CHAPTER 6

Inverses and Radical Functions and Relations

Then

- You simplified polynomial expressions.

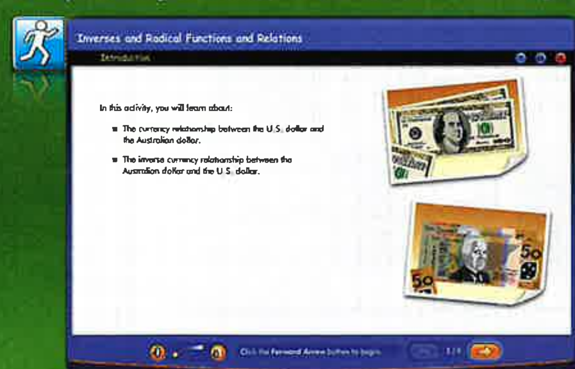
Now

You will:

- Find compositions and inverses of functions.
- Graph and analyze square root functions and inequalities.
- Simplify and solve equations involving roots, radicals, and rational exponents.

Why? ▲

FINANCE Connecting finances to mathematics is a skill that, once mastered, you will use your entire life. Learning to manage your finances entails creating a budget and living within that budget. In this chapter, you will explore financial topics such as saving for college, income, profit, inflation, and converting money when traveling.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

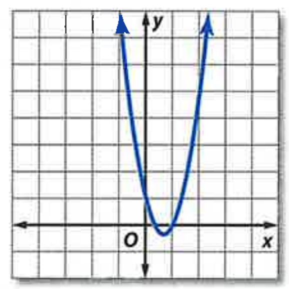
Use the related graph of each equation to determine its roots. If exact roots cannot be found, state the consecutive integers between which the roots are located.

1. $x^2 - 4x + 1 = 0$
2. $2x^2 + x - 6 = 0$
3. **PHYSICS** Allie drops a ball from the top of a 30-foot building. How long does it take for the ball to reach the ground, assuming there is no air resistance? Use the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet.

QuickReview

Example 1

Use the related graph of $0 = 3x^2 - 4x + 1$ to determine its roots. If exact roots cannot be found, state the consecutive integers between which the roots are located.



The roots are the x-coordinates where the graph crosses the x-axis.

The graph crosses the x-axis between 0 and 1 and at 1.

QuickCheck

Simplify each expression by using synthetic division.

4. $(5x^2 - 22x - 15) \div (x - 5)$
5. $(3x^2 + 14x - 12) \div (x + 4)$
6. $(2x^3 - 7x^2 - 36x + 36) \div (x - 6)$
7. $(3x^4 - 13x^3 + 17x^2 - 18x + 15) \div (x - 3)$
8. **FINANCE** The number of specialty coffee mugs sold at a coffee shop can be estimated by $n = \frac{4000x^2}{x^2 + 50}$, where x is the amount of money spent on advertising in hundreds of dollars and n is the number of mugs sold.
   a. Perform the division indicated by $\frac{4000x^2}{x^2 + 50}$.
   b. About how many mugs will be sold if $1000 is spent on advertising?

QuickReview

Example 2

Simplify $(3x^4 + 4x^3 + x^2 + 9x - 6) \div (x + 2)$ by using synthetic division.

$x - r = x + 2$, so $r = -2$.

−2	3	4	1	9	−6
	↓	−6	4	−10	2
	3	−2	5	−1	−4

The result is $3x^3 - 2x^2 + 5x - 1 - \frac{4}{x + 2}$.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

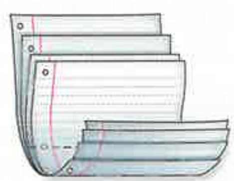
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

Inverses and Radical Functions Make this Foldable to help you organize your Chapter 6 notes about radical equations and inequalities. Begin with four sheets of notebook paper.

1 **Stack** four sheets of notebook paper so that each sheet is one inch higher than the sheet in front of it.

2 **Bring** the bottom of all the sheets upward and align the edges so that all of the layers or tabs are the same distance apart.



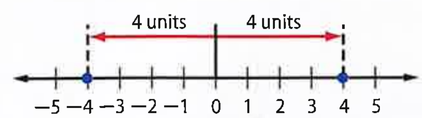
3 **When** all the tabs are an equal distance apart, fold the papers and crease well. Open the papers and staple them together along the valley or inner center fold. Label the pages with lesson titles.

NewVocabulary

English		Español
composition of functions	p. 387	composición de funciones
inverse relation	p. 393	relaciones inversas
inverse function	p. 393	función inversa
square root function	p. 400	función raíz cuadrada
radical function	p. 400	función radical
square root inequality	p. 402	desigualdad raíz cuadrada
*n*th root	p. 407	raíz enésima
radical sign	p. 407	signo radical
index	p. 407	índice
radicand	p. 407	radicando
principal root	p. 407	raíz principal
rationalizing the denominator	p. 416	racionalizar el denominador
conjugates	p. 418	conjugados
radical equation	p. 429	ecuación radical
extraneous solution	p. 429	solución extraña
radical inequality	p. 431	desigualdad radical

ReviewVocabulary

absolute value valor absoluto a number's distance from zero on the number line, represented by $|x|$



rational number número racional Any number $\frac{m}{n}$, where m and n are integers and n is not zero; the decimal form is either a terminating or repeating decimal.

relation relación a set of ordered pairs

LESSON 6-1 Operations on Functions

:: Then

- You performed operations on polynomials.

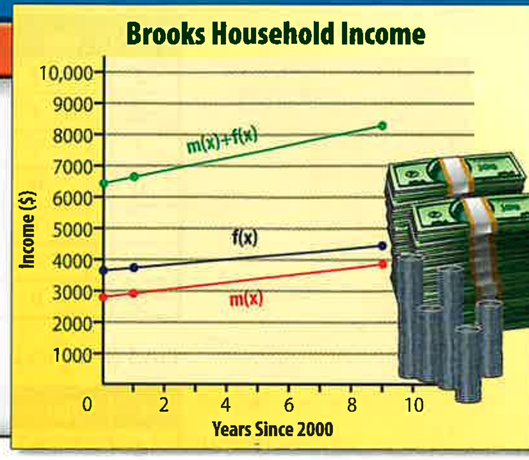
:: Now

1. Find the sum, difference, product, and quotient of functions.
2. Find the composition of functions.

:: Why?

- The graphs model the income for the Brooks family since 2000, where $m(x)$ represents Mr. Brooks' income and $f(x)$ represents Mrs. Brooks' income.

The total household income for the Brooks household can be represented by $f(x) + m(x)$.



NewVocabulary
composition of functions

Common Core State Standards

Content Standards

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F.BF.1.b Combine standard function types using arithmetic operations.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Arithmetic Operations

In Chapter 5, you performed arithmetic operations with polynomials. You can also use addition, subtraction, multiplication, and division with functions.

You can perform arithmetic operations according to the following rules.

KeyConcept Operations on Functions

Operation	Definition	Example Let $f(x) = 2x$ and $g(x) = -x + 5$.
Addition	$(f + g)(x) = f(x) + g(x)$	$2x + (-x + 5) = x + 5$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$2x - (-x + 5) = 3x - 5$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$	$2x(-x + 5) = -2x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$\frac{2x}{-x + 5}, x \neq 5$

Example 1 Add and Subtract Functions

Given $f(x) = x^2 - 4$ and $g(x) = 2x + 1$, find each function.

a. $(f + g)(x)$

$$
\begin{aligned}
(f + g)(x) &= f(x) + g(x) && \text{Addition of functions} \\
&= (x^2 - 4) + (2x + 1) && f(x) = x^2 - 4 \text{ and } g(x) = 2x + 1 \\
&= x^2 + 2x - 3 && \text{Simplify.}
\end{aligned}
$$

b. $(f - g)(x)$

$$
\begin{aligned}
(f - g)(x) &= f(x) - g(x) && \text{Subtraction of functions} \\
&= (x^2 - 4) - (2x + 1) && f(x) = x^2 - 4 \text{ and } g(x) = 2x + 1 \\
&= x^2 - 2x - 5 && \text{Simplify.}
\end{aligned}
$$

GuidedPractice

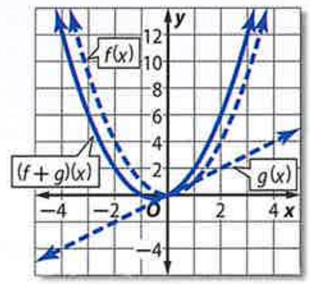
Given $f(x) = x^2 + 5x - 2$ and $g(x) = 3x - 2$, find each function.

1A. $(f + g)(x)$ **1B.** $(f - g)(x)$

You can graph sum and difference functions by graphing each function involved separately, then adding their corresponding functional values. Let $f(x) = x^2$ and $g(x) = x$. Examine the graphs of $f(x)$, $g(x)$, and their sum and difference.

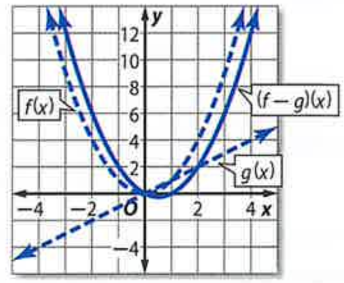
Find $(f + g)(x)$.

x	$f(x) = x^2$	$g(x) = x$	$(f + g)(x) = x^2 + x$
−3	9	−3	9 + (−3) = 6
−2	4	−2	4 + (−2) = 2
−1	1	−1	1 + (−1) = 0
0	0	0	0 + 0 = 0
1	1	1	1 + 1 = 2
2	4	2	4 + 2 = 6
3	9	3	9 + 3 = 12



Find $(f - g)(x)$.

x	$f(x) = x^2$	$g(x) = x$	$(f - g)(x) = x^2 - x$
−3	9	−3	9 − (−3) = 12
−2	4	−2	4 − (−2) = 6
−1	1	−1	1 − (−1) = 2
0	0	0	0 − 0 = 0
1	1	1	1 − 1 = 0
2	4	2	4 − 2 = 2
3	9	3	9 − 3 = 6



ReviewVocabulary

intersection the intersection of two sets is the set of elements common to both

In Example 1, the functions $f(x)$ and $g(x)$ have the same domain of all real numbers. The functions $(f + g)(x)$ and $(f - g)(x)$ also have domains that include all real numbers. For each new function, the domain consists of the intersection of the domains of $f(x)$ and $g(x)$. Under division, the domain of the new function is restricted by excluded values that cause the denominator to equal zero.

Example 2 Multiply and Divide Functions

Given $f(x) = x^2 + 7x + 12$ and $g(x) = 3x - 4$, find each function. Indicate any restrictions in the domain or range.

a. $(f \cdot g)(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$ — Multiplication of functions

$= (x^2 + 7x + 12)(3x - 4)$ — Substitution

$= 3x^3 + 21x^2 + 36x - 4x^2 - 28x - 48$ — Distributive Property

$= 3x^3 + 17x^2 + 8x - 48$ — Simplify.

b. $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ — Division of functions

$= \frac{x^2 + 7x + 12}{3x - 4}, x \neq \frac{4}{3}$ — Substitution

Because $x = \frac{4}{3}$ makes the denominator $3x - 4 = 0$, $\frac{4}{3}$ is excluded from the domain of $\left(\frac{f}{g}\right)(x)$.

GuidedPractice **Given $f(x) = x^2 - 7x + 2$ and $g(x) = x + 4$, find each function.**

2A. $(f \cdot g)(x)$

2B. $\left(\frac{f}{g}\right)(x)$

2 Composition of Functions

Another method used to combine functions is a composition of functions. In a **composition of functions**, the results of one function are used to evaluate a second function.

ReadingMath

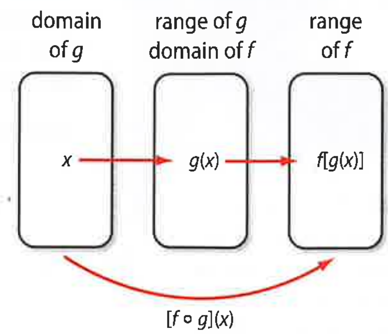
Composition of Functions The composition of f and g, denoted by $f \circ g$ or $f[g(x)]$, is read f of g.

KeyConcept Composition of Functions

Words Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composition function $f \circ g$ can be described by

$$[f \circ g](x) = f[g(x)].$$

Model



The composition of two functions may not exist. Given two functions f and g, $[f \circ g](x)$ is defined only if the range of $g(x)$ is a subset of the domain of f. Likewise, $[g \circ f](x)$ is defined only if the range of $f(x)$ is a subset of the domain of g.

StudyTip

Composition Be careful not to confuse a composition $f[g(x)]$ with multiplication of functions $(f \cdot g)(x)$.

Example 3 Compose Functions

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

a. $f = \{(1, 8), (0, 13), (15, 11), (14, 9)\}$, $g = \{(8, 15), (5, 1), (10, 14), (9, 0)\}$

To find $f \circ g$, evaluate $g(x)$ first. Then use the range to evaluate $f(x)$.

$f[g(8)] = f(15)$ or 11 $g(8) = 15$ $f[g(10)] = f(14)$ or 9 $g(10) = 14$

$f[g(5)] = f(1)$ or 8 $g(5) = 1$ $f[g(9)] = f(0)$ or 13 $g(9) = 0$

$f \circ g = \{(8, 11), (5, 8), (10, 9), (9, 13)\}$, D = {5, 8, 9, 10}, R = {8, 9, 11, 13}

To find $g \circ f$, evaluate $f(x)$ first. Then use the range to evaluate $g(x)$.

$g[f(1)] = g(8)$ or 15 $f(1) = 8$ $g[f(15)] = g(11)$ $g(11)$ is undefined.

$g[f(0)] = g(13)$ $g(13)$ is undefined. $g[f(14)] = g(9)$ or 0 $f(14) = 0$

Because 11 and 13 are not in the domain of g, $g \circ f$ is undefined for $x = 11$ and $x = 13$. However, $g[f(1)] = 15$ and $g[f(14)] = 0$, so $g \circ f = \{(1, 15), (14, 0)\}$, D = {5, 8, 9, 10}, and R = {8, 9, 11, 13}.

b. $f(x) = 2a - 5$, $g(x) = 4a$

$[f \circ g](x) = f[g(x)]$	Composition of functions	$[g \circ f](x) = g[f(x)]$
$= f(4a)$	Substitute.	$= g(2a - 5)$
$= 2(4a) - 5$	Substitute again.	$= 4(2a - 5)$
$= 8a - 5$	Simplify.	$= 8a - 20$

For $[f \circ g](x)$, D = {all real numbers} and R = {all real numbers}, and for $[g \circ f](x)$, D = {all real numbers} and R = {all real numbers}.

GuidedPractice

3A. $f(x) = \{(3, -2), (-1, -5), (4, 7), (10, 8)\}$, $g(x) = \{(4, 3), (2, -1), (9, 4), (3, 10)\}$

3B. $f(x) = x^2 + 2$ and $g(x) = x - 6$

Real-World Link

Adjusted for inflation, the average price of a new car declined from $23,014 in 1995 to $22,013 in 2005.

Source: U.S. Department of Commerce

Notice that in most cases, $f \circ g \neq g \circ f$. Therefore, the order in which two functions are composed is important.

Real-World Example 4 Use Composition of Functions

SHOPPING A new car dealer is discounting all new cars by 12%. At the same time, the manufacturer is offering a $1500 rebate on all new cars. Mr. Navarro is buying a car that is priced $24,500. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?

Understand Let x represent the original price of a new car, $d(x)$ represent the price of a car after the discount, and $r(x)$ the price of the car after the rebate.

Plan Write equations for $d(x)$ and $r(x)$.

The original price is discounted by 12%. $d(x) = x - 0.12x$

There is a $1500 rebate on all new cars. $r(x) = x - 1500$

Solve If the discount is applied *before* the rebate, then the final price of Mr. Navarro's new car is represented by $[r \circ d](24{,}500)$.

$$[r \circ d](x) = r[d(x)]$$

$$\begin{aligned}[r \circ d](24{,}500) &= r[24{,}500 - 0.12(24{,}500)] \\ &= r(24{,}500 - 2940) \\ &= r(21{,}560) \\ &= 21{,}560 - 1500 \\ &= 20{,}060\end{aligned}$$

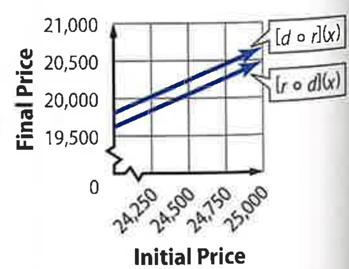
If the rebate is given *before* the discount is applied, then the final price of Mr. Navarro's car is represented by $[d \circ r](24{,}500)$.

$$[d \circ r](x) = d[r(x)]$$

$$\begin{aligned}[d \circ r](24{,}500) &= d(24{,}500 - 1500) \\ &= d(23{,}000) \\ &= 23{,}000 - 0.12(23{,}000) \\ &= 23{,}000 - 2760 \\ &= 20{,}240\end{aligned}$$

$[r \circ d](24{,}500) = 20{,}060$ and $[d \circ r](24{,}500) = 20{,}240$. So, the final price of the car is less when the discount is applied before the rebate.

Check The answer seems reasonable because the 12% discount is being applied to a greater amount. Thus, the dollar amount of the discount is greater.



Guided Practice

4. **SHOPPING** Sounds-to-Go offers both an in-store $35 rebate and a 15% discount on a digital audio player that normally sells for $300. Which provides the better price: taking the discount before the rebate or taking the discount after the rebate?

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in domain or range.**

1. $f(x) = x + 2$
$g(x) = 3x - 1$

2. $f(x) = x^2 - 5$
$g(x) = -x + 8$

Example 3 **For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.**

3. $f = \{(2, 5), (6, 10), (12, 9), (7, 6)\}$
$g = \{(9, 11), (6, 15), (10, 13), (5, 8)\}$

4. $f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}$
$g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

5. $f(x) = -3x$
$g(x) = 5x - 6$

6. $f(x) = x + 4$
$g(x) = x^2 + 3x - 10$

Example 4

7. CCSS **MODELING** Dora has 8% of her earnings deducted from her paycheck for a college savings plan. She can choose to take the deduction either before taxes are withheld, which reduces her taxable income, or after taxes are withheld. Dora's tax rate is 17.5%. If her pay before taxes and deductions is \$950, will she save more money if the deductions are taken before or after taxes are withheld? Explain.

Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–2 **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in domain or range.**

8. $f(x) = 2x$
$g(x) = -4x + 5$

9. $f(x) = x - 1$
$g(x) = 5x - 2$

10. $f(x) = x^2$
$g(x) = -x + 1$

11. $f(x) = 3x$
$g(x) = -2x + 6$

12. $f(x) = x - 2$
$g(x) = 2x - 7$

13. $f(x) = x^2$
$g(x) = x - 5$

14. $f(x) = -x^2 + 6$
$g(x) = 2x^2 + 3x - 5$

15. $f(x) = 3x^2 - 4$
$g(x) = x^2 - 8x + 4$

16. **POPULATION** In a particular county, the population of the two largest cities can be modeled by $f(x) = 200x + 25$ and $g(x) = 175x - 15$, where x is the number of years since 2000 and the population is in thousands.

a. What is the population of the two cities combined after any number of years?

b. What is the difference in the populations of the two cities?

Example 3 **For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.**

17. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
$g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$

18. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
$g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$

19. $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$
$g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$

20. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
$g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

21. $f = \{(-15, -5), (-4, 12), (1, 7), (3, 9)\}$
$g = \{(3, -9), (7, 2), (8, -6), (12, 0)\}$

22. $f = \{(-1, 11), (2, -2), (5, -7), (4, -4)\}$
$g = \{(5, -4), (4, -3), (-1, 2), (2, 3)\}$

23. $f = \{(7, -3), (-10, -3), (-7, -8), (-3, 6)\}$
$g = \{(4, -3), (3, -7), (9, 8), (-4, -4)\}$

24. $f = \{(1, -1), (2, -2), (3, -3), (4, -4)\}$
$g = \{(1, -4), (2, -3), (3, -2), (4, -1)\}$

25. $f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\}$
$g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\}$

26. $f = \{(12, -3), (9, -2), (8, -1), (6, 3)\}$
$g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

27. $f(x) = 2x$
$g(x) = x + 5$

28. $f(x) = -3x$
$g(x) = -x + 8$

29. $f(x) = x + 5$
$g(x) = 3x - 7$

30. $f(x) = x - 4$
$g(x) = x^2 - 10$

31. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

32. $f(x) = 2x^2 - x + 1$
$g(x) = 4x + 3$

33. $f(x) = 4x - 1$
$g(x) = x^3 + 2$

34. $f(x) = x^2 + 3x + 1$
$g(x) = x^2$

35. $f(x) = 2x^2$
$g(x) = 8x^2 + 3x$

36. **FINANCE** A ceramics store manufactures and sells coffee mugs. The revenue $r(x)$ from the sale of x coffee mugs is given by $r(x) = 6.5x$. Suppose the function for the cost of manufacturing x coffee mugs is $c(x) = 0.75x + 1850$.

a. Write the profit function.

b. Find the profit on 500, 1000, and 5000 coffee mugs.

37. **CCSS SENSE-MAKING** Ms. Smith wants to buy an HDTV, which is on sale for 35% off the original price of $2299. The sales tax is 6.25%.

a. Write two functions representing the price after the discount $p(x)$ and the price after sales tax $t(x)$.

b. Which composition of functions represents the price of the HDTV, $[p \circ t](x)$ or $[t \circ p](x)$? Explain your reasoning.

c. How much will Ms. Smith pay for the HDTV?

Perform each operation if $f(x) = x^2 + x - 12$ and $g(x) = x - 3$. State the domain of the resulting function.

38. $(f - g)(x)$

39. $2(g \cdot f)(x)$

40. $\left(\frac{f}{g}\right)(x)$

If $f(x) = 5x$, $g(x) = -2x + 1$, and $h(x) = x^2 + 6x + 8$, find each value.

41. $f[g(-2)]$

42. $g[h(3)]$

43. $h[f(-5)]$

44. $h[g(2)]$

45. $f[h(-3)]$

46. $h[f(9)]$

47. $f[g(3a)]$

48. $f[h(a + 4)]$

49. $g[f(a^2 - a)]$

50. **MULTIPLE REPRESENTATIONS** Let $f(x) = x^2$ and $g(x) = x$.

a. Tabular Make a table showing values for $f(x)$, $g(x)$, $(f + g)(x)$, and $(f - g)(x)$.

b. Graphical Graph $f(x)$, $g(x)$, and $(f + g)(x)$ on the same coordinate grid.

c. Graphical Graph $f(x)$, $g(x)$, and $(f - g)(x)$ on the same coordinate grid.

d. Verbal Describe the relationship among the graphs of $f(x)$, $g(x)$, $(f + g)(x)$, and $(f - g)(x)$.

51 **EMPLOYMENT** The number of women and men age 16 and over employed each year in the United States can be modeled by the following equations, where x is the number of years since 1994 and y is the number of people in thousands.

women: $y = 1086.4x + 56{,}610$ men: $y = 999.2x + 66{,}450$

a. Write a function that models the total number of men and women employed in the United States during this time.

b. If f is the function for the number of men, and g is the function for the number of women, what does $(f - g)(x)$ represent?

If $f(x) = x + 2$, $g(x) = -4x + 3$, and $h(x) = x^2 - 2x + 1$, find each value.

52. $(f \cdot g \cdot h)(3)$ **53.** $[(f + g) \cdot h](1)$ **54.** $\left(\frac{h}{fg}\right)(-6)$

55. $[f \circ (g \circ h)](2)$ **56.** $[g \circ (h \circ f)](-4)$ **57.** $[h \circ (f \circ g)](5)$

58. **MULTIPLE REPRESENTATIONS** You will explore $(f \cdot g)(x)$, $\left(\frac{f}{g}\right)(x)$, $[f \circ g](x)$, and $[g \circ f](x)$ if $f(x) = x^2 + 1$ and $g(x) = x - 3$.

a. Tabular Make a table showing values for $(f \cdot g)(x)$, $\left(\frac{f}{g}\right)(x)$, $[f \circ g](x)$, and $[g \circ f](x)$.

b. Graphical Use a graphing calculator to graph $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$ on the same coordinate plane.

c. Verbal Explain the relationship between $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$.

d. Graphical Use a graphing calculator to graph $[f \circ g](x)$, and $[g \circ f](x)$ on the same coordinate plane.

e. Verbal Explain the relationship between $[f \circ g](x)$, and $[g \circ f](x)$.

H.O.T. Problems Use Higher-Order Thinking Skills

59. **OPEN ENDED** Write two functions $f(x)$ and $g(x)$ such that $(f \circ g)(4) = 0$.

60. **CCSS CRITIQUE** Chris and Tobias are finding $(f \circ g)(x)$, where $f(x) = x^2 + 2x - 8$ and $g(x) = x^2 + 8$. Is either of them correct? Explain your reasoning.

Chris

$$(f \circ g)(x) = f[g(x)]$$
$$= (x^2 + 8)^2 + 2x - 8$$
$$= x^4 + 16x^2 + 64 + 2x - 8$$
$$= x^4 + 16x^2 + 2x + 58$$

Tobias

$$(f \circ g)(x) = f[g(x)]$$
$$= (x^2 + 8)^2 + 2(x^2 + 8) - 8$$
$$= x^4 + 16x^2 + 64 + 2x^2 + 16 - 8$$
$$= x^4 + 18x^2 + 72$$

61. **CHALLENGE** Given $f(x) = \sqrt{x^3}$ and $g(x) = \sqrt{x^6}$, determine the domain for each of the following.

a. $g(x) \cdot g(x)$ **b.** $f(x) \cdot f(x)$

62. **REASONING** State whether each statement is *sometimes*, *always*, or *never* true. Explain.

a. The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $f(x)$.

b. The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $g(x)$.

63. **WRITING IN MATH** In the real world, why would you ever perform a composition of functions?

Standardized Test Practice

64. What is the value of x in the equation $7(x - 4) = 44 - 11x$?

A 1
B 2
C 3
D 4

65. If $g(x) = x^2 + 9x + 21$ and $h(x) = 2(x + 5)^2$, which is an equivalent form of $h(x) - g(x)$?

F $k(x) = -x^2 - 11x - 29$
G $k(x) = x^2 + 11x + 29$
H $k(x) = x + 4$
J $k(x) = x^2 + 7x + 11$

66. GRIDDED RESPONSE In his first three years of coaching basketball at North High School, Coach Lucas' team won 8 games the first year, 17 games the second year, and 6 games the third year. How many games does the team need to win in the fourth year so the coach's average will be 10 wins per year?

67. SAT/ACT What is the value of $f[g(6)]$ if $f(x) = 2x + 4$ and $g(x) = x^2 + 5$?

A 38
B 43
C 57
D 86
E 261

Spiral Review

Find all of the rational zeros of each function. (Lesson 5-8)

68. $f(x) = 2x^3 - 13x^2 + 17x + 12$

69. $f(x) = x^3 - 3x^2 - 10x + 24$

70. $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

71. $f(x) = 2x^3 - 5x^2 - 28x + 15$

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function. (Lesson 5-7)

72. $f(x) = 2x^4 - x^3 + 5x^2 + 3x - 9$

73. $f(x) = -4x^4 - x^2 - x + 1$

74. $f(x) = 3x^4 - x^3 + 8x^2 + x - 7$

75. $f(x) = 2x^4 - 3x^3 - 2x^2 + 3$

76. MANUFACTURING A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. How much should be added to each dimension? (Lesson 5-7)

Solve each system of equations. (Lesson 3-4)

77. $x + 4y - z = 6$
$3x + 2y + 3z = 16$
$2x - y + z = 3$

78. $2a + b - c = 5$
$a - b + 3c = 9$
$3a - 6c = 6$

79. $y + z = 4$
$2x + 4y - z = -3$
$3y = -3$

80. INTERNET A webmaster estimates that the time, in seconds, to connect to the server when n people are connecting is given by $t(n) = 0.005n + 0.3$. Estimate the time to connect when 50 people are connecting. (Lesson 2-2)

Skills Review

Solve each equation or formula for the specified variable.

81. $5x - 7y = 12$, for x

82. $3x^2 - 6xy + 1 = 4$, for y

83. $4x + 8yz = 15$, for x

84. $D = mv$, for m

85. $A = k^2 + b$, for k

86. $(x + 2)^2 - (y + 5)^2 = 4$, for y

LESSON 6-2 Inverse Functions and Relations

Then

- You transformed and solved equations for a specific variable.

Now

1. Find the inverse of a function or relation.
2. Determine whether two functions or relations are inverses.

Why?

- The table shows the value of $1 (U.S.) compared to Canadian dollars and Mexican pesos.

The equation $p = 10.75d$ represents the number of pesos p you can receive for every U.S. dollar d. To determine how many U.S. dollars you can receive for one Mexican peso, solve the equation $p = 10.75d$ for d. The result, $d \approx 0.09p$, is the inverse function.

	U.S.	Canada	Mexico
U.S.		1.05	10.75
Canada	0.95		10.26
Mexico	0.09	0.10	

NewVocabulary

inverse relation
inverse function

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.BF.4.a Find inverse functions. - Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Mathematical Practices

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

1 Find Inverses

Recall that a relation is a set of ordered pairs. The **inverse relation** is the set of ordered pairs obtained by exchanging the coordinates of each ordered pair. The domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

Switch: resolve!!!

KeyConcept Inverse Relations

Words Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).

Example A and B are inverse relations.

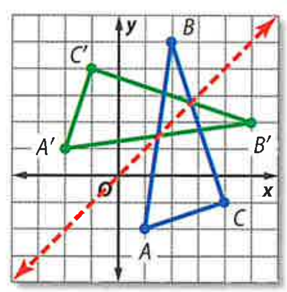
$A = \{(1, 5), (2, 6), (3, 7)\}$ $B = \{(5, 1), (6, 2), (7, 3)\}$

Example 1 Find an Inverse Relation

GEOMETRY **The vertices of $\triangle ABC$ can be represented by the relation $\{(1, -2), (2, 5), (4, -1)\}$. Find the inverse of this relation. Describe the graph of the inverse.**

Graph the relation. To find the inverse, exchange the coordinates of the ordered pairs. The inverse of the relation is $\{(-2, 1), (5, 2), (-1, 4)\}$.

Plotting these points shows that the ordered pairs describe the vertices of $\triangle A'B'C'$ as a reflection of $\triangle ABC$ in the line $y = x$.



GuidedPractice

1. **GEOMETRY** The ordered pairs of the relation $\{(-8, -3), (-8, -6), (-3, -6)\}$ are the coordinates of the vertices of a right triangle. Find the inverse of this relation. Describe the graph of the inverse.

As with relations, the ordered pairs of **inverse functions** are also related. We can write the inverse of the function $f(x)$ as $f^{-1}(x)$.

ReadingMath

CCSS Sense-Making f^{-1} is read *f inverse* or *the inverse of f*. Note that −1 is *not* an exponent.

KeyConcept Property of Inverses

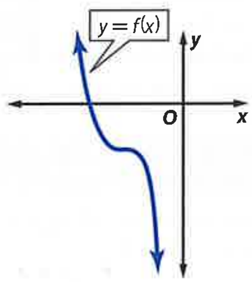
Words If f and f^{-1} are inverses, then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example Let $f(x) = x - 4$ and represent its inverse as $f^{-1}(x) = x + 4$.

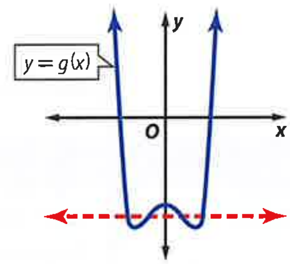
Evaluate $f(6)$.	Evaluate $f^{-1}(2)$.
$f(x) = x - 4$	$f^{-1}(x) = x + 4$
$f(6) = 6 - 4$ or 2	$f^{-1}(2) = 2 + 4$ or 6

Because $f(x)$ and $f^{-1}(x)$ are inverses, $f(6) = 2$ and $f^{-1}(2) = 6$.

When the inverse of a function is a function, the original function is one-to-one. Recall that the vertical line test can be used to determine whether a relation is a function. Similarly, the *horizontal line test* can be used to determine whether the inverse of a function is also a function.



No horizontal line can be drawn so that it passes through more than one point. The inverse of $y = f(x)$ is a function.



A horizontal line can be drawn that passes through more than one point. The inverse of $y = g(x)$ is not a function.

The inverse of a function can be found by exchanging the domain and the range.

PT

Example 2 Find and Graph an Inverse

Find the inverse of each function. Then graph the function and its inverse.

a. $f(x) = 2x - 5$

Step 1 Rewrite the function as an equation relating x and y.

$f(x) = 2x - 5 \rightarrow y = 2x - 5$

Step 2 Exchange x and y in the equation. $x = 2y - 5$

Step 3 Solve the equation for y.

$x = 2y - 5$ Inverse of $y = 2x - 5$

$x + 5 = 2y$ Add 5 to each side.

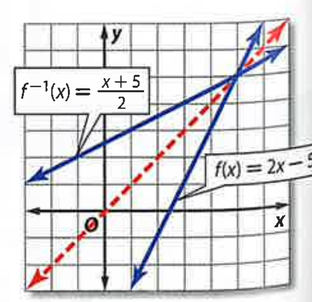
$\frac{x+5}{2} = y$ Divide each side by 2.

Step 4 Replace y with $f^{-1}(x)$.

$y = \frac{x+5}{2} \rightarrow f^{-1}(x) = \frac{x+5}{2}$

The inverse of $f(x) = 2x - 5$ is $f^{-1}(x) = \frac{x+5}{2}$.

The graph of $f^{-1}(x) = \frac{x+5}{2}$ is the reflection of the graph of $f(x) = 2x - 5$ in the line $y = x$.



b. $f(x) = x^2 + 1$

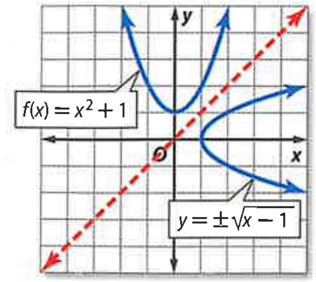
Step 1 $f(x) = x^2 + 1 \rightarrow y = x^2 + 1$

Step 2 $x = y^2 + 1$

Step 3
$$x = y^2 + 1$$
$$x - 1 = y^2$$
$$\pm\sqrt{x-1} = y \quad \text{Take the square root of each side.}$$

Step 4 $y = \pm\sqrt{x-1}$

Graph $y = \pm\sqrt{x-1}$ by reflecting the graph of $f(x) = x^2 + 1$ in the line $y = x$.



StudyTip

Functions The inverse of the function in part **b** is not a function since it does not pass the vertical line test.

GuidedPractice

Find the inverse of each function. Then graph the function and its inverse.

2A. $f(x) = \frac{x-3}{5}$ **2B.** $f(x) = 3x^2$

2 Verifying Inverses

You can determine whether two functions are inverses by finding both of their compositions. If both compositions equal the identity function $I(x) = x$, then the functions are inverse functions.

KeyConcept Inverse Functions

Words	Two functions f and g are inverse functions if and only if both of their compositions are the identity function.
Symbols	$f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 3 Verify that Two Functions are Inverses

PT

Determine whether each pair of functions are inverse functions. Explain your reasoning.

a. $f(x) = 3x + 9$ **and** $g(x) = \frac{1}{3}x - 3$

Verify that the compositions of $f(x)$ and $g(x)$ are identity functions.

$[f \circ g](x) = f[g(x)]$
$= f\left(\frac{1}{3}x - 3\right)$
$= 3\left(\frac{1}{3}x - 3\right) + 9$
$= x - 9 + 9$ or x

$[g \circ f](x) = g[f(x)]$
$= g(3x + 9)$
$= \frac{1}{3}(3x + 9) - 3$
$= x + 3 - 3$ or x

The functions are inverses because $[f \circ g](x) = [g \circ f](x) = x$.

b. $f(x) = 4x^2$ **and** $g(x) = 2\sqrt{x}$

$[f \circ g](x) = f(2\sqrt{x})$
$= 4(2\sqrt{x})^2$
$= 4(4x)$ or $16x$

Because $[f \circ g](x) \neq x$, $f(x)$ and $g(x)$ are not inverses.

WatchOut!

Inverse Functions Be sure to check both $[f \circ g](x)$ and $[g \circ f](x)$ to verify that functions are inverses. By definition, both compositions must be the identity function.

GuidedPractice

3A. $f(x) = 3x - 3$, $g(x) = \frac{1}{3}x + 4$ **3B.** $f(x) = 2x^2 - 1$, $g(x) = \sqrt{\frac{x+1}{2}}$

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Find the inverse of each relation.

1. $\{(-9, 10), (1, -3), (8, -5)\}$

2. $\{(-2, 9), (4, -1), (-7, 9), (7, 0)\}$

Example 2 Find the inverse of each function. Then graph the function and its inverse.

3. $f(x) = -3x$

4. $g(x) = 4x - 6$

5. $h(x) = x^2 - 3$

Example 3 Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

6. $f(x) = x - 7$
$g(x) = x + 7$

7. $f(x) = \frac{1}{2}x + \frac{3}{4}$
$g(x) = 2x - \frac{4}{3}$

8. $f(x) = 2x^3$
$g(x) = \frac{1}{3}\sqrt{x}$

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Find the inverse of each relation.

9. $\{(-8, 6), (6, -2), (7, -3)\}$

10. $\{(7, 7), (4, 9), (3, -7)\}$

11. $\{(8, -1), (-8, -1), (-2, -8), (2, 8)\}$

12. $\{(4, 3), (-4, -4), (-3, -5), (5, 2)\}$

13. $\{(1, -5), (2, 6), (3, -7), (4, 8), (5, -9)\}$

14. $\{(3, 0), (5, 4), (7, -8), (9, 12), (11, 16)\}$

Example 2 CCSS **SENSE-MAKING** Find the inverse of each function. Then graph the function and its inverse.

15. $f(x) = x + 2$

16. $g(x) = 5x$

17. $f(x) = -2x + 1$

18. $h(x) = \frac{x - 4}{3}$

19. $f(x) = -\frac{5}{3}x - 8$

20. $g(x) = x + 4$

21. $f(x) = 4x$

22. $f(x) = -8x + 9$

23. $f(x) = 5x^2$

24. $h(x) = x^2 + 4$

25. $f(x) = \frac{1}{2}x^2 - 1$

26. $f(x) = (x + 1)^2 + 3$

Example 3 Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

27. $f(x) = 2x + 3$
$g(x) = 2x - 3$

28. $f(x) = 4x + 6$
$g(x) = \frac{x - 6}{4}$

29. $f(x) = -\frac{1}{3}x + 3$
$g(x) = -3x + 9$

30. $f(x) = -6x$
$g(x) = \frac{1}{6}x$

31. $f(x) = \frac{1}{2}x + 5$
$g(x) = 2x - 10$

32. $f(x) = \frac{x + 10}{8}$
$g(x) = 8x - 10$

33. $f(x) = 4x^2$
$g(x) = \frac{1}{2}\sqrt{x}$

34. $f(x) = \frac{1}{3}x^2 + 1$
$g(x) = \sqrt{3x - 3}$

35. $f(x) = x^2 - 9$
$g(x) = x + 3$

36. $f(x) = \frac{2}{3}x^3$
$g(x) = \sqrt{\frac{2}{3}x}$

37. $f(x) = (x + 6)^2$
$g(x) = \sqrt{x} - 6$

38. $f(x) = 2\sqrt{x - 5}$
$g(x) = \frac{1}{4}x^2 - 5$

39. **FUEL** The average miles traveled for every gallon g of gas consumed by Leroy's car is represented by the function $m(g) = 28g$.

a. Find a function $c(g)$ to represent the cost per gallon of gasoline.

b. Use inverses to determine the function used to represent the cost per mile traveled in Leroy's car.

40. **SHOES** The shoe size for the average U.S. teen or adult male can be determined using the formula $M(x) = 3x - 22$, where x is length of a foot in measured inches. The shoe size for the average U.S. teen or adult female can be found by using the formula $F(x) = 3x - 21$.

a. Find the inverse of each function.

b. If Lucy wears a size $7\frac{1}{2}$ shoe, how long are her feet?

41 **GEOMETRY** The formula for the area of a circle is $A = \pi r^2$.

a. Find the inverse of the function.

b. Use the inverse to find the radius of a circle with an area of 36 square centimeters.

Use the horizontal line test to determine whether the inverse of each function is also a function.

42. $f(x) = 2x^2$

43. $f(x) = x^3 - 8$

44. $g(x) = x^4 - 6x^2 + 1$

45. $h(x) = -2x^4 - x - 2$

46. $g(x) = x^5 + x^2 - 4x$

47. $h(x) = x^3 + x^2 - 6x + 12$

48. **SHOPPING** Felipe bought a used car. The sales tax rate was 7.25% of the selling price, and he paid \$350 in processing and registration fees. Find the selling price if Felipe paid a total of \$8395.75.

49. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is $F(x) = \frac{9}{5}x + 32$.

a. Find the inverse $F^{-1}(x)$. Show that $F(x)$ and $F^{-1}(x)$ are inverses.

b. Explain what purpose $F^{-1}(x)$ serves.

50. **MEASUREMENT** There are approximately 1.852 kilometers in a nautical mile.

a. Write a function that converts nautical miles to kilometers.

b. Find the inverse of the function that converts kilometers back to nautical miles.

c. Using composition of functions, verify that these two functions are inverses.

51. **MULTIPLE REPRESENTATIONS** Consider the functions $y = x^n$ for $n = 0, 1, 2, \ldots$.

a. **Graphing** Use a graphing calculator to graph $y = x^n$ for $n = 0, 1, 2, 3$, and 4.

b. **Tabular** For which values of n is the inverse a function? Record your results in a table.

c. **Analytical** Make a conjecture about the values of n for which the inverse of $f(x) = x^n$ is a function. Assume that n is a whole number.

H.O.T. Problems Use Higher-Order Thinking Skills

52. **REASONING** If a relation is *not* a function, then its inverse is *sometimes*, *always*, or *never* a function. Explain your reasoning.

53. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses.

54. **CHALLENGE** Give an example of a function that is its own inverse.

55. CCSS **ARGUMENTS** Show that the inverse of a linear function $y = mx + b$, where $m \neq 0$ and $x \neq b$, is also a linear function.

56. **WRITING IN MATH** Suppose you have a composition of two functions that are inverses. When you put in a value of 5 for x, why is the result always 5?

Standardized Test Practice

57. SHORT RESPONSE If the length of a rectangular television screen is 24 inches and its height is 18 inches, what is the length of its diagonal in inches?

58. GEOMETRY If the base of a triangle is represented by $2x + 5$ and the height is represented by $4x$, which expression represents the area of the triangle?

A $(2x + 5) + (4x)$

B $(2x + 5)(4x)$

C $\frac{1}{2}(2x + 5) + (4x)$

D $\frac{1}{2}(2x + 5)(4x)$

59. Which expression represents $f[g(x)]$ if $f(x) = x^2 + 3$ and $g(x) = -x + 1$?

F $x^2 - x + 2$

G $-x^2 - 2$

H $-x^3 + x^2 - 3x + 3$

J $x^2 - 2x + 4$

60. SAT/ACT Which of the following is the inverse of $f(x) = \frac{3x - 5}{2}$?

A $g(x) = \frac{2x + 5}{3}$

B $g(x) = \frac{2x - 5}{3}$

C $g(x) = \frac{3x + 5}{2}$

D $g(x) = 2x + 5$

E $g(x) = \frac{3x - 5}{2}$

Spiral Review

If $f(x) = 3x + 5$, $g(x) = x - 2$, and $h(x) = x^2 - 1$, find each value. (Lesson 6-1)

61. $g[f(3)]$

62. $f[h(-2)]$

63. $h[g(1)]$

64. CONSTRUCTION A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base, and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet? (Lesson 5-8)

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square. (Lesson 4-5)

65. $x^2 + 34x + c$

66. $x^2 - 11x + c$

Simplify. (Lesson 4-4)

67. $(3 + 4i)(5 - 2i)$

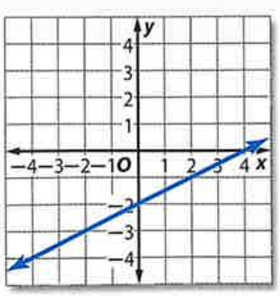
68. $(\sqrt{6} + i)(\sqrt{6} - i)$

69. $\frac{1 + i}{1 - i}$

70. $\frac{4 - 3i}{1 + 2i}$

Determine the rate of change of each graph. (Lesson 2-3)

71.



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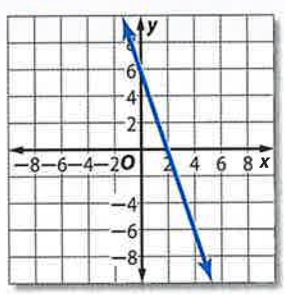

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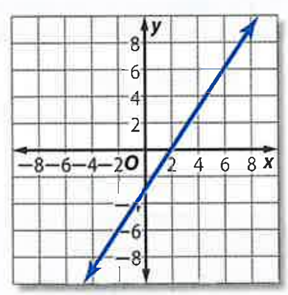


Skills Review

Graph each inequality.

74. $y > \frac{3}{4}x - 2$

75. $y \leq -3x + 2$

76. $y < -x - 4$

Graphing Technology Lab
6-2 Inverse Functions and Relations

You can use a TI-83/84 Plus graphing calculator to compare a function and its inverse using tables and graphs. Note that before you enter any values in the calculator, you should clear all lists.

Activity 1 Graph Inverses with Ordered Pairs

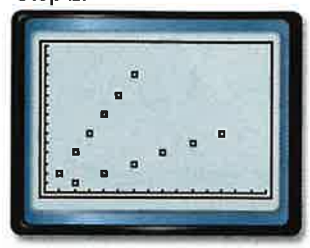
Graph $f(x) = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\}$ and its inverse.

Step 1 Enter the x-values in **L1** and the y-values in **L2**. Then graph the function.

KEYSTROKES: [STAT] [ENTER] 1 [ENTER] 2 [ENTER] 3 [ENTER] 4 [ENTER] 5 [ENTER] 6 [ENTER] [▶] 2 [ENTER] 4 [ENTER] 6 [ENTER] 8 [ENTER] 10 [ENTER] 12 [ENTER] [2nd] [STAT PLOT] [ENTER] [ENTER] [GRAPH]

Adjust the window to reflect the domain and range.

Step 2:



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Step 2 Define the inverse function by setting **Xlist** to **L2** and **Ylist** to **L1**. Then graph the inverse function.

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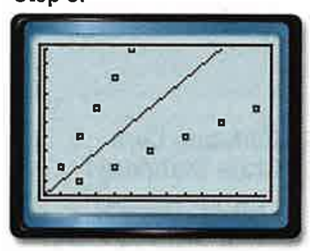
Step 3:



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Step 3 Graph the line $y = x$.

KEYSTROKES: [Y=] [X,T,θ,n] [GRAPH]

Activity 2 Graph Inverses with Function Notation

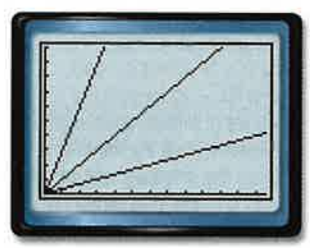
Graph $f(x) = 3x$ and its inverse $g(x) = \frac{x}{3}$.

Step 1 Clear the data from Activity 1.

KEYSTROKES: [2nd] [STAT PLOT] [ENTER] [▶] [ENTER] [▲] [▶] [ENTER] [▶] [ENTER] [2nd] [QUIT]

Step 2 Enter $f(x)$ as **Y1**, $g(x)$ as **Y2**, and $y = x$ as **Y3**. Then graph.

KEYSTROKES: [Y=] 3 [X,T,θ,n] [ENTER] [X,T,θ,n] [÷] 3 [ENTER] [X,T,θ,n] [GRAPH]



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Exercises

Graph each function $f(x)$ and its inverse $g(x)$. Then graph $(f \circ g)(x)$.

1. $f(x) = 5x$
2. $f(x) = x - 3$
3. $f(x) = 2x + 1$
4. $f(x) = \frac{1}{2}x + 3$
5. $f(x) = x^2$
6. $f(x) = x^2 - 3$
7. What is the relationship between the graphs of a function and its inverse?
8. **MAKE A CONJECTURE** For any function $f(x)$ and its inverse $g(x)$, what is $(f \circ g)(x)$?

LESSON 6-3 Square Root Functions and Inequalities

Then

- You simplified expressions with square roots.

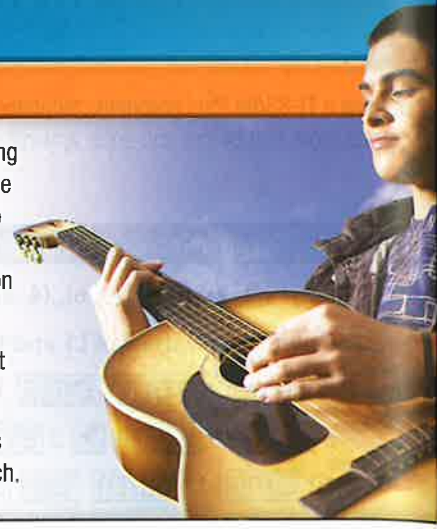
Now

1. Graph and analyze square root functions.
2. Graph square root inequalities.

Why?

- With guitars, pitch is dependent on string length and string tension. The longer the string, the higher the tension needed to produce a desired pitch. Likewise, the heavier the string, the higher the tension needed to reach a desired pitch.

This can be modeled by the square root function $f = \frac{1}{2L}\sqrt{\frac{T}{P}}$, where T is the tension, P is the mass of the string, L is the length of the string, and f is the pitch.



New Vocabulary

square root function
radical function
square root inequality

Common Core State Standards

Content Standards

F.IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

1 Square Root Functions

If a function contains the square root of a variable, it is called a **square root function**. The square root function is a type of **radical function**.

Key Concept Parent Function of Square Root Functions

Parent function: $f(x) = \sqrt{x}$

Domain: $\{x \mid x \geq 0\}$

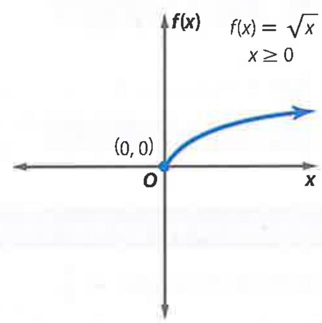
Range: $\{f(x) \mid f(x) \geq 0\}$

Intercepts: $x = 0$, $f(x) = 0$

Not defined: $x < 0$

End behavior: $x \to 0, f(x) \to 0$

$x \to +\infty, f(x) \to +\infty$



The domain of a square root function is limited to values for which the function is defined.

Example 1 Identify Domain and Range

Identify the domain and range of $f(x) = \sqrt{x + 4}$.

The domain only includes values for which the radicand is nonnegative.

$x + 4 \geq 0$ Write an inequality.

$x \geq -4$ Subtract 4 from each side.

Thus, the domain is $\{x \mid x \geq -4\}$.

Find $f(-4)$ to determine the lower limit of the range.

$f(-4) = \sqrt{-4 + 4}$ or 0

So, the range is $\{f(x) \mid f(x) \geq 0\}$.

Guided Practice

Identify the domain and range of each function.

1A. $f(x) = \sqrt{x - 3}$

1B. $f(x) = \sqrt{x + 6} + 2$

The same techniques used to transform the graph of other functions you have studied can be applied to the graphs of square root functions.

KeyConcept Transformations of Square Root Functions

$$f(x) = a\sqrt{x-h} + k$$

h—Horizontal Translation	*k*—Vertical Translation
h units right if *h* is positive $\|h\|$ units left if *h* is negative The domain is $\{x \mid x \geq h\}$.	*k* units up if *k* is positive $\|k\|$ units down if *k* is negative If $a > 0$, then the range is $\{f(x) \mid f(x) \geq k\}$. If $a < 0$, then the range is $\{f(x) \mid f(x) \leq k\}$.

a—Orientation and Shape

- If $a < 0$, the graph is reflected across the *x*-axis.
- If $|a| > 1$, the graph is stretched vertically.
- If $0 < |a| < 1$, the graph is compressed vertically.

StudyTip

Domain and Range The limits on the domain and range also represent the initial point of the graph of a square root function.

Example 2 Graph Square Root Functions

PT

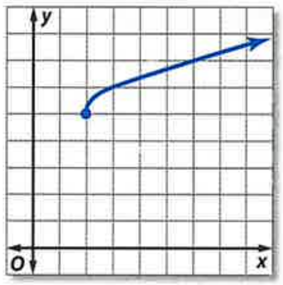
Graph each function. State the domain and range.

a. $y = \sqrt{x-2} + 5$

The minimum point is at $(h, k) = (2, 5)$. Make a table of values for $x \geq 2$, and graph the function. The graph is the same shape as $f(x) = \sqrt{x}$, but is translated 2 units right and 5 units up. Notice the end behavior. As *x* increases, *y* increases.

The domain is $\{x \mid x \geq 2\}$ and the range is $\{y \mid y \geq 5\}$.

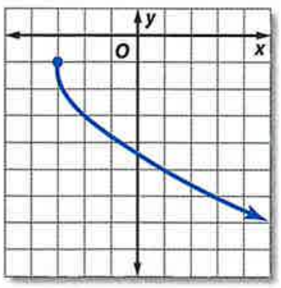
x	y
2	5
3	6
4	6.4
5	6.7
6	7
7	7.2
8	7.4



b. $y = -2\sqrt{x+3} - 1$

The minimum domain value is at *h* or −3. Make a table of values for $x \geq -3$, and graph the function. Because *a* is negative, the graph is similar to the graph of $f(x) = \sqrt{x}$, but is reflected in the *x*-axis. Because $|a| > 1$, the graph is vertically stretched. It is also translated 3 units left and 1 unit down.

x	y
−3	−1
−2	−3
−1	−3.8
0	−4.5
1	−5
2	−5.5
3	−5.9



The domain is $\{x \mid x \geq -3\}$ and the range is $\{y \mid y \leq -1\}$.

GuidedPractice

2A. $f(x) = 2\sqrt{x+4}$

2B. $f(x) = \frac{1}{4}\sqrt{x-5} + 3$

Real-WorldLink

On every string, the guitar player has an option of decreasing the length of the string in about 24 different ways. This will produce 24 different frequencies on each string.

Source: *Guitar World*

Real-World Example 3 Use Graphs to Analyze Square Root Functions

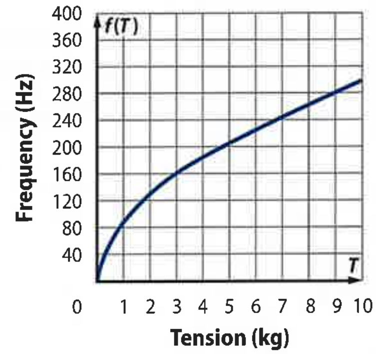
MUSIC **Refer to the application at the beginning of the lesson. The pitch, or frequency, measured in hertz (Hz) of a certain string can be modeled by $f(T) = \frac{1}{1.28}\sqrt{\frac{T}{0.0000708}}$, where T is tension in kilograms.**

a. Graph the function for tension in the domain $\{T \mid 0 \le T \le 10\}$.

Make a table of values for $0 \le T \le 10$ and graph.

T	$y(T)$
0	0
1	92.8
2	131.3
3	160.8
4	185.7
5	207.6

T	$f(T)$
6	227.4
7	245.7
8	262.6
9	278.5
10	293.6



b. How much tension is needed for a pitch of over 200 Hz?

According to the graph and the table, more than 4.5 kilograms of tension is needed for a pitch of more than 200 hertz.

GuidedPractice

3. **MUSIC** The frequency of vibrations for a certain guitar string when it is plucked can be determined by $F = 200\sqrt{T}$, where F is the number of vibrations per second and T is the tension measured in pounds. Graph the function for $0 \le T \le 10$. Then determine the frequency for $T = 3$, 6, and 9 pounds.

Problem-SolvingTip

CCSS Modeling Making a table is a good way to organize ordered pairs in order to see the general behavior of a graph.

2 Square Root Inequalities

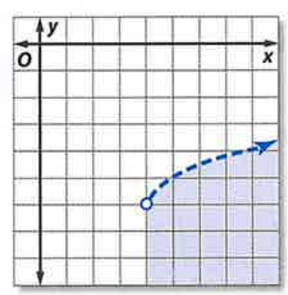
A **square root inequality** is an inequality involving square roots. They are graphed using the same method as other inequalities.

Example 4 Graph a Square Root Inequality

Graph $y < \sqrt{x-4} - 6$.

Graph the boundary $y = \sqrt{x-4} - 6$.

The domain is $\{x \mid x \ge 4\}$. Because y is *less than*, the shaded region should be *below* the boundary and within the domain.



CHECK Select a point in the shaded region, and verify that it is a solution of the inequality.

Test (7, −5): $-5 \stackrel{?}{<} \sqrt{7-4} - 6$

$-5 \stackrel{?}{<} \sqrt{3} - 6$

$-5 < -4.27$ ✓

GuidedPractice

4A. $f(x) \ge \sqrt{2x+1}$

4B. $f(x) < -\sqrt{x+2} - 4$

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 Identify the domain and range of each function.

1. $f(x) = \sqrt{4x}$
2. $f(x) = \sqrt{x - 5}$
3. $f(x) = \sqrt{x + 8} - 2$

Example 2 Graph each function. State the domain and range.

4. $f(x) = \sqrt{x} - 2$
5. $f(x) = 3\sqrt{x - 1}$
6. $f(x) = \frac{1}{2}\sqrt{x + 4} - 1$
7. $f(x) = -\sqrt{3x - 5} + 5$

Example 3

8. **OCEAN** The speed that a tsunami, or tidal wave, can travel is modeled by the equation $v = 356\sqrt{d}$, where v is the speed in kilometers per hour and d is the average depth of the water in kilometers. A tsunami is found to be traveling at 145 kilometers per hour. What is the average depth of the water? Round to the nearest hundredth of a kilometer.

Example 4 Graph each inequality.

9. $f(x) \geq \sqrt{x} + 4$
10. $f(x) \leq \sqrt{x - 6} + 2$
11. $f(x) < -2\sqrt{x + 3}$
12. $f(x) > \sqrt{2x - 1} - 3$

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Identify the domain and range of each function.

13. $f(x) = -\sqrt{2x} + 2$
14. $f(x) = \sqrt{x} - 6$
15. $f(x) = 4\sqrt{x - 2} - 8$
16. $f(x) = \sqrt{x + 2} + 5$
17. $f(x) = \sqrt{x - 4} - 6$
18. $f(x) = -\sqrt{x - 6} + 5$

Example 2 Graph each function. State the domain and range.

19. $f(x) = \sqrt{6x}$
20. $f(x) = -\sqrt{5x}$
21. $f(x) = \sqrt{x - 8}$
22. $f(x) = \sqrt{x + 1}$
23. $f(x) = \sqrt{x + 3} + 2$
24. $f(x) = \sqrt{x - 4} - 10$
25. $f(x) = 2\sqrt{x - 5} - 6$
26. $f(x) = \frac{3}{4}\sqrt{x + 12} + 3$
27. $f(x) = -\frac{1}{5}\sqrt{x - 1} - 4$
28. $f(x) = -3\sqrt{x + 7} + 9$

Example 3

29. **SKYDIVING** The approximate time t in seconds that it takes an object to fall a distance of d feet is given by $t = \sqrt{\frac{d}{16}}$. Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time?

30. **CCSS MODELING** The velocity of a roller coaster as it moves down a hill is $V = \sqrt{v^2 + 64h}$, where v is the initial velocity in feet per second and h is the vertical drop in feet. The designer wants the coaster to have a velocity of 90 feet per second when it reaches the bottom of the hill.
    a. If the initial velocity of the coaster at the top of the hill is 10 feet per second, write an equation that models the situation.
    b. How high should the designer make the hill?

Example 4 **Graph each inequality.**

31. $y < \sqrt{x-5}$

32. $y > \sqrt{x+6}$

33. $y \geq -4\sqrt{x+3}$

34. $y \leq -2\sqrt{x-6}$

35. $y > 2\sqrt{x+7} - 5$

36. $y \geq 4\sqrt{x-2} - 12$

37. $y \leq 6 - 3\sqrt{x-4}$

38. $y < \sqrt{4x-12} + 8$

39. **PHYSICS** The kinetic energy of an object is the energy produced due to its motion and mass. The formula for kinetic energy, measured in joules j, is $E = 0.5mv^2$, where m is the mass in kilograms and v is the velocity of the object in meters per second.

   a. Solve the above formula for v.

   b. If a 1500-kilogram vehicle is generating 1 million joules of kinetic energy, how fast is it traveling?

   c. *Escape velocity* is the minimum velocity at which an object must travel to escape the gravitational field of a planet or other object. Suppose a 100,000-kilogram ship must have a kinetic energy of 3.624×10^{14} joules to escape the gravitational field of Jupiter. Estimate the escape velocity of Jupiter.

40. **CCSS REASONING** After an accident, police can determine how fast a car was traveling before the driver put on his or her brakes by using the equation $v = \sqrt{30fd}$. In this equation, v represents the speed in miles per hour, f represents the coefficient of friction, and d represents the length of the skid marks in feet. The coefficient of friction varies depending on road conditions. Assume that $f = 0.6$.

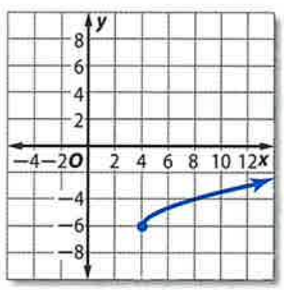
   a. Find the speed of a car that skids 25 feet.

   b. If your car is going 35 miles per hour, how many feet would it take you to stop?

   c. If the speed of a car is doubled, will the skid be twice as long? Explain.

Write the square root function represented by each graph.

41.



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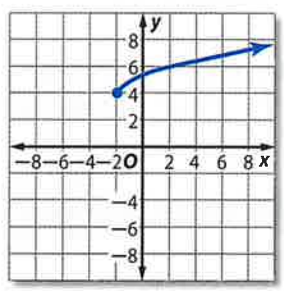

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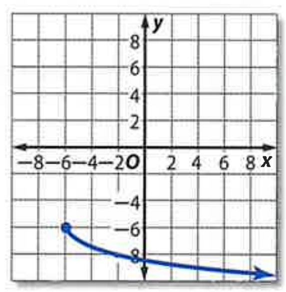


44. **MULTIPLE REPRESENTATIONS** In this problem, you will use the following functions to investigate transformations of square root functions.

$f(x) = 4\sqrt{x-6} + 3$	$g(x) = \sqrt{16x+1} - 6$	$h(x) = \sqrt{x+3} + 2$

   a. **Graphical** Graph each function on the same set of axes.

   b. **Analytical** Identify the transformation on the graph of the parent function. What values caused each transformation?

   c. **Analytical** Which functions appear to be stretched or compressed vertically? Explain your reasoning.

   d. **Verbal** The two functions that are stretched appear to be stretched by the same magnitude. How is this possible?

   e. **Tabular** Make a table of the rate of change for all three functions between 8 and 12 as compared to 12 and 16. What generalization about rate of change in square root functions can be made as a result of your findings?

45 **PENDULUMS** The period of a pendulum can be represented by $T = 2\pi\sqrt{\frac{L}{g}}$, where T is the time in seconds, L is the length in feet, and g is gravity, 32 feet per second squared.

a. Graph the function for $0 \le L \le 10$.

b. What is the period for lengths of 2, 5, and 8 feet?

46. **PHYSICS** Using the function $m = \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$, Einstein's theory of relativity states that the apparent mass m of a particle depends on its velocity v. An object that is traveling extremely fast, close to the speed of light c, will appear to have more mass compared to its mass at rest, m_0.

a. Use a graphing calculator to graph the function for a 10,000-kilogram ship for the domain $0 \le v \le 300{,}000{,}000$. Use 300 million meters per second for the speed of light.

b. What viewing window did you use to view the graph?

c. Determine the apparent mass m of the ship for speeds of 100 million, 200 million, and 299 million meters per second.

H.O.T. Problems Use Higher-Order Thinking Skills

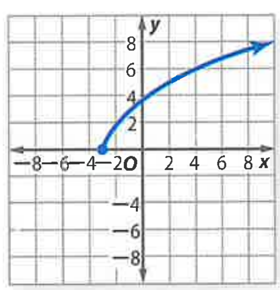
47. **CHALLENGE** Write an equation for a square root function with a domain of $\{x \mid x \ge -4\}$, a range of $\{y \mid y \le 6\}$, and that passes through (5, 3).

48. **REASONING** For what positive values of a are the domain and range of $f(x) = \sqrt[a]{x}$ the set of real numbers?

49. **OPEN ENDED** Write a square root function for which the domain is $\{x \mid x \ge 8\}$ and the range is $\{y \mid y \le 14\}$.

50. **WRITING IN MATH** Explain why there are limitations on the domain and range of square root functions.

51. **CCSS CRITIQUE** Cleveland thinks that the graph and the equation represent the same function. Molly disagrees. Who is correct? Explain your reasoning.



$y = \sqrt{5x + 10}$

52. **WRITING IN MATH** Explain why $y = \pm\sqrt{x}$ is not a function.

53. **OPEN ENDED** Write an equation of a relation that contains a radical and its inverse such that:

a. the original relation is a function, and its inverse is not a function.

b. the original relation is not a function, and its inverse is a function.

Standardized Test Practice

54. The expression $-\frac{64x^6}{8x^3}$, $x \neq 0$, is equivalent to

A $8x^2$ **C** $-8x^2$

B $8x^3$ **D** $-8x^3$

55. PROBABILITY For a game, Patricia must roll a standard die and draw a card from a deck of 26 cards, each card having a letter of the alphabet on it. What is the probability that Patricia will roll an odd number and draw a letter in her name?

F $\frac{2}{3}$ **H** $\frac{1}{13}$

G $\frac{3}{26}$ **J** $\frac{1}{26}$

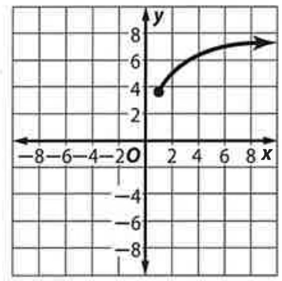
56. SHORT RESPONSE What is the product of $(d + 6)$ and $(d - 3)$?

57. SAT/ACT Given the graph of the square root function below, which must be true?

I. The domain is all real numbers.

II. The function is $y = \sqrt{x} + 3.5$.

III. The range is $\{y \mid y \geq 3.5\}$.



A I only **D** II only

B I, II, and III **E** III only

C II and III only

Spiral Review

Determine whether each pair of functions are inverse functions. Write *yes* or *no*. (Lesson 6-2)

58. $f(x) = 2x$
$g(x) = \frac{1}{2}x$

59. $f(x) = 3x - 7$
$g(x) = \frac{1}{3}x - \frac{7}{16}$

60. $f(x) = \frac{3x + 2}{5}$
$g(x) = \frac{5x - 2}{3}$

61. TIME The formula $h = \frac{m}{60}$ converts minutes m to hours h, and $d = \frac{h}{24}$ converts hours h to days d. Write a function that converts minutes to days. (Lesson 6-1)

62. CABLE TV The number of households in the United States with cable TV after 1985 can be modeled by the function $C(t) = -43.2t^2 + 1343t + 790$, where t represents the number of years since 1985. (Lesson 5-4)

a. Graph this equation for the years 1985 to 2005.

b. Describe the turning points of the graph and its end behavior.

c. What is the domain of the function? Use the graph to estimate the range for the function.

d. What trends in households with cable TV does the graph suggest? Is it reasonable to assume that the trend will continue indefinitely?

Skills Review

Determine whether each number is *rational* or *irrational*.

63. 6.34 **64.** 3.787887888... **65.** 5.333... **66.** 1.25

LESSON 6-4

nth Roots

Then

- You worked with square root functions.

Now

1. Simplify radicals.
2. Use a calculator to approximate radicals.

Why?

- According to a world-wide injury prevention study, the number of collisions between bicycles and automobiles increased as the number of bicycles per intersection increased. The relationship can be expressed using the equation $c = \sqrt[5]{b^2}$, where b is the number of bicycles and c is the number of collisions.

New Vocabulary

*n*th root
radical sign
index
radicand
principal root

Common Core State Standards

Content Standards
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Mathematical Practices
6 Attend to precision.

1 Simplify Radicals

Finding the square root of a number and squaring a number are inverse operations. To find the square root of a number a, you must find a number with a square of a. Similarly, the inverse of raising a number to the nth power is finding the **nth root** of a number.

Powers	Factors	Words	Roots
$x^3 = 64$	$4 \cdot 4 \cdot 4 = 64$	4 is a cube root of 64.	$\sqrt[3]{64} = 4$
$x^4 = 625$	$5 \cdot 5 \cdot 5 \cdot 5 = 625$	5 is a fourth root of 625.	$\sqrt[4]{625} = 5$
$x^5 = 32$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	2 is a fifth root of 32.	$\sqrt[5]{32} = 2$
$a^n = b$	$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a} = b$	a is an nth root of b.	$\sqrt[n]{b} = a$

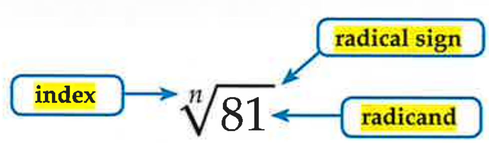
This pattern suggests the following formal definition of an nth root.

KeyConcept Definition of *n*th Root

Words For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.

Example Because $(-3)^4 = 81$, -3 is a fourth root of 81 and 3 is a principal root.

The symbol $\sqrt[n]{}$ indicates an nth root.



Some numbers have more than one real nth root. For example, 64 has two square roots, 8 and -8, since 8^2 and $(-8)^2$ both equal 64. When there is more than one real root and n is even, the nonnegative root is called the **principal root**.

Some examples of nth roots are listed below.

$\sqrt{25} = 5$ — $\sqrt{25}$ indicates the principal square root of 25.

$-\sqrt{25} = -5$ — $-\sqrt{25}$ indicates the opposite of the principal square root of 25.

$\pm\sqrt{25} = \pm 5$ — $\pm\sqrt{25}$ indicates both square roots of 25.

KeyConcept Real nth Roots

Suppose n is an integer greater than 1, and a is a real number.

a	n is even.	n is odd.
$a > 0$	1 unique positive and 1 unique negative real root: $\pm\sqrt[n]{a}$; positive root is principal root	1 unique positive and 0 negative real roots: $\sqrt[n]{a}$
$a < 0$	0 real roots	0 positive and 1 negative real root: $\sqrt[n]{a}$
$a = 0$	1 real root: $\sqrt[n]{0} = 0$	1 real root: $\sqrt[n]{0} = 0$

ReviewVocabulary

pure imaginary numbers Square roots of negative real numbers; for any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$, or bi, where i is the imaginary unit.

Example 1 Find Roots

Simplify.

a. $\pm\sqrt{16y^4}$

$$\pm\sqrt{16y^4} = \pm\sqrt{(4y^2)^2}$$
$$= \pm 4y^2$$

The square roots of $16y^4$ are $\pm 4y^2$.

b. $-\sqrt{(x^2-6)^8}$

$$-\sqrt{(x^2-6)^8} = -\sqrt{[(x^2-6)^4]^2}$$
$$= -(x^2-6)^4$$

The opposite of the principal square root of $(x^2-6)^8$ is $-(x^2-6)^4$.

c. $\sqrt[5]{243a^{20}b^{25}}$

$$\sqrt[5]{243a^{20}b^{25}} = \sqrt[5]{(3a^4b^5)^5}$$
$$= 3a^4b^5$$

The fifth root of $243a^{20}b^{25}$ is $3a^4b^5$.

d. $\sqrt{-16x^4y^8}$

$$\sqrt[2]{-16x^4y^8}$$

n is even. b is negative.

There are no real roots since $\sqrt{-16}$ is not a real number. However, there are two imaginary roots, $4ix^2y^4$ and $-4ix^2y^4$.

StudyTip

Odd Index If n is odd, there is only one real root. Therefore, there is no principal root when n is odd, and absolute value symbols are never needed.

GuidedPractice

1A. $\pm\sqrt{36x^{10}}$

1B. $-\sqrt{(y+7)^{16}}$

When you find an even root of an even power and the result is an odd power, you must use the absolute value of the result to ensure that the answer is nonnegative.

Example 2 Simplify Using Absolute Value

Simplify.

a. $\sqrt[4]{y^4}$

$$\sqrt[4]{y^4} = |y|$$

Since y could be negative, you must take the absolute value of y to identify the principal root.

b. $\sqrt[6]{64(x^2-3)^{18}}$

$$\sqrt[6]{64(x^2-3)^{18}} = 2|(x^2-3)^3|$$

Since the index 6 is even and the exponent 3 is odd, you must use absolute value.

GuidedPractice

2A. $\sqrt{36y^6}$

2B. $\sqrt[4]{16(x-3)^{12}}$

2 Approximate Radicals with a Calculator

Recall that real numbers that cannot be expressed as terminating or repeating decimals are irrational numbers. Approximations for irrational numbers are often used in real-world problems.

Real-WorldLink

77% of the employees in China commute by bicycle.

Source: International Bicycle Fund

Real-World Example 3 Approximate Radicals

INJURY PREVENTION **Refer to the beginning of the lesson.**

a. **If $c = \sqrt[5]{b^2}$ represents the number of collisions and b represents the number of bicycle riders per intersection, estimate the number of collisions at an intersection that has 1000 bicycle riders per week.**

Understand You want to find out how many collisions there are.

Plan Let 1000 be the number of bicycle riders. The number of collisions is c.

Solve

$c = \sqrt[5]{b^2}$ — Original formula

$= \sqrt[5]{1000^2}$ — $b = 1000$

≈ 15.85 — Use a calculator.

There are about 16 collisions per week at the intersection.

Check

$15.85 \stackrel{?}{=} \sqrt[5]{b^2}$ — $c = 15.85$

$15.85^5 \stackrel{?}{=} b^2$ — Raise each side to the fifth power.

$1{,}000{,}337 \stackrel{?}{=} b^2$ — Simplify.

$1000 \approx b$ ✓ — Take the positive square root of each side.

b. **If the total number of collisions reported in one week is 21, estimate the number of bicycle riders that passed through that intersection.**

$c = \sqrt[5]{b^2}$ — Original formula

$21 = \sqrt[5]{b^2}$ — $c = 21$

$21^5 = b^2$ — Raise each side to the fifth power.

$4{,}084{,}101 = b^2$ — Simplify.

$2021 \approx b$ — Take the positive square root of each side.

GuidedPractice

3A. The surface area of a sphere can be determined from the volume of the sphere using the formula $S = \sqrt[3]{36\pi V^2}$, where V is the volume. Determine the surface area of a sphere with a volume of 200 cubic inches.

3B. If the surface area of a sphere is about 214.5 square inches, determine the volume.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Examples 1–2 **Simplify.**

1. $\pm\sqrt{100y^8}$

2. $-\sqrt{49u^8v^{12}}$

3. $\sqrt{(y-6)^8}$

4. $\sqrt[4]{16g^{16}h^{24}}$

5. $\sqrt{-16y^4}$

6. $\sqrt[6]{64(2y+1)^{18}}$

Example 3 **Use a calculator to approximate each value to three decimal places.**

7. $\sqrt{58}$

8. $-\sqrt{76}$

9. $\sqrt[5]{-43}$

10. $\sqrt[4]{71}$

11. **CCSS PERSEVERANCE** The radius r of the orbit of a television satellite is given by $\sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of Earth, and t is the time it takes the satellite to complete one orbit. Find the radius of the satellite's orbit if G is $6.67 \times 10^{-11} N \cdot m^2/kg^2$, M is 5.98×10^{24} kg, and t is 2.6×10^6 seconds.

Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–2 Simplify.

12. $\pm\sqrt{121x^4y^{16}}$
13. $\pm\sqrt{225a^{16}b^{36}}$
14. $\pm\sqrt{49x^4}$
15. $-\sqrt{16c^4d^2}$
16. $-\sqrt{81a^{16}b^{20}c^{12}}$
17. $-\sqrt{400x^{32}y^{40}}$
18. $\sqrt{(x+15)^4}$
19. $\sqrt{(x^2+6)^{16}}$
20. $\sqrt{(a^2+4a)^{12}}$
21. $\sqrt[3]{8a^6b^{12}}$
22. $\sqrt[6]{d^{24}x^{36}}$
23. $\sqrt[3]{27b^{18}c^{12}}$
24. $-\sqrt{(2x+1)^6}$
25. $\sqrt{-(x+2)^8}$
26. $\sqrt[3]{-(y-9)^9}$
27. $\sqrt[6]{x^{18}}$
28. $\sqrt[4]{a^{12}}$
29. $\sqrt[3]{a^{12}}$
30. $\sqrt[4]{81(x+4)^4}$
31. $\sqrt[3]{(4x-7)^{24}}$
32. $\sqrt[3]{(y^3+5)^{18}}$
33. $\sqrt[4]{256(5x-2)^{12}}$
34. $\sqrt[8]{x^{16}y^8}$
35. $\sqrt[5]{32a^{15}b^{10}}$

Example 3

36. **SHIPPING** An online book store wants to increase the size of the boxes it uses to ship orders. The new volume N is equal to the old volume V times the scale factor F cubed, or $N = V \cdot F^3$. What is the scale factor if the old volume was 0.8 cubic feet and the new volume is 21.6 cubic feet?

37. **GEOMETRY** The side length of a cube is determined by $r = \sqrt[3]{V}$, where V is the volume in cubic units. Determine the side length of a cube with a volume of 512 cm^3.

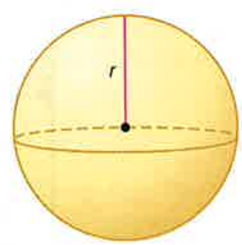
Use a calculator to approximate each value to three decimal places.

38. $\sqrt{92}$
39. $-\sqrt{150}$
40. $\sqrt{0.43}$
41. $\sqrt{0.62}$
42. $\sqrt[3]{168}$
43. $\sqrt[5]{-4382}$
44. $\sqrt[6]{(8912)^2}$
45. $\sqrt[5]{(4756)^2}$

46. **GEOMETRY** The radius r of a sphere with volume V can be found using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$.

    a. Determine the radius for volumes of 1000 cm^3, 8000 cm^3, and 64,000 cm^3.

    b. How does the volume of the sphere change if the radius is doubled? Explain.



Simplify.

47. $\sqrt{196c^6d^4}$
48. $\sqrt{-64y^8z^6}$
49. $\sqrt[3]{-27a^{15}b^9}$
50. $\sqrt[4]{-16x^{16}y^8}$
51. $\sqrt{400x^{16}y^6}$
52. $\sqrt[3]{8c^3d^{12}}$
53. $\sqrt[3]{64(x+y)^6}$
54. $\sqrt[5]{-(y-z)^{15}}$

55. **PHYSICS** Johannes Kepler developed the formula $d = \sqrt[3]{6t^2}$, where d is the distance of a planet from the Sun in millions of miles and t is the number of Earth-days that it takes for the planet to orbit the Sun. If the length of a year on Mars is 687 Earth-days, how far from the Sun is Mars?

56. **CCSS SENSE-MAKING** All matter is composed of atoms. The nucleus of an atom is the center portion of the atom that contains most of the mass of the atom. A theoretical formula for the radius r of the nucleus of an atom is $r = (1.3 \times 10^{-15})\sqrt[3]{A}$ meters, where A is the mass number of the nucleus. Find the radius of the nucleus for each atom in the table.

Atom	Mass Number
carbon	6
oxygen	8
sodium	11
aluminum	13
chlorine	17

57. **BIOLOGY** Kleiber's Law, $P = 73.3\sqrt[4]{m^3}$, shows the relationship between the mass m in kilograms of an organism and its metabolism P in Calories per day. Determine the metabolism for each of the animals listed at the right.

Animal	Mass (kg)
bald eagle	4.5
golden retriever	30
komodo dragon	72
bottlenose dolphin	156
Asian elephant	2300

58. **MULTIPLE REPRESENTATIONS** In this problem, you will use $f(x) = x^n$ and $g(x) = \sqrt[n]{x}$ to explore inverses.

a. Tabular Make tables for $f(x)$ and $g(x)$ using $n = 3$ and $n = 4$.

b. Graphical Graph the equations.

c. Analytical Which equations are functions? Which functions are one-to-one?

d. Analytical For what values of n are $g(x)$ and $f(x)$ inverses of each other?

e. Verbal What conclusions can you make about $g(x) = \sqrt[n]{x}$ and $f(x) = x^n$ for all positive even values of n? for odd values of n?

H.O.T. Problems Use Higher-Order Thinking Skills

59. **CCSS CRITIQUE** Ashley and Kimi are simplifying $\sqrt[4]{16x^4y^8}$. Is either of them correct? Explain your reasoning.

Ashley

$$\sqrt[4]{16x^4y^8} = \sqrt[4]{(2xy^2)^4} = 2|xy^2|$$

Kimi

$$\sqrt[4]{16x^4y^8} = \sqrt[4]{(2xy^2)^4} = 2y^2|x|$$

60. **CHALLENGE** Under what conditions is $\sqrt{x^2 + y^2} = x + y$ true?

61. **REASONING** Determine whether the statement $\sqrt[4]{(-x)^4} = x$ is *sometimes*, *always*, or *never* true.

62. **CHALLENGE** For what real values of x is $\sqrt[3]{x} > x$?

63. **OPEN ENDED** Write a number for which the principal square root and cube root are both integers.

64. **WRITING IN MATH** Explain when and why absolute value symbols are needed when taking an nth root.

65. **CHALLENGE** Write an equivalent expression for $\sqrt[3]{2x} \cdot \sqrt[3]{8y}$. Simplify the radical.

CHALLENGE Simplify each expression.

66. $\sqrt[4]{0.0016}$

67. $\sqrt[7]{-0.0000001}$

68. $\dfrac{\sqrt[5]{-0.00032}}{\sqrt[3]{-0.027}}$

69. **CHALLENGE** Solve $-\dfrac{5}{\sqrt{a}} = -125$ for a.

Standardized Test Practice

70. What is the value of w in the equation $\frac{1}{2}(4w + 36) = 3(4w - 3)$?

A 2

B 2.7

C 27

D 36

71. What is the product of the complex numbers $(5 + \boldsymbol{i})$ and $(5 - \boldsymbol{i})$?

F 24

G 26

H $25 - \boldsymbol{i}$

J $26 - 10\boldsymbol{i}$

72. EXTENDED RESPONSE A cylindrical cooler has a diameter of 9 inches and a height of 11 inches. Tate plans to use it for soda cans that have a diameter of 2.5 inches and a height of 4.75 inches.

a. Tate plans to place two layers consisting of 9 cans each into the cooler. What is the volume of the space that will not be filled with the cans?

b. Find the ratio of the volume of the cooler to the volume of the cans in part **a**.

73. SAT/ACT Which of the following is closest to $\sqrt[3]{7.32}$?

A 1.8

B 1.9

C 2.0

D 2.1

E 2.2

Spiral Review

Graph each function. (Lesson 6-3)

74. $y = \sqrt{x - 5}$

75. $y = \sqrt{x} - 2$

76. $y = 3\sqrt{x} + 4$

77. HEALTH The average weight of a baby born at a certain hospital is $7\frac{1}{2}$ pounds and the average length is 19.5 inches. One kilogram is about 2.2 pounds and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters. (Lesson 6-2)

Simplify. (Lesson 5-1)

78. $(4c - 5) - (c + 11) + (-6c + 17)$

79. $(11x^2 + 13x - 15) - (7x^2 - 9x + 19)$

80. $(d - 5)(d + 3)$

81. $(2a^2 + 6)^2$

82. GAS MILEAGE The gas mileage y in miles per gallon for a certain vehicle is given by the equation $y = 10 + 0.9x - 0.01x^2$, where x is the speed of the vehicle between 10 and 75 miles per hour. Find the range of speeds that would give a gas mileage of at least 25 miles per gallon. (Lesson 4-8)

Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function. (Lesson 4-7)

83. $y = -6(x + 2)^2 + 3$

84. $y = -\frac{1}{3}x^2 + 8x$

85. $y = (x - 2)^2 - 2$

86. $y = 2x^2 + 8x + 10$

Skills Review

Find each product.

87. $(x + 4)(x + 5)$

88. $(y - 3)(y + 4)$

89. $(a + 2)(a - 9)$

90. $(a - b)(a - 3b)$

91. $(x + 2y)(x - y)$

92. $2(w + z)(w - 4z)$

EXTEND 6-4

Graphing Technology Lab
Graphing *n*th Root Functions

You can use a graphing calculator to graph *n*th root functions.

CCSS Common Core State Standards

Content Standards

F.IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Mathematical Practices

5 Use appropriate tools strategically.

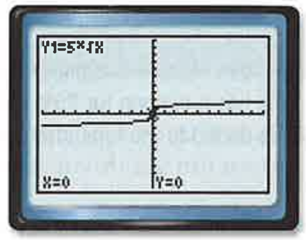
Example 1 Graph an *n*th Root Function

Graph $y = \sqrt[5]{x}$.

Enter the equation as **Y1** and graph.

KEYSTROKES: [Y=] 5 [MATH] 5 [X,T,θ,*n*] [GRAPH]

Another way to enter the equation is to use $y = x^{\frac{1}{5}}$. You will learn about this later in Chapter 6.



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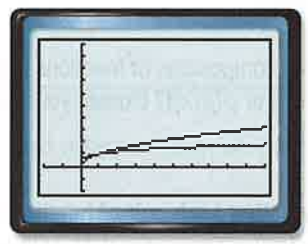
Example 2 *n*th Root Functions with Different Roots

Graph and compare $y = \sqrt{x}$ and $y = \sqrt[4]{x}$.

Enter $y = \sqrt{x}$ as **Y1** and $y = \sqrt[4]{x}$ as **Y2**. Change the viewing window. Then graph.

KEYSTROKES: [Y=] [CLEAR] [2nd] [√] [X,T,θ,*n*] [ENTER] 4 [MATH] 5 [X,T,θ,*n*] [GRAPH] [WINDOW] [(−)] 2 [ENTER] 10 [ENTER] 1 [ENTER] [(−)] 2 [ENTER] 10 [ENTER] 1



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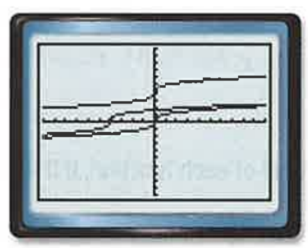
Example 3 *n*th Root Functions with Different Radicands

Graph and compare $y = \sqrt[3]{x}$, $y = \sqrt[3]{x + 4}$, and $y = \sqrt[3]{x} + 4$.

Enter $y = \sqrt[3]{x}$ as **Y1**, $y = \sqrt[3]{x + 4}$ as **Y2**, and $y = \sqrt[3]{x} + 4$ as **Y3**. Then graph in the standard viewing window.

KEYSTROKES: [Y=] [CLEAR] 3 [MATH] 5 [X,T,θ,*n*] [ENTER] [CLEAR] 3 [MATH] 5 [( ] [X,T,θ,*n*] [+] 4 [ )] [ENTER] 3 [MATH] 5 [X,T,θ,*n*] [+] 4 [ENTER] [ZOOM] 6



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Exercises

Graph each function.

1. $y = \sqrt[4]{x}$
2. $y = \sqrt[4]{x + 2}$
3. $y = \sqrt[4]{x} + 2$
4. $y = \sqrt[5]{x}$
5. $y = \sqrt[5]{x - 5}$
6. $y = \sqrt[5]{x} - 5$
7. What is the effect of adding or subtracting a constant under the radical sign?
8. What is the effect of adding or subtracting a constant outside the radical sign?

CHAPTER 6 Mid-Chapter Quiz

Lessons 6-1 through 6-4

Given $f(x) = 2x^2 + 4x - 3$ and $g(x) = 5x - 2$, find each function. (Lesson 6-1)

1. $(f + g)(x)$
2. $(f - g)(x)$
3. $(f \cdot g)(x)$
4. $\left(\frac{f}{g}\right)(x)$
5. $[f \circ g](x)$
6. $[g \circ f](x)$

7. **SHOPPING** Mrs. Ross is shopping for her children's school clothes. She has a coupon for 25% off her total. The sales tax of 6% is added to the total after the coupon is applied. (Lesson 6-1)

   a. Express the total price after the discount and the total price after the tax using function notation. Let x represent the price of the clothing, $p(x)$ represent the price after the 25% discount, and $g(x)$ represent the price after the tax is added.

   b. Which composition of functions represents the final price, $p[g(x)]$ or $g[p(x)]$? Explain your reasoning.

Determine whether each pair of functions are inverse functions. Write *yes* or *no*. (Lesson 6-2)

8. $f(x) = 2x + 16$
   $g(x) = \frac{1}{2}x - 8$
9. $g(x) = 4x + 15$
   $h(x) = \frac{1}{4}x - 15$
10. $f(x) = x^2 - 5$
    $g(x) = 5 + x^{-2}$
11. $g(x) = -6x + 8$
    $h(x) = \frac{8 - x}{6}$

Find the inverse of each function, if it exists. (Lesson 6-2)

12. $h(x) = \frac{2}{5}x + 8$
13. $f(x) = \frac{4}{9}(x - 3)$
14. $h(x) = -\frac{10}{3}(x + 5)$
15. $f(x) = \frac{x + 12}{7}$

16. **JOBS** Louise runs a lawn care service. She charges \$25 for supplies plus \$15 per hour. The function $f(h) = 15h + 25$ gives the cost $f(h)$ for h hours of work. (Lesson 6-2)

    a. Find $f^{-1}(h)$. What is the significance of $f^{-1}(h)$?

    b. If Louise charges a customer \$85, how many hours did she work?

Graph each inequality. (Lesson 6-3)

17. $y < \sqrt{x - 5}$
18. $y \leq -2\sqrt{x}$
19. $y > \sqrt{x + 9} + 3$
20. $y \geq \sqrt{x + 4} - 5$

Graph each function. State the domain and range of each function. (Lesson 6-3)

21. $y = 2 + \sqrt{x}$
22. $y = \sqrt{x + 4} - 1$

23. **MULTIPLE CHOICE** What is the domain of $f(x) = \sqrt{2x + 5}$? (Lesson 6-3)

    A $\left\{x \middle| x > \frac{5}{2}\right\}$

    B $\left\{x \middle| x > -\frac{5}{2}\right\}$

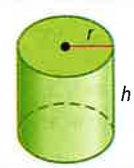
    C $\left\{x \middle| x \geq \frac{5}{2}\right\}$

    D $\left\{x \middle| x \geq -\frac{5}{2}\right\}$

Simplify. (Lesson 6-4)

24. $\pm\sqrt{121a^4b^{18}}$
25. $\sqrt{(x^4 + 3)^{12}}$
26. $\sqrt[3]{27(2x - 5)^{15}}$
27. $\sqrt[5]{-(y - 6)^{20}}$
28. $\sqrt[3]{8(x + 4)^6}$
29. $\sqrt[4]{16(y + x)^8}$

30. **MULTIPLE CHOICE** The radius of the cylinder below is equal to the height of the cylinder. The radius r can be found using the formula $r = \sqrt[3]{\frac{V}{\pi}}$. Find the radius of the cylinder if the volume is 500 cubic inches. (Lesson 6-4)



    F 2.53 inches

    G 5.42 inches

    H 7.94 inches

    J 24.92 inches

31. **PRODUCTION** The cost in dollars of producing p cell phones in a factory is represented by $C(p) = 5p + 60$. The number of cell phones produced in h hours is represented by $P(h) = 40h$. (Lesson 6-1)

    a. Find the composition function.

    b. Determine the cost of producing cell phones for 8 hours.

LESSON

6-5 Operations with Radical Expressions

Then

- You simplified expressions with nth roots.

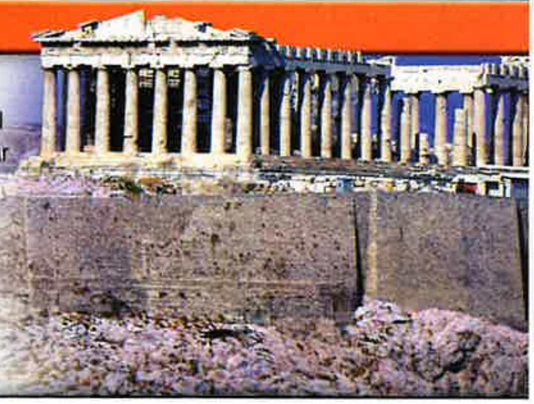
Now

- **1** Simplify radical expressions.
- **2** Add, subtract, multiply, and divide radical expressions.

Why?

- Golden rectangles have been used by artists and architects to create beautiful designs. Many golden rectangles appear in the Parthenon in Athens, Greece. The ratio of the lengths of the sides of a golden rectangle is $\frac{2}{\sqrt{5}-1}$.

  In this lesson, you will learn to simplify radical expressions like $\frac{2}{\sqrt{5}-1}$.



NewVocabulary

rationalizing the denominator
like radical expressions
conjugate

Common Core State Standards

Content Standards

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Simplify Radicals

The properties you have used to simplify radical expressions involving square roots also hold true for expressions involving nth roots.

KeyConcept Product Property of Radicals

Words	For any real numbers a and b and any integer $n > 1$, $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, if n is even and a and b are both nonnegative or if n is odd.
Examples	$\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$ or 4 and $\sqrt[3]{3} \cdot \sqrt[3]{9} = \sqrt[3]{27}$ or 3

In order for a radical to be in simplest form, the radicand must contain no factors that are nth powers of an integer or polynomial.

PT

Example 1 Simplify Expressions with the Product Property

Simplify.

a. $\sqrt{32x^8}$

$$\sqrt{32x^8} = \sqrt{4^2 \cdot 2 \cdot (x^4)^2} \quad \text{Factor into squares.}$$
$$= \sqrt{4^2} \cdot \sqrt{(x^4)^2} \cdot \sqrt{2} \quad \text{Product Property of Radicals}$$
$$= 4x^4\sqrt{2} \quad \text{Simplify.}$$

b. $\sqrt[4]{16a^{24}b^{13}}$

$$\sqrt[4]{16a^{24}b^{13}} = \sqrt[4]{2^4 \cdot (a^6)^4(b^3)^4 \cdot b} \quad \text{Factor into squares.}$$
$$= \sqrt[4]{2^4} \cdot \sqrt[4]{(a^6)^4} \cdot \sqrt[4]{(b^3)^4} \cdot \sqrt[4]{b} \quad \text{Product Property of Radicals}$$
$$= 2a^6|b^3|\sqrt[4]{b} \quad \text{Simplify.}$$

In this case, the absolute value symbols are not necessary because in order for $\sqrt[4]{16a^{24}b^{13}}$ to be defined, b must be nonnegative.

Thus, $\sqrt[4]{16a^{24}b^{13}} = 2a^6b^3\sqrt[4]{b}$.

GuidedPractice

1A. $\sqrt{12c^6d^3}$

1B. $\sqrt[3]{27y^{12}z^7}$

Photodisc/PunchStock

The Quotient Property of Radicals is another property used to simplify radicals.

KeyConcept Quotient Property of Radicals

Words For any real numbers a and $b \neq 0$ and any integer $n > 1$,
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

Examples $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9}$ or 3 $\qquad \sqrt[3]{\frac{x^6}{8}} = \frac{\sqrt[3]{x^6}}{\sqrt[3]{8}} = \frac{x^2}{2}$ or $\frac{1}{2}x^2$

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

If the denominator is:	Multiply the numerator and denominator by:	Examples
$\sqrt{b}$	$\sqrt{b}$	$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{5}{\sqrt[3]{2}} = \frac{5}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$ or $\frac{5\sqrt[3]{4}}{2}$

PT

Example 2 Simplify Expressions with the Quotient Property

Simplify.

a. $\sqrt{\frac{x^6}{y^7}}$

$\sqrt{\frac{x^6}{y^7}} = \frac{\sqrt{x^6}}{\sqrt{y^7}}$ Quotient Property

$= \frac{\sqrt{(x^3)^2}}{\sqrt{(y^3)^2 \cdot y}}$ Factor into squares.

$= \frac{\sqrt{(x^3)^2}}{\sqrt{(y^3)^2} \cdot \sqrt{y}}$ Product Property

$= \frac{|x^3|}{y^3\sqrt{y}}$ Simplify.

$= \frac{|x^3|}{y^3\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$ Rationalize the denominator.

$= \frac{|x^3|\sqrt{y}}{y^4}$ $\sqrt{y} \cdot \sqrt{y} = y$

b. $\sqrt[4]{\frac{6}{5x}}$

$\sqrt[4]{\frac{6}{5x}} = \frac{\sqrt[4]{6}}{\sqrt[4]{5x}}$ Quotient Property

$= \frac{\sqrt[4]{6}}{\sqrt[4]{5x}} \cdot \frac{\sqrt[4]{5^3x^3}}{\sqrt[4]{5^3x^3}}$ Rationalize the denominator.

$= \frac{\sqrt[4]{6 \cdot 5^3x^3}}{\sqrt[4]{5x \cdot 5^3x^3}}$ Product Property

$= \frac{\sqrt[4]{750x^3}}{\sqrt[4]{5^4x^4}}$ Multiply.

$= \frac{\sqrt[4]{750x^3}}{5x}$ $\sqrt[4]{5^4x^4} = 5x$

GuidedPractice

2A. $\frac{\sqrt{a^9}}{\sqrt{b^5}}$

2B. $\sqrt[5]{\frac{3}{4y}}$

Here is a summary of the rules used to simplify radicals.

ConceptSummary Simplifying Radical Expressions

A radical expression is in simplified form when the following conditions are met.

- The index n is as small as possible.
- The radicand contains no factors (other than 1) that are nth powers of an integer or polynomial.
- The radicand contains no fractions.
- No radicals appear in a denominator.

2 Operations with Radicals

You can use the Product and Quotient Properties to multiply and divide some radicals.

StudyTip

CCSS Regularity Exact roots occur when the powers of the constants and variables are all identical to or multiples of the index. For example, $\sqrt[3]{2} \cdot \sqrt[3]{2^2} = \sqrt[3]{2^3}$ or 2.

Example 3 Multiply Radicals

Simplify $5\sqrt[3]{-12ab^4} \cdot 3\sqrt[3]{18a^2b^2}$.

$5\sqrt[3]{-12ab^4} \cdot 3\sqrt[3]{18a^2b^2} = 5 \cdot 3 \cdot \sqrt[3]{-12ab^4 \cdot 18a^2b^2}$	Product Property of Radicals
$= 15 \cdot \sqrt[3]{-2^2 \cdot 3 \cdot ab^4 \cdot 2 \cdot 3^2 \cdot a^2b^2}$	Factor constants.
$= 15 \cdot \sqrt[3]{-2^3 \cdot 3^3 \cdot a^3b^6}$	Group into cubes if possible.
$= 15 \cdot \sqrt[3]{-2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{a^3} \cdot \sqrt[3]{b^6}$	Product Property of Radicals
$= 15 \cdot (-2) \cdot 3 \cdot a \cdot b^2$	Simplify.
$= -90ab^2$	Multiply.

GuidedPractice

Simplify.

3A. $6\sqrt{8c^3d^5} \cdot 4\sqrt{2cd^3}$

3B. $2\sqrt[4]{8x^3y^2} \cdot 3\sqrt[4]{2x^5y^2}$

Radicals can be added and subtracted in the same manner as monomials. In order to add or subtract, the radicals must be like terms. Radicals are **like radical expressions** if *both* the index and the radicand are identical.

Like: $\sqrt{3b}$ and $4\sqrt{3b}$ — Unlike: $\sqrt{3b}$ and $\sqrt[3]{3b}$ — Unlike: $\sqrt{2b}$ and $\sqrt{3b}$

StudyTip

Adding and Subtracting Radicals Simplify the individual radicals before attempting to combine like terms.

Example 4 Add and Subtract Radicals

Simplify $\sqrt{98} - 2\sqrt{32}$.

$\sqrt{98} - 2\sqrt{32} = \sqrt{2 \cdot 7^2} - 2\sqrt{4^2 \cdot 2}$	Factor using squares.
$= \sqrt{7^2} \cdot \sqrt{2} - 2 \cdot \sqrt{4^2} \cdot \sqrt{2}$	Product Property
$= 7\sqrt{2} - 2 \cdot 4 \cdot \sqrt{2}$	Simplify radicals.
$= 7\sqrt{2} - 8\sqrt{2}$	Multiply.
$= -\sqrt{2}$	$(7 - 8)\sqrt{2} = (-1)(\sqrt{2})$

GuidedPractice

4A. $4\sqrt{8} + 3\sqrt{50}$

4B. $5\sqrt{12} + 2\sqrt{27} - \sqrt{128}$

Just as you can add and subtract radicals like monomials, you can multiply radicals usin
the FOIL method as you do when multiplying binomials.

Example 5 Multiply Radicals

Simplify $(4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6)$.

$$(4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6) = \overset{F}{4\sqrt{3} \cdot 3\sqrt{2}} + \overset{O}{4\sqrt{3} \cdot (-6)} + \overset{I}{5\sqrt{2} \cdot 3\sqrt{2}} + \overset{L}{5\sqrt{2} \cdot (-6)}$$

$$= 12\sqrt{3 \cdot 2} - 24\sqrt{3} + 15\sqrt{2^2} - 30\sqrt{2} \quad \text{Product Property}$$

$$= 12\sqrt{6} - 24\sqrt{3} + 30 - 30\sqrt{2} \quad \text{Simplify.}$$

GuidedPractice

Simplify.

5A. $(6\sqrt{3} - 5)(2\sqrt{5} + 4\sqrt{2})$

5B. $(7\sqrt{2} - 3\sqrt{3})(7\sqrt{2} + 3\sqrt{3})$

StudyTip

Conjugates The product of conjugates is always a rational number.

Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates** of each other. You can use conjugates to rationalize denominators.

Real-World Example 6 Use a Conjugate to Rationalize a Denominator

ARCHITECTURE **Refer to the beginning of the lesson. Use a conjugate to rationalize the denominator and simplify $\frac{2}{\sqrt{5} - 1}$.**

$$\frac{2}{\sqrt{5} - 1} = \frac{2}{\sqrt{5} - 1} \cdot \frac{\sqrt{5} + 1}{\sqrt{5} + 1} \quad \sqrt{5} + 1 \text{ is the conjugate of } \sqrt{5} - 1.$$

$$= \frac{2\sqrt{5} + 2(1)}{(\sqrt{5})^2 + 1(\sqrt{5}) - 1(\sqrt{5}) - 1(1)} \quad \text{Multiply.}$$

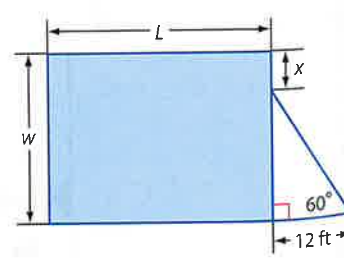
$$= \frac{2\sqrt{5} + 2}{5 + \sqrt{5} - \sqrt{5} - 1} \quad \text{Simplify.}$$

$$= \frac{2\sqrt{5} + 2}{4} \quad \text{Subtract.}$$

$$= \frac{\sqrt{5} + 1}{2} \quad \text{Simplify.}$$

GuidedPractice

6. **GEOMETRY** The area of the rectangle at the right is 900 ft^2. Write and simplify an equation for L in terms of x.



Math HistoryLink

Theano (c. 5th century B.C.) Theano is believed to have been the wife of Pythagoras. It is also believed that she directed a famous mathematics academy and carried on the work of Pythagoras after his death. Her most important work was on the idea of the golden mean, which is the irrational number $\frac{1 + \sqrt{5}}{2}$.

Source: The Granger Collection, New York

Check Your Understanding

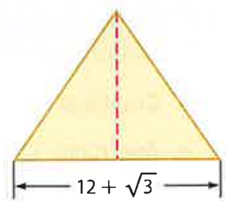
= Step-by-Step Solutions begin on page R14.

Examples 1–5 **CCSS PRECISION** Simplify.

1. $\sqrt{36ab^4c^5}$
2. $\sqrt{144x^7y^5}$
3. $\frac{\sqrt{c^5}}{\sqrt{d^9}}$
4. $\sqrt[4]{\frac{5x}{8y}}$
5. $5\sqrt{2x} \cdot 3\sqrt{8x}$
6. $4\sqrt{5a^5} \cdot \sqrt{125a^3}$
7. $3\sqrt[3]{36xy} \cdot 2\sqrt[3]{6x^2y^2}$
8. $\sqrt[4]{3x^3y^2} \cdot \sqrt[4]{27xy^2}$
9. $5\sqrt{32} + \sqrt{27} + 2\sqrt{75}$
10. $4\sqrt{40} + 3\sqrt{28} - \sqrt{200}$
11. $(4 + 2\sqrt{5})(3\sqrt{3} + 4\sqrt{5})$
12. $(8\sqrt{3} - 2\sqrt{2})(8\sqrt{3} + 2\sqrt{2})$
13. $\frac{5}{\sqrt{2} + 3}$
14. $\frac{8}{\sqrt{6} - 5}$
15. $\frac{4 + \sqrt{2}}{\sqrt{2} - 3}$
16. $\frac{6 - \sqrt{3}}{\sqrt{3} + 4}$

Example 6

17. **GEOMETRY** Find the altitude of the triangle if the area is $189 + 4\sqrt{3}$ square centimeters.

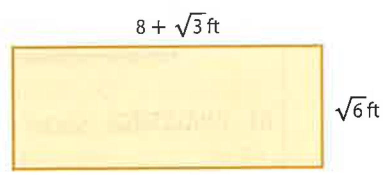


Practice and Problem Solving

Extra Practice is on page R6.

Examples 1–4 Simplify.

18. $\sqrt{72a^8b^5}$
19. $\sqrt{9a^{15}b^3}$
20. $\sqrt{24a^{16}b^8c}$
21. $\sqrt{18a^6b^3c^5}$
22. $\frac{\sqrt{5a^5}}{\sqrt{b^{13}}}$
23. $\sqrt{\frac{7x}{10y^3}}$
24. $\frac{\sqrt[3]{6x^2}}{\sqrt[3]{5y}}$
25. $\sqrt[4]{\frac{7x^3}{4b^2}}$
26. $3\sqrt{5y} \cdot 8\sqrt{10yz}$
27. $2\sqrt{32a^3b^5} \cdot \sqrt{8a^7b^2}$
28. $6\sqrt{3ab} \cdot 4\sqrt{24ab^3}$
29. $5\sqrt{x^8y^3} \cdot 5\sqrt{2x^5y^4}$
30. $3\sqrt{90} + 4\sqrt{20} + \sqrt{162}$
31. $9\sqrt{12} + 5\sqrt{32} - \sqrt{72}$
32. $4\sqrt{28} - 8\sqrt{810} + \sqrt{44}$
33. $3\sqrt{54} + 6\sqrt{288} - \sqrt{147}$
34. **GEOMETRY** Find the perimeter of the rectangle.
35. **GEOMETRY** Find the area of the rectangle.
36. **GEOMETRY** Find the exact surface area of a sphere with radius of $4 + \sqrt{5}$ inches.



Examples 5–6 Simplify.

37. $(7\sqrt{2} - 3\sqrt{3})(4\sqrt{6} + 3\sqrt{12})$
38. $(8\sqrt{5} - 6\sqrt{3})(8\sqrt{5} + 6\sqrt{3})$
39. $(12\sqrt{10} - 6\sqrt{5})(12\sqrt{10} + 6\sqrt{5})$
40. $(6\sqrt{3} + 5\sqrt{2})(2\sqrt{6} + 3\sqrt{8})$
41. $\frac{6}{\sqrt{3} - \sqrt{2}}$
42. $\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}}$
43. $\frac{9 - 2\sqrt{3}}{\sqrt{3} + 6}$
44. $\frac{2\sqrt{2} + 2\sqrt{5}}{\sqrt{5} + \sqrt{2}}$

Simplify.

45. $\sqrt[3]{16y^4z^{12}}$

46. $\sqrt[3]{-54x^6y^{11}}$

47. $\sqrt[4]{162a^6b^{13}c}$

48. $\sqrt[4]{48a^9b^3c^{16}}$

49. $\sqrt[4]{\dfrac{12x^3y^2}{5a^2b}}$

50. $\dfrac{\sqrt[3]{36xy^2}}{\sqrt[3]{10xz}}$

51. $\dfrac{x+1}{\sqrt{x}-1}$

52. $\dfrac{x-2}{\sqrt{x^2-4}}$

53. $\dfrac{\sqrt{x}}{\sqrt{x^2-1}}$

54. **APPLES** The diameter of an apple is related to its weight and can be modeled by the formula $d = \sqrt[3]{3w}$, where d is the diameter in inches and w is the weight in ounces. Find the diameter of an apple that weighs 6.47 ounces.

Simplify each expression if b is an even number.

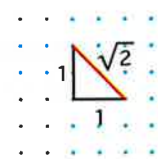
55. $\sqrt[b]{a^b}$

56. $\sqrt[b]{a^{4b}}$

57. $\sqrt[b]{a^{2b}}$

58. $\sqrt[b]{a^{3b}}$

59. **MULTIPLE REPRESENTATIONS** In this problem, you will explore operations with like radicals.



a. **Numerical** Copy the diagram at the right on dot paper. Use the Pythagorean Theorem to prove that the length of the red segment is $\sqrt{2}$ units.

b. **Graphical** Extend the segment to represent $\sqrt{2} + \sqrt{2}$.

c. **Analytical** Use your drawing to show that $\sqrt{2} + \sqrt{2} \neq \sqrt{2+2}$ or 2.

d. **Graphical** Use the dot paper to draw a square with side lengths $\sqrt{2}$ units.

e. **Numerical** Prove that the area of the square is $\sqrt{2} \cdot \sqrt{2} = 2$ square units.

H.O.T. Problems Use Higher-Order Thinking Skills

60. **ERROR ANALYSIS** Twyla and Ben are simplifying $4\sqrt{32} + 6\sqrt{18}$. Is either of them correct? Explain your reasoning.

Twyla

$4\sqrt{32} + 6\sqrt{18}$
$= 4\sqrt{4^2 \cdot 2} + 6\sqrt{3^2 \cdot 2}$
$= 16\sqrt{2} + 18\sqrt{2}$
$= 34\sqrt{2}$

Ben

$4\sqrt{32} + 6\sqrt{18}$
$= 4\sqrt{16 \cdot 2} + 6\sqrt{9 \cdot 2}$
$= 64\sqrt{2} + 54\sqrt{2}$
$= 118\sqrt{2}$

61. **CHALLENGE** Show that $\dfrac{-1 - i\sqrt{3}}{2}$ is a cube root of 1.

62. **CCSS ARGUMENTS** For what values of a is $\sqrt{a} \cdot \sqrt{-a}$ a real number? Explain.

63. **CHALLENGE** Find four combinations of whole numbers that satisfy $\sqrt[a]{256} = b$.

64. **OPEN ENDED** Find a number other than 1 that has a positive whole number for a square root, cube root, and fourth root.

65. **WRITING IN MATH** Explain why absolute values may be unnecessary when an nth root of an even power results in an odd power.

Standardized Test Practice

66. PROBABILITY A six-sided number cube has faces with the numbers 1 though 6 marked on it. What is the probability that a number less than 4 will occur on one toss of the number cube?

A $\frac{1}{2}$
C $\frac{1}{4}$
B $\frac{1}{3}$
D $\frac{1}{5}$

67. When the number of a year is divisible by 4, the year is a leap year. However, when the year is divisible by 100, the year is not a leap year, unless the year is divisible by 400. Which is *not* a leap year?

F 1884
H 1904
G 1900
J 1940

68. SHORT RESPONSE Which property is illustrated by $4x + 0 = 4x$?

69. SAT/ACT The expression $\sqrt{180a^2b^8}$ is equivalent to which of the following?

A $3\sqrt{10}\,|a|b^4$
B $5\sqrt{6}\,|a|b^4$
C $6\sqrt{5}\,|a|b^4$
D $18\sqrt{10}\,|a|b^4$
E $36\sqrt{5}\,|a|b^4$

Spiral Review

Simplify. (Lesson 6-4)

70. $\sqrt{81x^6}$
71. $\sqrt[3]{729a^3b^9}$
72. $\sqrt{(g+5)^2}$

73. Graph $y \leq \sqrt{x-2}$. (Lesson 6-3)

Solve each equation. (Lesson 5-5)

74. $x^4 - 34x^2 + 225 = 0$
75. $x^4 - 15x^2 - 16 = 0$
76. $x^4 + 6x^2 - 27 = 0$
77. $x^3 + 64 = 0$
78. $27x^3 + 1 = 0$
79. $8x^3 - 27 = 0$

80. MODELS A model car builder is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could be the width of the table? (Lesson 4-8)

81. CONSTRUCTION Cho charges \$1500 to build a small deck and \$2500 to build a large deck. During the spring and summer, she built 5 more small decks than large decks. If she earned \$23,500 how many of each type of deck did she build? (Lesson 3-8)

82. FOOD The Hot Dog Grille offers the lunch combinations shown. Assume that the price of a combo meal is the same price as purchasing each item separately. Find the prices for a hot dog, a soda, and a bag of potato chips. (Lesson 3-4)

Lunch Combo Meals	
1. Two hot dogs, one soda	\$5.40
2. One hot dog, potato chips, one soda	\$4.35
3. Two hot dogs, two bags of chips	\$5.70

Skills Review

Evaluate each expression.

83. $2\left(\frac{1}{6}\right)$
84. $3\left(\frac{1}{8}\right)$
85. $\frac{1}{4} + \frac{1}{3}$
86. $\frac{1}{2} + \frac{3}{8}$
87. $\frac{2}{3} - \frac{1}{4}$
88. $\frac{5}{6} - \frac{2}{5}$

LESSON 6-6 Rational Exponents

Then

- You used properties of exponents.

Now

- **1** Write expressions with rational exponents in radical form and vice versa.
- **2** Simplify expressions in exponential or radical form.

Why?

- The formula $C = c(1 + r)^n$ can be used to estimate the future cost of an item due to inflation. C represents the future cost, c represents the current cost, r is the rate of inflation, and n is the number of years for the projection.

  For example, $C = c(1 + r)^{\frac{1}{2}}$ can be used to estimate the cost of a video game system in six months.

CCSS Common Core State Standards

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Rational Exponents and Radicals

You know that squaring a number and taking the square root of a number are inverse operations. But how would you evaluate an expression that contains a fractional exponent such as the one above? You can investigate such an expression by assuming that fractional exponents behave as integral exponents.

$$\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \quad \text{Write as a multiplication expression.}$$

$$= b^{\frac{1}{2}+\frac{1}{2}} \quad \text{Add the exponents.}$$

$$= b^1 \text{ or } b \quad \text{Simplify.}$$

Thus, $b^{\frac{1}{2}}$ is a number with a square equal to b. So $b^{\frac{1}{2}} = \sqrt{b}$.

KeyConcept $b^{\frac{1}{n}}$

Words	For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even. When $b < 0$ and n is even, a complex root may exist.
Examples	$27^{\frac{1}{3}} = \sqrt[3]{27}$ or 3 $\quad (-16)^{\frac{1}{2}} = \sqrt{-16}$ or $4i$

PT

Example 1 Radical and Exponential Forms

Simplify.

a. Write $x^{\frac{1}{6}}$ in radical form.

$x^{\frac{1}{6}} = \sqrt[6]{x}$ Definition of $b^{\frac{1}{n}}$

b. Write $\sqrt[4]{z}$ in exponential form.

$\sqrt[4]{z} = z^{\frac{1}{4}}$ Definition of $b^{\frac{1}{n}}$

Guided Practice

1A. Write $a^{\frac{1}{5}}$ in radical form.

1B. Write $\sqrt[8]{c}$ in exponential form.

1C. Write $d^{\frac{7}{4}}$ in radical form.

1D. Write $\sqrt[3]{c^{-5}}$ in exponential form.

ReadingMath

Power Functions Square root functions are also *power functions* when the exponent is a fraction. In the case of square roots, the exponent is $\frac{1}{2}$.

The rules for negative exponents also apply to negative rational exponents.

Example 2 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. $81^{-\frac{1}{4}}$

$$81^{-\frac{1}{4}} = \frac{1}{81^{\frac{1}{4}}} \qquad b^{-n} = \frac{1}{b^n}$$

$$= \frac{1}{\sqrt[4]{81}} \qquad 81^{\frac{1}{4}} = \sqrt[4]{81}$$

$$= \frac{1}{\sqrt[4]{3^4}} \qquad 81 = 3^4$$

$$= \frac{1}{3} \qquad \text{Simplify.}$$

b. $216^{\frac{2}{3}}$

$$216^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} \qquad 216 = 6^3$$

$$= 6^{3 \cdot \frac{2}{3}} \qquad \text{Power of a Power}$$

$$= 6^2 \qquad \text{Multiply exponents.}$$

$$= 36 \qquad \text{Simplify.}$$

GuidedPractice

2A. $-3125^{-\frac{1}{5}}$

2B. $256^{\frac{3}{8}}$

Examples **2a** and **2b** use the definition of $b^{\frac{1}{n}}$ and the properties of powers to evaluate an expression. Both methods suggest the following general definition of rational exponents

KeyConcept Rational Exponents

Words For any real nonzero number b, and any integers x and y, with $y > 1$, $b^{\frac{x}{y}} = \sqrt[y]{b^x} = \left(\sqrt[y]{b}\right)^x$, except when $b < 0$ and y is even. When $b < 0$ and y is even, a complex root may exist.

Examples $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2$ or 9 $\qquad (-16)^{\frac{3}{2}} = \left(\sqrt{-16}\right)^3 = (4i)^3$ or $-64i$

Real-WorldCareer

Economist Economists are employed in banking, finance, accounting, commerce, marketing, and business administration. Politicians often consult economists before enacting policy. Economists use shopping as one way of judging the economy. A bachelor's or master's degree in economics is required.

Real-World Example 3 Solve Equations with Rational Exponents

FINANCIAL LITERACY **Refer to the beginning of the lesson. Suppose a video game system costs $390 now. How much would the price increase in six months with an annual inflation rate of 5.3%?**

$$C = c(1 + r)^n \qquad \text{Original formula}$$

$$= 390(1 + 0.053)^{\frac{1}{2}} \qquad c = 390,\ r = 0.053,\ \text{and } n = \frac{6 \text{ months}}{12 \text{ months}} \text{ or } \frac{1}{2}$$

$$\approx 400.20 \qquad \text{Use a calculator.}$$

In six months the price of the video game system will be $400.20 − $390.00 or $10.20 more than its current price.

GuidedPractice

3. Suppose a gallon of milk costs $2.99 now. How much would the price increase in 9 months with an annual inflation rate of 5.3%?

2 Simplify Expressions

All of the properties of powers you learned in Lesson 6-1 apply to rational exponents. Write each expression with all positive exponents. Also, any exponents in the denominator of a fraction must be positive *integers*. So, it may be necessary to rationalize a denominator.

StudyTip

Simplifying Expressions When simplifying expressions containing rational exponents, leave the exponent in rational form rather than writing the expression as a radical.

Example 4 Simplify Expressions with Rational Exponents

Simplify each expression.

a. $a^{\frac{2}{7}} \cdot a^{\frac{4}{7}}$

$a^{\frac{2}{7}} \cdot a^{\frac{4}{7}} = a^{\frac{2}{7}+\frac{4}{7}}$ Add powers.

$= a^{\frac{6}{7}}$ Add exponents.

b. $b^{-\frac{5}{6}}$

$b^{-\frac{5}{6}} = \dfrac{1}{b^{\frac{5}{6}}}$ $\quad b^{-n} = \dfrac{1}{b^n}$

$= \dfrac{1}{b^{\frac{5}{6}}} \cdot \dfrac{b^{\frac{1}{6}}}{b^{\frac{1}{6}}}$ $\quad \dfrac{b^{\frac{1}{6}}}{b^{\frac{1}{6}}} = 1$

$= \dfrac{b^{\frac{1}{6}}}{b^{\frac{6}{6}}}$ $\quad b^{\frac{5}{6}} \cdot b^{\frac{1}{6}} = b^{\frac{5}{6}+\frac{1}{6}}$

$= \dfrac{b^{\frac{1}{6}}}{b}$ $\quad b^{\frac{6}{6}} = b^1$ or b

GuidedPractice

4A. $p^{\frac{1}{4}} \cdot p^{\frac{9}{4}}$

4B. $r^{-\frac{4}{5}}$

When simplifying a radical expression, always use the least index possible. Using rational exponents makes this process easier, but the answer should be written in radical form.

Example 5 Simplify Radical Expressions

Simplify each expression.

a. $\dfrac{\sqrt[4]{27}}{\sqrt{3}}$

$\dfrac{\sqrt[4]{27}}{\sqrt{3}} = \dfrac{27^{\frac{1}{4}}}{3^{\frac{1}{2}}}$ Rational exponents

$= \dfrac{(3^3)^{\frac{1}{4}}}{3^{\frac{1}{2}}}$ $\quad 27 = 3^3$

$= \dfrac{3^{\frac{3}{4}}}{3^{\frac{1}{2}}}$ Power of a Power

$= 3^{\frac{3}{4}-\frac{1}{2}}$ Quotient of Powers

$= 3^{\frac{1}{4}}$ Simplify.

$= \sqrt[4]{3}$ Rewrite in radical form.

b. $\sqrt[3]{64z^6}$

$\sqrt[3]{64z^6} = (64z^6)^{\frac{1}{3}}$ Rational expone

$= (8^2 \cdot z^6)^{\frac{1}{3}}$ $\quad 64 = 8^2$

$= 8^{\frac{2}{3}} \cdot z^{\frac{6}{3}}$ Power of a Pow

$= 4z^2$ $\quad 8^{\frac{2}{3}} = 4$

c. $\dfrac{x^{\frac{1}{2}} - 2}{3x^{\frac{1}{2}} + 2}$

$\dfrac{x^{\frac{1}{2}} - 2}{3x^{\frac{1}{2}} + 2} = \dfrac{x^{\frac{1}{2}} - 2}{3x^{\frac{1}{2}} + 2} \cdot \dfrac{3x^{\frac{1}{2}} - 2}{3x^{\frac{1}{2}} - 2}$ — $3x^{\frac{1}{2}} - 2$ is the conjugate of $3x^{\frac{1}{2}} + 2$.

$= \dfrac{3x^{\frac{2}{2}} - 8x^{\frac{1}{2}} + 4}{9x^{\frac{2}{2}} - 4}$ — Multiply.

$= \dfrac{3x - 8x^{\frac{1}{2}} + 4}{9x - 4}$ — Simplify.

GuidedPractice

5A. $\dfrac{\sqrt[4]{32}}{\sqrt[3]{2}}$ **5B.** $\sqrt[3]{16x^4}$ **5C.** $\dfrac{y^{\frac{1}{2}} + 2}{y^{\frac{1}{2}} - 2}$

StudyTip

Radical Expressions Write the simplified expression in the same form as the beginning expression. When you start with a radical expression, end with a radical expression. When you start with an expression with rational exponents, end with an expression with rational exponents.

ConceptSummary Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no exponents that are not positive integers in the denominator.
- It is not a complex fraction.
- The index of any remaining radical is the least number possible.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 Write each expression in radical form, or write each radical in exponential form.

1. $10^{\frac{1}{4}}$ **2.** $x^{\frac{3}{5}}$ **3.** $\sqrt[3]{15}$ **4.** $\sqrt[4]{7x^6y^9}$

Example 2 Evaluate each expression.

5. $343^{\frac{1}{3}}$ **6.** $32^{-\frac{1}{5}}$ **7.** $125^{\frac{2}{3}}$ **8.** $\dfrac{24}{4^{\frac{3}{2}}}$

Example 3 **9** **GARDENING** If the area A of a square is known, then the lengths of its sides ℓ can be computed using $\ell = A^{\frac{1}{2}}$. You have purchased a 169 ft² share in a community garden for the season. What is the length of one side of your square garden?

Examples 4–5 **CCSS PRECISION** Simplify each expression.

10. $a^{\frac{3}{4}} \cdot a^{\frac{1}{2}}$ **11.** $\dfrac{x^{\frac{4}{5}}}{x^{\frac{1}{5}}}$ **12.** $\dfrac{b^3}{c^{\frac{1}{2}}} \cdot \dfrac{c}{b^{\frac{1}{3}}}$

13. $\sqrt[4]{9g^2}$ **14.** $\dfrac{\sqrt[5]{64}}{\sqrt[5]{4}}$ **15.** $\dfrac{g^{\frac{1}{2}} - 1}{g^{\frac{1}{2}} + 1}$

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Write each expression in radical form, or write each radical in exponential form.

16. $8^{\frac{1}{5}}$ **17.** $4^{\frac{2}{7}}$ **18.** $a^{\frac{3}{4}}$ **19.** $(x^3)^{\frac{3}{2}}$

20. $\sqrt{17}$ **21.** $\sqrt[4]{63}$ **22.** $\sqrt[3]{5xy^2}$ **23.** $\sqrt[4]{625x^2}$

Example 2 Evaluate each expression.

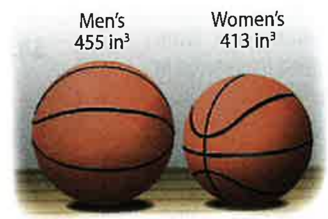
24. $27^{\frac{1}{3}}$ **25.** $256^{\frac{1}{4}}$ **26.** $16^{-\frac{1}{2}}$ **27.** $81^{-\frac{1}{4}}$

Example 3

28. CCSS **SENSE-MAKING** A women's regulation-sized basketball is slightly smaller than a men's basketball. The radius r of the ball that holds V cubic units of air is $\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$.

a. Find the radius of a women's basketball.

b. Find the radius of a men's basketball.



29. **GEOMETRY** The radius r of a sphere with volume V is given by $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$. Find the radius of a ball with a volume of 77 cm^3.

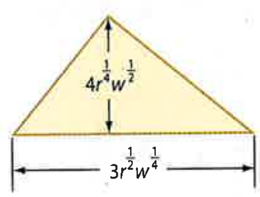
Examples 4–5 Simplify each expression.

30. $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$ **31.** $a^{\frac{4}{9}} \cdot a^{\frac{1}{4}}$ **32.** $b^{-\frac{3}{4}}$ **33.** $y^{-\frac{4}{5}}$ **34.** $\frac{\sqrt[8]{81}}{\sqrt[6]{3}}$

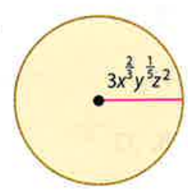
35. $\frac{\sqrt[4]{27}}{\sqrt[4]{3}}$ **36.** $\sqrt[4]{25x^2}$ **37.** $\sqrt[6]{81g^3}$ **38.** $\frac{h^{\frac{1}{2}} + 1}{h^{\frac{1}{2}} - 1}$ **39.** $\frac{x^{\frac{1}{4}} + 2}{x^{\frac{1}{4}} - 2}$

GEOMETRY Find the area of each figure.

40.



41.



42. Find the simplified form of $18^{\frac{1}{2}} + 2^{\frac{1}{2}} - 32^{\frac{1}{2}}$.

43. What is the simplified form of $64^{\frac{1}{3}} - 32^{\frac{1}{3}} + 8^{\frac{1}{3}}$?

Simplify each expression.

44. $a^{\frac{7}{4}} \cdot a^{\frac{5}{4}}$ **45.** $x^{\frac{2}{3}} \cdot x^{\frac{8}{3}}$ **46.** $\left(b^{\frac{3}{4}}\right)^{\frac{1}{3}}$ **47.** $\left(y^{-\frac{3}{5}}\right)^{-\frac{1}{4}}$

48. $\sqrt[4]{64}$ **49.** $\sqrt[6]{216}$ **50.** $d^{-\frac{5}{6}}$ **51.** $w^{-\frac{7}{8}}$

52. **WILDLIFE** A population of 100 deer is reintroduced to a wildlife preserve. Suppose the population does extremely well and the deer population doubles in two years. Then the number D of deer after t years is given by $D = 100 \cdot 2^{\frac{t}{2}}$.

a. How many deer will there be after $4\frac{1}{2}$ years?

b. Make a table that charts the population of deer every year for the next five years.

c. Make a graph using your table.

d. Using your table and graph, decide whether this is a reasonable trend over the long term. Explain.

Simplify each expression.

53. $\dfrac{f^{-\frac{1}{4}}}{4f^{\frac{1}{2}} \cdot f^{-\frac{1}{3}}}$

54. $\dfrac{g^{\frac{5}{2}}}{g^{\frac{1}{2}} + 2}$

55. $\dfrac{c^{\frac{2}{3}}}{c^{\frac{1}{6}}}$

56. $\dfrac{z^{\frac{4}{5}}}{z^{\frac{1}{2}}}$

57. $\sqrt{23} \cdot \sqrt[3]{23^2}$

58. $\sqrt[8]{36h^4j^4}$

59. $\sqrt{\sqrt{81}}$

60. $\sqrt[4]{\sqrt{256}}$

61. $\dfrac{ab}{\sqrt{c}}$

62. $\dfrac{xy}{\sqrt[3]{z}}$

63. $\dfrac{8^{\frac{1}{6}} - 9^{\frac{1}{4}}}{\sqrt{3} + \sqrt{2}}$

64. $\dfrac{x^{\frac{5}{3}} - x^{\frac{1}{3}}z^{\frac{4}{3}}}{x^{\frac{2}{3}} + z^{\frac{2}{3}}}$

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the functions $f(x) = x^3$ and $g(x) = x^{\frac{1}{3}}$.

a. **Tabular** Copy and complete the table to the right.

b. **Graphical** Graph $f(x)$ and $g(x)$.

c. **Verbal** Explain the transformation between $f(x)$ and $g(x)$.

x	$f(x)$	$g(x)$
−2		
−1		
0		
1		
2		

H.O.T. Problems Use Higher-Order Thinking Skills

66. **REASONING** Determine whether $-x^{-2} = (-x)^{-2}$ is *always, sometimes,* or *never* true. Explain your reasoning.

67. **CHALLENGE** Consider $\sqrt[4]{(-16)^3}$.

a. Explain why the expression is not a real number.

b. Find n such that $n\sqrt[4]{(-16)^3}$ is a real number.

68. **OPEN ENDED** Find two different expressions that equal 2 in the form $x^{\frac{1}{a}}$.

69. **WRITING IN MATH** Explain how it might be easier to simplify an expression using rational exponents rather than using radicals.

70. **CCSS CRITIQUE** Ayana and Kenji are simplifying $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}}$. Is either of them correct? Explain your reasoning.

Ayana

$$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}} = x^{\frac{3}{4} + \frac{1}{2}}$$
$$= x^{\frac{3}{4} + \frac{2}{4}}$$
$$= x^{\frac{5}{4}}$$

Kenji

$$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}} = x^{\frac{3}{4} \div \frac{1}{2}}$$
$$= x^{\frac{3}{4} \cdot \frac{2}{1}}$$
$$= x^{\frac{3}{2}}$$

Standardized Test Practice

71. The expression $\sqrt{56 - c}$ is equivalent to a positive integer when c is equal to

A -10 **B** -8 **C** 36 **D** 56

72. SAT/ACT Which of the following sentences is true about the graphs of $y = 2(x - 3)^2 + 1$ and $y = 2(x + 3)^2 + 1$?

F Their vertices are maximums.

G The graphs have the same shape with different vertices.

H The graphs have different shapes with the same vertices.

J The graphs have different shapes with different vertices.

K One graph has a vertex that is a maximum; the other has a vertex that is a minimum.

73. GEOMETRY What is the converse of the statement? *If it is summer, then it is hot outside.*

A If it is not hot outside, then it is not summer.

B If it is not summer, then it is not hot outside.

C If it is hot outside, then it is summer.

D If it is hot outside, it is not summer.

74. SHORT RESPONSE If $3^5 \cdot p = 3^3$, then find p.

Spiral Review

Simplify. (Lesson 6-5)

75. $\sqrt{243}$

76. $\sqrt[3]{16y^3}$

77. $3\sqrt[3]{56y^6z^3}$

78. PHYSICS The speed of sound in a liquid is $s = \sqrt{\frac{B}{d}}$, where B is the bulk modulus of the liquid and d is its density. For water, $B = 2.1 \times 10^9$ N/m^2 and $d = 10^3$ kg/m^3. Find the speed of sound in water to the nearest meter per second. (Lesson 6-4)

Find $p(-4)$ and $p(x + h)$ for each function. (Lesson 5-3)

79. $p(x) = x - 2$

80. $p(x) = -x + 4$

81. $p(x) = 6x + 3$

82. $p(x) = x^2 + 5$

83. $p(x) = x^2 - x$

84. $p(x) = 2x^3 - 1$

Solve each equation by factoring. (Lesson 4-3)

85. $x^2 - 11x = 0$

86. $x^2 + 6x - 16 = 0$

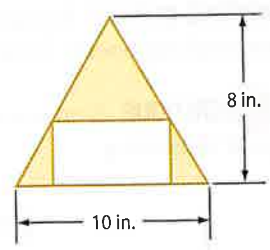
87. $4x^2 - 13x = 12$

88. $x^2 - 14x = -49$

89. $x^2 + 9 = 6x$

90. $x^2 - 3x = -\frac{9}{4}$

91. GEOMETRY A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (*Hint*: Use similar triangles.) (Lesson 4-1)



Skills Review

Find each power.

92. $(\sqrt{x - 3})^2$

93. $(\sqrt[3]{3x - 4})^3$

94. $(\sqrt[4]{7x - 1})^4$

95. $(\sqrt{x} - 4)^2$

96. $(2\sqrt{x} - 5)^2$

97. $(3\sqrt{x} + 1)^2$

LESSON 6-7

Solving Radical Equations and Inequalities

Then

- You solved polynomial equations.

Now

1. Solve equations containing radicals.
2. Solve inequalities containing radicals.

Why?

- When you jump, the time that you are in the air is your hang time. Hang time can be calculated in seconds t if you know the height h of the jump in feet. The formula for hang time is $t = 0.5\sqrt{h}$.

  Michael Jordan had a hang time of about 0.98 second. How would you calculate the height of Jordan's jump?

NewVocabulary

radical equation
extraneous solution
radical inequality

Common Core State Standards

Content Standards

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Mathematical Practices

4 Model with mathematics.

1 Solve Radical Equations

Radical equations include radical expressions. You can solve a radical equation by raising each side of the equation to a power.

KeyConcept Solving Radical Equations

Step 1 Isolate the radical on one side of the equation.

Step 2 Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Step 3 Solve the resulting polynomial equation. Check your results.

When solving radical equations, the result may be a number that does not satisfy the original equation. Such a number is called an **extraneous solution**.

Example 1 Solve Radical Equations

PT

Solve each equation.

a. $\sqrt{x+2} + 4 = 7$

$\sqrt{x+2} + 4 = 7$	Original equation
$\sqrt{x+2} = 3$	Subtract 4 from each side to isolate the radical.
$(\sqrt{x+2})^2 = 3^2$	Square each side to eliminate the radical.
$x + 2 = 9$	Find the squares.
$x = 7$	Subtract 2 from each side.

CHECK	$\sqrt{x+2} + 4 = 7$	Original equation
	$\sqrt{7+2} + 4 \stackrel{?}{=} 7$	Replace x with 7.
	$7 = 7$ ✓	Simplify.

b. $\sqrt{x-12} = 2 - \sqrt{x}$

$\sqrt{x-12} = 2 - \sqrt{x}$	Original equation
$(\sqrt{x-12})^2 = (2 - \sqrt{x})^2$	Square each side.
$x - 12 = 4 - 4\sqrt{x} + x$	Find the squares.
$-16 = -4\sqrt{x}$	Isolate the radical.
$4 = \sqrt{x}$	Divide each side by -4.
$16 = x$	Square each side.

StudyTip

Solution Check You can use a graphing calculator to check solutions of equations. Graph each side of the original equation, and examine the intersection.

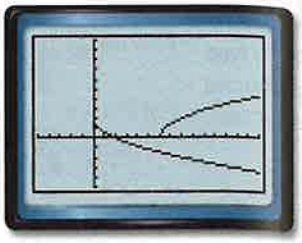
CHECK $\sqrt{x-12} = 2 - \sqrt{x}$

$\sqrt{16-12} \stackrel{?}{=} 2 - \sqrt{16}$

$\sqrt{4} \stackrel{?}{=} 2 - 4$

$2 \neq -2$ ✗

The solution does not check, so the equation has an extraneous solution. The graphs of $y = \sqrt{x-12}$ and $y = 2 - \sqrt{x}$ do not intersect, which confirms that there is no real solution.



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GuidedPractice

1A. $5 = \sqrt{x-2} - 1$

1B. $\sqrt{x+15} = 5 + \sqrt{x}$

To undo a square root, you square the expression. To undo a cube root, you must raise th expression to the third power.

Example 2 Solve a Cube Root Equation

Solve $2(6x-3)^{\frac{1}{3}} - 4 = 0$.

In order to remove the $\frac{1}{3}$ power, or cube root, you must first isolate it and then raise each side of the equation to the third power.

$2(6x-3)^{\frac{1}{3}} - 4 = 0$	Original equation
$2(6x-3)^{\frac{1}{3}} = 4$	Add 4 to each side.
$(6x-3)^{\frac{1}{3}} = 2$	Divide each side by 2.
$\left[(6x-3)^{\frac{1}{3}}\right]^3 = 2^3$	Cube each side.
$6x - 3 = 8$	Evaluate the cubes.
$6x = 11$	Add 3 to each side.
$x = \frac{11}{6}$	Divide each side by 6.

CHECK

$2(6x-3)^{\frac{1}{3}} - 4 = 0$	Original equation
$2\left(6 \cdot \frac{11}{6} - 3\right)^{\frac{1}{3}} - 4 \stackrel{?}{=} 0$	Replace x with $\frac{11}{6}$.
$2(8)^{\frac{1}{3}} - 4 \stackrel{?}{=} 0$	Simplify.
$2(2) - 4 \stackrel{?}{=} 0$	The cube root of 8 is 2.
$0 = 0$ ✓	Subtract.

GuidedPractice

Solve each equation.

2A. $(3n+2)^{\frac{1}{3}} + 1 = 0$

2B. $3(5y-1)^{\frac{1}{3}} - 2 = 0$

You can apply the methods used to solve square and cube root equations to solving equations with roots of any index. To undo an nth root, raise to the nth power.

Standardized Test Example 3 Solve a Radical Equation

What is the solution of $3(\sqrt[4]{2n+6}) - 6 = 0$?

A -1 **B** 1 **C** 5 **D** 11

$3(\sqrt[4]{2n+6}) - 6 = 0$ Original equation

$3(\sqrt[4]{2n+6}) = 6$ Add 6 to each side.

$\sqrt[4]{2n+6} = 2$ Divide each side by 3.

$(\sqrt[4]{2n+6})^4 = 2^4$ Raise each side to the fourth power.

$2n + 6 = 16$ Evaluate each side.

$2n = 10$ Subtract 6 from each side.

$n = 5$ The answer is C.

Test-Taking Tip

Substitute Values You could also solve the test question by substituting each answer for n in the equation to see if the solution is correct.

Guided Practice

3. What is the solution of $4(3x+6)^{\frac{1}{4}} - 12 = 0$?

F $x = 7$ **G** $x = 25$ **H** $x = 29$ **J** $x = 37$

2 Solve Radical Inequalities

A **radical inequality** has a variable in the radicand. To solve radical inequalities, complete the following steps.

Key Concept Solving Radical Inequalities

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.

Step 2 Solve the inequality algebraically.

Step 3 Test values to check your solution.

Example 4 Solve a Radical Inequality

Solve $3 + \sqrt{5x-10} \leq 8$.

Step 1 Since the radicand of a square root must be greater than or equal to zero, first solve $5x - 10 \geq 0$ to identify the values of x for which the left side of the inequality is defined.

$5x - 10 \geq 0$ Set the radicand ≥ 0.

$5x \geq 10$ Add 10 to each side.

$x \geq 2$ Divide each side by 5.

Step 2 Solve $3 + \sqrt{5x-10} \leq 8$.

$3 + \sqrt{5x-10} \leq 8$ Original inequality

$\sqrt{5x-10} \leq 5$ Isolate the radical.

$5x - 10 \leq 25$ Eliminate the radical.

$5x \leq 35$ Add 10 to each side.

$x \leq 7$ Divide each side by 5.

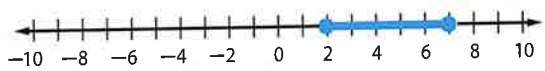
Study Tip

Radical Inequalities Since a principal square root is never negative, inequalities that simplify to the form $\sqrt{ax+b} \leq c$, where c is a negative number, have no solutions.

Step 3 It appears that $2 \leq x \leq 7$. You can test some x-values to confirm the solution. Use three test values: one less than 2, one between 2 and 7, and one greater than 7. Organize the test values in a table.

$x = 0$	$x = 4$	$x = 9$
$3 + \sqrt{5(0) - 10} \overset{?}{\leq} 8$ $3 + \sqrt{-10} \leq 8$ ✗ Since $\sqrt{-10}$ is not a real number, the inequality is not satisfied.	$3 + \sqrt{5(4) - 10} \overset{?}{\leq} 8$ $6.16 \leq 8$ ✓ Since $6.16 \leq 8$, the inequality is satisfied.	$3 + \sqrt{5(9) - 10} \overset{?}{\leq} 8$ $8.92 \leq 8$ ✗ Since $8.92 \not\leq 8$, the inequality is not satisfied.

The solution checks. Only values in the interval $2 \leq x \leq 7$ satisfy the inequality. You can summarize the solution with a number line.



Guided Practice

Solve each inequality.

4A. $\sqrt{2x + 2} + 1 \geq 5$

4B. $\sqrt{4x - 4} - 2 < 4$

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Examples 1–2 **Solve each equation.**

1. $\sqrt{x - 4} + 6 = 10$
2. $\sqrt{x + 13} - 8 = -2$
3. $8 - \sqrt{x + 12} = 3$
4. $\sqrt{x - 8} + 5 = 7$
5. $\sqrt[3]{x - 2} = 3$
6. $(x - 5)^{\frac{1}{3}} - 4 = -2$
7. $(4y)^{\frac{1}{3}} + 3 = 5$
8. $\sqrt[3]{n + 8} - 6 = -3$
9. $\sqrt{y} - 7 = 0$
10. $2 + 4z^{\frac{1}{2}} = 0$
11. $5 + \sqrt{4y - 5} = 12$
12. $\sqrt{2t - 7} = \sqrt{t + 2}$
13. **CCSS REASONING** The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared.
    a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?
    b. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

Example 3

14. **MULTIPLE CHOICE** Solve $(2y + 6)^{\frac{1}{4}} - 2 = 0$.

    A $y = 1$ **B** $y = 5$ **C** $y = 11$ **D** $y = 15$

Example 4 **Solve each inequality.**

15. $\sqrt{3x + 4} - 5 \leq 4$
16. $\sqrt{b - 7} + 6 \leq 12$
17. $2 + \sqrt{4y - 4} \leq 6$
18. $\sqrt{3a + 3} - 1 \leq 2$
19. $1 + \sqrt{7x - 3} > 3$
20. $\sqrt{3x + 6} + 2 \leq 5$
21. $-2 + \sqrt{9 - 5x} \geq 6$
22. $6 - \sqrt{2y + 1} < 3$

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Solve each equation. Confirm by using a graphing calculator.

23. $\sqrt{2x+5} - 4 = 3$

24. $6 + \sqrt{3x+1} = 11$

25. $\sqrt{x+6} = 5 - \sqrt{x+1}$

26. $\sqrt{x-3} = \sqrt{x+4} - 1$

27. $\sqrt{x-15} = 3 - \sqrt{x}$

28. $\sqrt{x-10} = 1 - \sqrt{x}$

29. $6 + \sqrt{4x+8} = 9$

30. $2 + \sqrt{3y-5} = 10$

31. $\sqrt{x-4} = \sqrt{2x-13}$

32. $\sqrt{7a-2} = \sqrt{a+3}$

33. $\sqrt{x-5} - \sqrt{x} = -2$

34. $\sqrt{b-6} + \sqrt{b} = 3$

35. CCSS **SENSE-MAKING** Isabel accidentally dropped her keys from the top of a Ferris wheel. The formula $t = \frac{1}{4}\sqrt{d-h}$ describes the time t in seconds at which the keys are h meters above the ground and Isabel is d meters above the ground. If Isabel was 65 meters high when she dropped the keys, how many meters above the ground will the keys be after 2 seconds?

Example 2 Solve each equation.

36. $(5n-6)^{\frac{1}{3}} + 3 = 4$

37. $(5p-7)^{\frac{1}{3}} + 3 = 5$

38. $(6q+1)^{\frac{1}{4}} + 2 = 5$

39. $(3x+7)^{\frac{1}{4}} - 3 = 1$

40. $(3y-2)^{\frac{1}{5}} + 5 = 6$

41. $(4z-1)^{\frac{1}{5}} - 1 = 2$

42. $2(x-10)^{\frac{1}{3}} + 4 = 0$

43. $3(x+5)^{\frac{1}{3}} - 6 = 0$

44. $\sqrt[3]{5x+10} - 5 = 0$

45. $\sqrt[3]{4n-8} - 4 = 0$

46. $\frac{1}{7}(14a)^{\frac{1}{3}} = 1$

47. $\frac{1}{4}(32b)^{\frac{1}{3}} = 1$

Example 3

48. **MULTIPLE CHOICE** Solve $\sqrt[4]{y+2} + 9 = 14$.

A 23 **B** 53 **C** 123 **D** 623

49. **MULTIPLE CHOICE** Solve $(2x-1)^{\frac{1}{4}} - 2 = 1$.

F 41 **G** 28 **H** 13 **J** 1

Example 4 Solve each inequality.

50. $1 + \sqrt{5x-2} > 4$

51. $\sqrt{2x+14} - 6 \geq 4$

52. $10 - \sqrt{2x+7} \leq 3$

53. $6 + \sqrt{3y+4} < 6$

54. $\sqrt{2x+5} - \sqrt{9+x} > 0$

55. $\sqrt{d+3} + \sqrt{d+7} > 4$

56. $\sqrt{3x+9} - 2 < 7$

57. $\sqrt{2y+5} + 3 \leq 6$

58. $-2 + \sqrt{8-4z} \geq 8$

59. $-3 + \sqrt{6a+1} > 4$

60. $\sqrt{2} - \sqrt{b+6} \leq -\sqrt{b}$

61. $\sqrt{c+9} - \sqrt{c} > \sqrt{3}$

62. **PENDULUMS** The formula $s = 2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where s is the time in seconds to swing back and forth, and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 1.5 seconds.

63. **FISH** The relationship between the length and mass of certain fish can be approximated by the equation $L = 0.46\sqrt[3]{M}$, where L is the length in meters and M is the mass in kilograms. Solve this equation for M.

64. **HANG TIME** Refer to the information at the beginning of the lesson regarding hang time. Describe how the height of a jump is related to the amount of time in the air. Write a step-by-step explanation of how to determine the height of Jordan's 0.98-second jump.

65. **CONCERTS** The organizers of a concert are preparing for the arrival of 50,000 people in the open field where the concert will take place. Each person is allotted 5 square feet of space, so the organizers rope off a circular area of 250,000 square feet. Using the formula $A = \pi r^2$, where A represents the area of the circular region and r represents the radius of the region, find the radius of this region.

66. **WEIGHTLIFTING** The formula $M = 512 - 146{,}230B^{-\frac{8}{5}}$ can be used to estimate the maximum total mass that a weightlifter of mass B kilograms can lift using the snatch and the clean and jerk. According to the formula, how much does a person weigh who can lift at most 470 kilograms?

H.O.T. Problems Use Higher-Order Thinking Skills

67. CCSS **ARGUMENTS** Which equation does not have a solution?

$\sqrt{x-1} + 3 = 4$	$\sqrt{x+1} + 3 = 4$
$\sqrt{x-2} + 7 = 10$	$\sqrt{x+2} - 7 = -10$

68. **CHALLENGE** Lola is working to solve $(x + 5)^{\frac{1}{4}} = -4$. She said that she could tell there was no real solution without even working the problem. Is Lola correct? Explain your reasoning.

69. **REASONING** Determine whether $\frac{\sqrt{(x^2)^2}}{-x} = x$ is *sometimes*, *always*, or *never* true when x is a real number. Explain your reasoning.

70. **OPEN ENDED** Select a whole number. Now work backward to write two radical equations that have that whole number as solutions. Write one square root equation and one cube root equation. You may need to experiment until you find a whole number you can easily use.

71. **WRITING IN MATH** Explain the relationship between the index of the root of a variable in an equation and the power to which you raise each side of the equation to solve the equation.

72. **OPEN ENDED** Write an equation that can be solved by raising each side of the equation to the given power.

a. $\frac{3}{2}$ power **b.** $\frac{5}{4}$ power **c.** $\frac{7}{8}$ power

73. **CHALLENGE** Solve $7^{3x-1} = 49^{x+1}$ for x. (*Hint:* $b^x = b^y$ if and only if $x = y$.)

REASONING **Determine whether the following statements are *sometimes*, *always*, or *never* true for $x^{\frac{1}{n}} = a$. Explain your reasoning.**

74. If n is odd, there will be extraneous solutions.

75. If n is even, there will be extraneous solutions.

Standardized Test Practice

76. What is an equivalent form of $\frac{4}{5+i}$?

A $\frac{10-2i}{13}$

B $\frac{5-i}{6}$

C $\frac{6-i}{6}$

D $\frac{6-i}{13}$

77. Which set of points describes a function?

F $\{(3, 0), (-2, 5), (2, -1), (2, 9)\}$

G $\{(-3, 5), (-2, 3), (-1, 5), (0, 7)\}$

H $\{(2, 5), (2, 4), (2, 3), (2, 2)\}$

J $\{(3, 1), (-3, 2), (3, 3), (-3, 4)\}$

78. SHORT RESPONSE The perimeter of an isosceles triangle is 56 inches. If one leg is 20 inches long, what is the measure of the base of the triangle?

79. SAT/ACT If $\sqrt{x+5}+1=4$, what is the value of x?

A 4

B 10

C 11

D 12

E 20

Spiral Review

Evaluate. (Lesson 6-6)

80. $27^{-\frac{2}{3}}$

81. $9^{\frac{1}{3}} \cdot 9^{\frac{5}{3}}$

82. $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

83. GEOMETRY The measures of the legs of a right triangle can be represented by the expressions $4x^2y^2$ and $8x^2y^2$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse. (Lesson 6-5)

Find the inverse of each function. (Lesson 6-2)

84. $y = 3x - 4$

85. $y = -2x - 3$

86. $y = x^2$

87. $y = (2x + 3)^2$

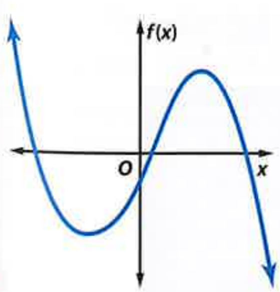
For each graph,

a. describe the end behavior,

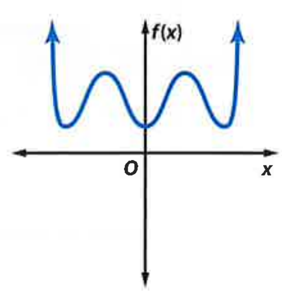
b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeros. (Lesson 5-3)

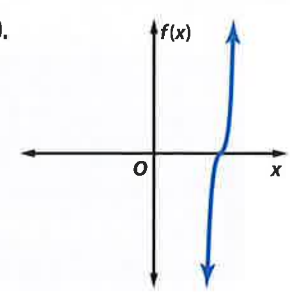
88.



89.



90.



Skills Review

Solve each equation. Write in simplest form.

91. $\frac{8}{5}x = \frac{4}{15}$

92. $\frac{27}{14}y = \frac{6}{7}$

93. $\frac{3}{10} = \frac{12}{25}a$

94. $\frac{6}{7} = 9m$

95. $\frac{9}{8}b = 18$

96. $\frac{6}{7}n = \frac{3}{4}$

97. $\frac{1}{3}p = \frac{5}{6}$

98. $\frac{2}{3}q = 7$

EXTEND 6-7

Graphing Technology Lab
Solving Radical Equations and Inequalities

You can use a TI-83/84 Plus graphing calculator to solve radical equations and inequalities. One way to do this is to rewrite the equation or inequality so that one side is 0. Then use the zero feature on the calculator.

CCSS Common Core State Standards
Content Standards
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Example 1 Radical Equation

Solve $\sqrt{x} + \sqrt{x+2} = 3$.

Step 1 Rewrite the equation.

- Subtract 3 from each side of the equation to get $\sqrt{x} + \sqrt{x+2} - 3 = 0$.
- Enter the function $y = \sqrt{x} + \sqrt{x+2} - 3$ in the **Y=** list.

KEYSTROKES: [Y=] [2nd] [√] [X,T,θ,n] [ )] [+] [2nd] [√] [X,T,θ,n] [+] 2 [ )] [−] 3 [ENTER]

Step 2 Use a table.

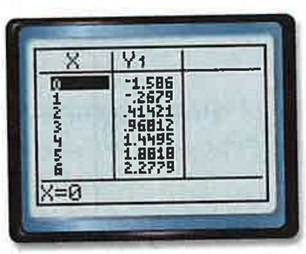
- You can use the **TABLE** function to locate intervals where the solution(s) lie. First, enter the starting value and the interval for the table.

KEYSTROKES: [2nd] [TBLSET] 0 [ENTER] 1 [ENTER]

Step 3 Estimate the solution.

- Complete the table and estimate the solution(s).

KEYSTROKES: [2nd] [TABLE]

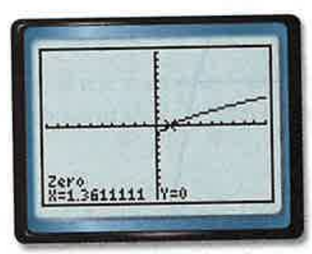


Since the function changes sign from negative to positive between $x = 1$ and $x = 2$, there is a solution between 1 and 2.

Step 4 Use the **ZERO** feature.

- Graph the function in the standard viewing window; then select **ZERO** from the **CALC** menu.

KEYSTROKES: [ZOOM] 6 [2nd] [CALC] 2



[−10, 10] scl: 1 by [−10, 10] scl: 1

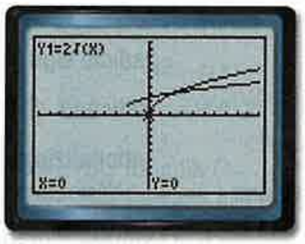
Place the cursor on a point at which $y < 0$ and press [ENTER] for the **LEFT BOUND**. Then place the cursor on a point at which $y > 0$ and press [ENTER] for the **RIGHT BOUND**. You can use the same point for the **GUESS**. The solution is about 1.36. This is consistent with the estimate made by using the **TABLE**.

Example 2 Radical Inequality

Solve $2\sqrt{x} > \sqrt{x+2} + 1$.

Step 1 **Graph each side of the inequality and use the TRACE feature.**

- In the **Y=** list, enter $y_1 = 2\sqrt{x}$ and $y_2 = \sqrt{x+2} + 1$. Then press [GRAPH].



[−10, 10] scl: 1 by [−10, 10] scl: 1

- Press [TRACE]. You can use [▲] or [▼] to switch the cursor between the two curves.

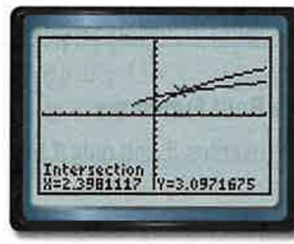
The calculator screen above shows that, for points to the left of where the curves cross, **Y1** < **Y2** or $2\sqrt{x} < \sqrt{x+2} + 1$. To solve the original inequality, you must find points for which **Y1** > **Y2**. These are the points to the right of where the curves cross.

Step 2 **Use the intersect feature.**

- You can use the **intersect** feature on the **CALC** menu to approximate the x-coordinate of the point at which the curves cross.

KEYSTROKES: [2nd] [CALC] 5

- Press [ENTER] for each of **FIRST CURVE?**, **SECOND CURVE?**, and **GUESS?**.



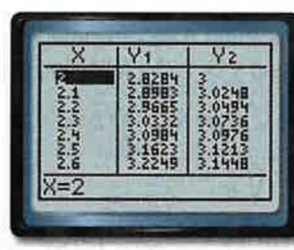
[−10, 10] scl: 1 by [−10, 10] scl: 1

The calculator screen shows that the x-coordinate of the point at which the curves cross is about 2.40. Therefore, the solution of the inequality is about $x > 2.40$. *Use the symbol > in the solution because the symbol in the original inequality is >.*

Step 3 **Use the TABLE feature to check your solution.**

- Start the table at 2 and show x-values in increments of 0.1. Scroll through the table.

KEYSTROKES: [2nd] [TBLSET] 2 [ENTER] .1 [ENTER] [2nd] [TABLE]



Notice that when x is less than or equal to 2.4, **Y1** < **Y2**. This verifies the solution $\{x \mid x > 2.40\}$.

Exercises

Use a graphical method to solve each equation or inequality.

1. $\sqrt{x+4} = 3$
2. $\sqrt{3x-5} = 1$
3. $\sqrt{x+5} = \sqrt{3x+4}$
4. $\sqrt{x+3} + \sqrt{x-2} = 4$
5. $\sqrt{3x-7} = \sqrt{2x-2} - 1$
6. $\sqrt{x+8} - 1 = \sqrt{x+2}$
7. $\sqrt{x-3} \geq 2$
8. $\sqrt{x+3} > 2\sqrt{x}$
9. $\sqrt{x} + \sqrt{x-1} < 4$
10. **WRITING IN MATH** Explain how you could apply the technique in the first example to solving an inequality.

CHAPTER 6 Study Guide and Review

Study Guide

KeyConcepts

Operations on Functions (Lesson 6-1)

Operation	Definition
Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
Composition	$[f \circ g](x) = f[g(x)]$

Inverse and Square Root Functions (Lessons 6-2 and 6-3)

- Two functions are inverses if and only if both their compositions are the identity function.

***n*th Roots** (Lesson 6-4)

Real *n*th roots of b, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$			
n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$\sqrt[n]{b}$ if $b = 0$
even	one positive root one negative root	no real roots	one real root, 0
odd	one positive root no negative roots	no positive roots one negative root	one real root, 0

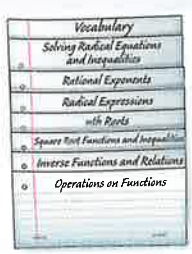
Radicals (Lessons 6-5 through 6-7)

For any real numbers a and b and any integers n, x, and y, with $b \neq 0$, $n > 1$, and $y > 1$, the following are true.

- Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- Rational Exponents: $b^{\frac{x}{y}} = \sqrt[y]{b^x} = \left(\sqrt[y]{b}\right)^x, b \geq 0$

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

composition of functions (p. 387)
conjugates (p. 418)
extraneous solution (p. 429)
index (p. 407)
inverse function (p. 393)
inverse relation (p. 393)
like radical expressions (p. 417)
*n*th root (p. 407)
principal root (p. 407)
radical equation (p. 429)
radical function (p. 400)
radical inequality (p. 431)
radical sign (p. 407)
radicand (p. 407)
rationalizing the denominator (p. 416)
square root function (p. 400)
square root inequality (p. 402)

VocabularyCheck

Choose a word or term that best completes each statement.

1. If both compositions result in the ________, then the functions are inverse functions.
2. Radicals are ________ if *both* the index and the radicand are identical.
3. In a(n) ________, the results of one function are used to evaluate a second function.
4. When there is more than one real root, the nonnegative roo is called the ________.
5. To eliminate radicals from a denominator or fractions from a radicand, you use a process called ________.
6. Equations with radicals that have variables in the radicand are called ________.
7. Two relations are ________ if and only if one relation contains the element (b, a) when the other relation contains the element (a, b).
8. When solving a radical equation, sometimes you will obtai a number that does not satisfy the original equation. Such a number is called a(n) ________.
9. The square root function is a type of ________.

Lesson-by-Lesson Review

6-1 Operations on Functions

Find $[f \circ g](x)$ and $[g \circ f](x)$.

10. $f(x) = 2x + 1$
$g(x) = 4x - 5$

11. $f(x) = x^2 + 1$
$g(x) = x - 7$

12. $f(x) = x^2 + 4$
$g(x) = -2x + 1$

13. $f(x) = 4x$
$g(x) = 5x - 1$

14. $f(x) = x^3$
$g(x) = x - 1$

15. $f(x) = x^2 + 2x - 3$
$g(x) = x + 1$

16. **MEASUREMENT** The formula $f = 3y$ converts yards y to feet f and $f = \frac{n}{12}$ converts inches n to feet f. Write a composition of functions that converts yards to inches.

Example 1

If $f(x) = x^2 + 3$ and $g(x) = 3x - 2$, find $g[f(x)]$ and $f[g(x)]$.

$g[f(x)] = 3(x^2 + 3) - 2$	Replace $f(x)$ with $x^2 + 3$.
$= 3x^2 + 9 - 2$	Distributive Property
$= 3x^2 + 7$	Simplify.
$f[g(x)] = (3x - 2)^2 + 3$	Replace $g(x)$ with $3x - 2$.
$= 9x^2 - 12x + 4 + 3$	Multiply.
$= 9x^2 - 12x + 7$	Simplify.

6-2 Inverse Functions and Relations

Find the inverse of each function. Then graph the function and its inverse.

17. $f(x) = 5x - 6$

18. $f(x) = -3x - 5$

19. $f(x) = \frac{1}{2}x + 3$

20. $f(x) = \frac{4x + 1}{5}$

21. $f(x) = x^2$

22. $f(x) = (2x + 1)^2$

23. **SHOPPING** Samuel bought a computer. The sales tax rate was 6% of the sale price, and he paid $50 for shipping. Find the sale price if Samuel paid a total of $1322.

Use the horizontal line test to determine whether the inverse of each function is also a function.

24. $f(x) = 3x^2$

25. $h(x) = x^3 - 3$

26. $g(x) = -3x^4 + 2x - 1$

27. $g(x) = 4x^3 - 5x$

28. $f(x) = -3x^5 + x^2 - 3$

29. $h(x) = 4x^4 + 7x$

30. **FINANCIAL LITERACY** During the last month, Jonathan has made two deposits of $45, made a deposit of double his original balance, and has withdrawn $35 five times. His balance is now $189. Write an equation that models this problem. How much money did Jonathan have in his account at the beginning of the month?

Example 2

Find the inverse of $f(x) = -2x + 7$.

Rewrite $f(x)$ as $y = -2x + 7$. Then interchange the variables and solve for y.

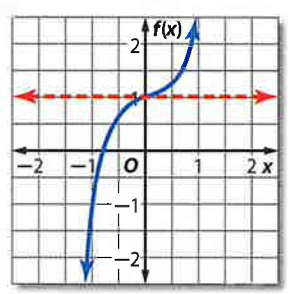
$x = -2y + 7$	Interchange the variables.
$2y = -x + 7$	Solve for y.
$y = \frac{-x + 7}{2}$	Divide each side by 2.
$f^{-1}(x) = \frac{-x + 7}{2}$	Rewrite using function notation.

Example 3

Use the horizontal line test to determine whether the inverse of $f(x) = 2x^3 + 1$ is also a function.

Graph the function.

No horizontal line can be drawn so that it passes through more than one point. The inverse of this function is a function.



Study Guide and Review *Continued*

6-3 Square Root Functions and Inequalities

Graph each function. State the domain and range.

31. $f(x) = \sqrt{3x}$

32. $f(x) = -\sqrt{6x}$

33. $f(x) = \sqrt{x - 7}$

34. $f(x) = \sqrt{x + 5} - 3$

35. $f(x) = \frac{3}{4}\sqrt{x - 1} + 5$

36. $f(x) = -\frac{1}{3}\sqrt{x + 4} - 1$

37. GEOMETRY The area of a circle is given by the formula $A = \pi r^2$. What is the radius of a circle with an area of 300 square inches?

Graph each inequality.

38. $y \geq \sqrt{x} + 3$

39. $y < 2\sqrt{x - 5}$

40. $y > -\sqrt{x - 1} + 2$

Example 4

Graph $f(x) = \sqrt{x + 1} - 2$. State the domain and range.

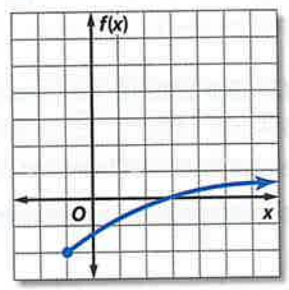
Identify the domain.

$x + 1 \geq 0$ Write the radicand as greater than or equal to 0.

$x \geq -1$ Subtract 1 from each side.

Make a table of values for $x \geq -1$ and graph the function.

x	$f(x)$
−1	−2
0	−1
1	−0.59
2	−0.27
3	0
4	0.24
5	0.45



The domain is $\{x | x \geq -1\}$, and the range is $\{f(x) | f(x) \geq -2\}$.

6-4 nth Roots

Simplify.

41. $\pm\sqrt{121}$

42. $\sqrt[3]{-125}$

43. $\sqrt{(-6)^2}$

44. $\sqrt{-(x + 3)^4}$

45. $\sqrt[6]{(x^2 + 2)^{18}}$

46. $\sqrt[3]{27(x + 3)^3}$

47. $\sqrt[4]{a^8 b^{12}}$

48. $\sqrt[5]{243x^{10}y^{25}}$

49. PHYSICS The velocity v of an object can be defined as $v = \sqrt{\frac{2K}{m}}$, where m is the mass of an object and K is the kinetic energy in joules. Find the velocity in meters per second of an object with a mass of 17 grams and a kinetic energy of 850 joules.

Example 5

Simplify $\sqrt{64x^6}$.

$\sqrt{64x^6} = \sqrt{(8x^3)^2}$ $64x^6 = (8x^3)^2$

$= 8|x^3|$ Simplify.

Use absolute value symbols because x could be negative.

Example 6

Simplify $\sqrt[6]{4096x^{12}y^{24}}$.

$\sqrt[6]{4096x^{12}y^{24}} = \sqrt[6]{(4x^2y^4)^6}$ $4096x^{12}y^{24} = (4x^2y^4)^6$

$= 4x^2y^4$ Simplify.

6-5 Operations with Radical Expressions

Simplify.

50. $\sqrt[3]{54}$

51. $\sqrt{144a^3b^5}$

52. $4\sqrt{6y} \cdot 3\sqrt{7x^2y}$

53. $6\sqrt{72} + 7\sqrt{98} - \sqrt{50}$

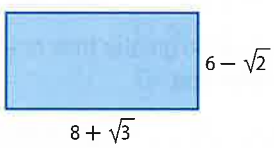
54. $(6\sqrt{5} - 2\sqrt{2})(3\sqrt{5} + 4\sqrt{2})$

55. $\dfrac{\sqrt{6m^5}}{\sqrt{p^{11}}}$

56. $\dfrac{3}{5 + \sqrt{2}}$

57. $\dfrac{\sqrt{3}}{\sqrt{5} - \sqrt{6}}$

58. **GEOMETRY** What are the perimeter and the area of the rectangle?



Example 7

Simplify $2\sqrt[3]{18a^2b} \cdot 3\sqrt[3]{12ab^5}$.

$2\sqrt[3]{18a^2b} \cdot 3\sqrt[3]{12ab^5}$

$= (2 \cdot 3)\sqrt[3]{18a^2b \cdot 12ab^5}$ — Product Property

$= 6\sqrt[3]{2^3 3^3 a^3 b^6}$ — Factor.

$= 6 \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{a^3} \cdot \sqrt[3]{b^6}$ — Product Property

$= 6 \cdot 2 \cdot 3 \cdot a \cdot b^2$ — Find cube roots.

$= 36ab^2$ — Simplify.

Example 8

Simplify $\sqrt{\dfrac{x^4}{y^5}}$.

$\sqrt{\dfrac{x^4}{y^5}} = \dfrac{\sqrt{x^4}}{\sqrt{y^5}}$ — Quotient Property

$= \dfrac{\sqrt{(x^2)^2}}{\sqrt{(y^2)^2} \cdot \sqrt{y}}$ — Factor into squares.

$= \dfrac{x^2}{y^2\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}}$ — Rationalize the denominator.

$= \dfrac{x^2\sqrt{y}}{y^3}$ — $\sqrt{y} \cdot \sqrt{y} = y$

6-6 Rational Exponents

Simplify each expression.

59. $x^{\frac{1}{2}} \cdot x^{\frac{2}{3}}$

60. $m^{-\frac{3}{4}}$

61. $\dfrac{d^{\frac{1}{6}}}{d^{\frac{3}{4}}}$

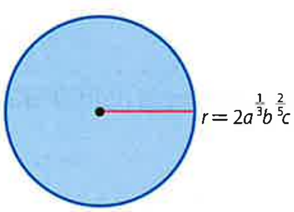
Simplify each expression.

62. $\dfrac{1}{y^{\frac{1}{4}}}$

63. $\sqrt[3]{\sqrt{729}}$

64. $\dfrac{x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}}}{x^{\frac{1}{3}}}$

65. **GEOMETRY** What is the area of the circle?



Example 9

Simplify $a^{\frac{2}{3}} \cdot a^{\frac{1}{5}}$.

$a^{\frac{2}{3}} \cdot a^{\frac{1}{5}} = a^{\frac{2}{3} + \frac{1}{5}}$ — Product of Powers

$= a^{\frac{13}{15}}$ — Add.

Example 10

Simplify $\dfrac{2a}{\sqrt[3]{b}}$.

$\dfrac{2a}{\sqrt[3]{b}} = \dfrac{2a}{b^{\frac{1}{3}}}$ — Rational exponents

$= \dfrac{2a}{b^{\frac{1}{3}}} \cdot \dfrac{b^{\frac{2}{3}}}{b^{\frac{2}{3}}}$ — Rationalize the denominator.

$= \dfrac{2ab^{\frac{2}{3}}}{b}$ or $\dfrac{2a\sqrt[3]{b^2}}{b}$ — Rewrite in radical form.

Study Guide and Review *Continued*

6-7 Solving Radical Equations and Inequalities

Solve each equation.

66. $\sqrt{x-3}+5=15$

67. $-\sqrt{x-11}=3-\sqrt{x}$

68. $4+\sqrt{3x-1}=8$

69. $\sqrt{m+3}=\sqrt{2m+1}$

70. $\sqrt{2x+3}=3$

71. $(x+1)^{\frac{1}{4}}=-3$

72. $a^{\frac{1}{3}}-4=0$

73. $3(3x-1)^{\frac{1}{3}}-6=0$

74. **PHYSICS** The formula $t=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where t is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 2.75 seconds.

Solve each inequality.

75. $2+\sqrt{3x-1}<5$

76. $\sqrt{3x+13}-5\geq 5$

77. $6-\sqrt{3x+5}\leq 3$

78. $\sqrt{-3x+4}-5\geq 3$

79. $5+\sqrt{2y-7}<5$

80. $3+\sqrt{2x-3}\geq 3$

81. $\sqrt{3x+1}-\sqrt{6+x}>0$

Example 11

Solve $\sqrt{2x+9}-2=5$.

$\sqrt{2x+9}-2=5$	Original equation
$\sqrt{2x+9}=7$	Add 2 to each side.
$(\sqrt{2x+9})^2=7^2$	Square each side.
$2x+9=49$	Evaluate the squares.
$2x=40$	Subtract 9 from each side.
$x=20$	Divide each side by 2.

Example 12

Solve $\sqrt{2x-5}+2>5$.

$\sqrt{2x-5}\geq 0$	Radicand must be ≥ 0.
$2x-5\geq 0$	Square each side.
$2x\geq 5$	Add 5 to each side.
$x\geq 2.5$	Divide each side by 2.

The solution must be greater than or equal to 2.5 to satisfy the domain restriction.

$\sqrt{2x-5}+2>5$	Original inequality
$\sqrt{2x-5}>3$	Subtract 2 from each side.
$(\sqrt{2x-5})^2>3^2$	Square each side.
$2x-5>9$	Evaluate the squares.
$2x>14$	Add 5 to each side.
$x>7$	Divide each side by 2.

Since $x\geq 2.5$ contains $x>7$, the solution of the inequality is $x>7$.

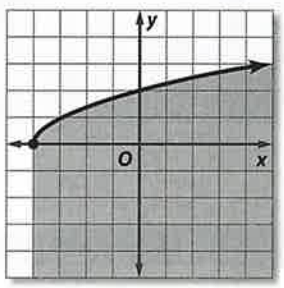
CHAPTER 6

Practice Test

Determine whether each pair of functions are inverse functions. Write *yes* or *no*. Explain your reasoning.

1. $f(x) = 3x + 8,\ g(x) = \frac{x - 8}{3}$
2. $f(x) = \frac{1}{3}x + 5,\ g(x) = 3x - 15$
3. $f(x) = x + 7,\ g(x) = x - 7$
4. $g(x) = 3x - 2,\ f(x) = \frac{x - 2}{3}$

5. **MULTIPLE CHOICE** Which inequality represents the graph below?



A $y \geq \sqrt{x + 4}$ C $y \geq \sqrt{x - 4}$
B $y \leq \sqrt{x + 4}$ D $y \leq \sqrt{x - 4}$

If $f(x) = 3x + 2$ and $g(x) = x^2 - 2x + 1$, find each function.

6. $(f + g)(x)$
7. $(f \cdot g)(x)$
8. $(f - g)(x)$
9. $\left(\frac{f}{g}\right)(x)$

Solve each equation.

10. $\sqrt{a + 12} = \sqrt{5a - 4}$
11. $\sqrt{3x} = \sqrt{x - 2}$
12. $4(\sqrt[4]{3x + 1}) - 8 = 0$
13. $\sqrt[3]{5m + 6} + 15 = 21$
14. $\sqrt{3x + 21} = \sqrt{5x + 27}$
15. $1 + \sqrt{x + 11} = \sqrt{2x + 15}$
16. $\sqrt{x - 5} = \sqrt{2x - 4}$
17. $\sqrt{x - 6} - \sqrt{x} = 3$

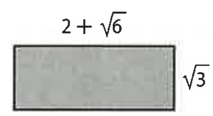
18. **MULTIPLE CHOICE** Which expression is equivalent to $125^{-\frac{1}{3}}$?

F -5 H $\frac{1}{5}$
G $-\frac{1}{5}$ J 5

Simplify.

19. $(2 + \sqrt{5})(6 - 3\sqrt{5})$
20. $(3 - 2\sqrt{2})(-7 + \sqrt{2})$
21. $\frac{12}{2 - \sqrt{3}}$
22. $\frac{m^{\frac{1}{2}} - 1}{2m^{\frac{1}{2}} + 1}$
23. $4\sqrt{3} - 8\sqrt{48}$
24. $5^{\frac{2}{3}} \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{5}{6}}$
25. $\sqrt[6]{729a^9b^{24}}$
26. $\sqrt[5]{32x^{15}y^{10}}$
27. $w^{-\frac{4}{5}}$
28. $\frac{r^{\frac{2}{3}}}{r^{\frac{1}{6}}}$
29. $\frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}} \cdot a^{-\frac{1}{4}}}$
30. $\frac{y^{\frac{3}{2}}}{y^{\frac{1}{2}} + 2}$

31. **MULTIPLE CHOICE** What is the area of the rectangle?

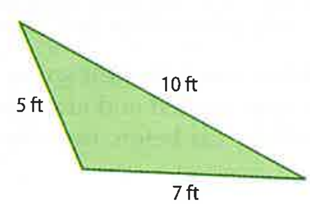


A $2\sqrt{3} + 3\sqrt{2}$ units^2
B $4 + 2\sqrt{6} + 2\sqrt{3}$ units^2
C $2\sqrt{3} + \sqrt{6}$ units^2
D $2\sqrt{3} + 3$ units^2

Solve each inequality.

32. $\sqrt{4x - 3} < 5$
33. $-2 + \sqrt{3m - 1} < 4$
34. $2 + \sqrt{4x - 4} \leq 6$
35. $\sqrt{2x + 3} - 4 \leq 5$
36. $\sqrt{b + 12} - \sqrt{b} > 2$
37. $\sqrt{y - 7} + 5 \geq 10$
38. $\sqrt{a - 5} - \sqrt{a + 7} \leq 4$
39. $\sqrt{c + 5} + \sqrt{c + 10} > 2$

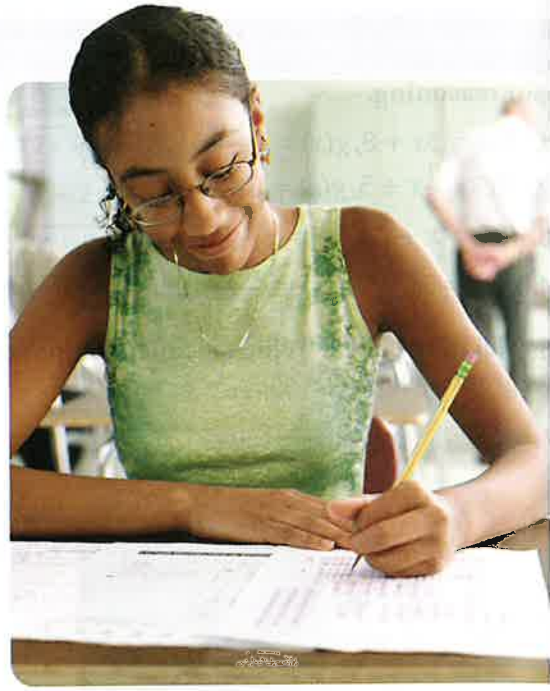
40. **GEOMETRY** The area of a triangle with sides of length *a*, *b*, and *c* is given by $A = \sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$. What is the area of the triangle expressed in radical form?



CHAPTER 6 Preparing for Standardized Tests

Work Backward

In certain math problems, you are given information about an end result, but you need to find out something that happened earlier. You can work backward to solve problems like this.



Strategies for Working Backward

Step 1

Read the problem statement carefully.

Ask yourself:

- What information am I given?
- What am I being asked to solve?
- Does any of the information given relate to an end result?
- Am I being asked to solve for a quantity that occurred "earlier" in the problem statement?
- What operations are being used in the problem?

Step 2

Model the problem situation with an equation, an inequality, or a graph as appropriate. Then work backward to solve the problem.

- If needed, sketch a flow of events to show the sequence described in the problem statement.
- Use inverse operations to undo any operations while working backward until you arrive at your answer.

Step 3

Check by beginning with your answer and seeing if you arrive at the same result given in the problem statement.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Maria bought a used car. The sales tax rate was 6.75% of the selling price, and she had to pay \$450 in processing, title, and registration fees. If Maria paid a total of \$15,768.63, what was the sale price of the car? Show your work.

Read the problem carefully. You know the total amount that Maria paid for the car after sales tax was applied and after she paid all of the other fees. You need to find the sale price of the car before taxes and fees.

Let x represent the sale price of the car and set up an equation. Use the work backward strategy to solve the problem.

Words	The sale price of the car plus the sales tax and other fees is equal to the final price.
Variable	Let x = sale price.
Equation	$1.0675x + 450 = 15{,}768.63$

Using the work backward strategy results in a simple equation. Use inverse operations to solve for x.

$1.0675x + 450 = 15{,}768.63$ Original equation

$1.0675x = 15{,}318.63$ Subtract 450 from each side.

$x \approx 14{,}350$ Divide each side by 1.0675.

Check your answer by working the problem forward. Begin with your answer and see if you get the same result as in the problem statement.

$14{,}350(1.0675) \approx 15{,}318.63$ Compute the sales tax.

$15{,}318.63 + 450 = 15{,}768.63$ Add the other fees.

$15{,}768.63 = 15{,}768.63$ The result is the same.

So, the sale price of the car was $14,350.

Exercises

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

1. The equation $d = \frac{s^2}{30f}$ can be used to model the length of the skid marks left by a car when a driver applies the brakes to come to a sudden stop. In the equation, d is the length (in feet) of the skid marks left on the road, s is the speed of the car in miles per hour, and f is a coefficient of friction that describes the condition of the road. Suppose a car left skid marks that are 120 feet long.

a. Solve the equation for s, the speed of the car.

b. If the coefficient of friction for the road is 0.75, about how fast was the car traveling?

c. How fast was the car traveling if the coefficient of friction for the road is 1.1?

2. An object is shot straight upward into the air with an initial speed of 800 feet per second. The height h that the object will be after t seconds is given by the equation $h = -16t^2 + 800t$. When will the object reach a height of 10,000 feet?

A 10 seconds

B 25 seconds

C 100 seconds

D 625 seconds

3. Pedro is creating a scale drawing of a car. He finds that the height of the car in the drawing is $\frac{1}{32}$ of the actual height of the car x. Which equation best represents this relationship?

F $y = x - \frac{1}{32}$

G $y = -\frac{1}{32}x$

H $y = \frac{1}{32}x$

J $y = x + \frac{1}{32}$

Standardized Test Practice

Cumulative, Chapters 1 through 6

CHAPTER 6

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A sporting goods store is discounting all camping equipment by 20% during the off-season. Charles also has a coupon good for \$5.00 off his next purchase from the store. If the coupon is applied *after* the store discount, which of the following functions can be used to find the final price of a tent that originally cost d dollars?

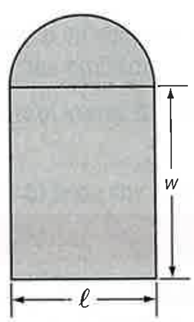
A $P(d) = 0.8 \times (d + 5)$

B $P(d) = (0.8 \times d) - 5$

C $P(d) = 0.2 \times (d - 5)$

D $P(d) = 0.8 \times (d - 5)$

2. Find the equation that can be used to determine the total area of the composite figure below.



F $A = \ell w + \frac{1}{2}\ell w$

G $A = \ell w + \pi\left(\frac{1}{2}\ell\right)^2$

H $A = \ell w + \frac{1}{2}\pi\ell^2$

J $A = \ell w + \pi\left(\frac{1}{2}\ell\right)^2\left(\frac{1}{2}\right)$

3. Which expression is equivalent to $3a(2a + 1) - (2a - 2)(a + 3)$?

A $2a^2 + 6a + 7$

B $4a^2 - a + 6$

C $4a^2 + 6a - 6$

D $4a^2 - 3a + 7$

4. Kay bought a used car. The sales tax rate was 6.5% of the selling price, and she also had to pay \$325 in registration fees. Find the selling price if Kay spent a total of \$15,501.25.

F \$13,850

G \$14,120

H \$14,250

J \$14,650

5. Simplify $\sqrt[3]{-27b^6c^{12}}$.

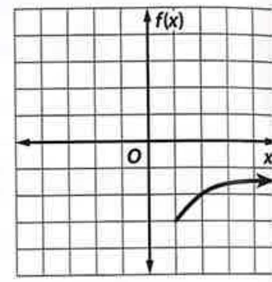
A $-3b^3c^6$

B $-3b^2c^4$

C $3b^2c^4$

D $3b^3c^6$

6. What is the equation of the square root function graphed at the right?



F $f(x) = \sqrt{x - 3} - 1$

G $f(x) = \sqrt{x + 1} - 3$

H $f(x) = \sqrt{x + 3} + 1$

J $f(x) = \sqrt{x - 1} - 3$

7. Which equation will produce the narrowest parabola when graphed?

A $y = 3x^2$

B $y = \frac{3}{4}x^2$

C $y = -6x^2$

D $y = -\frac{3}{4}x^2$

8. Find the inverse of $f(x) = x - 5$.

F $f(x) = x + 5$

G $f(x) = 5x$

H $f(x) = \frac{x}{5}$

J $f(x) = 5 - x$

9. The equations of two lines are $2x - y = 6$ and $4x - y = -2$. Which of the following describes the point of intersection?

A $(2, -2)$

B $(-8, -38)$

C $(-4, -14)$

D no intersection

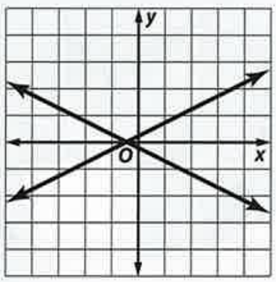
Test-Taking Tip

Question 4 You know the final price but need to know the sale price. Work backward to find the solution.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

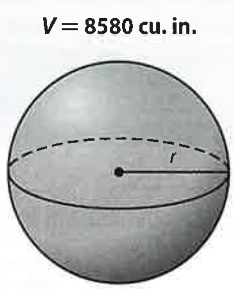
10. Which of the following terms does *not* describe the system of equations graphed below: consistent, dependent, independent, or intersecting?



11. Suppose a projectile is launched into the air from a platform. The formula $h = -16t^2 + 40t + 70$ relates the height h of the object (in feet) and the time t since it was launched (in seconds). What is the maximum height the object reaches?

12. The radius of a sphere with volume V can be found using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$.

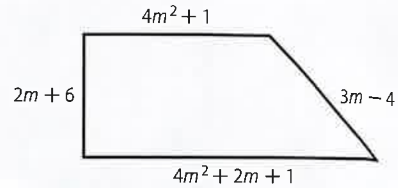
a. What is the radius of the sphere at the right? Round to the nearest tenth.



b. Solve the formula for V to find the formula for the volume of a sphere, given its radius.

c. What is the volume of a basketball that has a diameter of 9 inches? Round to the nearest tenth.

13. GRIDDED RESPONSE The perimeter of the quadrilateral below is 160. What is the value of m?



Extended Response

Record your answers on a sheet of paper. Show your work.

14. The amount that a retailer charges for shipping an electronics purchase is determined by the weight of the package. The charges for several different weights are given in the table.

Electronics Shipping Charges	
Weight (lb)	Shipping ($)
1	5.58
3	6.76
4	7.35
7	9.12
10	10.89
13	12.66
15	13.84

a. Find the rate of change of the shipping charge per pound.

b. Write an equation that could be used to find the shipping charge y for a package that weighs x pounds.

c. Find the shipping charge for a package that weighs 19 pounds.

15. Suppose $f(x)$ and $g(x)$ are inverse functions.

a. Describe how the graphs of $f(x)$ and $g(x)$ would appear on a coordinate grid.

b. What is the value of the composition $f[g(2)]$? Explain.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	6-1	1-1	5-1	6-2	6-4	6-3	4-7	6-2	3-1	3-1	6-7	6-4	5-1	2-4	6-2