

Statistics and Probability



Then

You calculated weighted averages.

Now

- You will:
- Evaluate surveys, studies, and experiments.
 - Create and use graphs of probability distributions.
 - Use the Empirical Rule to find probabilities.
 - Compare sample statistics and population statistics.

Why? ▲

EDUCATION Probability and statistics are used in all facets of education. Surveys and experiments are done to find out which teaching methods promote the most learning. Statistics are used to determine grades when classes are curved, or when college professors weight their grades.



Weighted averages take into account that one test is of more value in determining a grade than the others. Which grade is weighted as the most important grade in Todd's calculus class?

Midterm 1 :	$\frac{17}{50}$
Midterm 2 :	$\frac{16}{50}$
Final Examination :	$\frac{22}{100}$

How your grade is determined:

Grade	Weight of Each
Midterm 1	20%
Midterm 2	25%
Final Examination	55%

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- Self-Check Practice 
- Worksheets 

Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck	QuickReview
<p>Find the mean, median, and mode for each set of data.</p> <ol style="list-style-type: none"> number of customers at a store each day during the last two weeks: 78, 80, 101, 66, 73, 92, 97, 125, 110, 76, 89, 90, 82, 87 a student's quiz scores for the first grading period: 88, 70, 85, 92, 88, 77, 98, 88, 70, 82 the number of touchdowns scored by a player over the last 10 years: 7, 5, 10, 12, 4, 10, 11, 6, 9, 3 	<p>Example 1</p> <p>The number of days it rained in each month over the past year are shown below. Find the mean, median, and mode.</p> <p style="text-align: center;">4, 2, 9, 16, 13, 9, 8, 9, 7, 6, 8, 5</p> <p>Mean $\bar{x} = \frac{4 + 2 + 9 + 16 + 13 + 9 + 8 + 9 + 7 + 6 + 8 + 5}{12}$ or 8 days</p> <p>Median 2, 4, 5, 6, 7, 8, 8, 9, 9, 9, 13, 16 ↑ $\frac{8 + 8}{2}$ or 8 days</p> <p>Mode The value that occurs most often in the set is 9, so the mode of the data set is 9 days.</p>

A die is rolled and a coin is tossed. Find each probability.

- $P(4, \text{heads})$
- $P(\text{odds, tails})$
- $P(2 \text{ or } 4, \text{heads})$

Example 2
<p>A die is rolled and a coin is tossed. What is the probability that the die shows a 1 and the coin lands tails up?</p> <p>$P(1, \text{tails}) = \frac{1}{6} \cdot \frac{1}{2}$ or $\frac{1}{12}$</p>

Expand each binomial.

- | | |
|---------------------------|---------------------------|
| 7. $(a - 2)^4$ | 8. $(m - a)^5$ |
| 9. $(2b - x)^4$ | 10. $(2a + b)^6$ |
| 11. $(3x - 2y)^5$ | 12. $(3x + 2y)^4$ |
| 13. $(\frac{a}{2} + 2)^5$ | 14. $(3 + \frac{m}{3})^5$ |

Example 3
<p>Expand $(a + b)^4$.</p> <p>Replace n with 4 in the Binomial Theorem.</p> $(a + b)^4 = {}_4C_0 a^4 + {}_4C_1 a^3 b + {}_4C_2 a^2 b^2 + {}_4C_3 a b^3 + {}_4C_4 b^4$ $= \frac{4!}{(4-0)! \cdot 0!} a^4 + \frac{4!}{(4-1)! \cdot 1!} a^3 b + \frac{4!}{(4-2)! \cdot 2!} a^2 b^2$ $+ \frac{4!}{(4-3)! \cdot 3!} a b^3 + \frac{4!}{(4-4)! \cdot 4!} b^4$ $= \frac{24}{24} a^4 + \frac{24}{6 \cdot 1} a^3 b + \frac{24}{2 \cdot 2} a^2 b^2 + \frac{24}{1 \cdot 6} a b^3 + \frac{24}{1 \cdot 4} b^4$ $= a^4 + 4a^3 b + 6a^2 b^2 + 4a b^3 + b^4$

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.



Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 11. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES Study Organizer



Statistics and Probability Make this Foldable to help you organize your Chapter 11 notes about statistics and probability. Begin with a sheet of $8\frac{1}{2}$ " by 11" paper.

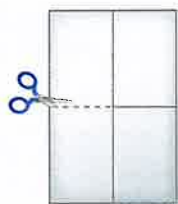
1 **Fold** in half lengthwise.



2 **Fold** the top to the bottom.



3 **Open.** Cut along the second fold to make two tabs.



4 **Label** each tab as shown.



New Vocabulary



English		Español
parameter	p. 723	parámetro
statistic	p. 723	estadística
survey	p. 723	exámenes
experiment	p. 723	experimento
observational study	p. 723	estudio de observación
random variable	p. 742	variable aleatoria
probability distribution	p. 743	distribución de probabilidad
expected value	p. 745	valor previsto
binomial experiment	p. 752	experimento binomio
binomial distribution	p. 754	distribución binomial
normal distribution	p. 760	distribución normal
z-value	p. 762	valor de z
confidence interval	p. 769	intervalo de la confianza
inferential statistics	p. 769	estadística deductiva
statistical inference	p. 769	inferencia estadística
hypothesis test	p. 771	prueba de hipótesis
null hypothesis	p. 771	hipótesis nula
alternative hypothesis	p. 771	hipótesis alternativa

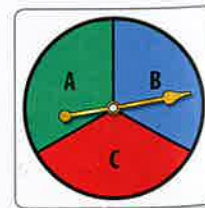
Review Vocabulary



combination **combinación** an arrangement or selection of objects in which order is not important

permutation **permutación** a group of objects or people arranged in a certain order

random **arbitrario** Unpredictable, or not based on any predetermined characteristics of the population; when a die is tossed, a coin is flipped, or a spinner is spun, the outcome is a random event.



11-1 Designing a Study

Then

- You identified various sampling techniques.

Now

- Classify study types.
- Design statistical studies.

Why?

- According to a recent study, 88% of teen cell phone users in the U.S. send text messages, and one in three teens sends more than 100 texts per day.

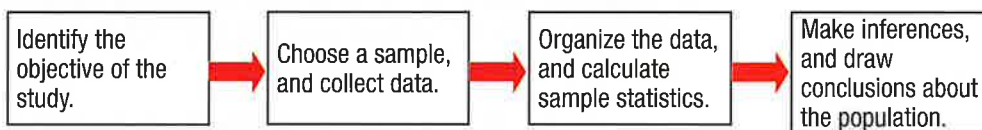


New Vocabulary

- parameter
- statistic
- bias
- random sample
- survey
- experiment
- observational study

1 Classifying Studies In a statistical study, data are collected and used to answer questions about a population characteristic or **parameter**. Due to time and money constraints, it may be impractical or impossible to collect data from each member of a population. Therefore, in many studies, a sample of the population is taken, and a measure called a **statistic** is calculated using the data. The sample statistic, such as the sample mean or sample standard deviation, is then used to make inferences about the population parameter.

The steps in a typical statistical study are shown below.



To obtain good information and draw accurate conclusions about a population, it is important to select an *unbiased* sample. A **bias** is an error that results in a misrepresentation of members of a population. A poorly chosen sample can cause biased results. To reduce the possibility of selecting a biased sample, a **random sample** can be taken, in which members of the population are selected entirely by chance.

You will review other sampling methods in Exercise 32.

The following study types can be used to collect sample information.

KeyConcept Study Types

Definition	Example
In a survey , data are collected from responses given by members of a population regarding their characteristics, behaviors, or opinions.	To determine whether the student body likes the new cafeteria menu, the student council asks a random sample of students for their opinion.
In an experiment , the sample is divided into two groups: <ul style="list-style-type: none"> an <i>experimental group</i> that undergoes a change, and a <i>control group</i> that does not undergo the change. The effect on the experimental group is then compared to the control group.	A restaurant is considering creating meals with chicken instead of beef. They randomly give half of a group of participants meals with chicken and the other half meals with beef. Then they ask how they like the meals.
In an observational study , members of a sample are measured or observed without being affected by the study.	Researchers at an electronics company observe a group of teenagers using different laptops and note their reactions.

Common Core State Standards

Content Standards

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.



Example 1 Classify Study Types

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

- a. **MUSIC** A record label wants to test three designs for an album cover. They randomly select 50 teenagers from local high schools to view the covers while they watch and record their reactions.



This is an observational study, because the company is going to observe the teens without them being affected by the study. The sample is the 50 teenagers selected, and the population is all potential purchasers of this album.

- b. **RECYCLING** The city council wants to start a recycling program. They send out a questionnaire to 200 random citizens asking what items they would recycle.

This is a survey, because the data are collected from participants' responses in the questionnaire. The sample is the 200 people who received the questionnaire, and the population is all of the citizens of the city.

StudyTip

Census

A *census* is a survey in which each member of a population is questioned. Therefore, when a census is conducted, there is no sample.

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- 1A. **RESEARCH** Scientists study the behavior of one group of dogs given a new heartworm treatment and another group of dogs given a false treatment or *placebo*.
- 1B. **YEARBOOKS** The yearbook committee conducts a study to determine whether students would prefer to have a print yearbook or both print and digital yearbooks.

To determine when to use a survey, experiment, or observational study, think about how the data will be obtained and whether or not the participants will be affected by the study.

Example 2 Choose a Study Type

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

- a. **MEDICINE** A pharmaceutical company wants to test whether a new medicine is effective.

The treatment will need to be tested on a sample group, which means that the members of the sample will be affected by the study. Therefore, this situation calls for an experiment.

- b. **ELECTIONS** A news organization wants to randomly call citizens to gauge opinions on a presidential election.

This situation calls for a survey because members of the sample population are asked for their opinion.

GuidedPractice

- 2A. **RESEARCH** A research company wants to study smokers and nonsmokers to determine whether 10 years of smoking affects lung capacity.
- 2B. **PETS** A national pet chain wants to know whether customers would pay a small annual fee to participate in a rewards program. They randomly select 200 customers and send them questionnaires.



2 Designing Studies The questions chosen for a survey or procedures used in an experiment can also introduce bias, and thus, affect the results of the study.

A survey question that is poorly written may result in a response that does not accurately reflect the opinion of the participant. Therefore, it is important to write questions that are clear and precise. Avoid survey questions that:

- are confusing or wordy
- encourage a certain response
- cause a strong reaction
- address more than one issue

Questions can also introduce bias if there is not enough information given for the participant to give an accurate response.



Example 3 Identify Bias in Survey Questions

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

a. Don't you agree that the cafeteria should serve healthier food?

This question is biased because it encourages a certain response. The phrase "don't you agree" encourages you to agree that the cafeteria should serve healthier food.

b. How often do you exercise?

This question is unbiased because it is clearly stated and does not encourage a certain response.

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3A. How many glasses of water do you drink a day?

3B. Do you prefer watching exciting action movies or boring documentaries?

When designing a survey, clearly state the objective, identify the population, and carefully choose unbiased survey questions.



Real-World Example 4 Design a Survey

TECHNOLOGY Jim is writing an article for his school newspaper about online courses. He wants to conduct a survey to determine how many students at his school would be interested in taking an online course from home. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Step 1 State the objective of the survey.

The objective of the survey is to determine students' interest in taking an online course from home.

Step 2 Identify the population.

The population is the student body.

Step 3 Write unbiased survey questions.

Possible survey questions:

- "Do you have Internet access at home?"
- "If offered, would you take an online course?"

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4. **TECHNOLOGY** In a follow-up article, Jim decides to conduct a survey to determine how many teachers from his school with at least five years of experience would be interested in teaching an online course. State the objective of the survey, suggest a population, and write two unbiased survey questions.



Real-WorldLink

Online Courses In 2009, about 1.2 million students took at least one online course.

Source: International Association for K-12 Online Learning



To avoid introducing bias in experiments, the experimental and control groups should be randomly selected and the experiment should be designed so that everything about the two groups is alike (except for the treatment or procedure).



Example 5 Identify Flaws in Experiments

StudyTip

Bias in Experiments

An experiment is biased when the participants know which group they are in.

Identify any flaws in the design of the experiment, and describe how they could be corrected.

Experiment: An electronics company wants to test whether using a new graphing calculator increases students' test scores. A random sample is taken. Calculus students in the experimental group are given the new calculator to use, and Algebra 2 students in the control group are asked to use their own calculator.

Results: When given the same test, the experimental group scored higher than the control group. The company concludes that the use of this calculator increases test scores.

Calculus students are more likely to score higher when given the same test as Algebra 2 students. Therefore, the flaw is that the experimental group consists of Calculus students and the control group consists of Algebra 2 students. This flaw could be corrected by selecting a random sample of all Calculus or all Algebra 2 students.

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5. **Experiment:** A research firm tests the effectiveness of a de-icer on car locks. They use a random sample of drivers in California and Minnesota for the control and experimental groups.

Results: They concluded that the de-icer is effective.

When designing an experiment, clearly state the objective, identify the population, determine the experimental and control groups, and define the procedure.



Real-World Example 6 Design an Experiment

PLANTS A research company wants to test the claim of the advertisement shown at the right. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Taller tomato plants in just 3 weeks!



Step 1 State the objective, and identify the population.

The objective of the experiment is to determine whether tomato plants given the plant food grow taller in three weeks than tomato plants not given the food. The population is all tomato plants.

Step 2 Determine the experimental and control groups.

The experimental group is the tomato plants given the food, and the control group is the tomato plants not given the food.

Step 3 Describe a sample procedure.

Measure the heights of the plants in each group, and give the experimental group the plant food. Then, wait three weeks, measure the heights of the plants again, and compare the heights for each group to see if the claim was valid.

GuidedPractice

6. **SPORTS** A company wants to determine whether wearing a new tennis shoe improves jogging time. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.





Example 1 Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

- SCHOOL** A group of high school students is randomly selected and asked to complete the form shown.
- DESIGN** An advertising company wants to test a new logo design. They randomly select 20 participants and watch them discuss the logo.

Do you agree with the new lunch rules?

- agree
 disagree
 don't care

Example 2 **CCSS ARGUMENTS** Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

- LITERACY** A literacy group wants to determine whether high school students that participated in a recent national reading program had higher standardized test scores than high school students that did not participate in the program.
- RETAIL** The research department of a retail company plans to conduct a study to determine whether a dye used on a new T-shirt will begin fading before 50 washes.

Example 3 Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

- Which student council candidate's platform do you support?
- How long have you lived at your current address?

Example 4 **7. HYBRIDS** A car manufacturer wants to determine what the demand in the U.S. is for hybrid vehicles. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Example 5 **8.** Identify any flaws in the experiment design, and describe how they could be corrected.

Experiment: A research company wants to determine whether a new vitamin boosts energy levels and decides to test the vitamin at a college campus. A random sample is taken. The experimental group consists of students who are given the vitamin, and the control group consists of instructors who are given a placebo.

Results: When given a physical test, the experimental group outperformed the control group. The company concludes that the vitamin is effective.

Example 6 **9. SPORTS** A research company wants to conduct an experiment to test the claim of the protein shake shown. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.

Helps athletes recover from intense exercise!



Practice and Problem Solving

Extra Practice is on page R11.

Example 1 Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

- FOOD** A grocery store conducts an online study in which customers are randomly selected and asked to provide feedback on their shopping experience.
- GRADES** A research group randomly selects 80 college students, half of whom took a physics course in high school, and compares their grades in a college physics course.
- HEALTH** A research group randomly chooses 100 people to participate in a study to determine whether eating blueberries reduces the risk of heart disease for adults.
- TELEVISION** A television network mails a questionnaire to randomly selected people across the country to determine whether they prefer watching sitcoms or dramas.



Example 2

Determine whether each situation calls for a *survey*, an *experiment*, or an *observational study*. Explain your reasoning.

- 14. **FASHION** A fashion magazine plans to poll 100 people in the U.S. to determine whether they would be more likely to buy a subscription if given a free issue.
- 15. **TRAVEL** A travel agency randomly calls 250 U.S. citizens and asks them what their favorite vacation destination is.
- 16. **FOOD** Chee wants to examine the eating habits of 100 random students at lunch to determine how many students eat in the cafeteria.
- 17. **ENGINEERING** An engineer is planning to test 50 metal samples to determine whether a new titanium alloy has a higher strength than a different alloy.

Example 3

Determine whether each survey question is *biased* or *unbiased*. If biased, explain your reasoning.

- 18. Do you think that the school needs a new gym and football field?
- 19. Which is your favorite football team, the Dallas Cowboys or the Pittsburgh Steelers?
- 20. Do you play any extracurricular sports?
- 21. Don't you agree that students should carpool to school?

Example 4

22. **COLLEGE** A school district wants to conduct a survey to determine the number of juniors in the district who are planning to attend college after high school. State the objective of the survey, suggest a population, and write two unbiased survey questions.

Example 5

23. Identify any flaws in the experiment design, and describe how they could be corrected.

Experiment: A supermarket chain wants to determine whether shoppers are more likely to buy sunscreen if it is located near the checkout line. The experimental group consists of a group of stores in the midwest in which the sunscreen was moved next to the checkout line, and the control group consists of stores in Arizona in which the sunscreen was not moved.

Results: The Arizona stores sold more sunscreen than the midwest stores. The company concluded that moving the sunscreen closer to the checkout line did not increase sales.

Example 6

24. **CCSS ARGUMENTS** In chemistry class, Pedro learned that copper objects become dull over time because the copper reacts with air to form a layer of copper oxide. He plans to use the supplies shown below to determine whether a mixture of lemon juice and salt will remove copper oxide from pennies.



2 lemons



1 teaspoon



30 dull pennies

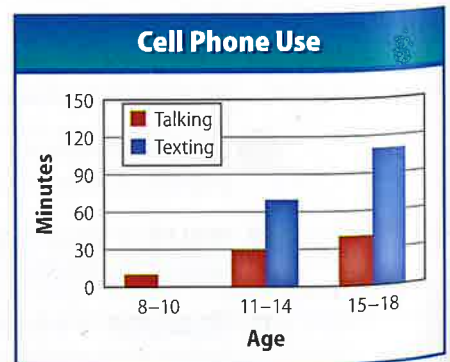


plastic bowl

- a. State the objective of the experiment, suggest a population, determine the experimental and control groups, and describe a sample procedure.
- b. What factors do you think should be considered when selecting pennies for the experiment? Explain your reasoning.

25. **REPORTS** The graph shown is from a report on the average number of minutes 8- to 18-year-olds in the U.S. spend on cell phones each day.

- a. Describe the sample and suggest a population.
- b. What type of sample statistic do you think was calculated for this report?
- c. Describe the results of the study for each age group.
- d. Who do you think would be interested in this type of report? Explain your reasoning.



26. **CCSS PERSEVERANCE** In 1936, the *Literary Digest* reported the results of a statistical study used to predict whether Alf Landon or Franklin D. Roosevelt would win the presidential election that year. The sample consisted of 2.4 million Americans, including subscribers to the magazine, registered automobile owners, and telephone users. The results concluded that Landon would win 57% of the popular vote. The actual election results are shown.

ELECTORAL VOTE



POPULAR VOTE

Roosevelt	60.8%
Landon	36.5%

- a. Describe the type of study performed, the sample taken, and the population.
 b. How do the predicted and actual results compare?
 c. Do you think that the survey was biased? Explain your reasoning.
27. **MULTIPLE REPRESENTATIONS** The results of two experiments concluded that Product A is 70% effective and Product B is 80% effective.

- a. **NUMERICAL** To simulate the experiment for Product A, use the random number generator on a graphing calculator to generate 30 integers between 0 and 9. Let 0–6 represent an effective outcome and 7–9 represent an ineffective outcome.
- b. **TABULAR** Copy and complete the frequency table shown using the results from part a. Then use the data to calculate the probability that Product A was effective. Repeat to find the probability for Product B.
- c. **ANALYTICAL** Compare the probabilities that you found in part b. Do you think that the difference in the effectiveness of each product is significant enough to justify selecting one product over the other? Explain.
- d. **LOGICAL** Suppose Product B costs twice as much as Product A. Do you think the probability of the product's effectiveness justifies the price difference to a consumer? Explain.



Product A	
Number	Frequency
0–6	
7–9	

H.O.T. Problems Use Higher-Order Thinking Skills

REASONING Determine whether each statement is *true* or *false*. If false, explain.

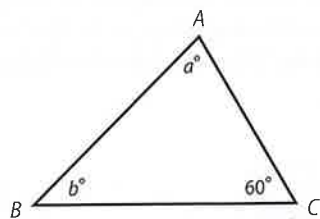
28. To save time and money, population parameters are used to estimate sample statistics.
29. Observational studies and experiments can both be used to study cause-and-effect relationships.
30. **OPEN ENDED** Design an observational study. Identify the objective of the study, define the population and sample, collect and organize the data, and calculate a sample statistic.
31. **CHALLENGE** What factors should be considered when determining whether a given statistical study is reliable?
32. **WRITING IN MATH** Research each of the following sampling methods. Then describe each method and discuss whether using the method could result in bias.
- | | |
|-------------------------|----------------------|
| a. convenience sample | c. stratified sample |
| b. self-selected sample | d. systematic sample |



Standardized Test Practice

33. **GEOMETRY** In $\triangle ABC$, $BC > AB$. Which of the following must be true?

- A $AB = BC$
 B $AC < AB$
 C $a > 60$
 D $a = b$



34. **SHORT RESPONSE** What is the solution set of $4^{4x^2 - 2x - 4} = 4^{-2}$?

35. **SAT/ACT** A pie is divided evenly between 3 boys and a girl. If one boy gives one half of his share to the girl and a second boy keeps two thirds of his share and gives the rest to the girl, what portion will the girl have in all?

- F $\frac{5}{24}$ H $\frac{1}{2}$ K $\frac{13}{12}$
 G $\frac{11}{24}$ J $\frac{13}{24}$

36. Which equation represents a hyperbola?

- A $y^2 = 49 - x^2$ C $y = 49x^2$
 B $y = 49 - x^2$ D $y = \frac{49}{x}$

Spiral Review

37. Prove that the statement $9^n - 1$ is divisible by 8 is true for all natural numbers. (Lesson 10-7)
38. **INTRAMURALS** Ofelia is taking ten shots in the intramural free-throw shooting competition. How many sequences of hits and misses are there that result in her making eight shots and missing two? (Lesson 10-6)

Solve each system of equations. (Lesson 9-7)

39. $y = x + 3$
 $y = 2x^2$
40. $x^2 + y^2 = 36$
 $y = x + 2$
41. $y^2 + x^2 = 9$
 $y = 7 - x$
42. $y + x^2 = 3$
 $x^2 + 4y^2 = 36$
43. $x^2 + y^2 = 64$
 $x^2 + 64y^2 = 64$
44. $y^2 = x^2 - 25$
 $x^2 - y^2 = 7$

Find the distance between each pair of points with the given coordinates. (Lesson 9-1)

45. $(9, -2), (12, -14)$
46. $(-4, -10), (-3, -11)$
47. $(1, -14), (-6, 10)$
48. $(-4, 9), (1, -3)$
49. $(2.3, -1.2), (-4.5, 3.7)$
50. $(0.23, 0.4), (0.68, -0.2)$

Simplify. Assume that no variable equals 0. (Lesson 5-1)

51. $(5cd^2)(-c^4d)$
52. $(7x^3y^{-5})(4xy^3)$
53. $\frac{a^2n^6}{an^5}$
54. $(n^4)^4$
55. $\frac{-y^5z^7}{y^2z^5}$
56. $(-2r^2t)^3(3rt^2)$

Write a quadratic equation with the given root(s). Write the equation in the form $ax^2 + bx + c = 0$, where a, b , and c are integers. (Lesson 4-3)

57. $-3, 9$
58. $-\frac{1}{3}, -\frac{3}{4}$
59. $4, -5$

Skills Review

60. **TESTS** Ms. Bonilla's class of 30 students took a biology test. If 20 of her students had an average of 83 on the test and the other students had an average score of 74, what was the average score of the whole class?
61. **DRIVING** During a 10-hour trip, Kwan drove 4 hours at 60 miles per hour and 6 hours at 65 miles per hour. What was her average rate, in miles per hour, for the entire trip?



Graphing Calculator Lab

Simulations and Margin of Error



The Pew Research Center conducted a survey of a random sample of teens and concluded that 43% of all teens who take their cell phones to school text in class on a daily basis. How accurately did their random sample represent all teens?

As you learned in the previous lesson, a survey of a random sample is a valuable tool for generalizing information about a larger population. The program in the following activity makes use of a random number generator ($\text{randInt}(a, b)$) to simulate the results of a random sampling survey.

CCSS Common Core State Standards Content Standards

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S.IC.6 Evaluate reports based on data.

Mathematical Practices

5 Use appropriate tools strategically.



Activity 1 Random Sampling Simulation

Use the following program that simulates the texting survey to measure the percent of teens who text in class for random sample sizes of 20, 50, and 100 students.

Step 1 Input the following program into a graphing calculator.

Program:SIMTEST	
:Input "SAMPLE SIZE ",S	:If C ≤ 43
:0→A	:B + 1→B
:0→B	:If A<S
:Lbl Z	:Goto Z
:A + 1→A	:100B/S→P
:randInt(1, 100) →C	:Disp "PERCENT WHO TEXT",P
	:Stop

Step 2 Run 10 trials of the program for each sample size of 20, 50, and 100. Press **ENTER** to run the program again each time.



Step 3 Record the percent who text for each trial in the table below.

Sample Size	1	2	3	4	5	6	7	8	9	10
20										
50										
100										

Analyze the Results

- Discard the percent that is farthest from the Pew survey result of 43% for each sample size. What is the range of the remaining nine percents for each sample size?
- What is the farthest any of these remaining trials is from the 43% for each sample size?
- The positive or negative of the result found in Exercise 2 is known as the **margin of error**. For your results, which sample size had the smallest margin of error?
- What would you expect to happen to the margin of error if we used a sample size of 500?

(continued on the next page)

Graphing Calculator Lab

Simulations and Margin of Error *Continued*

Statisticians have found that for large populations, the margin of error for a random sample of size n can be approximated by the following formula.

KeyConcept Margin of Error Formula

$$\text{Margin of error} = \pm \frac{1}{\sqrt{n}}(100)$$

Since n is in the denominator, the margin of error will decrease as the size of the random sample increases. This expression can also be used to determine the size of a random sample necessary to achieve a desired level of reliability.



Activity 2 Margin of Error and Sample Size

You are a member of a research team.

- a. You need to decide whether to conduct a survey with a margin of error of $\pm 3\%$ or $\pm 2\%$. What sample size would be needed to achieve each goal?

Set each percent equal to the margin of error formula and solve for n .

$\pm 3\% = \pm \frac{1}{\sqrt{n}}(100)$	Margin of error formula	$\pm 2\% = \pm \frac{1}{\sqrt{n}}(100)$
$0.03\sqrt{n} = 1$	Multiply by $\frac{\sqrt{n}}{100}$.	$0.02\sqrt{n} = 1$
$\sqrt{n} = 33.333$	Divide.	$\sqrt{n} = 50$
$n = 1111.11$	Square each side.	$n = 2500$

A random sample of about 1100 would have a margin of error of about $\pm 3\%$, while a random sample of 2500 would have a margin of error of $\pm 2\%$.

- b. Suppose the finance director would like to reduce the cost of the survey by using a random sample of 100. What would be the margin of error for this sample size?

Substitute 100 for n in the margin of error formula.

margin of error = $\pm \frac{1}{\sqrt{n}}(100)$	Margin of error formula
= $\pm \frac{1}{\sqrt{100}}(100)$ or $\pm 10\%$	$n = 100$

A random sample of 100 would have a margin of error of $\pm 10\%$.

Exercises

- What random sample size would produce a margin of error of $\pm 1\%$?
- What margin of error can be expected when using a sample size of 500?
- What are some reasons that a research center might decide that a survey with a margin of error of $\pm 3\%$ would be more desirable than one with a margin of error of $\pm 2\%$?
- What is the range for the percent of students that text in class that the research center can expect from any random survey they conduct with a sample size of 2500?
- If a survey with a random sample of 2500 students is conducted, is it possible that only 20% of the students could respond that they text during class? If so, how could this be possible?
- If Step 2 from Activity 1 is repeated using a sample size of 2500 and the range for the percents is found to be 19%–23%, would this result cause you to question the model?

Then

- You calculated measures of central tendency and variation.

Now

- Use the shapes of distributions to select appropriate statistics.
- Use the shapes of distributions to compare data.

Why?

- After four games as a reserve player, Craig joined the starting lineup and averaged 18 points per game over the *remaining* games. Craig's scoring average for the *entire* season was less than 18 points per game as a result of the lack of playing time in the first four games.



New Vocabulary

- distribution
- negatively skewed distribution
- symmetric distribution
- positively skewed distribution

1 Analyzing Distributions A **distribution** of data shows the observed or theoretical frequency of each possible data value. In Lesson 0-9, you described distributions of sample data using statistics. You used the mean or median to describe a distribution's center and standard deviation or quartiles to describe its spread. Analyzing the shape of a distribution can help you decide which measure of center or spread best describes a set of data.

The shape of the distribution for a set of data can be seen by drawing a curve over its histogram.



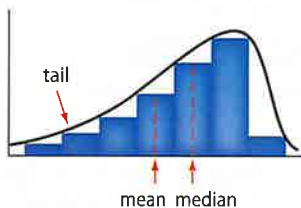
Common Core State Standards

Content Standards
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Mathematical Practices
1 Make sense of problems and persevere in solving them.

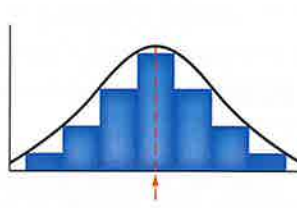
KeyConcept Symmetric and Skewed Distributions

Negatively Skewed Distribution



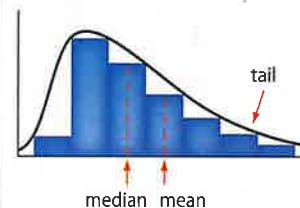
- The mean is less than the median.
- The majority of the data are on the right of the mean.

Symmetric Distribution



- The mean and median are approximately equal.
- The data are evenly distributed on both sides of the mean.

Positively Skewed Distribution



- The mean is greater than the median.
- The majority of the data are on the left of the mean.

When a distribution is symmetric, the mean and standard deviation accurately reflect the center and spread of the data. However, when a distribution is skewed, these statistics are not as reliable. Recall that outliers have a strong effect on the mean of a data set, while the median is less affected. Similarly, when a distribution is skewed, the mean lies away from the majority of the data toward the tail. The median is less affected, so it stays near the majority of the data.

When choosing appropriate statistics to represent a set of data, first determine the skewness of the distribution.

- If the distribution is relatively symmetric, the mean and standard deviation can be used.
- If the distribution is skewed or has outliers, use the five-number summary to describe the center and spread of the data.





Real-WorldLink

The first portable computer, the Osborne 1, was available for sale in 1981 for \$1795. The computer weighed 24 pounds and included a 5-inch display. Laptops can now be purchased for as little as \$250 and can weigh as little as 3 pounds.

Source: Computer History Museum

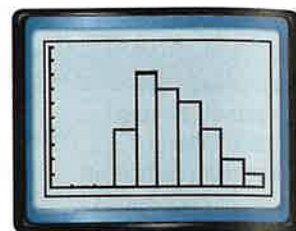
Real-World Example 1 Describe a Distribution Using a Histogram

COMPUTERS The prices for a random sample of personal computers are shown.

Price (dollars)							
723	605	847	410	440	386	572	523
374	915	734	472	420	508	613	659
706	463	470	752	671	618	538	425
811	502	490	552	390	512	389	621

a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.

First, press **STAT** **ENTER** and enter each data value. Then, press **2nd** **[STAT PLOT]** **ENTER** **ENTER** and choose **1:On**. Finally, adjust the window to the dimensions shown.



[0, 1000] scl: 100 by [0, 10] scl: 1

The majority of the computers cost between \$400 and \$700. Some of the computers are priced significantly higher, forming a tail for the distribution on the right. Therefore, the distribution is positively skewed.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is skewed, so use the five-number summary to describe the center and spread. Press **STAT** **▶** **ENTER** **ENTER** and scroll down to view the five-number summary.



The prices for this sample range from \$374 to \$915. The median price is \$530.50, and half of the computers are priced between \$451.50 and \$665.

GuidedPractice

1. **RAINFALL** The annual rainfall for a region over a 24-year period is shown below.

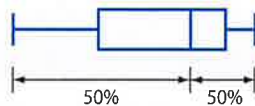
- A. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.
- B. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Annual Rainfall (in.)					
27.2	30.2	35.8	26.1	39.3	20.6
28.9	23.0	32.7	26.8	22.7	25.4
29.6	36.8	33.4	28.4	21.9	20.8
24.7	30.6	27.7	31.4	34.9	37.1

A box-and-whisker plot can also be used to identify the shape of a distribution. The position of the line representing the median indicates the center of the data. The "whiskers" show the spread of the data. If one whisker is considerably longer than the other and the median is closer to the shorter whisker, then the distribution is skewed.

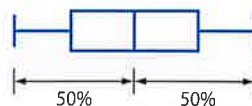
KeyConcept Box-and-Whisker Plots as Distributions

Negatively Skewed



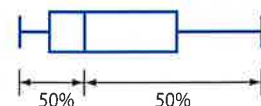
The data to the left of the median are distributed over a wider range than the data to the right. The data have a tail to the left.

Symmetric



The data are equally distributed to the left and right of the median.

Positively Skewed



The data to the right of the median are distributed over a wider range than the data to the left. The data have a tail to the right.

Example 2 Describe a Distribution Using a Box-and-Whisker Plot



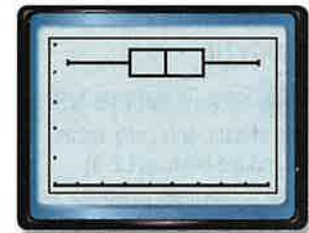
HOMEWORK The students in Mr. Fejis' language arts class found the average number of minutes that they each spent on homework each night.

Minutes per Night					
62	53	46	66	38	45
52	46	73	39	42	56
64	54	48	59	70	60
49	54	48	57	70	33

- a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

Enter the data as L1. Press $\boxed{2\text{nd}} \boxed{[\text{STAT PLOT}]} \boxed{\text{ENTER}} \boxed{\text{ENTER}}$ and choose $\boxed{\text{1-}} \boxed{\text{ENTER}}$. Adjust the window to the dimensions shown.

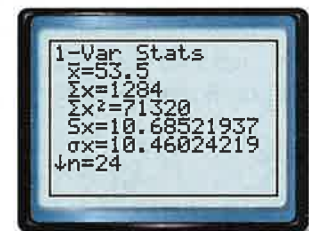
The lengths of the whiskers are approximately equal, and the median is in the middle of the data. This indicates that the data are equally distributed to the left and right of the median. Thus, the distribution is symmetric.



$[30, 75] \text{ scl: } 5$ by $[0, 5] \text{ scl: } 1$

- b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is symmetric, so use the mean and standard deviation to describe the center and spread. The average number of minutes that a student spent on homework each night was 53.5 with standard deviation of about 10.5.



WatchOut!

Standard Deviation Recall from Lesson 0-9 that the formulas for standard deviation for a population σ and for a sample s are slightly different. In Example 2, times for all of the students in Mr. Fejis' class are being analyzed, so use the population standard deviation.

GuidedPractice

2. **CELL PHONE** Janet's parents have given her a prepaid cell phone. The number of minutes she used each month for the last two years are shown in the table.

Minutes Used per Month			
582	608	670	620
667	598	671	613
537	511	674	627
638	661	642	641
668	673	680	695
658	653	670	688

- A. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
- B. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.



2 Comparing Distributions

To compare two sets of data, first analyze the shape of each distribution. Use the mean and standard deviation to compare two symmetric distributions. Use the five-number summaries to compare two skewed distributions or a symmetric distribution and a skewed distribution.



Example 3 Compare Data Using Histograms

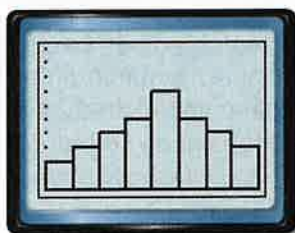
TEST SCORES Test scores from Mrs. Morash's class are shown below.

Chapter 3 Test Scores
81, 81, 92, 99, 61, 67, 86, 82, 76, 73, 62, 97, 97, 72, 72, 84, 77, 88, 92, 93, 76, 74, 66, 78, 76, 69, 84, 87, 83, 87, 92, 87, 82

Chapter 4 Test Scores
87, 73, 69, 83, 74, 86, 74, 69, 79, 84, 79, 74, 83, 74, 86, 69, 91, 73, 79, 83, 69, 79, 83, 74, 86, 79, 79, 78, 83, 79, 86, 79, 84

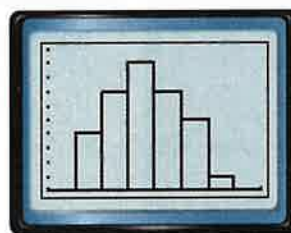
- a. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.

Chapter 3 Test Scores



[60, 100] scl: 5 by [0, 10] scl: 1

Chapter 4 Test Scores



[60, 100] scl: 5 by [0, 10] scl: 1

Both distributions are symmetric.

- b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

The distributions are symmetric, so use the means and standard deviations.

Chapter 3 Test Scores



Chapter 4 Test Scores



The Chapter 4 test scores, while lower in average, have a much smaller standard deviation, indicating that the scores are more closely grouped about the mean. Therefore, the mean for the Chapter 4 test scores is a better representation of the data than the mean for the Chapter 3 test scores.

StudyTip

CCSS Tools To compare two sets of data, enter one set as L1 and the other as L2. In order to calculate statistics for a set of data in L2, press

STAT **▶** **ENTER**
2nd **[L2]** **ENTER**.

GuidedPractice

3. **TYPING** The typing speeds of the students in two classes are shown below.

- A. Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
- B. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

3rd Period (wpm)
23, 38, 27, 28, 40, 45, 32, 33, 34, 27, 40, 22, 26, 34, 29, 31, 35, 33, 37, 38, 28, 29, 39, 42

6th Period (wpm)
38, 26, 43, 46, 23, 24, 27, 36, 22, 21, 26, 27, 31, 32, 27, 25, 23, 22, 28, 29, 28, 33, 23, 24



Box-and-whisker plots can be displayed alongside one another, making them useful for side-by-side comparisons of data.



Example 4 Compare Data Using Box-and-Whisker Plots

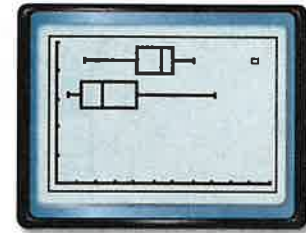
POINTS The points scored per game by a professional football team for the 2008 and 2009 football seasons are shown.

2008							
7	51	24	27	17	35	27	33
28	30	27	21	24	30	14	20

2009							
20	9	3	10	6	14	3	10
3	37	7	21	13	41	20	23

- a. Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.

Enter the 2008 scores as L1. Graph these data as Plot1 by pressing $\boxed{2\text{nd}} \boxed{[\text{STAT PLOT}]} \boxed{[\text{ENTER}]} \boxed{[\text{ENTER}]}$ and choosing $\boxed{1}$. Enter the 2009 scores as L2. Graph these data as Plot2 by pressing $\boxed{2\text{nd}} \boxed{[\text{STAT PLOT}]} \boxed{\nabla} \boxed{[\text{ENTER}]} \boxed{[\text{ENTER}]}$ and choosing $\boxed{2}$. For Xlist, enter L2. Adjust the window to the dimensions shown.



$[0, 55]$ scl: 5 by $[0, 5]$ scl: 1

For the 2008 scores, the left whisker is longer than the right and the median is closer to the right whisker. The distribution is negatively skewed.

For the 2009 scores, the right whisker is longer than the left and the median is closer to the left whisker. The distribution is positively skewed.

- b. Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

The distributions are skewed, so use the five-number summaries to compare the data.

The lower quartile for the 2008 season and the upper quartile for the 2009 season are both 20.5. This means that 75% of the scores from the 2008 season were greater than 20.5 and 75% of the scores from the 2009 season were less than 20.5.

The minimum of the 2008 season is approximately equal to the lower quartile for the 2009 season. This means that 25% of the scores from the 2009 season are lower than any score achieved in the 2008 season. Therefore, we can conclude that the team scored a significantly higher amount of points during the 2008 season than the 2009 season.

Guided Practice

4. **GOLF** Robert recorded his golf scores for his sophomore and junior seasons.
- Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.
 - Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Sophomore Season
42, 47, 43, 46, 50, 47, 52, 45, 53, 55, 48, 39, 40, 49, 47, 50

Junior Season
44, 38, 46, 48, 42, 41, 42, 46, 43, 40, 43, 43, 44, 45, 39, 44

Study Tip

Outliers Recall from Lesson 0-9 that outliers are data that are more than 1.5 times the interquartile range beyond the upper or lower quartile. All outliers should be plotted, but the whiskers should be drawn to the least and greatest values that are not outliers.





Example 1

- 1 **EXERCISE** The amount of time that James ran on a treadmill for the first 24 days of his workout is shown.

Time (minutes)											
23	10	18	24	13	27	19	7	25	30	15	22
10	28	23	16	29	26	26	22	12	23	16	27

- Use a graphing calculator to create a histogram. Then describe the shape of the distribution.
- Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Example 2

2. **RESTAURANTS** The total number of times that 20 random people either ate at a restaurant or bought fast food in a month are shown.

Restaurants or Fast Food									
4	7	5	13	3	22	13	6	5	10
7	18	4	16	8	5	15	3	12	6

- Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
- Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Example 3

3. **CCSS TOOLS** The total fundraiser sales for the students in two classes at Cantonville High School are shown.

Mrs. Johnson's Class (dollars)					
6	14	17	12	38	15
11	12	23	6	14	28
16	13	27	34	25	32
21	24	21	17	16	

Mr. Edmunds' Class (dollars)					
29	38	21	28	24	33
14	19	28	15	30	6
31	23	33	12	38	28
18	34	26	34	24	37

- Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
- Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Example 4

4. **RECYCLING** The weekly totals of recycled paper for the junior and senior classes are shown.

Junior Class (pounds)					
14	24	8	26	19	38
12	15	12	18	9	24
12	21	9	15	13	28

Senior Class (pounds)					
25	31	35	20	37	27
22	32	24	28	18	32
25	32	22	29	26	35

- Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.
- Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.



Examples 1–2 For Exercises 5 and 6, complete each step.

- Use a graphing calculator to create a histogram and a box-and-whisker plot. Then describe the shape of the distribution.
 - Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.
5. **FANTASY** The weekly total points of Kevin’s fantasy football team are shown.

Total Points							
165	140	88	158	101	137	112	127
53	151	120	156	142	179	162	79

6. **MOVIES** The students in one of Mr. Peterson’s classes recorded the number of movies they saw over the past month.

Movies Seen											
14	11	17	9	6	11	7	8	12	13	10	9
5	11	7	13	9	12	10	9	15	11	13	15

Example 3 **CCSS MODELING** For Exercises 7 and 8, complete each step.

- Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
 - Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.
7. **SAT** A group of students took the SAT their sophomore year and again their junior year. Their scores are shown.

Sophomore Year Scores					
1327	1663	1708	1583	1406	1563
1637	1521	1282	1752	1628	1453
1368	1681	1506	1843	1472	1560

Junior Year Scores					
1728	1523	1857	1789	1668	1913
1834	1769	1655	1432	1885	1955
1569	1704	1833	2093	1608	1753

8. **INCOME** The total incomes for 18 households in two neighboring cities are shown.

Yorkshire (thousands of dollars)					
68	59	61	78	58	66
56	72	86	58	63	53
68	58	74	60	103	64

Applewood (thousands of dollars)					
52	55	60	61	55	65
65	60	45	37	41	71
50	61	65	66	87	55

Example 4 **9 TUITION** The annual tuitions for a sample of public colleges and a sample of private colleges are shown. Complete each step.

- Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.
- Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Public Colleges (dollars)					
3773	3992	3004	4223	4821	3880
3163	4416	5063	4937	3321	4308
4006	3508	4498	3471	4679	3612

Private Colleges (dollars)					
10,766	13,322	12,995	15,377	16,792	9147
15,976	11,084	17,868	7909	12,824	10,377
14,304	10,055	12,930	16,920	10,004	11,806

10. **DANCE** The total amount of money that a random sample of seniors spent on prom is shown. Complete each step.
- Use a graphing calculator to create a box-and-whisker plot for each data set. Then describe the shape of each distribution.
 - Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Boys (dollars)					
253	288	304	283	348	276
322	368	247	404	450	341
291	260	394	302	297	272

Girls (dollars)					
682	533	602	504	635	541
489	703	453	521	472	368
562	426	382	668	352	587

11. **BASKETBALL** Refer to the beginning of the lesson. The points that Craig scored in the remaining games are shown.

Points Scored			
18	10	18	21
9	25	13	17
17	12	24	19
20	17	27	21

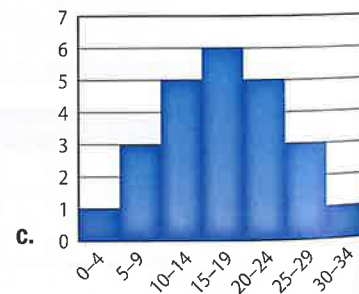
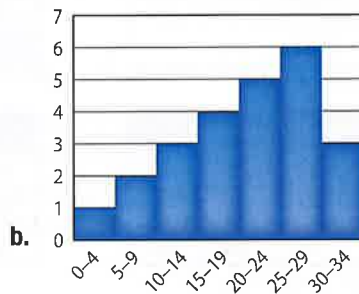
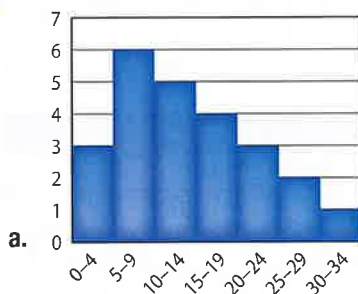
- Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.
 - Craig scored 0, 2, 1, and 0 points in the first four games. Use a graphing calculator to create a box-and-whisker plot that includes the new data. Then find the mean and median of the new data set.
 - What effect does adding the scores from the first four games have on the shape of the distribution and on how you should describe the center and spread?
12. **SCORES** Allison's quiz scores are shown.

Math Quiz Scores					
83	76	86	82	84	57
86	62	90	96	76	89
76	88	86	86	92	94

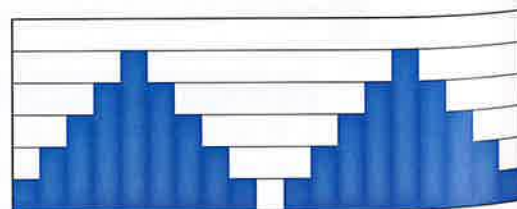
- Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread.
- Allison's teacher allows students to drop their two lowest quiz scores. Use a graphing calculator to create a box-and-whisker plot that reflects this change. Then describe the center and spread of the new data set.

H.O.T. Problems Use Higher-Order Thinking Skills

13. **CHALLENGE** Approximate the mean and median for each distribution of data.



14. **CCSS ARGUMENTS** Distributions of data are not always symmetric or skewed. If a distribution has a gap in the middle, like the one shown, two separate clusters of data may result, forming a *bimodal distribution*. How can the center and spread of a bimodal distribution be described?



15. **OPEN ENDED** Find a real-world data set that appears to represent a symmetric distribution and one that does not. Describe each distribution. Create a visual representation of each set of data.
16. **WRITING IN MATH** Explain the difference between positively skewed, negatively skewed, and symmetric sets of data, and give an example of each.



Standardized Test Practice

17. **DISTRIBUTIONS** Which of the following is a characteristic of a negatively skewed distribution?
- A The majority of the data are on the left of the mean.
 - B The mean and median are approximately equal.
 - C The mean is greater than the median.
 - D The mean is less than the median.

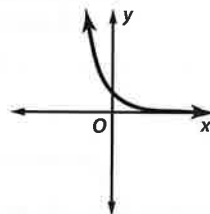
18. **SHORT RESPONSE** The average test score of a class of c students is 80, and the average test score of a class of d students is 85. When the scores of both classes are combined, the average score is 82. What is the value of $\frac{c}{d}$?

19. **SAT/ACT** What is the multiplicative inverse of $2i$?

- F $-2i$
- G -2
- H $-\frac{i}{2}$
- J $\frac{1}{2}$
- K $\frac{i}{2}$

20. Which equation best represents the graph?

- A $y = 4x$
- B $y = x^2 + 4$
- C $y = 4^{-x}$
- D $y = -4^x$



Spiral Review

Determine whether each survey question is biased. Explain your reasoning. (Lesson 11-1)

- 21. What toppings do you prefer on your pizza?
- 22. What is your favorite class, and what teacher gives the easiest homework?
- 23. Don't you hate how high gas prices are?
- 24. **PARTIES** Suppose each time a new guest arrives at a party, he or she shakes hands with each person already at the party. Prove that after n guests have arrived, a total of $\frac{n(n-1)}{2}$ handshakes have taken place. (Lesson 10-7)
- 25. **ASTRONOMY** The orbit of Pluto can be modeled by $\frac{x^2}{39.5^2} + \frac{y^2}{38.3^2} = 1$, where the units are astronomical units. Suppose a comet is following a path modeled by $x = y^2 + 20$. (Lesson 9-7)
 - a. Find the point(s) of intersection of the orbits of Pluto and the comet.
 - b. Will the comet necessarily hit Pluto? Explain.
 - c. Where do the graphs of $y = 2x + 1$ and $2x^2 + y^2 = 11$ intersect?
 - d. What are the coordinates of the points that lie on the graphs of both $x^2 + y^2 = 25$ and $2x^2 + 3y^2 = 66$?

Skills Review

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

- 26. the winner of the first, second, and third prizes in a contest with 8 finalists
- 27. selecting two of eight employees to attend a business seminar
- 28. an arrangement of the letters in the word *MATH*
- 29. placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf



Then

- You used statistics to describe symmetrical and skewed distributions of data.

Now

- Construct a probability distribution.
- Analyze a probability distribution and its summary statistics.

Why?

- Mutual funds are professionally managed investments that offer diversity to investors. An accurate analysis of the fund's current and expected performance can help an investor determine if the fund suits their needs.



New Vocabulary

- random variable
- discrete random variable
- continuous random variable
- probability distribution
- theoretical probability distribution
- experimental probability distribution
- Law of Large Numbers
- expected value



Common Core State Standards

Content Standards

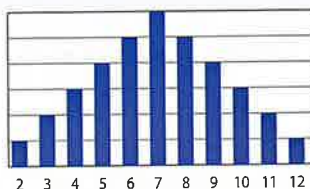
S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Mathematical Practices

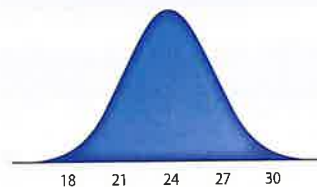
- Reason abstractly and quantitatively.

1 Construct a Probability Distribution A sample space is the set of all possible outcomes in a distribution. Consider a distribution of values represented by the sum of the values on two dice and a distribution of the miles per gallon for a sample of cars.

Sum of Two Dice



Miles Per Gallon



The sum of the values on the dice can be any integer from 2 to 12. So, the sample space is $[2, 3, \dots, 11, 12]$. This distribution is *discrete* because the number of possible values in the sample space can be counted.

The distribution of miles per gallon is *continuous*. While the sample space includes any positive value less than a certain maximum (around 100), the data can take on an infinite number of values within this range.

The value of a **random variable** is the numerical outcome of a random event. A random variable can be discrete or continuous. **Discrete random variables** represent countable values. **Continuous random variables** can take on any value.



Example 1 Identify and Classify Random Variables

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

a. the number of songs found on a random selection of mp3 players

The random variable X is the number of songs on any mp3 player in the random selection of players. The number of songs is countable, so X is discrete.

b. the weights of football helmets sent by a manufacturer

The random variable X is the weight of any particular helmet. The weight of any particular helmet can be anywhere within a certain range, typically 6 to 8 pounds. Therefore, X is continuous.

Guided Practice

- the exact distances of a sample of discus throws
- the ages of counselors at a summer camp



StudyTip

Discrete vs. Continuous

Variables representing height, weight, and capacity will always be continuous variables because they can take on any positive value.

A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. Probability distributions can be represented by tables, equations, or graphs. In this lesson, we will focus on discrete probability distributions.

A probability distribution has the following properties.

KeyConcept Probability Distribution

- A probability distribution can be determined theoretically or experimentally.
- A probability distribution can be discrete or continuous.
- The probability of each value of X must be at least 0 and not greater than 1.
- The sum of all the probabilities for all of the possible values of X must equal 1. That is, $\sum P(X) = 1$.

ReviewVocabulary

Theoretical and Experimental Probability

Theoretical probability is based on assumptions, and experimental probability is based on experiments.

(Lesson 0-5)

A **theoretical probability distribution** is based on what is expected to happen. For example, the distribution for flipping a fair coin is $P(\text{heads}) = 0.5$, $P(\text{tails}) = 0.5$.



Example 2 Construct a Theoretical Probability Distribution

X represents the sum of the values on two dice.

a. Construct a relative-frequency table.

The theoretical probabilities associated with rolling two dice can be described using a relative-frequency table. When two dice are rolled, 36 total outcomes are possible. To determine the relative frequency, or theoretical probability, of each outcome, divide the frequency by 36.

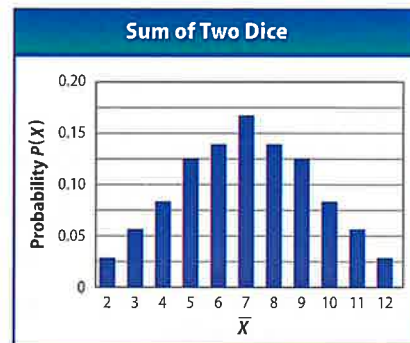
Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	2	3	4	5	6	5	4	3	2	1
Relative Frequency	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Sum: 36

b. Graph the theoretical probability distribution.

The graph shows the probability distribution for the sum of the values on two dice X . The bars are separated on the graph because the distribution is discrete (no other values of X are possible).

Each unique outcome of X is indicated on the horizontal axis, and the probability of each outcome occurring $P(X)$ is indicated on the vertical axis.



GuidedPractice

2. X represents the sum of the values of two spins of the wheel.

A. Construct a relative-frequency table.

B. Graph the theoretical probability distribution.



An **experimental probability distribution** is a distribution of probabilities estimated from experiments. Simulations can be used to construct an experimental probability distribution. When constructing this type of distribution, use the frequency of occurrences of each observed value to compute its probability.



Example 3 Construct an Experimental Probability Distribution

X represents the sum of the values found by rolling two dice.

a. Construct a relative-frequency table.

Roll two dice 100 times or use a random number generator to complete the simulation and create a simulation tally sheet.

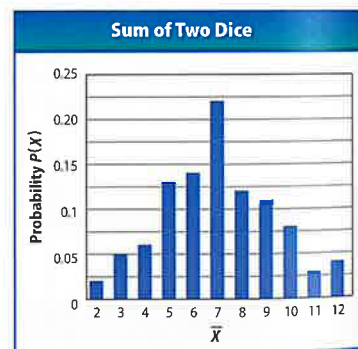
Sum	Tally	Frequency	Sum	Tally	Frequency
2		2	8		12
3		5	9		11
4		6	10		8
5		13	11		3
6		14	12		4
7		22			

Calculate the experimental probability of each value by dividing its frequency by the total number of trials, 100.

Sum	2	3	4	5	6	7	8	9	10	11	12
Relative Frequency	0.02	0.05	0.06	0.13	0.14	0.22	0.12	0.11	0.08	0.03	0.04

b. Graph the experimental probability distribution.

The graph shows the discrete probability distribution for the sum of the values shown on two dice X .



Review Vocabulary

Simulations and Random Number Generators

For more practice on simulations and random number generators, see Extend 12-1.

StudyTip

Random Number Generators and Proportions When using a random number generator to simulate events with different probabilities, set up a proportion. For example, suppose there are 3 possible outcomes with probabilities of A: 0.25, B:0.35, and C:0.40. Random numbers 1–25 can represent A, 26–60 represent B, and 61–100 represent C.

Guided Practice

- X represents the sum of the values of two spins of the wheel.
 - Construct a relative-frequency table for 100 trials.
 - Graph the experimental probability distribution.



Notice that this graph is different from the theoretical graph in Example 2. With small sample sizes, experimental distributions may vary greatly from their associated theoretical distributions. However, as the sample size increases, experimental probabilities will more closely resemble their associated theoretical probabilities. This is due to the **Law of Large Numbers**, which states that the variation in a data set decreases as the sample size increases.





Math HistoryLink

Christian Huygens

(1629–1695) This Dutchman was the first to discuss games of chance. “Although in a pure game of chance the results are uncertain, the chance that one player has to win or to lose depends on a determined value.” This became known as the *expected value*.

WatchOut!

Expected Value

The expected value is what you *expect* to happen in the long run, not necessarily what *will* happen.

2 Analyze a Probability Distribution Probability distributions are often used to analyze financial data. The two most common statistics used to analyze a discrete probability distribution are the mean, or expected value, and the standard deviation. The **expected value** $E(X)$ of a discrete random variable of a probability distribution is the weighted average of the variable.

KeyConcept Expected Value of a Discrete Random Variable

Words The expected value of a discrete random variable is the weighted average of the values of the variable. It is calculated by finding the sum of the products of every possible value of X and its associated probability $P(X)$.

Symbols $E(X) = \sum [X \cdot P(X)]$

Real-World Example 4 Expected Value



GAME SHOWS A game-show contestant has won one spin of the wheel at the right. Find the expected value of his winnings.



Each prize value represents a value of X and each percent represents the corresponding probability $P(X)$. Find $E(X)$.

$$\begin{aligned} E(X) &= \sum [X \cdot P(X)] \\ &= 0(0.20) + 25,000(0.08) + 15,000(0.14) + 10,000(0.22) + 5000(0.36) \\ &= 0 + 2000 + 2100 + 2200 + 1800 \\ &= 8100 \end{aligned}$$

The expected value of the contestant's winnings is \$8100.

GuidedPractice

4. PRIZES Curt won a ticket for a prize. The distribution of the values of the tickets and their relative frequencies are shown. Find the expected value of his winnings.

Value (\$)	1	10	100	1000	5000	25,000
Frequency	5000	100	25	5	1	1

Sometimes the expected value does not provide enough information to fully analyze a probability distribution. For example, suppose two wheels had roughly the same expected value. Which one would you choose? Which one is *riskier*? The standard deviation can provide more insight into the expected value of a probability distribution.

The formula for calculating the standard deviation of a probability distribution is similar to the one used for a set of data.

KeyConcept Standard Deviation of a Probability Distribution

Words For each value of X , subtract the mean from X and square the difference. Then multiply by the probability of X . The sum of each of these products is the variance. The standard deviation is the square root of the variance.

Symbols Variance: $\sigma^2 = \sum [(X - E(X))^2 \cdot P(X)]$
Standard Deviation: $\sigma = \sqrt{\sigma^2}$





Real-World Career

Mutual Fund Manager

Mutual fund managers buy and sell fund investments according to the investment objective of the fund.

Investment management includes financial statement analysis, asset and stock selection, and monitoring of investments. Certification beyond a bachelor's degree is required.

Source: International Financial Services, London

Real-World Example 5 Standard Deviation of a Distribution

DECISION MAKING Jimmy is thinking about investing \$10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below.

Fund A
50% chance of an \$800 profit
20% chance of a \$1200 profit
20% chance of a \$600 profit
10% chance of a \$100 loss

Fund B
30% chance of a \$2400 profit
10% chance of a \$1900 profit
40% chance of a \$200 loss
20% chance of a \$400 loss

a. Find the expected value of each investment.

$$\text{Fund A: } E(X) = 0.50(800) + 0.20(1200) + 0.20(600) + 0.10(-100) \text{ or } 750$$

$$\text{Fund B: } E(X) = 0.30(2400) + 0.10(1900) + 0.40(-200) + 0.20(-400) \text{ or } 750$$

An investment of \$10,000 in Fund A or Fund B will expect to yield \$750.

b. Find each standard deviation.

Fund A:

Profit, X	$P(X)$	$[X - E(X)]^2$	$[X - E(X)]^2 \cdot P(X)$
800	0.50	$(800 - 750)^2 = 2500$	$2500 \cdot 0.50 = 1250$
1200	0.20	$(1200 - 750)^2 = 202,500$	$202,500 \cdot 0.20 = 40,500$
600	0.20	$(600 - 750)^2 = 22,500$	$22,500 \cdot 0.20 = 4500$
-100	0.10	$(-100 - 750)^2 = 722,500$	$722,500 \cdot 0.10 = 72,250$
			$\Sigma[[X - E(X)]^2 \cdot P(X)] = 118,500$
			$\sqrt{118,500} \approx 344.2$

Fund B:

Profit, X	$P(X)$	$[X - E(X)]^2$	$[X - E(X)]^2 \cdot P(X)$
2400	0.30	$(2400 - 750)^2 = 2,722,500$	$2,722,500 \cdot 0.30 = 816,750$
1900	0.10	$(1900 - 750)^2 = 1,322,500$	$1,322,500 \cdot 0.10 = 132,250$
-200	0.40	$(-200 - 750)^2 = 902,500$	$902,500 \cdot 0.40 = 361,000$
-400	0.20	$(-400 - 750)^2 = 1,322,500$	$1,322,500 \cdot 0.20 = 264,500$
			$\Sigma[[X - E(X)]^2 \cdot P(X)] = 1,574,500$
			$\sqrt{1,574,500} \approx 1254.8$

c. Which investment would you advise Jimmy to choose, and why?

Jimmy should choose Fund A. While the funds have identical expected values, the standard deviation of Fund B is almost four times the standard deviation for Fund A. This means that the expected value for Fund B will have about four times the variability than Fund A and will be riskier with a greater chance for gains and losses.

Guided Practice

5. **DECISION MAKING** Compare a \$10,000 investment in the two funds. Which investment would you recommend, and why?

Fund C
30% chance of a \$1000 profit
40% chance of a \$500 profit
20% chance of a \$100 loss
10% chance of a \$300 loss

Fund D
40% chance of a \$1000 profit
30% chance of a \$600 profit
15% chance of a \$100 profit
15% chance of a \$200 loss

StudyTip

Return on Investment When investing \$1000 in a product that has a 6% expected return, the investor can expect a $0.06(1000)$ or \$60 profit.





Example 1 Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

1. the number of pages linked to a Web page
2. the number of stations in a cable package
3. the amount of precipitation in a city per month
4. the number of cars passing through an intersection in a given time interval

- Examples 2–5**
5. X represents the sum of the values of two spins of the wheel.
 - a. Construct a relative-frequency table showing the theoretical probabilities.
 - b. Graph the theoretical probability distribution.
 - c. Construct a relative-frequency table for 100 trials.
 - d. Graph the experimental probability distribution.
 - e. Find the expected value for the sum of two spins of the wheel.
 - f. Find the standard deviation for the sum of two spins of the wheel.



Practice and Problem Solving

Extra Practice is on page R11.

Example 1 Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

6. the number of texts received per week
7. the number of diggs (or “likes”) for a Web page
8. the height of a plant after a specific amount of time
9. the number of files infected by a computer virus

Examples 2–5 10. **CCSS PERSEVERANCE** A contestant has won a prize on a game show. The frequency table at the right shows the number of winners for 3200 hypothetical players.

- a. Construct a relative-frequency table showing the theoretical probability.
- b. Graph the theoretical probability distribution.
- c. Construct a relative-frequency table for 50 trials.
- d. Graph the experimental probability distribution.
- e. Find the expected value.
- f. Find the standard deviation.

Prize, X	Winners
\$100	1120
\$250	800
\$500	480
\$1000	320
\$2500	256
\$5000	128
\$7500	64
\$10,000	32

11 SNOW DAYS The following probability distribution lists the probable number of snow days per school year at North High School. Use this information to determine the expected number of snow days per year.

Number of Snow Days Per Year									
Days	0	1	2	3	4	5	6	7	8
Probability	0.1	0.1	0.15	0.15	0.25	0.1	0.08	0.05	0.02

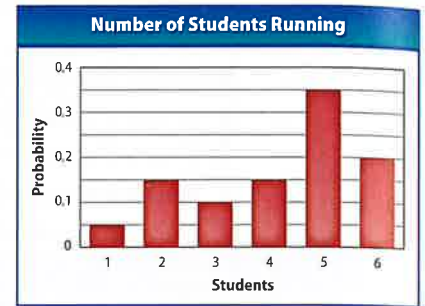
12. **CARDS** In a standard deck of 52 cards, there are 4 different suits.
 - a. If jacks = 11, queens = 12, kings = 13, and aces = 1, what is the expected value of a card that is drawn from a standard deck?
 - b. If you are dealt 7 cards with replacement, what is the expected number of spades?



13. **RAFFLES** The table shows the probability distribution for a raffle if 100 tickets are sold for \$1 each. There is 1 prize for \$20, 5 prizes for \$10, and 10 prizes for \$5.

Distribution of Prizes				
Prize	no prize	\$20	\$10	\$5
Probability	0.84	0.01	0.05	0.10

- a. Graph the theoretical probability distribution.
 b. Find the expected value.
 c. Interpret the results you found in part b. What can you conclude about the raffle?
14. **CCSS TOOLS** Based on previous data, the probability distribution of the number of students running for class president is shown.



- a. Determine the expected number of students who will run. Interpret your results.
 b. Construct a relative-frequency table for 50 trials.
 c. Graph the experimental probability distribution.

15. **BASKETBALL** The distribution below lists the probability of the number of major upsets in the first round of a basketball tournament each year.

Number of Upsets Per Year									
Upsets	0	1	2	3	4	5	6	7	8
Probability	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

- a. Determine the expected number of upsets. Interpret your results.
 b. Find the standard deviation.
 c. Construct a relative-frequency table for 50 trials.
 d. Graph the experimental probability distribution.
16. **RAFFLES** The French Club sold 500 raffle tickets for \$1 each. The first prize ticket will win \$100, 2 second prize tickets will each win \$10, and 5 third prize tickets each win \$5.
- a. What is the expected value of a single ticket?
 b. Calculate the standard deviation of the probability distribution.
 c. **DECISION MAKING** The Glee Club is offering a raffle with a similar expected value and a standard deviation of 2.2. In which raffle should you participate? Explain your reasoning.

17. **DECISION MAKING** Carmen is thinking about investing \$10,000 in two different investment funds. The expected rates of return and the corresponding probabilities for each fund are listed below. Compare the two investments using the expected value and standard deviation. Which investment would you advise Carmen to choose, and why?

Fund A
30% chance of a \$1900 profit
30% chance of a \$600 profit
15% chance of a \$200 loss
25% chance of a \$500 loss

Fund B
40% chance of a \$1600 profit
10% chance of a \$900 profit
10% chance of a \$300 loss
40% chance of a \$400 loss



18. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric probability.

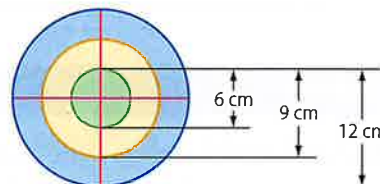
a. **Tabular** The spinner shown has a radius of 2.5 inches. Copy and complete the table below.



Color	Probability	Sector Area	Total Area	$\frac{\text{Sector Area}}{\text{Total Area}}$
red				
orange				
yellow				
green				
blue				

b. **Verbal** Make a conjecture about the relationship between the ratio of the area of the sector to the total area and the probability of the spinner landing on each color.

c. **Analytical** Consider the dartboard shown. Predict the probability of a dart landing in each area of the board. Assume that any dart thrown will land on the board and is equally likely to land at any point on the board.

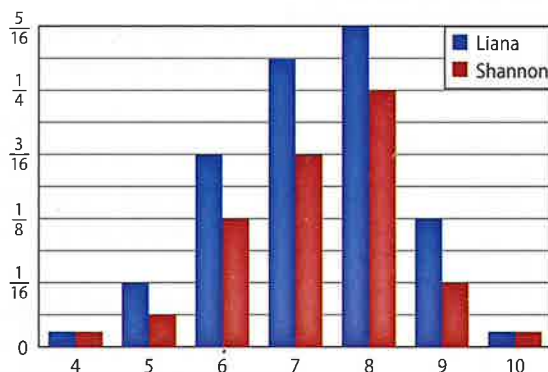


d. **Tabular** Construct a relative-frequency table for throwing 100 darts.

e. **Graphical** Graph the experimental probability distribution.

H.O.T. Problems Use Higher-Order Thinking Skills

19. **CCSS CRITIQUE** Liana and Shannon each created a probability distribution for the sum of two spins on the spinner at the right. Is either of them correct? Explain your reasoning.



20. **REASONING** Determine whether the following statement is *true* or *false*. Explain.

If you roll a die 10 times, you will roll the expected value at least twice.

21. **OPEN ENDED** Create a discrete probability distribution that shows five different outcomes and their associated probabilities.

22. **REASONING** Determine whether the following statement is *true* or *false*. Explain.
Random variables that can take on an infinite number of values are continuous.

23. **OPEN ENDED** Provide examples of a discrete probability distribution and a continuous probability distribution. Describe the differences between them.

24. **WRITING IN MATH** Compare and contrast two investments that have identical expected values and significantly different standard deviations.



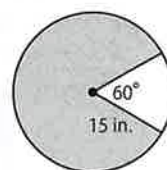
Standardized Test Practice

25. GRIDDED RESPONSE The height $f(x)$ of a bouncing ball after x bounces is represented by $f(x) = 140(0.8)^x$. How many times higher is the first bounce than the fifth bounce?

26. PROBABILITY Andres has a bag that contains 4 red, 6 yellow, 2 blue, and 4 green marbles. If he reaches into the bag and removes a marble without looking, what is the probability that it will not be yellow?

- A $\frac{1}{8}$ C $\frac{3}{8}$
 B $\frac{1}{4}$ D $\frac{5}{8}$

27. GEOMETRY Find the area of the shaded portion of the figure to the nearest square inch.



- F 79 H 589
 G 94 J 707

28. SAT/ACT If x and y are positive integers, which of the following expressions is equivalent to $\frac{(5^x)^y}{5^x}$?

- A 1^y D 5^{xy-1}
 B ± 1 E 5^{xy-x}
 C 5^y

Spiral Review

- 29. ARTICLES** Peter and Paul each write articles for an online magazine. Their employer keeps track of the number of *likes* received by each article. (Lesson 11-2)
- Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
 - Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.

Peter's Articles
16, 22, 19, 31, 24, 8, 40, 19, 33, 18, 36, 21, 55, 3, 16, 44, 22, 39, 12, 18, 13, 20, 67, 31, 13, 38, 31, 22, 26, 28

Paul's Articles
41, 38, 29, 33, 36, 55, 51, 19, 49, 56, 28, 52, 49, 19, 38, 33, 42, 61, 72, 55, 48, 39, 37, 43, 48, 45, 52, 43, 34, 29

Determine whether the situation calls for a *survey*, an *observational study*, or an *experiment*. Explain your reasoning. (Lesson 11-1)

- 30.** You want to test a drug that reverses male pattern baldness.
31. You want to find voters' opinions on recent legislation.

Find the first five terms of each geometric sequence described. (Lesson 10-3)

- 32.** $a_1 = 0.125, r = 1.5$ **33.** $a_1 = 0.5, r = 2.5$ **34.** $a_1 = 4, r = 0.5$
35. $a_1 = 12, r = \frac{1}{3}$ **36.** $a_1 = 21, r = \frac{2}{3}$ **37.** $a_1 = 80, r = \frac{5}{4}$

38. COMMUNICATION A microphone is placed at the focus of a parabolic reflector to collect sound for the television broadcast of a football game. Write an equation for the cross section, assuming that the focus is at the origin, the focus is 6 inches from the vertex, and the parabola opens to the right. (Lesson 9-2)

Solve each equation. Check your solutions. (Lesson 7-4)

- 39.** $\log_9 x = \frac{3}{2}$ **40.** $\log_{\frac{1}{10}} x = -3$ **41.** $\log_b 9 = 2$

Skills Review

Expand each power.

- 42.** $(a - b)^3$ **43.** $(m + n)^4$ **44.** $(r + n)^8$



Mid-Chapter Quiz

Lessons 11-1 through 11-3

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected. (Lesson 11-1)

- A high school principal wants to test five ideas for a new school mascot. He randomly selects 15 high school students to view pictures of the ideas while he watches and records their reactions.
- Half of the employees of a grocery store are randomly chosen for an extra hour lunch break. The managers then compare their attitudes with their co-workers.
- Students want to create a school yearbook. They send out a questionnaire to 100 students asking what they would like to showcase in the yearbook.
- The producers of a sitcom want to determine if a new character that they are planning to introduce will be well received. They show a clip of the show with the new character to 50 randomly chosen participants and then record the participants' reactions.
- MULTIPLE CHOICE** Which survey question is *unbiased*? (Lesson 11-1)
 - Do you like days like today?
 - Which is your favorite theme park, Park A or Park B?
 - Don't you think that carrots taste better than celery?
 - How often do you go to the movies?

- PARENTS** The table below shows the ages of parents who volunteered to assist in a neighborhood bake sale. (Lesson 11-2)

Ages of Parents (years)				
28	34	33	45	31
33	41	34	36	42
30	29	32	40	36
29	33	29	28	44
47	31	28	27	29

- Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
- Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

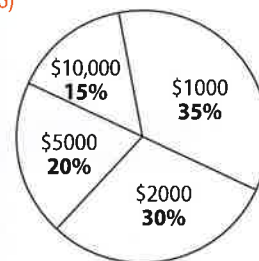
- TRAINING** Aiden and Mark's training times for the 40-meter dash are shown. (Lesson 11-2)

Aiden's 40-Meter Dash Times (seconds)					
4.84	4.94	4.87	4.78	5.04	4.98
4.83	5.03	4.74	5.15	4.82	4.91
4.62	4.83	4.76	4.93	4.85	4.82
4.76	4.98	4.94	5.05	4.94	5.04
4.86	4.85	4.71	4.66	4.91	4.82

Mark's 40-Meter Dash Times (seconds)					
5.03	4.76	4.69	4.52	4.81	4.78
4.65	4.66	4.83	4.95	4.64	4.76
4.43	4.64	4.50	4.58	4.68	4.65
4.83	4.78	4.71	4.81	4.76	4.84
4.61	4.63	4.33	4.46	4.74	4.63

- Use a graphing calculator to create a histogram for each data set. Then describe the shape of each distribution.
 - Compare the distributions using either the means and standard deviations or the five-number summaries. Justify your choice.
- MULTIPLE CHOICE** Find the expected value of winning one of the following prizes. (Lesson 11-3)

- \$1950
- \$2100
- \$3000
- \$3450



Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning. (Lesson 11-3)

- the number of calls received by an operator
- the number of books sold at a yard sale
- the height of students in a gym class
- the weight of animals on a farm

Then

- You used the Binomial Theorem.

Now

- 1 Identify and conduct a binomial experiment.
- 2 Find probabilities using binomial distributions.

Why?

- Jessica forgot to study for her civics quiz. The quiz consists of five multiple-choice questions with each question having four answer choices. Jessica randomly circles an answer for each question. In order to pass, she needs to answer at least four questions correctly.



New Vocabulary

binomial experiment
binomial distribution



Common Core State Standards

Content Standards

S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Mathematical Practices

4 Model with mathematics.

1 Binomial Experiments Each question on a multiple-choice quiz, like the one described above, can be thought of as a trial with two possible outcomes, correct or incorrect. If Jessica guesses on each question, the probability that she answers a question correctly is the same for all five questions.

Jessica's guessing on each question is an example of a binomial experiment. A **binomial experiment** is a probability experiment that satisfies the following conditions.

Key Concept Binomial Experiments

- There is a fixed number of independent trials n .
- Each trial has only two possible outcomes, success or failure.
- The probability of success p is the same in every trial. The probability of failure q is $1 - p$.
- The random variable X is the number of successes in n trials.

Many probability experiments are or can be reduced to binomial experiments.



Example 1 Identify a Binomial Experiment

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n , p , and q .

- a. The spinner at the right is spun 20 times to see how many times it lands on red.

This experiment can be reduced to a binomial experiment with success being that the spinner lands on red and failure being any other outcome. Thus, a trial is a spin, and the random variable X represents the number of reds spun. The number of trials n is 20, the probability of success p is $\frac{1}{4}$ or 0.25, and the probability of failure q is $1 - 0.25$ or 0.75.



- b. One hundred students are randomly asked their favorite food.

This is not a binomial experiment because there are many possible outcomes.

Guided Practice

- Seventy-five students are randomly asked if they own a car.
- Four cards are removed from a deck to see how many aces are selected.



Use the following guidelines when conducting a binomial experiment.

KeyConcept Conducting Binomial Experiments

- Step 1** Describe a trial for the situation, and determine the number of trials to be conducted.
- Step 2** Define a success, and calculate the theoretical probabilities of success and failure.
- Step 3** Describe the random variable X .
- Step 4** Design and conduct a simulation to determine the experimental probability.

A binomial experiment can be conducted to compare experimental and theoretical probabilities.



Example 2 Design a Binomial Experiment

Conduct a binomial experiment to determine the probability of drawing an odd-numbered card from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

- Step 1** A trial is drawing a card from a deck. The number of trials conducted can be any number greater than 0. We will use 52.
- Step 2** A success is drawing an odd-numbered card. The odd-numbered cards in a deck are 3, 5, 7, and 9, and they occur once in each of the four suits. Therefore, there are $4 \cdot 4$ or 16 odd-numbered cards in the deck. The probability of drawing an odd-numbered card, or the probability of success, is $\frac{16}{52}$ or $\frac{4}{13}$. The probability of failure is $1 - \frac{4}{13}$ or $\frac{9}{13}$.
- Step 3** The random variable X represents the number of odd-numbered cards drawn in 52 trials.
- Step 4** Use the random number generator on a calculator to create a simulation. Assign the integers 0–12 to accurately represent the probability data.

Odd-numbered cards 0, 1, 2, 3
 Other cards 4, 5, 6, 7, 8, 9, 10, 11, 12

Make a frequency table and record the results as you run the generator.

Outcome	Tally	Frequency
Odd-Numbered Card		12
Other Cards		40

An odd-numbered card was drawn 12 times, so the experimental probability is $\frac{12}{52}$ or about 23.1%. This is less than the theoretical probability of $\frac{16}{52}$ or about 30.8%.

GuidedPractice

2. Conduct a binomial experiment to determine the probability of drawing an even-numbered card from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

StudyTip

Random Number Generator

To generate all 52 random numbers using a graphing calculator, press **MATH** \leftarrow 5 and then enter the desired range followed by the number of trials. For example, type (0, 12, 52) for Example 2.



2 Binomial Distribution In the binomial experiment in Example 2, there were 12 successes in 52 trials. If you conducted that same experiment again, there may be any number of successes from 0 to 52. This situation can be represented by a binomial distribution. A **binomial distribution** is a frequency distribution of the probability of each value of X , where the random variable X represents the number of successes in n trials. Because X is a discrete random variable, a binomial distribution is a *discrete probability distribution*.

The probabilities in a binomial distribution can be calculated using the following formula.

StudyTip

Binomial Probability Formula

In the Binomial Probability Formula, X represents the number of successes in n trials. Thus, the exponent for q , $n - X$, represents the number of failures in n trials.

KeyConcept Binomial Probability Formula

The probability of X successes in n independent trials is

$$P(X) = {}_n C_X p^X q^{n-X},$$

where p is the probability of success of an individual trial and q is the probability of failure on that same individual trial ($q = 1 - p$).

Notice that the Binomial Probability Formula is an adaptation of the Binomial Theorem you have already studied. The expression ${}_n C_X p^X q^{n-X}$ represents the $p^X q^{n-X}$ term in the binomial expansion of $(p + q)^n$.

Test-TakingTip

CCSS Precision A common error when using the Binomial Probability Formula is to focus on only the successes and to forget the failures. Notice that $P(6) \neq (0.4)^6$ and $P(6) \neq {}_{10} C_6 (0.4)^6$.

Standardized Test Example 3 Find a Probability



Garrett is selling items from a catalog to raise money for school. He has a 40% chance of making a sale each time he solicits a potential customer. Garrett asks 10 people to purchase an item. Find the probability that 6 people make a purchase.

- A 8.6% B 11.2% C 24% D 40%

Read the Test Item

We need to find the probability that 6 people purchase an item. A success is making a sale, so $p = 0.4$, $q = 1 - 0.4$ or 0.6 , and $n = 10$.

Solve the Test Item

$$P(X) = {}_n C_X p^X q^{n-X}$$

Binomial Probability Formula

$$P(6) = {}_{10} C_6 (0.4)^6 (0.6)^{10-6}$$

$n = 10$, $X = 6$, $p = 0.4$, and $q = 0.6$

$$\approx 0.111$$

Simplify.

The probability of Garrett making six sales is about 0.111 or 11.1%. So, the correct answer is B.

GuidedPractice

3. TELEMARKETING At Jenny's telemarketing job, 15% of the calls that she makes to potential customers result in a sale. She makes 20 calls in a given hour. What is the probability that 5 calls result in a sale?

- F 6.7% G 8.3% H 10.3% J 11.9%

If, on average, 40% of the people Garrett solicits make a purchase and he solicits 10 people, he can probably expect to make $10(0.40)$ or 4 sales. This value represents the mean of the binomial distribution. In general, the mean of a binomial distribution can be calculated by the following formula.

KeyConcept Mean of a Binomial Distribution

The mean μ of a binomial distribution is given by $\mu = np$, where n is the number of trials and p is the probability of success.



You can find the probability distribution for a binomial experiment by fully expanding the binomial $(p + q)^n$. A probability distribution can be helpful when solving for problems that allow multiple numbers of successes.



Real-World Example 4 Full Probability Distribution

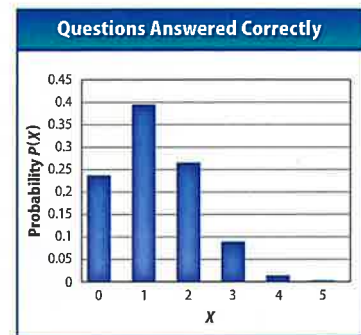
TEST TAKING Refer to the beginning of the lesson.

- a. Determine the probabilities associated with the number of questions Jessica answered correctly by calculating the probability distribution.

If there are four answer choices for each question, then the probability that Jessica guesses and answers a question correctly is $\frac{1}{4}$ or 0.25. In this binomial experiment, $n = 5$, $p = 0.25$, and $q = 1 - 0.25$ or 0.75. Expand the binomial $(p + q)^n$.

$$\begin{aligned}
 (p + q)^n &= 1p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + 1q^5 \\
 &= (0.25)^5 + 5(0.25)^4(0.75) + 10(0.25)^3(0.75)^2 + 10(0.25)^2(0.75)^3 + 5(0.25)(0.75)^4 + (0.75)^5 \\
 &\approx 0.001 + 0.015 + 0.089 + 0.264 + 0.396 + 0.237 \\
 &\quad \begin{array}{cccccc}
 0.1\% & 1.5\% & 8.9\% & 26.4\% & 39.6\% & 23.7\% \\
 5 \text{ correct} & 4 \text{ correct} & 3 \text{ correct} & 2 \text{ correct} & 1 \text{ correct} & 0 \text{ correct}
 \end{array}
 \end{aligned}$$

The graph shows the binomial probability distribution for the number of questions that Jessica answered correctly.



- b. What is the probability that Jessica passes the quiz?

Jessica must answer at least four questions correctly to pass the quiz. The probability that Jessica answers *at least* four correct is the sum of the probabilities that she answers four or five correct and is about $1.5\% + 0.1\%$ or 1.6% . So, Jessica has about a 1.6% chance of passing, which is not likely.

- c. How many questions should Jessica expect to answer correctly?

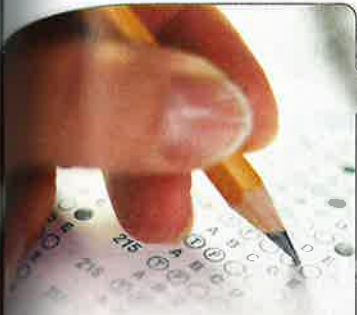
Find the mean.

$$\begin{aligned}
 \mu &= np && \text{Mean of a Binomial Distribution} \\
 &= 5(0.25) \text{ or } 1.25 && n = 5 \text{ and } p = 0.25
 \end{aligned}$$

The mean of the distribution is 1.25. On average, Jessica should expect to answer one question correctly when she guesses on five.

GuidedPractice

4. **TEST TAKING** Suppose Jessica's civics quiz consisted of five true-or-false questions instead of multiple-choice questions.
- Determine the probabilities associated with the number of answers Jessica answered correctly by calculating the probability distribution.
 - What is the probability that Jessica passes the quiz?
 - How many questions should Jessica expect to answer correctly?



Real-WorldLink

ACT The math portion of the ACT college entrance exam includes 60 multiple-choice questions that each have five answer choices.

Source: ACT

StudyTip

Mean and Expected Value

The mean of a binomial distribution can be any positive rational number. The expected value of a binomial distribution, however, should be rounded to the nearest whole number since a fraction of a success is not possible.





Example 1 Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n , p , and q .

1. A study finds that 58% of people have pets. You ask 100 people how many pets they have.
2. You roll a die 15 times and find the sum of all of the rolls.
3. A poll found that 72% of students plan on going to the homecoming dance. You ask 30 students if they are going to the homecoming dance.

Example 2 4. Conduct a binomial experiment to determine the probability of drawing an ace or a king from a deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Example 3 5 **GAMES** Aiden has earned five spins of the wheel on the right. He will receive a prize each time the spinner lands on WIN. What is the probability that he receives three prizes?

- A 4.2% C 7.1%
 B 5.8% D 8.8%



Example 4 6. **CCSS PRECISION** A poll at Steve's high school was taken to see if students are in favor of spending class money to expand the junior-senior parking lot. Steve surveyed 6 random students from the population.

Expand the Parking Lot	
favor	85%
oppose	15%

- a. Determine the probabilities associated with the number of students that Steve asked who are in favor of expanding the parking lot by calculating the probability distribution.
- b. What is the probability that no more than 2 people are in favor of expanding the parking lot?
- c. How many students should Steve expect to find who are in favor of expanding the parking lot?

Practice and Problem Solving

Extra Practice is on page R11.

Example 1 Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n , p , and q .

7. There is a 35% chance that it rains each day in a given month. You record the number of days that it rains for that month.
8. A survey found that on a scale of 1 to 10, a movie received a 7.8 rating. A movie theater employee asks 200 patrons to rate the movie on a scale of 1 to 10.
9. A ball is hidden under one of the hats shown below. A hat is chosen, one at a time, until the ball is found.



Example 2 10. **DICE** Conduct a binomial experiment to determine the probability of rolling a 7 with two dice. Then compare the experimental and theoretical probabilities of the experiment.

11. **MARBLES** Conduct a binomial experiment to determine the probability of pulling a red marble from the bag. Then compare the experimental and theoretical probabilities of the experiment.



12. **SPINNER** Conduct a binomial experiment to determine the probability of the spinner stopping on an even number. Then compare the experimental and theoretical probabilities of the experiment.



13. **CARDS** Conduct a binomial experiment to determine the probability of drawing a face card out of a standard deck of cards. Then compare the experimental and theoretical probabilities of the experiment.

Example 3

14. **PERSONAL MEDIA PLAYERS** According to a recent survey, 85% of high school students own a personal media player. What is the probability that 6 out of 10 random high school students own a personal media player?
15. **CARS** According to a recent survey, 92% of high school seniors drive their own car. What is the probability that 10 out of 12 random high school students drive their own car?
16. **SENIOR PROM** According to a recent survey, 25% of high school upperclassmen think that the junior-senior prom is the most important event of the school year. What is the probability that 3 out of 15 random high school upperclassmen think this way?
17. **FOOTBALL** A certain football team has won 75.7% of their games. Find the probability that they win 7 of their next 12 games.
18. **GARDENING** Peter is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Peter knows that the probability of having a blue flower is 75%. What is the probability that 20 of the flowers will be blue?

19. **FOOTBALL** A field goal kicker is accurate 75% of the time from within 35 yards. What is the probability that he makes exactly 7 of his next 10 kicks from within 35 yards?

Range (yd)	Accuracy (%)
0–35	75
35–45	62
45+	20

20. **BABIES** Mr. and Mrs. Davis are planning to have 3 children. The probability of each child being a boy is 50%. What is the probability that they will have 2 boys?

Example 4

21. **CCSS SENSE-MAKING** According to a recent survey, 52% of high school students own a laptop. Ten random students are chosen.
- Determine the probabilities associated with the number of students that own a laptop by calculating the probability distribution.
 - What is the probability that at least 8 of the 10 students own a laptop?
 - How many students should you expect to own a laptop?

22. **ATHLETICS** A survey was taken to see the percent of students that participate in sports for their school. Six random students are chosen.

Student Athletics	
0 sports	20%
1 sport	55%
2 sports	20%
3+ sports	5%

- Determine the probabilities associated with the number of students playing in at least one sport by calculating the probability distribution.
- What is the probability that no more than 2 of the students participated in a sport?
- How many students should you expect to have participated in at least one sport?



23. **CCSS MODELING** An online poll showed that 57% of adults still own vinyl records. Moe surveyed 8 random adults from the population.
- Determine the probabilities associated with the number of adults that still own vinyl records by calculating the probability distribution.
 - What is the probability that no less than 6 of the people surveyed still own vinyl records?
 - How many people should Moe expect to still own vinyl records?

A binomial distribution has a 60% rate of success. There are 18 trials.

- What is the probability that there will be at least 12 successes?
- What is the probability that there will be 12 failures?
- What is the expected number of successes?
27. **DECISION MAKING** Six roommates randomly select someone to wash the dishes each day.
 - What is the probability that the same person has to wash the dishes 3 times in a given week?
 - What method can the roommates use to select who washes the dishes each day?
28. **DECISION MAKING** A committee of five people randomly selects someone to take the notes of each meeting.
 - What is the probability that a person takes notes less than twice in 10 meetings?
 - What method can the committee use to select the notetaker each meeting?
 - If the method described in part b results in the same person being notetaker for nine straight meetings, would this result cause you to question the method?

Each binomial distribution has n trials and p probability of success. Determine the most likely number of successes.

29. $n = 8, p = 0.6$ 30. $n = 10, p = 0.4$ 31. $n = 6, p = 0.8$
 32. $n = 12, p = 0.55$ 33. $n = 9, p = 0.75$ 34. $n = 11, p = 0.35$

35. **SWEEPSTAKES** A beverage company is having a sweepstakes. The probabilities of winning selected prizes are shown at the right. If Ernesto purchases 8 beverages, what is the probability that he wins at least one prize?

Odds of Winning	
beverage	1 in 10
CD	1 in 200
hat	1 in 250
MP3 player	1 in 20,000
car	1 in 25,000,000

Each binomial distribution has n trials and p probability of success. Determine the probability of s successes.

36. $n = 8, p = 0.3, s \geq 2$ 37. $n = 10, p = 0.2, s > 2$ 38. $n = 6, p = 0.6, s \leq 4$
 39. $n = 9, p = 0.25, s \leq 5$ 40. $n = 10, p = 0.75, s \geq 8$ 41. $n = 12, p = 0.1, s < 3$

H.O.T. Problems Use Higher-Order Thinking Skills

42. **CHALLENGE** A poll of students determined that 88% wanted to go to college. Eight random students are chosen. The probability that at least x students want to go to college is about 0.752 or 75.2%. Solve for x .
43. **WRITING IN MATH** What should you consider when using a binomial distribution to make a decision?
44. **OPEN ENDED** Describe a real-world setting within your school or community activities that seems to fit a binomial distribution. Identify the key components of your setting that connect to binomial distributions.
45. **WRITING IN MATH** Describe how binomial distributions are connected to Pascal's triangle.
46. **WRITING IN MATH** Explain the relationship between a binomial experiment and a binomial distribution.



Standardized Test Practice

47. EXTENDED RESPONSE Carly is taking a 10 question multiple-choice test, in which each question has four choices. If she guesses on each question, what is the probability that she will get

- a. 7 questions correct?
- b. 9 questions correct?
- c. 0 questions correct?
- d. 3 questions correct?

48. What is the maximum point of the graph of the equation $y = -2x^2 + 16x + 5$?

- A $(-4, -59)$
- B $(-4, -91)$
- C $(4, 37)$
- D $(4, 101)$

49. GEOMETRY On a number line, point X has coordinate -8 and point Y has coordinate 4 . Point P is $\frac{2}{3}$ of the way from X to Y . What is the coordinate of P ?

- F -4
- G -2
- H 0
- J 2

50. SAT/ACT The cost of 4 CDs is d dollars. At this rate, what is the cost, in dollars, of 36 CDs?

- A $9d$
- B $144d$
- C $\frac{9d}{4}$
- D $\frac{d}{36}$
- E $\frac{36}{d}$

Spiral Review

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning. (Lesson 11-3)

- 51. the number of customers at an amusement park
- 52. the running time of a movie
- 53. the number of hot dogs sold at a sporting event
- 54. the distance between two cities

55. FINANCIAL LITERACY The prices of entrees offered by a restaurant are shown. (Lesson 11-2)

Prices (dollars)			
11.25	14.75	9.00	17.25
19.75	9.75	20.25	15.50
16.50	21.50	10.25	22.75
12.75	18.50	23.00	13.50

- a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
- b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

Find the missing value for each arithmetic sequence. (Lesson 10-1)

- 56. $a_5 = 12, a_{16} = 133, d = ?$
- 57. $a_9 = -34, a_{22} = 44, d = ?$
- 58. $a_4 = 18, a_n = 95, d = 7, n = ?$
- 59. $a_8 = ?, a_{19} = 31, d = 8$
- 60. $a_6 = ?, a_{20} = 64, d = 7$
- 61. $a_7 = -28, a_n = 76, d = 8, n = ?$

62. ASTRONOMY The table at the right shows the closest and farthest distances of Venus and Jupiter from the Sun in millions of miles. (Lesson 9-4)

Planet	Closest	Farthest
Venus	66.8	67.7
Jupiter	460.1	507.4

- a. Write an equation for the orbit of each planet. Assume that the center of the orbit is the origin and the center of the Sun is a focus that lies on the x -axis.
- b. Which planet has an orbit that is closer to a circle?

Write an equivalent exponential or logarithmic function. (Lesson 7-7)

- 63. $e^{-x} = 5$
- 64. $e^2 = 6x$
- 65. $\ln e = 1$
- 66. $\ln 5.2 = x$
- 67. $e^{x+1} = 9$
- 68. $e^{-1} = x^2$
- 69. $\ln \frac{7}{3} = 2x$
- 70. $\ln e^x = 3$

Skills Review

71. MUSIC Tina owns 11 pop, 6 country, 16 rock, and 7 rap CDs. Find each probability if she randomly selects 4 CDs.

- a. $P(2 \text{ rock})$
- b. $P(1 \text{ rap})$
- c. $P(1 \text{ rock and } 2 \text{ country})$



11-5 The Normal Distribution

Then

- You constructed and analyzed discrete probability distributions.

Now

- Use the Empirical Rule to analyze normally distributed variables.
- Apply the standard normal distribution and z-values.

Why?

- Extensive observations of Swiss cherry trees found that the mean flowering date is April 21 with a standard deviation of about 10 days. Therefore, 95% of the time, a Swiss cherry tree will have a flowering date between April 1 and May 3.



New Vocabulary

- normal distribution
- Empirical Rule
- z-value
- standard normal distribution



Common Core State Standards

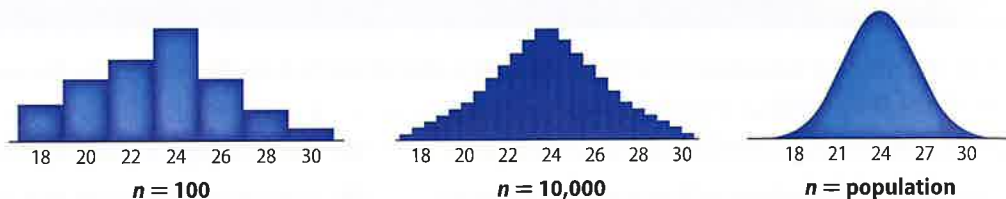
Content Standards

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Mathematical Practices

- Attend to precision.
- Look for and express regularity in repeated reasoning.

1 The Normal Distribution Distributions of mileages of different sample sizes of cars are shown below. As the sample size increases, the distributions become more and more symmetrical and resemble the curve at the right, due to the Law of Large Numbers.



The curve at the right is a **normal distribution**, a continuous, symmetric, bell-shaped distribution of a random variable. It is the most common *continuous probability distribution*. The characteristics of the normal distribution are as follows.

KeyConcept The Normal Distribution

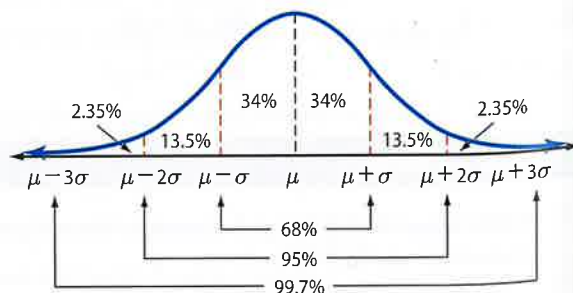
- The graph of the curve is continuous, bell-shaped, and symmetric with respect to the mean.
- The mean, median, and mode are equal and located at the center.
- The curve approaches, but never touches, the x-axis.
- The total area under the curve is equal to 1 or 100%.

The area under the normal curve represents the amount of data within a certain interval or the probability that a random data value falls within that interval. The **Empirical Rule** can be used to determine the area under the normal curve at specific intervals.

KeyConcept The Empirical Rule

In a normal distribution with mean μ and standard deviation σ ,

- approximately 68% of the data fall within 1σ of the mean,
- approximately 95% of the data fall within 2σ of the mean, and
- approximately 99.7% of the data fall within 3σ of the mean.



StudyTip

Normal Distributions In all of these cases, the number of data values must be large for the distribution to be approximately normal.

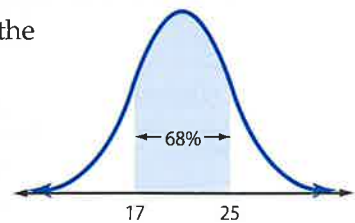
Example 1 Use the Empirical Rule to Analyze Data



A normal distribution has a mean of 21 and a standard deviation of 4.

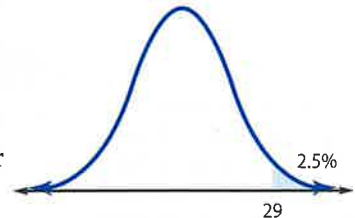
- a. Find the range of values that represent the middle 68% of the distribution.

The middle 68% of data in a normal distribution is the range from $\mu - \sigma$ to $\mu + \sigma$. Therefore, the range of values in the middle 68% is $17 < X < 25$.



- b. What percent of the data will be greater than 29?

29 is 2σ more than μ . 95% of the data fall between $\mu - 2\sigma$ and $\mu + 2\sigma$, so the remaining data values represented by the two tails covers 5% of the distribution. We are only concerned with the upper tail, so 2.5% of the data will be greater than 29.



GuidedPractice

- A normal distribution has a mean of 8.2 and a standard deviation of 1.3.
 - Find the range of values that represent the middle 95% of the distribution.
 - What percent of the data will be less than 4.3?

Real-World Example 2 Use the Empirical Rule to Analyze a Distribution

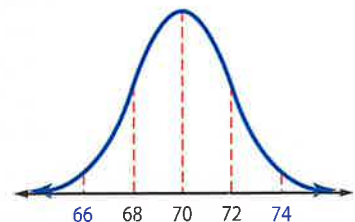


HEIGHTS The heights of 1800 adults are normally distributed with a mean of 70 inches and a standard deviation of 2 inches.

- a. About how many adults are between 66 and 74 inches?

66 and 74 are 2σ away from the mean. Therefore, about 95% of the data are between 66 and 74.

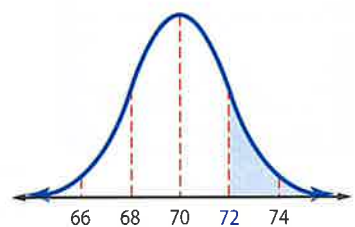
Since $1800 \times 95\% = 1710$, we know that about 1710 of the adults are between 66 and 74 inches tall.



- b. What is the probability that a random adult is more than 72 inches tall?

From the curve, values greater than 72 are more than 1σ from the mean. 13.5% are between 1σ and 2σ , 2.35% are between 2σ and 3σ , and 0.15% are greater than 3σ .

So, the probability that an adult selected at random has a height greater than 72 inches is $13.5 + 2.35 + 0.15$ or 16%.



GuidedPractice

- NETWORKING SITES** The number of friends per member in a sample of 820 members is normally distributed with a mean of 38 and a standard deviation of 12.
 - About how many members have between 26 and 50 friends?
 - What is the probability that a random member will have more than 14 friends?



Real-WorldLink

While the average adult American male is 5 feet 10 inches, the average height of adult males in the Netherlands is the highest worldwide, at almost 6 feet 1 inch.

Source: Eurostats Statistical Yearbook



2 Standard Normal Distribution The Empirical Rule is only useful for evaluating specific values, such as $\mu + \sigma$. Once the data set is *standardized*, however, any data value can be evaluated. Data are standardized by converting them to z-values, also known as z-scores. The **z-value** represents the number of standard deviations that a given data value is from the mean. Therefore, z-values can be used to determine the position of any data value within a set of data.

KeyConcept Formula for z-Values

The z-value for a data value X in a set of normally distributed data is given by $z = \frac{X - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation.

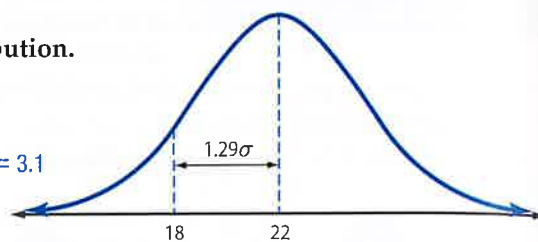
StudyTip

Symmetry The normal distribution is symmetrical, so when you are asked for the middle or outside set of data, the z-values will be opposites.

Example 3 Use z-Values to Locate Position

Find z if $X = 18$, $\mu = 22$, and $\sigma = 3.1$.
Indicate the position of X in the distribution.

$$\begin{aligned} z &= \frac{X - \mu}{\sigma} && \text{Formula for z-values} \\ &= \frac{18 - 22}{3.1} && X = 18, \mu = 22, \sigma = 3.1 \\ &\approx -1.29 && \text{Simplify.} \end{aligned}$$



The z-value that corresponds to $X = 18$ is approximately -1.29 . Therefore, 18 is about 1.29 standard deviations less than the mean of the distribution.

GuidedPractice

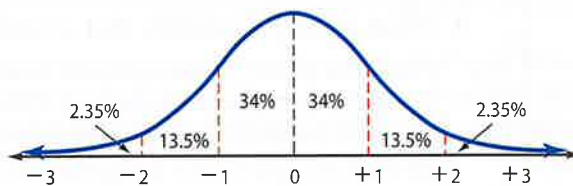
3. Find X if $\mu = 39$, $\sigma = 8.2$, and $z = 0.73$. Indicate the position of X in the distribution.

Any combination of mean and standard deviation is possible for a normally distributed set of data. As a result, there are infinitely many normal probability distributions. This makes comparing two individual distributions difficult. Different distributions *can* be compared, however, once they are standardized using z-values. The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

StudyTip

Standard Normal Distribution The standard normal distribution is the set of all z-values.

KeyConcept Characteristics of the Standard Normal Distribution



- The total area under the curve is equal to 1 or 100%.
- Almost all of the area is between $z = -3$ and $z = 3$.
- The distribution is symmetric.
- The mean is 0, and the standard deviation is 1.

The standard normal distribution allows us to assign actual areas to the intervals created by z-values. The area under the normal curve corresponds to the proportion of data values in an interval as well as the probability of a random data value falling within the interval. For example, the area between $z = 0$ and $z = 1$ is 0.34. Therefore, the probability of a z-value being in this interval is 34%.



Real-World Example 4 Find Probabilities

VIDEOS The number of videos uploaded daily to a video sharing site is normally distributed with $\mu = 181,099$ videos and $\sigma = 35,644$ videos. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

a. $P(180,000 < X < 200,000)$

The question is asking for the percent of days when between 180,000 and 200,000 videos are uploaded. First, find the corresponding z-values for $X = 180,000$ and $X = 200,000$.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for z-values}$$

$$= \frac{180,000 - 181,099}{35,644} \text{ or about } -0.03 \quad X = 180,000, \mu = 181,099, \text{ and } \sigma = 35,644$$

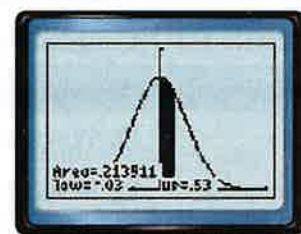
Use 200,000 to find the other z-value.

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for z-values}$$

$$= \frac{200,000 - 181,099}{35,644} \text{ or about } 0.53 \quad X = 200,000, \mu = 181,099, \text{ and } \sigma = 35,644$$

The range of z-values that corresponds to $180,000 < X < 200,000$ is $-0.03 < z < 0.53$. Find the area under the normal curve within this interval.

You can use a graphing calculator to display the area that corresponds to any z-value by selecting **2nd** **[DISTR]**. Then, under the **DRAW** menu, select **ShadeNorm(lower z value, upper z value)**. The area between $z = -0.03$ and $z = 0.53$ is about 0.21 as shown in the graph.



$[-4, 4]$ scl: 1 by $[0, 0.5]$ scl: 0.125

Therefore, about 21% of the time, there will be between 180,000 and 200,000 video uploads on a given day.

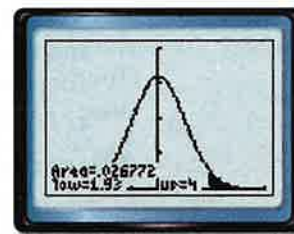
b. $P(X > 250,000)$

$$z = \frac{X - \mu}{\sigma} \quad \text{Formula for z-values}$$

$$= \frac{250,000 - 181,099}{35,644} \text{ or about } 1.93 \quad X = 250,000, \mu = 181,099, \text{ and } \sigma = 35,644$$

Using a graphing calculator, you can find the area between $z = 1.93$ and $z = 4$ to be about 0.027.

Therefore, the probability that more than 250,000 videos will be uploaded is about 2.7%.



$[-4, 4]$ scl: 1 by $[0, 0.5]$ scl: 0.125

Guided Practice

4. TIRES The life spans of a certain tread of tire are normally distributed with $\mu = 31,066$ miles and $\sigma = 1644$ miles. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.

- A. $P(30,000 < X < 32,000)$** **B. $P(X > 35,000)$**

Another method for calculating the area between two z-values is **2nd** **[DISTR]** **normalcdf(lower z value, upper z value)**.

Real-WorldLink

Video Uploading According to a recent study, 52% of people who said they upload videos to the Web do it through sites such as Facebook and MySpace. The rest use video-sharing sites like YouTube and Google Video.

Source: Pew Internet and American Life Project

StudyTip

Range of z-values The majority of data values are within $\pm 4\sigma$ of the mean, so setting the maximum value of z equal to 4 is sufficient in part **b**. Use the window $[-4, 4]$ by $[0, 0.5]$ when using **ShadeNorm**.





- Example 1** A normal distribution has a mean of 416 and a standard deviation of 55.
- Find the range of values that represent the middle 99.7% of the distribution.
 - What percent of the data will be less than 361?
- Example 2** 3. **CCSS TOOLS** The number of texts sent per day by a sample of 811 teens is normally distributed with a mean of 38 and a standard deviation of 7.
- About how many teens sent between 24 and 38 texts?
 - What is the probability that a teen selected at random sent less than 45 texts?
- Example 3** Find the missing variable. Indicate the position of X in the distribution.
- z if $\mu = 89$, $X = 81$, and $\sigma = 11.5$
 - X if $z = -1.38$, $\mu = 68.9$, and $\sigma = 6.6$
 - z if $\mu = 13.3$, $X = 17.2$, and $\sigma = 1.9$
 - σ if $\mu = 21.1$, $X = 13.7$, and $z = -2.40$
- Example 4** 8. **CONCERTS** The number of concerts attended per year by a sample of 925 teens is normally distributed with a mean of 1.8 and a standard deviation of 0.5. Find each probability. Then use a graphing calculator to sketch the area under each curve.
- $P(X < 2)$
 - $P(1 < X < 3)$

Practice and Problem Solving

Extra Practice is on page R11.

- Example 1** A normal distribution has a mean of 29.3 and a standard deviation of 6.7.
- Find the range of values that represent the outside 5% of the distribution.
 - What percent of the data will be between 22.6 and 42.7?
- Example 2** 11. **GYMS** The number of visits to a gym per year by a sample of 522 members is normally distributed with a mean of 88 and a standard deviation of 19.
- About how many members went to the gym at least 50 times?
 - What is the probability that a member selected at random went to the gym more than 145 times?
- Example 3** Find the missing variable. Indicate the position of X in the distribution.
- z if $\mu = 3.3$, $X = 3.8$, and $\sigma = 0.2$
 - μ if $z = -0.92$, $X = 44.2$, and $\sigma = 8.3$
 - z if $\mu = 19.9$, $X = 18.7$, and $\sigma = 0.9$
 - X if $\mu = 138.8$, $\sigma = 22.5$, and $z = 1.73$
- Example 4** 16. **VENDING** A vending machine dispenses about 8.2 ounces of coffee. The amount varies and is normally distributed with a standard deviation of 0.3 ounce. Find each probability. Then use a graphing calculator to sketch the corresponding area under the curve.
- $P(X < 8)$
 - $P(X > 7.5)$
17. **CAR BATTERIES** The useful life of a certain car battery is normally distributed with a mean of 113,627 miles and a standard deviation of 14,266 miles. The company makes 20,000 batteries a month.
- About how many batteries will last between 90,000 and 110,000 miles?
 - About how many batteries will last more than 125,000 miles?
 - What is the probability that if you buy a car battery at random, it will last less than 100,000 miles?
18. **FOOD** The shelf life of a particular snack chip is normally distributed with a mean of 173.3 days and a standard deviation of 23.6 days.
- About what percent of the product lasts between 150 and 200 days?
 - About what percent of the product lasts more than 225 days?
 - What range of values represents the outside 5% of the distribution?



- 19 FINANCIAL LITERACY** The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual's insurance rate. Suppose insurance rates for a sample population are normally distributed.
- If the mean annual cost per person is \$829 and the standard deviation is \$115, what is the range of rates you would expect the middle 68% of the population to pay annually?
 - If 900 people were sampled, how many would you expect to pay more than \$1000 annually?
 - Where on the distribution would you expect a person with several traffic citations to lie? Explain your reasoning.
 - How do you think auto insurance companies use each factor to calculate an individual's insurance rate?
- 20. STANDARDIZED TESTS** Nikki took three national standardized tests and scored an 86 on all three. The table shows the mean and standard deviation of each test.

	Math	Science	Social Studies
μ	76	81	72
σ	9.7	6.2	11.6

- Calculate the z-values that correspond to her score on each test.
- What is the probability of a student scoring an 86 or *lower* on each test?
- On which test was Nikki's standardized score the highest? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

- 21. CCSS CRITIQUE** A set of normally distributed tree diameters have mean 11.5 centimeters, standard deviation 2.5, and range from 3.6 to 19.8. Monica and Hiroko are to find the range that represents the middle 68% of the data. Is either of them correct? Explain.

Monica

The data span 16.2 cm. 68% of 16.2 is about 11 cm. Center this 11-cm range around the mean of 11.5 cm. This 68% group will range from about 6 cm to about 17 cm.

Hiroko

The middle 68% span from $\mu + \sigma$ to $\mu - \sigma$. So we move 2.5 cm below 11.5 and then 2.5 cm above 11.5. The 68% group will range from 9 cm to 14 cm.

- 22. CHALLENGE** A case of portable media players has an average battery life of 8.2 hours with a standard deviation of 0.7 hour. Eight of the players have a battery life greater than 9.3 hours. If the sample is normally distributed, how many portable media players are in the case?
- 23. REASONING** The term *six sigma process* comes from the notion that if one has six standard deviations between the mean of a process and the nearest specification limit, there will be practically no items that fail to meet the specifications. Is this a true assumption? Explain.
- 24. REASONING** *True or false:* According to the Empirical Rule, in a normal distribution, most of the data will fall within one standard deviation of the mean. Explain.
- 25. OPEN ENDED** Find a set of real-world data that appears to be normally distributed. Calculate the range of values that represent the middle 68%, the middle 95%, and the middle 99.7% of the distribution.
- 26. WRITING IN MATH** Describe the relationship between the z-value, the position of an interval of X in the normal distribution, the area under the normal curve, and the probability of the interval occurring. Use an example to explain your reasoning.

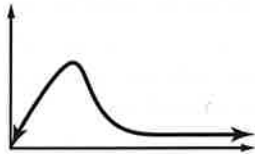


Standardized Test Practice

27. The lifetimes of 10,000 light bulbs are normally distributed. The mean lifetime is 300 days, and the standard deviation is 40 days. How many light bulbs will last between 260 and 340 days?

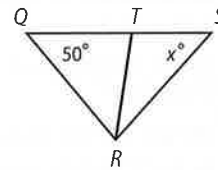
- A 2500 C 5000
B 3400 D 6800

28. Which description best represents the graph?



- F negatively skewed H normal distribution
G no correlation J positively skewed

29. **SHORT RESPONSE** In the figure below, $RT = TS$ and $QR = QT$. What is the value of x ?



30. **SAT/ACT** The integer 99 can be expressed as a sum of n consecutive positive integers. The value of n could be which of the following?

- I. 2
II. 3
III. 6

- A I only D I and II
B II only E I, II, and III
C III

Spiral Review

31. **SNOW** There is a 25% chance that it snows each day during a given week. Find the probability that it snows 3 out of the next 7 days. (Lesson 11-4)

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning. (Lesson 11-3)

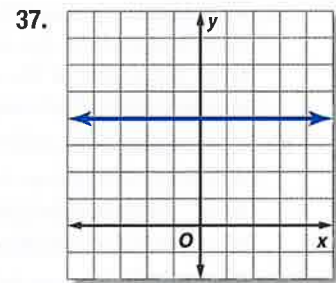
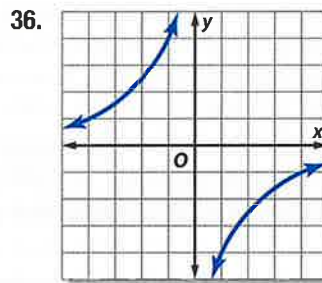
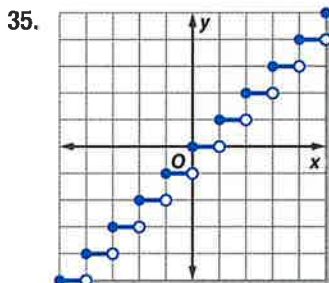
32. the number of pages in a newspaper

33. the amount of precipitation in a city per month

34. **BRIDGES** The Sydney Harbour Bridge connects the central business district to northern metropolitan Sydney. It has an arch in the shape of a parabola that opens downward. Write an equation of a parabola to model the arch, assuming that the origin is at the surface of the water, beneath the vertex of the arch. (Lesson 9-2)



Identify the type of function represented by each graph. (Lesson 2-7)



Skills Review

38. Calculate the standard deviation of the population of data.

13	18	17	21	16	9	11	28	8	10
7	19	16	16	12	19	21	11	8	13



11-5 Spreadsheet Lab

Normal Approximation of Binomial Distributions



In Lesson 11-4, you used a binomial expansion to find a full probability distribution. A spreadsheet can be used to quickly find and graph a full distribution for any number of trials.



Common Core State Standards

Content Standards

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.



Activity 1 Full Probability Distribution

PLAYING CARDS Tom randomly selects a card from a deck of 52 playing cards, records its suit, and replaces it. Use a spreadsheet to construct and graph a full probability distribution for X , the number of hearts that Tom selects if he chooses 4, 20, or 100 cards.

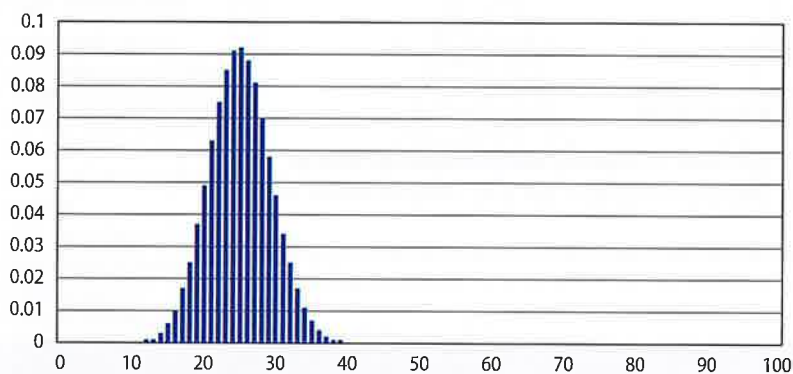
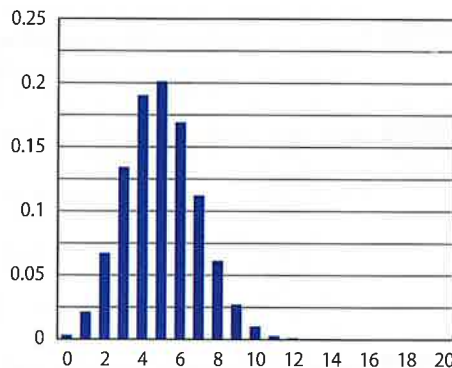
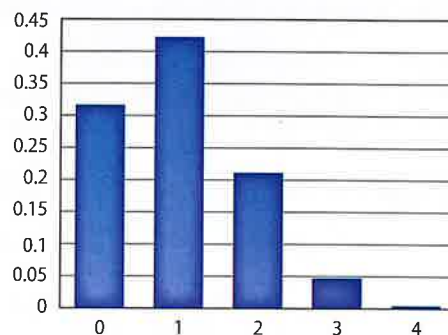
Since one fourth of the cards in a standard deck are hearts, the probability of success is 25% or 0.25, and the probability of failure is 75% or 0.75.

Step 1 Enter the numbers 0 to 4 in column A. In B1, enter the binomial probability formula as $=\text{COMBIN}(4,A1)*(0.25)^{A1}*(0.75)^{(4-A1)}$. Copy and paste this formula in cells B2:B5.

Step 2 Select cells B1:B5, and insert a clustered column bar graph. Use the values in column A for Category (X) axis labels.

Step 3 Select cells A1:A5, and autofill the through A21. In C1, enter the formula $=\text{COMBIN}(20,A1)*(0.25)^{A1}*(0.75)^{(20-A1)}$. Copy this formula in cells C2:C21. Repeat Step 2 to graph this distribution.

Step 4 Autofill column A through A101. In D1, enter the formula $=\text{COMBIN}(100,A1)*(0.25)^{A1}*(0.75)^{(100-A1)}$. Copy this formula in cells D2:C101. Repeat Step 2 to graph this distribution.



Notice that the number of trials increases, the shape of the graph of the distribution becomes more symmetric with its center at the mean, $\mu = np$. If the number of trials becomes large enough, the shape of the distribution approaches a normal curve. Therefore, a normal distribution can be used to approximate the binomial distribution.

KeyConcept Normal Approximation of a Binomial Distribution

In a binomial distribution with n trials, a probability of success p , and a probability of failure q , such that $np \geq 5$ and $nq \geq 5$, a binomial distribution can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$.

(continued on the next page)

Normal Approximation of Binomial Distributions *Continued*

Once the mean and standard deviation are calculated, a z-value can be determined, and the corresponding probability can be found as demonstrated in the next activity.

Activity 2 Normal Approximation of a Binomial Distribution

JURY DUTY According to a poll, 60% of the registered voters in a city have never been called for jury duty. Mariah conducts a random survey of 300 registered voters. What is the probability that at least 170 of those voters have never been called for jury duty?

This is a binomial experiment with $n = 300$, $p = 0.6$, and $q = 0.4$. Since $np = 300(0.6)$ or 180 and $nq = 300(0.4)$ or 120 are both greater than 5, the normal distribution can be used to approximate the binomial distribution.

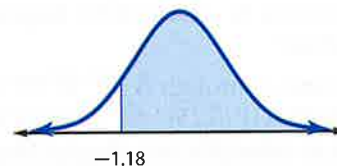
Step 1 The mean μ is np or 180. Find the standard deviation σ .

$$\begin{aligned} \sigma &= \sqrt{npq} && \text{Standard deviation of a binomial distribution} \\ &= \sqrt{300(0.6)(0.4)} && n = 300, p = 0.6, \text{ and } q = 0.4 \\ &\approx 8.49 && \text{Simplify.} \end{aligned}$$

Step 2 Write the problem in probability notation using X . The probability that at least 170 people have never been called for jury duty is $P(X \geq 170)$.

Step 3 Find the corresponding z-value for X .

$$\begin{aligned} z &= \frac{X - \mu}{\sigma} && \text{Formula for z-values} \\ &= \frac{170 - 180}{8.49} && X = 170, \mu = 180, \text{ and } \sigma = 8.49 \\ &\approx -1.18 && \text{Simplify.} \end{aligned}$$



Step 4 Use a calculator to find the area under the normal curve to the right of z .

KEYSTROKES: `2nd` `[DISTR]` `2` `(-)` `1.18` `,` `4` `)` `ENTER`

The approximate area to the right of z is 0.881. Therefore, the probability that at least 170 of the registered voters have never been called for jury duty is about 88.1%.

Exercises

- Use a spreadsheet to construct the graph of the full probability distribution for X , the number of times a 3 is rolled from rolling a die 25 times.
- SUMMER JOBS** According to an online poll, 80% of high school upperclassmen have summer jobs. Tadeo thinks the number should be lower so he conducts a survey of 480 random upperclassmen. What is the probability that no more than 380 of the surveyed upperclassmen have summer jobs?
- WORK** According to an online poll, 28% of adults feel that the standard 40-hour work week should be increased. Sheila interviews 300 adults at the mall. What is the probability that more than 80 but fewer than 100 of those surveyed will say that the work week should be increased?

11-6 Confidence Intervals and Hypothesis Testing

Then

- You applied the standard normal distribution and z-values.

Now

- Find confidence intervals for normally distributed data.
- Perform hypothesis tests on normally distributed data.

Why?

- In a recent Gallup Poll, 1514 teens who owned a portable media player had an average of 1033 songs. The poll had the following disclaimer: "For results based on the total sample of national teens, one can say with 95% confidence that the margin of sampling error is ± 31 songs."



New Vocabulary

inferential statistics
 statistical inference
 confidence interval
 maximum error of estimate
 hypothesis test
 null hypothesis
 alternative hypothesis
 critical region
 left-tailed test
 two-tailed test
 right-tailed test

Common Core State Standards

Content Standards
 S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

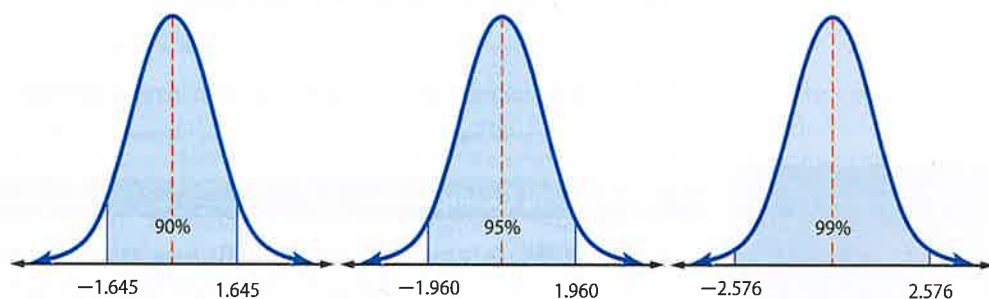
Mathematical Practices

- 7 Look for and make use of structure.

1 Confidence Intervals Inferential statistics are used to draw conclusions or statistical inferences about a population using a sample. For example, the sample mean of 1033 songs per portable media player can be used to estimate the population mean.

A confidence interval is an estimate of a population parameter stated as a range with a specific degree of certainty. Typically, statisticians use 90%, 95%, and 99% confidence intervals, but any percentage can be considered. In the opening example, we are 95% confident that the population mean is within 31 songs of 1033.

The confidence interval for a normal distribution is equivalent to the area under the standard normal curve between $-z$ and z , as shown. A 95% confidence interval for a population mean implies that we are 95% sure that the mean will fall within the range of z-values.



Suppose you want 95% confidence when conducting an experiment. The corresponding z-value is 1.960, where 2.5% of the area lies to the left of $-z$ and 2.5% lies to the right of z .

To find the confidence interval, use the maximum error of estimate and the sample mean. The maximum error of estimate E is the maximum difference between the estimate of the population mean μ and its actual value.

KeyConcept Maximum Error of Estimate

The maximum error of estimate E for a population mean is given by

$$E = z \cdot \frac{s}{\sqrt{n}}$$

where z is the z-value that corresponds to a particular confidence level, s is the standard deviation of the sample, and n is the sample size; $n \geq 30$.





Example 1 Maximum Error of Estimate

SOCIAL NETWORKING A poll of 218 randomly selected members of a social networking Web site showed that they spent an average of 14 minutes per day on the site with a standard deviation of 3.1 minutes. Use a 95% confidence interval to find the maximum error of estimate for the time spent on the site.

In a 95% confidence interval, 2.5% of the area lies in each tail. The corresponding z-value is 1.960.

$$\begin{aligned}
 E &= z \cdot \frac{s}{\sqrt{n}} && \text{Maximum Error of Estimate} \\
 &= 1.960 \cdot \frac{3.1}{\sqrt{218}} && z = 1.960, s = 3.1, \text{ and } n = 218 \\
 &\approx 0.41 && \text{Simplify.}
 \end{aligned}$$

This means that you can be 95% confident that the population mean time spent on the site will be within 0.41 minute of the sample mean of 14 minutes.

Guided Practice

- TEACHING** A poll of 184 randomly selected high school teachers showed that they spend an average of 16.8 hours per week grading, planning, and preparing for class. The standard deviation is 2.9 hours. Use a 90% confidence interval to find the maximum error of estimate for the amount of time spent per week.

Once the maximum error of estimate E is found, a confidence interval CI for the population mean can be determined by adding $\pm E$ to the sample mean.

KeyConcept Confidence Interval for the Population Mean

A confidence interval for a population mean is given by

$$CI = \bar{x} \pm E \text{ or } \bar{x} \pm z \cdot \frac{s}{\sqrt{n}}$$

where \bar{x} is the sample mean and E is the maximum error of estimate.



Real-World Example 2 Confidence Interval

SCHOOL WORK A sample of 200 students was asked the average time they spent on homework each weeknight. The mean time was 52.5 minutes with a standard deviation of 5.1 minutes. Determine a 99% confidence interval for the population mean.

$$\begin{aligned}
 CI &= \bar{x} \pm z \cdot \frac{s}{\sqrt{n}} && \text{Confidence Interval for Population Mean} \\
 &= 52.5 \pm 2.576 \cdot \frac{5.1}{\sqrt{200}} && \bar{x} = 52.5, z = 2.576, s = 5.1, \text{ and } n = 200 \\
 &\approx 52.5 \pm 0.93 && \text{Use a calculator.}
 \end{aligned}$$

The 99% confidence interval is $51.57 \leq \mu \leq 53.43$. Therefore, we are 99% confident that the population mean time is between 51.57 and 53.43 minutes.

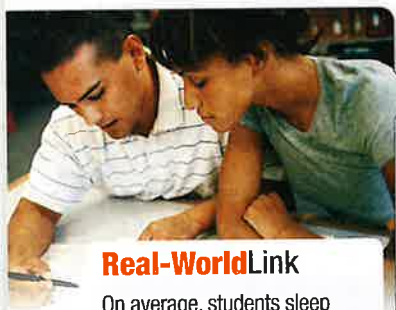
Guided Practice

- SCHOOL ATHLETICS** A sample of 224 students showed that they attend an average of 2.6 school athletic events per year with a standard deviation of 0.8. Determine a 90% confidence interval for the population mean.

StudyTip

CCSS Perseverance The z-value that corresponds to a particular confidence level is known as the *critical value*. The most commonly used levels and their corresponding z-value are shown below.

Confidence Level	z-Value
90%	1.645
95%	1.960
99%	2.576



Real-WorldLink

On average, students sleep 8.1 hours per weekday and spend 7.5 hours on educational activities, such as attending class or doing homework.

Source: Bureau of Labor Statistics



2 Hypothesis Testing While a confidence interval provides an estimate of the population mean, a **hypothesis test** is used to assess a specific claim about the mean. Typical claims are that the mean is equal to, is greater than, or is less than a specific value. There are two parts to a hypothesis test: the null hypothesis and the alternative hypothesis. The **null hypothesis** H_0 is a statement of equality to be tested. The **alternative hypothesis** H_a is a statement of inequality that is the complement of the null hypothesis. A *claim* can be part of the null or alternative hypothesis.



Example 3 Claims and Hypotheses

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

- a. A school administrator thinks it takes less than 3 minutes to evacuate the entire building for a fire drill.

less than 3 minutes not less than three minutes
 $\mu < 3$ $\mu \geq 3$

The claim is $\mu < 3$, and it is the alternative hypothesis because it does not include equality. The null hypothesis is $\mu \geq 3$, which is the complement of $\mu < 3$.

$$H_0: \mu \geq 3 \qquad H_a: \mu < 3 \text{ (claim)}$$

- b. The owner of a deli says that there are 2 ounces of ham in a sandwich.

$$H_0: \mu = 2 \text{ (claim)}; H_a: \mu \neq 2$$

The claim is $\mu = 2$, and it is the null hypothesis because it is an equality. The alternative hypothesis is $\mu \neq 2$, which is the complement of $\mu = 2$.

Guided Practice

- 3A. Jill thinks it takes longer than 10 minutes for the school bus to reach school from her stop.
 3B. Mindy thinks there aren't 35 peanuts in every package.

Reading Math

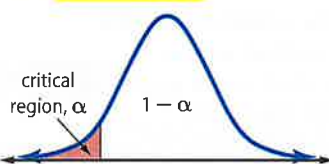
Significance Level

A significance level of 5% means that we need the data to provide evidence against H_0 so strong that the data would occur no more than 5% of the time when H_0 is true.

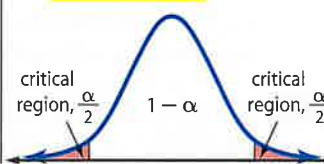
Once a claim is made, a sample is collected and analyzed, and the null hypothesis is either *rejected* or *not rejected*. From this information, the claim is then rejected or not rejected. This is determined by whether the sample mean falls within the critical region. The **critical region** is the range of values that suggests a significant enough difference to reject the null hypothesis. The critical region is determined by the significance level α , most commonly 1%, 5%, or 10%, and the inequality sign of the alternate hypothesis.

Concept Summary Tests of Significance

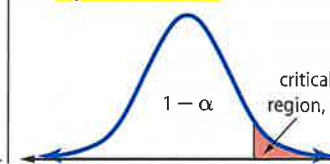
If $H_a: \mu < k$, the hypothesis test is a **left-tailed test**.



If $H_a: \mu \neq k$, the hypothesis test is a **two-tailed test**.



If $H_a: \mu > k$, the hypothesis test is a **right-tailed test**.



The value k represents the claim about the population mean μ .

The type of hypothesis test that we are using in this lesson is known as a z-test. Once the area corresponding to the significance level is determined, a z-statistic for the sample mean is calculated. The z-statistic is the z-value for the sample. When testing at 5% significance, it is only 5% likely for the z-statistic to fall within the critical region and for H_0 to be true. Therefore, when the z-statistic falls within the critical region, H_0 is rejected.

Study Tip

z-Values and z-Statistics In hypothesis testing, a z-value is used to determine the critical region. A z-statistic is calculated to see if the sample mean falls within that critical region, and thus determines whether or not to reject the null hypothesis.



Use these steps to test a hypothesis.

KeyConcept Steps for Hypothesis Testing

- Step 1** State the hypotheses and identify the claim.
- Step 2** Determine the critical value(s) and critical region.
- Step 3** Calculate the z-statistic using $z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.
- Step 4** Reject or fail to reject the null hypothesis.
- Step 5** Make a conclusion about the claim.



Real-World Career

Politician Politicians are elected officials in the federal, state, and local governments. They influence public decision making and make laws that affect everyone. Many politicians have business, teaching, or legal experience prior to getting involved in politics.

Real-World Example 4 One-Sided Hypothesis Test



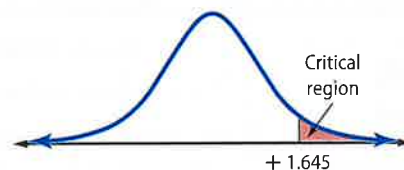
STUDENT COUNCIL Lindsey, the president of the student council, has heard complaints that the cafeteria lunch line moves too slowly. The dining services coordinator assures her that the average wait time is 6 minutes or less. Using a sample of 65 customers, Lindsey calculated a mean wait time of 6.3 minutes and a standard deviation of 1.1 minutes. Test the hypothesis at 5% significance.

- Step 1** State the claim and hypotheses. $H_0: \mu \leq 6$ (claim) and $H_a: \mu > 6$.
- Step 2** Determine the critical region.

The alternative hypothesis is $\mu > 6$, so this is a right-tailed test. We are testing at 5% significance, so we need to identify the z-value that corresponds with the upper 5% of the distribution.



KEYSTROKES: **2nd** [DISTR] 3 .95 **ENTER**



- Step 3** Calculate the z-statistic for the sample data.

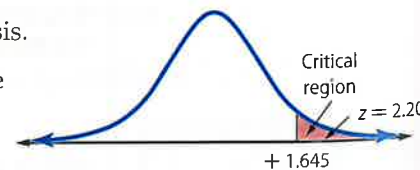
$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{Formula for z-statistic}$$

$$= \frac{6.3 - 6}{\frac{1.1}{\sqrt{65}}} \quad \bar{x} = 6.3, \mu = 6, s = 1.1, \text{ and } n = 65$$

$$\approx 2.20 \quad \text{Simplify.}$$

- Step 4** Decide whether to reject the null hypothesis.

H_0 is rejected because the z-statistic for the sample falls within the critical region.



- Step 5** Make a conclusion about the claim.

Therefore, there is enough evidence to reject the claim that the average wait time is 6 minutes or less.

Guided Practice

- 4. **EXERCISE** Julian thinks it takes less than 60 minutes for people to get in a full workout at the gym. Using a sample of 128 members, he calculated a mean workout time of 58.4 minutes with a standard deviation of 8.9 minutes. Test the hypothesis at 10% significance.

StudyTip

Rejecting the Null Hypothesis When the null hypothesis is rejected, it is implied that the alternative hypothesis is accepted.



For a two-sided test, the significance level α must be divided by 2 in order to determine the critical value at each tail.



Example 5 Two-Sided Hypothesis Test

ADVERTISEMENTS Jerrod wants to determine if the advertisement shown is accurate. Using a sample of 42 pizzas, Jerrod calculates a mean of 99.8 pieces and a standard deviation of 0.8. Test the hypothesis at 5% significance.



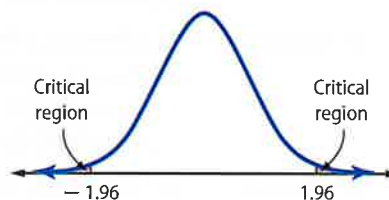
Step 1 State the claim and hypotheses.

$$H_0: \mu = 100 \text{ (claim) and } H_a: \mu \neq 100.$$

Step 2 Determine the critical region.

The alternative hypothesis is $\mu \neq 100$, so this is a two-tailed test. We are testing at 5% significance, so we need to identify the z-value that corresponds with upper and lower 2.5% of the distribution.

KEYSTROKES: **2nd** [DISTR] 3 0.25 **ENTER** and **2nd** [DISTR] 3 .975 **ENTER**



StudyTip

z-Test The z-test can be used when σ is known or when $n \geq 30$.

Step 3 Calculate the z-statistic for the sample data.

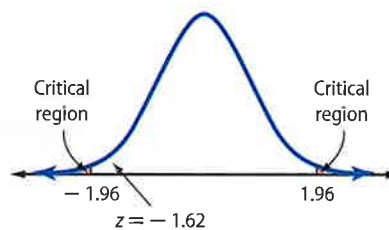
$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{Formula for z-statistic}$$

$$= \frac{99.8 - 100}{\frac{0.8}{\sqrt{42}}} \quad \bar{x} = 99.8, \mu = 100, s = 0.8, \text{ and } n = 42$$

$$\approx -1.62 \quad \text{Simplify.}$$

Step 4 Reject or fail to reject the null hypothesis.

H_0 is not rejected because the z-value for the sample does not fall within the critical region.



Step 5 Make a conclusion about the claim.

Therefore, there is not enough evidence to reject the claim that there are 100 pieces of pepperoni on every large pizza.

WatchOut!

Inferential Statistics

Inferential statistics are not designed to definitely prove a hypothesis. They depend on probability statements and therefore, may not be correct.

GuidedPractice

5. **JUICE** A manufacturer claims there are 12 ounces of juice in every can. Using a sample of 68 cans, Susan calculated a mean of 11.95 ounces with a standard deviation of 0.15 ounce. Test the hypothesis at 1% significance.





Example 1

1. **LUNCH** A sample of 145 high school seniors was asked how many times they go out for lunch per week. The mean number of times was 2.4 with a standard deviation of 0.7. Use a 90% confidence level to calculate the maximum error of estimate.

Example 2

2. **PRACTICE** A poll of 233 randomly chosen high school athletes showed that they spend an average of 1.6 hours practicing their sport during the off-season. The standard deviation is 0.5 hour. Determine a 99% confidence interval for the population mean.

Example 3

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

3. Lori thinks it takes a fast-food restaurant less than 2 minutes to serve her meal after she orders it.
4. A snack label states that one serving contains one gram of fat.
5. Mrs. Hart's review game takes at least 20 minutes to complete.
6. The tellers at a bank can complete no more than 18 transactions per hour.

CCSS REASONING Identify the hypotheses and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

Example 4

7. **COMPACT DISCS** A manufacturer of blank compact discs claims that each disc can hold at least 84 minutes of music. Using a sample of 219 compact discs, Cayla calculated a mean time of 84.1 minutes per disc with a standard deviation of 1.9 minutes. Test the hypothesis at 5% significance.

Example 5

8. **GOLF TEES** A company claims that each golf tee they produce is 5 centimeters in length. Using a sample of 168 tees, Angelene calculated a mean of 5.1 centimeters with a standard deviation of 0.3. Test the hypothesis at 10% significance.

Practice and Problem Solving

Extra Practice is on page R11.

Example 1

9. **MUSIC** A sample of 76 albums had a mean run time of 61.3 minutes with a standard deviation of 5.2 minutes. Use a 95% confidence level to calculate the maximum error of estimate.

Example 2

10. **COLLEGE** A poll of 218 students at a university showed that they spend 11.8 hours per week studying. The standard deviation is 3.7 hours. Determine a 90% confidence interval for the population mean.

Example 3

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

11. Julian sends at least six text messages to his best friend every day.
12. A car company states that one of their vehicles gets 27 miles per gallon.
13. A company advertisement states that it takes no more than 2 hours to paint a 200-square-foot room.
14. A singer plays at least 18 songs at every concert.

Identify the hypotheses and claim, decide whether to reject the null hypothesis, and make a conclusion about the claim.

Example 4

15. **PIZZA** A pizza chain promises a delivery time of less than 30 minutes. Using a sample of 38 deliveries, Chelsea calculated a mean delivery time of 29.6 minutes with a standard deviation of 3.9 minutes. Test the hypothesis at 1% significance.



Example 5

16. **CHEESE** A company claims that each package of cheese contains exactly 24 slices. Using a sample of 93 packages, Mr. Matthews calculated a mean of 24.1 slices with a standard deviation of 0.5. Test the hypothesis at 5% significance.
17. **DECISION MAKING** The number of peaches in 40 random cans is shown below. Should the manufacturer place a label on the can promising exactly 12 peaches in every can? Explain your reasoning.
- 13, 14, 13, 14, 12, 12, 12, 11, 15, 12, 13, 13, 14, 13, 14, 12, 15, 11, 11, 14, 13, 14, 14, 13, 12, 12, 12, 12, 13, 13, 11, 14, 14, 13, 14, 13, 13, 14, 12, 12
18. **CCSS ARGUMENTS** The number of chocolate chips in 40 random cookies is shown below. Should the manufacturer place a label on the package promising exactly 20 chips on every cookie? Explain your reasoning.
- 21, 19, 20, 20, 19, 19, 18, 21, 19, 17, 19, 18, 18, 20, 20, 19, 18, 20, 19, 20, 21, 21, 19, 17, 17, 18, 19, 19, 20, 17, 22, 21, 21, 20, 19, 18, 19, 17, 17, 21
19. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how the confidence interval is affected by the sample size and the confidence level. Consider a sample of data where $\bar{x} = 25$ and $s = 3$.
- GRAPHICAL** Graph the 90% confidence interval for $n = 50, 100,$ and 250 on a number line.
 - ANALYTICAL** How does the sample size affect the confidence interval?
 - GRAPHICAL** Graph the 90%, 95%, and 99% confidence intervals for $n = 150$.
 - ANALYTICAL** How does the confidence level affect the confidence interval?
 - ANALYTICAL** How does decreasing the size of the confidence interval affect the accuracy of the confidence interval?

H.O.T. Problems Use Higher-Order Thinking Skills

20. **ERROR ANALYSIS** Tim and Judie want to test whether a delivery service meets their promised time of 45 minutes or less. Their hypotheses are shown below. Is either of them correct?

<i>Tim</i>
$H_0: \mu < 45$ (claim)
$H_a: \mu \geq 45$

<i>Judie</i>
$H_0: \mu \leq 45$
$H_a: \mu > 45$ (claim)

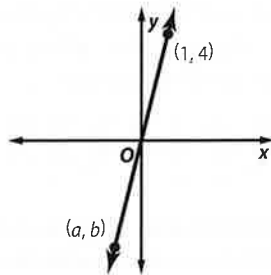
21. **CHALLENGE** A 95% confidence interval for the mean weight of a 20-ounce box of cereal was $19.932 \leq \mu \leq 20.008$ with a sample standard deviation of 0.128 ounces. Determine the sample size that led to this interval.
22. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
- If a confidence interval contains the H_0 value of μ , then it is not rejected.*
23. **WRITING IN MATH** How can a statistical test be used in a decision-making process?
24. **OPEN ENDED** Design and conduct your own research study, and draw conclusions based on the results of a hypothesis test. Write a brief summary of your findings.



Standardized Test Practice

25. **GEOMETRY** In the graph below, line l passes through the origin. What is the value of $\frac{a}{b}$?

- A -4
 B $-\frac{1}{4}$
 C $\frac{1}{4}$
 D 4



26. **SAT/ACT** If $5 + i$ and $5 - i$ are the roots of the equation $x^2 - 10x + c = 0$, what is the value of c ?

- F -26 G -25 H 25 J 26

27. The Service Club at Jake's school was founded 8 years ago. The number of members of the club by year is shown in the table. Which linear equation best models the data?

- A $y = 1.4x$
 B $y = 1.4x + 10.4$
 C $y = 1.6x$
 D $y = 1.6x + 11.1$

Year	Participation
0	11
2	13
4	15
6	19
8	22

28. **SHORT RESPONSE** Solve for x : $\log_2(x - 6) = 3$.

Spiral Review

29. **HEALTH** The heights of students at Madison High School are normally distributed with a mean of 66 inches and a standard deviation of 2 inches. Of the 1080 students in the school, how many would you expect to be less than 62 inches tall? (Lesson 11-5)
30. **CAR WASH** The Spanish Club is washing cars to raise money. They have determined that 65% of the customers donate more than the minimum amount for the car wash. What is the probability that at least 4 of the next 5 customers will donate more than the minimum? (Lesson 11-4)

Find a_n for each geometric sequence. (Lesson 10-3)

31. $a_1 = \frac{1}{3}, r = 3, n = 8$

32. $a_1 = \frac{1}{64}, r = 4, n = 9$

33. $a_4 = 16, r = 0.5, n = 8$

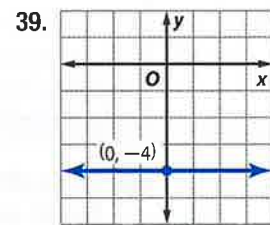
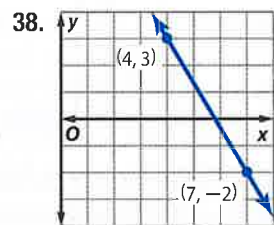
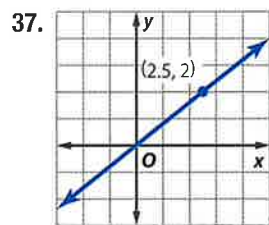
Write each equation in standard form. State whether the graph of the equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation. (Lesson 9-6)

34. $4x^2 + 2y^2 = 8$

35. $x^2 = 8y$

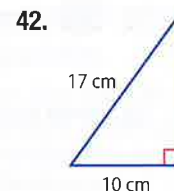
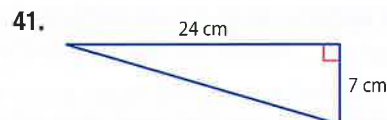
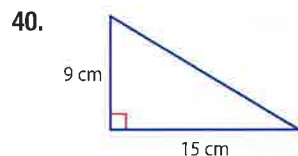
36. $(x - 1)^2 - 9(y - 4)^2 = 36$

Write an equation in slope-intercept form for each graph. (Lesson 2-4)



Skills Review

Find each missing measure. Round to the nearest tenth, if necessary.



Study Guide and Review

Study Guide

Key Concepts

Designing a Study (Lesson 11-1)

- A survey, an experiment, or an observational study can be used to collect information.
- A bias is an error that results in a misrepresentation of members of a population.

Distributions of Data (Lesson 11-2)

- Use the mean and standard deviation to describe a symmetric distribution.
- Use the five-number summary to describe a skewed distribution.

Probability Distributions (Lesson 11-3)

- A theoretical probability distribution is based on what is expected to happen. An experimental probability distribution is a distribution of probabilities estimated from experiments.
- The expected value of a discrete random variable is the weighted average of the values of the variable.

Binomial Distributions (Lesson 11-4)

- A binomial experiment has a fixed number of independent trials, only two possible outcomes for each trial, and the same probability of success for every trial.
- A binomial distribution is a frequency distribution of the probability of each value of a random variable.

Normal Distributions (Lesson 11-5)

- The graph of a normal distribution is bell-shaped.
- The z-value represents the number of standard deviations that a given data value is from the mean.

Hypothesis Testing (Lesson 11-6)

- A confidence interval for a population mean μ is given by $CI = \bar{x} \pm E$ or $\bar{x} \pm z \cdot \frac{s}{\sqrt{n}}$, where \bar{x} is the sample mean and E is the maximum error of estimate.
- A hypothesis test is used to assess a specific claim.

Key Vocabulary



- alternative hypothesis H_a (p. 771)
- bias (p. 723)
- binomial distribution (p. 754)
- confidence interval (p. 769)
- continuous random variable (p. 742)
- discrete random variable (p. 742)
- Empirical Rule (p. 760)
- expected value $E(X)$ (p. 745)
- experiment (p. 723)
- experimental probability distribution (p. 744)
- hypothesis test (p. 771)
- inferential statistics (p. 769)
- maximum error of estimate (p. 769)
- negatively skewed distribution (p. 733)
- normal distribution (p. 760)
- null hypothesis H_0 (p. 771)
- observational study (p. 723)
- parameter (p. 723)
- positively skewed distribution (p. 733)
- probability distribution (p. 743)
- random variable (p. 742)
- standard normal distribution (p. 762)
- statistic (p. 723)
- statistical inference (p. 769)
- survey (p. 723)
- symmetric distribution (p. 733)
- theoretical probability distribution (p. 743)
- z-value (p. 762)

Vocabulary Check

Choose a term from the list above that best completes each statement.

1. A(n) _____ for a particular random variable is a function that maps the sample space to the probabilities of the outcomes of the sample space.
2. A(n) _____ is an error that results in a misrepresentation of members of a population.
3. In a statistical study, data are collected and used to answer questions about a population characteristic or _____.
4. The _____ can be used to determine the area under the normal curve at specific intervals.
5. In a(n) _____, members of a sample are measured or observed without being affected by the study.
6. A _____ is an estimate of a population parameter stated as a range with a specific degree of certainty.

FOLDABLES Study Organizer

Be sure the following Key Concepts are noted in your Foldable.



Lesson-by-Lesson Review 

11-1 Designing a Study

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

7. **SHOPPING** Every tenth shopper coming out of a store is asked questions about his or her satisfaction with the store.
8. **MILK SHAKE** A fast food restaurant gives 25 of their customers a sample of a new milk shake and employees monitor their reactions as they taste it.
9. **SCHOOL** Every fifth person coming out of a high school is asked what their favorite class is.

Example 1

A dealership wants to test different promotions. They randomly select 100 customers and ask them which promotion they prefer. Does this situation describe a *survey*, an *experiment*, or an *observational study*? Identify the sample, and suggest a population from which it may have been selected.

This is a survey, because the data are collected from participants' responses. The sample is the 100 customers that were selected, and the population is all potential customers.

11-2 Distributions of Data

10. **DOGSLED** The Iditarod is a race across Alaska. The table shows the winning times, in days, for recent years.

Iditarod Winning Times

9.1, 9.4, 10.3, 9.3, 9.6, 8.7, 9.5, 9.4, 9.2, 17.3, 15.4,
15.5, 14.2, 12.0, 16.6, 13.5, 13.0, 18.1, 12.4, 11.6,
11.5, 11.3, 11.3, 13.1, 11.2, 11.6, 11.6, 9.7

- a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.
 - b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.
11. **SWIMMING** Kelly's practice times in the 400-meter individual medley are shown in the table.

Times in Seconds

301, 311, 320, 308, 312, 307, 303, 305, 309, 308,
304, 302, 311, 313, 313, 316, 314, 306, 329, 326,
319, 310, 306, 309, 320, 318, 315, 318, 314, 309

- a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
- b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

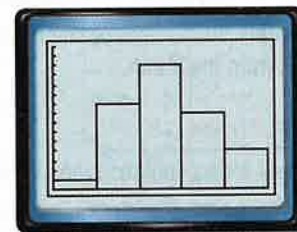
Example 2

Data collected from a group of third graders is shown.

Number of Years Playing an Instrument

2.5, 2.4, 3.1, 2.9, 4.2, 1.3, 2.6, 2.4, 3.3, 1.9, 3.4, 4.8,
2.3, 1.7, 3.2, 2.3, 3.5, 2.2, 3.6, 1.2, 4.4, 2.1, 3.4, 4.5,
1.9, 1.5, 1.4, 0.7, 1.2, 2.5, 1.9, 2.0, 2.4, 2.5, 3.4

- a. Use a graphing calculator to create a histogram. Then describe the shape of the distribution.



[0, 5] scl: 1 by [0, 15] scl: 1

The distribution is symmetric.

- b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

The distribution is symmetric, so use the mean and standard deviation. The mean number of years is about 2.6 with standard deviation of about 1 year.

11-3 Probability Distributions

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

- the number of ice cream sandwiches sold at an ice cream shop
- the time it takes to run a 5-kilometer race
- DANCE RECITALS** The probability distribution lists the probable number of dance recitals per year at Rena's Dance School. Determine the expected number of dance recitals per year.

Number of Dance Recitals Per Year					
Recitals	0	1	2	3	4
Probability	0.3	0.3	0.13	0.13	0.14

- SNOW DAYS** The distribution lists the number of snow days per year at Washington Elementary over the past 26 years. Determine the expected number of snow days this year.

Number of Snow Days Per Year					
Snow Days	0	1	2	3	4
Frequency	4	8	6	3	5

Example 3

Identify the random variable in a distribution of the number of DVDs on display at a store. Classify it as *discrete* or *continuous*. Explain your reasoning.

The random variable X is the number of DVDs on display. The number of DVDs is countable, so X is discrete.

Example 4

MEDICINE The probability distribution lists the probable number of drops of medicine that a veterinarian administers to her sick patients. Find the expected number of drops of medicine.

Number of Drops of Medicine				
Drops	1	2	3	4
Probability	0.5	0.3	0.1	0.1

Each drop of medicine represents a value of X , and each decimal represents the corresponding probability $P(X)$.

$$\begin{aligned}
 E(X) &= \sum[X \cdot P(X)] \\
 &= 1(0.5) + 2(0.3) + 3(0.1) + 4(0.1) \\
 &= 0.5 + 0.6 + 0.3 + 0.4 \text{ or } 1.8
 \end{aligned}$$

The expected number of drops of medicine is 1.8.

11-4 The Binomial Distribution

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n , p , and q .

- A survey found that 30% of adults like chocolate ice cream more than any other flavor. You ask 35 adults if they prefer chocolate ice cream more than any other flavor.
- Thirty random guests from Sheila's birthday party are asked their favorite song.
- WATCHES** According to an online poll, 74% of adults wear watches. Timmy surveyed 25 random adults. What is the probability that 20 of the adults surveyed wear a watch?
- SEASONS** Of 1108 people surveyed, 68% say that summer is their favorite season. What is the probability that at least 15 of 20 randomly selected people will prefer summer?

Example 5

WORK According to an online poll, 40% of adults feel that the standard 40-hour workweek should be increased. Sheila conducts a survey of 10 random adults. What is the probability that 3 of the surveyed adults feel that the standard 40-hour workweek should be increased?

A success is an adult agreeing that the 40-hour workweek should be increased, so $p = 0.4$, $X = 3$, $q = 1 - 0.4$ or 0.6 , and $n = 10$.

$$\begin{aligned}
 P(X) &= {}_n C_x p^x q^{n-x} \\
 P(6) &= {}_{10} C_3 (0.4)^3 (0.6)^{10-3} \\
 &\approx 0.215
 \end{aligned}$$

The probability that three surveyed adults will feel that the standard 40-hour workweek should be increased is about 0.215 or 21.5%.

11-5 The Normal Distribution

20. **RUNNING TIMES** The times in the 40-meter dash for a select group of professional football players are normally distributed with a mean of 4.74 seconds and a standard deviation of 0.13 second.
- About what percent of players have times between 4.6 and 4.8 seconds?
 - About how many of a sample of 800 players will have times below 4.5 seconds?
21. **ATTENDANCE** The number of tickets sold at high school basketball games in a particular conference are normally distributed with a mean of 68.7 and a standard deviation of 13.1.
- About what percent of the games sell fewer than 75 tickets?
 - About how many of a sample of 200 games will sell more than 100 tickets?
22. **COMMUTING** The number of minutes it takes Phil to commute to work each day are normally distributed with a mean of 18.6 and a standard deviation of 3.5.
- About what percent of the time will it take Phil more than 20 minutes to commute to work?
 - About how many of a sample of 50 days will it take Phil less than 15 minutes to commute to work?

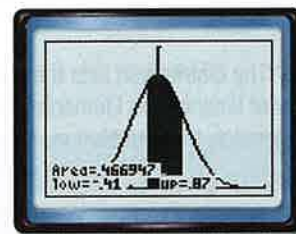
Example 6

TEST SCORES The midterm test scores for the students in Mrs. Hendrix's classes are normally distributed with a mean of 73.2 and standard deviation of 7.8. About how many test scores are between 70 and 80?

Find the corresponding z -values for $X = 70$ and 80.

$$\begin{aligned} z &= \frac{X - \mu}{\sigma} & z &= \frac{X - \mu}{\sigma} \\ &= \frac{70 - 73.2}{7.8} & &= \frac{80 - 73.2}{7.8} \\ &\approx -0.41 & &\approx 0.87 \end{aligned}$$

Using a graphing calculator, you can find the area between $-0.41 < z < 0.87$ to be about 0.47.



$[-4, 4]$ scl: 1 by $[0, 0.5]$ scl: 0.125

11-6 Confidence Intervals and Hypothesis Testing

23. **INTERNET** A sample of 300 students was asked the average amount of time they spend online during a weeknight. The mean time was 64.3 minutes with a standard deviation of 7.3 minutes. Determine a 95% confidence interval.

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

- A cupcake shop owner says that they sell 200 cupcakes everyday.
- A technician says that it takes at least 45 minutes to diagnose a computer problem.
- A member of a swim team says that she practices no more than 35 minutes on Mondays.

Example 7

TELEVISION WATCHING A sample of 100 teenagers was asked the average amount of time they spent watching television on the weekends. The mean time was 65 minutes with a standard deviation of 1.6 minutes. Determine a 95% confidence interval for the population.

$$\begin{aligned} CI &= \bar{x} \pm 1.96 \cdot \frac{s}{\sqrt{n}} && \text{Confidence Interval Formula} \\ &= 65 \pm 1.96 \cdot \frac{1.6}{\sqrt{100}} && \bar{x} = 65, s = 1.6, n = 100 \\ &= 65 \pm 0.31 && \text{Use a calculator.} \end{aligned}$$

The 95% confidence interval is $64.69 \leq \mu \leq 65.31$. Therefore, we are 95% confident that the population mean time is between 64.69 and 65.31 minutes.

Practice Test

1. **BUTTERFLIES** Students in a biology class are learning about the monarch butterfly's life cycle. Each student is given a caterpillar. When a caterpillar turns into chrysalis, it is placed in a glass enclosure with food and a heat lamp and examined.
- Determine whether the situation describes a *survey*, an *experiment* or an *observational study*.
 - Identify the sample, and suggest a population from which it was selected.

2. **HEIGHTS** The heights of Ms. Joy's dance students are shown.

Height (Inches)				
60	64	62	69	64
63	65	64	66	73
74	63	62	65	64
68	70	66	63	61

- Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.
 - Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.
3. A binomial distribution has a 65% rate of success. There are 15 trials.
- What is the probability that there will be exactly 12 successes?
 - What is the probability that there will be at least 10 successes?

Determine whether each experiment is a binomial experiment or can be reduced to a binomial experiment. If so, describe a trial, determine the random variable, and state n , p , and q .

- A poll found that 65% of high school teachers own a pet. You ask 15 high school teachers if they own a pet.
- A study finds that 20% of families in a town have a land line telephone. You ask 55 families how many land line telephones they have.
- A survey found that on a scale of 1 to 5, a sandwich received a 3.5 rating. A restaurant manager asks 150 customers to rate the sandwich on a scale of 1 to 5.

Identify the random variable in each distribution, and classify it as *discrete* or *continuous*. Explain your reasoning.

- the number of laps that Jesse swims
- the body temperatures of patients in a hospital
- the weights of pets in a pet store

10. **MULTIPLE CHOICE** The table shows the number of gift cards previously won in a mall contest. What is the expected value of the gift card that is won?

- A \$250.00
 B \$223.15
 C \$143.25
 D \$100.23

Amount, X	Winners
\$100	495
\$125	405
\$150	285
\$200	180
\$250	90
\$300	45

11. **WEIGHTS** The weights of 1500 bodybuilders are normally distributed with a mean of 190.6 pounds and a standard deviation of 5.8 pounds.
- About how many bodybuilders are between 180 and 190 pounds?
 - What is the probability that a bodybuilder selected at random has a weight greater than 195 pounds?

A normal distribution has a mean of 16.4 and a standard deviation of 2.6.

- Find the range of values that represent the middle 95% of the distribution.
- What percent of the data will be less than 19?
- MULTIPLE CHOICE** A survey of 300 randomly selected members of a fitness club showed that they spent an average of 25.3 minutes per visit in the gym with a standard deviation of 8.6 minutes. Which is the maximum error of estimate for the time spent in the gym using a 95% confidence interval?
 F 0.82 G 0.97 H 1.28 J 2.86

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

- A restaurant owner says that each milk shake contains 5 strawberries.
- A valet says that no more than 56 cars can be parked in the parking lot at one time.



Solve Multi-Step Problems

Some problems that you will encounter on standardized tests require you to solve multiple parts in order to come up with the final solution. Use this lesson to practice these types of problems.

Strategies for Solving Multi-Step Problems

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve? What information is given?
- Are there any intermediate steps that need to be completed before I can solve the problem?

Step 2

Organize your approach.

- List the steps you will need to complete in order to solve the problem.
- Remember that there may be more than one possible way to solve the problem.

Step 3

Solve and check.

- Work as efficiently as possible to complete each step and solve.
- If time permits, check your answer.



Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

There are 15 boys and 12 girls in Mrs. Lawrence's homeroom. Suppose a committee is to be made up of 6 randomly selected students. What is the probability that the committee will contain 3 boys and 3 girls? Round your answer to the nearest tenth of a percent.

- | | |
|---------|---------|
| A 27.2% | C 31.5% |
| B 29.6% | D 33.8% |

Read the problem statement carefully. You are asked to find the probability that a committee will be made up of 3 boys and 3 girls. Finding this probability involves successfully completing several steps.

Step 1 Find the number of possible successes.

There are $C(15, 3)$ ways to choose 3 boys from 15, and there are $C(12, 3)$ to choose 3 girls from 12. Use the Fundamental Counting Principle to find s , the number of possible successes.

$$s = C(15, 3) \times C(12, 3) = \frac{15!}{12!3!} \times \frac{12!}{9!3!} \text{ or } 100,100$$

Step 2 Find the total number of possible outcomes.

Compute the number of ways 6 people can be chosen from a group of 27 students.

$$C(27, 6) = 296,010$$

Step 3 Compute the probability.

Find the probability by comparing the number of successes to the number of possible outcomes.

$$P(3 \text{ boys}, 3 \text{ girls}) = \frac{100,100}{296,010} \approx 0.33816$$

So, there is about a 33.8% chance of selecting 3 boys and 3 girls for the committee. The answer is D.

Exercises

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

- There are 52 cards in a standard deck. Of these, 4 of the cards are aces. What is the probability of a randomly dealt 5-card hand containing a pair of aces? Round your answer to the nearest whole percent.
 - 4%
 - 5%
 - 6%
 - 7%

- According to the table, what is the probability that a randomly selected camper went on the horse ride, given that the camper is an 8th grader?

Camp Activities			
Grade	Canoe Trip	Horse Ride	Nature Hike
6th	8	6	3
7th	5	4	7
8th	11	9	6

F 0.731

H 0.346

G 0.441

J 0.153

Standardized Test Practice

Cumulative, Chapters 1 through 11

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

- Suppose the test scores on a final exam are normally distributed with a mean of 74 and a standard deviation of 3. What is the probability that a randomly selected test has a score higher than 77?

A 2.5% C 16%

B 13.5% D 34%
- A diameter of a circle has endpoints $A(4, 6)$ and $B(-3, -1)$. Find the approximate length of the radius.

F 2.5 units H 5.1 units

G 4.9 units J 9.9 units
- An equation can be used to find the total cost of a pizza with a certain diameter. Using the table below, find the equation that best represents y , the total cost, as a function of x , the diameter in inches.

Diameter, x (in.)	Total Cost, y
9	\$10.80
12	\$14.40
20	\$24.00

- A $y = 1.2x$ C $y = 0.83x$
- B $x = 1.2y$ D $y = x + 1.80$
- Which shows the functions correctly listed in order from widest to narrowest graph?

F $y = 8x^2, y = 2x^2, y = \frac{1}{2}x^2, y = -\frac{4}{5}x^2$

G $y = -\frac{4}{5}x^2, y = \frac{1}{2}x^2, y = 2x^2, y = 8x^2$

H $y = \frac{1}{2}x^2, y = -\frac{4}{5}x^2, y = 2x^2, y = 8x^2$

J $y = 8x^2, y = 2x^2, y = -\frac{4}{5}x^2, y = \frac{1}{2}x^2$

Test-Taking Tip

Question 5 You can use a scientific calculator to find the standard deviation. Enter the data values as a list and calculate the 1-Var statistics.

- The table at the right shows the grades earned by students on a science test. Calculate the standard deviation of the test scores.

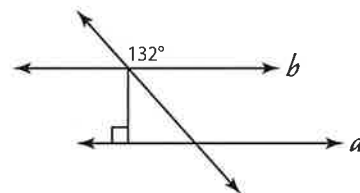
76	84	91	75	83
82	65	94	90	71
92	84	83	88	80
78	84	89	95	93

- A 7.82 B 8.03 C 8.23 D 8.75
- Which expression is equivalent to $(6a - 2b) - \frac{1}{4}(4a + 12b)$?

F $5a + 10b$ H $5a + b$

G $10a + 10b$ J $5a - 5b$
 - Simplify $\sqrt[6]{27x^3}$.

A $3x^2$ B $3x$ C $\sqrt{3x}$ D $\sqrt{3x}$
 - In the figure below, lines a and b are parallel. What are the measures of the angles in the triangle?



- F 42, 48, 90 H 48, 52, 90
- G 42, 90, 132 J 48, 90, 132
- Which of the following functions represents exponential decay?

A $y = 0.2(7)^x$ C $y = 4(9)^x$

B $y = (0.5)^x$ D $y = 5\left(\frac{4}{3}\right)^x$
 - Using the table below, which expression can be used to determine the n th term of the sequence?

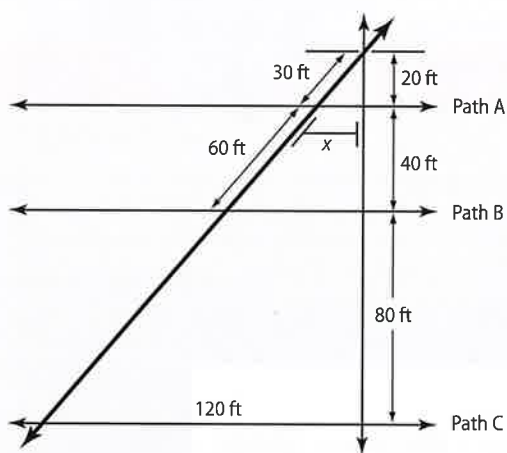
n	1	2	3	4
y	6	10	14	18

- F $y = 6n$ H $y = 2n + 1$
- G $y = n + 5$ J $y = 2(2n + 1)$

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Determine whether each of the following situations calls for a *survey*, an *observational study*, or an *experiment*. Explain the process.
- Caroline wants to find out if a particular plant food helps plants to grow faster than just water.
 - Allison wants to find opinions on favorite candidates in the upcoming student council elections.
 - Manuel wants to find out if people who get regular exercise sleep better at night.
12. **GRIDDED RESPONSE** Carla received a map of some walking paths through her college campus. Paths A, B, and C are parallel. What is the length x to the nearest tenth of a foot?



13. Alex wants to find the area of a triangle. He draws the triangle on a coordinate plane and finds that it has vertices at $(2, 1)$, $(3, 4)$, and $(1, 4)$. Find the area of the triangle using determinants.

14. **GRIDDED RESPONSE** Perry drove to the gym at an average rate of 30 miles per hour. It took him 45 minutes. Going home, he took the same route, but drove at a rate of 45 miles per hour. How many miles is it to his house from the gym?

Extended Response

Record your answers on a sheet of paper. Show your work.

15. Martin is taking a multiple choice test that has 8 questions. Each question has four possible answers: A, B, C, or D. Martin forgot to study for the test, so he must guess at each answer.
- What is the probability of guessing a correct answer on the test?
 - What is the expected number of correct answers if Martin guesses at each question?
 - What is the probability that Martin will get at least half of the questions correct? Round your answer to the nearest tenth of a percent.
16. Christine had one dress and three sweaters cleaned at the dry cleaner and the charge was \$19.50. The next week, she had two dresses and two sweaters cleaned for a total charge of \$23.00.
- Let d represent the price of cleaning a dress and s represent the price of cleaning a sweater. Write a system of linear equations to represent the prices of cleaning each item.
 - Solve the system of equations using substitution or elimination. Explain your choice of method.
 - What will the charge be if Christine takes two dresses and four sweaters to be cleaned?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Go to Lesson...	11-5	9-1	2-4	4-7	0-9	5-1	6-6	3-1	7-1	10-2	11-1	1-4	3-8	8-5	11-4	3-1

