing Recursive Formulas

To write a recursive formula for an arithmetic or geometric sequence, complete the following steps.

KeyConcept Writing Recursive Formulas

Step 1 Determine if the sequence is arithmetic or geometric by finding a common difference or a common ratio.

Step 2 Write a recursive formula.

Arithmetic Sequences $a_n = a_{n-1} + d$, where d is the common difference

Geometric Sequences $a_n = r \cdot a_{n-1}$, where r is the common ratio

Step 3 State the first term and domain for n.

StudyTip

Defining n For the nth term of a sequence, the value of n must be a positive integer. Although we must still state the domain of n, from this point forward, we will assume that n is an integer.

Example 2 Write Recursive Formulas

Write a recursive formula for each sequence.

a. 17, 13, 9, 5, ...

Step 1 First subtract each term from the term that follows it.

$13 - 17 = -4 \qquad 9 - 13 = -4 \qquad 5 - 9 = -4$

There is a common difference of -4. The sequence is arithmetic.

Step 2 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ Recursive formula for arithmetic sequence

$a_n = a_{n-1} + (-4)$ $d = -4$

Step 3 The first term a_1 is 17, and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 17$, $a_n = a_{n-1} - 4$, $n \geq 2$.

StudyTip

Domain The domain for n is decided by the given terms. Since the first term is already given, it makes sense that the first term to which the formula would apply is the 2nd term of the sequence, or when $n = 2$.

b. 6, 24, 96, 384, ...

Step 1 First subtract each term from the term that follows it.

$24 - 6 = 18 \qquad 96 - 24 = 72 \qquad 384 - 96 = 288$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$\frac{24}{6} = 4 \qquad \frac{96}{24} = 4 \qquad \frac{384}{96} = 4$

There is a common ratio of 4. The sequence is geometric.

Step 2 Use the formula for a geometric sequence.

$a_n = r \cdot a_{n-1}$ Recursive formula for geometric sequence

$a_n = 4a_{n-1}$ $r = 4$

Step 3 The first term a_1 is 6, and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 6$, $a_n = 4a_{n-1}$, $n \geq 2$.

GuidedPractice

2A. 4, 10, 25, 62.5, ...

2B. 9, 36, 63, 90, ...

Real-World Career

Transportation The number of jobs in the transportation industry is expected to grow by an estimated 1.1 million between 2004 and 2014. The specific fields dictate the educational requirements, which include a high school diploma and some form of specialized training.

Source: United States Department of Labor

A sequence can be represented by both an explicit formula and a recursive formula.

Example 3 Write Recursive and Explicit Formulas

COST Refer to the beginning of the lesson. Let n be the number of customers.

a. Write a recursive formula for the sequence.

Steps 1 and 2 First subtract each term from the term that follows it.

$35 - 25 = 10 \qquad 45 - 35 = 10 \qquad 55 - 45 = 10$

There is a common difference of 10. The sequence is arithmetic.

Step 3 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ — Recursive formula for arithmetic sequence

$a_n = a_{n-1} + 10$ — $d = 10$

Step 4 The first term a_1 is 25, and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 25$, $a_n = a_{n-1} + 10$, $n \geq 2$.

b. Write an explicit formula for the sequence.

Step 1 The common difference is 10.

Step 2 Use the formula for the nth term of an arithmetic sequence.

$a_n = a_1 + (n-1)d$ — Formula for the nth term

$= 25 + (n-1)10$ — $a_1 = 25$ and $d = 10$

$= 25 + 10n - 10$ — Distributive Property

$= 10n + 15$ — Simplify.

An explicit formula for the sequence is $a_n = 10n + 15$.

Guided Practice

3. SAVINGS The money that Ronald has in his savings account earns interest each year. He does not make any withdrawals or additional deposits. The account balance at the beginning of each year is \$10,000, \$10,300, \$10,609, \$10,927.27, and so on. Write a recursive formula and an explicit formula for the sequence.

If several successive terms of a sequence are needed, a recursive formula may be useful, whereas if just the nth term of a sequence is needed, an explicit formula may be useful. Thus, it is sometimes beneficial to translate between the two forms.

Example 4 Translate between Recursive and Explicit Formulas

a. Write a recursive formula for $a_n = 6n + 3$.

$a_n = 6n + 3$ is an explicit formula for an arithmetic sequence with $d = 6$ and $a_1 = 6(1) + 3$ or 9. Therefore, a recursive formula for a_n is $a_1 = 9$, $a_n = a_{n-1} + 6$, $n \geq 2$.

b. Write an explicit formula for $a_1 = 120$, $a_n = 0.8a_{n-1}$, $n \geq 2$.

$a_n = 0.8a_{n-1}$ is a recursive formula for a geometric sequence with $a_1 = 120$ and $r = 0.8$. Therefore, an explicit formula for a_n is $a_n = 120(0.8)^{n-1}$.

Study Tip

Geometric Sequence Recall that the formula for the nth term of a geometric sequence is $a_n = a_1 r^{n-1}$.

Guided Practice

4A. Write a recursive formula for $a_n = 4(3)^{n-1}$.

4B. Write an explicit formula for $a_1 = -16$, $a_n = a_{n-1} - 7$, $n \geq 2$.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Find the first five terms of each sequence.**

1. $a_1 = 16, a_n = a_{n-1} - 3, n \geq 2$

2. $a_1 = -5, a_n = 4a_{n-1} + 10, n \geq 2$

Example 2 **Write a recursive formula for each sequence.**

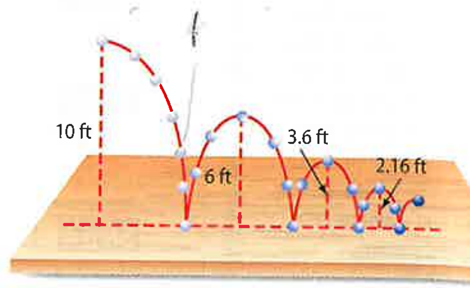
3. 1, 6, 11, 16, …

4. 4, 12, 36, 108, …

Example 3 **5. BALL** A ball is dropped from an initial height of 10 feet. The maximum heights the ball reaches on the first three bounces are shown.

a. Write a recursive formula for the sequence.

b. Write an explicit formula for the sequence.



Example 4 **For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.**

6. $a_1 = 4, a_n = a_{n-1} + 16, n \geq 2$

7 $a_n = 5n + 8$

8. $a_n = 15(2)^{n-1}$

9. $a_1 = 22, a_n = 4a_{n-1}, n \geq 2$

Practice and Problem Solving

Extra Practice is on page

Example 1 **Find the first five terms of each sequence.**

10. $a_1 = 23, a_n = a_{n-1} + 7, n \geq 2$

11. $a_1 = 48, a_n = -0.5a_{n-1} + 8, n \geq 2$

12. $a_1 = 8, a_n = 2.5a_{n-1}, n \geq 2$

13. $a_1 = 12, a_n = 3a_{n-1} - 21, n \geq 2$

14. $a_1 = 13, a_n = -2a_{n-1} - 3, n \geq 2$

15. $a_1 = \frac{1}{2}, a_n = a_{n-1} + \frac{3}{2}, n \geq 2$

Example 2 **Write a recursive formula for each sequence.**

16. 12, −1, −14, −27, …

17. 27, 41, 55, 69, …

18. 2, 11, 20, 29, …

19. 100, 80, 64, 51.2, …

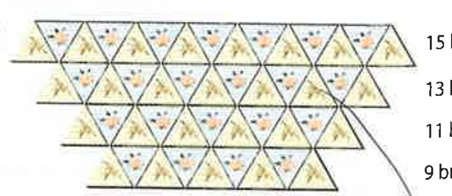
20. 40, −60, 90, −135, …

21. 81, 27, 9, 3, …

Example 3 **22.** CCSS **MODELING** A landscaper is building a brick patio. Part of the patio includes a pattern constructed from triangles. The first four rows of the pattern are shown.

a. Write a recursive formula for the sequence.

b. Write an explicit formula for the sequence.



Example 4 **For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.**

23. $a_n = 3(4)^{n-1}$

24. $a_1 = -2, a_n = a_{n-1} - 12, n \geq 2$

25. $a_1 = 38, a_n = \frac{1}{2}a_{n-1}, n \geq 2$

26. $a_n = -7n + 52$

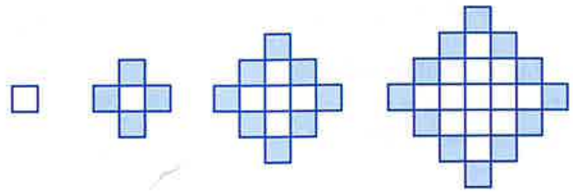
27 **TEXTING** Barbara received a chain text that she forwarded to five of her friends. Each of her friends forwarded the text to five more friends, and so on.

a. Find the first five terms of the sequence representing the number of people who receive the text in the nth round.

b. Write a recursive formula for the sequence.

c. If Barbara represents a_1, find a_8.

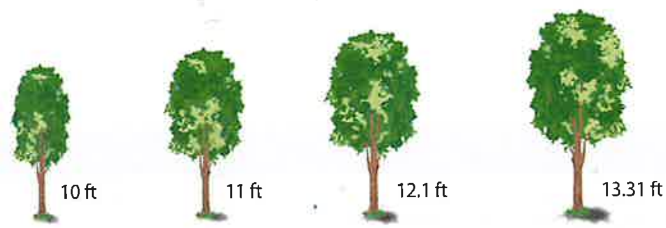
28. **GEOMETRY** Consider the pattern below. The number of blue boxes increases according to a specific pattern.



a. Write a recursive formula for the sequence of the number of blue boxes in each figure.

b. If the first box represents a_1, find the number of blue boxes in a_8.

29. **TREE** The growth of a certain type of tree slows as the tree continues to age. The heights of the tree over the past four years are shown.



a. Write a recursive formula for the height of the tree.

b. If the pattern continues, how tall will the tree be in two more years? Round your answer to the nearest tenth of a foot.

30. **MULTIPLE REPRESENTATIONS** The Fibonacci sequence is neither arithmetic nor geometric and can be defined by a recursive formula. The first terms are 1, 1, 2, 3, 5, 8, …

a. Logical Determine the relationship between the terms of the sequence. What are the next five terms in the sequence?

b. Algebraic Write a formula for the nth term if $a_1 = 1$, $a_2 = 1$, and $n \geq 3$.

c. Algebraic Find the 15th term.

d. Analytical Explain why the Fibonacci sequence is not an arithmetic sequence.

H.O.T. Problems Use Higher-Order Thinking Skills

31. **ERROR ANALYSIS** Patrick and Lynda are working on a math problem that involves the sequence 2, −2, 2, −2, 2, … . Patrick thinks that the sequence can be written as a recursive formula. Lynda believes that the sequence can be written as an explicit formula. Is either of them correct? Explain.

32. **CHALLENGE** Find a_1 for the sequence in which $a_4 = 1104$ and $a_n = 4a_{n-1} + 16$.

33. **CCSS ARGUMENTS** Determine whether the following statement is *true* or *false.* Justify your reasoning.

There is only one recursive formula for every sequence.

34. **CHALLENGE** Find a recursive formula for 4, 9, 19, 39, 79, …

35. **WRITING IN MATH** Explain the difference between an explicit formula and a recursive formula.

Standardized Test Practice

36. Find a recursive formula for the sequence 12, 24, 36, 48, … .

A $a_1 = 12, a_n = 2a_{n-1}, n \geq 2.$

B $a_1 = 12, a_n = 4a_{n-1} - 24, n \geq 2.$

C $a_1 = 12, a_n = a_{n-1} + 12, n \geq 2.$

D $a_1 = 12, a_n = 12a_{n-1} + 12, n \geq 2.$

37. GEOMETRY The area of a rectangle is $36m^4n^6$ square feet. The length of the rectangle is $6m^3n^3$ feet. What is the width of the rectangle?

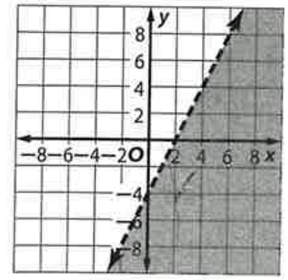
F $216m^7n^9$ ft

G $6mn^3$ ft

H $42m^7n^3$ ft

J $30mn^3$ ft

38. Find an inequality for the graph shown.



A $y > 2x - 4$

B $y \geq 2x - 4$

C $y < 2x - 4$

D $y \leq 2x - 4$

39. Write an equation of the line that passes through (−2, −20) and (4, 58).

F $y = 13x + 6$

G $y = 19x - 18$

H $y = 19x + 18$

J $y = 13x - 6$

Spiral Review

Find the next three terms in each geometric sequence. (Lesson 7-7)

40. 675, 225, 75, …

41. 16, −24, 36, …

42. 6, 18, 54, …

43. 512, −256, 128, …

44. 125, 25, 5, …

45. 12, 60, 300, …

46. INVESTMENT Nicholas invested \$2000 with a 5.75% interest rate compounded monthly. How much money will Nicholas have after 5 years? (Lesson 7-6)

47. TOURS The Snider family and the Rollins family are traveling together on a trip to visit a candy factory. The number of people in each family and the total cost are shown in the table below. Find the adult and children's admission prices. (Lesson 6-3)

Family	Number of Adults	Number of Children	Total Cost
Snider	2	3	\$58
Rollins	2	1	\$38

Write each equation in standard form. (Lesson 4-3)

48. $y + 6 = -3(x + 2)$

49. $y - 12 = 4(x - 7)$

50. $y + 9 = 5(x - 3)$

51. $y - 1 = \frac{1}{3}(x + 15)$

52. $y + 10 = \frac{2}{5}(x - 6)$

53. $y - 4 = -\frac{2}{7}(x + 1)$

Skills Review

Simplify each expression. If not possible, write *simplified*.

54. $8x + 3y^2 + 7x - 2y$

55. $4(x - 16) + 6x$

56. $4n - 3m + 9m - n$

57. $6r^2 + 7r$

58. $-2(4g - 5h) - 6g$

59. $9x^2 - 7x + 16y^2$

CHAPTER 7

Study Guide and Review

Study Guide

KeyConcepts

Multiplication and Division Properties of Exponents (Lessons 7-1 and 7-2)

For any nonzero real numbers a and b and any integers m, n, and p, the following are true.

- Product of Powers: $a^m \cdot a^n = a^{m+n}$
- Power of a Power: $(a^m)^n = a^{m \cdot n}$
- Power of a Product: $(ab)^m = a^m b^m$
- Quotient of Powers: $\frac{a^m}{a^p} = a^{m-p}$
- Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- Zero Exponent: $a^0 = 1$
- Negative Exponent: $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Rational Exponents (Lesson 7-3)

For any positive real number b and any integers m and $n > 1$, the following are true.

$b^{\frac{1}{2}} = \sqrt{b}$ $\qquad b^{\frac{1}{n}} = \sqrt[n]{b}$ $\qquad b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$ or $\sqrt[n]{b^m}$

Scientific Notation (Lesson 7-4)

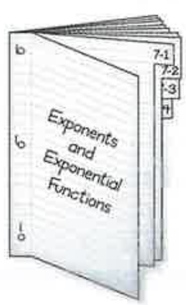
- A number is in scientific notation if it is in the form $a \times 10^n$, where $1 \le a < 10$.
- To write in standard form:
  - If $n > 0$, move the decimal n places right.
  - If $n < 0$, move the decimal n places left.

Exponential Functions (Lessons 7-5 and 7-6)

- The equation for exponential growth is $y = a(1 + r)^t$, where $r > 0$. The equation for exponential decay is $y = a(1 - r)^t$, where $0 < r < 1$. y is the final amount, a is the initial amount, r is the rate of change, and t is the time in years.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

common ratio (p. 438)	monomial (p. 391)
compound interest (p. 433)	negative exponent (p. 400)
constant (p. 391)	*n*th root (p. 407)
cube root (p. 407)	order of magnitude (p. 401)
exponential decay (p. 424)	rational exponent (p. 406)
exponential equation (p. 409)	recursive formula (p. 445)
exponential function (p. 424)	scientific notation (p. 414)
exponential growth (p. 424)	zero exponent (p. 399)
geometric sequence (p. 438)	

VocabularyCheck

Choose the word or term that best completes each sentence.

1. $7xy^4$ is an example of a(n) ______________.
2. The ______________ of 95,234 is 10^5.
3. 2 is a(n) ______________ of 8.
4. The rules for operations with exponents can be extended to apply to expressions with a(n) ______________ such as $7^{\frac{2}{3}}$.
5. A number written in ______________ is of the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.
6. $f(x) = 3^x$ is an example of a(n) ______________.
7. $a_1 = 4$ and $a_n = 3a_{n-1} - 12$, if $n \ge 2$, is a(n) ______________ for the sequence 4, −8, −20, −32, ...
8. $2^{3x-1} = 16$ is an example of a(n) ______________.
9. The equation for ______________ is $y = C(1 - r)^t$.
10. If $a^n = b$ for a positive integer n, then a is a(n) ______________ of b.

Lesson-by-Lesson Review

7-1 Multiplication Properties of Exponents

Simplify each expression.

11. $x \cdot x^3 \cdot x^5$

12. $(2xy)(-3x^2y^5)$

13. $(-4ab^4)(-5a^5b^2)$

14. $(6x^3y^2)^2$

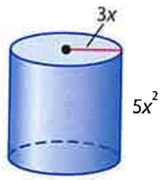
15. $\left[(2r^3t)^3\right]^2$

16. $(-2u^3)(5u)$

17. $(2x^2)^3(x^3)^3$

18. $\frac{1}{2}(2x^3)^3$

19. GEOMETRY Use the formula $V = \pi r^2 h$ to find the volume of the cylinder.



Example 1

Simplify $(5x^2y^3)(2x^4y)$.

$(5x^2y^3)(2x^4y)$

$= (5 \cdot 2)(x^2 \cdot x^4)(y^3 \cdot y)$ Commutative Property

$= 10x^6y^4$ Product of Powers

Example 2

Simplify $(3a^2b^4)^3$.

$(3a^2b^4)^3 = 3^3(a^2)^3(b^4)^3$ Power of a Product

$= 27a^6b^{12}$ Simplify.

7-2 Division Properties of Exponents

Simplify each expression. Assume that no denominator equals zero.

20. $\frac{(3x)^0}{2a}$

21. $\left(\frac{3xy^3}{2z}\right)^3$

22. $\frac{12y^{-4}}{3y^{-5}}$

23. $a^{-3}b^0c^6$

24. $\frac{-15x^7y^8z^4}{-45x^3y^5z^3}$

25. $\frac{(3x^{-1})^{-2}}{(3x^2)^{-2}}$

26. $\left(\frac{6xy^{11}z^9}{48x^6yz^{-7}}\right)^0$

27. $\left(\frac{12}{2}\right)\left(\frac{x}{y^5}\right)\left(\frac{y^4}{x^4}\right)$

28. GEOMETRY The area of a rectangle is $25x^2y^4$ square feet. The width of the rectangle is $5xy$ feet. What is the length of the rectangle?

Example 3

Simplify $\frac{2k^4m^3}{4k^2m}$. Assume that no denominator equals zero.

$\frac{2k^4m^3}{4k^2m} = \left(\frac{2}{4}\right)\left(\frac{k^4}{k^2}\right)\left(\frac{m^3}{m}\right)$ Group powers with the same base.

$= \left(\frac{1}{2}\right)k^{4-2}m^{3-1}$ Quotient of Powers

$= \frac{k^2m^2}{2}$ Simplify.

Example 4

Simplify $\frac{t^4uv^{-2}}{t^{-3}u^7}$. Assume that no denominator equals zero.

$\frac{t^4uv^{-2}}{t^{-3}u^7} = \left(\frac{t^4}{t^{-3}}\right)\left(\frac{u}{u^7}\right)(v^{-2})$ Group the powers with the same base.

$= (t^{4+3})(u^{1-7})(v^{-2})$ Quotient of Powers

$= t^7u^{-6}v^{-2}$ Simplify.

$= \frac{t^7}{u^6v^2}$ Simplify.

7-3 Rational Exponents

Simplify.

29. $\sqrt[3]{343}$

30. $\sqrt[6]{729}$

31. $625^{\frac{1}{4}}$

32. $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

33. $256^{\frac{3}{4}}$

34. $32^{\frac{2}{5}}$

35. $343^{\frac{4}{3}}$

36. $\left(\frac{4}{49}\right)^{\frac{3}{2}}$

Solve each equation.

37. $6^x = 7776$

38. $4^{4x-1} = 32$

Example 5

Simplify $125^{\frac{2}{3}}$.

$125^{\frac{2}{3}} = \left(\sqrt[3]{125}\right)^2$ — $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$

$= \left(\sqrt[3]{5 \cdot 5 \cdot 5}\right)^2$ — $125 = 5^3$

$= 5^2$ or 25 — Simplify.

Example 6

Solve $9^{x-1} = 729$.

$9^{x-1} = 729$ — Original equation

$9^{x-1} = 9^3$ — Rewrite 729 as 9^3.

$x - 1 = 3$ — Power Property of Equality

$x = 4$ — Add 1 to each side.

7-4 Scientific Notation

Express each number in scientific notation.

39. 2,300,000

40. 0.0000543

41. **ASTRONOMY** Earth has a diameter of about 8000 miles. Jupiter has a diameter of about 88,000 miles. Write in scientific notation the ratio of Earth's diameter to Jupiter's diameter.

Example 7

Express 300,000,000 in scientific notation.

Step 1 300,000,000 ⟶ 3.00000000

Step 2 The decimal point moved 8 places to the left, so $n = 8$.

Step 3 $300{,}000{,}000 = 3 \times 10^8$

7-5 Exponential Functions

Graph each function. Find the *y*-intercept, and state the domain and range.

42. $y = 2^x$

43. $y = 3^x + 1$

44. $y = 4^x + 2$

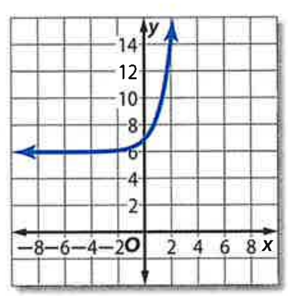
45. $y = 2^x - 3$

46. **BIOLOGY** The population of bacteria in a petri dish increases according to the model $p = 550(2.7)^{0.008t}$, where t is the number of hours and $t = 0$ corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5:00 P.M.

Example 8

Graph $y = 3^x + 6$. Find the *y*-intercept, and state the domain and range.

x	$3^x + 6$	y
−3	$3^{-3} + 6$	6.04
−2	$3^{-2} + 6$	6.11
−1	$3^{-1} + 6$	6.33
0	$3^0 + 6$	7
1	$3^1 + 6$	9



The *y*-intercept is (0, 7). The domain is all real numbers, and the range is all real numbers greater than 6.

Study Guide and Review *Continued*

7-6 Growth and Decay

47. Find the final value of \$2500 invested at an interest rate of 2% compounded monthly for 10 years.

48. COMPUTERS Zita's computer is depreciating at a rate of 3% per year. She bought the computer for \$1200.

a. Write an equation to represent this situation.

b. What will the computer's value be after 5 years?

Example 9

Find the final value of \$2000 invested at an interest rate of 3% compounded quarterly for 8 years.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$= 2000\left(1 + \frac{0.03}{4}\right)^{4(8)}$ $P = 2000$, $r = 0.03$, $n = 4$, and $t = 8$

$\approx \$2540.22$ Use a calculator.

7-7 Geometric Sequences as Exponential Functions

Find the next three terms in each geometric sequence.

49. $-1, 1, -1, 1, \ldots$

50. $3, 9, 27, \ldots$

51. $256, 128, 64, \ldots$

Write the equation for the nth term of each geometric sequence.

52. $-1, 1, -1, 1, \ldots$

53. $3, 9, 27, \ldots$

54. $256, 128, 64, \ldots$

55. SPORTS A basketball is dropped from a height of 20 feet. It bounces to $\frac{1}{2}$ its height after each bounce. Draw a graph to represent the situation.

Example 10

Find the next three terms in the geometric sequence 2, 6, 18, ...

Step 1 Find the common ratio. Each number is 3 times the previous number, so $r = 3$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$18 \times 3 = 54$, $54 \times 3 = 162$, $162 \times 3 = 486$

The next three terms are 54, 162, and 486.

Example 11

Write the equation for the nth term of the geometric sequence $-3, 12, -48, \ldots$.

The common ratio is -4. So $r = -4$.

$a_n = a_1 r^{n-1}$ Formula for the nth term

$a_n = -3(-4)^{n-1}$ $a_1 = -3$ and $r = -4$

7-8 Recursive Formulas

Find the first five terms of each sequence.

56. $a_1 = 11$, $a_n = a_{n-1} - 4$, $n \geq 2$

57. $a_1 = 3$, $a_n = 2a_{n-1} + 6$, $n \geq 2$

Write a recursive formula for each sequence.

58. $2, 7, 12, 17, \ldots$

59. $32, 16, 8, 4, \ldots$

60. $2, 5, 11, 23, \ldots$

Example 12

Write a recursive formula for $3, 1, -1, -3, \ldots$.

Step 1 First subtract each term from the term that follows it.

$1 - 3 = -2$, $-1 - 1 = -2$, $-3 - (-1) = -2$

There is a common difference of -2. The sequence is arithmetic.

Step 2 Use the formula for an arithmetic sequence.

$a_n = a_{n-1} + d$ Recursive formula

$a_n = a_{n-1} + (-2)$ $d = -2$

Step 3 The first term a_1 is 3, and $n \geq 2$.

A recursive formula is $a_1 = 3$, $a_n = a_{n-1} - 2$, $n \geq 2$.

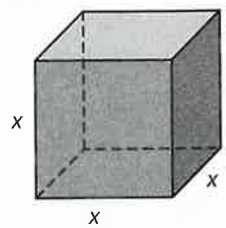
CHAPTER 7 Practice Test

Simplify each expression.

1. $(x^2)(7x^8)$

2. $(5a^7bc^2)(-6a^2bc^5)$

3. **MULTIPLE CHOICE** Express the volume of the solid as a monomial.



A x^3
B $6x$
C $6x^3$
D x^6

Simplify each expression. Assume that no denominator equals 0.

4. $\dfrac{x^6y^8}{x^2}$

5. $\left(\dfrac{2a^4b^3}{c^6}\right)^0$

6. $\dfrac{2xy^{-7}}{8x}$

Simplify.

7. $\sqrt[3]{1000}$

8. $\sqrt[5]{3125}$

9. $1728^{\frac{1}{3}}$

10. $\left(\dfrac{16}{81}\right)^{\frac{1}{2}}$

11. $27^{\frac{2}{3}}$

12. $10{,}000^{\frac{3}{4}}$

13. $27^{\frac{5}{3}}$

14. $\left(\dfrac{1}{121}\right)^{\frac{3}{2}}$

Solve each equation.

15. $12^x = 1728$

16. $7^{x-1} = 2401$

17. $9^{x-3} = 729$

Express each number in scientific notation.

18. 0.00021

19. 58,000

Express each number in standard form.

20. 2.9×10^{-5}

21. 9.1×10^6

Evaluate each product or quotient. Express the results in scientific notation.

22. $(2.5 \times 10^3)(3 \times 10^4)$

23. $\dfrac{8.8 \times 10^2}{4 \times 10^{-4}}$

24. **ASTRONOMY** The average distance from Mercury to the Sun is 35,980,000 miles. Express this distance in scientific notation.

Graph each function. Find the y-intercept, and state the domain and range.

25. $y = 2(5)^x$

26. $y = -3(11)^x$

27. $y = 3^x + 2$

Find the next three terms in each geometric sequence.

28. 2, −6, 18, …

29. 1000, 500, 250, …

30. 32, 8, 2, …

31. **MULTIPLE CHOICE** Lynne invested \$500 into an account with a 6.5% interest rate compounded monthly. How much will Lynne's investment be worth in 10 years?

F \$600.00
G \$938.57
H \$956.09
J \$957.02

32. **INVESTMENTS** Shelly's investment of \$3000 has been losing value at a rate of 3% each year. What will her investment be worth in 6 years?

Find the first five terms of each sequence.

33. $a_1 = 18, a_n = a_{n-1} - 4, n \geq 2$

34. $a_1 = -2, a_n = 4a_{n-1} + 5, n \geq 2$

CHAPTER 7

Preparing for Standardized Tests

Using a Scientific or Graphing Calculator

Scientific and graphing calculators are powerful problem-solving tools. There are times when a calculator can be used to make computations faster and easier, such as computations with very large numbers. However, there are times when using a calculator is necessary, like the estimation of irrational numbers.

Strategies for Using a Scientific or Graphing Calculator

Step 1

Familiarize yourself with the various functions of a scientific or graphing calculator as well as when they should be used:

- **Exponents** scientific notation, calculating with large or small numbers
- **Pi** solving circle problems, like circumference and area
- **Square roots** distance on a coordinate plane, Pythagorean theorem
- **Graphs** analyzing paired data in a scatter plot, graphing functions, finding roots of equations

Step 2

Use your scientific or graphing calculator to solve the problem.

- Remember to work as efficiently as possible. Some steps may be done mentally or by hand, while others should be completed using your calculator.
- If time permits, check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The distance from the Sun to Jupiter is approximately 7.786×10^{11} meters. If the speed of light is about 3×10^{8} meters per second, how long does it take for light from the Sun to reach Jupiter? Round to the nearest minute.

A about 43 minutes

B about 51 minutes

C about 1876 minutes

D about 2595 minutes

Read the problem carefully. You are given the approximate distance from the Sun to Jupiter as well as the speed of light. Both quantities are given in scientific notation. You are asked to find how many minutes it takes for light from the Sun to reach Jupiter. Use the relationship distance = rate × time to find the amount of time.

$d = r \times t$

$\frac{d}{r} = t$

To find the amount of time, divide the distance by the rate. Notice, however, that the units for time will be seconds.

$\frac{7.786 \times 10^{11} \text{ m}}{3 \times 10^{8} \text{ m/s}} = t \text{ seconds}$

Use a scientific calculator to quickly find the quotient. On most scientific calculators, the EE key is used to enter numbers in scientific notation.

KEYSTROKES: [( ] 7.786 [2nd] [EE] 11 [ )] [÷] [( ] 3 [2nd] [EE] 8 [ )] [ENTER]

The result is 2595.33333333 seconds. To convert this number to minutes, use your calculator to divide the result by 60. This gives an answer of about 43.2555 minutes. The answer is A.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Since its creation 5 years ago, approximately 2.504×10^7 items have been sold or traded on a popular online website. What is the average daily number of items sold or traded over the 5-year period?

A about 9640 items per day

B about 13,720 items per day

C about 1,025,000 items per day

D about 5,008,000 items per day

2. Evaluate $\sqrt{ab}$ if $a = 121$ and $b = 23$.

F about 5.26

G about 9.90

H about 12

J about 52.75

3. The population of the United States is about 3.034×10^8 people. The land area of the country is about 3.54×10^6 square miles. What is the average *population density* (number of people per square mile) of the United States?

A about 136.3 people per square mile

B about 112.5 people per square mile

C about 94.3 people per square mile

D about 85.7 people per square mile

4. Eleece is making a cover for the marching band's bass drum. The drum has a diameter of 20 inches. Estimate the area of the face of the bass drum.

F 31.41 square inches

G 62.83 square inches

H 78.54 square inches

J 314.16 square inches

CHAPTER 7

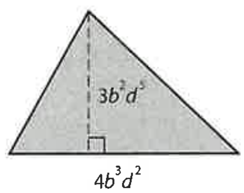
Standardized Test Practice

Cumulative, Chapters 1 through 7

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Express the area of the triangle below as a monomial.



A $12b^5d^7$

B $12b^6d^{10}$

C $6b^6d^{10}$

D $6b^5d^7$

2. Simplify the following expression.

$$\left(\frac{2w^2z^5}{3y^4}\right)^3$$

F $\frac{2w^5z^8}{3y^7}$

G $\frac{8w^6z^{15}}{27y^{12}}$

H $\frac{8w^5z^8}{27y^7}$

J $\frac{2w^6z^{15}}{3y^{12}}$

3. Which equation of a line is perpendicular to $y = \frac{3}{5}x - 3$?

A $y = -\frac{5}{3}x + 2$

B $y = -\frac{3}{5}x + 2$

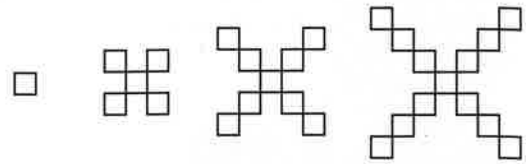
C $y = \frac{5}{3}x - 2$

D $y = \frac{3}{5}x - 2$

Test-TakingTip

Question 2 Use the laws of exponents to simplify the expression. Remember, to find the power of a power, multiply the exponents.

4. Write a recursive formula for the sequence of the number of squares in each figure.



F $a_1 = 1, a_n = 4a_{n-1} - 3, n \geq 1$

G $a_1 = 1, a_n = 4a_{n-1}, n \geq 2$

H $a_1 = 1, a_n = a_{n-1} + 4, n \geq 2$

J $a_1 = 1, a_n = 4a_{n-1} + 4, n \geq 2$

5. Evaluate $(4.2 \times 10^6)(5.7 \times 10^8)$.

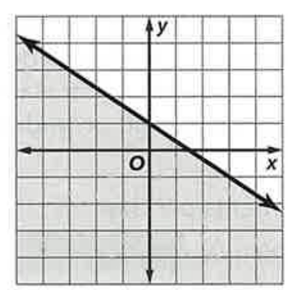
A 2.394×10^{15}

B 23.94×10^{14}

C 9.9×10^{14}

D 2.394×10^{48}

6. Which inequality is shown in the graph?



F $y \leq -\frac{2}{3}x - 1$

G $y \leq -\frac{3}{4}x - 1$

H $y \leq -\frac{2}{3}x + 1$

J $y \leq -\frac{3}{4}x + 1$

Short Response/Gridded Response

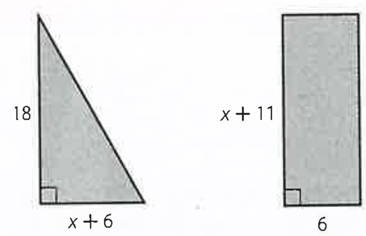
7. Jaden created a Web site for the Science Olympiad team. The total number of hits the site has received is shown.

Day	Total Hits	Day	Total Hits
3	5	17	27
6	7	21	33
10	12	26	40
13	17	34	55

a. Find an equation for the regression line.

b. Predict the number of total hits that the Web site will have received on day 46.

8. Find the value of x so that the figures have the same area.



9. What is the solution to the following system of equations? Show your work.

$$\begin{cases} y = 6x - 1 \\ y = 6x + 4 \end{cases}$$

10. **GRIDDED RESPONSE** At a family fun center, the Wilson and Sanchez families each bought video game tokens and batting cage tokens as shown in the table.

Family	Wilson	Sanchez
Number of Video Game Tokens	25	30
Number of Batting Cage Tokens	8	6
Total Cost	\$26.50	\$25.50

What is the cost in dollars of a batting cage token at the family fun center?

Extended Response

Record your answers on a sheet of paper. Show your work.

11. The table below shows the distances from the Sun to Mercury, Earth, Mars, and Saturn. Use the data to answer each question.

Planet	Distance from Sun (km)
Mercury	5.79×10^7
Earth	1.50×10^8
Mars	2.28×10^8
Saturn	1.43×10^9

a. Of the planets listed, which one is the closest to the Sun?

b. About how many times as far from the Sun is Mars as Earth?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson...	7-1	7-2	4-4	6-6	7-3	5-6	4-6	2-4	6-2	6-4	7-4