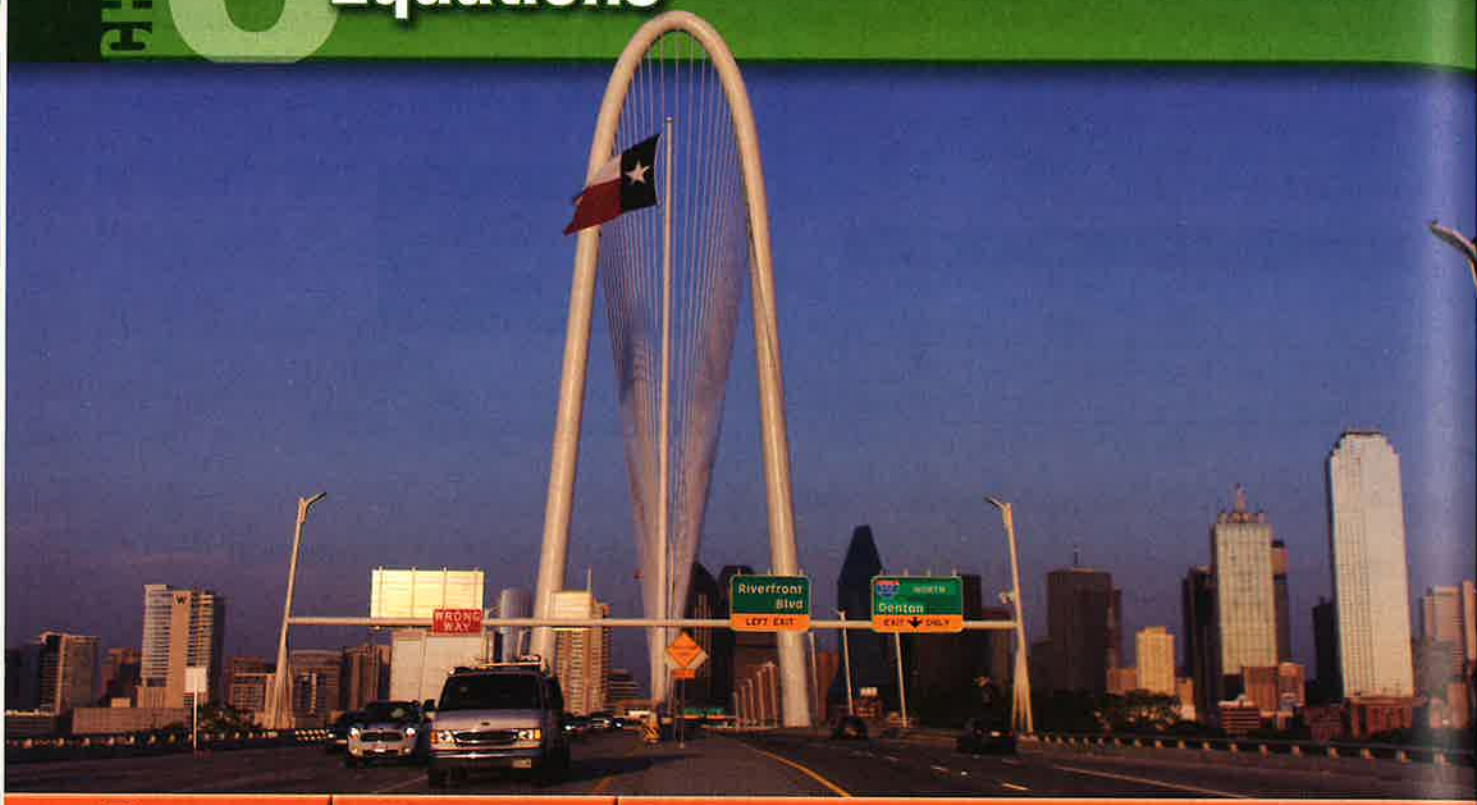
CHAPTER 8

Quadratic Expressions and Equations



Then

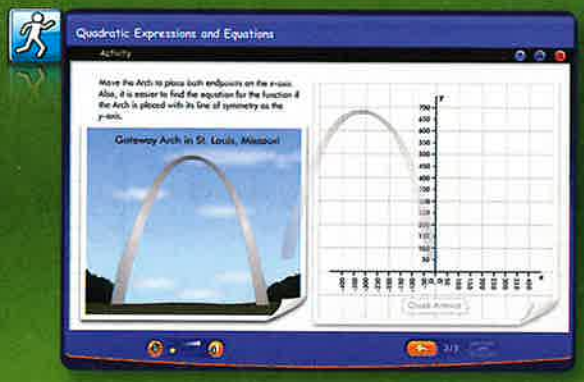
- You applied the laws of exponents and explored exponential functions.

Now

- In this chapter, you will:
  - Add, subtract, and multiply polynomials.
  - Factor trinomials.
  - Factor differences of squares.
  - Graph quadratic functions.
  - Solve quadratic equations.

Why? ▲

- **ARCHITECTURE** Quadratic equations can be used to model the shape of architectural structures.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Rewrite each expression using the Distributive Property. Then simplify.

1. $a(a + 5)$
2. $2(3 + x)$
3. $n(n - 3n^2 + 2)$
4. $-6(x^2 - 5x + 6)$
5. **FINANCIAL LITERACY** Five friends will pay \$9 per ticket, \$3 per drink, and \$6 per popcorn at the movies. Write an expression that could be used to determine the cost for them to go to the movies.

Simplify each expression. If not possible, write *simplified.*

6. $3u + 10u$
7. $5a - 2 + 6a$
8. $6m^2 - 8m$
9. $4w^2 + w + 15w^2$
10. $2x^2 + 5 - 11x^2$
11. $8v^3 - 27$
12. $4k^2 + 2k - 2k + 1$
13. $a^2 - 4a - 4a + 16$
14. $6y^2 + 2y - 3y - 1$
15. $9g^2 - 3g - 6g + 2$

Simplify.

16. $b(b^6)$
17. $4n^3(n^2)$
18. $8m(4m^2)$
19. $-5z^4(3z^5)$
20. $5xy(4x^3y)$
21. $(-2a^4c^5)(7ac^4)$
22. **GEOMETRY** A square is $6x^3$ inches on each side. What is the area of the square?

QuickReview

Example 1

Rewrite $6x(-3x - 5x - 5x^2 + x^3)$ using the Distributive Property. Then simplify.

$$\begin{aligned} 6x(-3x - 5x - 5x^2 + x^3) &= 6x(-3x) + 6x(-5x) + 6x(-5x^2) + 6x(x^3) \\ &= -18x^2 - 30x^2 - 30x^3 + 6x^4 \\ &= -48x^2 - 30x^3 + 6x^4 \end{aligned}$$

Example 2

Simplify $8c + 6 - 4c + 2c^2$.

$$\begin{aligned} 8c + 6 - 4c + 2c^2 &= 2c^2 + 8c - 4c + 6 \\ &= 2c^2 + (8 - 4)c + 6 \\ &= 2c^2 + 4c + 6 \end{aligned}$$

Example 3

Simplify $(-2y^3)(9y^4)$.

$$\begin{aligned} (9y^3)(-2y^4) &= (-2 \cdot 9)(y^3 \cdot y^4) \\ &= (-2 \cdot 9)(y^{3+4}) \\ &= -18y^7 \end{aligned}$$

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

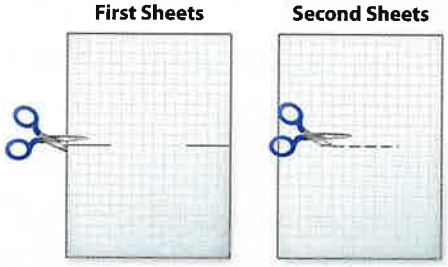
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 8. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

Quadratic Expressions and Equations Make this Foldable to help you organize your Chapter 8 notes about quadratic expressions and equations. Begin with five sheets of grid paper.

1 **Fold** in half along the width. On the first three sheets, cut 5 centimeters along the fold at the ends. On the second two sheets cut in the center, stopping 5 centimeters from the ends.



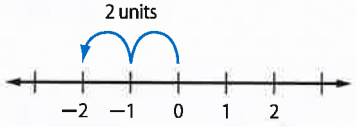
2 **Insert** the first sheets through the second sheets and align the folds. Label the front Chapter 8, Quadratic Expressions and Equations. Label the pages with lesson numbers and the last page with vocabulary.

NewVocabulary

English		Español
polynomial	p. 465	polinomio
binomial	p. 465	binomio
trinomial	p. 465	trinomio
degree of a monomial	p. 465	grado de un monomio
degree of a polynomial	p. 465	grado de un polinomio
standard form of a polynomial	p. 466	forma estándar de polinomio
leading coefficient	p. 466	coeficiente lider
FOIL method	p. 481	método foil
quadratic expression	p. 481	expression cuadrática
factoring	p. 494	factorización
factoring by grouping	p. 495	factorización por agrupamiento
Zero Product Property	p. 496	propiedad del producto de cero
quadratic equation	p. 506	ecuación cuadrática
prime polynomial	p. 512	polinomio primo
difference of two squares	p. 516	diferencia de cuadrados
perfect square trinomial	p. 522	trinomio cuadrado perfecto
Square Root Property	p. 525	Propiedad de la raíz cuadrada

ReviewVocabulary

absolute value valor absoluto the absolute value of any number n is the distance the number is from zero on a number line and is written $|n|$



The absolute value of −2 is 2 because it is 2 units from 0.

perfect square cuadrado perfecto a number with a square root that is a rational number

EXPLORE

8-1 Algebra Lab
Adding and Subtracting Polynomials

Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram below shows the models.

Common Core State Standards
Content Standards
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Polynomial Models

- Polynomials are modeled using three types of tiles.
- Each tile has an opposite.

Activity 1 Model Polynomials

Use algebra tiles to model each polynomial.

- $5x$

  To model this polynomial, you will need 5 green x-tiles.

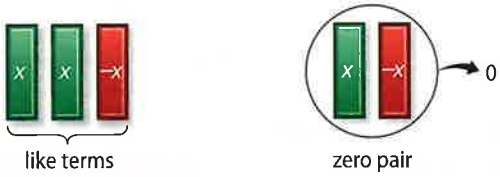
- $-2x^2 + x + 3$

  To model this polynomial, you will need 2 red $-x^2$-tiles, 1 green x-tile, and 3 yellow 1-tiles.

Monomials such as $3x$ and $-2x$ are called *like terms* because they have the same variable to the same power.

Polynomial Models

- Like terms are represented by tiles that have the same shape and size.
- A *zero pair* may be formed by pairing one tile with its opposite. You can remove or add zero pairs without changing the polynomial.

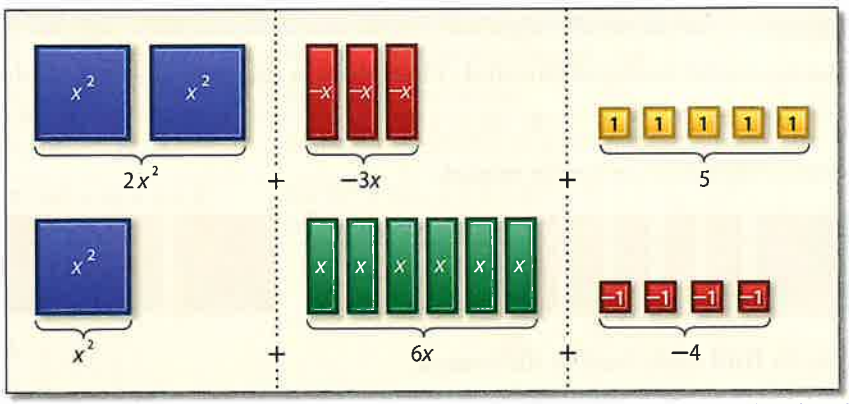


Activity 2 Add Polynomials

Use algebra tiles to find $(2x^2 - 3x + 5) + (x^2 + 6x - 4)$.

Step 1

Model each polynomial.



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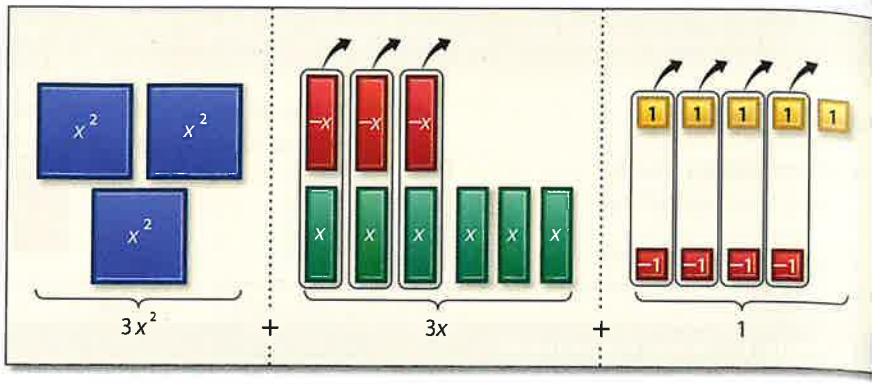
Adding and Subtracting Polynomials *Continued*

Step 2

Combine like terms and remove zero pairs.

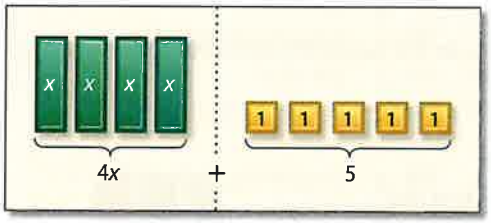


Step 3 Write the polynomial. $(2x^2 - 3x + 5) + (x^2 + 6x - 4) = 3x^2 + 3x + 1$

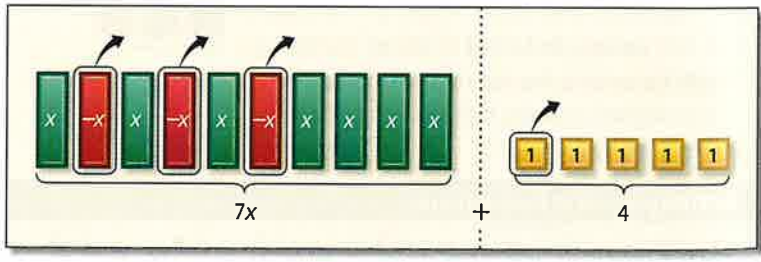
Activity 3 Subtract Polynomials

Use algebra tiles to find $(4x + 5) - (-3x + 1)$.

Step 1 Model the polynomial $4x + 5$.



Step 2 To subtract $-3x + 1$, remove 3 red $-x$-tiles and 1 yellow 1-tile. You can remove the 1-tile, but there are no $-x$-tiles. Add 3 zero pairs of x-tiles. Then remove the 3 red $-x$-tiles.



Step 3 Write the polynomial.
$(4x + 5) - (-3x + 1) = 7x + 4$

Model and Analyze

Use algebra tiles to model each polynomial. Then draw a diagram of your model.

1. $-2x^2$ **2.** $5x - 4$ **3.** $x^2 - 4x$

Write an algebraic expression for each model.

4.

5.

Use algebra tiles to find each sum or difference.

6. $(x^2 + 5x - 2) + (3x^2 - 2x + 6)$ **7.** $(2x^2 + 8x + 1) - (x^2 - 4x - 2)$ **8.** $(-4x^2 + x) - (x^2 + 5x)$

LESSON 8-1

Adding and Subtracting Polynomials

Then

- You identified monomials and their characteristics.

Now

1. Write polynomials in standard form.
2. Add and subtract polynomials.

Why?

- In 2017, sales of digital audio players are expected to reach record numbers. The sales data can be modeled by the equation $U = -2.7t^2 + 49.4t + 128.7$, where U is the number of units shipped in millions and t is the number of years since 2005.

The expression $-2.7t^2 + 49.4t + 128.7$ is an example of a polynomial. Polynomials can be used to model situations.

New Vocabulary

- polynomial
- binomial
- trinomial
- degree of a monomial
- degree of a polynomial
- standard form of a polynomial
- leading coefficient

Common Core State Standards

Content Standards

A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

1 Polynomials in Standard Form

A **polynomial** is a monomial or the sum of monomials, each called a *term* of the polynomial. Some polynomials have special names. A **binomial** is the sum of *two* monomials, and a **trinomial** is the sum of *three* monomials.

Monomial	Binomial	Trinomial
$5x$	$2x^2 + 7$	$x^3 - 10x + 1$

The **degree of a monomial** is the sum of the exponents of all its variables. A nonzero constant term has degree 0, and zero has no degree.

The **degree of a polynomial** is the greatest degree of any term in the polynomial. You can find the degree of a polynomial by finding the degree of each term. Polynomials are named based on their degree.

Degree	Name
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic
6 or more	6th degree, 7th degree, and so on

Example 1 Identify Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

Expression	Is it a polynomial?	Degree	Monomial, binomial, or trinomial?
a. $4y - 5xz$	Yes; $4y - 5xz$ is the sum of $4y$ and $-5xz$.	2	binomial
b. -6.5	Yes; -6.5 is a real number.	0	monomial
c. $7a^{-3} + 9b$	No; $7a^{-3} = \frac{7}{a^3}$, which is not a monomial.	—	———
d. $6x^3 + 4x + x + 3$	Yes; $6x^3 + 4x + x + 3 = 6x^3 + 5x + 3$, the sum of three monomials.	3	trinomial

Guided Practice

1A. x

1B. $-3y^2 - 2y + 4y - 1$

1C. $5rx + 7tuv$

1D. $10x^{-4} - 8x^a$

The terms of a polynomial can be written in any order. However, polynomials in one variable are usually written in standard form. The **standard form of a polynomial** has the terms in order from greatest to least degree. In this form, the coefficient of the first term is called the **leading coefficient**.

leading coefficient — greatest degree

Standard form: $4x^3 - 5x^2 + 2x + 7$

Example 2 Standard Form of a Polynomial

Write each polynomial in standard form. Identify the leading coefficient.

a. $3x^2 + 4x^5 - 7x$

Find the degree of each term.

Degree: 2 5 1

Polynomial: $3x^2 + 4x^5 - 7x$

The greatest degree is 5. Therefore, the polynomial can be rewritten as $4x^5 + 3x^2 - 7x$, with a leading coefficient of 4.

b. $5y - 9 - 2y^4 - 6y^3$

Find the degree of each term.

Degree: 1 0 4 3

Polynomial: $5y - 9 - 2y^4 - 6y^3$

The greatest degree is 4. Therefore, the polynomial can be rewritten as $-2y^4 - 6y^3 + 5y - 9$, with a leading coefficient of -2.

GuidedPractice

2A. $8 - 2x^2 + 4x^4 - 3x$

2B. $y + 5y^3 - 2y^2 - 7y^6 + 10$

2 Add and Subtract Polynomials

Adding polynomials involves adding like terms. You can group like terms by using a horizontal or vertical format.

Example 3 Add Polynomials

Find each sum.

a. $(2x^2 + 5x - 7) + (3 - 4x^2 + 6x)$

Horizontal Method

Group and combine like terms.

$$(2x^2 + 5x - 7) + (3 - 4x^2 + 6x)$$
$$= [2x^2 + (-4x^2)] + [5x + 6x] + [-7 + 3] \quad \text{Group like terms.}$$
$$= -2x^2 + 11x - 4 \quad \text{Combine like terms.}$$

b. $(3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8)$

Vertical Method

Align like terms in columns and combine.

$$\begin{array}{r} y^3 + 0y^2 + 3y - 5 \\ (+)\ 2y^3 + 4y^2 - 4y + 8 \\ \hline 3y^3 + 4y^2 - y + 3 \end{array}$$

Insert a placeholder to help align the terms.

Align and combine like terms.

StudyTip

Vertical Method Notice that the polynomials are written in standard form with like terms aligned. Since there is no y^2-term in the first polynomial, $0y^2$ is used as a placeholder.

GuidedPractice

3A. $(5x^2 - 3x + 4) + (6x - 3x^2 - 3)$

3B. $(y^4 - 3y + 7) + (2y^3 + 2y - 2y^4 - 11)$

StudyTip

Additive Inverse When finding the additive inverse of a polynomial, you are multiplying every term by −1.

You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, write the opposite of each term, as shown.

$$-(3x^2 + 2x - 6) = \underbrace{-3x^2 - 2x + 6}_{\text{Additive Inverse}}$$

Example 4 Subtract Polynomials

Find each difference.

a. $(3 - 2x + 2x^2) - (4x - 5 + 3x^2)$

Horizontal Method

Subtract $4x - 5 + 3x^2$ by adding its additive inverse.

$(3 - 2x + 2x^2) - (4x - 5 + 3x^2)$

$= (3 - 2x + 2x^2) + (-4x + 5 - 3x^2)$ The additive inverse of $4x - 5 + 3x^2$ is $-4x + 5 - 3x^2$.

$= [2x^2 + (-3x^2)] + [(-2x) + (-4x)] + [3 + 5]$ Group like terms.

$= -x^2 - 6x + 8$ Combine like terms.

b. $(7p + 4p^3 - 8) - (3p^2 + 2 - 9p)$

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 4p^3 + 0p^2 + 7p - 8 \\ (-) & 3p^2 - 9p + 2 \end{array} \quad \xrightarrow{\text{Add the opposite.}} \quad \begin{array}{rr} & 4p^3 + 0p^2 + 7p - 8 \\ (+) & -3p^2 + 9p - 2 \\ \hline & 4p^3 - 3p^2 + 16p - 10 \end{array}$$

GuidedPractice

4A. $(4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)$

4B. $(8y - 10 + 5y^2) - (7 - y^3 + 12y)$

Adding or subtracting integers results in an integer, so the set of integers is closed under addition and subtraction. Similarly, adding or subtracting polynomials results in a polynomial, so the set of polynomials is closed under addition and subtraction.

Digital Vision/Alamy

Real-WorldLink

Sales of digital cameras recently increased by 42% in one year. Sales are expected to increase by at least 15% each year as consumers upgrade their cameras.

Source: Big Planet Marketing Company

Real-World Example 5 Add and Subtract Polynomials

ELECTRONICS **The equations $P = 7m + 137$ and $C = 4m + 78$ represent the number of cell phones P and digital cameras C sold in m months at an electronics store. Write an equation for the total monthly sales T of phones and cameras. Then predict the number of phones and cameras sold in 10 months.**

To write an equation that represents the total sales T, add the equations that represent the number of cell phones P and digital cameras C.

$T = 7m + 137 + 4m + 78$

$= 11m + 215$

Substitute 10 for m to predict the number of phones and cameras sold in 10 months.

$T = 11(10) + 215$

$= 110 + 215$ or 325

Therefore, a total of 325 cell phones and digital cameras will be sold in 10 months.

GuidedPractice

5. Use the information above to write an equation that represents the difference in the monthly sales of cell phones and the monthly sales of digital cameras. Use the equation to predict the difference in monthly sales in 24 months.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial, binomial,* or *trinomial.***

1. $7ab + 6b^2 - 2a^3$
2. $2y - 5 + 3y^2$
3. $3x^2$
4. $\frac{4m}{3p}$
5. $5m^2p^3 + 6$
6. $5q^{-4} + 6q$

Example 2 **Write each polynomial in standard form. Identify the leading coefficient.**

7. $2x^5 - 12 + 3x$
8. $-4d^4 + 1 - d^2$
9. $4z - 2z^2 - 5z^4$
10. $2a + 4a^3 - 5a^2 - 1$

Examples 3–4 **Find each sum or difference.**

11. $(6x^3 - 4) + (-2x^3 + 9)$
12. $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$
13. $(4 + 2a^2 - 2a) - (3a^2 - 8a + 7)$
14. $(8y - 4y^2) + (3y - 9y^2)$
15. $(-4z^3 - 2z + 8) - (4z^3 + 3z^2 - 5)$
16. $(-3d^2 - 8 + 2d) + (4d - 12 + d^2)$
17. $(y + 5) + (2y + 4y^2 - 2)$
18. $(3n^3 - 5n + n^2) - (-8n^2 + 3n^3)$

Example 5

19. CCSS **SENSE-MAKING** The total number of students T who traveled for spring break consists of two groups: students who flew to their destinations F and students who drove to their destination D. The number (in thousands) of students who flew and the total number of students who flew or drove can be modeled by the following equations, where n is the number of years since 1995.

$$T = 14n + 21 \qquad F = 8n + 7$$

   a. Write an equation that models the number of students who drove to their destination for this time period.
   b. Predict the number of students who will drive to their destination in 2012.
   c. How many students will drive or fly to their destination in 2015?

Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial, binomial,* or *trinomial.***

20. $\frac{5y^3}{x^2} + 4x$
21. 21
22. $c^4 - 2c^2 + 1$
23. $d + 3d^c$
24. $a - a^2$
25. $5n^3 + nq^3$

Example 2 **Write each polynomial in standard form. Identify the leading coefficient.**

26. $5x^2 - 2 + 3x$
27. $8y + 7y^3$
28. $4 - 3c - 5c^2$
29. $-y^3 + 3y - 3y^2 + 2$
30. $11t + 2t^2 - 3 + t^5$
31. $2 + r - r^3$
32. $\frac{1}{2}x - 3x^4 + 7$
33. $-9b^2 + 10b - b^6$

Examples 3–4 **Find each sum or difference.**

34. $(2c^2 + 6c + 4) + (5c^2 - 7)$

35. $(2x + 3x^2) - (7 - 8x^2)$

36. $(3c^3 - c + 11) - (c^2 + 2c + 8)$

37. $(z^2 + z) + (z^2 - 11)$

38. $(2x - 2y + 1) - (3y + 4x)$

39. $(4a - 5b^2 + 3) + (6 - 2a + 3b^2)$

40. $(x^2y - 3x^2 + y) + (3y - 2x^2y)$

41. $(-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)$

42. $(5n - 2p^2 + 2np) - (4p^2 + 4n)$

43. $(4rxt - 8r^2x + x^2) - (6rx^2 + 5rxt - 2x^2)$

Example 5

44. PETS From 1999 through 2009, the number of dogs D and the number of cats C (in hundreds) adopted from animal shelters in the United States are modeled by the equations $D = 2n + 3$ and $C = n + 4$, where n is the number of years since 1999.

a. Write a function that models the total number T of dogs and cats adopted in hundreds for this time period.

b. If this trend continues, how many dogs and cats will be adopted in 2013?

Classify each polynomial according to its degree and number of terms.

45. $4x - 3x^2 + 5$

46. $11z^3$

47. $9 + y^4$

48. $3x^3 - 7$

49. $-2x^5 - x^2 + 5x - 8$

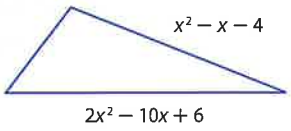
50. $10t - 4t^2 + 6t^3$

51. ENROLLMENT In a rapidly growing school system, the numbers (in hundreds) of total students is represented by N and the number of students in Kindergarten through 5th grade is represented by P. The equations $N = 1.25t^2 - t + 7.5$ and $P = 0.7t^2 - 0.95t + 3.8$, models the number of students enrolled from 2000 to 2009, where t is the number of years since 2000.

a. Write an equation modeling the number of students S in grades 6 through 12 enrolled for this time period.

b. How many students were enrolled in grades 6 through 12 in the school system in 2007?

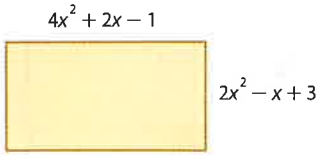
52. CCSS REASONING The perimeter of the triangle can be represented by the expression $3x^2 - 7x + 2$. Write a polynomial that represents the measure of the third side.



53. GEOMETRY Consider the rectangle.

a. What does $(4x^2 + 2x - 1)(2x^2 - x + 3)$ represent?

b. What does $2(4x^2 + 2x - 1) + 2(2x^2 - x + 3)$ represent?



Find each sum or difference.

54. $(4x + 2y - 6z) + (5y - 2z + 7x) + (-9z - 2x - 3y)$

55. $(5a^2 - 4) + (a^2 - 2a + 12) + (4a^2 - 6a + 8)$

56. $(3c^2 - 7) + (4c + 7) - (c^2 + 5c - 8)$

57. $(3n^3 + 3n - 10) - (4n^2 - 5n) + (4n^3 - 3n^2 - 9n + 4)$

58. FOOTBALL The National Football League is divided into two conferences, the American A and the National N. From 2002 through 2009, the total attendance T (in thousands) for both conferences and for the American Conference games can be modeled by the following equations, where x is the number of years since 2002.

$T = -0.69x^3 + 55.83x^2 + 643.31x + 10{,}538$ $A = -3.78x^3 + 58.96x^2 + 265.96x + 5257$

Estimate how many people attended National Conference football games in 2009.

59. **CAR RENTAL** The cost to rent a car for a day is $15 plus $0.15 for each mile driven.

a. Write a polynomial that represents the cost of renting a car for m miles.

b. If a car is driven 145 miles, how much would it cost to rent?

c. If a car is driven 105 miles each day for four days, how much would it cost to rent a car?

d. If a car is driven 220 miles each day for seven days, how much would it cost to rent a car?

60. **MULTIPLE REPRESENTATIONS** In this problem, you will explore perimeter and area.

a. **Geometric** Draw three rectangles that each have a perimeter of 400 feet.

b. **Tabular** Record the width and length of each rectangle in a table like the one shown below. Find the area of each rectangle.

Rectangle	Length	Width	Area
1	100 ft		
2	50 ft		
3	75 ft		
4	x ft		

c. **Graphical** On a coordinate system, graph the area of rectangle 4 in terms of the length, x. Use the graph to determine the largest area possible.

d. **Analytical** Determine the length and width that produce the largest area.

H.O.T. Problems Use Higher-Order Thinking Skills

61. **CCSS CRITIQUE** Cheyenne and Sebastian are finding $(2x^2 - x) - (3x + 3x^2 - 2)$. Is either of them correct? Explain your reasoning.

Cheyenne

$(2x^2 - x) - (3x + 3x^2 - 2)$

$= (2x^2 - x) + (-3x + 3x^2 - 2)$

$= 5x^2 - 4x - 2$

Sebastian

$(2x^2 - x) - (3x + 3x^2 - 2)$

$= (2x^2 - x) + (-3x - 3x^2 - 2)$

$= -x^2 - 4x - 2$

62. **REASONING** Determine whether each of the following statements is *true* or *false*. Explain your reasoning.

a. A binomial can have a degree of zero.

b. The order in which polynomials are subtracted does not matter.

63. **CHALLENGE** Write a polynomial that represents the sum of an odd integer $2n + 1$ and the next two consecutive odd integers.

64. **WRITING IN MATH** Why would you add or subtract equations that represent real-world situations? Explain.

65. **WRITING IN MATH** Describe how to add and subtract polynomials using both the vertical and horizontal formats.

Standardized Test Practice

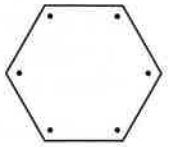
66. Three consecutive integers can be represented by x, $x + 1$, and $x + 2$. What is the sum of these three integers?

A $x(x + 1)(x + 2)$ C $3x + 3$

B $x^3 + 3$ D $x + 3$

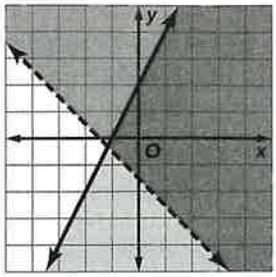
67. SHORT RESPONSE What is the perimeter of a square with sides that measure $2x + 3$ units?

68. Jim cuts a board in the shape of a regular hexagon and pounds in a nail at each vertex, as shown. How many rubber bands will he need to stretch a rubber band across every possible pair of nails?



F 15 G 14 H 12 J 9

69. Which ordered pair is in the solution set of the system of inequalities shown in the graph?



A $(-3, 0)$ C $(5, 0)$

B $(0, -3)$ D $(0, 5)$

Spiral Review

70. COMPUTERS A computer technician charges by the hour to fix and repair computer equipment. The total cost of the technician for one hour is \$75, for two hours is \$125, for three hours is \$175, for four hours is \$225, and so on. Write a recursive formula for the sequence. (Lesson 7-8)

Determine whether each sequence is *arithmetic*, *geometric*, or *neither*. Explain. (Lesson 7-7)

71. 8, −32, 128, −512, …

72. 25, 8, −9, −26, …

73. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

74. 43, 52, 61, 70, …

75. −27, −16, −5, 6, …

76. 200, 100, 50, 25, …

77. JOBS Kimi received an offer for a new job. She wants to compare the offer with her current job. What is total amount of sales that Kimi must get each month to make the same income at either job? (Lesson 6-2)

New Offer
\$600/mo 2% commission

Current Job
\$1000/mo 1.5% commission

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference. (Lesson 3-5)

78. 24, 16, 8, 0, …

79. $3\frac{1}{4}, 6\frac{1}{2}, 13, 26, \ldots$

80. 7, 6, 5, 4, …

81. 10, 12, 15, 18, …

82. −15, −11, −7, −3, …

83. −0.3, 0.2, 0.7, 1.2, …

Skills Review

Simplify.

84. $t(t^5)(t^7)$

85. $n^3(n^2)(-2n^3)$

86. $(5t^5v^2)(10t^3v^4)$

87. $(-8u^4z^5)(5uz^4)$

88. $[(3)^2]^3$

89. $[(2)^3]^2$

90. $(2m^4k^3)^2(-3mk^2)^3$

91. $(6xy^2)^2(2x^2y^2z^2)^3$

LESSON 8-2

Multiplying a Polynomial by a Monomial

Then

- You multiplied monomials.

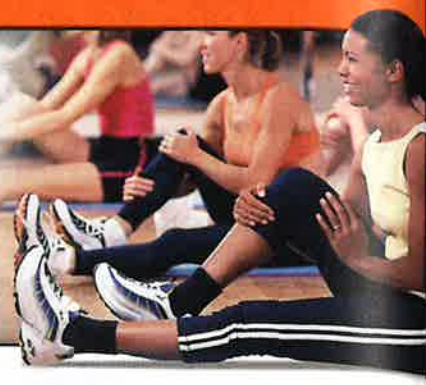
Now

- 1 Multiply a polynomial by a monomial.
- 2 Solve equations involving the products of monomials and polynomials.

Why?

- Charmaine Brooks is opening a fitness club. She tells the contractor that the length of the fitness room should be three times the width plus 8 feet.

To cover the floor with mats for exercise classes, Ms. Brooks needs to know the area of the floor. So she multiplies the width times the length, $w(3w + 8)$.



Common Core State Standards

Content Standards

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mathematical Practices

5 Use appropriate tools strategically.

1 Polynomial Multiplied by Monomial

To find the product of a polynomial and a monomial, you can use the Distributive Property.

Example 1 Multiply a Polynomial by a Monomial

Find $-3x^2(7x^2 - x + 4)$.

Horizontal Method

$-3x^2(7x^2 - x + 4)$ — Original expression

$= -3x^2(7x^2) - (-3x^2)(x) + (-3x^2)(4)$ — Distributive Property

$= -21x^4 - (-3x^3) + (-12x^2)$ — Multiply.

$= -21x^4 + 3x^3 - 12x^2$ — Simplify.

Vertical Method

$$\begin{array}{rr} & 7x^2 - x + 4 \\ (\times) & -3x^2 \\ \hline & -21x^4 + 3x^3 - 12x^2 \end{array}$$

Distributive Property

Multiply.

GuidedPractice

Find each product.

1A. $5a^2(-4a^2 + 2a - 7)$

1B. $-6d^3(3d^4 - 2d^3 - d + 9)$

We can use this same method more than once to simplify large expressions.

Example 2 Simplify Expressions

Simplify $2p(-4p^2 + 5p) - 5(2p^2 + 20)$.

$2p(-4p^2 + 5p) - 5(2p^2 + 20)$ — Original expression

$= (2p)(-4p^2) + (2p)(5p) + (-5)(2p^2) + (-5)(20)$ — Distributive Property

$= -8p^3 + 10p^2 - 10p^2 - 100$ — Multiply.

$= -8p^3 + (10p^2 - 10p^2) - 100$ — Commutative and Associative Properties

$= -8p^3 - 100$ — Combine like terms.

GuidedPractice

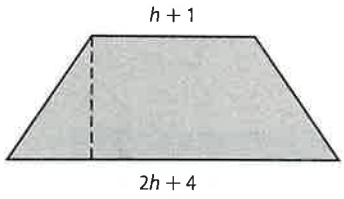
Simplify each expression.

2A. $3(5x^2 + 2x - 4) - x(7x^2 + 2x - 3)$ **2B.** $15t(10y^3t^5 + 5y^2t) - 2y(yt^2 + 4y^2)$

We can use the Distributive Property to multiply monomials by polynomials and solve real world problems.

Standardized Test Example 3 Write and Evaluate a Polynomial Expression

GRIDDED RESPONSE **The theme for a school dance is "Solid Gold." For one decoration, Kana is covering a trapezoid-shaped piece of poster board with metallic gold paper to look like a bar of gold. If the height of the poster board is 18 inches, how much metallic paper will Kana need in square inches?**



Test-TakingTip

CCSS Tools Many standardized tests provide formula sheets with commonly used formulas. If you are unsure of the correct formula, check the sheet before beginning to solve the problem.

Read the Test Item

The question is asking you to find the area of the trapezoid with a height of h and bases of $h + 1$ and $2h + 4$.

Solve the Test Item

Write an equation to represent the area of the trapezoid.
Let $b_1 = h + 1$, let $b_2 = 2h + 4$ and let h = height of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Area of a trapezoid}$$

$$= \frac{1}{2}h[(h + 1) + (2h + 4)] \quad b_1 = h + 1 \text{ and } b_2 = 2h + 4$$

$$= \frac{1}{2}h(3h + 5) \quad \text{Add and simplify.}$$

$$= \frac{3}{2}h^2 + \frac{5}{2}h \quad \text{Distributive Property}$$

$$= \frac{3}{2}(18)^2 + \frac{5}{2}(18) \quad h = 18$$

$$= 531 \quad \text{Simplify.}$$

5	3	1		
	/	/	/	
.	.	.	.	.
0	0	0	0	0
1	1	●	1	1
2	2	2	2	2
3	●	3	3	3
4	4	4	4	4
●	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Kana will need 531 square inches of metallic paper.
Grid in your response of 531.

Real-WorldLink

In a recent year, the pet supply business hit an estimated $7.05 billion in sales. This business ranges from gourmet food to rhinestone tiaras, pearl collars, and cashmere coats.

Source: *Entrepreneur Magazine*

Noryuki Yamaguchi/ANYONE/amana

David Schmidt/Masterfile

GuidedPractice

3. SHORT RESPONSE Kachima is making triangular bandanas for the dogs and cats in her pet club. The base of the bandana is the length of the collar with 4 inches added to each end to tie it on. The height is $\frac{1}{2}$ of the collar length.

A. If Kachima's dog has a collar length of 12 inches, how much fabric does she need in square inches?

B. If Kachima makes a bandana for her friend's cat with a 6-inch collar, how much fabric does Kachima need in square inches?

2 Solve Equations with Polynomial Expressions

We can use the Distributive Property to solve equations that involve the products of monomials and polynomials.

StudyTip

Combining Like Terms When simplifying a long expression, it may be helpful to put a circle around one set of like terms, a rectangle around another set, a triangle around another set, and so on.

Example 4 Equations with Polynomials on Both Sides

Solve $2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50$.

$2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50$	Original equation
$10a^2 - 4a + 6a^2 + 18a + 8 = 4a^2 + a + 12a^2 - 8a + 50$	Distributive Property
$16a^2 + 14a + 8 = 16a^2 - 7a + 50$	Combine like terms.
$14a + 8 = -7a + 50$	Subtract $16a^2$ from each side.
$21a + 8 = 50$	Add $7a$ to each side.
$21a = 42$	Subtract 8 from each side.
$a = 2$	Divide each side by 21.

CHECK

$$2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50$$
$$2(2)[5(2) - 2] + 3(2)[2(2) + 6] + 8 \stackrel{?}{=} 2[4(2) + 1] + 2(2)[6(2) - 4] + 50$$
$$4(8) + 6(10) + 8 \stackrel{?}{=} 2(9) + 4(8) + 50 \quad \text{Simplify.}$$
$$32 + 60 + 8 \stackrel{?}{=} 18 + 32 + 50 \quad \text{Multiply.}$$
$$100 = 100 \checkmark \quad \text{Add and subtract.}$$

GuidedPractice

Solve each equation.

4A. $2x(x + 4) + 7 = (x + 8) + 2x(x + 1) + 12$

4B. $d(d + 3) - d(d - 4) = 9d - 16$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Find each product.**

1. $5w(-3w^2 + 2w - 4)$

2. $6g^2(3g^3 + 4g^2 + 10g - 1)$

3. $4km^2(8km^2 + 2k^2m + 5k)$

4. $-3p^4r^3(2p^2r^4 - 6p^6r^3 - 5)$

5 $2ab(7a^4b^2 + a^5b - 2a)$

6. $c^2d^3(5cd^7 - 3c^3d^2 - 4d^3)$

Example 2 **Simplify each expression.**

7. $t(4t^2 + 15t + 4) - 4(3t - 1)$

8. $x(3x^2 + 4) + 2(7x - 3)$

9. $-2d(d^3c^2 - 4dc^2 + 2d^2c) + c^2(dc^2 - 3d^4)$

10. $-5w^2(8w^2x - 11wx^2) + 6x(9wx^4 - 4w - 3x^2)$

Example 3

11. GRIDDED RESPONSE Marlene is buying a new plasma television. The height of the screen of the television is one half the width plus 5 inches. The width is 30 inches. Find the height of the screen in inches.

Example 4 **Solve each equation.**

12. $-6(11 - 2c) = 7(-2 - 2c)$

13. $t(2t + 3) + 20 = 2t(t - 3)$

14. $-2(w + 1) + w = 7 - 4w$

15. $3(y - 2) + 2y = 4y + 14$

16. $a(a + 3) + a(a - 6) + 35 = a(a - 5) + a(a + 7)$

17. $n(n - 4) + n(n + 8) = n(n - 13) + n(n + 1) + 16$

Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Find each product.**

18. $b(b^2 - 12b + 1)$

19. $f(f^2 + 2f + 25)$

20. $-3m^3(2m^3 - 12m^2 + 2m + 25)$

21. $2j^2(5j^3 - 15j^2 + 2j + 2)$

22. $2pr^2(2pr + 5p^2r - 15p)$

23. $4t^3u(2t^2u^2 - 10tu^4 + 2)$

Example 2 **Simplify each expression.**

24. $-3(5x^2 + 2x + 9) + x(2x - 3)$

25. $a(-8a^2 + 2a + 4) + 3(6a^2 - 4)$

26. $-4d(5d^2 - 12) + 7(d + 5)$

27. $-9g(-2g + g^2) + 3(g^2 + 4)$

28. $2j(7j^2k^2 + jk^2 + 5k) - 9k(-2j^2k^2 + 2k^2 + 3j)$

29. $4n(2n^3p^2 - 3np^2 + 5n) + 4p(6n^2p - 2np^2 + 3p)$

Example 3

30. DAMS A new dam being built has the shape of a trapezoid. The length of the base at the bottom of the dam is 2 times the height. The length of the base at the top of the dam is $\frac{1}{5}$ times the height minus 30 feet.

a. Write an expression to find the area of the trapezoidal cross section of the dam.

b. If the height of the dam is 180 feet, find the area of this cross section.

Example 4 **Solve each equation.**

31. $7(t^2 + 5t - 9) + t = t(7t - 2) + 13$

32. $w(4w + 6) + 2w = 2(2w^2 + 7w - 3)$

33. $5(4z + 6) - 2(z - 4) = 7z(z + 4) - z(7z - 2) - 48$

34. $9c(c - 11) + 10(5c - 3) = 3c(c + 5) + c(6c - 3) - 30$

35. $2f(5f - 2) - 10(f^2 - 3f + 6) = -8f(f + 4) + 4(2f^2 - 7f)$

36. $2k(-3k + 4) + 6(k^2 + 10) = k(4k + 8) - 2k(2k + 5)$

Simplify each expression.

37. $\frac{2}{3}np^2(30p^2 + 9n^2p - 12)$

38. $\frac{3}{5}r^2t(10r^3 + 5rt^3 + 15t^2)$

39. $-5q^2w^3(4q + 7w) + 4qw^2(7q^2w + 2q) - 3qw(3q^2w^2 + 9)$

40. $-x^2z(2z^2 + 4xz^3) + xz^2(xz + 5x^3z) + x^2z^3(3x^2z + 4xz)$

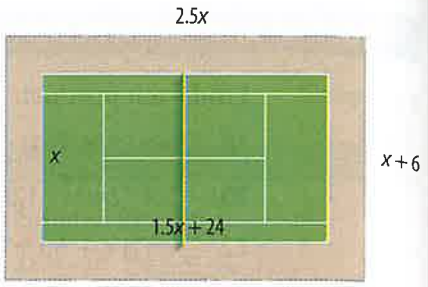
41. PARKING A parking garage charges \$30 per month plus \$0.50 per daytime hour and \$0.25 per hour during nights and weekends. Suppose Trent parks in the garage for 47 hours in January and h of those are night and weekend hours.

a. Find an expression for Trent's January bill.

b. Find the cost if Trent had 12 hours of night and weekend hours.

42. CCSS MODELING Che is building a dog house for his new puppy. The upper face of the dog house is a trapezoid. If the height of the trapezoid is 12 inches, find the area of the face of this piece of the dog house.

43. **TENNIS** The tennis club is building a new tennis court with a path around it.



a. Write an expression for the area of the tennis court.

b. Write an expression for the area of the path.

c. If $x = 36$ feet, what is the perimeter of the outside of the path?

44. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the degree of the product of a monomial and a polynomial.

a. Tabular Write three monomials of different degrees and three polynomials of different degrees. Determine the degree of each monomial and polynomial. Multiply the monomials by the polynomials. Determine the degree of each product. Record your results in a table like the one shown below.

Monomial	Degree	Polynomial	Degree	Product of Monomial and Polynomial	Degree

b. Verbal Make a conjecture about the degree of the product of a monomial and a polynomial. What is the degree of the product of a monomial of degree a and a polynomial of degree b?

H.O.T. Problems Use Higher-Order Thinking Skills

45. **ERROR ANALYSIS** Pearl and Ted both worked on this problem. Is either of them correct? Explain your reasoning.

Pearl

$2x^2(3x^2 + 4x + 2)$

$6x^4 + 8x^2 + 4x^2$

$6x^4 + 12x^2$

Ted

$2x^2(3x^2 + 4x + 2)$

$6x^4 + 8x^3 + 4x^2$

46. **PERSEVERANCE** Find p such that $3x^p(4x^{2p+3} + 2x^{3p-2}) = 12x^{12} + 6x^{10}$.

47. **CHALLENGE** Simplify $4x^{-3}y^2(2x^5y^{-4} + 6x^{-7}y^6 - 4x^0y^{-2})$.

48. **REASONING** Is there a value for x that makes the statement $(x + 2)^2 = x^2 + 2^2$ true? If so, find a value for x. Explain your reasoning.

49. **OPEN ENDED** Write a monomial and a polynomial using n as the variable. Find their product.

50. **WRITING IN MATH** Describe the steps to multiply a polynomial by a monomial.

Standardized Test Practice

51. Every week a store sells j jeans and t T-shirts. The store makes \$8 for each T-shirt and \$12 for each pair of jeans. Which of the following expressions represents the total amount of money, in dollars, the store makes every week?

A $8j + 12t$
B $12j + 8t$
C $20(j + t)$
D $96jt$

52. If $a = 5x + 7y$ and $b = 2y - 3x$, what is $a + b$?

F $2x - 9y$
G $3y + 4x$
H $2x + 9y$
J $2x - 5y$

53. GEOMETRY A triangle has sides of length 5 inches and 8.5 inches. Which of the following cannot be the length of the third side?

A 3.5 inches
B 4 inches
C 5.5 inches
D 12 inches

54. SHORT RESPONSE Write an equation in which x varies directly as the cube of y and inversely as the square of z.

Spiral Review

Find each sum or difference. (Lesson 8-1)

55. $(2x^2 - 7) + (8 - 5x^2)$
56. $(3z^2 + 2z - 1) + (z^2 - 6)$
57. $(2a - 4a^2 + 1) - (5a^2 - 2a - 6)$
58. $(a^3 - 3a^2 + 4) - (4a^2 + 7)$
59. $(2ab - 3a + 4b) + (5a + 4ab)$
60. $(8c^3 - 3c^2 + c - 2) - (3c^3 + 9)$

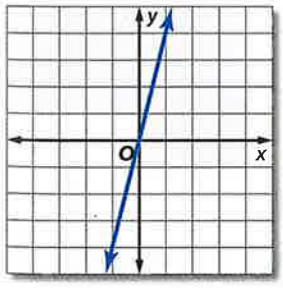
Write a recursive formula for each sequence. (Lesson 7-8)

61. 16, 2, −12, −26, …
62. −5, 3, 11, 19, …
63. 27, 43, 59, 75, …
64. 80, −200, 500, −1250, …
65. 100, 60, 36, 21.6, …
66. $\frac{1}{16}, \frac{1}{4}, 1, 4, \ldots$

67. TRAVEL In 2003, about 9.5 million people took cruises. Between 2003 and 2008, the number increased by about 740,000 each year. Write the point-slope form of an equation to find the total number of people y taking a cruise for any year x. Estimate the number of people who took a cruise in 2010. (Lesson 4-3)

Write an equation in function notation for each relation. (Lesson 3-6)

68.



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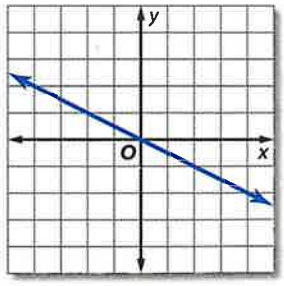


Skills Review

Simplify.

70. $b(b^2)(b^3)$
71. $2y(3y^2)$
72. $-y^4(-2y^3)$
73. $-3z^3(-5z^4 + 2z)$
74. $2m(-4m^4) - 3(-5m^3)$
75. $4p^2(-2p^3) + 2p^4(5p^6)$

EXPLORE

8-3 Algebra Lab
Multiplying Polynomials

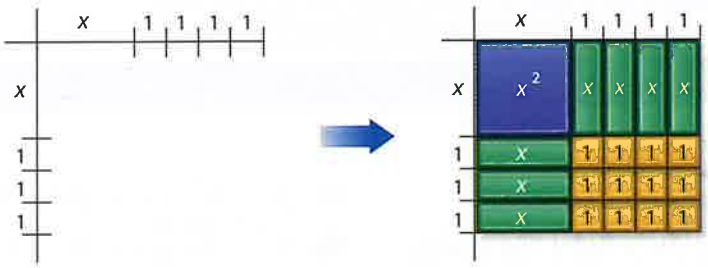
You can use algebra tiles to find the product of two binomials.

Common Core State Standards
Content Standards
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Activity 1 Multiply Binomials

Use algebra tiles to find $(x + 3)(x + 4)$.

The rectangle will have a width of $x + 3$ and a length of $x + 4$. Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.

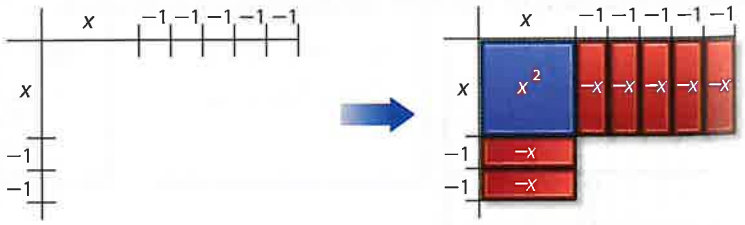


The rectangle consists of 1 blue x^2-tile, 7 green x-tiles, and 12 yellow 1-tiles. The area of the rectangle is $x^2 + 7x + 12$. So, $(x + 3)(x + 4) = x^2 + 7x + 12$.

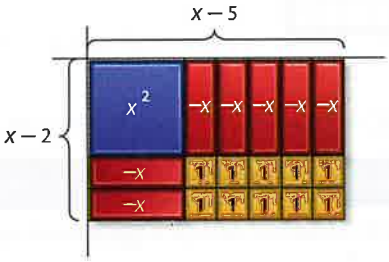
Activity 2 Multiply Binomials

Use algebra tiles to find $(x - 2)(x - 5)$.

Step 1 The rectangle will have a width of $x - 2$ and a length of $x - 5$. Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.



Step 2 Determine whether to use 10 yellow 1-tiles or 10 red -1-tiles to complete the rectangle. The area of each yellow tile is the product of -1 and -1. Fill in the space with 10 yellow 1-tiles to complete the rectangle.

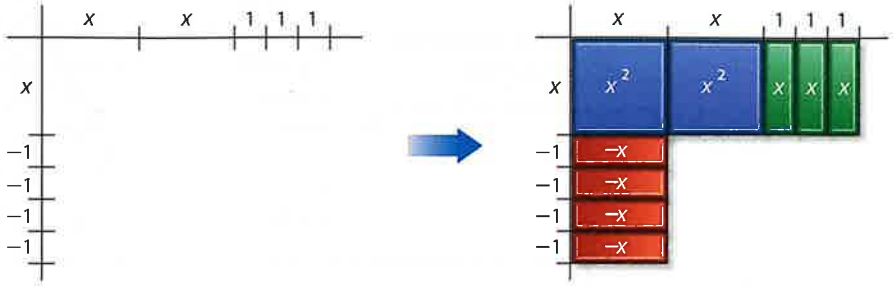


The rectangle consists of 1 blue x^2-tile, 7 red $-x$-tiles, and 10 yellow 1-tiles. The area of the rectangle is $x^2 - 7x + 10$. So, $(x - 2)(x - 5) = x^2 - 7x + 10$.

Activity 3 Multiply Binomials

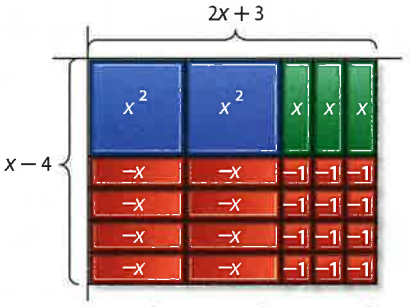
Use algebra tiles to find $(x - 4)(2x + 3)$.

Step 1 The rectangle will have a width of $x - 4$ and a length of $2x + 3$. Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.



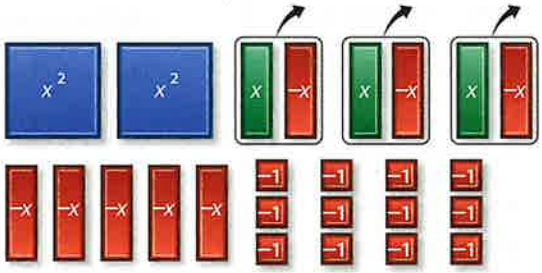
Step 2 Determine what color x-tiles and what color 1-tiles to use to complete the rectangle. The area of each red x-tile is the product of x and -1. The area of each red -1-tile is represented by $1(-1)$ or -1.

Complete the rectangle with 4 red x-tiles and 12 red -1-tiles.



Step 3 Rearrange the tiles to simplify the polynomial you have formed. Notice that a 3 zero pair are formed by three positive and three negative x-tiles.

There are 2 blue x^2-tiles, 5 red $-x$-tiles, and 12 red -1-tiles left. In simplest form, $(x - 4)(2x + 3) = 2x^2 - 5x - 12$.



Model and Analyze

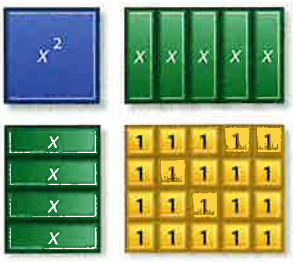
Use algebra tiles to find each product.

1. $(x + 1)(x + 4)$
2. $(x - 3)(x - 2)$
3. $(x + 5)(x - 1)$
4. $(x + 2)(2x + 3)$
5. $(x - 1)(2x - 1)$
6. $(x + 4)(2x - 5)$

Is each statement *true* or *false*? Justify your answer with a drawing of algebra tiles.

7. $(x - 4)(x - 2) = x^2 - 6x + 8$
8. $(x + 3)(x + 5) = x^2 + 15$

9. **WRITING IN MATH** You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for $(x + 4)(x + 5)$ separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.



LESSON 8-3

Multiplying Polynomials

Then

- You multiplied polynomials by monomials.

Now

1. Multiply binomials by using the FOIL method.
2. Multiply polynomials by using the Distributive Property.

Why?

- Bodyboards, which are used to ride waves, are made of foam and are more rectangular than surfboards. A bodyboard's dimensions are determined by the height and skill level of the user.

The length of Ann's bodyboard should be Ann's height h minus 32 inches or $h - 32$. The board's width should be half of Ann's height plus 11 inches or $\frac{1}{2}h + 11$. To approximate the area of the bodyboard, you need to find $(h - 32)\left(\frac{1}{2}h + 11\right)$.

New Vocabulary

- **FOIL method**
- **quadratic expression**

Common Core State Standards

Content Standards

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mathematical Practices

7 Look for and make use of structure.

1 Multiply Binomials

To multiply two binomials such as $h - 32$ and $\frac{1}{2}h + 11$, the Distributive Property is used. Binomials can be multiplied horizontally or vertically.

Example 1 The Distributive Property

Find each product.

a. $(2x + 3)(x + 5)$

Vertical Method

Multiply by 5.

$$\begin{array}{r} 2x + 3 \\ (\times)\ x + 5 \\ \hline 10x + 15 \end{array}$$

$5(2x + 3) = 10x + 15$

Multiply by x.

$$\begin{array}{r} 2x + 3 \\ (\times)\ x + 5 \\ \hline 10x + 15 \\ 2x^2 + 3x \end{array}$$

$x(2x + 3) = 2x^2 + 3x$

Combine like terms.

$$\begin{array}{r} 2x + 3 \\ (\times)\ x + 5 \\ \hline 10x + 15 \\ 2x^2 + 3x \\ \hline 2x^2 + 13x + 15 \end{array}$$

Horizontal Method

$$\begin{aligned} (2x + 3)(x + 5) &= 2x(x + 5) + 3(x + 5) && \text{Rewrite as the sum of two products.} \\ &= 2x^2 + 10x + 3x + 15 && \text{Distributive Property} \\ &= 2x^2 + 13x + 15 && \text{Combine like terms.} \end{aligned}$$

b. $(x - 2)(3x + 4)$

Vertical Method

Multiply by 4.

$$\begin{array}{r} x - 2 \\ (\times)\ 3x + 4 \\ \hline 4x - 8 \end{array}$$

$4(x - 2) = 4x - 8$

Multiply by $3x$.

$$\begin{array}{r} x - 2 \\ (\times)\ 3x + 4 \\ \hline 4x - 8 \\ 3x^2 - 6x \end{array}$$

$3x(x - 2) = 3x^2 - 6x$

Combine like terms.

$$\begin{array}{r} x - 2 \\ (\times)\ 3x + 4 \\ \hline 4x - 8 \\ 3x^2 - 6x \\ \hline 3x^2 - 2x - 8 \end{array}$$

Horizontal Method

$$\begin{aligned} (x - 2)(3x + 4) &= x(3x + 4) - 2(3x + 4) && \text{Rewrite as the difference of two products.} \\ &= 3x^2 + 4x - 6x - 8 && \text{Distributive Property} \\ &= 3x^2 - 2x - 8 && \text{Combine like terms.} \end{aligned}$$

GuidedPractice

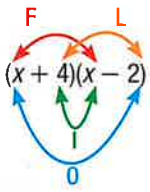
1A. $(3m + 4)(m + 5)$ **1B.** $(5y - 2)(y + 8)$

A shortcut version of the Distributive Property for multiplying binomials is called the **FOIL method**.

KeyConcept FOIL Method

Words To multiply two binomials, find the sum of the products of ***F*** the *First* terms, ***O*** the *Outer* terms, ***I*** the *Inner* terms, ***L*** and the *Last* terms.

Example



Product of First Terms: $(x)(x)$ + Product of Outer Terms: $(x)(-2)$ + Product of Inner Terms: $(4)(x)$ + Product of Last Terms: $(4)(-2)$

$= x^2 - 2x + 4x - 8$

$= x^2 + 2x - 8$

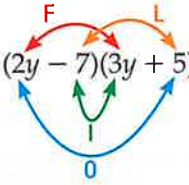
ReadingMath

Polynomials as Factors The expression $(x + 4)(x - 2)$ is read *the quantity x plus 4 times the quantity x minus 2.*

Example 2 FOIL Method

Find each product.

a. $(2y - 7)(3y + 5)$



$= (2y)(3y) + (2y)(5) + (-7)(3y) + (-7)(5)$ FOIL method

$= 6y^2 + 10y - 21y - 35$ Multiply.

$= 6y^2 - 11y - 35$ Combine like terms.

b. $(4a - 5)(2a - 9)$

$(4a - 5)(2a - 9)$

$= (4a)(2a) + (4a)(-9) + (-5)(2a) + (-5)(-9)$ FOIL method

$= 8a^2 - 36a - 10a + 45$ Multiply.

$= 8a^2 - 46a + 45$ Combine like terms.

GuidedPractice

2A. $(x + 3)(x - 4)$ **2B.** $(4b - 5)(3b + 2)$

2C. $(2y - 5)(y - 6)$ **2D.** $(5a + 2)(3a - 4)$

Notice that when two linear expressions are multiplied, the result is a quadratic expression. A **quadratic expression** is an expression in one variable with a degree of 2. When three linear expressions are multiplied, the result has a degree of 3.

The FOIL method can be used to find an expression that represents the area of a rectangular object when the lengths of the sides are given as binomials.

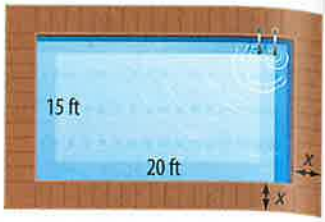
Real-WorldLink

The cost of a swimming pool depends on many factors, including the size of the pool, whether the pool is an above-ground or an in-ground pool, and the material used.

Source: American Dream Homes

Real-World Example 3 FOIL Method

SWIMMING POOL **A contractor is building a deck around a rectangular swimming pool. The deck is x feet from every side of the pool. Write an expression for the total area of the pool and deck.**



Understand We need to find an expression for the total area of the pool and deck.

Plan Find the product of the length and width of the pool with the deck.

Solve Since the deck is the same distance from every side of the pool, the length and width of the pool are $2x$ longer. So, the length can be represented by $2x + 20$ and the width can be represented by $2x + 15$.

$$\begin{aligned}
\text{Area} &= \text{length} \cdot \text{width} && \text{Area of a rectangle}\\
&= (2x + 20)(2x + 15) && \text{Substitution}\\
&= (2x)(2x) + (2x)(15) + (20)(2x) + (20)(15) && \text{FOIL Method}\\
&= 4x^2 + 30x + 40x + 300 && \text{Multiply.}\\
&= 4x^2 + 70x + 300 && \text{Combine like terms.}
\end{aligned}$$

So, the total area of the deck and pool is $4x^2 + 70x + 300$.

Check Choose a value for x. Substitute this value into $(2x + 20)(2x + 15)$ and $4x^2 + 70x + 300$. The result should be the same for both expressions.

GuidedPractice

3. If the pool is 25 feet long and 20 feet wide, find the area of the pool and deck.

2 Multiply Polynomials

The Distributive Property can also be used to multiply any two polynomials.

Example 4 The Distributive Property

Find each product.

a. $(6x + 5)(2x^2 - 3x - 5)$

$$\begin{aligned}
&(6x + 5)(2x^2 - 3x - 5)\\
&= 6x(2x^2 - 3x - 5) + 5(2x^2 - 3x - 5) && \text{Distributive Property}\\
&= 12x^3 - 18x^2 - 30x + 10x^2 - 15x - 25 && \text{Multiply.}\\
&= 12x^3 - 8x^2 - 45x - 25 && \text{Combine like terms.}
\end{aligned}$$

b. $(2y^2 + 3y - 1)(3y^2 - 5y + 2)$

$$\begin{aligned}
&(2y^2 + 3y - 1)(3y^2 - 5y + 2)\\
&= 2y^2(3y^2 - 5y + 2) + 3y(3y^2 - 5y + 2) - 1(3y^2 - 5y + 2) && \text{Distributive Property}\\
&= 6y^4 - 10y^3 + 4y^2 + 9y^3 - 15y^2 + 6y - 3y^2 + 5y - 2 && \text{Multiply.}\\
&= 6y^4 - y^3 - 14y^2 + 11y - 2 && \text{Combine like terms.}
\end{aligned}$$

StudyTip

Multiplying Polynomials
If a polynomial with c terms and a polynomial with d terms are multiplied together, there will be $c \cdot d$ terms before simplifying. In Example 4a, there are $2 \cdot 3$ or 6 terms before simplifying.

GuidedPractice

4A. $(3x - 5)(2x^2 + 7x - 8)$ **4B.** $(m^2 + 2m - 3)(4m^2 - 7m + 5)$

Check Your Understanding

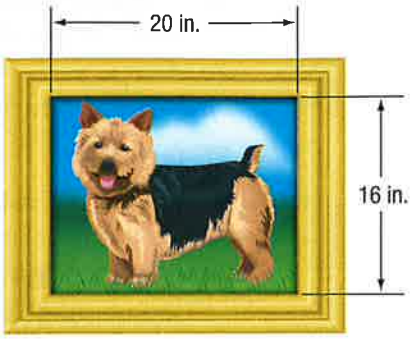
= Step-by-Step Solutions begin on page R13.

Examples 1–2 Find each product.

1. $(x + 5)(x + 2)$
2. $(y - 2)(y + 4)$
3. $(b - 7)(b + 3)$
4. $(4n + 3)(n + 9)$
5. $(8h - 1)(2h - 3)$
6. $(2a + 9)(5a - 6)$

Example 3

7. **FRAME** Hugo is designing a frame as shown at the right. The frame has a width of x inches all the way around. Write an expression that represents the total area of the picture and frame.



Example 4 Find each product.

8. $(2a - 9)(3a^2 + 4a - 4)$
9. $(4y^2 - 3)(4y^2 + 7y + 2)$
10. $(x^2 - 4x + 5)(5x^2 + 3x - 4)$
11. $(2n^2 + 3n - 6)(5n^2 - 2n - 8)$

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–2 Find each product.

12. $(3c - 5)(c + 3)$
13. $(g + 10)(2g - 5)$
14. $(6a + 5)(5a + 3)$
15. $(4x + 1)(6x + 3)$
16. $(5y - 4)(3y - 1)$
17. $(6d - 5)(4d - 7)$
18. $(3m + 5)(2m + 3)$
19. $(7n - 6)(7n - 6)$
20. $(12t - 5)(12t + 5)$
21. $(5r + 7)(5r - 7)$
22. $(8w + 4x)(5w - 6x)$
23. $(11z - 5y)(3z + 2y)$

Example 3

24. **GARDEN** A walkway surrounds a rectangular garden. The width of the garden is 8 feet, and the length is 6 feet. The width x of the walkway around the garden is the same on every side. Write an expression that represents the total area of the garden and walkway.

Example 4 Find each product.

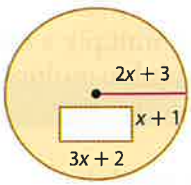
25. $(2y - 11)(y^2 - 3y + 2)$
26. $(4a + 7)(9a^2 + 2a - 7)$
27. $(m^2 - 5m + 4)(m^2 + 7m - 3)$
28. $(x^2 + 5x - 1)(5x^2 - 6x + 1)$
29. $(3b^3 - 4b - 7)(2b^2 - b - 9)$
30. $(6z^2 - 5z - 2)(3z^3 - 2z - 4)$

Simplify.

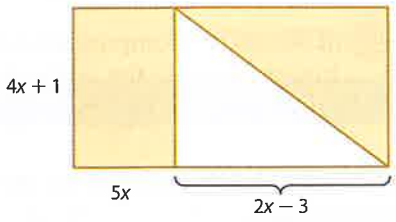
31. $(m + 2)[(m^2 + 3m - 6) + (m^2 - 2m + 4)]$
32. $[(t^2 + 3t - 8) - (t^2 - 2t + 6)](t - 4)$

CCSS **STRUCTURE** **Find an expression to represent the area of each shaded region.**

33.



34.



35 **VOLLEYBALL** The dimensions of a sand volleyball court are represented by a width of $6y - 5$ feet and a length of $3y + 4$ feet.

a. Write an expression that represents the area of the court.

b. The length of a sand volleyball court is 31 feet. Find the area of the court.

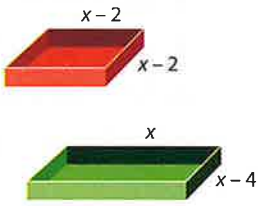
36. GEOMETRY Write an expression for the area of a triangle with a base of $2x + 3$ and a height of $3x - 1$.

Find each product.

37. $(a - 2b)^2$ **38.** $(3c + 4d)^2$ **39.** $(x - 5y)^2$

40. $(2r - 3t)^3$ **41.** $(5g + 2h)^3$ **42.** $(4y + 3z)(4y - 3z)^2$

43. CONSTRUCTION A sandbox kit allows you to build a square sandbox or a rectangular sandbox as shown.



a. What are the possible values of x? Explain.

b. Which shape has the greater area?

c. What is the difference in areas between the two?

44. MULTIPLE REPRESENTATIONS In this problem, you will investigate the square of a sum.

a. Tabular Copy and complete the table for each sum.

Expression	(Expression)^2
$x + 5$	
$3y + 1$	
$z + q$	

b. Verbal Make a conjecture about the terms of the square of a sum.

c. Symbolic For a sum of the form $a + b$, write an expression for the square of the sum.

H.O.T. Problems Use Higher-Order Thinking Skills

45. REASONING Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The FOIL method can be used to multiply a binomial and a trinomial.

46. CHALLENGE Find $(x^m + x^p)(x^{m-1} - x^{1-p} + x^p)$.

47. OPEN ENDED Write a binomial and a trinomial involving a single variable. Then find their product.

48. CCSS REGULARITY Compare and contrast the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.

49. WRITING IN MATH Summarize the methods that can be used to multiply polynomials.

Standardized Test Practice

50. What is the product of $2x - 5$ and $3x + 4$?

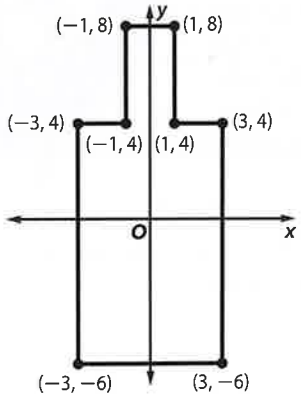
A $5x - 1$

B $6x^2 - 7x - 20$

C $6x^2 - 20$

D $6x^2 + 7x - 20$

51. Which statement is correct about the symmetry of this design?



F The design is symmetrical only about the y-axis.

G The design is symmetrical only about the x-axis.

H The design is symmetrical about both the y- and the x-axes.

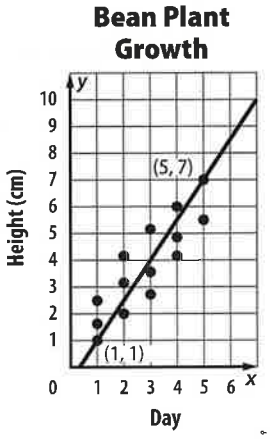
J The design has no symmetry.

52. Which point on the number line represents a number that, when cubed, will result in a number greater than itself?

A P C R

B Q D T

53. SHORT RESPONSE For a science project, Jodi selected three bean plants of equal height. Then, for five days, she measured their heights in centimeters and plotted the values on the graph below.



She drew a line of best fit on the graph. What is the slope of the line that she drew?

Spiral Review

54. SAVINGS Carrie has \$6000 to invest. She puts x dollars of this money into a savings account that earns 2% interest per year. She uses the rest of the money to purchase a certificate of deposit that earns 4% interest. Write an equation for the amount of money that Carrie will have in one year. (Lesson 8-2)

Find each sum or difference. (Lesson 8-1)

55. $(7a^2 - 5) + (-3a^2 + 10)$

56. $(8n - 2n^2) + (4n - 6n^2)$

57. $(4 + n^3 + 3n^2) + (2n^3 - 9n^2 + 6)$

58. $(-4u^2 - 9 + 2u) + (6u + 14 + 2u^2)$

59. $(b + 4) + (c + 3b - 2)$

60. $(3a^3 - 6a) - (3a^3 + 5a)$

61. $(-4m^3 - m + 10) - (3m^3 + 3m^2 - 7)$

62. $(3a + 4ab + 3b) - (2b + 5a + 8ab)$

Skills Review

Simplify.

63. $(-2t^4)^3 - 3(-2t^3)^4$

64. $(-3h^2)^3 - 2(-h^3)^2$

65. $2(-5y^3)^2 + (-3y^3)^3$

66. $3(-6n^4)^2 + (-2n^2)^2$

LESSON 8-4

Special Products

Then

- You multiplied binomials by using the FOIL method.

Now

1. Find squares of sums and differences.
2. Find the product of a sum and a difference.

Why?

- Colby wants to attach a dartboard to a square piece of corkboard. If the radius of the dartboard is $r + 12$, how large does the square corkboard need to be?

Colby knows that the diameter of the dartboard is $2(r + 12)$ or $2r + 24$. Each side of the square also measures $2r + 24$. To find how much corkboard is needed, Colby must find the area of the square: $A = (2r + 24)^2$.

Common Core State Standards

Content Standards

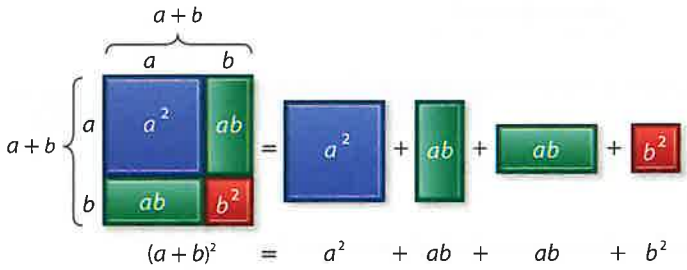
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Squares of Sums and Differences

Some pairs of binomials, such as squares like $(2r + 24)^2$, have products that follow a specific pattern. Using the pattern can make multiplying easier. The square of a sum, $(a + b)^2$ or $(a + b)(a + b)$, is one of those products.



KeyConcept Square of a Sum

Words	The square of $a + b$ is the square of a plus twice the product of a and b plus the square of b.
Symbols	$(a + b)^2 = (a + b)(a + b)$ $= a^2 + 2ab + b^2$
Example	$(x + 4)^2 = (x + 4)(x + 4)$ $= x^2 + 8x + 16$

Example 1 Square of a Sum

Find $(3x + 5)^2$.

$(a + b)^2 = a^2 + 2ab + b^2$ Square of a sum

$(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2$ $a = 3x, b = 5$

$= 9x^2 + 30x + 25$ Simplify. Use FOIL to check your solution.

GuidedPractice

Find each product.

1A. $(8c + 3d)^2$

1B. $(3x + 4y)^2$

There is also a pattern for the *square of a difference*. Write $a - b$ as $a + (-b)$ and square it using the square of a sum pattern.

$$\begin{aligned}(a - b)^2 &= [a + (-b)]^2 \\ &= a^2 + 2(a)(-b) + (-b)^2 && \text{Square of a sum} \\ &= a^2 - 2ab + b^2 && \text{Simplify.}\end{aligned}$$

KeyConcept Square of a Difference

Words The square of $a - b$ is the square of a minus twice the product of a and b plus the square of b.

Symbols $(a - b)^2 = (a - b)(a - b)$
$= a^2 - 2ab + b^2$

Example $(x - 3)^2 = (x - 3)(x - 3)$
$= x^2 - 6x + 9$

WatchOut!

CCSS Regularity Remember that $(x - 7)^2$ does not equal $x^2 - 7^2$, or $x^2 - 49$.

$(x - 7)^2$
$= (x - 7)(x - 7)$
$= x^2 - 14x + 49$

Example 2 Square of a Difference

Find $(2x - 5y)^2$.

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 && \text{Square of a difference} \\ (2x - 5y)^2 &= (2x)^2 - 2(2x)(5y) + (5y)^2 && a = 2x \text{ and } b = 5y \\ &= 4x^2 - 20xy + 25y^2 && \text{Simplify.}\end{aligned}$$

GuidedPractice

Find each product.

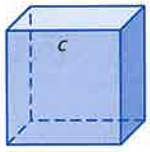
2A. $(6p - 1)^2$

2B. $(a - 2b)^2$

The product of the square of a sum or the square of a difference is called a *perfect square trinomial*. We can use these to find patterns to solve real-world problems.

Real-World Example 3 Square of a Difference

PHYSICAL SCIENCE Each edge of a cube of aluminum is 4 centimeters less than each edge of a cube of copper. Write an equation to model the surface area of the aluminum cube.



Let c = the length of each edge of the cube of copper.
So, each edge of the cube of aluminum is $c - 4$.

$$\begin{aligned}SA &= 6s^2 && \text{Formula for surface area of a cube} \\ SA &= 6(c - 4)^2 && \text{Replace } s \text{ with } c - 4. \\ SA &= 6[c^2 - 2(4)(c) + 4^2] && \text{Square of a difference} \\ SA &= 6(c^2 - 8c + 16) && \text{Simplify.}\end{aligned}$$

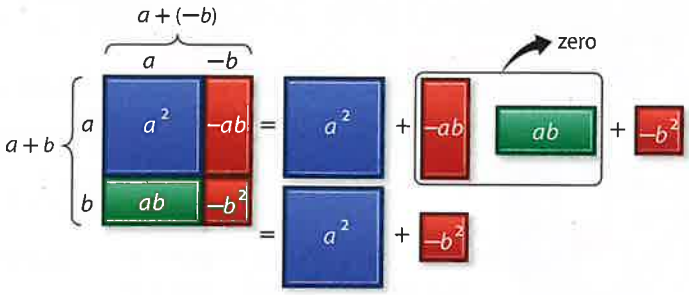
GuidedPractice

3. GARDENING Alano has a garden that is g feet long and g feet wide. He wants to add 3 feet to the length and the width.

A. Show how the new area of the garden can be modeled by the square of a binomial.

B. Find the square of this binomial.

2 Product of a Sum and a Difference Now we will see what the result is when we multiply a sum and a difference, or $(a + b)(a - b)$. Recall that $a - b$ can be written as $a + (-b)$.



Notice that the middle terms are opposites and add to a zero pair.
So $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$.

StudyTip
Patterns When using any of these patterns, a and b can be numbers, variables, or expressions with numbers and variables.

KeyConcept Product of a Sum and a Difference

Words	The product of $a + b$ and $a - b$ is the square of a minus the square of b.
Symbols	$(a + b)(a - b) = (a - b)(a + b)$ $= a^2 - b^2$

Example 4 Product of a Sum and a Difference

Find $(2x^2 + 3)(2x^2 - 3)$.

$(a + b)(a - b) = a^2 - b^2$ Product of a sum and a difference

$(2x^2 + 3)(2x^2 - 3) = (2x^2)^2 - (3)^2$ $a = 2x^2$ and $b = 3$

$= 4x^4 - 9$ Simplify.

GuidedPractice

Find each product.

4A. $(3n + 2)(3n - 2)$

4B. $(4c - 7d)(4c + 7d)$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–2 **Find each product.**

1. $(x + 5)^2$ **2.** $(11 - a)^2$ **3.** $(2x + 7y)^2$

4. $(3m - 4)(3m - 4)$ **5.** $(g - 4h)(g - 4h)$ **6.** $(3c + 6d)^2$

Example 3

7. GENETICS The color of a Labrador retriever's fur is genetic. Dark genes D are dominant over yellow genes y. A dog with genes DD or Dy will have dark fur. A dog with genes yy will have yellow fur. Pepper's genes for fur color are Dy, and Ramiro's are yy.

	D	y
D	DD	Dy
y	Dy	yy

a. Write an expression for the possible fur colors of Pepper's and Ramiro's puppies.

b. What is the probability that a puppy will have yellow fur?

Example 4 **Find each product.**

8. $(a - 3)(a + 3)$

9. $(x + 5)(x - 5)$

10. $(6y - 7)(6y + 7)$

11. $(9t + 6)(9t - 6)$

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–2 **Find each product.**

12. $(a + 10)(a + 10)$

13. $(b - 6)(b - 6)$

14. $(h + 7)^2$

15. $(x + 6)^2$

16. $(8 - m)^2$

17. $(9 - 2y)^2$

18. $(2b + 3)^2$

19. $(5t - 2)^2$

20. $(8h - 4n)^2$

Example 3

21. GENETICS The ability to roll your tongue is inherited genetically from parents if either parent has the dominant trait T. Children of two parents without the trait will not be able to roll their tongues.

	T	t
T	TT	Tt
t	Tt	tt

a. Show how the combinations can be modeled by the square of a sum.

b. Predict the percent of children that will have both dominant genes, one dominant gene, and both recessive genes.

Example 4 **Find each product.**

22. $(u + 3)(u - 3)$

23. $(b + 7)(b - 7)$

24. $(2 + x)(2 - x)$

25. $(4 - x)(4 + x)$

26. $(2q + 5r)(2q - 5r)$

27. $(3a^2 + 7b)(3a^2 - 7b)$

28. $(5y + 7)^2$

29. $(8 - 10a)^2$

30. $(10x - 2)(10x + 2)$

31. $(3t + 12)(3t - 12)$

32. $(a + 4b)^2$

33. $(3q - 5r)^2$

34. $(2c - 9d)^2$

35. $(g + 5h)^2$

36. $(6y - 13)(6y + 13)$

37. $(3a^4 - b)(3a^4 + b)$

38. $(5x^2 - y^2)^2$

39. $(8a^2 - 9b^3)(8a^2 + 9b^3)$

40. $\left(\frac{3}{4}k + 8\right)^2$

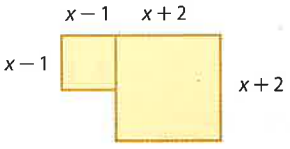
41. $\left(\frac{2}{5}y - 4\right)^2$

42. $(7z^2 + 5y^2)(7z^2 - 5y^2)$

43. $(2m + 3)(2m - 3)(m + 4)$

44. $(r + 2)(r - 5)(r - 2)(r + 5)$

45. CCSS **SENSE-MAKING** Write a polynomial that represents the area of the figure at the right.



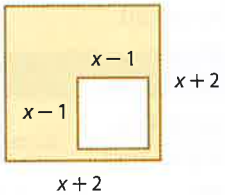
46. FLYING DISKS A flying disk shaped like a circle has a radius of $x + 3$ inches.

a. Write an expression representing the area of the flying disk.

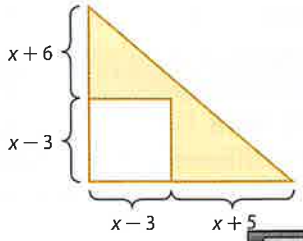
b. A hole with a radius of $x - 1$ inches is cut in the center of the disk. Write an expression for the remaining area.

GEOMETRY **Find the area of each shaded region.**

47.



48.

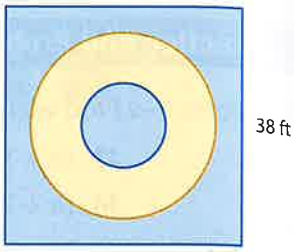


Find each product.

49. $(c + d)(c + d)(c + d)$ **50.** $(2a - b)^3$ **51.** $(f + g)(f - g)(f + g)$

52. $(k - m)(k + m)(k - m)$ **53.** $(n - p)^2(n + p)$ **54.** $(q + r)^2(q - r)$

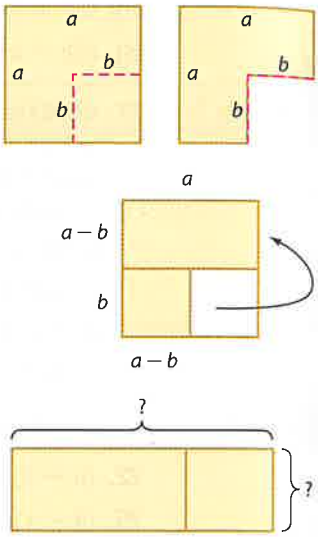
55. WRESTLING A high school wrestling mat must be a square with 38-foot sides and contain two circles as shown. Suppose the inner circle has a radius of r feet, and the radius of the outer circle is nine feet longer than the inner circle.



a. Write an expression for the area of the larger circle.

b. Write an expression for the area of the portion of the square outside the larger circle.

56. MULTIPLE REPRESENTATIONS In this problem, you will investigate a pattern. Begin with a square piece of construction paper. Label each edge of the paper a. In any of the corners, draw a smaller square and label the edges b.



a. Numerical Find the area of each of the squares.

b. Concrete Cut the smaller square out of the corner. What is the area of the shape?

c. Analytical Remove the smaller rectangle on the bottom. Turn it and slide it next to the top rectangle. What is the length of the new arrangement? What is the width? What is the area?

d. Analytical What pattern does this verify?

H.O.T. Problems Use Higher-Order Thinking Skills

57. WHICH ONE DOESN'T BELONG? Which expression does not belong? Explain.

$(2c - d)(2c - d)$	$(2c + d)(2c - d)$	$(2c + d)(2c + d)$	$(c + d)(c + d)$

58. CCSS STRUCTURE Does a pattern exist for the cube of a sum $(a + b)^3$?

a. Investigate this question by finding the product $(a + b)(a + b)(a + b)$.

b. Use the pattern you discovered in part **a** to find $(x + 2)^3$.

c. Draw a diagram of a geometric model for $(a + b)^3$.

d. What is the pattern for the cube of a difference, $(a - b)^3$?

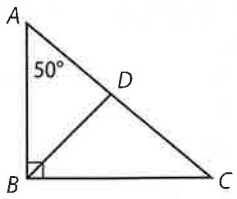
59. REASONING Find c that makes $25x^2 - 90x + c$ a perfect square trinomial.

60. OPEN ENDED Write two binomials with a product that is a binomial. Then write two binomials with a product that is not a binomial.

61. WRITING IN MATH Describe how to square the sum of two quantities, square the difference of two quantities, and how to find the product of a sum of two quantities and a difference of two quantities.

Standardized Test Practice

62. GRIDDED RESPONSE In the right triangle, $\overline{DB}$ bisects $\angle B$. What is the measure of $\angle ADB$ in degrees?



63. What is the product of $(2a - 3)$ and $(2a - 3)$?

A $4a^2 + 12a + 9$ **C** $4a^2 - 12a - 9$
B $4a^2 + 9$ **D** $4a^2 - 12a + 9$

64. Myron can drive 4 miles in m minutes. At this rate, how many minutes will it take him to drive 19 miles?

F $76m$ **H** $\frac{4m}{19}$
G $\frac{19m}{4}$ **J** $\frac{4}{19m}$

65. What property is illustrated by the equation $2x + 0 = 2x$?

A Commutative Property of Addition
B Additive Inverse Property
C Additive Identity Property
D Associative Property of Addition

Spiral Review

Find each product. (Lesson 8-3)

66. $(y - 4)(y - 2)$ **67.** $(2c - 1)(c + 3)$ **68.** $(d - 9)(d + 5)$

69. $(4h - 3)(2h - 7)$ **70.** $(3x + 5)(2x + 3)$ **71.** $(5m + 4)(8m + 3)$

Simplify. (Lesson 8-2)

72. $x(2x - 7) + 5x$ **73.** $c(c - 8) + 2c(c + 3)$ **74.** $8y(-3y + 7) - 11y^2$

75. $-2d(5d) - 3d(d + 6)$ **76.** $5m(2m^3 + m^2 + 8) + 4m$ **77.** $3p(6p - 4) + 2\left(\frac{1}{2}p^2 - 3p\right)$

Use substitution to solve each system of equations. (Lesson 6-2)

78. $4c = 3d + 3$
$c = d - 1$

79. $c - 5d = 2$
$2c + d = 4$

80. $5r - t = 5$
$-4r + 5t = 17$

81. BIOLOGY Each type of fish thrives in a specific range of temperatures. The best temperatures for sharks range from 18°C to 22°C, inclusive. Write a compound inequality to represent temperatures where sharks will not thrive. (Lesson 5-4)

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

82. (1, 1), (7, 4) **83.** (5, 7), (0, 6) **84.** (5, 1), (8, −2)

85. COFFEE A coffee store wants to create a mix using two coffees. How many pounds of coffee A should be mixed with 9 pounds of coffee B to get a mixture that can sell for $6.95 per pound? (Lesson 2-9)

Skills Review

Write each polynomial in standard form. Identify the leading coefficient.

86. $2x^2 - x^4 - 8 + x$ **87.** $-5p^4 + p^2 + 12 + 2p^5$ **88.** $-10 + a^3 - a + 6a^2$

CHAPTER 8

Mid-Chapter Quiz

Lessons 8-1 through 8-4

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*. (Lesson 8-1)

1. $3y^2 - 2$
2. $4t^5 + 3t^2 + t$
3. $\frac{3x}{5y}$
4. ax^{-3}
5. $3b^2$
6. $2x^{-3} - 4x + 1$
7. **POPULATION** The table shows the population density for Nevada for various years. (Lesson 8-1)

Year	Years Since 1930	People/ Square Mile
1930	0	0.8
1960	30	2.6
1980	50	7.3
1990	60	10.9
2000	70	18.2

   a. The population density d of Nevada from 1930 to 2000 can be modeled by $d = 0.005n^2 - 0.127n + 1$, where n represents the number of years since 1930. Identify the type of polynomial for $0.005n^2 - 0.127n + 1$.

   b. What is the degree of the polynomial?

   c. Predict the population density of Nevada for 2020 and for 2030. Explain your method.

Find each sum or difference. (Lesson 8-1)

8. $(y^2 + 2y + 3) + (y^2 + 3y - 1)$
9. $(3n^3 - 2n + 7) - (n^2 - 2n + 8)$
10. $(5d + d^2) - (4 - 4d^2)$
11. $(x + 4) + (3x + 2x^2 - 7)$
12. $(3a - 3b + 2) - (4a + 5b)$
13. $(8x - y^2 + 3) + (9 - 3x + 2y^2)$

Find each product. (Lesson 8-2)

14. $6y(y^2 + 3y + 1)$
15. $3n(n^2 - 5n + 2)$
16. $d^2(-4 - 3d + 2d^2)$
17. $-2xy(3x^2 + 2xy - 4y^2)$
18. $ab^2(12a + 5b - ab)$
19. $x^2y^4(3xy^2 - x + 2y^2)$
20. **MULTIPLE CHOICE** Simplify $x(4x + 5) + 3(2x^2 - 4x + 1)$. (Lesson 8-2)

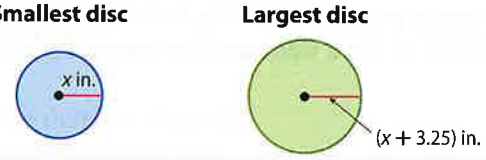
   A $10x^2 + 17x + 3$ **C** $2x^2 - 7x + 3$
   B $10x^2 - 7x + 3$ **D** $2x^2 + 17x + 3$

Find each product. (Lesson 8-3)

21. $(x + 2)(x + 5)$
22. $(3b - 2)(b - 4)$
23. $(n - 5)(n + 3)$
24. $(4c - 2)(c + 2)$
25. $(k - 1)(k - 3k^2)$
26. $(8d - 3)(2d^2 + d + 1)$
27. **MANUFACTURING** A company is designing a box for dry pasta in the shape of a rectangular prism. The length is 2 inches more than twice the width, and the height is 3 inches more than the length. Write an expression, in terms of the width, for the volume of the box. (Lesson 8-3)

Find each product. (Lesson 8-4)

28. $(x + 2)^2$
29. $(n - 11)^2$
30. $(4b - 2)^2$
31. $(6c + 3)^2$
32. $(5d - 3)(5d + 3)$
33. $(9k + 1)(9k - 1)$
34. **DISC GOLF** The discs approved for use in disc golf vary in size. (Lesson 8-4)



   a. Write two different expressions for the area of the largest disc.

   b. If x is 10.5, what are the areas of the smallest and largest discs?

EXPLORE

8-5 Algebra Lab
Factoring Using the Distributive Property

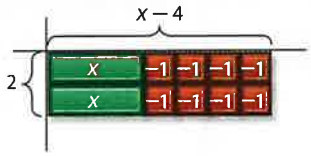
When two or more numbers are multiplied, these numbers are *factors* of the product. Sometimes you know the product of binomials and are asked to find the factors. This is called factoring. You can use algebra tiles and a product mat to factor binomials.

Common Core State Standards
Content Standards
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Activity 1 Use Algebra Tiles to Factor $2x - 8$

Step 1 Model $2x - 8$.

Step 2 Arrange the tiles into a rectangle. The total area of the rectangle represents the product, and its length and width represent the factors.

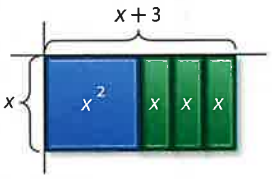


The rectangle has a width of 2 and a length of $x - 4$. Therefore, $2x - 8 = 2(x - 4)$.

Activity 2 Use Algebra Tiles to Factor $x^2 + 3x$

Step 1 Model $x^2 + 3x$.

Step 2 Arrange the tiles into a rectangle.



The rectangle has a width of x and a length of $x + 3$. Therefore, $x^2 + 3x = x(x + 3)$.

Model and Analyze

Use algebra tiles to factor each binomial.

1. $4x + 12$
2. $4x - 6$
3. $3x^2 + 4x$
4. $10 - 2x$

Determine whether each binomial can be factored. Justify your answer with a drawing.

5. $6x - 9$
6. $5x - 4$
7. $4x^2 + 7$
8. $x^2 + 3x$

9. **WRITING IN MATH** Write a paragraph that explains how you can use algebra tiles to determine whether a binomial can be factored. Include an example of one binomial that can be factored and one that cannot.

LESSON 8-5

Using the Distributive Property

Then
- Used the Distributive Property to evaluate expressions.

Now
1. Use the Distributive Property to factor polynomials.
2. Solve equations of the form $ax^2 + bx = 0$.

Why?
- The cost of rent for Mr. Cole's store is determined by the square footage of the space. The area of the store can be modeled by the equation $A = 1.6w^2 + 6w$, where w is the width of the store in feet. We can use factoring and the Zero Product Property to find possible dimensions of the store.

New Vocabulary
- factoring
- factoring by grouping
- Zero Product Property

Common Core State Standards

Content Standards

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Use the Distributive Property to Factor

You have used the Distributive Property to multiply a monomial by a polynomial. You can work backward to express a polynomial as the product of a monomial factor and a polynomial factor.

$$\begin{aligned} 1.6w^2 + 6w &= 1.6w(w) + 6(w) \\ &= w(1.6w + 6) \end{aligned}$$

So, $w(1.6w + 6)$ is the *factored form* of $1.6w^2 + 6w$. **Factoring** a polynomial involves finding the *completely* factored form.

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $27y^2 + 18y$

Find the GCF of each term.

$27y^2 = 3 \cdot 3 \cdot 3 \cdot y \cdot y$ — Factor each term.

$18y = 2 \cdot 3 \cdot 3 \cdot y$ — Circle common factors.

GCF $= 3 \cdot 3 \cdot y$ or $9y$

Write each term as the product of the GCF and the remaining factors. Use the Distributive Property to *factor out* the GCF.

$27y^2 + 18y = 9y(3y) + 9y(2)$ — Rewrite each term using the GCF.

$= 9y(3y + 2)$ — Distributive Property

b. $-4a^2b - 8ab^2 + 2ab$

$-4a^2b = -1 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b$ — Factor each term.

$-8ab^2 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b$ — Circle common factors.

$2ab = 2 \cdot a \cdot b$

GCF $= 2 \cdot a \cdot b$ or $2ab$

$-4a^2b - 8ab^2 + 2ab = 2ab(-2a) - 2ab(4b) + 2ab(1)$ — Rewrite each term using the GCF.

$= 2ab(-2a - 4b + 1)$ — Distributive Property

Guided Practice

1A. $15w - 3v$

1B. $7u^2t^2 + 21ut^2 - ut$

Using the Distributive Property to factor polynomials with four or more terms is called **factoring by grouping** because terms are put into groups and then factored. The Distributive Property is then applied to a common binomial factor.

KeyConcept Factoring by Grouping

Words A polynomial can be factored by grouping only if all of the following conditions exist.

- There are four or more terms.
- Terms have common factors that can be grouped together.
- There are two common factors that are identical or additive inverses of each other.

Symbols
$$\begin{aligned} ax + bx + ay + by &= (ax + bx) + (ay + by) \\ &= x(a + b) + y(a + b) \\ &= (x + y)(a + b) \end{aligned}$$

Example 2 Factor by Grouping

Factor $4qr + 8r + 3q + 6$.

$4qr + 8r + 3q + 6$	Original expression
$= (4qr + 8r) + (3q + 6)$	Group terms with common factors.
$= 4r(q + 2) + 3(q + 2)$	Factor the GCF from each group.

Notice that $(q + 2)$ is common in both groups, so it becomes the GCF.

$= (4r + 3)(q + 2)$	Distributive Property

GuidedPractice

Factor each polynomial.

2A. $rn + 5n - r - 5$

2B. $3np + 15p - 4n - 20$

It can be helpful to recognize when binomials are additive inverses of each other. For example $6 - a = -1(a - 6)$.

StudyTip

Check To check your factored answers, multiply your factors out. You should get your original expression as a result.

Example 3 Factor by Grouping with Additive Inverses

Factor $2mk - 12m + 42 - 7k$.

$2mk - 12m + 42 - 7k$	
$= (2mk - 12m) + (42 - 7k)$	Group terms with common factors.
$= 2m(k - 6) + 7(6 - k)$	Factor the GCF from each group.
$= 2m(k - 6) + 7[(-1)(k - 6)]$	$6 - k = -1(k - 6)$
$= 2m(k - 6) - 7(k - 6)$	Associative Property
$= (2m - 7)(k - 6)$	Distributive Property

GuidedPractice

Factor each polynomial.

3A. $c - 2cd + 8d - 4$

3B. $3p - 2p^2 - 18p + 27$

2 Solve Equations by Factoring

Some equations can be solved by factoring. Consider the following.

$3(0) = 0$ $\quad 0(2 - 2) = 0$ $\quad -312(0) = 0$ $\quad 0(0.25) = 0$

Notice that in each case, at least one of the factors is 0. These examples are demonstrations of the **Zero Product Property**.

KeyConcept Zero Product Property

Words	If the product of two factors is 0, then at least one of the factors must be 0.
Symbols	For any real numbers a and b, if $ab = 0$, then $a = 0$, $b = 0$, or both a and b equal zero.

Recall that a solution or root of an equation is any value that makes the equation true.

WatchOut!

Unknown Value It may be tempting to solve an equation by dividing each side by the variable. However, the variable has an unknown value, so you may be dividing by 0, which is undefined.

Example 4 Solve Equations

Solve each equation. Check your solutions.

a. $(2d + 6)(3d - 15) = 0$

$(2d + 6)(3d - 15) = 0$ — Original equation

$2d + 6 = 0$ or $3d - 15 = 0$ — Zero Product Property

$2d = -6$ $\quad 3d = 15$ — Solve each equation.

$d = -3$ $\quad d = 5$ — Divide.

The roots are -3 and 5.

CHECK Substitute -3 and 5 for d in the original equation.

$(2d + 6)(3d - 15) = 0$ $\quad (2d + 6)(3d - 15) = 0$

$[2(-3) + 6][3(-3) - 15] \stackrel{?}{=} 0$ $\quad [2(5) + 6][3(5) - 15] \stackrel{?}{=} 0$

$(-6 + 6)(-9 - 15) \stackrel{?}{=} 0$ $\quad (10 + 6)(15 - 15) \stackrel{?}{=} 0$

$(0)(-24) \stackrel{?}{=} 0$ $\quad 16(0) \stackrel{?}{=} 0$

$0 = 0$ ✓ $\quad 0 = 0$ ✓

b. $c^2 = 3c$

$c^2 = 3c$ — Original equation

$c^2 - 3c = 0$ — Subtract $3c$ from each side to get 0 on one side of the equation.

$c(c - 3) = 0$ — Factor by using the GCF to get the form $ab = 0$.

$c = 0$ or $c - 3 = 0$ — Zero Product Property

$c = 3$ — Solve each equation.

The roots are 0 and 3. — Check by substituting 0 and 3 for c.

GuidedPractice

4A. $3n(n + 2) = 0$ $\quad$ **4B.** $8b^2 - 40b = 0$ $\quad$ **4C.** $x^2 = -10x$

Real-WorldLink

Dog agility tests a person's skills as a trainer and handler. Competitors race through an obstacle course that includes hurdles, tunnels, a see-saw, and line poles.

Source: United States Dog Agility Association

Real-World Example 5 Use Factoring

AGILITY **Penny is a Fox Terrier who competes with her trainer in the agility course. Within the course, Penny must leap over a hurdle. Penny's jump can be modeled by the equation $h = -16t^2 + 20t$, where h is the height of the leap in inches at t seconds. Find the values of t when $h = 0$.**

$h = -16t^2 + 20t$	Original equation
$0 = -16t^2 + 20t$	Substitution, $h = 0$
$0 = 4t(-4t + 5)$	Factor by using the GCF.
$4t = 0$ or $-4t + 5 = 0$	Zero Product Property
$t = 0$ $\quad -4t = -5$	Solve each equation.
$t = \frac{5}{4}$ or 1.25	Divide each side by -4.

Penny's height is 0 inches at 0 seconds and 1.25 seconds into the jump.

GuidedPractice

5. KANGAROOS The hop of a kangaroo can be modeled by $h = 24t - 16t^2$ where h represents the height of the hop in meters and t is the time in seconds. Find the values of t when $h = 0$.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Use the Distributive Property to factor each polynomial.**

1. $21b - 15a$

2. $14c^2 + 2c$

3. $10g^2h^2 + 9gh^2 - g^2h$

4. $12jk^2 + 6j^2k + 2j^2k^2$

Examples 2–3 **Factor each polynomial.**

5. $np + 2n + 8p + 16$

6. $xy - 7x + 7y - 49$

7. $3bc - 2b - 10 + 15c$

8. $9fg - 45f - 7g + 35$

Example 4 **Solve each equation. Check your solutions.**

9. $3k(k + 10) = 0$

10. $(4m + 2)(3m - 9) = 0$

11. $20p^2 - 15p = 0$

12. $r^2 = 14r$

Example 5

13. SPIDERS Jumping spiders can commonly be found in homes and barns throughout the United States. A jumping spider's jump can be modeled by the equation $h = 33.3t - 16t^2$, where t represents the time in seconds and h is the height in feet.

a. When is the spider's height at 0 feet?

b. What is the spider's height after 1 second? after 2 seconds?

14. REASONING At a Fourth of July celebration, a rocket is launched straight up with an initial velocity of 125 feet per second. The height h of the rocket in feet above sea level is modeled by the formula $h = 125t - 16t^2$, where t is the time in seconds after the rocket is launched.

a. What is the height of the rocket when it returns to the ground?

b. Let $h = 0$ in the equation and solve for t.

c. How many seconds will it take for the rocket to return to the ground?

Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Use the Distributive Property to factor each polynomial.**

15. $16t - 40y$

16. $30v + 50x$

17. $2k^2 + 4k$

18. $5z^2 + 10z$

19. $4a^2b^2 + 2a^2b - 10ab^2$

20. $5c^2v - 15c^2v^2 + 5c^2v^3$

Examples 2–3 **Factor each polynomial.**

21. $fg - 5g + 4f - 20$

22. $a^2 - 4a - 24 + 6a$

23. $hj - 2h + 5j - 10$

24. $xy - 2x - 2 + y$

25. $45pq - 27q - 50p + 30$

26. $24ty - 18t + 4y - 3$

27. $3dt - 21d + 35 - 5t$

28. $8r^2 + 12r$

29. $21th - 3t - 35h + 5$

30. $vp + 12v + 8p + 96$

31. $5br - 25b + 2r - 10$

32. $2nu - 8u + 3n - 12$

33. $5gf^2 + g^2f + 15gf$

34. $rp - 9r + 9p - 81$

35. $27cd^2 - 18c^2d^2 + 3cd$

36. $18r^3t^2 + 12r^2t^2 - 6r^2t$

37. $48tu - 90t + 32u - 60$

38. $16gh + 24g - 2h - 3$

Example 4 **Solve each equation. Check your solutions.**

39. $3b(9b - 27) = 0$

40. $2n(3n + 3) = 0$

41. $(8z + 4)(5z + 10) = 0$

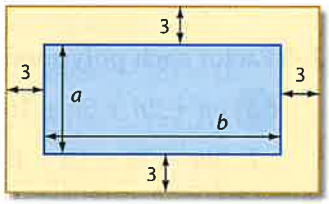
42. $(7x + 3)(2x - 6) = 0$

43. $b^2 = -3b$

44. $a^2 = 4a$

Example 5

45. CCSS **SENSE-MAKING** Use the drawing at the right.



a. Write an expression in factored form to represent the area of the blue section.

b. Write an expression in factored form to represent the area of the region formed by the outer edge.

c. Write an expression in factored form to represent the yellow region.

46. **FIREWORKS** A ten-inch fireworks shell is fired from ground level. The height of the shell in feet is given by the formula $h = 263t - 16t^2$, where t is the time in seconds after launch.

a. Write the expression that represents the height in factored form.

b. At what time will the height be 0? Is this answer practical? Explain.

c. What is the height of the shell 8 seconds and 10 seconds after being fired?

d. At 10 seconds, is the shell rising or falling?

47. **ARCHITECTURE** The frame of a doorway is an arch that can be modeled by the graph of the equation $y = -3x^2 + 12x$, where x and y are measured in feet. On a coordinate plane, the floor is represented by the x-axis.

a. Make a table of values for the height of the arch if $x = 0, 1, 2, 3$, and 4 feet.

b. Plot the points from the table on a coordinate plane and connect the points to form a smooth curve to represent the arch.

c. How high is the doorway?

48. **RIDES** Suppose the height of a rider after being dropped can be modeled by $h = -16t^2 - 96t + 160$, where h is the height in feet and t is time in seconds.
   a. Write an expression to represent the height in factored form.
   b. From what height is the rider initially dropped?
   c. At what height will the rider be after 3 seconds of falling? Is this possible? Explain.

49. **ARCHERY** The height h in feet of an arrow can be modeled by the equation $h = 64t - 16t^2$, where t is time in seconds. Ignoring the height of the archer, how long after the arrow is released does it hit the ground?

50. **TENNIS** A tennis player hits a tennis ball upward with an initial velocity of 80 feet per second. The height h in feet of the tennis ball can be modeled by the equation $h = 80t - 16t^2$, where t is time in seconds. Ignoring the height of the tennis player, how long does it take the ball to hit the ground?

51. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the *box method* of factoring. To factor $x^2 + x - 6$, write the first term in the top left-hand corner of the box, and then write the last term in the lower right-hand corner.

	?	?
?	x^2	?
?	?	-6

   a. **Analytical** Determine which two factors have a product of -6 and a sum of 1.
   b. **Symbolic** Write each factor in an empty square in the box. Include the positive or negative sign and variable.
   c. **Analytical** Find the factor for each row and column of the box. What are the factors of $x^2 + x - 6$?
   d. **Verbal** Describe how you would use the box method to factor $x^2 - 3x - 40$.

H.O.T. Problems Use Higher-Order Thinking Skills

52. **CCSS CRITIQUE** Hernando and Rachel are solving $2m^2 = 4m$. Is either of them correct? Explain your reasoning.

Hernando

$2m^2 = 4m$

$\frac{2m^2}{m} = \frac{4m^2}{2m}$

$2m = 2$

$m = 1$

Rachel

$2m^2 = 4m$

$2m^2 - 4m = 0$

$2m(m - 2) = 0$

$2m = 0$ or $m - 2 = 0$

$m = 0$ or 2

53. **CHALLENGE** Given the equation $(ax + b)(ax - b) = 0$, solve for x. What do we know about the values of a and b?

54. **OPEN ENDED** Write a four-term polynomial that can be factored by grouping. Then factor the polynomial.

55. **REASONING** Given the equation $c = a^2 - ab$, for what values of a and b does $c = 0$?

56. **WRITING IN MATH** Explain how to solve a quadratic equation by using the Zero Product Property.

Standardized Test Practice

57. Which is a factor of $6z^2 - 3z - 2 + 4z$?

A $2z + 1$ C $z + 2$
B $3z - 2$ D $2z - 1$

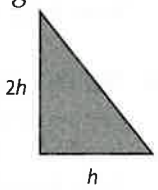
58. PROBABILITY Hailey has 10 blocks: 2 red, 4 blue, 3 yellow, and 1 green. What is the probability that a randomly chosen block will be either red or yellow?

F $\frac{3}{10}$ H $\frac{1}{2}$
G $\frac{1}{5}$ J $\frac{7}{10}$

59. GRIDDED RESPONSE Cho is making a 140-inch by 160-inch quilt with quilt squares that measure 8 inches on each side. How many will be needed to make the quilt?

60. GEOMETRY The area of the right triangle shown below is $5h$ square centimeters. What is the height of the triangle?

A 2 cm
B 5 cm
C 8 cm
D 10 cm



Spiral Review

61. GENETICS Brown genes B are dominant over blue genes b. A person with genes BB or Bb has brown eyes. Someone with genes bb has blue eyes. Elisa has brown eyes with Bb genes, and Bob has blue eyes. Write an expression for the possible eye coloring of Elisa and Bob's children. Determine the probability that their child would have blue eyes. (Lesson 8-4)

Find each product. (Lesson 8-2)

62. $n(n^2 - 4n + 3)$
63. $2b(b^2 + b - 5)$
64. $-c(4c^2 + 2c - 2)$
65. $-4x(x^3 + x^2 + 2x - 1)$
66. $2ab(4a^2b + 2ab - 2b^2)$
67. $-3xy(x^2 + xy + 2y^2)$

Simplify. (Lesson 7-1)

68. $(ab^4)(ab^2)$
69. $(p^5r^4)(p^2r)$
70. $(-7c^3d^4)(4cd^3)$
71. $(9xy^7)^2$
72. $[(3^2)^4]^2$
73. $[(4^2)^3]^2$

74. BASKETBALL In basketball, a free throw is 1 point and a field goal is either 2 or 3 points. In a season, Tim Duncan of the San Antonio Spurs scored a total of 1342 points. The total number of 2-point field goals and 3-point field goals was 517, and he made 305 of the 455 free throws that he attempted. Find the number of 2-point field goals and 3-point field goals Duncan made that season. (Lesson 6-4)

Solve each inequality. Check your solution. (Lesson 5-3)

75. $3y - 4 > -37$
76. $-5q + 9 > 24$
77. $-2k + 12 < 30$
78. $5q + 7 \leq 3(q + 1)$
79. $\frac{z}{4} + 7 \geq -5$
80. $8c - (c - 5) > c + 17$

Skills Review

Find each product.

81. $(a + 2)(a + 5)$
82. $(d + 4)(d + 10)$
83. $(z - 1)(z - 8)$
84. $(c + 9)(c - 3)$
85. $(x - 7)(x - 6)$
86. $(g - 2)(g + 11)$

EXPLORE

8-6 Algebra Lab
Factoring Trinomials

You can use algebra tiles to factor trinomials. If a polynomial represents the area of a rectangle formed by algebra tiles, then the rectangle's length and width are *factors* of the area. If a rectangle cannot be formed to represent the trinomial, then the trinomial is not factorable.

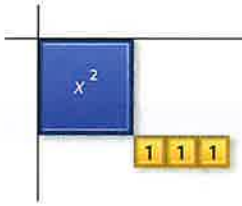
Common Core State Standards
Content Standards
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Activity 1 Factor $x^2 + bx + c$

Use algebra tiles to factor $x^2 + 4x + 3$.

Step 1 Model $x^2 + 4x + 3$.

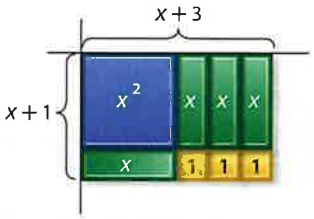
Step 2 Place the x^2-tile at the corner of the product mat. Arrange the 1-tiles into a rectangular array. Because 3 is prime, the 3 tiles can be arranged in a rectangle in one way, a 1-by-3 rectangle.



Step 3 Complete the rectangle with the x-tiles.

The rectangle has a width of $x + 1$ and a length of $x + 3$.

Therefore, $x^2 + 4x + 3 = (x + 1)(x + 3)$.

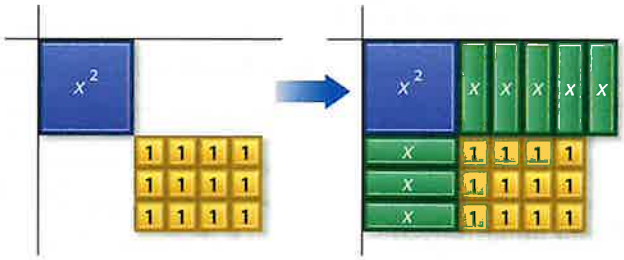


Activity 2 Factor $x^2 + bx + c$

Use algebra tiles to factor $x^2 + 8x + 12$.

Step 1 Model $x^2 + 8x + 12$.

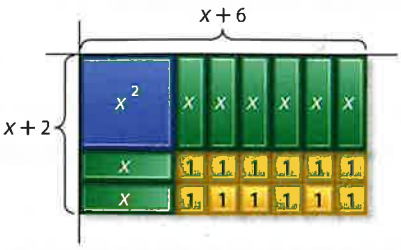
Step 2 Place the x^2-tile at the corner of the product mat. Arrange the 1-tiles into a rectangular array. Since $12 = 3 \times 4$, try a 3-by-4 rectangle. Try to complete the rectangle. Notice that there is an extra x-tile.



Step 3 Arrange the 1-tiles into a 2-by-6 rectangular array. This time you can complete the rectangle with the x-tiles.

The rectangle has a width of $x + 2$ and a length of $x + 6$.

Therefore, $x^2 + 8x + 12 = (x + 2)(x + 6)$.



Algebra Lab

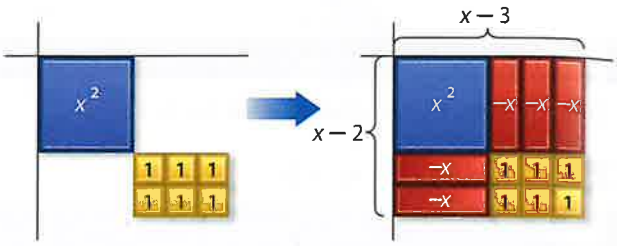
Factoring Trinomials *Continued*

Activity 3 Factor $x^2 - bx + c$

Use algebra tiles to factor $x^2 - 5x + 6$.

Step 1 Model $x^2 - 5x + 6$.

Step 2 Place the x^2-tile at the corner of the product mat. Arrange the 1-tiles into a 2-by-3 rectangular array as shown.



Step 3 Complete the rectangle with the x-tiles. The rectangle has a width of $x - 2$ and a length of $x - 3$.

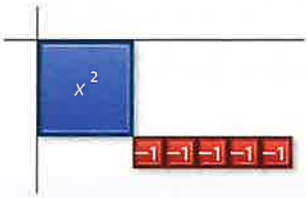
Therefore, $x^2 - 5x + 6 = (x - 2)(x - 3)$.

Activity 4 Factor $x^2 - bx - c$

Use algebra tiles to factor $x^2 - 4x - 5$.

Step 1 Model $x^2 - 4x - 5$.

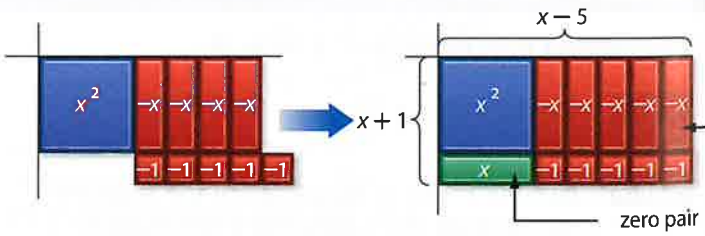
Step 2 Place the x^2-tile at the corner of the product mat. Arrange the 1-tiles into a 1-by-5 rectangular array as shown.



Step 3 Place the x-tile as shown. Recall that you can add zero pairs without changing the value of the polynomial. In this case, add a zero pair of x-tiles.

The rectangle has a width of $x + 1$ and a length of $x - 5$.

Therefore, $x^2 - 4x - 5 = (x + 1)(x - 5)$.



Model and Analyze

Use algebra tiles to factor each trinomial.

1. $x^2 + 3x + 2$
2. $x^2 + 6x + 8$
3. $x^2 + 3x - 4$
4. $x^2 - 7x + 12$
5. $x^2 + 7x + 10$
6. $x^2 - 2x + 1$
7. $x^2 + x - 12$
8. $x^2 - 8x + 15$

Tell whether each trinomial can be factored. Justify your answer with a drawing.

9. $x^2 + 3x + 6$
10. $x^2 - 5x - 6$
11. $x^2 - x - 4$
12. $x^2 - 4$
13. **WRITING IN MATH** How can you use algebra tiles to determine whether a trinomial can be factored?

LESSON 8-6

Solving $x^2 + bx + c = 0$

Then

- You multiplied binomials by using the FOIL method.

Now

1. Factor trinomials of the form $x^2 + bx + c$.
2. Solve equations of the form $x^2 + bx + c = 0$.

Why?

- Diana is having a rectangular in-ground swimming pool installed and she wants to include a 24-foot fence around the pool. The pool requires a space of 36 square feet. What dimensions should the pool have?

To solve this problem, the landscape architect needs to find two numbers that have a product of 36 and a sum of 12, half the perimeter of the pool.

New Vocabulary
quadratic equation

Common Core State Standards

Content Standards

A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Mathematical Practices

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

1 Factor $x^2 + bx + c$ You have learned how to multiply two binomials using the FOIL method. Each of the binomials was a factor of the product. The pattern for multiplying two binomials can be used to factor certain types of trinomials.

$$\begin{aligned}(x + 3)(x + 4) &= x^2 + 4x + 3x + 3 \cdot 4 && \text{Use the FOIL method.}\\ &= x^2 + (4 + 3)x + 3 \cdot 4 && \text{Distributive Property}\\ &= x^2 + 7x + 12 && \text{Simplify.}\end{aligned}$$

Notice that the coefficient of the middle term, $7x$, is the sum of 3 and 4, and the last term, 12, is the product of 3 and 4.

Observe the following pattern in this multiplication.

$$\begin{aligned}(x + 3)(x + 4) &= x^2 + (4 + 3)x + (3 \cdot 4)\\ (x + m)(x + p) &= x^2 + (p + m)x + mp && \text{Let } 3 = m \text{ and } 4 = p.\\ &= x^2 + \underbrace{(m + p)}_{b}x + \underbrace{mp}_{c} && \text{Commutative (+)}\\ x^2 &+ bx + c && b = m + p \text{ and } c = mp\end{aligned}$$

Notice that the coefficient of the middle term is the sum of m and p, and the last term is the product of m and p. This pattern can be used to factor trinomials of the form $x^2 + bx + c$.

KeyConcept Factoring $x^2 + bx + c$

Words	To factor trinomials in the form $x^2 + bx + c$, find two integers, m and p, with a sum of b and a product of c. Then write $x^2 + bx + c$ as $(x + m)(x + p)$.
Symbols	$x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$.
Example	$x^2 + 6x + 8 = (x + 2)(x + 4)$, because $2 + 4 = 6$ and $2 \cdot 4 = 8$.

When c is positive, its factors have the same signs. Both of the factors are positive or negative based upon the sign of b. If b is positive, the factors are positive. If b is negative, the factors are negative.

Problem-Solving Tip

Guess and Check When factoring a trinomial, make an educated guess, check for reasonableness, and then adjust the guess until the correct answer is found.

Example 1 b and c are Positive

Factor $x^2 + 9x + 20$.

In this trinomial, $b = 9$ and $c = 20$. Since c is positive and b is positive, you need to find two positive factors with a sum of 9 and a product of 20. Make an organized list of the factors of 20, and look for the pair of factors with a sum of 9.

Factors of 20	Sum of Factors
1, 20	21
2, 10	12
4, 5	9

The correct factors are 4 and 5.

$x^2 + 9x + 20 = (x + m)(x + p)$ Write the pattern.

$= (x + 4)(x + 5)$ $m = 4$ and $p = 5$

CHECK You can check this result by multiplying the two factors. The product should be equal to the original expression.

$(x + 4)(x + 5) = x^2 + 5x + 4x + 20$ FOIL Method

$= x^2 + 9x + 20$ ✓ Simplify.

GuidedPractice

Factor each polynomial.

1A. $d^2 + 11d + 24$

1B. $9 + 10t + t^2$

When factoring a trinomial in which b is negative and c is positive, use what you know about the product of binomials to narrow the list of possible factors.

Example 2 b is Negative and c is Positive

Factor $x^2 - 8x + 12$. Confirm your answer using a graphing calculator.

In this trinomial, $b = -8$ and $c = 12$. Since c is positive and b is negative, you need to find two negative factors with a sum of -8 and a product of 12.

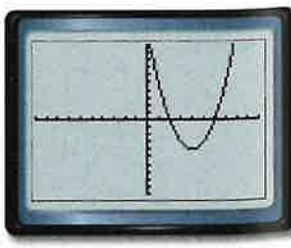
Factors of 12	Sum of Factors
$-1, -12$	-13
$-2, -6$	-8
$-3, -4$	-7

The correct factors are -2 and -6.

$x^2 - 8x + 12 = (x + m)(x + p)$ Write the pattern.

$= (x - 2)(x - 6)$ $m = -2$ and $p = -6$

CHECK Graph $y = x^2 - 8x + 12$ and $y = (x - 2)(x - 6)$ on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly. ✓



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Study Tip

CCSS **Regularity** Once the correct factors are found, it is not necessary to test any other factors. In Example 2, -2 and -6 are the correct factors, so -3 and -4 do not need to be tested.

GuidedPractice

Factor each polynomial.

2A. $21 - 22m + m^2$

2B. $w^2 - 11w + 28$

ReviewVocabulary
absolute value the distance a number is from zero on a number line, written $|n|$

When c is negative, its factors have opposite signs. To determine which factor is positive and which is negative, look at the sign of b. The factor with the greater absolute value has the same sign as b.

Example 3 c is Negative

Factor each polynomial. Confirm your answers using a graphing calculator.

a. $x^2 + 2x - 15$

In this trinomial, $b = 2$ and $c = -15$. Since c is negative, the factors m and p have opposite signs. So either m or p is negative, but not both. Since b is positive, the factor with the greater absolute value is also positive.

List the factors of -15, where one factor of each pair is negative. Look for the pair of factors with a sum of 2.

Factors of -15	Sum of Factors
$-1, 15$	14
$-3, 5$	2

The correct factors are -3 and 5.

$x^2 + 2x - 15 = (x + m)(x + p)$ Write the pattern.

$= (x - 3)(x + 5)$ $m = -3$ and $p = 5$

CHECK $(x - 3)(x + 5) = x^2 + 5x - 3x - 15$ FOIL Method

$= x^2 + 2x - 15$ ✓ Simplify.

b. $x^2 - 7x - 18$

In this trinomial, $b = -7$ and $c = -18$. Either m or p is negative, but not both. Since b is negative, the factor with the greater absolute value is also negative.

List the factors of -18, where one factor of each pair is negative. Look for the pair of factors with a sum of -7.

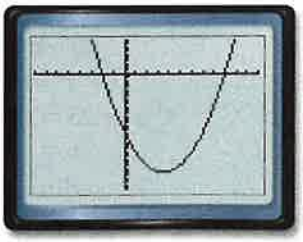
Factors of -18	Sum of Factors
$1, -18$	-17
$2, -9$	-7
$3, -6$	-3

The correct factors are 2 and -9.

$x^2 - 7x - 18 = (x + m)(x + p)$ Write the pattern.

$= (x + 2)(x - 9)$ $m = 2$ and $p = -9$

CHECK Graph $y = x^2 - 7x - 18$ and $y = (x + 2)(x - 9)$ on the same screen.



[−10, 15] scl: 1 by [−40, 20] scl: 1

The graphs coincide. Therefore, the trinomial has been factored correctly. ✓

GuidedPractice

3A. $y^2 + 13y - 48$ **3B.** $r^2 - 2r - 24$

2 Solve Equations by Factoring

A **quadratic equation** can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. Some equations of the form $x^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 4 Solve an Equation by Factoring

Solve $x^2 + 6x = 27$. Check your solutions.

$x^2 + 6x = 27$	Original equation
$x^2 + 6x - 27 = 0$	Subtract 27 from each side.
$(x - 3)(x + 9) = 0$	Factor.
$x - 3 = 0$ or $x + 9 = 0$	Zero Product Property
$x = 3$ $\quad x = -9$	Solve each equation.

The roots are 3 and -9.

CHECK Substitute 3 and -9 for x in the original equation.

$$x^2 + 6x = 27$$
$$(3)^2 + 6(3) \stackrel{?}{=} 27$$
$$9 + 18 \stackrel{?}{=} 27$$
$$27 = 27 \checkmark$$

$$x^2 + 6x = 27$$
$$(-9)^2 + 6(-9) \stackrel{?}{=} 27$$
$$81 - 54 \stackrel{?}{=} 27$$
$$27 = 27 \checkmark$$

StudyTip

Solving an Equation By Factoring Remember to get 0 on one side of the equation before factoring.

GuidedPractice

Solve each equation. Check your solutions.

4A. $z^2 - 3z = 70$

4B. $x^2 + 3x - 18 = 0$

Factoring can be useful when solving real-world problems.

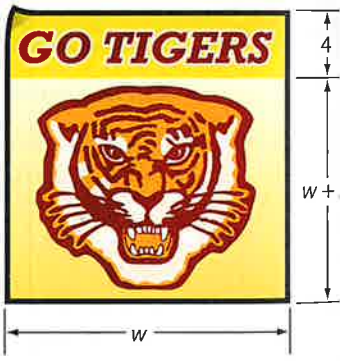
Real-WorldLink

A company that produces event signs recommends foamcore boards for event signs that will be used only once. For signs used more than once, use a stronger type of foamcore board.

Source: MegaPrint Inc.

Real-World Example 5 Solve a Problem by Factoring

DESIGN Ling is designing a poster. The top of the poster is 4 inches long and the rest of the poster is 2 inches longer than the width. If the poster requires 616 square inches of poster board, find the width w of the poster.



Understand You want to find the width of the poster.

Plan Since the poster is a rectangle, width · length = area.

Solve Let w = the width of the poster. The length is $w + 2 + 4$ or $w + 6$.

$w(w + 6) = 616$	Write the equation.
$w^2 + 6w = 616$	Multiply.
$w^2 + 6w - 616 = 0$	Subtract 616 from each side.
$(w + 28)(w - 22) = 0$	Factor.
$w + 28 = 0$ or $w - 22 = 0$	Zero Product Property
$w = -28$ $\quad w = 22$	Solve each equation.

Since dimensions cannot be negative, the width is 22 inches.

Check If the width is 22 inches, then the area of the poster is 22 · (22 + 6) or 616 square inches, which is the amount the poster requires. ✓

GuidedPractice

5. GEOMETRY The height of a parallelogram is 18 centimeters less than its base. If the area is 175 square centimeters, what is its height?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 **Factor each polynomial. Confirm your answers using a graphing calculator.**

1. $x^2 + 14x + 24$

2. $y^2 - 7y - 30$

3. $n^2 + 4n - 21$

4. $m^2 - 15m + 50$

Example 4 **Solve each equation. Check your solutions.**

5. $x^2 - 4x - 21 = 0$

6. $n^2 - 3n + 2 = 0$

7. $x^2 - 15x + 54 = 0$

8. $x^2 + 12x = -32$

9. $x^2 - x - 72 = 0$

10. $x^2 - 10x = -24$

Example 5

11. FRAMING Tina bought a frame for a photo, but the photo is too big for the frame. Tina needs to reduce the width and length of the photo by the same amount. The area of the photo should be reduced to half the original area. If the original photo is 12 inches by 16 inches, what will be the dimensions of the smaller photo?

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–3 **Factor each polynomial. Confirm your answers using a graphing calculator.**

12. $x^2 + 17x + 42$

13. $y^2 - 17y + 72$

14. $a^2 + 8a - 48$

15. $n^2 - 2n - 35$

16. $44 + 15h + h^2$

17. $40 - 22x + x^2$

18. $-24 - 10x + x^2$

19. $-42 - m + m^2$

Example 4 **Solve each equation. Check your solutions.**

20. $x^2 - 7x + 12 = 0$

21. $y^2 + y = 20$

22. $x^2 - 6x = 27$

23. $a^2 + 11a = -18$

24. $c^2 + 10c + 9 = 0$

25. $x^2 - 18x = -32$

26. $n^2 - 120 = 7n$

27. $d^2 + 56 = -18d$

28. $y^2 - 90 = 13y$

29. $h^2 + 48 = 16h$

Example 5

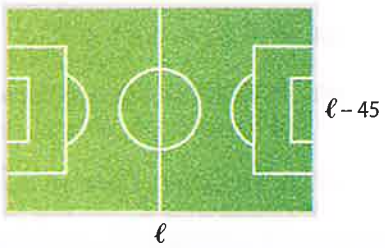
30. GEOMETRY A triangle has an area of 36 square feet. If the height of the triangle is 6 feet more than its base, what are its height and base?

31. GEOMETRY A rectangle has an area represented by $x^2 - 4x - 12$ square feet. If the length is $x + 2$ feet, what is the width of the rectangle?

32. SOCCER The width of a high school soccer field is 45 yards shorter than its length.

a. Define a variable, and write an expression for the area of the field.

b. The area of the field is 9000 square yards. Find the dimensions.



CCSS **STRUCTURE** **Factor each polynomial.**

33. $q^2 + 11qr + 18r^2$

34. $x^2 - 14xy - 51y^2$

35. $x^2 - 6xy + 5y^2$

36. $a^2 + 10ab - 39b^2$

37. **SWIMMING** The length of a rectangular swimming pool is 20 feet greater than its width. The area of the pool is 525 square feet.

a. Define a variable and write an equation for the area of the pool.

b. Solve the equation.

c. Interpret the solutions. Do both solutions make sense? Explain.

GEOMETRY Find an expression for the perimeter of a rectangle with the given area.

38. $A = x^2 + 24x - 81$

39. $A = x^2 + 13x - 90$

40. **MULTIPLE REPRESENTATIONS** In this problem, you will explore factoring when the leading coefficient is not 1.

a. **Tabular** Copy and complete the table below.

Product of Two Binomials	$ax^2 + mx + px + c$	$ax^2 + bx + c$	$m \times p$	$a \times c$
$(2x + 3)(x + 4)$	$2x^2 + 8x + 3x + 12$	$2x^2 + 11x + 12$	24	24
$(x + 1)(3x + 5)$				
$(2x - 1)(4x + 1)$				
$(3x + 5)(4x - 2)$				

b. **Analytical** How are m and p related to a and c?

c. **Analytical** How are m and p related to b?

d. **Verbal** Describe a process you can use for factoring a polynomial of the form $ax^2 + bx + c$.

H.O.T. Problems Use Higher-Order Thinking Skills

41. **ERROR ANALYSIS** Jerome and Charles have factored $x^2 + 6x - 16$. Is either of them correct? Explain your reasoning.

Jerome

$x^2 + 6x - 16 = (x + 2)(x - 8)$

Charles

$x^2 + 6x - 16 = (x - 2)(x + 8)$

CCSS **ARGUMENTS** **Find all values of k so that each polynomial can be factored using integers.**

42. $x^2 + kx - 19$

43. $x^2 + kx + 14$

44. $x^2 - 8x + k, k > 0$

45. $x^2 - 5x + k, k > 0$

46. **REASONING** For any factorable trinomial, $x^2 + bx + c$, will the absolute value of b *sometimes*, *always*, or *never* be less than the absolute value of c? Explain.

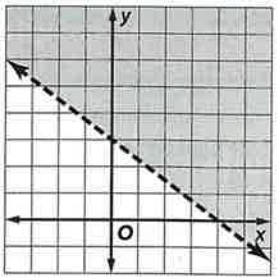
47. **OPEN ENDED** Give an example of a trinomial that can be factored using the factoring techniques presented in this lesson. Then factor the trinomial.

48. **CHALLENGE** Factor $(4y - 5)^2 + 3(4y - 5) - 70$.

49. **WRITING IN MATH** Explain how to factor trinomials of the form $x^2 + bx + c$ and how to determine the signs of the factors of c.

Standardized Test Practice

50. Which inequality is shown in the graph below?



A $y \leq -\frac{3}{4}x + 3$

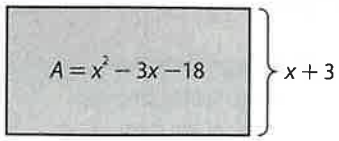
B $y < -\frac{3}{4}x + 3$

C $y > -\frac{3}{4}x + 3$

D $y \geq -\frac{3}{4}x + 3$

51. SHORT RESPONSE Olivia must earn more than $254 from selling candy bars in order to go on a trip with the National Honor Society. If each candy bar is sold for $1.25, what is the fewest candy bars she must sell?

52. GEOMETRY Which expression represents the length of the rectangle?



F $x + 5$

G $x + 6$

H $x - 6$

J $x - 5$

53. The difference of 21 and a number n is 6. Which equation shows the relationship?

A $21 - n = 6$

B $21 + n = 6$

C $21n = 6$

D $6n = -21$

Spiral Review

Factor each polynomial. (Lesson 8-5)

54. $10a^2 + 40a$

55. $11x + 44x^2y$

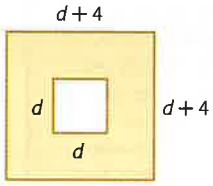
56. $2m^3p^2 - 16mp^2 + 8mp$

57. $2ax + 6xc + ba + 3bc$

58. $8ac - 2ad + 4bc - bd$

59. $x^2 - xy - xy + y^2$

60. Write a polynomial that represents the area of the shaded region in the figure at the right. (Lesson 8-4)



Use elimination to solve each system of equations. (Lesson 6-3)

61. $-x + y = 9$
$x + 2y = 30$

62. $5a + 2b = 4$
$-5a - b = -7$

63. $2c + d = 12$
$c - d = -3$

64. $6x + 2y = 14$
$5x - 2y = 8$

65. LANDSCAPING Kendrick is planning a circular flower garden with a low fence around the border. He has 38 feet of fence. What is the radius of the largest garden he can make? (*Hint:* $C = 2\pi r$) (Lesson 5-2)

Skills Review

Factor each polynomial.

66. $6mx - 4m + 3rx - 2r$

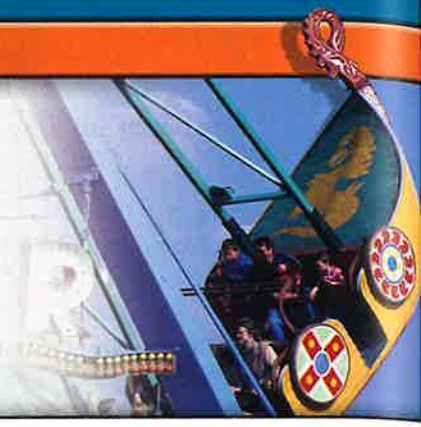
67. $3ax - 6bx + 8b - 4a$

68. $2d^2g + 2fg + 4d^2h + 4fh$

LESSON 8-7

Solving $ax^2 + bx + c = 0$

Then	Now	Why?
You factored trinomials of the form $x^2 + bx + c$.	1 Factor trinomials of the form $ax^2 + bx + c$. 2 Solve equations of the form $ax^2 + bx + c = 0$.	The path of a rider on the amusement park ride shown at the right can be modeled by $16t^2 - 5t + 120$. Factoring this expression can help the ride operators determine how long a rider rides on the initial swing.



New Vocabulary
prime polynomial

Common Core State Standards

Content Standards
A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

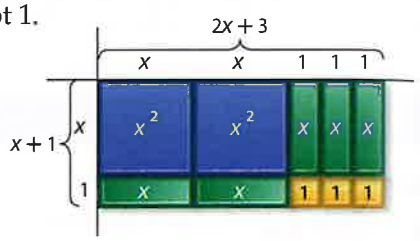
A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Mathematical Practices
4 Model with mathematics.

1 Factor $ax^2 + bx + c$

In the last lesson, you factored quadratic expressions of the form $ax^2 + bx + c$, where $a = 1$. In this lesson, you will apply the factoring methods to quadratic expressions in which a is not 1.

The dimensions of the rectangle formed by the algebra tiles are the factors of $2x^2 + 5x + 3$. The factors of $2x^2 + 5x + 3$ are $x + 1$ and $2x + 3$.



You can also use the method of factoring by grouping to solve this expression.

Step 1 Apply the pattern: $2x^2 + 5x + 3 = 2x^2 + mx + px + 3$.

Step 2 Find two numbers that have a product of $2 \cdot 3$ or 6 and a sum of 5.

Factors of 6	Sum of Factors
1, 6	7
2, 3	5

Step 3 Use grouping to find the factors.

$2x^2 + 5x + 3 = 2x^2 + mx + px + 3$ Write the pattern.

$= 2x^2 + 2x + 3x + 3$ $m = 2$ and $p = 3$

$= (2x^2 + 2x) + (3x + 3)$ Group terms with common factors.

$= 2x(x + 1) + 3(x + 1)$ Factor the GCF.

$= (2x + 3)(x + 1)$ $x + 1$ is the common factor.

Therefore, $2x^2 + 5x + 3 = (2x + 3)(x + 1)$.

KeyConcept Factoring $ax^2 + bx + c$

Words To factor trinomials of the form $ax^2 + bx + c$, find two integers, m and p, with a sum of b and a product of ac. Then write $ax^2 + bx + c$ as $ax^2 + mx + px + c$, and factor by grouping.

Example $5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$ $m = -10$ and $p = -3$

$= 5x(x - 2) + (-3)(x - 2)$

$= (5x - 3)(x - 2)$

Example 1 Factor $ax^2 + bx + c$

Factor each trinomial.

a. $7x^2 + 29x + 4$

In this trinomial, $a = 7$, $b = 29$, and $c = 4$. You need to find two numbers with a sum of 29 and a product of $7 \cdot 4$ or 28. Make a list of the factors of 28 and look for the pair of factors with the sum of 29.

Factors of 28	Sum of Factors
1, 28	29

The correct factors are 1 and 28.

$7x^2 + 29x + 4 = 7x^2 + mx + px + 4$ Write the pattern.

$= 7x^2 + 1x + 28x + 4$ $m = 1$ and $p = 28$

$= (7x^2 + 1x) + (28x + 4)$ Group terms with common factors.

$= x(7x + 1) + 4(7x + 1)$ Factor the GCF.

$= (x + 4)(7x + 1)$ $7x + 1$ is the common factor.

b. $3x^2 + 15x + 18$

The GCF of the terms $3x^2$, $15x$, and 18 is 3. Factor this first.

$3x^2 + 15x + 18 = 3(x^2 + 5x + 6)$ Distributive Property

$= 3(x + 3)(x + 2)$ Find two factors of 6 with a sum of 5.

StudyTip

Greatest Common Factor Always look for a GCF of the terms of a polynomial before you factor.

GuidedPractice

1A. $5x^2 + 13x + 6$ **1B.** $6x^2 + 22x - 8$

LWA/Dann Tardif/Blend Images/Getty Images

Real-WorldCareer

Urban Planner Urban planners design the layout of an area. They take into consideration the available land and geographical and environmental factors to design an area that benefits the community the most. City planners have a bachelor's degree in planning and almost half have a master's degree.

Sometimes the coefficient of the x-term is negative.

Example 2 Factor $ax^2 - bx + c$

Factor $3x^2 - 17x + 20$.

In this trinomial, $a = 3$, $b = -17$, and $c = 20$. Since b is negative, $m + p$ will be negative. Since c is positive, mp will be positive.

To determine m and p, list the negative factors of ac or 60. The sum of m and p should be -17.

Factors of 60	Sum of Factors
$-2, -30$	-32
$-3, -20$	-23
$-4, -15$	-19
$-5, -12$	-17

The correct factors are -5 and -12.

$3x^2 - 17x + 20 = 3x^2 - 12x - 5x + 20$ $m = -12$ and $p = -5$

$= (3x^2 - 12x) + (-5x + 20)$ Group terms with common factors.

$= 3x(x - 4) + (-5)(x - 4)$ Factor the GCF.

$= (3x - 5)(x - 4)$ Distributive Property

GuidedPractice

2A. $2n^2 - n - 1$ **2B.** $10y^2 - 35y + 30$

A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**.

Example 3 Determine Whether a Polynomial is Prime

Factor $4x^2 - 3x + 5$, if possible. If the polynomial cannot be factored using integers, write *prime*.

In this trinomial, $a = 4$, $b = -3$, and $c = 5$. Since b is negative, $m + p$ is negative. Since c is positive, mp is positive. So, m and p are both negative. Next, list the factors of 20. Look for the pair with a sum of -3.

Factors of 20	Sum of Factors
$-20, -1$	-21
$-4, -5$	-9
$-2, -10$	-12

There are no factors with a sum of -3. So the quadratic expression cannot be factored using integers. Therefore, $4x^2 - 3x + 5$ is prime.

GuidedPractice

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

3A. $4r^2 - r + 7$

3B. $2x^2 + 3x - 5$

Real-WorldLink

Cheetahs are the fastest land animals in the world, reaching speeds of up to 70 mph. It can accelerate from 0 to 40 mph in 3 strides. It takes just seconds for the cheetah to reach the full speed of 70 mph.

Source: Cheetah Conservation Fund

2 Solve Equations by Factoring

A model for the height of a projectile is given by $h = -16t^2 + vt + h_0$, where h is the height in feet, t is the time in seconds, v is the initial upward velocity in feet per second, and h_0 is the initial height in feet. Equations of the form $ax^2 + bx + c = 0$ can be solved by factoring and by using the Zero Product Property.

Real-World Example 4 Solve Equations by Factoring

WILDLIFE Suppose a cheetah pouncing on an antelope leaps with an initial upward velocity of 19 feet per second. How long is the cheetah in the air if it lands on the antelope's hind quarter, 3 feet from the ground?

$h = -16t^2 + vt + h_0$	Equation for height
$3 = -16t^2 + 19t + 0$	$h = 3$, $v = 19$, and $h_0 = 0$
$0 = -16t^2 + 19t - 3$	Subtract 3 from each side.
$0 = 16t^2 - 19t + 3$	Multiply each side by -1.
$0 = (16t - 3)(t - 1)$	Factor $16t^2 - 19t + 3$.
$16t - 3 = 0$ or $t - 1 = 0$	Zero Product Property
$16t = 3$ $\quad$ $t = 1$	Solve each equation.
$t = \frac{3}{16}$	

The solutions are $\frac{3}{16}$ and 1 seconds. It takes the cheetah $\frac{3}{16}$ second to reach a height of 3 feet on his way up. It takes the cheetah 1 second to reach a height of 3 feet on his way down. So, the cheetah is in the air 1 second before he catches the antelope.

WatchOut!

Keep the -1 Do not forget to carry the -1 that was factored out through the rest of the steps or multiply both sides by -1.

GuidedPractice

4. PHYSICAL SCIENCE A person throws a ball upward from a 506-foot tall building. The ball's height h in feet after t seconds is given by the equation $h = -16t^2 + 48t + 506$. The ball lands on a balcony that is 218 feet above the ground. How many seconds was it in the air?

Check Your Understanding

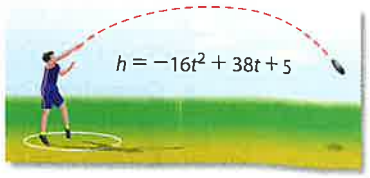
= Step-by-Step Solutions begin on page R13.

Examples 1–3 **Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.**

1. $3x^2 + 17x + 10$
2. $2x^2 + 22x + 56$
3. $5x^2 - 3x + 4$
4. $3x^2 - 11x - 20$

Example 4 **Solve each equation. Confirm your answers using a graphing calculator.**

5. $2x^2 + 9x + 9 = 0$
6. $3x^2 + 17x + 20 = 0$
7. $3x^2 - 10x + 8 = 0$
8. $2x^2 - 17x + 30 = 0$
9. CCSS **MODELING** Ken throws the discus at a school meet.
   a. What is the initial height of the discus?
   b. After how many seconds does the discus hit the ground?



Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–3 **Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.**

10. $5x^2 + 34x + 24$
11. $2x^2 + 19x + 24$
12. $4x^2 + 22x + 10$
13. $4x^2 + 38x + 70$
14. $2x^2 - 3x - 9$
15. $4x^2 - 13x + 10$
16. $2x^2 + 3x + 6$
17. $5x^2 + 3x + 4$
18. $12x^2 + 69x + 45$
19. $4x^2 - 5x + 7$
20. $5x^2 + 23x + 24$
21. $3x^2 - 8x + 15$

Example 4

22. **SHOT PUT** An athlete throws a shot put with an initial upward velocity of 29 feet per second and from an initial height of 6 feet.
    a. Write an equation that models the height of the shot put in feet with respect to time in seconds.
    b. After how many seconds will the shot put hit the ground?

Solve each equation. Confirm your answers using a graphing calculator.

23. $2x^2 + 9x - 18 = 0$
24. $4x^2 + 17x + 15 = 0$
25. $-3x^2 + 26x = 16$
26. $-2x^2 + 13x = 15$
27. $-3x^2 + 5x = -2$
28. $-4x^2 + 19x = -30$
29. **BASKETBALL** When Jerald shoots a free throw, the ball is 6 feet from the floor and has an initial upward velocity of 20 feet per second. The hoop is 10 feet from the floor.
    a. Use the vertical motion model to determine an equation that models Jerald's free throw.
    b. How long is the basketball in the air before it reaches the hoop?
    c. Raymond shoots a free throw that is 5 foot 9 inches from the floor with the same initial upward velocity. Will the ball be in the air more or less time? Explain.
30. **DIVING** Ben dives from a 36-foot platform. The equation $h = -16t^2 + 14t + 36$ models the dive. How long will it take Ben to reach the water?

31. **NUMBER THEORY** Six times the square of a number x plus 11 times the number equals 2. What are possible values of x?

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

32. $-6x^2 - 23x - 20$
33. $-4x^2 - 15x - 14$
34. $-5x^2 + 18x + 8$
35. $-6x^2 + 31x - 35$
36. $-4x^2 + 5x - 12$
37. $-12x^2 + x + 20$

38. **URBAN PLANNING** The city has commissioned the building of a rectangular park. The area of the park can be expressed as $660x^2 + 524x + 85$. Factor this expression to find binomials with integer coefficients that represent possible dimensions of the park. If $x = 8$, what is a possible perimeter of the park?

39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore factoring a special type of polynomial.
    a. **Geometric** Draw a square and label the sides a. Within this square, draw a smaller square that shares a vertex with the first square. Label the sides b. What are the areas of the two squares?
    b. **Geometric** Cut and remove the small square. What is the area of the remaining region?
    c. **Analytical** Draw a diagonal line between the inside corner and outside corner of the figure, and cut along this line to make two congruent pieces. Then rearrange the two pieces to form a rectangle. What are the dimensions?
    d. **Analytical** Write the area of the rectangle as the product of two binomials.
    e. **Verbal** Complete this statement: $a^2 - b^2 = \ldots$ Why is this statement true?

H.O.T. Problems Use Higher-Order Thinking Skills

40. **CCSS CRITIQUE** Zachary and Samantha are solving $6x^2 - x = 12$. Is either of them correct? Explain your reasoning.

Zachary	Samantha
$6x^2 - x = 12$	$6x^2 - x = 12$
$x(6x - 1) = 12$	$6x^2 - x - 12 = 0$
$x = 12$ or $6x - 1 = 12$	$(2x - 3)(3x + 4) = 0$
$6x = 13$	$2x - 3 = 0$ or $3x + 4 = 0$
$x = \frac{13}{6}$	$x = \frac{3}{2}$ $\quad x = -\frac{4}{3}$

41. **REASONING** A square has an area of $9x^2 + 30xy + 25y^2$ square inches. The dimensions are binomials with positive integer coefficients. What is the perimeter of the square? Explain.

42. **CHALLENGE** Find all values of k so that $2x^2 + kx + 12$ can be factored as two binomials using integers.

43. **WRITING IN MATH** What should you consider when solving a quadratic equation that models a real-world situation?

44. **WRITING IN MATH** Explain how to determine which values should be chosen for m and p when factoring a polynomial of the form $ax^2 + bx + c$.

Standardized Test Practice

45. Gridded Response Savannah has two sisters. One sister is 8 years older than her and the other sister is 2 years younger than her. The product of Savannah's sisters' ages is 56. How old is Savannah?

46. What is the product of $\frac{2}{3}a^3b^5$ and $\frac{3}{5}a^5b^2$?

A $\frac{2}{5}a^8b^7$

B $\frac{2}{5}a^2b^3$

C $\frac{2}{5}a^8b^3$

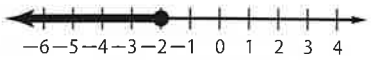
D $\frac{2}{5}a^2b^7$

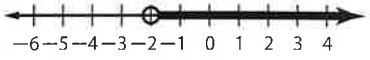
47. What is the solution set of $x^2 + 2x - 24 = 0$?

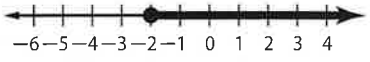
F $\{-4, 6\}$ H $\{-3, 8\}$

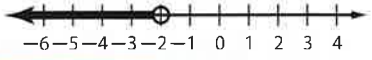
G $\{3, -8\}$ J $\{4, -6\}$

48. Which is the solution set of $x \geq -2$?

A 

B 

C 

D 

Spiral Review

Factor each polynomial. (Lesson 8-6)

49. $x^2 - 9x + 14$ **50.** $n^2 - 8n + 15$ **51.** $x^2 - 5x - 24$

52. $z^2 + 15z + 36$ **53.** $r^2 + 3r - 40$ **54.** $v^2 + 16v + 63$

Solve each equation. Check your solutions. (Lesson 8-5)

55. $a(a - 9) = 0$ **56.** $(2y + 6)(y - 1) = 0$ **57.** $10x^2 - 20x = 0$

58. $8b^2 - 12b = 0$ **59.** $15a^2 = 60a$ **60.** $33x^2 = -22x$

61. ART A painter has 32 units of yellow dye and 54 units of blue dye to make two shades of green. The units needed to make a gallon of light green and a gallon of dark green are shown. Make a graph showing the numbers of gallons of the two greens she can make, and list three possible solutions. (Lesson 6-6)

Color	Units of Yellow Dye	Units of Blue Dye
light green	4	1
dark green	1	6

Solve each compound inequality. Then graph the solution set. (Lesson 5-4)

62. $k + 2 > 12$ and $k + 2 \leq 18$ **63.** $d - 4 > 3$ or $d - 4 \leq 1$ **64.** $3 < 2x - 3 < 15$

65. $3t - 7 \geq 5$ and $2t + 6 \leq 12$ **66.** $h - 10 < -21$ or $h + 3 < 2$ **67.** $4 < 2y - 2 < 10$

68. FINANCIAL LITERACY A home security company provides security systems for \$5 per week, plus an installation fee. The total cost for installation and 12 weeks of service is \$210. Write the point-slope form of an equation to find the total fee y for any number of weeks x. What is the installation fee? (Lesson 4-3)

Skills Review

Find the principal square root of each number.

69. 16 **70.** 36 **71.** 64

72. 81 **73.** 121 **74.** 100

LESSON 8-8

Differences of Squares

Then	Now	Why?
• You factored trinomials into two binomials.	• 1 Factor binomials that are the difference of squares. • 2 Use the difference of squares to solve equations.	• Computer graphics designers use a combination of art and mathematics skills to design images and videos. They use equations to form shapes and lines on computers. Factoring can help to determine the dimensions and shapes of the figures.

NewVocabulary
difference of two squares

Common Core State Standards

Content Standards

A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Factor Differences of Squares

You have previously learned about the product of the sum and difference of two quantities. This resulting product is referred to as the **difference of two squares**. So, the factored form of the difference of squares is called the product of the sum and difference of the two quantities.

KeyConcept Factoring Differences of Squares

Symbols	$a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$
Examples	$x^2 - 25 = (x + 5)(x - 5)$ or $(x - 5)(x + 5)$
	$t^2 - 64 = (t + 8)(t - 8)$ or $(t - 8)(t + 8)$

Example 1 Factor Differences of Squares

Factor each polynomial.

a. $16h^2 - 9a^2$

$16h^2 - 9a^2 = (4h)^2 - (3a)^2$ Write in the form of $a^2 - b^2$.

$= (4h + 3a)(4h - 3a)$ Factor the difference of squares.

b. $121 - 4b^2$

$121 - 4b^2 = (11)^2 - (2b)^2$ Write in the form of $a^2 - b^2$.

$= (11 - 2b)(11 + 2b)$ Factor the difference of squares.

c. $27g^3 - 3g$

Because the terms have a common factor, factor out the GCF first. Then proceed with other factoring techniques.

$27g^3 - 3g = 3g(9g^2 - 1)$ Factor out the GCF of $3g$.

$= 3g[(3g)^2 - (1)^2]$ Write in the form $a^2 - b^2$.

$= 3g(3g - 1)(3g + 1)$ Factor the difference of squares.

GuidedPractice

1A. $81 - c^2$

1B. $64g^2 - h^2$

1C. $9x^3 - 4x$

1D. $-4y^3 + 9y$

To factor a polynomial completely, a technique may need to be applied more than once. This also applies to the difference of squares pattern.

Example 2 Apply a Technique More than Once

Factor each polynomial.

a. $b^4 - 16$

$b^4 - 16 = (b^2)^2 - (4)^2$ Write $b^4 - 16$ in $a^2 - b^2$ form.

$= (b^2 + 4)(b^2 - 4)$ Factor the difference of squares.

$= (b^2 + 4)(b^2 - 2^2)$ $b^2 - 4$ is also a difference of squares.

$= (b^2 + 4)(b + 2)(b - 2)$ Factor the difference of squares.

b. $625 - x^4$

$625 - x^4 = (25)^2 - (x^2)^2$ Write $625 - x^4$ in $a^2 - b^2$ form.

$= (25 + x^2)(25 - x^2)$ Factor the difference of squares.

$= (25 + x^2)(5^2 - x^2)$ Write $25 - x^2$ in $a^2 - b^2$ form.

$= (25 + x^2)(5 - x)(5 + x)$ Factor the difference of squares.

GuidedPractice

2A. $y^4 - 1$

2B. $4a^4 - b^4$

2C. $81 - x^4$

2D. $16y^4 - 1$

WatchOut!

Sum of Squares The sum of squares, $a^2 + b^2$, does not factor into $(a + b)(a + b)$. The sum of squares is a prime polynomial and cannot be factored.

Sometimes more than one factoring technique needs to be applied to ensure that a polynomial is factored completely.

Example 3 Apply Different Techniques

Factor each polynomial.

a. $5x^5 - 45x$

$5x^5 - 45x = 5x(x^4 - 9)$ Factor out GCF.

$= 5x[(x^2)^2 - (3)^2]$ Write $x^4 - 9$ in the form $a^2 - b^2$.

$= 5x(x^2 - 3)(x^2 + 3)$ Factor the difference of squares.

$x^2 - 3$ is not a difference of squares because 3 is not a perfect square.

b. $7x^3 + 21x^2 - 7x - 21$

$7x^3 + 21x^2 - 7x - 21$ Original expression

$= 7(x^3 + 3x^2 - x - 3)$ Factor out GCF.

$= 7[(x^3 + 3x^2) - (x + 3)]$ Group terms with common factors.

$= 7[x^2(x + 3) - 1(x + 3)]$ Factor each grouping.

$= 7(x + 3)(x^2 - 1)$ $x + 3$ is the common factor.

$= 7(x + 3)(x + 1)(x - 1)$ Factor the difference of squares.

GuidedPractice

3A. $2y^4 - 50$

3B. $6x^4 - 96$

3C. $2m^3 + m^2 - 50m - 25$

3D. $r^3 + 6r^2 + 11r + 66$

2 Solve Equations by Factoring

After factoring, you can apply the Zero Product Property to an equation that is written as the product of factors set equal to 0.

Test-Taking Tip

CCSS Sense-Making

Another method that can be used to solve this equation is to substitute each answer choice into the equation.

Standardized Test Example 4 Solve an Equation by Factoring

In the equation $y = x^2 - \frac{9}{16}$, which is a value of x when $y = 0$?

A $-\frac{9}{4}$ B 0 C $\frac{3}{4}$ D $\frac{9}{4}$

Read the Test Item

Replace y with 0 and then solve.

Solve the Test Item

$y = x^2 - \frac{9}{16}$ Original equation

$0 = x^2 - \frac{9}{16}$ Replace y with 0.

$0 = x^2 - \left(\frac{3}{4}\right)^2$ Write in the form $a^2 - b^2$.

$0 = \left(x + \frac{3}{4}\right)\left(x - \frac{3}{4}\right)$ Factor the difference of squares.

$0 = x + \frac{3}{4}$ or $0 = x - \frac{3}{4}$ Zero Product Property

$x = -\frac{3}{4}$ $x = \frac{3}{4}$ The correct answer is C.

Guided Practice

4. Which are the solutions of $18x^3 = 50x$?

F $0, \frac{5}{3}$ G $-\frac{5}{3}, \frac{5}{3}$ H $-\frac{5}{3}, \frac{5}{3}, 0$ J $-\frac{5}{3}, \frac{5}{3}, 1$

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 **Factor each polynomial.**

1. $x^2 - 9$
2. $4a^2 - 25$
3. $9m^2 - 144$
4. $2p^3 - 162p$
5. $u^4 - 81$
6. $2d^4 - 32f^4$
7. $20r^4 - 45n^4$
8. $256n^4 - c^4$
9. $2c^3 + 3c^2 - 2c - 3$
10. $f^3 - 4f^2 - 9f + 36$
11. $3t^3 + 2t^2 - 48t - 32$
12. $w^3 - 3w^2 - 9w + 27$

Example 4 **EXTENDED RESPONSE** **During an accident, skid marks may result from sudden breaking. The formula $\frac{1}{24}s^2 = d$ approximates a vehicle's speed s in miles per hour given the length d in feet of the skid marks on dry concrete.**

13. If skid marks on dry concrete are 54 feet long, how fast was the car traveling when the brakes were applied?
14. If the skid marks on dry concrete are 150 feet long, how fast was the car traveling when the brakes were applied?

Practice and Problem Solving

Extra Practice is on page R8.

Examples 1–3 **Factor each polynomial.**

15. $q^2 - 121$

16. $r^4 - k^4$

17. $6n^4 - 6$

18. $w^4 - 625$

19. $r^2 - 9t^2$

20. $2c^2 - 32d^2$

21. $h^3 - 100h$

22. $h^4 - 256$

23. $2x^3 - x^2 - 162x + 81$

24. $x^2 - 4y^2$

25. $7h^4 - 7p^4$

26. $3c^3 + 2c^2 - 147c - 98$

27. $6k^2h^4 - 54k^4$

28. $5a^3 - 20a$

29. $f^3 + 2f^2 - 64f - 128$

30. $3r^3 - 192r$

31. $10q^3 - 1210q$

32. $3xn^4 - 27x^3$

33. $p^3r^5 - p^3r$

34. $8c^3 - 8c$

35. $r^3 - 5r^2 - 100r + 500$

36. $3t^3 - 7t^2 - 3t + 7$

37. $a^2 - 49$

38. $4m^3 + 9m^2 - 36m - 81$

39. $3m^4 + 243$

40. $3x^3 + x^2 - 75x - 25$

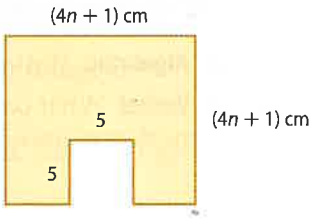
41. $12a^3 + 2a^2 - 192a - 32$

42. $x^4 + 6x^3 - 36x^2 - 216x$

43. $15m^3 + 12m^2 - 375m - 300$

Example 4

44. GEOMETRY The drawing at the right is a square with a square cut out of it.



a. Write an expression that represents the area of the shaded region.

b. Find the dimensions of a rectangle with the same area as the shaded region in the drawing. Assume that the dimensions of the rectangle must be represented by binomials with integral coefficients.

45. DECORATIONS An arch decorated with balloons was used to decorate the gym for the spring dance. The shape of the arch can be modeled by the equation $y = -0.5x^2 + 4.5x$, where x and y are measured in feet and the x-axis represents the floor.

a. Write the expression that represents the height of the arch in factored form.

b. How far apart are the two points where the arch touches the floor?

c. Graph this equation on your calculator. What is the highest point of the arch?

46. CCSS SENSE-MAKING Zelda is building a deck in her backyard. The plans for the deck show that it is to be 24 feet by 24 feet. Zelda wants to reduce one dimension by a number of feet and increase the other dimension by the same number of feet. If the area of the reduced deck is 512 square feet, what are the dimensions of the deck?

47. SALES The sales of a particular CD can be modeled by the equation $S = -25m^2 + 125m$, where S is the number of CDs sold in thousands, and m is the number of months that it is on the market.

a. In what month should the music store expect the CD to stop selling?

b. In what month will CD sales peak?

c. How many copies will the CD sell at its peak?

Solve each equation by factoring. Confirm your answers using a graphing calculator.

48. $36w^2 = 121$

49. $100 = 25x^2$

50. $64x^2 - 1 = 0$

51. $4y^2 - \frac{9}{16} = 0$

52. $\frac{1}{4}b^2 = 16$

53. $81 - \frac{1}{25}x^2 = 0$

54. $9d^2 - 81 = 0$

55. $4a^2 = \frac{9}{64}$

56. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate perfect square trinomials.

a. Tabular Copy and complete the table below by factoring each polynomial. Then write the first and last terms of the given polynomials as perfect squares.

b. Analytical Write the middle term of each polynomial using the square roots of the perfect squares of the first and last terms.

Polynomial	Factored Polynomial	First Term	Last Term	Middle Term
$4x^2 + 12x + 9$	$(2x + 3)(2x + 3)$	$4x^2 = (2x)^2$	$9 = 3^2$	
$9x^2 - 24x + 16$				
$4x^2 - 20x + 25$				
$16x^2 + 24x + 9$				
$25x^2 + 20x + 4$				

c. Algebraic Write the pattern for a perfect square trinomial.

d. Verbal What conditions must be met for a trinomial to be classified as a perfect square trinomial?

H.O.T. Problems Use Higher-Order Thinking Skills

57. **ERROR ANALYSIS** Elizabeth and Lorenzo are factoring an expression. Is either of them correct? Explain your reasoning.

Elizabeth

$16x^4 - 25y^2 =$

$(4x - 5y)(4x + 5y)$

Lorenzo

$16x^4 - 25y^2 =$

$(4x^2 - 5y)(4x^2 + 5y)$

58. **CHALLENGE** Factor and simplify $9 - (k + 3)^2$, a difference of squares.

59. **PERSEVERANCE** Factor $x^{16} - 81$.

60. **REASONING** Write and factor a binomial that is the difference of two perfect squares and that has a greatest common factor of $5mk$.

61. **REASONING** Determine whether the following statement is *true* or *false*. Give an example or counterexample to justify your answer.

All binomials that have a perfect square in each of the two terms can be factored.

62. **OPEN ENDED** Write a binomial in which the difference of squares pattern must be repeated to factor it completely. Then factor the binomial.

63. **WRITING IN MATH** Describe why the difference of squares pattern has no middle term with a variable.

Standardized Test Practice

64. One of the roots of $2x^2 + 13x = 24$ is -8. What is the other root?

A $-\frac{3}{2}$ C $\frac{2}{3}$

B $-\frac{2}{3}$ D $\frac{3}{2}$

65. Which of the following is the sum of both solutions of the equation $x^2 + 3x = 54$?

F -21 H 3

G -3 J 21

66. What are the x-intercepts of the graph of $y = -3x^2 + 7x + 20$?

A $\frac{5}{3}, -4$ C $-\frac{5}{3}, 4$

B $-\frac{5}{3}, -4$ D $\frac{5}{3}, 4$

67. EXTENDED RESPONSE Two cars leave Cleveland at the same time from different parts of the city and both drive to Cincinnati. The distance in miles of the cars from the center of Cleveland can be represented by the two equations below, where t represents the time in hours.

Car A: $65t + 15$ Car B: $60t + 25$

a. Which car is faster? Explain.

b. Find an expression that models the distance between the two cars.

c. How far apart are the cars after $2\frac{1}{2}$ hours?

Spiral Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-7)

68. $5x^2 - 17x + 14$ **69.** $5a^2 - 3a + 15$ **70.** $10x^2 - 20xy + 10y^2$

Solve each equation. Check your solutions. (Lesson 8-6)

71. $n^2 - 9n = -18$ **72.** $10 + a^2 = -7a$ **73.** $22x - x^2 = 96$

74. SAVINGS Victoria and Trey each want to buy a scooter. In how many weeks will Victoria and Trey have saved the same amount of money, and how much will each of them have saved? (Lesson 6-2)

Solve each inequality. Graph the solution set on a number line. (Lesson 5-1)

75. $t + 14 \geq 18$ **76.** $d + 5 \leq 7$ **77.** $-5 + k > -1$

78. $5 < 3 + g$ **79.** $2 \leq -1 + m$ **80.** $2y > -8 + y$

81. FITNESS Silvia is beginning an exercise program that calls for 20 minutes of walking each day for the first week. Each week thereafter, she has to increase her daily walking for a week by 7 minutes. In which week will she first walk over an hour a day? (Lesson 3-5)

Skills Review

Find each product.

82. $(x - 6)^2$ **83.** $(x - 2)(x - 2)$ **84.** $(x + 3)(x + 3)$

85. $(2x - 5)^2$ **86.** $(6x - 1)^2$ **87.** $(4x + 5)(4x + 5)$

LESSON 8-9

Perfect Squares

Then
- You found the product of a sum and difference.

Now
1. Factor perfect square trinomials.
2. Solve equations involving perfect squares.

Why?
- In a vacuum, a feather and a piano would fall at the same speed, or velocity. To find about how long it takes an object to hit the ground if it is dropped from an initial height of h_0 feet above ground, you would need to solve the equation $0 = -16t^2 + h_0$, where t is time in seconds after the object is dropped.

New Vocabulary
perfect square trinomial

Common Core State Standards

Content Standards
A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Mathematical Practices
6 Attend to precision.

1 Factor Perfect Square Trinomials

You have learned the patterns for the products of the binomials $(a + b)^2$ and $(a - b)^2$. Recall that these are special products that follow specific patterns.

$$(a + b)^2 = (a + b)(a + b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

These products are called **perfect square trinomials**, because they are the squares of binomials. The above patterns can help you factor perfect square trinomials.

For a trinomial to be factorable as a perfect square, the first and last terms must be perfect squares and the middle term must be two times the square roots of the first and last terms.

The trinomial $16x^2 + 24x + 9$ is a perfect square trinomial, as illustrated below.

$16x^2 + 24x + 9$

- Is the first term a perfect square? Yes, because $16x^2 = (4x)^2$.
- Is the middle term twice the product of the square roots of the first and last terms? Yes, because $24x = 2(4x)(3)$.
- Is the last term a perfect square? Yes, because $9 = 3^2$.

KeyConcept Factoring Perfect Square Trinomials

Symbols	$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
	$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$
Examples	$x^2 + 8x + 16 = (x + 4)(x + 4)$ or $(x + 4)^2$
	$x^2 - 6x + 9 = (x - 3)(x - 3)$ or $(x - 3)^2$

StudyTip

Recognizing Perfect Square Trinomials If the constant term of the trinomial is negative, the trinomial is not a perfect square trinomial, so it is not necessary to check the other conditions.

Example 1 Recognize and Factor Perfect Square Trinomials

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

a. $4y^2 + 12y + 9$

1. Is the first term a perfect square? Yes, $4y^2 = (2y)^2$.
2. Is the last term a perfect square? Yes, $9 = 3^2$.
3. Is the middle term equal to $2(2y)(3)$? Yes, $12y = 2(2y)(3)$

Since all three conditions are satisfied, $4y^2 + 12y + 9$ is a perfect square trinomial.

$4y^2 + 12y + 9 = (2y)^2 + 2(2y)(3) + 3^2$ Write as $a^2 + 2ab + b^2$.

$= (2y + 3)^2$ Factor using the pattern.

b. $9x^2 - 6x + 4$

1. Is the first term a perfect square? Yes, $9x^2 = (3x)^2$.
2. Is the last term a perfect square? Yes, $4 = 2^2$.
3. Is the middle term equal to $-2(3x)(2)$? No, $-6x \neq -2(3x)(2)$.

Since the middle term does not satisfy the required condition, $9x^2 - 6x + 4$ is not a perfect square trinomial.

GuidedPractice

1A. $9y^2 + 24y + 16$ **1B.** $2a^2 + 10a + 25$

A polynomial is completely factored when it is written as a product of prime polynomials. More than one method might be needed to factor a polynomial completely. When completely factoring a polynomial, the Concept Summary can help you decide where to start.

Remember, if the polynomial does not fit any pattern or cannot be factored, the polynomial is prime.

ConceptSummary Factoring Methods

Steps	Number of Terms	Examples
Step 1 Factor out the GCF.	any	$4x^3 + 2x^2 - 6x = 2x(2x^2 + x - 3)$
Step 2 Check for a difference of squares or a perfect square trinomial.	2 or 3	$9x^2 - 16 = (3x + 4)(3x - 4)$ $16x^2 + 24x + 9 = (4x + 3)^2$
Step 3 Apply the factoring patterns for $x^2 + bx + c$ or $ax^2 + bx + c$ (general trinomials), or factor by grouping.	3 or 4	$x^2 - 8x + 12 = (x - 2)(x - 6)$ $2x^2 + 13x + 6 = (2x + 1)(x + 6)$ $12y^2 + 9y + 8y + 6$ $= (12y^2 + 9y) + (8y + 6)$ $= 3y(4y + 3) + 2(4y + 3)$ $= (4y + 3)(3y + 2)$

Example 2 Factor Completely

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

a. $5x^2 - 80$

Step 1 The GCF of $5x^2$ and -80 is 5, so factor it out.

Step 2 Since there are two terms, check for a difference of squares.

$$\begin{aligned} 5x^2 - 80 &= 5(x^2 - 16) && \text{5 is the GCF of the terms.} \\ &= 5(x^2 - 4^2) && x^2 = x \cdot x \text{ and } 16 = 4 \cdot 4 \\ &= 5(x - 4)(x + 4) && \text{Factor the difference of squares.} \end{aligned}$$

b. $9x^2 - 6x - 35$

Step 1 The GCF of $9x^2$, $-6x$, and -35 is 1.

Step 2 Since 35 is not a perfect square, this is not a perfect square trinomial.

Step 3 Factor using the pattern $ax^2 + bx + c$. Are there two numbers with a product of $9(-35)$ or -315 and a sum of -6? Yes, the product of 15 and -21 is -315, and the sum is -6.

$$\begin{aligned} 9x^2 - 6x - 35 &= 9x^2 + mx + px - 35 && \text{Write the pattern.} \\ &= 9x^2 + 15x - 21x - 35 && m = 15 \text{ and } n = -21 \\ &= (9x^2 + 15x) + (-21x - 35) && \text{Group terms with common factors.} \\ &= 3x(3x + 5) - 7(3x + 5) && \text{Factor out the GCF from each grouping.} \\ &= (3x + 5)(3x - 7) && 3x + 5 \text{ is the common factor.} \end{aligned}$$

StudyTip

Check Your Answer You can check your answer by:

- Using the FOIL method.
- Using the Distributive Property.
- Graphing the original expression and factored expression and comparing the graphs.

If the product of the factors does not match the original expression exactly, the answer is incorrect.

GuidedPractice

2A. $2x^2 - 32$

2B. $12x^2 + 5x - 25$

2 Solve Equations with Perfect Squares

When solving equations involving repeated factors, it is only necessary to set one of the repeated factors equal to zero.

Example 3 Solve Equations with Repeated Factors

Solve $9x^2 - 48x = -64$.

$$\begin{aligned} 9x^2 - 48x &= -64 && \text{Original equation} \\ 9x^2 - 48x + 64 &= 0 && \text{Add 64 to each side.} \\ (3x)^2 - 2(3x)(8) + (8)^2 &= 0 && \text{Recognize } 9x^2 - 48x + 64 \text{ as a perfect square trinomial.} \\ (3x - 8)^2 &= 0 && \text{Factor the perfect square trinomial.} \\ (3x - 8)(3x - 8) &= 0 && \text{Write } (3x - 8)^2 \text{ as two factors.} \\ 3x - 8 &= 0 && \text{Set the repeated factor equal to zero.} \\ 3x &= 8 && \text{Add 8 to each side.} \\ x &= \frac{8}{3} && \text{Divide each side by 3.} \end{aligned}$$

GuidedPractice

Solve each equation. Check your solutions.

3A. $a^2 + 12a + 36 = 0$

3B. $y^2 - \frac{4}{3}y + \frac{4}{9} = 0$

You have solved equations like $x^2 - 16 = 0$ by factoring. You can also use the definition of a square root to solve the equation.

$x^2 - 16 = 0$	Original equation
$x^2 = 16$	Add 16 to each side.
$x = \pm\sqrt{16}$	Take the square root of each side.

ReadingMath
Square Root Solutions
$\pm\sqrt{16}$ is read as plus or minus the square root of 16.

Remember that there are two square roots of 16, namely 4 and -4. Therefore, the solution set is $\{-4, 4\}$. You can express this as $\{\pm 4\}$.

KeyConcept Square Root Property

Words	To solve a quadratic equation in the form $x^2 = n$, take the square root of each side.
Symbols	For any number $n \geq 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.
Example	$x^2 = 25$ $x = \pm\sqrt{25}$ or ± 5

In the equation $x^2 = n$, if n is not a perfect square, you need to approximate the square root. Use a calculator to find an approximation. If n is a perfect square, you will have an exact answer.

Example 4 Use the Square Root Property

StudyTip
Solving by Inspection
Equations involving square roots can often be solved mentally. For $x^2 = n$, think: *The square of what number is n?* When n is a perfect square, x is rational. Otherwise, x is irrational.

Solve each equation. Check your solutions.

a. $(y - 6)^2 = 81$

$(y - 6)^2 = 81$	Original equation
$y - 6 = \pm\sqrt{81}$	Square Root Property
$y - 6 = \pm 9$	$81 = 9 \cdot 9$
$y = 6 \pm 9$	Add 6 to each side.
$y = 6 + 9$ or $y = 6 - 9$	Separate into two equations.
$= 15$ $\quad = -3$	Simplify.

The roots are 15 and -3. Check in the original equation.

b. $(x + 6)^2 = 12$

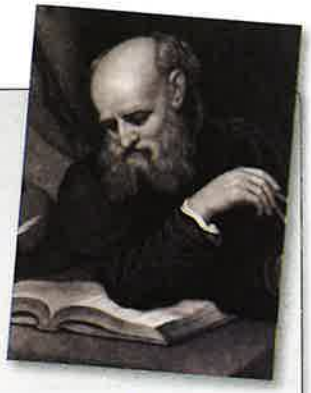
$(x + 6)^2 = 12$	Original equation
$x + 6 = \pm\sqrt{12}$	Square Root Property
$x = -6 \pm \sqrt{12}$	Subtract 6 from each side.

The roots are $-6 \pm \sqrt{12}$ or $-6 + \sqrt{12}$ and $-6 - \sqrt{12}$.

Using a calculator, $-6 + \sqrt{12} \approx -2.54$ and $-6 - \sqrt{12} \approx -9.46$.

GuidedPractice

4A. $(a - 10)^2 = 121$ **4B.** $(z + 3)^2 = 26$



Math History Link

Galileo Galilei (1564–1642) Galileo was the first person to prove that objects of different weights fall at the same velocity by dropping two objects of different weights from the top of the Leaning Tower of Pisa in 1589.

Real-World Example 5 Solve an Equation

PHYSICAL SCIENCE **During an experiment, a ball is dropped from a height of 205 feet. The formula $h = -16t^2 + h_0$ can be used to approximate the number of seconds t it takes for the ball to reach height h from an initial height of h_0 in feet. Find the time it takes the ball to reach the ground.**

At ground level, $h = 0$ and the initial height is 205, so $h_0 = 205$.

$h = -16t^2 + h_0$	Original Formula
$0 = -16t^2 + 205$	Replace h with 0 and h_0 with 205.
$-205 = -16t^2$	Subtract 205 from each side.
$12.8125 = t^2$	Divide each side by -16.
$\pm 3.6 \approx t$	Use the Square Root Property.

Since a negative number does not make sense in this situation, the solution is 3.6. It takes about 3.6 seconds for the ball to reach the ground.

Guided Practice

5. Find the time it takes a ball to reach the ground if it is dropped from a bridge that is half as high as the one described above.

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.**

1. $25x^2 + 60x + 36$

2. $6x^2 + 30x + 36$

Example 2 **Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.**

3. $2x^2 - x - 28$

4. $6x^2 - 34x + 48$

5. $4x^2 + 64$

6. $4x^2 + 9x - 16$

Examples 3–4 **Solve each equation. Confirm your answers using a graphing calculator.**

7. $4x^2 = 36$

8. $25a^2 - 40a = -16$

9. $64y^2 - 48y + 18 = 9$

10. $(z + 5)^2 = 47$

Example 5

11. CCSS **REASONING** While painting his bedroom, Nick drops his paintbrush off his ladder from a height of 6 feet. Use the formula $h = -16t^2 + h_0$ to approximate the number of seconds it takes for the paintbrush to hit the floor.

Practice and Problem Solving

Extra Practice is on page R8.

Example 1 **Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.**

12. $4x^2 - 42x + 110$

13. $16x^2 - 56x + 49$

14. $81x^2 - 90x + 25$

15 $x^2 + 26x + 168$

Example 2 **Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.**

16. $24d^2 + 39d - 18$

17. $8x^2 + 10x - 21$

18. $2b^2 + 12b - 24$

19. $8y^2 - 200z^2$

20. $16a^2 - 121b^2$

21. $12m^3 - 22m^2 - 70m$

22. $8c^2 - 88c + 242$

23. $12x^2 - 84x + 147$

24. $w^4 - w^2$

25. $12p^3 - 3p$

26. $16q^3 - 48q^2 + 36q$

27. $4t^3 + 10t^2 - 84t$

28. $x^3 + 2x^2y - 4x - 8y$

29. $2a^2b^2 - 2a^2 - 2ab^3 + 2ab$

30. $2r^3 - r^2 - 72r + 36$

31. $3k^3 - 24k^2 + 48k$

32. $4c^4d - 10c^3d + 4c^2d^3 - 10cd^3$

33. $g^2 + 2g - 3h^2 + 4h$

Examples 3–4 **Solve each equation. Confirm your answers using a graphing calculator.**

34. $4m^2 - 24m + 36 = 0$

35. $(y - 4)^2 = 7$

36. $a^2 + \frac{10}{7}a + \frac{25}{49} = 0$

37. $x^2 - \frac{3}{2}x + \frac{9}{16} = 0$

38. $x^2 + 8x + 16 = 25$

39. $5x^2 - 60x = -180$

40. $4x^2 = 80x - 400$

41. $9 - 54x = -81x^2$

42. $4c^2 + 4c + 1 = 15$

43. $x^2 - 16x + 64 = 6$

44. PHYSICAL SCIENCE For an experiment in physics class, a water balloon is dropped from the window of the school building. The window is 40 feet high. How long does it take until the balloon hits the ground? Round to the nearest hundredth.

45. SCREENS The area A in square feet of a projected picture on a movie screen can be modeled by the equation $A = 0.25d^2$, where d represents the distance from a projector to a movie screen. At what distance will the projected picture have an area of 100 square feet?

Example 5

46. GEOMETRY The area of a square is represented by $9x^2 - 42x + 49$. Find the length of each side.

47. GEOMETRY The area of a square is represented by $16x^2 + 40x + 25$. Find the length of each side.

48. GEOMETRY The volume of a rectangular prism is represented by the expression $8y^3 + 40y^2 + 50y$. Find the possible dimensions of the prism if the dimensions are represented by polynomials with integer coefficients.

49. POOLS Ichiro wants to buy an above-ground swimming pool for his yard. Model A is 42 inches deep and holds 1750 cubic feet of water. The length of the rectangular pool is 5 feet more than the width.

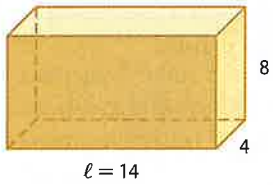
a. What is the surface area of the water?

b. What are the dimensions of the pool?

c. Model B pool holds twice as much water as Model A. What are some possible dimensions for this pool?

d. Model C has length and width that are both twice as long as Model A, but the height is the same. What is the ratio of the volume of Model A to Model C?

50. GEOMETRY Use the rectangular prism at the right.



a. Write an expression for the height and width of the prism in terms of the length, ℓ.

b. Write a polynomial for the volume of the prism in terms of the length.

51. CCSS PRECISION A zoo has an aquarium shaped like a rectangular prism. It has a volume of 180 cubic feet. The height of the aquarium is 9 feet taller than the width, and the length is 4 feet shorter than the width. What are the dimensions of the aquarium?

52 ELECTION For the student council elections, Franco is building the voting box shown with a volume of 96 cubic inches. What are the dimensions of the voting box?

H.O.T. Problems Use Higher-Order Thinking Skills

53. ERROR ANALYSIS Debbie and Adriano are factoring the expression $x^8 - x^4$ completely. Is either of them correct? Explain your reasoning.

Debbie	Adriano
$x^8 - x^4 = x^4(x^2 + 1)(x^2 - 1)$	$x^8 - x^4 = x^4(x^2 + 1)(x - 1)(x + 1)$

54. CHALLENGE Factor $x^{n+6} + x^{n+2} + x^n$ completely.

55. OPEN ENDED Write a perfect square trinomial equation in which the coefficient of the middle term is negative and the last term is a fraction. Solve the equation.

56. REASONING A counterexample is a specific case in which a statement is false. Find a counterexample to the following statement.

A polynomial equation of degree three always has three real solutions.

57. CCSS REGULARITY Explain how to factor a polynomial completely.

58. WHICH ONE DOESN'T BELONG? Identify the trinomial that does not belong. Explain.

$4x^2 - 36x + 81$	$25x^2 + 10x + 1$	$4x^2 + 10x + 4$	$9x^2 - 24x + 16$

59. OPEN ENDED Write a binomial that can be factored using the difference of two squares twice. Set your binomial equal to zero and solve the equation.

60. WRITING IN MATH Explain how to determine whether a trinomial is a perfect square trinomial.

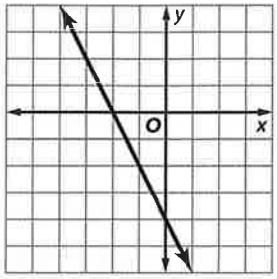
Standardized Test Practice

61. What is the solution set for the equation $(x - 3)^2 = 25$?

A $\{-8, 2\}$ C $\{4, 14\}$

B $\{-2, 8\}$ D $\{-4, 14\}$

62. SHORT RESPONSE Write an equation in slope-intercept form for the graph shown below.



63. At an amphitheater, the price of 2 lawn seats and 2 pavilion seats is \$120. The price of 3 lawn seats and 4 pavilion seats is \$225. How much do lawn and pavilion seats cost?

F \$20 and \$41.25

G \$10 and \$50

H \$15 and \$45

J \$30 and \$30

64. GEOMETRY The circumference of a circle is $\frac{6\pi}{5}$ units. What is the area of the circle?

A $\frac{9\pi}{25}$ units^2 B $\frac{3\pi}{5}$ units^2

C $\frac{6\pi}{5}$ units^2 D $\frac{12\pi}{5}$ units^2

Spiral Review

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-8)

65. $x^2 - 16$ **66.** $4x^2 - 81y^2$ **67.** $1 - 100p^2$

68. $3a^2 - 20$ **69.** $25n^2 - 1$ **70.** $36 - 9c^2$

Solve each equation. Confirm your answers using a graphing calculator. (Lesson 8-7)

71. $4x^2 - 8x - 32 = 0$ **72.** $6x^2 - 48x + 90 = 0$ **73.** $14x^2 + 14x = 28$

74. $2x^2 - 10x = 48$ **75.** $5x^2 - 25x = -30$ **76.** $8x^2 - 16x = 192$

SOUND The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds. (Lesson 7-2)

77. How many times more intense is the sound from busy street traffic than sound from normal conversation?

78. Which sound is 10,000 times as loud as a busy street traffic?

79. How does the intensity of a whisper compare to that of normal conversation?

Watts Per Square Meter	Common Sounds
10^{-11}	rustling leaves
10^{-10}	whisper
10^{-6}	normal conversation
10^{-5}	busy street traffic
10^{-4}	vacuum cleaner
10^{-1}	front rows of rock concert
10^{1}	threshold of pain
10^{2}	military jet takeoff

Skills Review

Find the slope of the line that passes through each pair of points.

80. $(5, 7), (-2, -3)$ **81.** $(2, -1), (5, -3)$ **82.** $(-4, -1), (-3, -3)$

83. $(-3, -4), (5, -1)$ **84.** $(-2, 3), (8, 3)$ **85.** $(-5, 4), (-5, -1)$

CHAPTER 8

Study Guide and Review

Study Guide

KeyConcepts

Operations with Polynomials (Lessons 8-1 through 8-4)

- To add or subtract polynomials, add or subtract like terms.
- To multiply polynomials, use the Distributive Property.
- Special products: $(a + b)^2 = a^2 + 2ab + b^2$
  $(a - b)^2 = a^2 - 2ab + b^2$
  $(a + b)(a - b) = a^2 - b^2$

Factoring Using the Distributive Property (Lesson 8-5)

- Using the Distributive Property to factor polynomials with four or more terms is called factoring by grouping.
  $ax + bx + ay + by = x(a + b) + y(a + b)$
  $= (a + b)(x + y)$

Solving Quadratic Equations by Factoring (Lessons 8-6 through 8-8)

- To factor $x^2 + bx + c$, find m and p with a sum of b and a product of c. Then write $x^2 + bx + c$ as $(x + m)(x + p)$.
- To factor $ax^2 + bx + c$, find m and p with a sum of b and a product of ac. Then write as $ax^2 + mx + px + c$ and factor by grouping.
- $a^2 - b^2 = (a - b)(a + b)$

Perfect Squares and Factoring (Lesson 8-9)

- For a trinomial to be a perfect square, the first and last terms must be perfect squares, and the middle term must be twice the product of the square roots of the first and last terms.
- For any number $n \geq 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

binomial (p. 465)
degree of a monomial (p. 465)
degree of a polynomial (p. 465)
difference of two squares (p. 516)
factoring (p. 494)
factoring by grouping (p. 495)
FOIL method (p. 481)
leading coefficient (p. 466)
perfect square trinomial (p. 522)
polynomial (p. 465)
prime polynomial (p. 512)
quadratic equation (p. 506)
quadratic expression (p. 481)
Square Root Property (p. 525)
standard form of a polynomial (p. 466)
trinomial (p. 465)
Zero Product Property (p. 496)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined phrase or expression to make a true sentence.

1. $x^2 + 5x + 6$ is an example of a prime polynomial.
2. $(x + 5)(x - 5)$ is the factorization of a difference of squares.
3. $4x^2 - 2x + 7$ is a polynomial of degree 2.
4. $(x + 5)(x - 2)$ is the factored form of $x^2 - 3x - 10$.
5. Expressions with four or more unlike terms can sometimes be factored by grouping.
6. The Zero Product Property states that if $ab = 1$, then a or b is 1.
7. $x^2 - 12x + 36$ is an example of a perfect square trinomial.
8. The leading coefficient of $1 + 6a + 9a^2$ is 1.
9. $x^2 - 16$ is an example of a perfect square trinomial.
10. The FOIL method is used to multiply two trinomials.

Lesson-by-Lesson Review

8-1 Adding and Subtracting Polynomials

Write each polynomial in standard form.

11. $x + 2 + 3x^2$

12. $1 - x^4$

13. $2 + 3x + x^2$

14. $3x^5 - 2 + 6x - 2x^2 + x^3$

Find each sum or difference.

15. $(x^3 + 2) + (-3x^3 - 5)$

16. $a^2 + 5a - 3 - (2a^2 - 4a + 3)$

17. $(4x - 3x^2 + 5) + (2x^2 - 5x + 1)$

18. PICTURE FRAMES Jean is framing a painting that is a rectangle. What is the perimeter of the frame?

Example 1

Write $3 - x^2 + 4x$ in standard form.

Step 1 Find the degree of each term.

3: degree 0

$-x^2$: degree 2

$4x$: degree 1

Step 2 Write the terms in descending order of degree.

$3 - x^2 + 4x = -x^2 + 4x + 3$

Example 2

Find $(8r^2 + 3r) - (10r^2 - 5)$.

$(8r^2 + 3r) - (10r^2 - 5)$

$= (8r^2 + 3r) + (-10r^2 + 5)$ Use the additive inverse.

$= (8r^2 - 10r^2) + 3r + 5$ Group like terms.

$= -2r^2 + 3r + 5$ Add like terms.

8-2 Multiplying a Polynomial by a Monomial

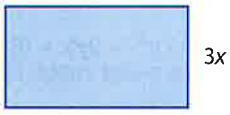
Solve each equation.

19. $x^2(x + 2) = x(x^2 + 2x + 1)$

20. $2x(x + 3) = 2(x^2 + 3)$

21. $2(4w + w^2) - 6 = 2w(w - 4) + 10$

22. GEOMETRY Find the area of the rectangle.



Example 3

Solve $m(2m - 5) + m = 2m(m - 6) + 16$.

$m(2m - 5) + m = 2m(m - 6) + 16$

$2m^2 - 5m + m = 2m^2 - 12m + 16$

$2m^2 - 4m = 2m^2 - 12m + 16$

$-4m = -12m + 16$

$8m = 16$

$m = 2$

8-3 Multiplying Polynomials

Find each product.

23. $(x - 3)(x + 7)$

24. $(3a - 2)(6a + 5)$

25. $(3r - 7t)(2r + 5t)$

26. $(2x + 5)(5x + 2)$

27. PARKING LOT The parking lot shown is to be paved. What is the area to be paved?

Example 4

Find $(6x - 5)(x + 4)$.

$(6x - 5)(x + 4)$

F O I L

$= (6x)(x) + (6x)(4) + (-5)(x) + (-5)(4)$

$= 6x^2 + 24x - 5x - 20$ Multiply.

$= 6x^2 + 19x - 20$ Combine like terms.

8-4 Special Products

Find each product.

28. $(x + 5)(x - 5)$

29. $(3x - 2)^2$

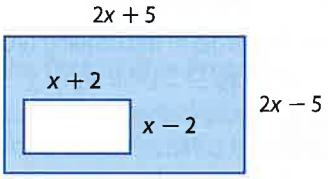
30. $(5x + 4)^2$

31. $(2x - 3)(2x + 3)$

32. $(2r + 5t)^2$

33. $(3m - 2)(3m + 2)$

34. GEOMETRY Write an expression to represent the area of the shaded region.



Example 5

Find $(x - 7)^2$.

$(a - b)^2 = a^2 - 2ab + b^2$ Square of a Difference

$(x - 7)^2 = x^2 - 2(x)(7) + (-7)^2$ $a = x$ and $b = 7$

$= x^2 - 14x + 49$ Simplify.

Example 6

Find $(5a - 4)(5a + 4)$.

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and Difference

$(5a - 4)(5a + 4) = (5a)^2 - (4)^2$ $a = 5a$ and $b = 4$

$= 25a^2 - 16$ Simplify.

8-5 Using the Distributive Property

Use the Distributive Property to factor each polynomial.

35. $12x + 24y$

36. $14x^2y - 21xy + 35xy^2$

37. $8xy - 16x^3y + 10y$

38. $a^2 - 4ac + ab - 4bc$

39. $2x^2 - 3xz - 2xy + 3yz$

40. $24am - 9an + 40bm - 15bn$

Solve each equation. Check your solutions.

41. $x(3x - 6) = 0$

42. $6x^2 = 12x$

43. $x^2 = 3x$

44. $3x^2 = 5x$

45. GEOMETRY The area of the rectangle shown is $x^3 - 2x^2 + 5x$ square units. What is the length?

Example 7

Factor $12y^2 + 9y + 8y + 6$.

$12y^2 + 9y + 8y + 6$

$= (12y^2 + 9y) + (8y + 6)$ Group terms with common factors.

$= 3y(4y + 3) + 2(4y + 3)$ Factor the GCF from each group.

$= (4y + 3)(3y + 2)$ Distributive Property

Example 8

Solve $x^2 - 6x = 0$. Check your solutions.

Write the equation so that it is of the form $ab = 0$.

$x^2 - 6x = 0$ Original equation

$x(x - 6) = 0$ Factor by using the GCF.

$x = 0$ or $x - 6 = 0$ Zero Product Property

$x = 6$ Solve.

The roots are 0 and 6. Check by substituting 0 and 6 for x in the original equation.

8-6 Solving $x^2 + bx + c = 0$

Factor each trinomial. Confirm your answers using a graphing calculator.

46. $x^2 - 8x + 15$ **47.** $x^2 + 9x + 20$

48. $x^2 - 5x - 6$ **49.** $x^2 + 3x - 18$

Solve each equation. Check your solutions.

50. $x^2 + 5x - 50 = 0$

51. $x^2 - 6x + 8 = 0$

52. $x^2 + 12x + 32 = 0$

53. $x^2 - 2x - 48 = 0$

54. $x^2 + 11x + 10 = 0$

55. ART An artist is working on a painting that is 3 inches longer than it is wide. The area of the painting is 154 square inches. What is the length of the painting?

Example 9

Factor $x^2 + 10x + 21$

$b = 10$ and $c = 21$, so $m + p$ is positive and mp is positive. Therefore, m and p must both be positive. List the positive factors of 21, and look for the pair of factors with a sum of 10.

Factors of 21	Sum of 10
1, 21	22
3, 7	10

The correct factors are 3 and 7.

$x^2 + 10x + 21 = (x + m)(x + p)$ Write the pattern.

$= (x + 3)(x + 7)$ $m = 3$ and $p = 7$

8-7 Solving $ax^2 + bx + c = 0$

Factor each trinomial, if possible. If the trinomial cannot be factored, write *prime*.

56. $12x^2 + 22x - 14$

57. $2y^2 - 9y + 3$

58. $3x^2 - 6x - 45$

59. $2a^2 + 13a - 24$

Solve each equation. Confirm your answers using a graphing calculator.

60. $40x^2 + 2x = 24$

61. $2x^2 - 3x - 20 = 0$

62. $-16t^2 + 36t - 8 = 0$

63. $6x^2 - 7x - 5 = 0$

64. GEOMETRY The area of the rectangle shown is $6x^2 + 11x - 7$ square units. What is the width of the rectangle?

$2x - 1$

Example 10

Factor $12a^2 + 17a + 6$

$a = 12$, $b = 17$, and $c = 6$. Since b is positive, $m + p$ is positive. Since c is positive, mp is positive. So, m and p are both positive. List the factors of 12(6) or 72, where both factors are positive.

Factors of 72	Sum of 17
1, 72	73
2, 36	38
3, 24	27
4, 18	22
6, 12	18
8, 9	17

The correct factors are 8 and 9.

$12a^2 + 17a + 6 = 12a^2 + ma + pa + 6$

$= 12a^2 + 8a + 9a + 6$

$= (12a^2 + 8a) + (9a + 6)$

$= 4a(3a + 2) + 3(3a + 2)$

$= (3a + 2)(4a + 3)$

So, $12a^2 + 17a + 6 = (3a + 2)(4a + 3)$.

8-8 Differences of Squares

Factor each polynomial.

65. $y^2 - 81$

66. $64 - 25x^2$

67. $16a^2 - 21b^2$

68. $3x^2 - 3$

Solve each equation by factoring. Confirm your answers using a graphing calculator.

69. $a^2 - 25 = 0$

70. $9x^2 - 25 = 0$

71. $81 - y^2 = 0$

72. $x^2 - 5 = 20$

73. EROSION A boulder falls down a mountain into water 64 feet below. The distance d that the boulder falls in t seconds is given by the equation $d = 16t^2$. How long does it take the boulder to hit the water?

Example 11

Solve $x^2 - 4 = 12$ by factoring.

$x^2 - 4 = 12$	Original equation
$x^2 - 16 = 0$	Subtract 12 from each side.
$x^2 - (4)^2 = 0$	$16 = 4^2$
$(x + 4)(x - 4) = 0$	Factor the difference of squares.
$x + 4 = 0$ or $x - 4 = 0$	Zero Product Property
$x = -4$ $\quad x = 4$	Solve each equation.

The solutions are -4 and 4.

8-9 Perfect Squares

Factor each polynomial, if possible. If the polynomial cannot be factored write *prime*.

74. $x^2 + 12x + 36$

75. $x^2 + 5x + 25$

76. $9y^2 - 12y + 4$

77. $4 - 28a + 49a^2$

78. $x^4 - 1$

79. $x^4 - 16x^2$

Solve each equation. Confirm your answers using a graphing calculator.

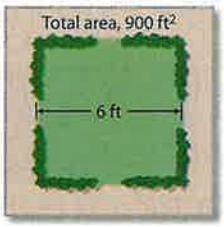
80. $(x - 5)^2 = 121$

81. $4c^2 + 4c + 1 = 9$

82. $4y^2 = 64$

83. $16d^2 + 40d + 25 = 9$

84. LANDSCAPING A sidewalk of equal width is being built around a square yard. What is the width of the sidewalk?



Example 12

Solve $(x - 9)^2 = 144$.

$(x - 9)^2 = 144$	Original equation
$x - 9 = \pm\sqrt{144}$	Square Root Property
$x - 9 = \pm 12$	$144 = 12 \cdot 12$
$x = 9 \pm 12$	Add 9 to each side.
$x = 9 + 12$ or $x = 9 - 12$	Zero Product Property
$x = 21$ $\quad x = -3$	Solve.

CHECK

$(x - 9)^2 = 144$ $\quad$ $(x - 9)^2 = 144$

$(21 - 9)^2 \stackrel{?}{=} 144$ $\quad$ $(-3 - 9)^2 \stackrel{?}{=} 144$

$(12)^2 \stackrel{?}{=} 144$ $\quad$ $(-12)^2 \stackrel{?}{=} 144$

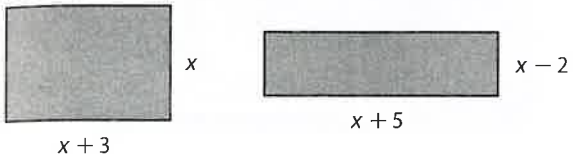
$144 = 144$ ✓ $\quad$ $144 = 144$ ✓

CHAPTER 8 Practice Test

Find each sum or difference.

1. $(x + 5) + (x^2 - 3x + 7)$
2. $(7m - 8n^2 + 3n) - (-2n^2 + 4m - 3n)$

3. **MULTIPLE CHOICE** Antonia is carpeting two of the rooms in her house. The dimensions are shown. Which expression represents the total area to be carpeted?



A $x^2 + 3x$
C $x^2 + 3x - 5$
B $2x^2 + 6x - 10$
D $8x + 12$

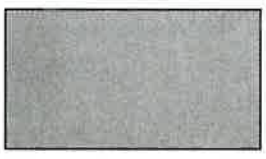
Find each product.

4. $a(a^2 + 2a - 10)$
5. $(2a - 5)(3a + 5)$
6. $(x - 3)(x^2 + 5x - 6)$
7. $(x + 3)^2$
8. $(2b - 5)(2b + 5)$

9. **FINANCIAL LITERACY** Suppose you invest \$4000 in a 2-year certificate of deposit (CD).
   a. If the interest rate is 5% per year, the expression $4000(1 + 0.05)^2$ can be evaluated to find the total amount of money after two years. Explain the numbers in this expression.
   b. Find the amount at the end of two years.
   c. Suppose you invest \$10,000 in a CD for 4 years at an annual rate of 6.25%. What is the total amount of money you will have after 4 years?

10. **MULTIPLE CHOICE** The area of the rectangle shown below is $2x^2 - x - 15$ square units. What is the width of the rectangle?

F $x - 5$
G $x + 3$
H $x - 3$
J $2x - 5$



Solve each equation.

11. $5(t^2 - 3t + 2) = t(5t - 2)$
12. $3x(x + 2) = 3(x^2 - 2)$

Factor each polynomial.

13. $5xy - 10x$
14. $7ab + 14ab^2 + 21a^2b$
15. $4x^2 + 8x + x + 2$
16. $10a^2 - 50a - a + 5$

Solve each equation. Confirm your answers using a graphing calculator.

17. $y(y - 14) = 0$
18. $3x(x + 6) = 0$
19. $a^2 = 12a$

20. **MULTIPLE CHOICE** Chantel is carpeting a room that has an area of $x^2 - 100$ square feet. If the width of the room is $x - 10$ feet, what is the length of the room?

A $x - 10$ ft
B $x + 10$ ft
C $x - 100$ ft
D 10 ft

Factor each trinomial.

21. $x^2 + 7x + 6$
22. $x^2 - 3x - 28$
23. $10x^2 - x - 3$
24. $15x^2 + 7x - 2$
25. $x^2 - 25$
26. $4x^2 - 81$
27. $9x^2 - 12x + 4$
28. $16x^2 + 40x + 25$

Solve each equation. Confirm your answers using a graphing calculator.

29. $x^2 - 4x = 21$
30. $x^2 - 2x - 24 = 0$
31. $6x^2 - 5x - 6 = 0$
32. $2x^2 - 13x + 20 = 0$

33. **MULTIPLE CHOICE** Which choice is a factor of $x^4 - 1$ when it is factored completely?

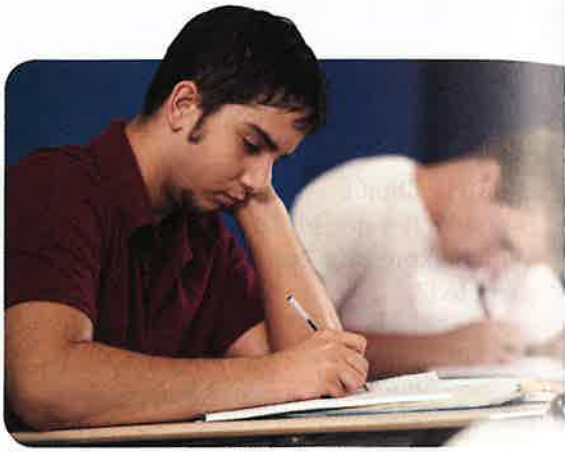
F $x^2 - 1$
H x
G $x - 1$
J 1

CHAPTER 8

Preparing for Standardized Tests

Solve Multi-Step Problems

Some problems that you will encounter on standardized tests require you to solve multiple parts in order to come up with the final solution. Use this lesson to practice these types of problems.



Strategies for Solving Multi-Step Problems

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve? What information is given?
- Are there any intermediate steps that need to be completed before I can solve the problem?

Step 2

Organize your approach.

- List the steps you will need to complete in order to solve the problem.
- Remember that there may be more than one possible way to solve the problem.

Step 3

Solve and check.

- Work as efficiently as possible to complete each step and solve.
- If time permits, check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

A florist has 80 roses, 50 tulips, and 20 lilies that he wants to use to create bouquets. He wants to create the maximum number of bouquets possible and use all of the flowers. Each bouquet should have the same number of each type of flower. How many roses will be in each bouquet?

A 4 roses

B 8 roses

C 10 roses

D 15 roses

Read the problem carefully. You are given the number of roses, tulips, and lilies and told that bouquets will be made using the same number of flowers in each. You need to find the number of roses that will be in each bouquet.

Step 1 Find the GCF of the number of roses, tulips, and lilies.

Step 2 Use the GCF to determine how many bouquets will be made.

Step 3 Divide the total number of roses by the number of bouquets.

Step 1 Write the prime factorization of each number of flowers to find the GCF.

$80 = \mathbf{2} \cdot 2 \cdot 2 \cdot 2 \cdot \mathbf{5}$

$50 = \mathbf{2} \cdot \mathbf{5} \cdot 5$

$20 = \mathbf{2} \cdot 2 \cdot \mathbf{5}$

$\text{GCF} = 2 \cdot 5 = 10$

Step 2 The GCF of the number of roses, tulips, and lilies tells you how many bouquets can be made because each bouquet will contain the same number of flowers. So, the florist can make a total of 10 bouquets.

Step 3 Divide the number of roses by the number of bouquets to find the number of roses in each bouquet.

$\frac{80}{10} = 8$

So, there will be 8 roses in each bouquet. The answer is B.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Which of the following values is not a solution to $x^3 - 3x^2 - 25x + 75 = 0$?

A $x = 5$ **C** $x = -3$

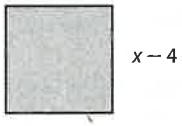
B $x = 3$ **D** $x = -5$

2. There are 12 teachers, 90 students, and 36 parent volunteers going on a field trip. Mrs. Bartholomew wants to divide everyone into equal groups with the same number of teachers, students, and parents in each group. If she makes as many groups as possible, how many students will be in each group?

F 6 **H** 12

G 9 **J** 15

3. What is the area of the square?



A $x^2 + 16$

B $4x - 16$

C $x^2 - 8x - 16$

D $x^2 - 8x + 16$

4. Students are selling magazines to raise money for a field trip. They make $2.75 for each magazine they sell. If they want to raise $600, what is the least amount of magazines they need to sell?

F 121 **H** 202

G 177 **J** 219

CHAPTER 8

Standardized Test Practice

Cumulative, Chapters 1 through 8

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Each year a local country club sponsors a tennis tournament. Play starts with 256 participants. During each round, half of the players are eliminated. How many players remain after 6 rounds?

A 128

B 64

C 16

D 4

2. Evaluate $\frac{5^6 - 5^5}{4}$.

F 5^6

G 5^5

H $\frac{5}{4}$

J $\frac{25}{4}$

3. Factor $mn + 5m - 3n - 15$.

A $(mn - 3)(5)$

B $(n - 3)(m + 5)$

C $(m - 5)(n + 3)$

D $(m - 3)(n + 5)$

4. Which of the following is a solution to $x^2 + 6x - 112 = 0$?

F -14

G -8

H 6

J 12

5. Which of the following polynomials is prime?

A $5x^2 + 34x + 24$

B $4x^2 + 22x + 10$

C $4x^2 + 38x + 70$

D $5x^2 + 3x + 4$

6. Which of the following is not a factor of the polynomial $45a^2 - 80b^2$?

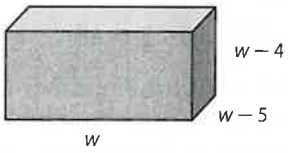
F 5

G $3a - 4b$

H $2a - 5b$

J $3a + 4b$

7. A rectangular gift box has dimensions that can b represented as shown in the figure. The volume of the box is $56w$ cubic inches. Which of the following is *not* a dimension of the box?



A 6 in.

B 7 in.

C 8 in.

D 12 in.

8. Factor the polynomial $y^2 - 9y + 20$.

F $(y - 2)(y - 10)$

G $(y - 4)(y - 5)$

H $(y - 2)(y - 7)$

J $(y - 5)(y + 2)$

9. Which of the following numbers is less than zero?

A 1.03×10^{-21}

B 7.5×10^2

C 8.21543×10^{10}

D none of the above

Test-Taking Tip

Question 4 If time permits, be sure to check your answer. Substitute it into the equation to see if you get a true number sentence.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. GRIDDED RESPONSE Mr. Branson bought a total of 9 tickets to the zoo. He bought children tickets at the rate of $6.50 and adult tickets for $9.25 each. If he spent $69.50 altogether, how many adult tickets did Mr. Branson purchase?

11. What is the domain of the following relation? $\{(2, -1), (4, 3), (7, 6)\}$

12. Ken just added 15 more songs to his digital media player, making the total number of songs more than 84. Draw a number line that represents the original number of songs he had on his digital media player.

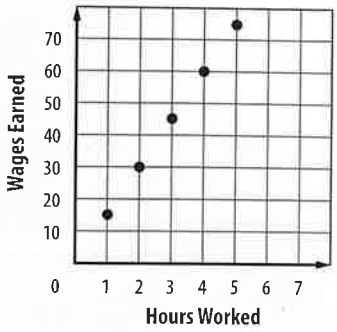
13. Carlos bought a rare painting in 1995 for $14,200. By 2003, the painting was worth $17,120. Assuming that a linear relationship exists, write an equation in slope-intercept form that represents the value V of the painting after t years.

14. The equation $h = -16t^2 + 40t + 3$ models the height h in feet of a soccer ball after t seconds. What is the height of the ball after 2 seconds?

15. Marcel spent $24.50 on peanuts and walnuts for a party. He bought 1.5 pounds more peanuts than walnuts. How many pounds of peanuts and walnuts did he buy?

Product	Price per pound
Peanuts p	$3.80
Cashews c	$6.90
Walnuts w	$5.60

16. GRIDDED RESPONSE The amount of money that Humberto earns varies directly as the number of hours that he works as shown in the graph. How much money will he earn for working 40 hours next week? Express your answer in dollars.



Extended Response

Record your answers on a sheet of paper. Show your work.

17. The height in feet of a model rocket t seconds after being launched into the air is given by the function $h(t) = -16t^2 + 200t$.

a. Write the expression that shows the height of the rocket in factored form.

b. At what time(s) is the height of the rocket equal to zero feet above the ground? Describe the real world meaning of your answer.

c. What is the greatest height reached by the model rocket? When does this occur?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Go to Lesson...	7-7	7-2	8-5	8-6	8-3	8-7	8-9	8-6	7-4	6-5	1-6	5-1	4-2	8-7	2-9	3-4	8-5