CHAPTER 1

Equations and Inequalities

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Then

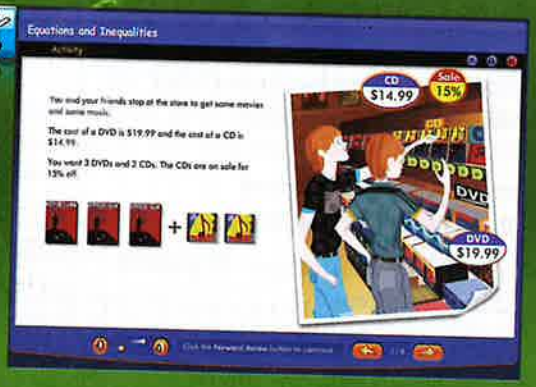
- You wrote expressions with variables.

Now

- You will:
  - Simplify and evaluate algebraic expressions.
  - Solve linear and absolute value equations.
  - Solve and graph inequalities.

Why? ▲

- **MONEY** Connecting money to mathematics is one of the most practical skills you can learn. As long as you use money, you will be using mathematics. In this chapter, you will explore money topics such as sales tax, income, and budgeting for your first apartment.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Simplify.

1. $15.7 + (-3.45)$
2. $-18.54 - (-32.05)$
3. $-9.8 \cdot 6.75$
4. $4 \div (-0.5)$
5. $3\frac{2}{3} + \left(-1\frac{4}{5}\right)$
6. $\frac{54}{7} - \frac{26}{6}$
7. $\left(\frac{6}{5}\right)\left(-\frac{10}{9}\right)$
8. $-3 \div \frac{7}{8}$
9. **CRAFTS** Felisa needs $\frac{7}{8}$ yard of one type of material to make a quilt. How much of this material will she need to make 12 quilts?

Evaluate each power.

10. 6^3
11. $(-4)^3$
12. $-(0.6)^2$
13. $-(-2.5)^3$
14. $\left(\frac{4}{5}\right)^2$
15. $\left(\frac{7}{3}\right)^4$
16. $\left(-\frac{7}{10}\right)^2$
17. $-\left(\frac{15}{2}\right)^3$
18. **FOOD** Nate's Deli offers 3 types of bread, 3 types of meat, and 3 types of cheese. How many different sandwiches can be made with 1 type each of bread, meat, and cheese?

Identify each statement as *true* or *false*.

19. $-6 \geq -7$
20. $8 > -5$
21. $\frac{1}{7} \leq \frac{1}{9}$
22. $\frac{5}{6} \leq \frac{25}{30}$
23. **MEASUREMENT** Christy has a board that is 0.6 yard long. Marissa has a board that is $\frac{2}{3}$ yard long. Marissa states that $\frac{2}{3} > 0.6$. Is she correct?

QuickReview

Example 1

Simplify $\left(\frac{3}{16}\right)\left(-\frac{4}{5}\right)$.

$\left(\frac{3}{16}\right)\left(-\frac{4}{5}\right) = -\frac{3(4)}{16(5)}$ Multiply the numerators and the denominators.

$= -\frac{12}{80}$ Simplify.

$= -\frac{12 \div 4}{80 \div 4}$ Divide the numerator and denominator by the GCF, 4.

$= -\frac{3}{20}$ Simplify.

Example 2

Evaluate $(-1.5)^3$.

$(-1.5)^3 = (-1.5)(-1.5)(-1.5)$ $(-1.5)^3$ means 1.5 is a factor 3 times.

$= -3.375$ Simplify.

Example 3

Identify $\frac{3}{8} > \frac{12}{24}$ as *true* or *false*.

$\frac{3}{8} \stackrel{?}{>} \frac{12 \div 3}{24 \div 3}$ Divide 12 and 24 by 3 to get a denominator of 8.

$\frac{3}{8} \not> \frac{4}{8}$ Simplify.

False; $\frac{3}{8} \not> \frac{4}{8}$ because $\frac{3}{8} < \frac{4}{8}$.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

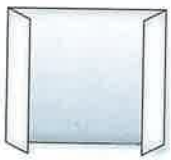
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 1. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

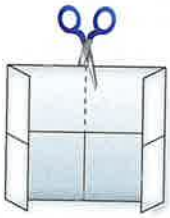
FOLDABLES StudyOrganizer

Equations and Inequalities Make this Foldable to help you organize your Chapter 1 notes about equations and inequalities. Begin with one sheet of 11" × 17" paper.

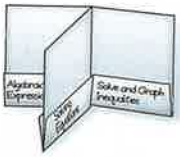
1 **Fold** 2" tabs on each of the short sides.



2 **Then** fold in half in both directions. Open and cut as shown.



3 **Refold** along the width. Staple each pocket. Label pockets as *Algebraic Expressions, Properties of Real Numbers, Solving Equations,* and *Solve and Graph Inequalities.* Place index cards for notes in each pocket.



NewVocabulary

English		Español
variable	p. 5	variable
algebraic expression	p. 5	expresión algebraica
order of operations	p. 5	orden de las operaciones
formula	p. 6	fórmula
real numbers	p. 11	números reales
rational numbers	p. 11	números racional
irrational numbers	p. 11	números irracional
integers	p. 11	enteros
whole numbers	p. 11	números enteros
natural numbers	p. 11	números naturales
open sentence	p. 18	enunciado abierto
equation	p. 18	ecuación
solution	p. 18	solución
absolute value	p. 27	valor absoluto
empty set	p. 28	conjunto vacío
set-builder notation	p. 35	notación de construcción de conjuntos
compound inequality	p. 41	desigualdad compuesta
intersection	p. 41	intersección
union	p. 42	unión

ReviewVocabulary

evaluate evaluar to find the value of an expression

inequality desigualdad an open sentence that contains the symbol $<$, $\leq$, $>$, or $\geq$

power potencia an expression of the form x^n, read *x to the nth power*

The *base* is the number that is multiplied. → x^n ← The *exponent* tells how many times the base is used as a factor.

The number that can be expressed using an exponent is called a *power.*

LESSON 1-1

Expressions and Formulas

Then

- You used the rules of exponents.

Now

1. Use the order of operations to evaluate expressions.
2. Use formulas.

Why?

- The following formula can be used to calculate a baseball player's on-base percentage x.

$$x = \frac{h + w + p}{b + w + p + s}$$

  - h is the number of hits.
  - w is the number of walks.
  - p is the number of times the player has been hit by a pitch.
  - b is the number of times at bat.
  - s is the number of sacrifice flies and sacrifice bunts.

During the first twenty games of a season, Ian has 9 hits, 2 walks, 38 at bats, 5 sacrifice flies, and he is hit by 1 pitch. The expression $\frac{9 + 2 + 1}{38 + 2 + 1 + 5}$ gives Ian's on-base percentage.

NewVocabulary

variables
algebraic expressions
order of operations
formula

Common Core State Standards

Content Standards

A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Order of Operations

Variables are letters used to represent unknown quantities. Expressions that contain at least one variable are called **algebraic expressions**. You can evaluate an algebraic expression by replacing each variable with a number and then applying the **order of operations**.

KeyConcept Order of Operations

Step 1 Evaluate the expressions inside grouping symbols.

Step 2 Evaluate all powers.

Step 3 Multiply and/or divide from left to right.

Step 4 Add and/or subtract from left to right.

Example 1 Evaluate Algebraic Expressions

Evaluate $m + (p - 1)^2$ if $m = 3$ and $p = -4$.

$m + (p - 1)^2 = 3 + (-4 - 1)^2$ Replace m with 3 and p with -4.

$= 3 + (-5)^2$ Add -4 and -1.

$= 3 + 25$ Evaluate $(-5)^2$.

$= 28$ Add 3 and 25.

GuidedPractice

Evaluate each expression if $m = 12$ and $q = -1$.

1A. $m + (3 - q)^2$

1B. $m \div 2q + 4$

Taxi/Getty Images

Example 2 Evaluate Algebraic Expressions

a. Evaluate $a + b^2(b - a)$ if $a = 5$ and $b = -3.2$.

$$\begin{aligned} a + b^2(b - a) &= 5 + (-3.2)^2(-3.2 - 5) && a = 5 \text{ and } b = -3.2 \\ &= 5 + (-3.2)^2(-8.2) && \text{Subtract 5 from } -3.2. \\ &= 5 + 10.24(-8.2) && \text{Evaluate } (-3.2)^2. \\ &= 5 + (-83.968) && \text{Multiply 10.24 and } -8.2. \\ &= -78.968 && \text{Add 5 and } -83.968. \end{aligned}$$

b. Evaluate $\dfrac{x^4 - 3wy}{y^3 + 2w}$ if $w = 4$, $x = -3$, and $y = -5$.

$$\begin{aligned} \frac{x^4 - 3wy}{y^3 + 2w} &= \frac{(-3)^4 - 3(4)(-5)}{(-5)^3 + 2(4)} && w = 4, x = -3, \text{ and } y = -5 \\ &= \frac{81 - 3(4)(-5)}{-125 + 2(4)} && \text{Evaluate the numerator and denominator separately.} \\ &= \frac{81 - (-60)}{-125 + 8} && \text{Multiply in the numerator and denominator.} \\ &= \frac{141}{-117} \text{ or } -\frac{47}{39} && \text{Simplify the numerator and denominator. Then simplify the fraction.} \end{aligned}$$

StudyTip

CCSS Structure

Remember that a fraction bar is a type of grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

GuidedPractice

Evaluate each expression if $h = 4$, $j = -1$, and $k = 0.5$.

2A. $h^2k + h(h - k)$ **2B.** $j + (3 - h)^2$ **2C.** $\dfrac{j^2 - 3h^2k}{j^3 + 2}$

2 Formulas

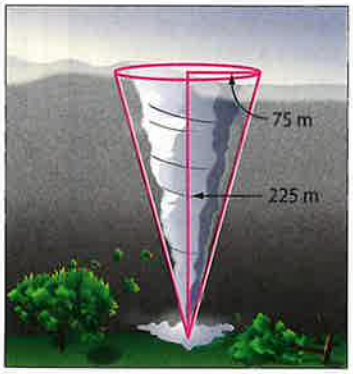
A formula is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

ReadingMath

Exponents x^2 may be read as *x squared* or *x to the second power*. x^3 may be read as *x cubed* or *x to the third power*.

Real-World Example 3 Use a Formula

TORNADOES The formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$, can be used to approximate the volume of a tornado. Find the approximate volume of the tornado at the right.



$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Volume of a cone} \\ &= \frac{1}{3}\pi(75)^2(225) && r = 75 \text{ and } h = 225 \\ &= \frac{1}{3}\pi(5625)(225) && \text{Evaluate } 75^2. \\ &\approx 1{,}325{,}359 && \text{Multiply.} \end{aligned}$$

The approximate volume of the tornado is about 1,325,359 cubic meters.

GuidedPractice

3. GEOMETRY The formula for the volume V of a rectangular prism is $V = \ell wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the volume of a rectangular prism with a length of 4 feet, a width of 2 feet, and a height of 3.5 feet.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Evaluate each expression if $a = -2$, $b = 3$, and $c = 4.2$.

1. $a - 2b + 3c$
2. $2a + (b + 3)^2$
3. $a + 3[b^2 - (a + c)]$

Example 2

4. $5c - 2[(b - a) + c]$
5. $4(2a + 3b) - 2c$
6. $\frac{a^2 + 4c}{3b + 2a}$
7. $\frac{b^3 + ac}{ab + 2bc}$
8. $\frac{3b + 2a}{5 - c}$
9. $\frac{3a - 2c}{4ab}$

Example 3

10. **VOLLEYBALL** A player's attack percentage A is calculated using the formula $A = \frac{k - e}{t}$, where k represents the number of kills, e represents the number of attack errors including blocks, and t represents the total attacks attempted. Find the attack percentage given each set of values.

    a. $k = 22$, $e = 11$, $t = 35$

    b. $k = 33$, $e = 9$, $t = 50$

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 Evaluate each expression if $w = -3$, $x = 4$, $y = 2.6$, and $z = \frac{1}{3}$.

11. $y + x - z$
12. $w - 2x + y \div 2$
13. $4(x - w)$
14. $6(y + x)$
15. $9z - 4y + 2w$
16. $3y - 4z + x$

17. **GAS MILEAGE** The gasoline used by a car is measured in miles per gallon and is related to the distance traveled by the following formula.

    miles per gallon × number of gallons = distance traveled

    a. During a trip your car used a total of 46.2 gallons of gasoline. If your car gets 33 miles to the gallon, how far did you travel?

    b. Your friend has decided to buy a hybrid car that gets 60 miles per gallon. The gasoline tank holds 12 gallons. How far can the car go on one tank of gasoline?

Example 2 Evaluate each expression if $a = -4$, $b = -0.8$, $c = 5$, and $d = \frac{1}{5}$.

18. $\frac{a + b}{c - d}$
19. $\frac{a - b}{bd}$
20. $\frac{ac}{d + b}$
21. $\frac{b^2c^2}{ad}$
22. $\frac{b + 6}{4(d + c)}$
23. $\frac{5(d + a)}{2ab^2}$

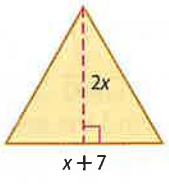
24. CCSS **SENSE-MAKING** The formula $C = \frac{5(F - 32)}{9}$ can be used to convert temperatures in degrees Fahrenheit to degrees Celsius.

    a. Room temperature commonly ranges from 64°F to 73°F. Determine the room temperature range in degrees Celsius.

    b. The normal average human body temperature is 98.6°F. A temperature above this indicates a fever. If your temperature is 42°C, do you have a fever? Explain your reasoning.

Example 3

25. **GEOMETRY** The formula for the area A of a triangle with height h and base b is $A = \frac{1}{2}bh$. Write an expression to represent the area of the triangle.



26. **FINANCIAL LITERACY** The profit that a business made during a year is $536,897,000. If the business divides the profit evenly for each share, estimate how much each share made if there are 10,995,000 shares.

27. CCSS **REASONING** The radius of Earth's orbit is 93,000,000 miles.

a. Find the circumference of Earth's orbit assuming that the orbit is a circle. The formula for the circumference of a circle is $2\pi r$.

b. Earth travels at a speed of 66,698 miles per hour around the Sun. Use the formula $T = \frac{C}{V}$, where T is time in hours, C is circumference, and V is velocity to find the number of hours it takes Earth to revolve around the Sun.

c. Did you prove that it takes 1 year for Earth to go around the Sun? Explain.

28. **ANCIENT PYRAMID** The Great Pyramid in Cairo, Egypt, is approximately 146.7 meters high, and each side of its base is approximately 230 meters.

a. Find the area of the base of the pyramid. Remember $A = \ell w$.

b. The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height. What is the volume of the Great Pyramid?

Evaluate each expression if $w = \frac{3}{4}$, $x = 8$, $y = -2$, and $z = 0.4$.

29. $x^3 + 2y^4$

30. $(x - 6z)^2$

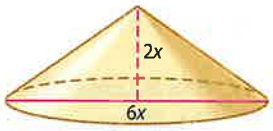
31. $2(6w - 2y) - 8z$

32. $\frac{(y + z)^2}{xw}$

33. $\frac{12w - 6y}{z^2}$

34. $\frac{wx + yz}{wx - yz}$

35. **GEOMETRY** The formula for the volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Write an expression for the volume of the cone at the right.



36. **SEARCH ENGINES** Page rank is a numerical value that represents how important a page is on the Web. One formula used to calculate the page rank for a page is $PR = 0.15 + 0.85L$, where L is the page rank of the linking page divided by the number of outbound links on the page. Determine the page rank of a page in which $L = 10$.

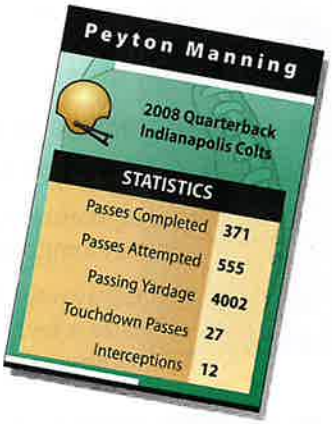
37. **WEATHER** In 1898, A.E. Dolbear studied various species of crickets to determine their "chirp rate" based on temperatures. He determined that the formula $t = 50 + \frac{n - 40}{4}$, where n is the number of chirps per minute, could be used to find the temperature t in degrees Fahrenheit. What is the temperature if the number of chirps is 120?

38. **FOOTBALL** The following formula can be used to calculate a quarterback efficiency rating.

$$\left(\frac{\frac{C}{A} - 0.3}{0.2} + \frac{\frac{Y}{A} - 3}{4} + \frac{\frac{T}{A}}{0.05} + \frac{0.095 - \frac{I}{A}}{0.04}\right) \cdot \frac{100}{6}$$

- C is the number of passes completed.
- A is the number of passes attempted.
- Y is passing yardage.
- T is the number of touchdown passes.
- I is the number of interceptions.

Find Peyton Manning's efficiency rating to the nearest tenth for the season statistics shown.

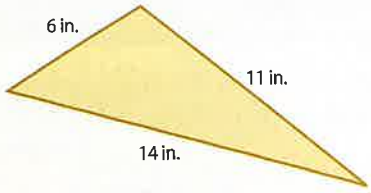


39. **MOVIES** The average price for a movie ticket can be represented by $P = \frac{y^2}{400} + \frac{7y}{100} + 2.96$ where y is the number of years since 1980.

a. Find the average price of a ticket in 1990, 2000, and 2010.

b. Another equation that can be used to represent ticket prices is $P = \frac{y^3}{2500} - \frac{y^2}{100} + \frac{6y}{25} + 2.62$. Find the price of a ticket in 1990, 2000, and 2010. How do these values compare to those you found in part a?

40. GEOMETRY The area of a triangle can be found using Heron's Formula, $A = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the three sides of the triangle, and $s = \frac{a+b+c}{2}$. Find the area of the triangle at the right.



41 Evaluate $y = \sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}$ if $a = 6$, $b = 8$, and $x = 3$. Round to the nearest tenth.

42. MULTIPLE REPRESENTATIONS You will write expressions using the formula for the volume of a cylinder. Recall that the volume of a cylinder can be found using the formula $v = \pi r^2 h$, in which v = volume, r = radius, and h = height.

a. Geometric Draw two cylinders of different sizes.

b. Tabular Use a ruler to measure the radius and height of each cylinder. Organize the measures for each cylinder into a table. Include a column in your table to calculate the volume of each cylinder.

c. Verbal Write a verbal expression for the difference in volume of the two cylinders.

d. Algebraic Write and solve an algebraic expression for the difference in volume of the two cylinders.

H.O.T. Problems Use Higher-Order Thinking Skills

43. CCSS CRITIQUE Lauren and Rico are evaluating $\frac{-3d-4c}{2ab}$ for $a = -2$, $b = -3$, $c = 5$, and $d = 4$. Is either of them correct? Explain your reasoning.

Lauren

$$\frac{-3d-4c}{2ab} = \frac{-3(4)-4(5)}{2(-2)(-3)}$$

$$= \frac{-12-20}{12} = \frac{-32}{12} = -\frac{8}{3}$$

Rico

$$\frac{-3d-4c}{2ab} = \frac{-3(4)-4(5)}{2(-2)(-3)}$$

$$= \frac{-12-20}{12} = \frac{8}{12} = \frac{2}{3}$$

44. CHALLENGE For any three distinct numbers a, b, and c, $a\$b\$c$ is defined as $a\$b\$c = \frac{-a-b-c}{c-b-a}$. Find $-2\$(-4)\$5$.

45. REASONING The following equivalent expressions represent the height in feet of a stone thrown downward off a bridge where t is the time in seconds after release. Which do you find most useful for finding the maximum height of the stone? Explain.

a. $-4t^2 - 2t + 6$ **b.** $-2t(2t+1) + 6$ **c.** $-2(t-1)(2t+3)$

46. CHALLENGE Let m, n, p, and q represent nonzero positive integers. Find a number in terms of m, n, p, and q that is halfway between $\frac{m}{n}$ and $\frac{p}{q}$.

47. OPEN ENDED Write an algebraic expression using $x = -2$, $y = -3$, and $z = 4$ and all four operations for which the value of the expression is 10.

48. WRITING IN MATH Provide an example of a formula used in everyday situations. Explain its usefulness and what happens if the formula is not used correctly.

49. WRITING IN MATH Use the information for on-base percentage given at the beginning of the lesson to explain why a formula for on-base percentage is more useful than a table of specific percentages.

Standardized Test Practice

50. SAT/ACT If the area of a square with side x is 9, what is the area of a square of side $4x$?

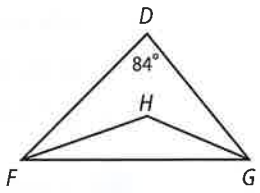
A 36
B 144
C 212
D 324
E 1296

51. SHORT RESPONSE A coffee shop owner wants to open a second shop when his daily customer average reaches 800 people. He has calculated the daily customer average in the table below for each month since he has opened.

Month	Daily Customer Average
1	225
2	298
3	371
4	444

If the trend continues, during what month can he open a second shop?

52. GEOMETRY In $\triangle DFG$, $\overline{FH}$ and $\overline{HG}$ are angle bisectors and $m\angle D = 84°$. How many degrees are in $\angle FHG$?



F 96
G 132
H 145
J 192

53. A skydiver in a computer game free-falls from a height of 3000 meters at a rate of 55 meters per second. Which equation can be used to find h, the height of the skydiver after t seconds of free fall?

A $h = -55t - 3000$
B $h = -55t + 3000$
C $h = 3000t - 55$
D $h = 3000t + 55$

Spiral Review

54. The lengths of the three sides of a triangle are 10, 14, and 18 inches. Determine whether this triangle is a right triangle. (Lesson 0-8)

55. The legs of a right triangle measure 6 centimeters and 8 centimeters. Find the length of the hypotenuse. (Lesson 0-8)

56. MAPS On a map of the U.S., the cities of Milwaukee, Wisconsin, and Charlotte, North Carolina, are $6\frac{1}{2}$ inches apart. The actual distance between Milwaukee and Charlotte is 670 miles. If Birmingham, Alabama, and St. Petersburg, Florida, are 465 miles apart, how far apart are they on the map? (Lesson 0-7)

57. Factor $6x^2 + 12x$. (Lesson 0-3)

58. Find the product of $(a + 2)(a - 4)$. (Lesson 0-2)

59. NUMBER An integer is 2 less than a number, and another integer is 1 greater than double that same number. What are the two integers if their sum is 14? (Lesson 0-2)

Skills Review

Evaluate each expression.

60. $\sqrt{4}$
61. $\sqrt{25}$
62. $\sqrt{81}$
63. $\sqrt{121}$
64. $-\sqrt{9}$
65. $-\sqrt{16}$
66. $\sqrt{\frac{49}{100}}$
67. $\sqrt{\frac{25}{64}}$

LESSON 1-2

Properties of Real Numbers

Then	Now	Why?
You identified and used the arithmetic properties of real numbers.	1 Classify real numbers. 2 Use the properties of real numbers to evaluate expressions.	The Central High School Boosters sell snacks and beverages at school functions. The items are priced the same to make determining the total cost easy. You can use the Distributive Property to calculate the total cost when multiple items are purchased.

New Vocabulary
real numbers
rational numbers
irrational numbers
integers
whole numbers
natural numbers

Common Core State Standards

Content Standards
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

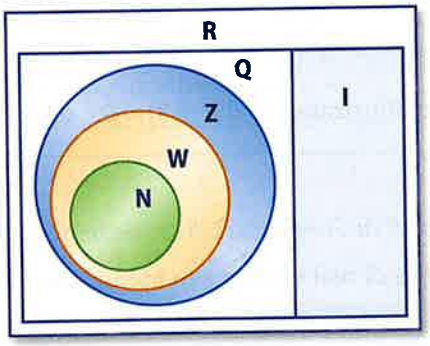
Mathematical Practices
2 Reason abstractly and quantitatively.
7 Look for and make use of structure.

1 Real Numbers

Real numbers consist of several different kinds of numbers.

- **Rational numbers** can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and b is not zero. The decimal form of a rational number is either a terminating or repeating decimal.
- The decimal form of an **irrational number** neither terminates nor repeats. Square roots of numbers that are not perfect squares are irrational numbers.
- The sets of **integers**, {…, −3, −2, −1, 0, 1, 2, 3, …}, **whole numbers**, {0, 1, 2, 3, 4, …}, and **natural numbers**, {1, 2, 3, 4, 5, …}, are subsets of the rational numbers. These numbers are subsets of the rational numbers because every integer n is equal to $\frac{n}{1}$.

KeyConcept Real Numbers (R)



Letter	Set	Examples
Q	rationals	$0.125, -\frac{7}{8}, \frac{2}{3} = 0.66\ldots$
I	irrationals	$\pi = 3.14159\ldots$ $\sqrt{3} = 1.73205\ldots$
Z	integers	−5, 17, −23, 8
W	wholes	$2, 96, 0, \sqrt{36}$
N	naturals	3, 17, 6, 86

PT

Example 1 Classify Numbers

Name the sets of numbers to which each number belongs.

a. −23 integers (Z), rationals (Q), reals (R)

b. $\sqrt{50}$ irrationals (I), reals (R)

c. $-\frac{4}{9}$ rationals (Q), reals (R)

Guided Practice

1A. −185 **1B.** $-\sqrt{49}$ **1C.** $\sqrt{95}$ **1D.** $-\frac{7}{8}$

StudyTip

Real Numbers A number can belong to more than one set of numbers. For example, if a number is natural, it is also whole, an integer, rational, and real.

2 Properties of Real Numbers

Some of the properties of real numbers are summarized below.

ConceptSummary Real Number Properties

For any real numbers a, b, and c:

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	$a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a, a \neq 0$
Closure	$a + b$ is a real number.	$a \cdot b$ is a real number.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Example 2 Name Properties of Real Numbers

Name the property illustrated by $5 \cdot (4 \cdot 13) = (5 \cdot 4) \cdot 13$.

Associative Property of Multiplication

The Associative Property of Multiplication states that the way in which you group factors does not affect the product.

GuidedPractice

2. Name the property illustrated by $2(x + 3) = 2x + 6$.

StudyTip

Additive and Multiplicative Inverses The additive inverse of a number has the opposite sign as the number. The multiplicative inverse of a number has the same sign as the number.

You can use the properties of real numbers to identify related values.

Example 3 Additive and Multiplicative Inverses

Find the additive inverse and multiplicative inverse for $-\frac{5}{8}$.

Since $-\frac{5}{8} + \frac{5}{8} = 0$, the additive inverse of $-\frac{5}{8}$ is $\frac{5}{8}$.

Since $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1$, the multiplicative inverse of $-\frac{5}{8}$ is $-\frac{8}{5}$.

GuidedPractice

Find the additive and multiplicative inverse for each number.

3A. 1.25

3B. $2\frac{1}{2}$

Many real-world applications involve working with real numbers.

Real-WorldCareer

Retail Store Manager Store managers are responsible for the day-to-day operations of a retail store. A store manager may range from being a high school graduate to having a 4-year degree, depending on the business.

Real-World Example 4 Distributive Property

MONEY The prices of the components of a computer package offered by Computer Depot are shown in the table. If a 6% sales tax is added to the purchase price, how much sales tax is charged for this computer package?

Component	Price ($)
Computer	359.95
Monitor	219.99
Printer	79.00
Digital Camera	149.50
Software Bundle	99.00

There are two ways to determine the total sales tax.

Method 1 Multiply, then add.

Multiply each dollar amount by 6% or 0.06 and then add.

$$T = 0.06(359.95) + 0.06(219.99) + 0.06(79.00) + 0.06(149.50) + 0.06(99.00)$$

$$= 21.60 + 13.20 + 4.74 + 8.97 + 5.94$$

$$= 54.45$$

Method 2 Add, then multiply.

Find the total cost of the computer package, and then multiply the total by 0.06.

$$T = 0.06(359.95 + 219.99 + 79.00 + 149.50 + 99.00)$$

$$= 0.06(907.44)$$

$$= 54.45$$

The sales tax charged is $54.45. Notice that both methods result in the same answer.

GuidedPractice

4. JOBS Kayla makes $8 per hour working at a grocery store. The number of hours Kayla worked each day in one week are 3, 2.5, 2, 1, and 4. How much money did Kayla earn this week?

The properties of real numbers can be used to simplify algebraic expressions.

Example 5 Simplify an Expression

Simplify $3(2q + r) + 5(4q - 7r)$.

$3(2q + r) + 5(4q - 7r)$

$= 3(2q) + 3(r) + 5(4q) - 5(7r)$ Distributive Property

$= 6q + 3r + 20q - 35r$ Multiply.

$= 6q + 20q + 3r - 35r$ Commutative Property (+)

$= (6 + 20)q + (3 - 35)r$ Distributive Property

$= 26q - 32r$ Simplify.

GuidedPractice

5. Simplify $3(4x - 2y) - 2(3x + y)$.

Jon Riley/Stone/Getty Images

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Name the sets of numbers to which each number belongs.

1. 62
2. $\frac{5}{4}$
3. $\sqrt{11}$
4. -12

Example 2 Name the property illustrated by each equation.

5. $(6 \cdot 8) \cdot 5 = 6 \cdot (8 \cdot 5)$
6. $7(9 - 5) = 7 \cdot 9 - 7 \cdot 5$
7. $84 + 16 = 16 + 84$
8. $(12 + 5)6 = 12 \cdot 6 + 5 \cdot 6$

Example 3 Find the additive inverse and multiplicative inverse for each number.

9. -7
10. $\frac{4}{9}$
11. 3.8
12. $\sqrt{5}$

Example 4

13. CCSS REASONING Melba is mowing lawns for $22 each to earn money for a video game console that costs $550.
    a. Write an expression to represent the total amount of money Melba earned during this week.
    b. Evaluate the expression from part a by using the Distributive Property.
    c. When do you think Melba will earn enough for the video game console? Is this reasonable? Explain.

Lawns Mowed in One Week

Day	Lawns Mowed
Monday	2
Tuesday	4
Wednesday	3
Thursday	1
Friday	5
Saturday	6
Sunday	7

Example 5 Simplify each expression.

14. $5(3x + 6y) + 4(2x - 9y)$
15. $6(6a + 5b) - 3(4a + 7b)$
16. $-4(6c - 3d) - 5(-2c - 4d)$
17. $-5(8x - 2y) - 4(-6x - 3y)$

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 Name the sets of numbers to which each number belongs.

18. $-\frac{4}{3}$
19. -8.13
20. $\sqrt{25}$
21. $0.\overline{61}$
22. $\frac{9}{3}$
23. $-\sqrt{144}$
24. $\frac{21}{7}$
25. $\sqrt{17}$

Example 2 Name the property illustrated by each equation.

26. $-7y + 7y = 0$
27. $8\sqrt{11} + 5\sqrt{11} = (8 + 5)\sqrt{11}$
28. $(16 + 7) + 23 = 16 + (7 + 23)$
29. $\left(\frac{22}{7}\right)\left(\frac{7}{22}\right) = 1$

Example 3 Find the additive inverse and multiplicative inverse for each number.

30. -8
31. 12.1
32. -0.25
33. $\frac{6}{13}$
34. $-\frac{3}{8}$
35. $\sqrt{15}$

Example 4

36. CONSTRUCTION Jorge needs two different kinds of concrete: quick drying and slow drying. The quick-drying concrete mix calls for $2\frac{1}{2}$ pounds of dry cement, and the slow-drying concrete mix calls for $1\frac{1}{4}$ pounds of dry cement. He needs 5 times more quick-drying concrete and 3 times more slow-drying concrete than the mixes make.
    a. How many pounds of dry cement mix will he need?
    b. Use the properties of real numbers to show how Jorge could compute this amount mentally. Justify each step.

Example 5 **Simplify each expression.**

37. $8b - 3c + 4b + 9c$

38. $-2a + 9d - 5a - 6d$

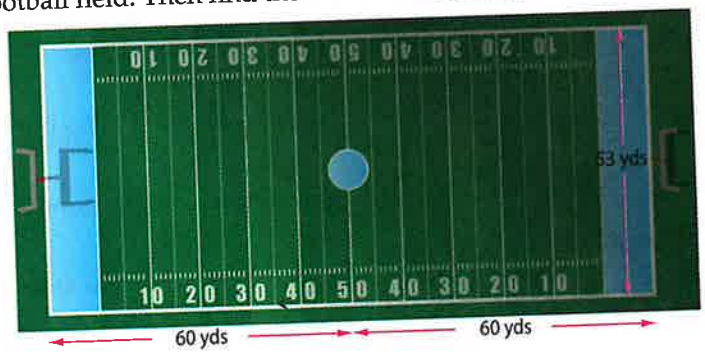
39. $4(4x - 9y) + 8(3x + 2y)$

40. $6(9a - 3b) - 8(2a + 4b)$

41. $-2(-5g + 6k) - 9(-2g + 4k)$

42. $-5(10x + 8z) - 6(4x - 7z)$

43. FOOTBALL Illustrate the Distributive Property by writing two expressions for the area of a college football field. Then find the area of the football field.



44. PETS The chart shows the percent of dogs registered with the American Kennel Club that are of the eight most popular breeds.

a. Illustrate the Distributive Property by writing two expressions to represent the number of registered dogs of the top four breeds.

b. Evaluate the expressions you wrote to find the number of registered dogs of the top four breeds.

Top Dogs	
Breed	**Percent of Registered Dogs**
Labrador Retrievers	14.2
Yorkshire Terriers	5.6
German Shepherds	5.0
Golden Retrievers	4.9
Beagles	4.5
Dachshunds	4.1
Boxers	4.1
Poodles	3.4
Total Registered Dogs	**870,192**

Source: American Kennel Club

45 FINANCIAL LITERACY Billie is given $20 in lunch money by her parents once every two weeks. On some days, she packs her lunch, and on other days, she buys her lunch. A hot lunch from the cafeteria costs $4.50, and a cold sandwich from the lunch line costs $2.

a. Billie decides that she wants to buy a hot lunch on Thursday and Friday of the first week and on Wednesday of the second week. Use the Distributive Property to determine how much that will cost.

b. How many cold sandwiches can Billie buy with the amount left over?

c. Assuming that both weeks are Monday through Friday, how many times will Billie have to pack her lunch?

Simplify each expression.

46. $\frac{1}{3}(5x + 8y) + \frac{1}{4}(6x - 2y)$

47. $\frac{2}{5}(6c - 8d) + \frac{3}{4}(4c - 9d)$

48. $-6(3a + 5b) - 3(6a - 8c)$

49. $-9(3x + 8y) - 3(5x + 10z)$

50. CCSS MODELING Mary is making curtains out of the same fabric for 5 windows. The two larger windows are the same size, and the three smaller windows are the same size. One larger window requires $3\frac{3}{4}$ yards of fabric, and one smaller window needs $2\frac{1}{3}$ yards of fabric.

a. How many yards of material will Mary need?

b. Use the properties of real numbers to show how Mary could compute this amount mentally.

51 **MULTIPLE REPRESENTATIONS** Consider the following real numbers.

$$-\sqrt{6}, 3, \frac{-15}{3}, 4.1, \pi, 0, \frac{3}{8}, \sqrt{36}$$

a. **Tabular** Organize the numbers into a table according to the sets of numbers to which each belongs.

b. **Algebraic** Convert each number to decimal form. Then list the numbers from least to greatest.

c. **Graphical** Graph the numbers on a number line.

d. **Verbal** Make a conjecture about using decimal form to list real numbers in order.

52. **CLOTHING** A department store sells shirts for $12.50 each. Dalila buys 2, Latisha buys 3, and Pilar buys 1.

a. Illustrate the Distributive Property by writing two expressions to represent the cost of these shirts.

b. Use the Distributive Property to find how much money the store received from selling these shirts.

H.O.T. Problems Use Higher-Order Thinking Skills

53. **WHICH ONE DOESN'T BELONG?** Identify the number that does not belong with the other three. Explain your reasoning.

$\sqrt{21}$	$\sqrt{35}$	$\sqrt{67}$	$\sqrt{81}$

54. **CHALLENGE** If $12(5r + 6t) = w$, then in terms of w, what is $48(30r + 36t)$?

55. CCSS **CRITIQUE** Luna and Sophia are simplifying $4(14a - 10b) - 6(b + 4a)$. Is either of them correct? Explain your reasoning.

Luna	*Sophia*
$4(14a - 10b) - 6(b + 4a)$	$4(14a - 10b) - 6(b + 4a)$
$56a - 40b - 6b + 24a$	$56a - 40b - 6a - 24b$
$80a - 46b$	$50a - 64b$

56. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

An irrational number is a real number within a radical sign.

57. **OPEN ENDED** Determine whether the Closure Property of Multiplication applies to irrational numbers. If not, provide a counterexample.

OPEN ENDED The set of all real numbers is *dense*, meaning between any two distinct members of the set there lies infinitely many other members of the set. Find an example of (a) a rational number, and (b) an irrational number between the given numbers.

58. 2.45 and 2.5

59. π and $\frac{10}{3}$

60. $1.\overline{9}$ and 2.01

61. **WRITING IN MATH** Explain and provide examples to show why the Commutative Property does not hold true for subtraction or division.

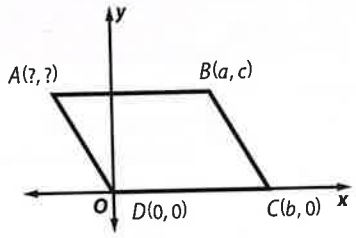
Standardized Test Practice

62. EXTENDED RESPONSE Lenora bought several pounds of cashews and several pounds of almonds for a party. The cashews cost \$8 per pound, and the almonds cost \$6 per pound. Lenora bought a total of 7 pounds and paid a total of \$48. Write and solve a system of equations to determine the pounds of cashews and the pounds of almonds that Lenora purchased.

63. SAT/ACT Find the 10th term in the series 2, 4, 7, 11, 16, … .

A 41
B 46
C 56
D 67
E 72

64. GEOMETRY What are the coordinates of point A in the parallelogram?



F $(b - a, c)$
G $(a - b, c)$
H (b, c)
J (c, c)

65. What is the domain of the function that contains the points $(-3, 0)$, $(0, 4)$, $(-2, 5)$, and $(6, 4)$?

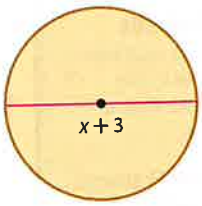
A $\{-3, 6\}$
B $\{-3, -2, 0, 6\}$
C $\{0, 4, 5, 6\}$
D $\{-3, -2, 0, 4, 5, 6\}$

Spiral Review

66. Evaluate $8(4 - 2)^3$. (Lesson 1-1)

67. Evaluate $a + 3(b + c) - d$, if $a = 5$, $b = 4$, $c = 3$, and $d = 2$. (Lesson 1-1)

68. GEOMETRY The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle. (Lesson 1-1)



69. CONSTRUCTION A 10-meter ladder leans against a building so that the top is 9.64 meters above the ground. How far from the base of the wall is the bottom of the ladder? (Lesson 0-8)

Factor each polynomial. (Lesson 0-3)

70. $14x^2 + 10x - 8$
71. $9x^2 - 3x + 18$
72. $8x^2 + 16x + 12$
73. $10x^2 - 20x$
74. $7x^2 - 14x - 21$
75. $12x^2 - 18x - 24$

Find each product. (Lesson 0-2)

76. $(x + 2)(x - 3)$
77. $(y + 2)(y - 1)$
78. $(a - 5)(a + 4)$
79. $(b - 7)(b - 3)$
80. $(n + 6)(n + 8)$
81. $(p - 9)(p + 1)$

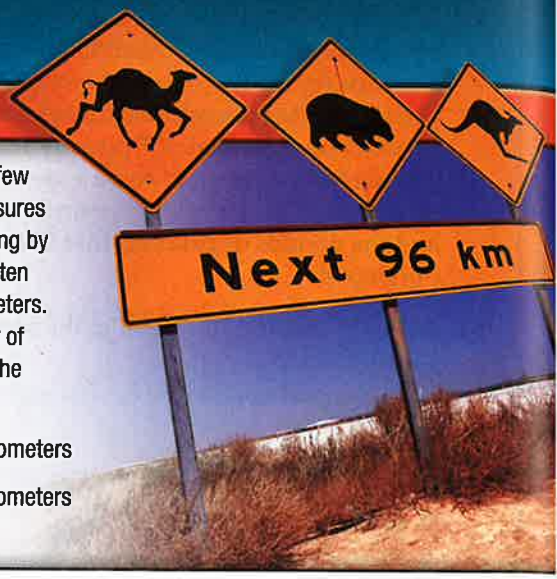
Skills Review

Evaluate each expression if $a = 3$, $b = \frac{2}{3}$, and $c = -1.7$.

82. $6b - 5$
83. $\frac{1}{6}b + 1$
84. $2.3c - 7$
85. $-8(a - 4)$
86. $a + b + c$
87. $\frac{a \cdot b}{c}$
88. $a^2 - c$
89. $\frac{a \cdot c}{a}$

LESSON 1-3

Solving Equations



Then

- You used properties of real numbers to evaluate expressions.

Now

- **1** Translate verbal expressions into algebraic expressions and equations, and vice versa.
- **2** Solve equations using the properties of equality.

Why?

- The United States is one of the few countries in the world that measures distances in miles. When traveling by car in different countries, it is often useful to convert miles to kilometers. To find the approximate number of kilometers k in miles m, divide the number of miles by 0.62137.

$$m \text{ miles} \times \frac{1 \text{ kilometer}}{0.62137 \text{ mile}} \approx k \text{ kilometers}$$

$$\frac{m}{0.62137} \approx k \text{ kilometers}$$

NewVocabulary

- open sentence
- equation
- solution

Common Core State Standards

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

8 Look for and express regularity in repeated reasoning.

1 Verbal Expressions and Algebraic Expressions

Verbal expressions can be translated into algebraic expressions by using the language of algebra.

Example 1 Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. 2 more than 4 times the cube of a number $4x^3 + 2$

b. the quotient of 5 less than a number and 12 $\frac{n-5}{12}$

GuidedPractice

1A. the cube of a number increased by 4 times the same number

1B. three times the difference of a number and 8

A mathematical sentence containing one or more variables is called an **open sentence**. A mathematical sentence stating that two mathematical expressions are equal is called an **equation**.

Example 2 Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. $6x = 72$ The product of 6 and a number is 72.

b. $n + 15 = 91$ The sum of a number and 15 is ninety-one.

GuidedPractice

2A. $g - 5 = -2$

2B. $2c = c^2 - 4$

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a **solution** of the open sentence.

2 Properties of Equality

To solve equations, we can use properties of equality. Some of these properties are listed below.

akg-images

Math HistoryLink

Diophantus of Alexandria (c. 200–284)

Diophantus was famous for his work in algebra. His main work was titled *Arithmetica* and introduced symbolism to Greek algebra as well as propositions in number theory and polygonal numbers.

KeyConcept Properties of Equality

Property	Symbols	Examples
Reflexive	For any real number a, $a = a$.	$b + 12 = b + 12$
Symmetric	For all real numbers a and b, if $a = b$, then $b = a$.	If $18 = -2n + 4$, then $-2n + 4 = 18$.
Transitive	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $5p + 3 = 48$ and $48 = 7p - 15$, then $5p + 3 = 7p - 15$.
Substitution	If $a = b$, then a may be replaced by b and b may be replaced by a.	If $(6 + 1)x = 21$, then $7x = 21$.

PT

Example 3 Identify Properties of Equality

Name the property illustrated by each statement.

a. If $3a - 4 = b$, and $b = a + 17$, then $3a - 4 = a + 17$.

Transitive Property of Equality

b. If $2g - h = 62$, and $h = 24$, then $2g - 24 = 62$.

Substitution Property of Equality

GuidedPractice

3. If $-11a + 2 = -3a$, then $-3a = -11a + 2$.

Solving most equations requires assuming that the original equation has a solution, and performing the same operations on each side of the equals sign. The properties of equality allow for the equation to be solved in this way.

KeyConcept

Addition and Subtraction Properties of Equality	
Symbols	For any real numbers, a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Examples	If $x - 6 = 14$, then $x - 6 + 6 = 14 + 6$. If $n + 5 = -32$, then $n + 5 - 5 = -32 - 5$.
Multiplication and Division Properties of Equality	
Symbols	For any real numbers, a, b, and c, $c \neq 0$, if $a = b$, then $a \cdot c = b \cdot c$ and $\frac{a}{c} = \frac{b}{c}$.
Examples	If $\frac{m}{8} = -7$, then $8 \cdot \frac{m}{8} = 8 \cdot (-7)$. If $-2y = 12$, then $\frac{-2y}{-2} = \frac{12}{-2}$.

Example 4 Solve One-Step Equations

Solve each equation. Check your solution.

a. $n - 3.24 = 42.1$

$n - 3.24 = 42.1$ Original equation

$n - 3.24 + 3.24 = 42.1 + 3.24$ Add 3.24 to each side.

$n = 45.34$ Simplify.

The solution is 45.34.

CHECK $n - 3.24 = 42.1$ Original equation

$45.34 - 3.24 \stackrel{?}{=} 42.1$ Substitute 45.34 for n.

$42.1 = 42.1$ ✓ Simplify.

b. $-\frac{5}{8}x = 20$

$-\frac{5}{8}x = 20$ Original equation

$-\frac{8}{5}\left(-\frac{5}{8}\right)x = -\frac{8}{5}(20)$ Multiply each side by $-\frac{8}{5}$.

$x = -32$ Simplify.

The solution is -32.

CHECK $-\frac{5}{8}x = 20$ Original equation

$-\frac{5}{8}(-32) \stackrel{?}{=} 20$ Replace x with -32.

$20 = 20$ ✓ Simplify.

StudyTip

CCSS Regularity

In Example 4b, notice that multiplying both sides of the equation by $-\frac{8}{5}$ is the same as dividing both sides by $-\frac{5}{8}$.

GuidedPractice

4A. $x - 14.29 = 25$

4B. $\frac{2}{3}y = -18$

To solve an equation with more than one operation, undo operations by working backward.

Example 5 Solve a Multi-Step Equation

Solve $5(x + 3) + 2(1 - x) = 14$.

$5(x + 3) + 2(1 - x) = 14$ Original equation

$5x + 15 + 2 - 2x = 14$ Apply the Distributive Property.

$3x + 17 = 14$ Simplify the left side.

$3x = -3$ Subtract 17 from each side.

$x = -1$ Divide each side by 3.

StudyTip

Checking Answers

When solving for a variable, you can use substitution to check your answer by replacing the variable in the original equation with your answer.

GuidedPractice

Solve each equation.

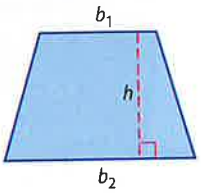
5A. $-10x + 3(4x - 2) = 6$

5B. $2(2x - 1) - 4(3x + 1) = 2$

You can use properties to solve an equation for a variable.

Example 6 Solve for a Variable

GEOMETRY **The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h represents the height, and b_1 and b_2 represent the measures of the bases. Solve the formula for b_2.**



$A = \frac{1}{2}h(b_1 + b_2)$	Area formula
$2A = 2\left[\frac{1}{2}h(b_1 + b_2)\right]$	Multiply each side by 2.
$2A = h(b_1 + b_2)$	Simplify.
$\frac{2A}{h} = \frac{h(b_1 + b_2)}{h}$	Divide each side by h.
$\frac{2A}{h} = b_1 + b_2$	Simplify.
$\frac{2A}{h} - b_1 = b_1 + b_2 - b_1$	Subtract b_1 from each side.
$\frac{2A}{h} - b_1 = b_2$	Simplify.

Guided Practice

6. The formula for the surface area S of a cylinder is $S = 2\pi r^2 + 2\pi rh$, where r is the radius of the base and h is the height of the cylinder. Solve the formula for h.

There are often many ways to solve a problem. Using the properties of equality can help you find a simpler way.

Standardized Test Example 7 Use Properties of Equality

If $6x - 12 = 18$, what is the value of $6x + 5$?

A 5 B 11 C 35 D 41

Test-Taking Tip

Read the Question Read the question carefully before solving the equation. In Example 7, you are to find the value of $6x + 5$, not the value of x.

Read the Test Item

You are asked to find the value of $6x + 5$. Note that you do not have to find the value of x. Instead, you can use the Addition Property of Equality to make the left side of the equation $6x + 5$.

Solve the Test Item

$6x - 12 = 18$	Original equation
$6x - 12 + 17 = 18 + 17$	Add 17 to each side because $-12 + 17 = 5$.
$6x + 5 = 35$	Simplify.

The answer is C.

Guided Practice

7. If $5y + 2 = \frac{8}{3}$, what is the value of $5y - 6$?

F $\frac{-20}{3}$ G $\frac{-16}{3}$ H $\frac{16}{3}$ J $\frac{32}{3}$

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Write an algebraic expression to represent each verbal expression.**

1. the product of 12 and the sum of a number and negative 3
2. the difference between the product of 4 and a number and the square of the number

Example 2 **Write a verbal sentence to represent each equation.**

3. $5x + 7 = 18$
4. $x^2 - 9 = 27$
5. $5y - y^3 = 12$
6. $\frac{x}{4} + 8 = -16$

Example 3 **Name the property illustrated by each statement.**

7. $(8x - 3) + 12 = (8x - 3) + 12$
8. If $a = -3$ and $-3 = d$, then $a = d$.

Examples 4–5 CCSS **PRECISION** **Solve each equation. Check your solution.**

9. $z - 19 = 34$
10. $x + 13 = 7$
11. $-y = 8$
12. $-6x = 42$
13. $5x - 3 = -33$
14. $-6y - 8 = 16$
15. $3(2a + 3) - 4(3a - 6) = 15$
16. $5(c - 8) - 3(2c + 12) = -84$
17. $-3(-2x + 20) + 8(x + 12) = 92$
18. $-4(3m - 10) - 6(-7m - 6) = -74$

Example 6 **Solve each equation or formula for the specified variable.**

19. $8r - 5q = 3$, for q
20. $Pv = nrt$, for n

Example 7

21. **MULTIPLE CHOICE** If $\frac{y}{5} + 8 = 7$, what is the value of $\frac{y}{5} - 2$?

A -10 B -3 C 1 D 5

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **Write an algebraic expression to represent each verbal expression.**

22. the difference between the product of four and a number and 6
23. the product of the square of a number and 8
24. fifteen less than the cube of a number
25. five more than the quotient of a number and 4

Example 2 **Write a verbal sentence to represent each equation.**

26. $8x - 4 = 16$
27. $\frac{x + 3}{4} = 5$
28. $4y^2 - 3 = 13$

29. **BASEBALL** During a recent season, Miguel Cabrera and Mike Jacobs hit a combined total of 46 home runs. Cabrera hit 6 more home runs than Jacobs. How many home runs did each player hit? Define a variable, write an equation, and solve the problem.

Example 3 **Name the property illustrated by each statement.**

30. If $x + 9 = 2$, then $x + 9 - 9 = 2 - 9$
31. If $y = -3$, then $7y = 7(-3)$
32. If $g = 3h$ and $3h = 16$, then $g = 16$
33. If $-y = 13$, then $-(-y) = -13$

34. **MONEY** Aiko and Kendra arrive at the state fair with $32.50. What is the total number of rides they can go on if they each pay the entrance fee?

Examples 4–5 **PRECISION** **Solve each equation. Check your solution.**

35. $3y + 4 = 19$

36. $-9x - 8 = 55$

37. $7y - 2y + 4 + 3y = -20$

38. $5g + 18 - 7g + 4g = 8$

39. $5(-2x - 4) - 3(4x + 5) = 97$

40. $-2(3y - 6) + 4(5y - 8) = 92$

41. $\frac{2}{3}(6c - 18) + \frac{3}{4}(8c + 32) = -18$

42. $\frac{3}{5}(15d + 20) - \frac{1}{6}(18d - 12) = 38$

43. **GEOMETRY** The perimeter of a regular pentagon is 100 inches. Find the length of each side.

44. **MEDICINE** For Nina's illness her doctor gives her a prescription for 28 pills. The doctor says that she should take 4 pills the first day and then 2 pills each day until her prescription runs out. For how many days does she take 2 pills?

Example 6 **Solve each equation or formula for the specified variable.**

45. $E = mc^2$, for m

46. $c(a + b) - d = f$, for a

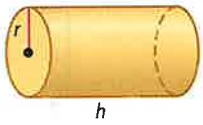
47. $z = \pi q^3 h$, for h

48. $\frac{x + y}{z} - a = b$, for y

49. $y = ax^2 + bx + c$, for a

50. $wx + yz = bc$, for z

51. **GEOMETRY** The formula for the volume of a cylinder with radius r and height h is π times the radius times the radius times the height.



a. Write this as an algebraic expression.

b. Solve the expression in part a for h.

52. **AWARDS BANQUET** A banquet room can seat a maximum of 69 people. The coach, principal, and vice principal have invited the award-winning girls' tennis team to the banquet. If the tennis team consists of 22 girls, how many guests can each student bring?

PRECISION **Solve each equation. Check your solution.**

53. $5x - 9 = 11x + 3$

54. $\frac{1}{x} + \frac{1}{4} = \frac{7}{12}$

55. $5.4(3k - 12) + 3.2(2k + 6) = -136$

56. $8.2p - 33.4 = 1.7 - 3.5p$

57. $\frac{4}{9}y + 5 = -\frac{7}{9}y - 8$

58. $\frac{3}{4}z - \frac{1}{3} = \frac{2}{3}z + \frac{1}{5}$

59. **FINANCIAL LITERACY** Benjamin spent $10,734 on his living expenses last year. Most of these expenses are listed at the right. Benjamin's only other expense last year was rent. If he paid rent 12 times last year, how much was Benjamin's rent each month?

Expense	Annual Cost
Electric	$622
Gas	$428
Water	$240
Renter's Insurance	$144

60 CCSS **SENSE-MAKING** The Sunshine Skyway Bridge spans Tampa Bay, Florida. Suppose one crew began building south from St. Petersburg, and another crew began building north from Bradenton. The two crews met 10,560 feet south of St. Petersburg approximately 5 years after construction began.

a. Suppose the St. Petersburg crew built an average of 176 feet per month. Together the two crews built 21,120 feet of bridge. Determine the average number of feet built per month by the Bradenton crew.

b. About how many miles of bridge did each crew build?

c. Is this answer reasonable? Explain.

61 **MULTIPLE REPRESENTATIONS** The absolute value of a number describes the distance of the number from zero.

a. Geometric Draw a number line. Label the integers from -5 to 5.

b. Tabular Create a table of the integers on the number line and their distance from zero.

c. Graphical Make a graph of each integer x and its distance from zero y using the data points in the table.

d. Verbal Make a conjecture about the integer and its distance from zero. Explain the reason for any changes in sign.

H.O.T. Problems Use Higher-Order Thinking Skills

62. **ERROR ANALYSIS** Steven and Jade are solving $A = \frac{1}{2}h(b_1 + b_2)$ for b_2. Is either of them correct? Explain your reasoning.

Steven

$A = \frac{1}{2}h(b_1 + b_2)$

$\frac{2A}{h} = (b_1 + b_2)$

$\frac{2A - b_1}{h} = b_2$

Jade

$A = \frac{1}{2}h(b_1 + b_2)$

$\frac{2A}{h} = (b_1 + b_2)$

$\frac{2A}{h} - b_1 = b_2$

63. **CHALLENGE** Solve $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for y_1.

64. **REASONING** Use what you have learned in this lesson to explain why the following number trick works.

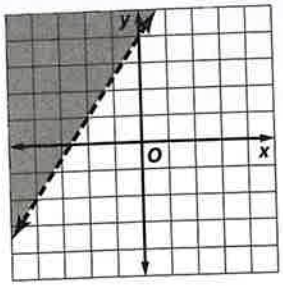
- Take any number.
- Multiply it by ten.
- Subtract 30 from the result.
- Divide the new result by 5.
- Add 6 to the result.
- Your new number is twice your original.

65. **OPEN ENDED** Provide one example of an equation involving the Distributive Property that has no solution and another example that has infinitely many solutions.

66. **WRITING IN MATH** Compare and contrast the Substitution Property of Equality and the Transitive Property of Equality.

Standardized Test Practice

67. The graph shows the solution of which inequality?



A $y < \frac{2}{3}x + 4$

B $y > \frac{2}{3}x + 4$

C $y < \frac{3}{2}x + 4$

D $y > \frac{3}{2}x + 4$

68. SAT/ACT What is $1\frac{1}{3}$ subtracted from its reciprocal?

F $-2\frac{2}{3}$

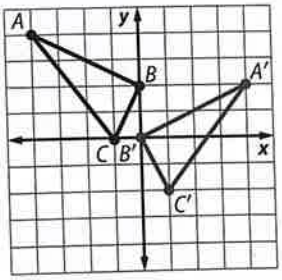
G $-\frac{7}{12}$

H $-\frac{1}{12}$

J $\frac{1}{4}$

K $\frac{3}{4}$

69. GEOMETRY Which of the following describes the transformation of $\triangle ABC$ to $\triangle A'B'C'$?



A a reflection across the y-axis and a translation down 2 units

B a reflection across the x-axis and a translation down 2 units

C a rotation 90° to the right and a translation down 2 units

D a rotation 90° to the right and a translation right 2 units

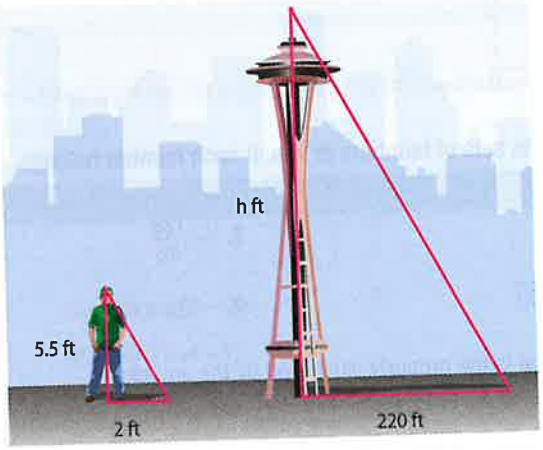
70. SHORT RESPONSE A local theater sold 1200 tickets during the opening weekend of a movie. On the following weekend, 840 tickets were sold. What was the percent decrease of tickets sold?

Spiral Review

71. Simplify $3x + 8y + 5z - 2y - 6x + z$. (Lesson 1-2)

72. BAKING Tamera is making two types of bread. The first type of bread needs $2\frac{1}{2}$ cups of flour, and the second needs $1\frac{3}{4}$ cups of flour. Tamera wants to make 2 loaves of the first recipe and 3 loaves of the second recipe. How many cups of flour does she need? (Lesson 1-2)

73. LANDMARKS Suppose the Space Needle in Seattle, Washington, casts a 220-foot shadow at the same time a nearby tourist casts a 2-foot shadow. If the tourist is $5\frac{1}{2}$ feet tall, how tall is the Space Needle? (Lesson 0-7)



74. Evaluate $a - [c(b - a)]$, if $a = 5$, $b = 7$, and $c = 2$. (Lesson 1-1)

Skills Review

Identify the additive inverse for each number or expression.

75. $-4\frac{1}{5}$

76. 3.5

77. $-2x$

78. $6 - 7y$

79. $3\frac{2}{3}$

80. -1.25

81. $5x$

82. $4 - 9x$

CHAPTER 1

Mid-Chapter Quiz

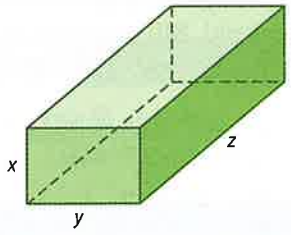
Lessons 1-1 through 1-3

1. Evaluate $3c - 4(a + b)$ if $a = -1$, $b = 2$ and $c = \frac{1}{3}$. (Lesson 1-1)

2. **TRAVEL** The distance that Maurice traveled in 2.5 hours riding his bicycle can be found by using the formula $d = rt$, where d is the distance traveled, r is the rate, and t is the time. How far did Maurice travel if he traveled at a rate of 16 miles per hour? (Lesson 1-1)

3. Evaluate $(5 - m)^3 + n(m - n)$ if $m = 6$ and $n = -3$. (Lesson 1-1)

4. **GEOMETRY** The formula for the surface area of the rectangular prism below is given by the formula $S = 2xy + 2yz + 2xz$. What is the surface area of the prism if $x = 2.2$, $y = 3.5$, and $z = 5.1$? (Lesson 1-1)



5. **MULTIPLE CHOICE** What is the value of $\frac{q^2 + rt}{qr - 2t}$ if $q = -4$, $r = 3$, and $t = 8$? (Lesson 1-1)

   A $-\frac{17}{6}$

   B $-\frac{10}{7}$

   C $-\frac{2}{7}$

   D $-\frac{1}{6}$

Name the sets of numbers to which each number belongs. (Lesson 1-2)

6. $\frac{25}{11}$

7. $-\frac{128}{32}$

8. $\sqrt{50}$

9. -32.4

10. What is the property illustrated by the equation $(4 + 15)7 = 4 \cdot 7 + 15 \cdot 7$? (Lesson 1-2)

11. Simplify $-3(7a - 4b) + 2(-3a + b)$. (Lesson 1-2)

12. **CLOTHES** Brittany is buying T-shirts and jeans for her new job. T-shirts cost \$10.50, and jeans cost \$26.50. She buys 3 T-shirts and 3 pairs of jeans. Illustrate the Distributive Property by writing two expressions representing how much Brittany spent. (Lesson 1-2)

13. **MULTIPLE CHOICE** Which expression is equivalent to $\frac{2}{3}(4m - 5n) + \frac{1}{5}(2m + n)$? (Lesson 1-2)

   F $\frac{46}{15}m - \frac{47}{15}n$

   G $46m - 47n$

   H $-\frac{mn}{15}$

   J $\frac{5}{4}m - \frac{9}{8}n$

14. Identify the additive inverse and the multiplicative inverse for $\frac{7}{6}$. (Lesson 1-2)

15. Write a verbal sentence to represent the equation $\frac{a}{a - 3} = 1$. (Lesson 1-3)

16. Solve $6x + 4y = -1$ for x. (Lesson 1-3)

17. **MULTIPLE CHOICE** Which algebraic expression represents the verbal expression, the product of 4 and the difference of a number and 13? (Lesson 1-3)

   A $4n - 13$

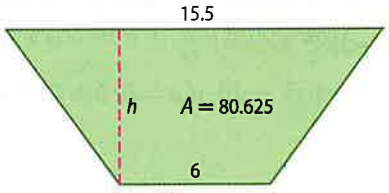
   B $4(n - 13)$

   C $\frac{4}{n - 13}$

   D $\frac{4n}{13}$

18. Solve $-3(6x + 5) + 2(4x) = 20$. (Lesson 1-3)

19. What is the height of the trapezoid below? (Lesson 1-3)



20. **GEOMETRY** The formula for the surface area of a sphere is $SA = 4\pi r^2$, and the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. (Lesson 1-3)

   a. Find the volume and surface area of a sphere with radius 2 inches. Write your answers in terms of π.

   b. Is it possible for a sphere to have the same numerical value for the surface area and volume? If so, find the radius of such a sphere.

LESSON 1-4

Solving Absolute Value Equations

Then

- You solved equations using properties of equality.

Now

1. Evaluate expressions involving absolute values.
2. Solve absolute value equations.

Why?

- Sailors sometimes use a laser range finder to determine distances. Suppose one such range finder is accurate to within ±0.5 yard. This means that if a sailor estimating the distance to shore reads 323.1 yards on the laser range finder, the distance to shore might actually be as close as 322.6 or as far away as 323.6 yards. These extremes can be described by the equation $|E - 323.1| = 0.5$.

OLIVIER MORIN/AFP/Getty Images

New Vocabulary

- absolute value
- empty set
- constraint
- extraneous solution

Common Core State Standards

Content Standards

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

Mathematical Practices

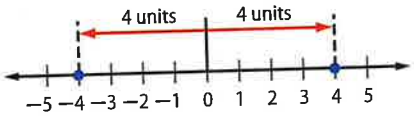
6 Attend to precision.

1 Absolute Value Expressions

The **absolute value** of a number is its distance from 0 on a number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent the absolute value of a number x.

KeyConcept Absolute Value

Words	For any real number a, if a is positive or zero, the absolute value of a is a. If a is negative, the absolute value of a is the opposite of a.
Symbols	For any real number a, $\lvert a\rvert = a$ if $a \geq 0$, and $\lvert a\rvert = -a$ if $a < 0$.
Model	$\lvert -4\rvert = 4$ and $\lvert 4\rvert = 4$



When evaluating expressions, absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

Example 1 Evaluate an Expression with Absolute Value

Evaluate $8.4 - |2n + 5|$ if $n = -7.5$.

$8.4 - \lvert 2n + 5\rvert = 8.4 - \lvert 2(-7.5) + 5\rvert$	Replace n with -7.5.
$= 8.4 - \lvert -15 + 5\rvert$	Multiply 2 and -7.5.
$= 8.4 - \lvert -10\rvert$	Add -15 and 5.
$= 8.4 - 10$	$\lvert -10\rvert = 10$
$= -1.6$	Subtract 10 from 8.4.

GuidedPractice

1A. Evaluate $|4x + 3| - 3\frac{1}{2}$ if $x = -2$.

1B. Evaluate $1\frac{1}{3} - |2y + 1|$ if $y = -\frac{2}{3}$.

2 Absolute Value Equations

Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$. This second case is often written as $a = -b$.

Real-WorldLink

Originally, players used leather gloves to hit tennis balls. Soon after, the glove was placed at the end of a stick to extend the reach of the "hand."

Source: The Cliff Richard Tennis Foundation

Real-World Example 2 Solve an Absolute Value Equation

TENNIS **A standard adult tennis racket has a 100-square-inch head, plus or minus 20 square inches. Write and solve an absolute value equation to determine the least and greatest possible sizes for the head of an adult tennis racket.**

Understand We need to determine the greatest and least possible sizes for the head of a tennis racket given the middle size and the range in sizes.

Plan When writing an absolute value equation, the middle or *central* value is always placed inside the absolute value symbols. The *range* is always placed on the other side of the equality symbol.

central value, range

$$|x - c| = r$$

Solve

$|x - c| = r$ Absolute value equation

$|x - 100| = 20$ $c = 100$, and $r = 20$

Case 1

$a = b$

$x - 100 = 20$

$x - 100 + 100 = 20 + 100$

$x = 120$

Case 2

$a = -b$

$x - 100 = -20$

$x - 100 + 100 = -20 + 100$

$x = 80$

Check

$|x - 100| = 20$

$|120 - 100| \stackrel{?}{=} 20$

$|20| \stackrel{?}{=} 20$

$20 = 20$ ✓

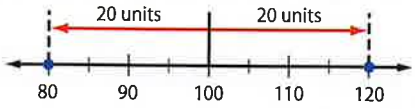
$|x - 100| = 20$

$|80 - 100| \stackrel{?}{=} 20$

$|-20| \stackrel{?}{=} 20$

$20 = 20$ ✓

On a number line, you can see that both solutions are 20 units away from 100.



The solutions are 120 and 80. The greatest size is 120 square inches and the least is 80 square inches.

Problem-SolvingTip

Write an Equation

Frequently, the best way to solve a problem is to use the given information to write and solve an equation.

GuidedPractice

Solve each equation. Check your solutions.

2A. $9 = |x + 12|$

2B. $8 = |y + 5|$

Because the absolute value of a number is always positive or zero, an equation like $|x| = -4$ is never true. Thus, it has no solution. The solution set for this type of equation is the **empty set**, symbolized by $\{\}$ or $\varnothing$.

Example 3 No Solution

Solve $|3x - 2| + 8 = 1$.

$|3x - 2| + 8 = 1$ Original equation

$|3x - 2| + 8 - 8 = 1 - 8$ Subtract 8 from each side.

$|3x - 2| = -7$ Simplify.

This sentence is *never* true. The solution set is $\varnothing$.

GuidedPractice Solve each equation. Check your solutions.

3A. $-2|3a| = 6$

3B. $|4b + 1| + 8 = 0$

In mathematics, a **constraint** is a condition that a solution must satisfy. Equations can be viewed as constraints in a problem situation. The solutions of the equation meet the constraints of the problem.

Even if the correct procedure for solving the equation is used, the answers may not be actual solutions to the original equation. Such a number is called an **extraneous solution**.

StudyTip

CCSS Precision

It is possible for an absolute value equation to have only one solution. Remember to set up two cases. Then check your solutions.

Example 4 One Solution

Solve $|x + 10| = 4x - 8$. Check your solutions.

Case 1

$a = b$

$x + 10 = 4x - 8$

$10 = 3x - 8$

$18 = 3x$

$6 = x$

Case 2

$a = -b$

$x + 10 = -(4x - 8)$

$x + 10 = -4x + 8$

$5x + 10 = 8$

$5x = -2$

$x = -\frac{2}{5}$

There appear to be two solutions, 6 and $-\frac{2}{5}$.

CHECK Substitute each value in the original equation.

$|x + 10| = 4x - 8$

$|6 + 10| \stackrel{?}{=} 4(6) - 8$

$|16| \stackrel{?}{=} 24 - 8$

$16 = 16$ ✓

$|x + 10| = 4x - 8$

$\left|-\frac{2}{5} + 10\right| \stackrel{?}{=} 4\left(-\frac{2}{5}\right) - 8$

$\left|9\frac{3}{5}\right| \stackrel{?}{=} -1\frac{3}{5} - 8$

$9\frac{3}{5} \neq -9\frac{3}{5}$ ✗

Because $9\frac{3}{5} \neq -9\frac{3}{5}$, the only solution is 6. The solution set is {6}.

GuidedPractice Solve each equation. Check your solutions.

4A. $2|x + 1| - x = 3x - 4$

4B. $3|2x + 2| - 2x = x + 3$

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Evaluate each expression if $x = -4$ and $y = -9$.

1. $|x - 8|$
2. $|7y|$
3. $-3|xy|$
4. $-2|3x + 8| - 4$

5. CCSS **MODELING** Most freshwater tropical fish thrive if the water is within 2°F of 78°F.
   a. Write an equation to determine the least and greatest optimal temperatures.
   b. Solve the equation you wrote in part a.
   c. If your aquarium's thermometer is accurate to within plus or minus 1°F, what should the temperature of the water be to ensure that it reaches the minimum temperature? Explain.

Examples 2–4 Solve each equation. Check your solutions.

6. $|x + 8| = 12$
7. $|y - 4| = 11$
8. $|a - 5| + 4 = 9$
9. $|b - 3| + 8 = 3$
10. $3|2x - 3| - 5 = 4$
11. $-2|5y - 1| = -10$
12. $|a - 4| = 3a - 6$
13. $|b + 5| = 2b + 3$

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 Evaluate each expression if $a = -3$, $b = -5$, and $c = 4.2$.

14. $|-3c|$
15. $|5b|$
16. $|a - b|$
17. $|b - c|$
18. $|3b - 4a|$
19. $2|4a - 3c|$
20. $-|3c - a|$
21. $-|abc|$

22. **FOOD** To make cocoa powder, cocoa beans are roasted. The ideal temperature for roasting is 300°F, plus or minus 25°. Write and solve an equation describing the maximum and minimum roasting temperatures for cocoa beans.

Examples 2–4 Solve each equation. Check your solutions.

23. $|z - 13| = 21$
24. $|w + 9| = 17$
25. $9 = |d + 5|$
26. $35 = |x - 6|$
27. $5|q + 6| = 20$
28. $-3|r + 4| = -21$
29. $3|2a - 4| = 0$
30. $8|5w - 1| = 0$
31. $2|3x - 4| + 8 = 6$
32. $4|7y + 2| - 8 = -7$
33. $-3|3t - 2| - 12 = -6$
34. $-5|3z + 8| - 5 = -20$

35. **MONEY** The U.S. Mint produces quarters that weigh about 5.67 grams each. After the quarters are produced, a machine weighs them. If the quarter weighs 0.02 gram more or less than the desired weight, the quarter is rejected. Write and solve an equation to find the heaviest and lightest quarters the machine will approve.

Evaluate each expression if $q = -8$, $r = -6$, and $t = 3$.

36. $12 - t|3r + 2|$
37. $2q + |2rt + q|$
38. $-5t - q|8r - t|$

Solve each equation. Check your solutions.

39. $8x = 2|6x - 2|$

40. $-6y + 4 = |4y + 12|$

41. $8z + 20 = -|2z + 4|$

42. $-3y - 2 = |6y + 25|$

43 **SEA LEVEL** Florida is on average 100 feet above sea level. This level varies by as much as 245 feet depending on precipitation and your location. Write and solve an equation describing the maximum and minimum sea levels for Florida. Is this solution reasonable? Explain.

44. **MULTIPLE REPRESENTATIONS** Draw a number line.

a. Geometric Label any 5 integers on the number line points A, B, C, D, and F.

b. Tabular Fill in each blank in the table with either $>$ or $<$ using the points from the number line.

A ___ B	$A + C$ ___ $B + C$ $A + D$ ___ $B + D$ $A + F$ ___ $B + F$
B ___ A	$B + C$ ___ $A + C$ $B + D$ ___ $A + D$ $B + F$ ___ $A + F$

A ___ B	$A - C$ ___ $B - C$ $A - D$ ___ $B - D$ $A - F$ ___ $B - F$
B ___ A	$B - C$ ___ $A - C$ $B - D$ ___ $A - D$ $B - F$ ___ $A - F$

c. Verbal Describe the patterns in the table.

d. Algebraic Describe the patterns algebraically, using the variable x to replace C, D, and F.

H.O.T. Problems Use Higher-Order Thinking Skills

45. CCSS **CRITIQUE** Ana and Ling are solving $|3x + 14| = -6x$. Is either of them correct? Explain your reasoning.

Ana

$|3x + 14| = -6x$

$3x + 14 = -6x$ or $3x + 14 = 6x$

$9x = -14$ $\qquad$ $14 = 3x$

$x = -\frac{14}{9}$ ✓ $\qquad$ $x = \frac{14}{3}$ ✓

Ling

$|3x + 14| = -6x$

$3x + 14 = -6x$ or $3x + 14 = 6x$

$9x = -14$ $\qquad$ $14 = 3x$

$x = -\frac{14}{9}$ ✓ $\qquad$ $x = \frac{14}{3}$ ✗

46. **CHALLENGE** Solve $|2x - 1| + 3 = |5 - x|$. List all cases and resulting equations. (*Hint*: There are four possible cases to examine as potential solutions.)

REASONING **If a, x, and y are real numbers, determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.**

47. If $|a| > 7$, then $|a + 3| > 10$.

48. If $|x| < 3$, then $|x| + 3 > 0$.

49. If y is between 1 and 5, then $|y - 3| \leq 2$.

50. **OPEN ENDED** Write an absolute value equation of the form $|ax + b| = cx + d$ that has no solution. Assume that a, b, c, and $d \neq 0$.

51. **WRITING IN MATH** How are symbols used to represent mathematical ideas? Use an example to justify your reasoning.

Standardized Test Practice

52. If $4x - y = 3$ and $2x + 3y = 19$, what is the value of y?

A 2
B 3
C 4
D 5

53. GRIDDED RESPONSE Two male and 2 female students from each of the 9th, 10th, 11th, and 12th grades comprise the Student Council. If a Student Council representative is chosen at random to attend a board meeting, what is the probability that the student will be either an 11th grader or male?

54. Which equation is equivalent to $4(9 - 3x) = 7 - 2(6 - 5x)$?

F $8x = 41$
G $22x = 41$
H $8x = 24$
J $22x = 24$

55. SAT/ACT A square with side length 4 units has one vertex at the point (1, 2). Which of the following points *cannot* be diagonally opposite that vertex?

A (−3, −2)
B (−3, 6)
C (5, −2)
D (5, 6)
E (1, 6)

Spiral Review

Solve each equation. Check your solution. (Lesson 1-3)

56. $4x + 6 = 30$

57. $5p - 10 = 4(7 + 6p)$

58. $\frac{3}{5}y - 7 = \frac{2}{5}y + 3$

59. MONEY Nhu is saving to buy a car. In the first 6 months, his savings were \$80 less than $\frac{3}{4}$ the price of the car. In the second six months, Nhu saved \$50 more than $\frac{1}{5}$ the price of the car. He still needs \$370. (Lesson 1-3)

a. What is the price of the car?

b. What is the average amount of money Nhu saved each month?

c. If Nhu continues to save the average amount each month, in how many months will he be able to afford the car?

Name the property illustrated by each equation. (Lesson 1-2)

60. $(1 + 8) + 11 = 11 + (1 + 8)$

61. $z(9 - 4) = z \cdot 9 - z \cdot 4$

Simplify each expression. (Lesson 1-2)

62. $7a + 3b - 4a - 5b$

63. $3x + 5y + 7x - 3y$

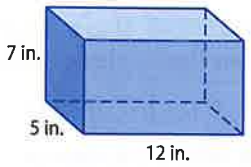
64. $3(15x - 9y) + 5(4y - x)$

65. $2(10m - 7a) + 3(8a - 3m)$

66. $8(r + 7t) - 4(13t + 5r)$

67. $4(14c - 10d) - 6(d + 4c)$

68. GEOMETRY The formula for the surface area of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism at the right. (Lesson 1-1)



Skills Review

Solve each equation.

69. $15x + 5 = 35$

70. $2.4y + 4.6 = 20$

71. $8a + 9 = 6a - 7$

72. $3(w - 1) = 2w - 6$

73. $\frac{1}{2}(2b - 4) = 2 + 8b$

74. $\frac{1}{3}(6p - 24) = 18 + 3p$

LESSON 1-5

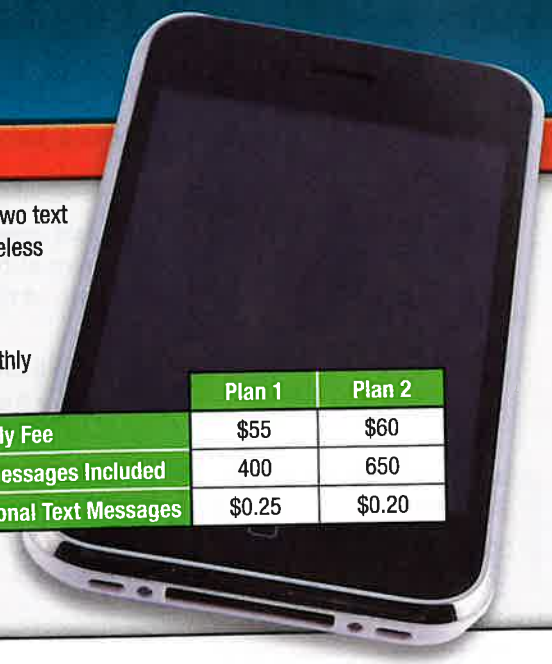
Solving Inequalities

Then
- You solved equations involving absolute values.

Now
1. Solve one-step inequalities.
2. Solve multi-step inequalities.

Why?
- Josh is trying to decide between two text messaging plans offered by a wireless telephone company.

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, \$55 < \$60. However, the additional text messaging fee for Plan 1 is greater than that of Plan 2, \$0.25 > \$0.20.



	Plan 1	Plan 2
Monthly Fee	\$55	\$60
Text Messages Included	400	650
Additional Text Messages	\$0.25	\$0.20

NewVocabulary
set-builder notation

Common Core State Standards

Content Standards
A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Mathematical Practices
4 Model with mathematics.

1 One-Step Inequalities

For any two real numbers, a and b, exactly one of the following statements is true.

$$a < b \qquad a = b \qquad a > b$$

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

KeyConcept

Addition Property of Inequality

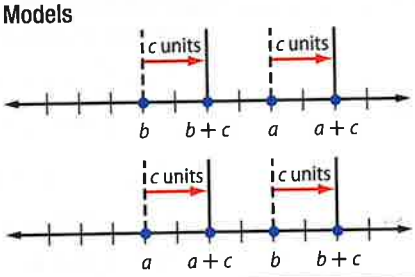
Words

For any real numbers, a, b, and c:

If $a > b$, then $a + c > b + c$.

If $a < b$, then $a + c < b + c$.

Models



Subtraction Property of Inequality

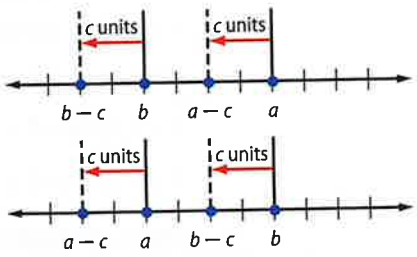
Words

For any real numbers, a, b, and c:

If $a > b$, then $a - c > b - c$.

If $a < b$, then $a - c < b - c$.

Models



These properties are also true for ≤, ≥, and ≠.

These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines.

ReviewVocabulary

Inequality Symbols

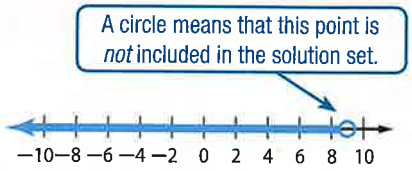
- $>$ greater than; is more than
- $<$ less than; is fewer than
- $\geq$ greater than or equal to; is at least; is no less than
- $\leq$ less than or equal to; is at most; is no more than

Example 1 Solve an Inequality Using Addition or Subtraction

Solve $y - 6 < 3$. Graph the solution set on a number line.

$y - 6 < 3$	Original inequality
$y - 6 + 6 < 3 + 6$	Add 6 to each side.
$y < 9$	Simplify.

Any real number less than 9 is a solution of this inequality. The graph of the solution set is shown at the right.



CHECK Substitute 8 and then 10 for y in $y - 6 < 3$. The inequality should be true for $y = 8$ and false for $y = 10$. ✓

StudyTip

Graphing Inequalities

A circle is used for $<$ and $>$. A dot is used for $\leq$ and $\geq$.

GuidedPractice

Solve each inequality. Graph the solution set on a number line.

1A. $5w + 3 > 4w + 9$

1B. $5x - 3 > 4x + 2$

Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a *negative* number requires that the order of the inequality be *reversed*. For example, to reverse $\leq$, replace it with $\geq$.

KeyConcept

Multiplication Property of Inequality

Words	Examples
For any real numbers, a, b, and c,	$-5 < -3$
where c is positive:	$-5(6) < -3(6)$
If $a > b$, then $ac > bc$.	$-30 < -18$
If $a < b$, then $ac < bc$.	
where c is negative:	$12 > -7$
If $a > b$, then $ac < bc$.	$12(-4) < -7(-4)$
If $a < b$, then $ac > bc$.	$-48 < 28$

Division Property of Inequality

Words	Examples
For any real numbers, a, b, and c,	$-12 < -8$
where c is positive:	$\frac{-12}{4} < \frac{-8}{4}$
If $a > b$, then $\frac{a}{c} > \frac{b}{c}$.	$-3 < -2$
If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	
where c is negative:	$-21 < -14$
If $a > b$, then $\frac{a}{c} < \frac{b}{c}$.	$\frac{-21}{-7} > \frac{-14}{-7}$
If $a < b$, then $\frac{a}{c} > \frac{b}{c}$.	$3 > 2$

These properties are also true for $\leq$, $\geq$, and $\neq$.

ReadingMath
Set-Builder Notation
$\{y \mid y < 9\}$ is read *the set of all numbers y such that y is less than 9.*

The solution set of an inequality can be expressed by using **set-builder notation**. For example, the solution set in Example 1 can be expressed as $\{y \mid y < 9\}$.

Example 2 Solve an Inequality Using Multiplication or Division

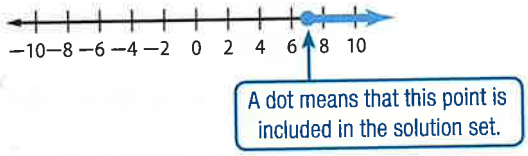
Solve $-4.2x \leq -29.4$. Graph the solution set on a number line.

$-4.2x \leq -29.4$ Original inequality

$\frac{-4.2x}{-4.2} \geq \frac{-29.4}{-4.2}$ Divide each side by -4.2, reversing the inequality symbol.

$x \geq 7$ Simplify.

The solution set is $\{x|x \geq 7\}$. The graph of the solution is shown below.



CHECK Substitute 6 and then 8 for x in $-4.2x \leq -29.4$. The inequality should be true for $x = 8$ and false for $x = 6$. ✓

GuidedPractice

Solve each inequality. Graph the solution set on a number line.

2A. $-4x \geq -24$

2B. $-9.2y < 23$

2 Multi-Step Inequalities

Solving multi-step inequalities is similar to solving multi-step equations.

Example 3 Solve Multi-Step Inequalities

Solve $-4c \leq \frac{5c + 58}{6}$. Graph the solution set on a number line.

StudyTip
When multiplying or dividing by a negative number, remember to reverse the inequality symbol.

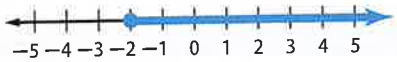
$-4c \leq \frac{5c + 58}{6}$ Original inequality

$-24c \leq 5c + 58$ Multiply each side by 6.

$-29c \leq 58$ Add $-5c$ to each side.

$c \geq -2$ Divide each side by -29, reversing the inequality symbol.

The solution set is $\{c|c \geq -2\}$ and is graphed below.



CHECK Substitute -3 and then -1 for x in $-4c \leq \frac{5c + 58}{6}$. The inequality should be true for $x = -1$ and false for $x = -3$. ✓

GuidedPractice

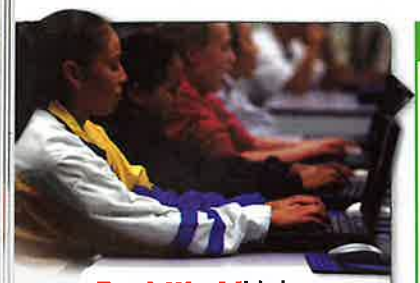
Solve each inequality. Graph the solution set on a number line.

3A. $-3x \leq \frac{-4x + 22}{5}$

3B. $8y \geq \frac{-5y + 9}{-4}$

3C. $-6(-4v + 3) \leq 2(10v + 3)$

3D. $-5(3d - 7) > 3(2d + 14)$



Real-WorldLink

In 2007, the Netcraft Web Server Survey found over 108,000,000 distinct Web sites.

Source: Netcraft

Real-World Example 4 Write and Solve an Inequality

WEB SITES **Enrique's company pays Salim to advertise on Salim's Web site. Salim's Web site earns $15 per month plus $0.05 every time a visitor clicks on the advertisement. What is the least number of clicks per month that Salim needs in order to earn $50 per month or more?**

Understand Let c = the number of clicks on the advertisement. Salim earns $15 per month and $0.05 per click, and he wants to earn a minimum of $50 for the advertisement.

Plan Write an inequality.

Words	The monthly income is	$15 plus	$0.05 per	click, and the total should be	at least $50.
Variable	Let c represent the number of	clicks	per month.		
Inequality	Flat fee	plus	fee per click	is at least	$50.
	15	+	$0.05c$	$\geq$	50

Solve

$15 + 0.05c \geq 50$ Original inequality

$0.05c \geq 35$ Subtract 15 from each side.

$c \geq 700$ Divide each side by 0.05.

Check

$15 + 0.05c \geq 50$ Original inequality

$5 + 0.05(700) \stackrel{?}{\geq} 50$ Replace c with 700.

$15 + 35 \stackrel{?}{\geq} 50$ Multiply.

$50 \geq 50$ ✓ Add.

Visitors to Salim's Web site need to click on Enrique's advertisement at least 700 times per month in order for Salim to earn $50 or more from Enrique's company.

GuidedPractice

4. Rosa's cell phone plan costs her $50 per month plus $0.25 for each minute she goes beyond her free minutes. How many minutes can she go beyond her free minutes and still pay less than a total of $70?

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–3 **Solve each inequality. Then graph the solution set on a number line.**

1. $b + 6 < 14$

2. $12 - d > -8$

3. $18 \leq -3x$

4. $-5y \geq -35$

5. $-4w - 13 > -21$

6. $8z - 9 \geq -15$

7. $s \geq \frac{s+6}{5}$

8. $\frac{2x-9}{4} \leq x + 2$

Example 4

9. CCSS **MODELING** Tara is delivering bags of mulch. Each bag weighs 48 pounds, and the push cart weighs 65 pounds. If her flat-bed truck is capable of hauling 2000 pounds, how many bags of mulch can Tara safely take on each trip?

Practice and Problem Solving

Extra Practice is on page R1.

Examples 1–3 **Solve each inequality. Then graph the solution set on a number line.**

10. $m - 8 > -12$

11. $n + 6 \leq 3$

12. $6r < -36$

13. $-12t \geq -6$

14. $-\frac{w}{4} \leq -7$

15. $\frac{k}{3} - 14 < -5$

16. $4x - 15 \leq 21$

17. $-6z - 14 > -32$

18. $-16 \geq 5(2z - 11)$

19. $12 < -4(3c - 6)$

20. $\frac{3y - 4}{0.2} - 8 > 12$

21. $\frac{9z + 5}{4} + 18 < 26$

Example 4

22. GYMNASTICS In a gymnastics competition, an athlete's final score is calculated by taking 75% of the average technical score and adding 25% of the artistic score. All scores are out of 10, and one gymnast has a 7.6 average technical score. What artistic score does the gymnast need to have a final score of at least 8.0?

Define a variable and write an inequality for each problem. Then solve.

23. Twelve less than the product of three and a number is less than 21.

24. The quotient of three times a number and 4 is at least -16.

25. The difference of 5 times a number and 6 is greater than the number.

26. The quotient of the sum of 3 and a number and 6 is less than -2.

27. HIKING Danielle can hike 3 miles in an hour, but she has to take a one-hour break for lunch and a one-hour break for dinner. If Danielle wants to hike at least 18 miles, solve $3(x - 2) \geq 18$ to determine how many hours the hike should take.

Solve each inequality. Then graph the solution set on a number line.

28. $18 - 3x < 12$

29. $-8(4x + 6) < -24$

30. $\frac{1}{4}n + 12 \geq \frac{3}{4}n - 4$

31. $0.24y - 0.64 > 3.86$

32. $10x - 6 \leq 4x + 42$

33. $-6v + 8 > -14v - 28$

34. $n > \frac{-3n - 15}{8}$

35. $-2r < \frac{6 - 2r}{5}$

36. $\frac{9z - 4}{5} \leq \frac{7z + 2}{4}$

37. MONEY Jin is selling advertising space in *Central City Magazine* to local businesses. Jin earns 3% commission for every advertisement he sells plus a salary of $250 a week. If the average amount of money that a business spends on an advertisement is $500, how many advertisements must he sell each week to make a salary of at least $700 that week?

a. Write an inequality to describe this situation.

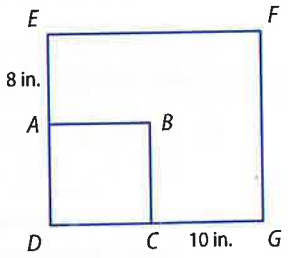
b. Solve the inequality and interpret the solution.

Define a variable and write an inequality for each problem. Then solve.

38. One third of the sum of 5 times a number and 3 is less than one fourth the sum of six times that number and 5.

39. The sum of one third a number and 4 is at most the sum of twice that number and 12.

40. CCSS SENSE-MAKING The sides of square $ABCD$ are extended to form rectangle $DEFG$. If the perimeter of the rectangle is at least twice the perimeter of the square, what is the maximum length of a side of square $ABCD$?



41. **MARATHONS** Jamie wants to be able to run at least the standard marathon distance of 26.2 miles. A good rule for training is that runners generally have enough endurance to finish a race that is up to 3 times his or her average daily distance.

   a. If the length of her current daily run is 5 miles, write an inequality to find the amount by which she needs to increase her daily run to have enough endurance to finish a marathon.

   b. Solve the inequality and interpret the solution.

42. **MODELING** The costs for renting a car from Ace Car Rental and from Basic Car Rental are shown in the table. For what mileage does Basic have the better deal? Use the inequality $38 + 0.1x > 42 + 0.05x$. Explain why this inequality works.

Rental Car Costs		
Company	Cost per Day	Cost per Mile
Ace	$38	$0.10
Basic	$42	$0.05

43. **MULTIPLE REPRESENTATIONS** In this exercise, you will explore graphing inequalities on a coordinate plane.

   a. Tabular Organize the following into a table. Substitute 5 points into the inequality $y \geq -\frac{1}{2}x + 3$. State whether the resulting statement is *true* or *false*.

   b. Graphical Graph $y = -\frac{1}{2}x + 3$. Also graph the 5 points from the table. Label all points that resulted in a true statement with a T. Label all points that resulted in a false statement with an F.

   c. Verbal Describe the pattern produced by the points you have labeled. Make a conjecture about which points on the coordinate plane would result in true and false statements.

H.O.T. Problems Use Higher-Order Thinking Skills

44. **CHALLENGE** If $-4 < x < 5$ and $0.25 < y < 4$, then $a < \frac{x}{y} < b$. What is $a + b$?

45. **ERROR ANALYSIS** Madlynn and Emilie were comparing their homework. Is either of them correct? Explain your reasoning.

Madlynn

$$\frac{4x + 5}{-2} - 1 > -3$$
$$\frac{4x + 5}{-2} < -2$$
$$4x + 5 > 4$$
$$4x > -1$$
$$x > -\frac{1}{4}$$

Emilie

$$\frac{4x + 5}{-2} - 1 > -3$$
$$\frac{4x + 5}{-2} > -2$$
$$4x + 5 > 4$$
$$4x > -1$$
$$x > -\frac{1}{4}$$

46. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

   The opposite of the absolute value of a negative number is less than the opposite of that number.

47. **CHALLENGE** Given $\triangle ABC$ with sides $AB = 3x + 4$, $BC = 2x + 5$, and $AC = 4x$, determine the values of x such that $\triangle ABC$ exists.

48. **OPEN ENDED** Write an inequality for which the solution is all real numbers in the form $ax + b > c(x + d)$. Explain how you know this.

49. **WRITING IN MATH** Why does the inequality symbol need to be reversed when multiplying or dividing by a negative number?

Standardized Test Practice

50. SHORT RESPONSE Rogelio found a cookie recipe that requires $\frac{3}{4}$ cup of sugar and 2 cups of flour. How many cups of sugar would he need if he used 6 cups of flour?

51. STATISTICS The mean score for Samantha's first six algebra quizzes was 88. If she scored a 95 on her next quiz, what will her mean score be for all 7 quizzes?

A 89
B 90
C 91
D 92

52. SAT/ACT The average of five numbers is 9. The average of 7 other numbers is 8. What is the average of all 12 numbers?

F $8\frac{5}{12}$
G $8\frac{1}{2}$
H $8\frac{7}{12}$
J $8\frac{3}{4}$
K $8\frac{11}{12}$

53. What is the complete solution of the equation $|8 - 4x| = 40$?

A $x = 8; x = 12$
B $x = 8; x = -12$
C $x = -8; x = -12$
D $x = -8; x = 12$

Spiral Review

Solve each equation. Check your solutions. (Lesson 1-4)

54. $|x - 5| = 12$

55. $7|3y - 4| = 35$

56. $|a + 6| = a$

57. ASTRONOMY Pluto travels in a path that is not circular. Pluto's farthest distance from the Sun is 4539 million miles, and its closest distance is 2756 million miles. Write an equation that can be solved to find the minimum and maximum distances from the Sun to Pluto. (Lesson 1-4)

58. POPULATION In 2005, the population of Bay City was 19,611. For each of the next five years, the population decreased by an average of 715 people per year. (Lesson 1-3)

a. What was the population in 2010?

b. If the population continues to decline at the same rate as from 2005 to 2010, what would you expect the population to be in 2025?

59. GEOMETRY The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. (Lesson 1-2)

a. Use the Distributive Property to rewrite the formula by factoring out the greatest common factor of the two terms.

b. Find the surface area for a cylinder with radius 3 centimeters and height 10 centimeters using both formulas. Leave the answer in terms of π.

c. Which formula do you prefer? Explain your reasoning.

60. CONSTRUCTION The Sawyers are adding a family room to their house. The dimensions of the room are 26 feet by 28 feet. Show how to use the Distributive Property to mentally calculate the area of the room. (Lesson 1-2)

Skills Review

Solve each equation. Check your solutions.

61. $|x| = 9$

62. $|x + 3| = 10$

63. $|4y - 15| = 13$

64. $18 = |3x - 9|$

65. $16 = 4|w + 2|$

66. $|y + 3| + 4 = 20$

EXPLORE

1-6 Algebra Lab
Interval Notation

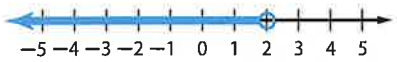
The solution set of an inequality can be described by using **interval notation**. The **infinity** symbols below are used to indicate that a set is unbounded in the positive or negative direction, respectively.

Read as *positive infinity.* → $+\infty$ $\quad -\infty$ ← Read as *negative infinity.*

Common Core State Standards
Content Standards
A.CED.1 Create equations and inequalities in one variable and use them to solve problems.
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

To indicate that an endpoint is *not* included in the set, a parenthesis, (or), is used. Parentheses are always used with the symbols $+\infty$ and $-\infty$, because they do not include endpoints.

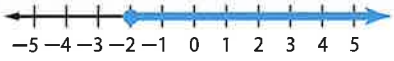
$x < 2$



interval notation
$(-\infty, 2)$

A bracket is used to indicate that the endpoint, -2, *is* included in the solution set below.

$x \geq -2$

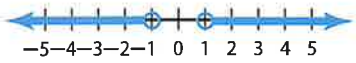


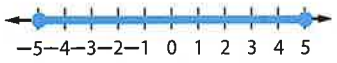
interval notation
$[-2, +\infty)$

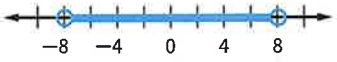
In interval notation, the symbol for the union of the two sets is $\cup$. The compound inequality $y \leq -7$ or $y > -1$ is written as $(-\infty, -7] \cup (-1, +\infty)$.

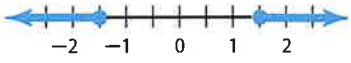
Exercises

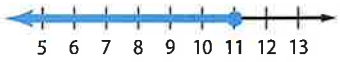
Write each inequality using interval notation.

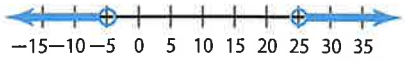
1. $\{a \mid a \leq -3\}$
2. $\{n \mid n > -8\}$
3. $\{y \mid y < 2 \text{ or } y \geq 14\}$
4. $\{b \mid b \leq -9 \text{ or } b > 1\}$
5. $\{t \mid 1 < t < 3\}$
6. $\{m \mid m \geq 4 \text{ or } m \leq -7\}$
7. $\{x \mid x \geq 0\}$
8. $\{r \mid -3 < r < 4\}$
9. 

10. 

11. 

12. 

13. 

14. 

Graph each solution set on a number line.

15. $(-1, -\infty)$
16. $(-\infty, 4]$
17. $(-\infty, 5] \cup (7, +\infty)$

18. **WRITING IN MATH** Write in words the meaning of $(-\infty, 3) \cup [10, +\infty)$. Then write the compound inequality that this notation represents.

19. **WRITING IN MATH** How are symbols used to write solution sets for inequalities? Explain.

LESSON 1-6

Solving Compound and Absolute Value Inequalities

Then

- You solved one-step and multi-step inequalities.

Now

1. Solve compound inequalities.
2. Solve absolute value inequalities.

Why?

- Marine biologists often have to transplant a dolphin from its natural habitat to a pool. Dolphins prefer the temperature of water to be at least 22°C but no more than 29°C. The acceptable temperature of water t for dolphins can be described by the following compound inequality.

$t \geq 22$ and $t \leq 29$

Paul Katz/Photodisc/Getty Images

New Vocabulary

- compound inequality
- intersection
- union

Common Core State Standards

Mathematical Practices

5 Use appropriate tools strategically.

1 Compound Inequalities

A **compound inequality** consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part of the inequality. The graph of a compound inequality containing *and* is the **intersection** of the solution sets of the two inequalities.

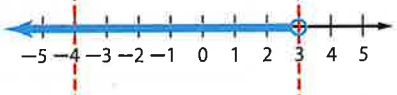
KeyConcept "And" Compound Inequalities

Words A compound inequality containing the word *and* is true if and only if *both* inequalities are true.

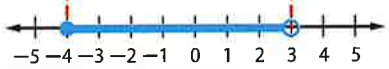
Example

$x \geq -4$

$x < 3$



$x \geq -4$ and $x < 3$



Another way of writing $x \geq -4$ and $x < 3$ is $-4 \leq x < 3$.
Both forms are read *x is greater than or equal to −4 and less than 3.*

Example 1 Solve an "And" Compound Inequality

Solve $8 < 3y - 7 \leq 23$. Graph the solution set on a number line.

Method 1 Solve separately.

Write the compound inequality using the word *and*. Then solve each inequality.

$8 < 3y - 7$ and $3y - 7 \leq 23$

$15 < 3y$ $\qquad 3y \leq 30$

$5 < y$ $\qquad y \leq 10$

$5 < y \leq 10$

Method 2 Solve both together.

Solve both parts at the same time by adding 7 to each part. Then divide each part by 3.

$8 < 3y - 7 \leq 23$

$15 < 3y \leq 30$

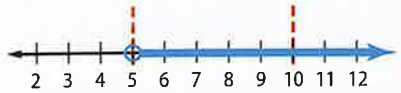
$5 < y \leq 10$

$5 < y \leq 10$

(continued on the next page)

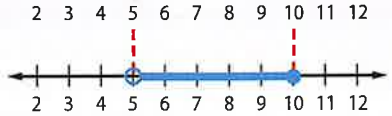
Graph the solution set for each inequality and find their intersection.

$5 < y$



$y \leq 10$

$5 < y \leq 10$



The solution set is $\{y \mid 5 < y \leq 10\}$ or $(5, 10]$.

GuidedPractice

Solve each inequality. Graph the solution set on a number line.

1A. $-12 \leq 4x + 8 \leq 32$

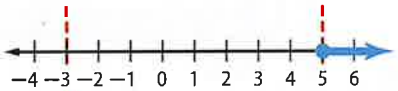
1B. $-5 \geq 3z - 2 > -14$

The graph of a compound inequality containing *or* is the **union** of the solution sets of the two inequalities.

KeyConcept "Or" Compound Inequalities

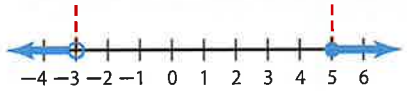
Words A compound inequality containing the word *or* is true if one or more of the inequalities is true.

Example $x \geq 5$



$x < -3$

$x \geq 5$ or $x < -3$



Example 2 Solve an "Or" Compound Inequality

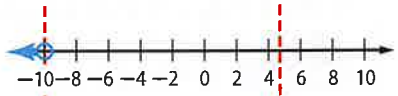
Solve $k + 6 < -4$ or $3k \geq 14$. Graph the solution set.

Solve each inequality separately.

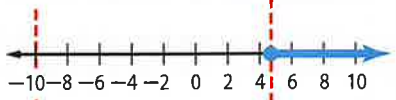
$k + 6 < -4$ or $3k \geq 14$

$k < -10$ $k \geq \frac{14}{3}$

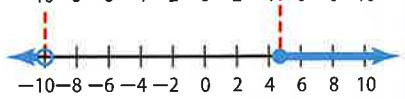
$k < -10$



$k \geq \frac{14}{3}$



$k < -10$ or $k \geq \frac{14}{3}$



GuidedPractice

Solve each inequality. Graph the solution set on a number line.

2A. $5j \geq 15$ or $-3j \geq 21$

2B. $g - 6 > -11$ or $2g + 4 < -15$

2 Absolute Value Inequalities

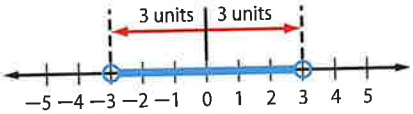
In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.

Example 3 Solve Absolute Value Inequalities

Solve each inequality. Graph the solution set on a number line.

a. $|x| < 3$

$|x| < 3$ means that the distance between x and 0 on a number line is less than 3 units. To make $|x| < 3$ true, substitute numbers for x that are fewer than 3 units from 0.

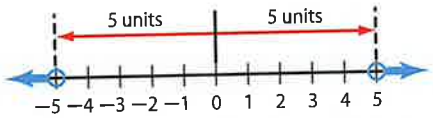


Notice that the graph of $|x| < 3$ is the same as the graph of $x > -3$ and $x < 3$.

All of the numbers between -3 and 3 are less than 3 units from 0. The solution set is $\{x | -3 < x < 3\}$ or $(-3, 3)$.

b. $|x| > 5$

$|x| > 5$ means that the distance between x and 0 on a number line is more than 5 units. To make $|x| > 5$ true, substitute numbers for x that are more than 5 units from 0.



Notice that the graph of $|x| > 5$ is the same as the graph of $x < -5$ or $x > 5$.

All of the numbers between and including -5 and 5 are no more than 5 units from 0, so they are not part of the solution set. The solution set is $\{x | -5 > x \text{ or } x > 5\}$ or $(-\infty, -5) \cup (5, \infty)$.

ReadingMath

within and *between* When solving problems involving inequalities, *within* is meant to be inclusive. Use $\leq$ or $\geq$.

Between is meant to be exclusive. Use $<$ or $>$.

StudyTip

Absolute Value Inequalities

Because the absolute value of a number is never negative, solutions involving negative numbers are as follows.

$|x| < -5$ is the empty set.

$|x| > -5$ is infinite solutions.

GuidedPractice

Solve each inequality. Graph the solution set on a number line.

3A. $|t| < 6$

3B. $|u| < -3$

3C. $|t| > 3$

3D. $|u| > -2$

An absolute value inequality can be solved by rewriting it as a compound inequality.

KeyConcept Absolute Value Inequalities

For all real numbers a, b, c, and x, $c > 0$, the following statements are true.

Absolute Value Inequality	Compound Inequality	Example
$\|ax + b\| > c$	$ax + b > c$ or $ax + b < -c$	If $\|4x + 5\| > 7$, then $4x + 5 > 7$ or $4x + 5 < -7$.
$\|ax + b\| < c$	$-c < ax + b < c$	If $\|4x + 5\| < 7$, then $-7 < 4x + 5 < 7$.

These statements are also true for $\leq$ and $\geq$, respectively.

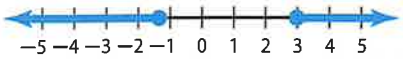
Example 4 Solve a Multi-Step Absolute Value Inequality

Solve $|6y - 5| \geq 13$. Graph the solution set on a number line.

$|6y - 5| \geq 13$ is equivalent to $6y - 5 \geq 13$ or $6y - 5 \leq -13$. Solve the inequality.

$6y - 5 \geq 13$	or	$6y - 5 \leq -13$	Rewrite the inequality.
$6y \geq 18$		$6y \leq -8$	Add 5 to each side.
$y \geq 3$		$y \leq -\frac{8}{6}$ or $-\frac{4}{3}$	Divide each side by 6.

The solution set is $\left\{y \mid y \leq -\frac{4}{3} \text{ or } y \geq 3\right\}$ or $(-\infty, -\frac{4}{3}] \cup [3, \infty)$.



GuidedPractice

Solve each inequality. Graph the solution set on a number line.

4A. $|4x - 7| > 13$

4B. $|5z + 2| \leq 17$

An inequality can be viewed as a constraint in a problem situation. Each solution of the inequality represents a combination that meets the constraint.

In real-world problems, the domain and range are often restricted to nonnegative or whole numbers.

Real-WorldLink

Apartment costs vary greatly depending on location. Of the major U.S. cities, New York has the highest average monthly rent of $2400, while Oklahoma City is lowest at $543.

Source: MSN

Real-World Example 5 Write and Solve an Absolute Value Inequality

MONEY **Amanda is apartment hunting in a specific area. She discovers that the average monthly rent for a 2-bedroom apartment is $600 a month, but the actual price could differ from the average as much as $225 a month.**

a. Write an absolute value inequality to describe this situation.

Let r = average monthly rent. $|600 - r| \leq 225$

b. Solve the inequality to find the range of monthly rent.

Rewrite the absolute value inequality as a compound inequality. Then solve for r.

$$-225 \leq 600 - r \leq 225$$

$$-225 - 600 \leq 600 - r - 600 \leq 225 - 600$$

$$-825 \leq -r \leq -375$$

$$825 \geq r \geq 375$$

The solution set is $\{r \mid 375 \leq r \leq 825\}$ or $[375, 825]$. Thus, monthly rent could fall between $375 and $825, inclusive.

GuidedPractice

5. TUITION Rachel is considering colleges to attend and determines that the average tuition among her choices is $3725 per year, but the tuition at a school could differ by as much as $1650 from the average. Write and solve an absolute value inequality to find the range of tuition.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–4 Solve each inequality. Graph the solution set on a number line.

1. $-4 < g + 8 < 6$
2. $-9 \leq 4y - 3 \leq 13$
3. $z + 6 > 3$ or $2z < -12$
4. $m - 7 \geq -3$ or $-2m + 1 \geq 11$
5. $|c| \geq 8$
6. $|q| \geq -1$
7. $|z| < 6$
8. $|x| \leq -4$
9. $|3v + 5| > 14$
10. $|4t - 3| \leq 7$

Example 5

11. **MONEY** Khalid is considering several types of paint for his bedroom. He estimates that he will need between 2 and 3 gallons. The table at the right shows the price per gallon for each type of paint Khalid is considering. Write a compound inequality and determine how much he could be spending.

Paint Type	Price per Gallon
Flat	$21.98
Satin	$23.98
Semi-Gloss	$24.98
Gloss	$25.98

Practice and Problem Solving

Extra Practice is on page R1.

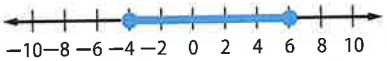
Examples 1–4 Solve each inequality. Graph the solution set on a number line.

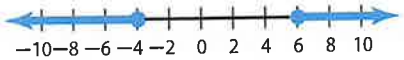
12. $8 < 2v - 4 < 16$
13. $-7 \leq 4d - 3 \leq -1$
14. $4r + 3 < -6$ or $3r - 7 > 2$
15. $6y - 3 < -27$ or $-4y + 2 < -26$
16. $|6h| < 12$
17. $|-4k| > 16$
18. $|3x - 4| > 10$
19. $|8t + 3| \leq 4$
20. $|-9n - 3| < 6$
21. $|-5j - 4| \geq 12$

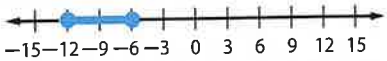
Example 5

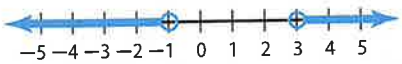
22. **MODELING** Forensic scientists use the equation $h = 2.6f + 47.2$ to estimate the height h of a woman given the length in centimeters f of her femur bone.
   a. Suppose the equation has a margin of error of ±3 centimeters. Write an inequality to represent the height of a woman given the length of her femur bone.
   b. If the length of a female skeleton's femur is 50 centimeters, write and solve an absolute value inequality that describes the woman's height in centimeters.

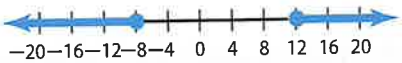
Write an absolute value inequality for each graph.

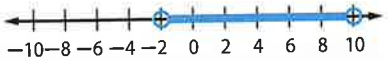
23. 

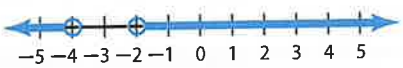
24. 

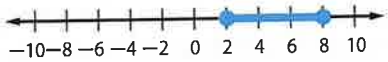
25. 

26. 

27. 

28. 

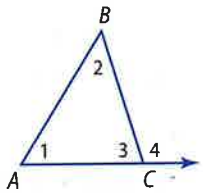
29. 

30. 

31 DOGS The Labrador retriever is one of the most recognized and popular dogs kept as a pet. Using the information given, write a compound inequality to describe the range of healthy weights for a fully grown female Labrador retriever.

Healthy Heights and Weights for Labrador Retrievers		
Gender	Height (in.)	Weight (lb)
Male	22.5–24.5	65–80
Female	21.5–23.5	55–70

32. GEOMETRY The *Exterior Angle Inequality Theorem* states that an exterior angle measure is greater than the measure of either of its corresponding remote interior angles. Write two inequalities to express the relationships among the measures of the angles of $\triangle ABC$.



Solve each inequality. Graph the solution set on a number line.

33. $28 > 6k + 4 > 16$

34. $m - 7 > -12$ or $-3m + 2 > 38$

35. $|-6h| > 90$

36. $-|-5k| > 15$

37. $3|2z - 4| - 6 > 12$

38. $6|4p + 2| - 8 < 34$

39. $\dfrac{|5f - 2|}{6} > 4$

40. $\dfrac{|2w + 8|}{5} \geq 3$

Write an algebraic expression to represent each verbal expression.

41. numbers that are at least 4 units from -5

42. numbers that are no more than $\frac{3}{8}$ unit from 1

43. numbers that are at least 6 units but no more than 10 units from 2

44. CCSS REASONING NASCAR rules stipulate that a car must conform to a set of 32 templates, each shaped to fit a different contour of the car. When a template is placed on a car, the gap between it and the car cannot exceed the specified tolerance. Each template is marked on its edge with a colored line that indicates the tolerance for the template.

Line Color	Tolerance (in.)
Red	0.07
Blue	0.25
Green	0.5

a. Suppose a certain template is 24.42 inches long. Use the information in the table at the right to write an absolute value inequality for templates with each line color.

b. Find the acceptable lengths for that part of a car if the template has each line color.

c. Graph the solution set for each line color on a number line.

d. The tolerance of which line color includes the tolerances of the other line colors? Explain your reasoning.

Solve each inequality. Graph the solution set on a number line.

45. $n + 6 > 2n + 5 > n - 2$

46. $y + 7 < 2y + 2 < 0$

47. $2x + 6 < 3(x - 1) \leq 2(x + 3)$

48. $a - 16 \leq 2(a - 4) < a + 2$

49. $4g + 8 \geq g + 6$ or $7g - 14 \geq 2g - 4$

50. $5t + 7 > 2t + 4$ and $3t + 3 < 24 - 4t$

51. HEALTH Hypoglycemia (low blood sugar) and hyperglycemia (high blood sugar) are potentially dangerous and occur when a person's blood sugar fluctuates by more than 38 mg from the normal blood sugar level of 88 mg. Write and solve an absolute value inequality to describe blood sugar levels that are considered potentially dangerous.

52. AIR TRAVEL The airline on which Drew is flying has weight restrictions for checked baggage. Drew is checking one bag.

a. Describe the ranges of weights that would classify Drew's bag as free, $25, $50, and unacceptable.

b. If Drew's bag weighs 68 pounds, how much will he pay to take it on the plane?

Cost for Checked Baggage	
Weight	**Cost**
Up to 50 lb limit	free
20 lb over limit	$25
More than 20, but less than 50 lb over limit	$50
More than 50 lb over limit	not accepted

H.O.T. Problems Use Higher-Order Thinking Skills

53. CCSS ARGUMENTS David and Sarah are solving $4|-5x-3|-6 \geq 34$. Is either of them correct? Explain your reasoning.

David

$4|-5x-3|-6 \geq 34$

$|-5x-3| \geq 10$

$-5x-3 \geq 10$ or $-5x-3 \leq -10$

$-5x \geq 13$ $\qquad -5x \leq -7$

$x \leq -\frac{13}{5}$ $\qquad x \geq \frac{7}{5}$

Sarah

$4|-5x-3|-6 \geq 34$

$|-5x-3| \geq 10$

$-5x-3 \leq 10$ or $-5x-3 \geq -10$

$-5x \leq 13$ $\qquad -5x \geq -7$

$x \geq -\frac{13}{5}$ $\qquad x \leq \frac{7}{5}$

54. CHALLENGE Solve $|x-2|-|x+2| > x$.

REASONING Determine whether each statement is *true* or *false*. If false, provide a counterexample.

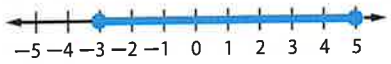
55. The graph of a compound inequality involving an *and* statement is bounded on the left and right by two values of x.

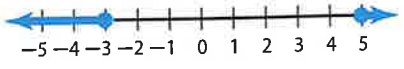
56. The graph of a compound inequality involving an *or* statement contains a region of values that are not solutions.

57. The graph of a compound inequality involving an *and* statement includes values that make all parts of the given statement true.

58. WRITING IN MATH An alternate definition of absolute value is to define $|a-b|$ as the distance between a and b on the number line. Explain how this definition can be used to solve inequalities of the form $|x-c| < r$.

59. REASONING The graphs of the solutions of two different absolute value inequalities are shown. Compare and contrast the absolute value inequalities.





60. OPEN ENDED Write an absolute value inequality with a solution of $a \leq x \leq b$.

61. WHICH ONE DOESN'T BELONG? Identify the compound inequality that is not the same as the other three. Explain your reasoning.

$-3 < x < 5$	$x > 2$ and $x < 3$	$x > 5$ and $x < 1$	$x > -4$ and $x > -2$

62. WRITING IN MATH Summarize the difference between *and* compound inequalities and *or* compound inequalities.

Standardized Test Practice

63. Which of the following best describes the graph of the equations below?

$$24y = 8x + 11$$
$$36y = 12x + 11$$

A The lines have the same x-intercept.

B The lines have the same y-intercept.

C The lines are parallel.

D The lines are perpendicular.

64. SAT/ACT Find an expression equivalent to $\left(\frac{3x^3}{y^3}\right)^3$.

F $\frac{9x^6}{3y}$

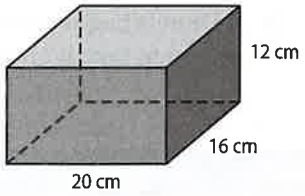
G $\frac{9x^9}{y^3}$

H $\frac{9x^6}{y^3}$

J $\frac{27x^6}{3y}$

K $\frac{27x^9}{y^3}$

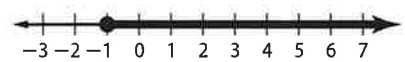
65. GRIDDED RESPONSE How many cubes that measure 4 centimeters on each side can be placed completely inside the box below?



66. Which graph represents the solution set for $|3x - 6| + 8 \geq 17$?

A


B


C


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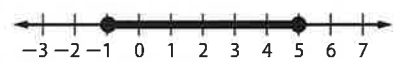


Spiral Review

67. HEALTH The National Heart Association recommends that less than 30% of a person's total daily caloric intake come from fat. One gram of fat yields nine Calories. Consider a healthy 21-year-old whose average caloric intake is between 2500 and 3300 Calories. (Lesson 1-5)

a. Write an inequality that represents the suggested fat intake for the person.

b. What is the greatest suggested fat intake for the person?

68. TRAVEL Maggie is planning a 5-day, 5-night, trip to a family reunion. She wants to spend no more than $1000. Her plane ticket is $375, and the hotel is $85 per night. (Lesson 1-5)

a. Let f represent the cost of food for one day. Write an inequality to represent this situation.

b. Solve the inequality and interpret the solution.

Solve each equation. Check your solutions. (Lesson 1-4)

69. $4|x - 5| = 20$

70. $|3y + 10| = 25$

71. $|7z + 8| = -9$

Skills Review

Name the property illustrated by each statement.

72. If $5x = 7$, then $5x + 3 = 7 + 3$.

73. If $-3x + 9 = 11$ and $6x + 2 = 11$, then $-3x + 9 = 6x + 2$.

74. If $[x + (-2)] + (-4) = 5$, then $x + [-2 + (-4)] = 5$.

Chapter 1 Study Guide and Review

Study Guide

KeyConcepts

Expressions and Formulas (Lesson 1-1)

- Use the order of operations to solve equations.

Properties of Real Numbers (Lesson 1-2)

- Real numbers can be classified as rational (Q) or irrational (I). Rational numbers can be classified as integers (Z), whole numbers (W), natural numbers (N), and/or quotients of these.

Solving Equations (Lessons 1-3 and 1-4)

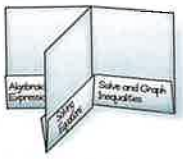
- Verbal expressions can be translated into algebraic expressions.
- The absolute value of a number is the number of units it is from 0 on a number line.
- For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $a = -b$.

Solving Inequalities (Lessons 1-5 and 1-6)

- Adding or subtracting the same number from each side of an inequality does not change the truth of the inequality.
- When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be reversed.
- The graph of an *and* compound inequality is the intersection of the solution sets of the two inequalities. The graph of an *or* compound inequality is the union of the solution sets of the two inequalities.
- An *and* compound inequality can be expressed in two different ways. For example, $-2 \leq x \leq 3$ is equivalent to $x \geq -2$ and $x \leq 3$.
- For all real numbers a and b, where $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

absolute value (p. 27)
algebraic expressions (p. 5)
compound inequality (p. 41)
constraint (p. 29)
empty set (p. 28)
equation (p. 18)
extraneous solution (p. 29)
formula (p. 6)
infinity (p. 40)
integers (p. 11)
intersection (p. 41)
interval notation (p. 40)
irrational numbers (p. 11)
natural numbers (p. 11)
open sentence (p. 18)
order of operations (p. 5)
rational numbers (p. 11)
real numbers (p. 11)
set-builder notation (p. 35)
solution (p. 18)
union (p. 42)
variables (p. 5)
whole numbers (p. 11)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The absolute value of a number is always negative.
2. $\sqrt{12}$ belongs to the set of rational numbers.
3. An equation is a statement that two expressions have the same value.
4. A solution of an equation is a value that makes the equation false.
5. The empty set contains no elements.
6. A mathematical sentence containing one or more variables is called an open sentence.
7. The graph of a compound inequality containing *and* is the union of the solution sets of the two inequalities.
8. Variables are used to represent unknown quantities.
9. The set of rational numbers includes terminating and repeating decimals.
10. Expressions that contain at least one variable are called algebraic expressions.

Study Guide and Review *Continued*

Lesson-by-Lesson Review

1-1 Expressions and Formulas

Evaluate each expression.

11. $[28 - (16 + 3)] \div 3$

12. $\frac{2}{3}(3^3 + 12)$

13. $\frac{15(9 - 7)}{3}$

Evaluate each expression if $w = 0.2$, $x = 10$, $y = \frac{1}{2}$, and $z = -4$.

14. $4w - 8y$

15. $z^2 + xy$

16. $\frac{5w - xy}{z}$

17. GEOMETRY The formula for the volume of a cylinder is $V = \pi r^2 h$, where V is the volume, r is the radius, and h is the height. What is the volume of a cylinder that is 6 inches high and has a radius of 3 inches?

Example 1

Evaluate $(12 - 15) \div 3^2$.

$(12 - 15) \div 3^2 = -3 \div 3^2$ Subtract.

$= -3 \div 9$ $3^2 = 9$

$= -\frac{1}{3}$ Divide.

Example 2

Evaluate $\frac{a^2}{2ac - b}$ if $a = -6$, $b = 5$, and $c = 0.25$.

$\frac{a^2}{2ac - b} = \frac{(-6)^2}{2(-6)(0.25) - 5}$ $a = -6$, $b = 5$, and $c = 0.25$

$= \frac{36}{2(-1.5) - 5}$ Evaluate the numerator and denominator separately.

$= \frac{36}{-8}$ or $-\frac{9}{2}$ Simplify.

1-2 Properties of Real Numbers

Name the sets of numbers to which each value belongs.

18. $1.\overline{3}$ **19.** $\sqrt{4}$ **20.** $-\frac{3}{4}$

Simplify each expression.

21. $4x - 3y + 7x + 5y$

22. $2(a + 3) - 4a + 8b$

23. $4(2m + 5n) - 3(m - 7n)$

24. MONEY At Fun City Amusement Park, hot dogs sell for \$3.50 and sodas sell for \$2.50. Dion bought 3 hot dogs and 3 sodas during one day at the park.

a. Illustrate the Distributive Property by writing two expressions to represent the cost of the hot dogs and the sodas.

b. Use the Distributive Property to find how much money Dion spent on food and drinks.

Example 3

Name the sets of numbers to which $\sqrt{50}$ belongs.

$\sqrt{50} = 5\sqrt{2}$ Irrationals (I), and reals (R)

Example 4

Simplify $-4(a + 3b) + 5b$.

$-4(a + 3b) + 5b$ Original expression

$= -4(a) + -4(3b) + 5b$ Distributive Property

$= -4a - 12b + 5b$ Multiply.

$= -4a - 7b$ Simplify.

1-3 Solving Equations

Solve each equation. Check your solution.

25. $8 + 5r = -27$

26. $4w + 10 = 6w - 13$

27. $\frac{x}{6} + \frac{x}{3} = \frac{3}{4}$

28. $6b - 5 = 3(b + 2)$

29. **MONEY** It cost Lori $14 to go to the movies. She bought popcorn for $3.50 and a soda for $2.50. How much was her ticket?

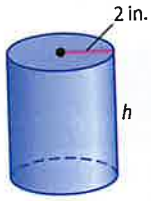
Solve each equation or formula for the specified variable.

30. $2k - 3m = 16$ for k

31. $\frac{r + 5}{mn} = p$ for m

32. $A = \frac{1}{2}h(a + b)$ for h

33. **GEOMETRY** Yu-Jun wants to fill the water container at the right. He knows that the radius is 2 inches and the volume is 100.48 cubic inches. What is the height of the water bottle? Use the formula for the volume of a cylinder, $V = \pi r^2 h$, to find the height of the bottle.



Example 5

Solve $-3(a - 3) + 2(3a - 2) = 14$.

$-3(a - 3) + 2(3a - 2) = 14$	Original equation
$-3a + 9 + 6a - 4 = 14$	Distributive Property
$-3a + 6a + 9 - 4 = 14$	Commutative Property
$3a + 5 = 14$	Substitution Property
$3a = 9$	Subtraction Property
$a = 3$	Division Property

Example 6

Solve each equation or formula for the specified variable.

a. $y = 2x + 3z$ for x

$y = 2x + 3z$	Original equation
$y - 3z = 2x$	Subtract $3z$ from each side.
$\frac{y - 3z}{2} = x$	Divide each side by 2.

b. $V = \frac{\pi r^2 h}{3}$ for h

$V = \frac{\pi r^2 h}{3}$	Original equation
$3V = \pi r^2 h$	Multiply each side by 3.
$\frac{3V}{\pi r^2} = h$	Divide each side by πr^2.

1-4 Solving Absolute Value Equations

Solve each equation. Check your solution.

34. $|r + 5| = 12$

35. $4|a - 6| = 16$

36. $|3x + 7| = -15$

37. $|b + 5| = 2b - 9$

38. **MEASUREMENT** Marcos is cutting ribbons for a craft project. Each ribbon needs to be $\frac{3}{4}$ yard long. If each piece is always within plus or minus $\frac{1}{16}$ yard, how long are the shortest and longest pieces of ribbon?

Example 7

Solve $|3m + 7| = 13$.

Case 1

$a = b$

$3m + 7 = 13$

$3m = 6$

$m = 2$

Case 2

$a = -b$

$3m + 7 = -13$

$3m = -20$

$m = -\frac{20}{3}$

The solutions are 2 and $-\frac{20}{3}$.

Study Guide and Review *Continued*

1-5 Solving Inequalities

Solve each inequality. Then graph the solution set on a number line.

39. $-4a \leq 24$

40. $\frac{r}{5} - 8 > 3$

41. $4 - 7x \geq 2(x + 3)$

42. $-p - 13 < 3(5 + 4p) - 2$

43. MONEY Ms. Hawkins is taking her science class on a field trip to a museum. She has \$572 to spend on the trip. There are 52 students that will go to the museum. The museum charges \$5 per student, and Ms. Hawkins gets in for free. If the students will have slices of pizza for lunch that cost \$2 each, how many slices can each student have?

Example 8

Solve $2m - 7 < -11$. Graph the solution set on a number line.

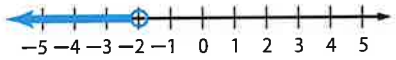
$2m - 7 < -11$ Original inequality

$2m < -4$ Add 7 to each side.

$m < -2$ Divide each side by 2.

The solution set is $\{m \mid m < -2\}$.

The graph of the solution set is shown below.



1-6 Solving Compound and Absolute Value Inequalities

Solve each inequality. Graph the solution set on a number line.

44. $2m + 4 < 7$ or $3m + 5 > 14$

45. $-5 < 4x + 3 < 19$

46. $6y - 1 > 17$ or $8y - 6 \leq -10$

47. $-2 \leq 5(m - 3) < 9$

48. $|a| + 2 < 15$

49. $|p - 14| \leq 19$

50. $|6k - 1| < 15$

51. $|2r + 7| < -1$

52. $\frac{1}{3}|8q + 5| \geq 7$

53. MONEY Cara is making a beaded necklace for a gift. She wants to spend between \$20 and \$30 on the necklace. The bead store charges \$2.50 for large beads and \$1.25 for small beads. If she buys 3 large beads, how many small beads can she buy to stay within her budget? Write and solve a compound inequality to describe the range of possible beads.

Example 9

Solve each inequality. Graph the solution set on a number line.

a. $-14 \leq 3x - 8 < 16$

$-14 \leq 3x - 8 < 16$ Original inequality

$-6 \leq 3x < 24$ Add 8 to each part.

$-2 \leq x < 8$ Divide each part by 3.

The solution set is $\{x \mid -2 \leq x < 8\}$.

b. $|3a - 5| > 13$

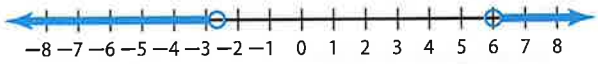
$|3a - 5| > 13$ is equivalent to $3a - 5 > 13$ or $3a - 5 < -13$.

$3a - 5 > 13$ or $3a - 5 < -13$

$3a > 18$ $3a < -8$ Subtract.

$a > 6$ $a < -\frac{8}{3}$ Divide.

The solution set is $\left\{a \mid a > 6 \text{ or } a < -\frac{8}{3}\right\}$.



CHAPTER 1

Practice Test

1. Evaluate $x + y^2(2 + x)$ if $x = 3$ and $y = -1$.

2. Simplify $-4(3a + b) - 2(a - 5b)$.

3. **MULTIPLE CHOICE** If $3m + 5 = 23$, what is the value of $2m - 3$?

   A 105

   B 9

   C $\frac{47}{3}$

   D 6

4. Solve $r = \frac{1}{2}m^2p$ for p.

Write an algebraic expression to represent each verbal expression.

5. twice the difference of a number and 11

6. the product of the square of a number and 5

7. Evaluate $2|3y - 8| + y$ if $y = 2.5$.

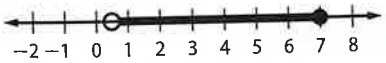
8. Solve $-2b > \frac{18 - b}{5}$. Graph the solution set on a number line.

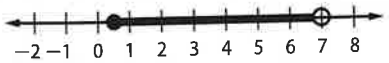
9. **MONEY** Carson has $35 to spend at the water park. The admission price is $25 and each soda is $2.50. Write an inequality to show how many sodas he can buy.

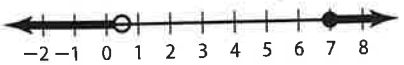
10. Solve $r - 3 < -5$ or $4r + 1 > 15$. Graph the solution set.

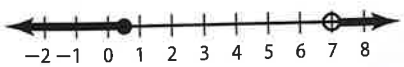
11. Solve $|p - 4| \leq 11$. Graph the solution set on a number line.

12. **MULTIPLE CHOICE** Which graph represents the solution set for $4 < 6t + 1 \leq 43$?

F 

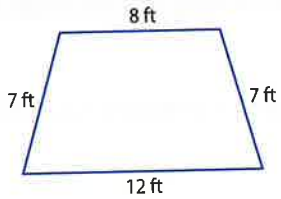
G 

H 

J 

13. **MONEY** Sofia is buying new skis. She finds that the average price of skis is $500 but the actual price could differ from the average by as much as $250. Write and solve an absolute value inequality to describe this situation.

14. **GARDENING** Andy is making 3 trapezoidal garden boxes for his backyard. Each trapezoid will be the size of the trapezoid below. He will place stone blocks around the borders of the boxes. How many feet of stones will Andy need?



Solve each equation.

15. $|x + 4| = 3$

16. $|3m + 2| = 1$

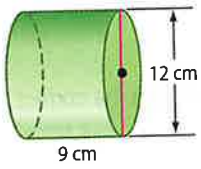
17. $|3a + 2| = -4$

18. $|2t + 5| - 7 = 4$

19. $|5n - 2| - 6 = -3$

20. $|p + 6| + 9 = 8$

21. **GEOMETRY** The volume of a cylinder is given by the formula $V = \pi r^2 h$. What is the volume of the cylinder below?



22. Solve $-3b - 5 \geq -6b - 13$. Graph the solution set on a number line.

23. Evaluate $\frac{3(x + y)}{4xy^2}$ if $x = \frac{2}{3}$ and $y = -2$.

24. Name the set(s) of numbers to which $-\frac{1}{3}$ belongs.

25. **MONEY** The costs for making necklaces at two craft stores are shown in the table. For what quantity of beads does The Accessory Store have a better deal? Use the inequality $15 + 3.25b < 20 + 2.50b$.

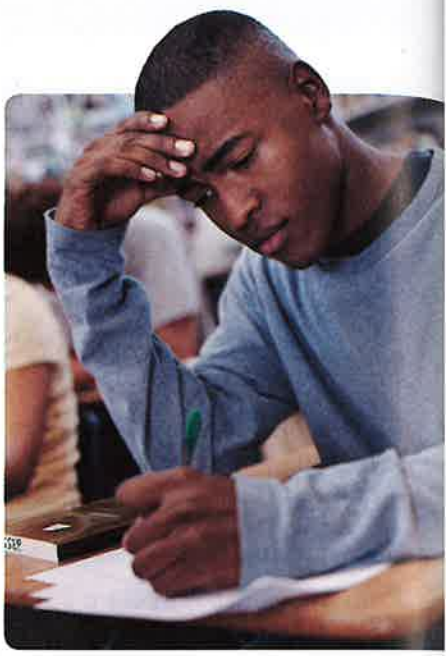
Shop	Cost per Chain	Cost per Bead
The Accessory Store	$15	$3.25
Finishing Touch	$20	$2.50

CHAPTER 1

Preparing for Standardized Tests

Eliminate Unreasonable Answers

You can eliminate unreasonable answers to help you find the correct answer when solving multiple-choice test items.



Strategies for Eliminating Unreasonable Answers

Step 1

Read the problem statement carefully to determine exactly what you are being asked to find.

Ask yourself:

- What am I being asked to solve?
- In what format (that is, fraction, number, decimal, percent, type of graph) will the correct answer be?
- What units (if any) will the correct answer have?

Step 2

Carefully look over each possible answer choice and evaluate for reasonableness.

- Identify any answer choices that are clearly incorrect and eliminate them.
- Eliminate any answer choices that are not in the proper format.
- Eliminate any answer choices that do not have the correct units.

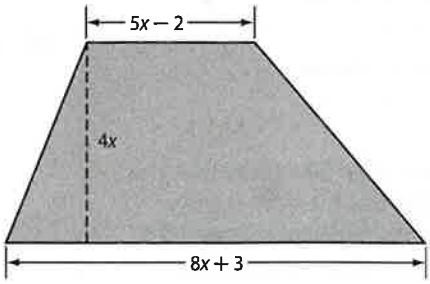
Step 3

Solve the problem and choose the correct answer from those remaining. Check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The formula for the area A of a trapezoid with height h and bases b_1 and b_2 is $A = \frac{h}{2}(b_1 + b_2)$. Write an expression to represent the area of the trapezoid at the right.



A $26x^2 + 2x$

B $52x^2 + 4x$

C $13x + 1$

D $28x + 10$

To compute the area of the trapezoid, you need to multiply half the height, $2x$, by another linear factor in x. So, the correct answer will contain an x^2 term. Since choices C and D are both linear, they can be eliminated. The correct answer is either A or B. Multiply to find the expression for the area.

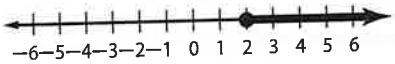
$$A = \frac{h}{2}(b_1 + b_2)$$
$$= \frac{4x}{2}(8x + 3 + 5x - 2)$$
$$= 2x(13x + 1)$$
$$= 26x^2 + 2x$$

The correct answer is A.

Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

1. The graph below shows the solution to which inequality?



A $8x - 9 \leq 5x - 3$

B $8x - 9 < 5x - 3$

C $8x - 9 \geq 5x - 3$

D $8x - 9 > 5x - 3$

2. Einstein's theory of relativity relates the energy E of an object to its mass m and the speed of light c. This relationship can be represented by the formula $E = mc^2$. Solve the formula for m.

F $m = \frac{c^2}{E}$

G $m = \frac{E}{c^2}$

H $m = \frac{c}{E^2}$

J $m = \frac{E^2}{c}$

3. A rectangle has a width of 8 inches and a perimeter of 30 inches. What is the perimeter, in inches, of a similar rectangle with a width of 12 inches?

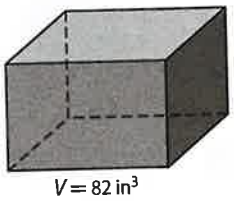
A 40

B 45

C 48

D 360

4. The rectangular prism below has a volume of 82 cubic inches. What will the volume be if the length, width, and height of the prism are all doubled?



F 41 in^3

G 164 in^3

H 482 in^3

J 656 in^3

5. Evaluate $a + (b + 1)^2$ if $a = 3$ and $b = 2$.

A −6

B −1

C 12

D 15

6. At a veterinarian's office, 2 cats and 4 dogs are seen in a random order. What is the probability that the 2 cats are seen in a row?

F $\frac{1}{3}$

G $\frac{2}{3}$

H $\frac{1}{2}$

J $\frac{3}{5}$

CHAPTER 1

Standardized Test Practice

Cumulative

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Evaluate $\frac{m^2 + 2mn}{n^2 - 1}$ if $m = -3$ and $n = 2$.

A -3

B -1

C 2

D 4

2. The volume of a cone with height h and radius r can be found by multiplying one-third π by the product of the height and the square of the radius. Which equation represents the volume of a cone?

F $V = \frac{1}{3}\pi r^2 h$

G $V = 3\pi r^2 h$

H $V = \frac{1}{3}\pi rh$

J $V = \frac{1}{3}\pi rh^2$

3. Which property of equality is illustrated by the equation below?

$$a + 2 = 4 \quad \rightarrow \quad 4 = a + 2$$

A Reflexive

B Substitution

C Symmetric

D Transitive

Test-Taking Tip

Question 1 Substitute -3 for m and 2 for n in the expression. Then use the order of operations to evaluate the expression.

4. Suppose a thermometer is accurate to within plus or minus 0.2°F. If the thermometer reads 81.5°F, which absolute value inequality represents the actual temperature T?

F $|T - 81.5| < 0.2$

G $|T - 81.5| \leq 0.2$

H $|T - 0.2| < 81.5$

J $|T - 0.2| \leq 81.5$

5. To which set of numbers does -25 *not* belong?

A integers

B rationals

C reals

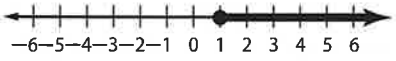
D wholes

6. Which number line shows the solution of the inequality $2n - 3 \geq 5n - 6$?

F

G

H



J

7. Write an algebraic expression to represent the verbal expression below.

two more than the product of a number and 5

A $\frac{n}{5} + 2$

B $2n + 5$

C $5n + 2$

D $\frac{n}{2} + 5$

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Use the absolute value equation below to answer each question.

$$|x - 3| - 2 = 0$$

a. How many solutions are there of the absolute value equation?

b. Solve the equation.

9. GRIDDED RESPONSE The table below shows the fill amounts and tolerances of different size soft drinks at a fountain drink vending machine. What is the maximum acceptable fill amount, in fluid ounces, for a medium drink?

Size	Amount (fl. oz)	Tolerance (fl. oz)
small	16	0.25
medium	21	0.35
large	32	0.4

10. Simplify the expression below. Show your work.

$$-4(3a - b) + 3(-2a + 5b)$$

11. While grilling steaks, Washington likes to keep the grill temperature at 425°, plus or minus 15°.

a. Write an absolute value inequality to model this situation. Let t represent the temperature of the grill.

b. Within what range of temperatures does Washington like the grill to be when he cooks his steaks?

12. GRIDDED RESPONSE Cameron uses a laser range finder to determine distances on the golf course. Her range finder is accurate to within 0.5 yard. If Cameron measures the distance from the tee to the flag on a par 3 to be 136 yards, what is the minimum number of yards that the distance could actually be?

Extended Response

Record your answers on a sheet of paper. Show your work.

13. Cindy is evaluating the expression $\frac{-5m - 3n}{-2p + r}$ for $m = 1$, $n = -4$, $p = -3$, and $r = -2$. Her work is shown below.

$$\frac{-5m - 3n}{-2p + r} = \frac{-5(1) - 3(-4)}{-2(-3) + (-2)}$$

$$= \frac{-5 - 12}{6 - 2} = -\frac{17}{4} = -4\frac{1}{4}$$

a. What error did Cindy make in her computation?

b. What is the correct answer?

14. The table at the right shows Ricardo's scores on the first 5 math quizzes this quarter. Each quiz is worth 100 points. There will be 1 more quiz this quarter.

Quiz	Score
1	86
2	79
3	80
4	85
5	77

a. In order to receive a B, Ricardo must have a quiz average of 82 or better. Write an inequality that can be solved to find the minimum score he must earn on Quiz 6.

b. Solve the inequality you wrote in part **a**.

c. What does the solution mean?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Go to Lesson...	1-1	1-3	1-3	1-6	1-2	1-5	1-3	1-4	1-6	1-2	1-6	1-4	1-1	1-5