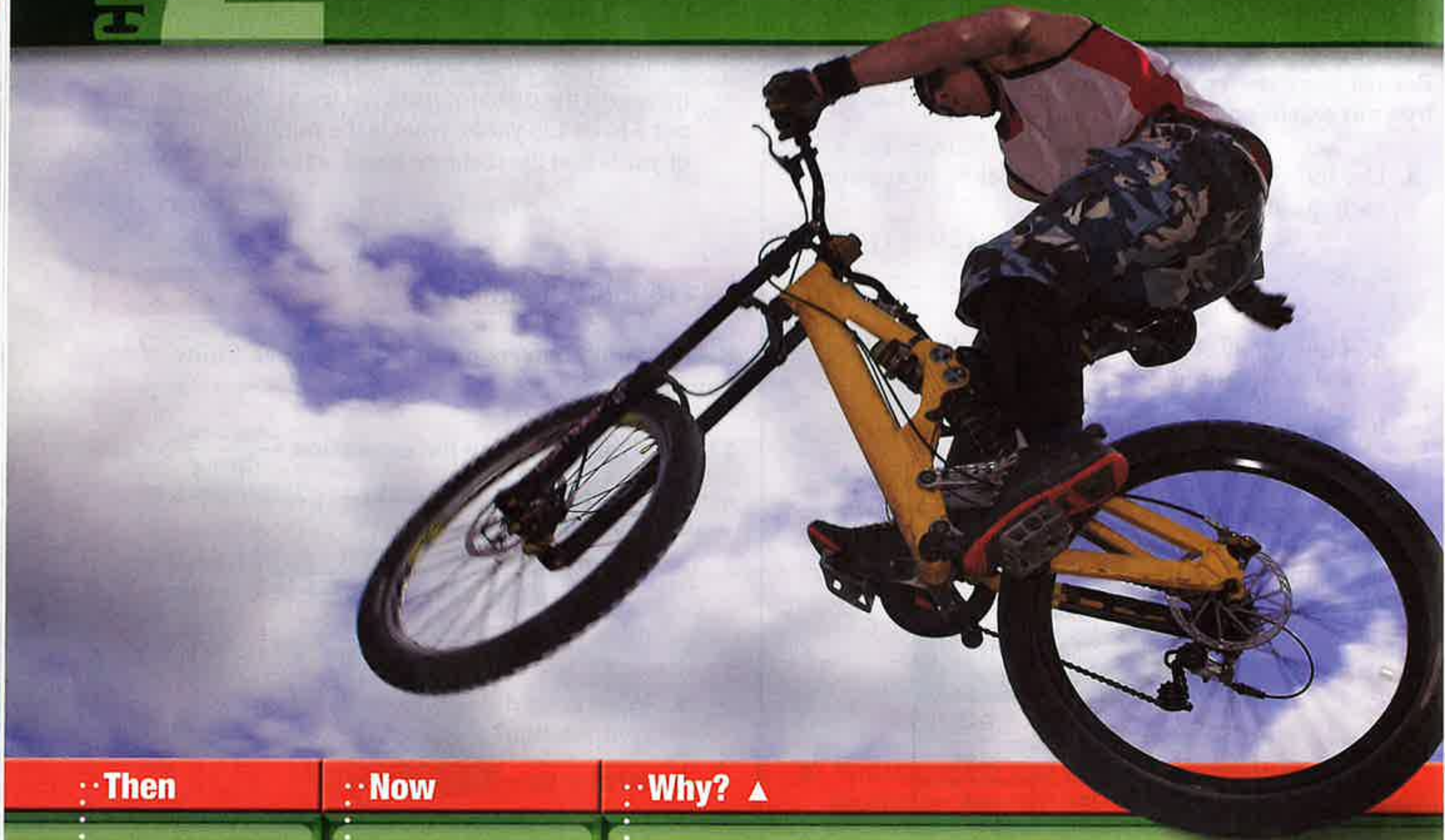
CHAPTER 2

Linear Relations and Functions



Then

You solved equations and inequalities.

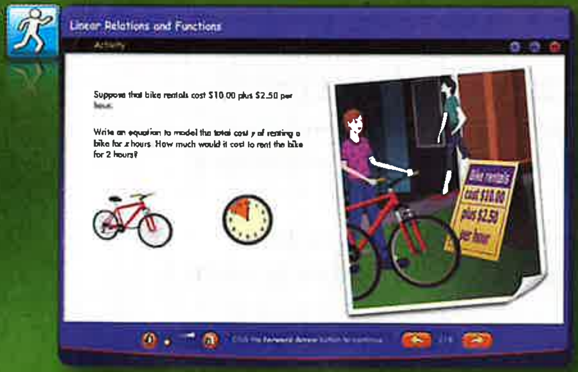
Now

You will:

- Use equations of relations and functions.
- Determine the slope of a line.
- Use scatter plots and prediction equations.
- Graph linear inequalities.

Why? ▲

RECREATION **Linear functions** can be used to model many aspects of recreational activities such as distance ridden on a bicycle, the amount of money a group of people would spend at a state fair, the height of a water slide at various points, or the amount of money you could earn from a hobby.



Suppose that bike rentals cost $10.00 plus $2.50 per hour.

Write an equation to model the total cost y of renting a bike for x hours. How much would it cost to rent the bike for 2 hours?

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Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

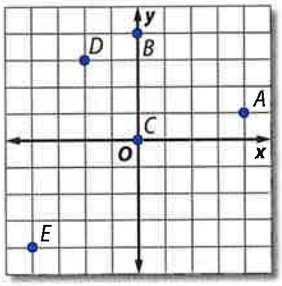
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Write the ordered pair for each point. Then name the quadrant in which it is located.



1. *A*
2. *B*
3. *C*
4. *D*
5. *E*

6. **BABYSITTING** Aliza earns \$6 per hour babysitting. Make a table in which the *x*-coordinate represents the number of hours Aliza babysits, and the *y*-coordinate represents the amount of money she earns.

Evaluate each expression if $a = -3$, $b = 4$, and $c = -2$.

7. $4a - 3$
8. $2b - 5c$
9. $b^2 - 3b + 6$
10. $\frac{2a + 4b}{c}$

11. **PHONE SERVICE** A cell phone company uses the expression $20 + 0.25m$ to determine the monthly charge for *m* minutes of air time. Find the monthly charge for 80 minutes of air time.

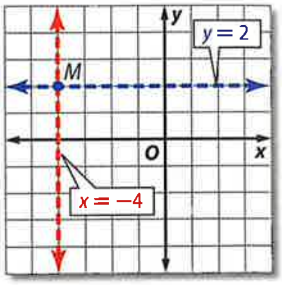
Solve each equation for the given variable.

12. $4x + 2y = 12$ for *y*
13. $a = 3b + 9$ for *b*
14. $15w - 10 = 5v$ for *v*
15. $3x - 4y = 8$ for *x*
16. $\frac{d}{6} + \frac{f}{3} = 4$ for *d*

QuickReview

Example 1

Write the ordered pair for point *M*. Then name the quadrant in which it is located.



Step 1 Follow a vertical line through the point to find the *x*-coordinate on the *x*-axis.

Step 2 Follow a horizontal line through the point to find the *y*-coordinate on the *y*-axis.

Step 3 The ordered pair for point *M* is $(-4, 2)$. It can also be written as $M(-4, 2)$.

The *x*-coordinate of *M* is negative, while the *y*-coordinate is positive. So *M* lies in Quadrant II.

Example 2

Evaluate $3a^2 - 2ab + b^2$ if $a = 4$ and $b = -3$.

$$\begin{aligned} 3a^2 - 2ab + b^2 &= 3(4^2) - 2(4)(-3) + (-3)^2 \\ &= 3(16) - 2(4)(-3) + 9 \\ &= 48 - (-24) + 9 \\ &= 48 + 24 + 9 \\ &= 81 \end{aligned}$$

Example 3

Solve $3x + 6y = 24$ for *y*.

$$\begin{aligned} 3x + 6y &= 24 && \text{Original equation} \\ 3x + 6y - 3x &= 24 - 3x && \text{Subtract } 3x \text{ from each side.} \\ 6y &= 24 - 3x && \text{Simplify.} \\ \frac{6y}{6} &= \frac{24}{6} - \frac{3x}{6} && \text{Divide each side by 6.} \\ y &= 4 - \frac{1}{2}x && \text{Simplify.} \end{aligned}$$

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Roine Magnusson/Photodisc/Getty Images

Get Started on the Chapter

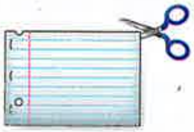
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 2. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

Linear Relations and Functions Make this Foldable to help you organize your Chapter 2 notes about linear relations and functions. Begin with four sheets of notebook paper.

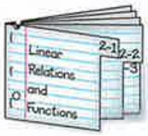
1 **Fold** each sheet of paper in half from top to bottom.

2 **Cut** along the fold. Staple the eight half-sheets together to form a booklet.



3 **Cut** tabs into the margin. The top tab is 2 lines deep, the next tab is 6 lines deep, and so on.

4 **Label** each of the tabs with a lesson number.



NewVocabulary

English		Español
one-to-one function	p. 61	función biunívoca
onto function	p. 61	función
discrete relation	p. 62	relación discreta
continuous relation	p. 62	relación continua
vertical line test	p. 62	prueba de la recta vertical
independent variable	p. 64	variable independiente
dependent variable	p. 64	variable dependiente
linear equation	p. 69	ecuación lineal
linear function	p. 69	función lineal
rate of change	p. 76	tasa de cambio
bivariate data	p. 92	datos bivariados
positive correlation	p. 92	correlación positiva
negative correlation	p. 92	correlación negativa
line of fit	p. 92	recta de ajuste
regression line	p. 94	línea de regresión
piecewise-linear function	p. 102	función a intervalos lineal
absolute value function	p. 103	función del valor absoluto
parent function	p. 109	función madre
quadratic function	p. 109	función cuadrática
linear inequality	p. 117	desigualdad lineal

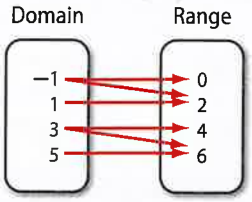
ReviewVocabulary

equation ecuación a mathematical sentence stating that two mathematical expressions are equal

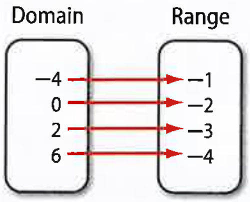
function función a relation in which each x-coordinate is paired with exactly one y-coordinate

relation relación a set of ordered pairs

Relation



Function



LESSON 2-1 Relations and Functions

Then

- You identified domains and ranges for given situations.

Now

- **1** Analyze relations and functions.

  2 Use equations of relations and functions.

Why?

- The table shows the monthly average low and high temperatures for Charlotte, North Carolina. Each month's average temperatures can be represented by the ordered pair (average low, average high). For example, January's average temperatures can be expressed as (32, 51).

Monthly Average Temperature (°F) Charlotte, NC

Month	Jan	Feb	Mar	Apr	May	Jun
Low	32	34	42	49	58	66
High	51	56	64	73	80	87
Month	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Low	71	69	63	51	42	35
High	90	88	82	73	63	54

Source: The Weather Channel

NewVocabulary

one-to-one function
onto function
discrete relation
continuous relation
vertical line test
independent variable
dependent variable
function notation

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Mathematical Practices

1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

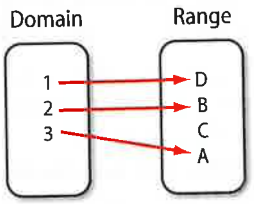
1 Relations and Functions

Relations and Functions Recall that a function is a relation in which each element of the domain is paired with exactly one element in the range. All functions map elements of the domain to elements of the range, but they may differ in the way the elements of the domain and range are paired.

KeyConcept Functions

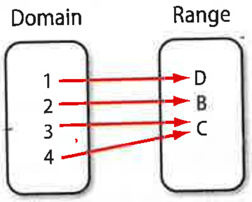
one-to-one function

Each element of the domain pairs to exactly one unique element of the range.



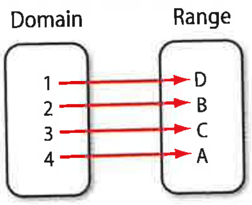
onto function

Each element of the range corresponds to an element of the domain.



both one-to-one and onto

Each element of the domain is paired to exactly one element of the range, and each element of the range corresponds to a unique element of the domain.



Example 1 Domain and Range

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.

a. **{(−6, −1), (−5, −9), (−3, −7), (−1, 7), (6, −9)}**

Domain: {−6, −5, −3, −1, 6} Range: {−9, −7, −1, 7}

function:	Yes, because each element of the domain is paired with one element of the range.
one-to-one:	No, because each element of the domain is not paired with a unique element of the range.
onto:	Yes, because each element of the range corresponds to an element of the domain.

b.

x	2	−1	−2	−1	2
y	−2	−1	0	1	2

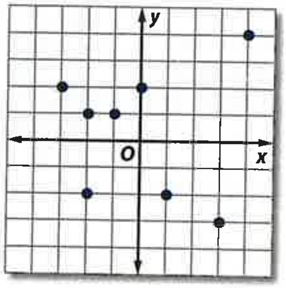
Domain: $\{-2, -1, 2\}$ Range: $\{-2, -1, 0, 1, 2\}$

The relation is not a function because 2 is mapped to both −2 and 2, and −1 is mapped to both −1 and 1.

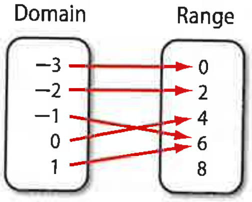
GuidedPractice

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.

1A.



1B.



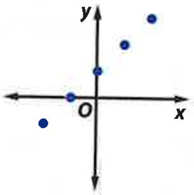
StudyTip

CCSS Structure

Notice that Graph A is composed of individual, or discrete, points while Graph B continues from one point to the next with no gaps.

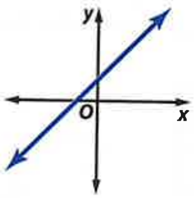
A relation in which the domain is a set of individual points, like the relation in Graph A, is said to be a **discrete relation**. Notice that its graph consists of points that are not connected. When the domain of a relation has an infinite number of elements and the relation can be graphed with a line or smooth curve, the relation is a **continuous relation**.

Graph A



discrete relation

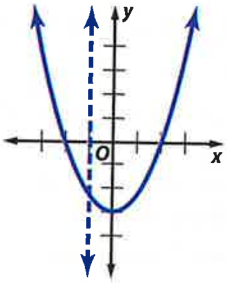
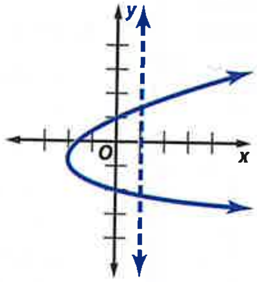
Graph B

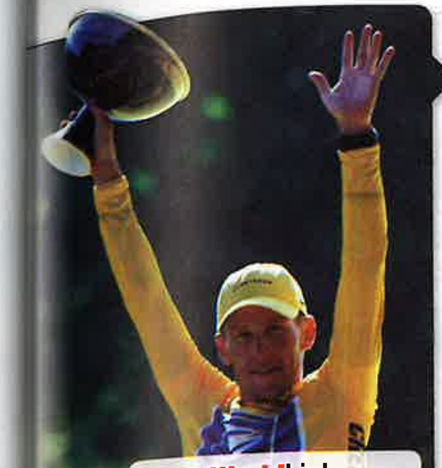


continuous relation

With both discrete and continuous graphs, you can use the **vertical line test** to determine whether the relation is a function.

KeyConcept Vertical Line Test

Words	If no vertical line intersects a graph in more than one point, the graph represents a function.	If a vertical line intersects a graph in two or more points, the graph does not represent a function.
Models		



Real-WorldLink

Lance Armstrong has won the Tour de France more than any other cyclist, having won 7 consecutive races from 1999 through 2005.

Source: *USA Cycling*

BERND THISSEN/DPA/epa/Corbis

Real-World Example 2

BICYCLING The graph shows the length of the Tour de France in kilometers each year from 2000 through 2009. Is the relation *discrete* or *continuous*? Does the graph represent a function?

Because the graph consists of distinct points, the function is discrete. Use the vertical line test. No vertical line can be drawn that contains more than one of the data points. Therefore, the relation is a function.

GuidedPractice

2. The number of employees a company had in each year from 2004 to 2009 were 25, 28, 34, 31, 27, and 29. Graph this information and determine whether the relation is *discrete* or *continuous*. Does the graph represent a function?

2 Equations of Relations and Functions

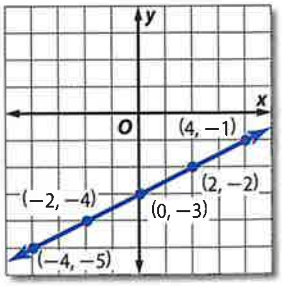
Relations and functions can also be represented by equations. The solutions of an equation in x and y are the set of ordered pairs (x, y) that make the equation true. To determine whether an equation represents a function, it is often simplest to look at the graph of the relation.

Example 3 Graph a Relation

Graph $y = \frac{1}{2}x - 3$, and determine the domain and range. Then determine whether the equation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. State whether it is *discrete* or *continuous*.

Make a table of values that satisfy the equation. Then graph the equation.

x	y
−4	−5
−2	−4
0	−3
2	−2
4	−1



Every real number is the x-coordinate of some point on the line, and every real number is the y-coordinate of some point on the line. So the domain and range are both all real numbers.

The graph passes the vertical line test, so the equation is a function. Every x-value is paired with exactly one unique y-value, and every y-value corresponds to an x-value. Thus, the function is both one-to-one and onto.

Because the graph is a solid line without breaks, the function is continuous.

GuidedPractice

3. Graph $y = x^2 + 1$, and determine the domain and range. Then determine whether the equation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. State whether it is *discrete* or *continuous*.

When an equation represents a function, the variable, often x, with values making up the domain is called the **independent variable**. The other variable, often y, is called the **dependent variable** because its values depend on x.

Equations that represent functions are often written in **function notation**. The equation $y = 5x - 1$ can be written as $f(x) = 5x - 1$. Suppose you want to find the value in the range that corresponds to the element -6 in the domain of the function. The value $f(-6)$ is found by substituting -6 for each x in the equation. Therefore, $f(-6) = 5(-6) - 1$ or -31.

ReadingMath

Function Notation The symbol $f(x)$ replaces the y and is read "f of x." The f is just the name of the function. It is not a variable that is multiplied by x.

Example 4 Evaluate a Function

Given $f(x) = 2x^2 - 8$, find each value.

a. $f(6)$

$f(x) = 2x^2 - 8$	Original function
$f(6) = 2(6)^2 - 8$	Substitute.
$= 2(36) - 8$	Evaluate 6^2.
$= 72 - 8$ or 64	Simplify.

b. $f(2y)$

$f(x) = 2x^2 - 8$	Original function
$f(2y) = 2(2y)^2 - 8$	Substitute.
$= 2(4y^2) - 8$	$(2y)^2 = 2^2y^2$
$= 8y^2 - 8$	Simplify.

GuidedPractice

Given $g(x) = 0.5x^2 - 5x + 3.5$, find each value.

4A. $g(2.8)$

4B. $g(4a)$

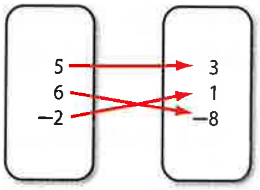
Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1

STRUCTURE **State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.**

1.



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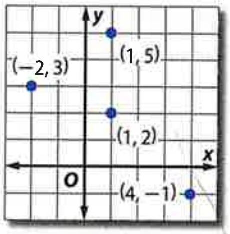


3.

x	y
−2	−4
1	−4
4	−2
8	6

Example 2

4. BASKETBALL **The table shows the average points per game for Dwayne Wade of the Miami Heat for four seasons.**

Season	Dwayne Wade's Age	Average Points Per Game
2005–2006	24	27.2
2006–2007	25	27.4
2007–2008	26	24.6
2008–2009	27	30.2

Source: *Basketball-Reference*

a. Assume that the ages are the domain. Identify the domain and range.

b. Write a relation of ordered pairs for the data.

c. State whether the relation is *discrete* or *continuous*.

d. Graph the relation. Is this relation a function?

Example 3

Graph each equation, and determine the domain and range. Determine whether the equation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is *discrete* or *continuous*.

5. $y = 5x + 4$

6. $y = -4x - 2$

7. $y = 3x^2$

8. $x = 7$

Jed Jacobsohn/Getty Images Sport/Getty Images

Example 4 **Evaluate each function.**

9. $f(-3)$ if $f(x) = -4x - 8$

10. $g(5)$ if $g(x) = -2x^2 - 4x + 1$

Practice and Problem Solving

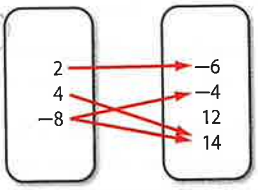
Extra Practice is on page R2.

Example 1 **State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.**

11.

x	y
−0.3	−6
0.4	−3
1.2	−1
1.2	4

12.



13. $\{(-3, -4), (-1, 0), (3, 0), (5, 3)\}$

Example 2 **14.** **POLITICS** The table below shows the population of several states and the number of U.S. representatives from those states.

a. Make a graph of the data with population on the horizontal axis and representatives on the vertical axis.

b. Identify the domain and range.

c. Is the relation *discrete* or *continuous*?

d. Does the graph represent a function? Explain your reasoning.

State	Population (millions)	Number of Representatives
California	33.93	53
Florida	16.03	25
Illinois	12.44	19
New York	19.00	29
North Carolina	8.07	13
Texas	20.90	32

Source: U.S. Bureau of the Census

Example 3 **CCSS STRUCTURE** **Graph each equation, and determine the domain and range. Determine whether the equation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is *discrete* or *continuous*.**

15. $y = -3x + 2$

16. $y = 0.5x - 3$

17. $y = 2x^2$

18. $y = -5x^2$

19. $y = 4x^2 - 8$

20. $y = -3x^3 - 1$

Example 4 **Evaluate each function.**

21 $f(-8)$ if $f(x) = 5x^3 + 1$

22. $f(2.5)$ if $f(x) = 16x^2$

23. **DIVING** The table below shows the pressure on a diver at various depths.

Depth (ft)	0	20	40	60	80	100
Pressure (atm)	1	1.6	2.2	2.8	3.4	4

a. Write a relation to represent the data.

b. Graph the relation.

c. Identify the domain and range. Is the relation *discrete* or *continuous*?

d. Is the relation a function? Explain your reasoning.

Find each value if $f(x) = 3x + 2$, $g(x) = -2x^2$, and $h(x) = -4x^2 - 2x + 5$.

24. $f(-5)$

25. $f(9)$

26. $g(-3)$

27. $g(-6)$

28. $h(3)$

29. $h(8)$

30. $f\left(\frac{2}{3}\right)$

31. $g\left(\frac{3}{2}\right)$

32. $h\left(\frac{1}{5}\right)$

33. **PODCASTS** Chaz has a collection of 15 podcasts downloaded on his digital audio player. He decides to download 3 more podcasts each month. The function $P(t) = 15 + 3t$ counts the number of podcasts $P(t)$ he has after t months. How many podcasts will he have after 8 months?

34. **MULTIPLE REPRESENTATIONS** In this problem you will investigate one-to-one and onto functions.

    a. **Graphical** Graph each function on a separate graphing calculator screen.

    $f(x) = x^2$ $\qquad g(x) = 2^x$ $\qquad h(x) = x^3 - 3x^2 - 5x + 6$ $\qquad j(x) = x^3$

    b. **Tabular** Use the graphs to create a table showing the number of times a horizontal line could intersect the graph of each function. List all possibilities.

    c. **Analytical** For a function to be one-to-one, a horizontal line on the graph of the function can intersect the function at most once. Which functions meet this condition? Which do not? Explain your reasoning.

    d. **Analytical** For a function to be onto, every possible horizontal line on the graph of the function must intersect the function at least once. Which functions meet this condition? Which do not? Explain your reasoning.

    e. **Graphical** Create a table showing whether each function is one-to-one and/or onto.

H.O.T. Problems Use Higher-Order Thinking Skills

35. **CRITIQUE** Omar and Madison are finding $f(3d)$ for the function $f(x) = -4x^2 - 2x + 1$. Is either of them correct? Explain your reasoning.

Omar

$f(3d) = -4(3d)^2 - 2(3d) + 1$

$= -4(9d^2) - 6d + 1$

$= -36d^2 - 6d + 1$

Madison

$f(3d) = -4(3d)^2 - 2(3d) + 1$

$= 12d^2 - 6d + 1$

36. **CHALLENGE** Consider the functions $f(x)$ and $g(x)$. $f(a) = 19$ and $g(a) = 33$, while $f(b) = 31$ and $g(b) = 51$. If $a = 5$ and $b = 8$, find two possible functions to represent $f(x)$ and $g(x)$.

37. **REASONING** If the graph of a relation crosses the y-axis at more than one point, is the relation *sometimes*, *always*, or *never* a function? Explain your reasoning.

38. **OPEN ENDED** Graph a relation that can be used to represent each of the following.

    a. the height of a baseball that is hit into the outfield

    b. the speed of a car that travels to the store, stopping at two lights along the way

    c. the height of a person from age 5 to age 80

    d. the temperature on a typical day from 6 A.M. to 11 P.M.

39. **REASONING** Determine whether the following statement is *true* or *false*. Explain your reasoning.

    If a function is onto, then it must be one-to-one as well.

40. **WRITING IN MATH** Explain why the vertical line test can determine if a relation is a function.

Standardized Test Practice

41. Patricia's swimming pool contains 19,500 gallons of water. She drains the pool at a rate of 6 gallons per minute. Which of these equations represents the number of gallons of water g remaining in the pool after m minutes?

A $g = 19{,}500 - 6m$

B $g = 19{,}500 + 6m$

C $g = \frac{19{,}500}{6m}$

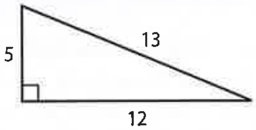
D $g = \frac{6m}{19{,}500}$

42. SHORT RESPONSE Look at the pattern below.

$$-\frac{5}{2}, -2, -\frac{3}{2}, -1, \ldots$$

If the pattern continues, what will the next term be?

43. GEOMETRY Which set of dimensions represents a triangle similar to the triangle shown below?



F 1 unit, 2 units, 3 units

G 7 units, 11 units, 12 units

H 10 units, 23 units, 24 units

J 20 units, 48 units, 52 units

44. SAT/ACT If $g(x) = x^2$, which expression is equal to $g(x + 1)$?

A 1

B $x^2 + 1$

C $x^2 + 2x + 1$

D $x^2 - x$

E $x^2 + x + 1$

Spiral Review

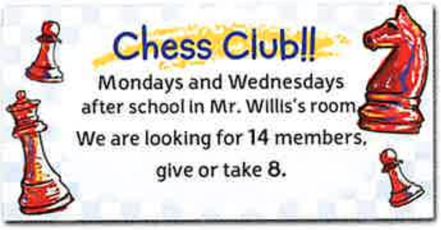
Solve each inequality. (Lesson 1-6)

45. $48 > 7y + 6 > 20$

46. $z + 12 > 18$ or $-2z + 16 > 12$

47. $2|4x + 2| + 3 > 21$

48. CLUBS Mr. Willis is starting a chess club at his high school. He sent the advertisement at the right to all of the homerooms. Write an absolute value inequality representing the situation. (Lesson 1-6)



Chess Club!!

Mondays and Wednesdays after school in Mr. Willis's room.

We are looking for 14 members, give or take 8.

49. SALES Ling can spend no more than \$120 at the summer sale of a department store. She wants to buy shirts on sale for \$15 each. Write and solve an inequality to determine the number of shirts she can buy. (Lesson 1-5)

Solve each equation. Check your solutions. (Lesson 1-4)

50. $18 = 2|2a + 6| - 2$

51. $2 = -3|4c - 5| + 8$

52. $-5 = 2|3b + 4| - 9$

Simplify each expression. (Lesson 1-2)

53. $6(3a - 2b) + 3(5a + 4b)$

54. $-4(5x - 3y) + 2(y + 3x)$

55. $-7(2c - 4d) + 8(3c + d)$

Skills Review

Solve each equation. Check your solutions.

56. $5x + 2 = 32$

57. $6a - 3 = 21$

58. $-2x + 5 = 5x + 19$

59. $6b + 4 = -2b - 28$

60. $2(x + 5) - 3(x - 4) = 19$

61. $4(2y - 3) + 5(3y + 1) = -99$

62. $5c - 8 + 2c = 4c + 10$

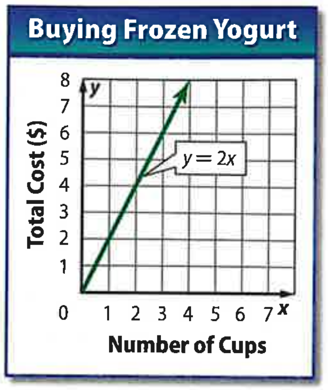
63. $8d - 4 + 3d = 2d - 100 - 7d$

64. $10y - 5 - 3y = 4(2y + 3) - 20$

EXTEND 2-1

Algebra Lab
Discrete and Continuous Functions

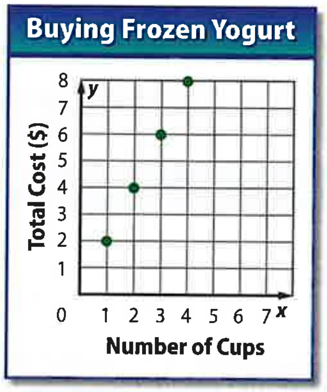
A cup of frozen yogurt costs \$2 at the Yogurt Shack. We might describe the cost of x cups of yogurt using the continuous function $y = 2x$, where y is the total cost in dollars. The graph of that function is shown at the right.



CCSS Common Core State Standards
Content Standards
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

From the graph, you can see that 2 cups of yogurt cost \$4, 3 cups cost \$6, and so on. The graph also shows that 1.5 cups of yogurt cost 2(1.5) or \$3. However, the Yogurt Shack probably will not sell partial cups of yogurt. This function is more accurately modeled with a discrete function.

The graph of the discrete function at the right also models the cost of buying cups of frozen yogurt. The domain in this graph makes sense in this situation.

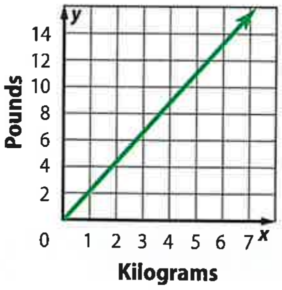


When choosing a discrete function or a continuous function to model a real-world situation, consider whether all real numbers make sense as part of the domain.

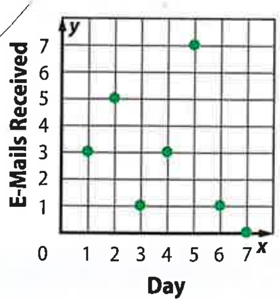
Exercises

Determine whether each function is correctly modeled using a discrete or continuous function. Explain your reasoning.

1. **Converting Units**



2. **E-Mails Received**



3. y represents the distance a car travels in x hours.

4. y represents the total number of riders who have ridden on a roller coaster after x rides.

5. **WRITING IN MATH** Give an example of a real-world function that is discrete and a real-world function that is continuous. Explain your reasoning.

LESSON 2-2

Linear Relations and Functions

Then

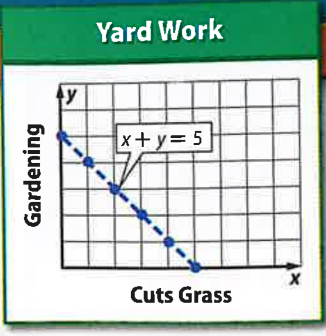
- You analyzed relations and functions.

Now

1. Identify linear relations and functions.
2. Write linear equations in standard form.

Why?

- Laura does yard work to earn money during the summer. She either cuts grass x or does general gardening y, and she schedules 5 jobs per day. The equation $x + y = 5$ can be used to relate how many of each task Laura can do in a day.


Yard Work

NewVocabulary

linear relation
nonlinear relation
linear equation
linear function
standard form
y-intercept
x-intercept

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

1 Linear Relations and Functions

The points on the graph above lie along a straight line. Relations that have straight line graphs are called **linear relations**. Relations that are not linear are called **nonlinear relations**.

An equation such as $x + y = 5$ is called a linear equation. A **linear equation** has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

Linear equations	Nonlinear equations
$4x - 5y = 16$	$2x + 6y^2 = -25$
$x = 10$	$y = \sqrt{x} + 2$
$y = -\frac{2}{3}x - 1$	$x + xy = -\frac{5}{8}$
$y = \frac{1}{2}x$	$y = \frac{1}{x}$

A **linear function** is a function with ordered pairs that satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers.

PT

Example 1 Identify Linear Functions

State whether each function is a linear function. Write *yes* or *no*. Explain.

a. $f(x) = 8 - \frac{3}{4}x$

Yes; it can be written as $f(x) = -\frac{3}{4}x + 8$.

$m = -\frac{3}{4}, b = 8$

b. $f(x) = \frac{2}{x}$

No; the expression includes division by the variable.

c. $g(x, y) = 3xy - 4$

No; the two variables are multiplied together.

GuidedPractice

1A. $f(x) = \frac{5}{x + 6}$

1B. $g(x) = -\frac{3}{2}x + \frac{1}{3}$

You can evaluate linear functions by substituting values for x.

Real-WorldLink

The largest member of the grass family, bamboo, is capable of growing from 1 to 4 feet per day.

Source: Infoplease

Real-World Example 2 Evaluate a Linear Function

PLANTS **The growth rate of a sample of Bermuda grass is given by the function $f(x) = 5.9x + 3.25$, where $f(x)$ is the total height in inches x days after an initial measurement.**

a. How tall is the sample after 3 days?

$f(x) = 5.9x + 3.25$	Original function
$f(3) = 5.9(3) + 3.25$	Substitute 3 for x.
$= 20.95$	Simplify.

The height of the sample after 3 days is 20.95 inches.

b. The term 3.25 in the function represents the height of the grass when it was initially measured. The sample is how many times as tall after 3 days?

Divide the height after 3 days by the initial height. $\frac{20.95}{3.25} \approx 6.4$

The height after 3 days is about 6.4 times as great as the initial height.

GuidedPractice

2A. If the Bermuda grass is 50.45 inches tall, how many days has it been since it was last cut?

2B. Is it reasonable to think that this rate of growth can be maintained for long periods of time? Explain.

2 Standard Form

Any linear equation can be written in **standard form**, $Ax + By = C$, where A, B, and C are integers with a greatest common factor of 1.

KeyConcept Standard Form of a Linear Equation

Words	The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers with a greatest common factor of 1, $A \geq 0$, and A and B are not both zero.
Example	$3x + 5y = 12$; $A = 3$, $B = 5$, and $C = 12$

Example 3 Standard Form

Write $-\frac{3}{10}x = 8y - 15$ in standard form. Identify A, B, and C.

$-\frac{3}{10}x = 8y - 15$	Original equation
$-\frac{3}{10}x - 8y = -15$	Subtract $8y$ from each side.
$3x + 80y = 150$	Multiply each side by -10.

$A = 3$, $B = 80$, and $C = 150$

GuidedPractice

Write each equation in standard form. Identify A, B, and C.

3A. $2y = 4x + 5$

3B. $3x - 6y - 9 = 0$

Since two points determine a line, one way to graph a linear function is to find the points at which the graph intersects each axis and connect them with a line. The y-coordinate of the point at which a graph crosses the y-axis is called the **y-intercept**. Likewise, the x-coordinate of the point at which it crosses the x-axis is called the **x-intercept**.

StudyTip

Vertical and Horizontal Lines When C represents a constant, an equation of the form $x = C$ represents a vertical line with only an x-intercept. The equation $y = C$ represents a horizontal line with only a y-intercept.

Example 4 Use Intercepts to Graph a Line

Find the x-intercept and the y-intercept of the graph of $2x - 3y + 8 = 0$. Then graph the equation.

The x-intercept is the value of x when $y = 0$.

$2x - 3y + 8 = 0$ Original equation

$2x - 3(0) + 8 = 0$ Substitute 0 for y.

$2x = -8$ Subtract 8 from each side.

$x = -4$ Divide each side by 2.

The x-intercept is -4.

Likewise, the y-intercept is the value of y when $x = 0$.

$2x - 3y + 8 = 0$ Original equation

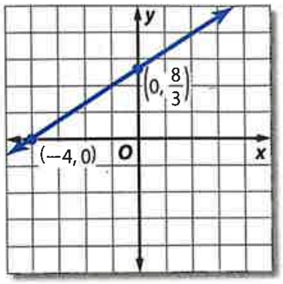
$2(0) - 3y + 8 = 0$ Substitute 0 for x.

$-3y = -8$ Subtract 8 from each side.

$y = \frac{8}{3}$ Divide each side by -3.

The y-intercept is $\frac{8}{3}$.

Use these ordered pairs to graph the equation.



GuidedPractice

4. Find the x-intercept and the y-intercept of the graph of $2x + 5y - 10 = 0$. Then graph the equation.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **State whether each function is a linear function. Write *yes* or *no*. Explain.**

1. $f(x) = \frac{x + 12}{5}$
2. $g(x) = \frac{7 - x}{x}$
3. $p(x) = 3x^2 - 4$
4. $q(x) = -8x - 21$

Example 2 **5** **RECREATION** You want to make sure that you have enough music for a car trip. If each CD is an average of 45 minutes long, the linear function $m(x) = 0.75x$ could be used to find out how many CDs you need to bring.

a. How many hours of music are there on 4 CDs?

b. If the trip you are taking is 6 hours, how many CDs should you bring?

Example 3 **CCSS STRUCTURE** **Write each equation in standard form. Identify A, B, and C.**

6. $y = -4x - 7$
7. $y = 6x + 5$
8. $3x = -2y - 1$
9. $-8x = 9y - 6$
10. $12y = 4x + 8$
11. $4x - 6y = 24$

Example 4 **Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation using the intercepts.**

12. $y = 5x + 12$
13. $y = 4x - 10$
14. $2x + 3y = 12$
15. $3x - 4y - 6 = 15$

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **State whether each equation or function is a linear function. Write *yes* or *no*. Explain.**

16. $3y - 4x = 20$

17. $y = x^2 - 6$

18. $h(x) = 6$

19. $j(x) = 2x^2 + 4x + 1$

20. $g(x) = 5 + \frac{6}{x}$

21. $f(x) = \sqrt{7 - x}$

22. $4x + \sqrt{y} = 12$

23. $\frac{1}{x} + \frac{1}{y} = 1$

24. $f(x) = \frac{4x}{5} + \frac{8}{3}$

Example 2 **25. ROLLER COASTERS** The speed of the Steel Dragon 2000 roller coaster in Mie Prefecture, Japan, can be modeled by $y = 10.4x$, where y is the distance traveled in meters in x seconds.

a. How far does the coaster travel in 25 seconds?

b. The speed of the Kingda Ka roller coaster in Jackson, New Jersey, can be described by $y = 33.9x$. Which coaster travels faster? Explain your reasoning.

Example 3 **Write each equation in standard form. Identify *A*, *B*, and *C*.**

26. $-7x - 5y = 35$

27. $8x + 3y + 6 = 0$

28. $10y - 3x + 6 = 11$

29. $-6x - 3y - 12 = 21$

30. $3y = 9x - 12$

31. $2.4y = -14.4x$

32. $\frac{2}{3}y - \frac{3}{4}x + \frac{1}{6} = 0$

33. $\frac{4}{5}y + \frac{1}{8}x = 4$

34. $-0.08x = 1.24y - 3.12$

Example 4 **Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation using the intercepts.**

35. $y = -8x - 4$

36. $5y = 15x - 90$

37. $-4y + 6x = -42$

38. $-9x - 7y = -30$

39. $\frac{1}{3}x - \frac{2}{9}y = 4$

40. $\frac{3}{4}y - \frac{2}{3}x = 12$

41. **CCSS MODELING** Latonya earns a commission of \$1.75 for each magazine subscription that she sells and \$1.50 for each newspaper subscription that she sells. Her goal is to earn a total of \$525 in commissions in the next two weeks.

a. Write an equation that is a model for the different numbers of magazine and newspaper subscriptions that can be sold to meet the goal.

b. Graph the equation. Does this equation represent a function? Explain.

c. If Latonya sells 100 magazine subscriptions and 200 newspaper subscriptions, will she meet her goal? Explain.

42. SNAKES Suppose the body length L in inches of a baby snake is given by $L(m) = 1.5 + 2m$, where m is the age of the snake in months until it becomes 12 months old.

a. Find the length of an 8-month-old snake.

b. Find the snake's age if the length of the snake is 25.5 inches.

43. STATE FAIR The Ohio State Fair charges \$8 for admission and \$5 for parking. After Joey pays for admission and parking, he plans to spend all of his remaining money at the ring game, which costs \$3 per game.

a. Write an equation representing the situation.

b. How much did Joey spend at the fair if he paid \$6 for food and drinks and played the ring game 4 times?

Write each equation in standard form. Identify A, B, and C.

44. $\frac{x+5}{3} = -2y + 4$

45. $\frac{4x-1}{5} = 8y - 12$

46. $\frac{-2x-8}{3} = -12y + 18$

Find the x-intercept and the y-intercept of the graph of each equation.

㊼ $\frac{6x+15}{4} = 3y - 12$

48. $\frac{-8x+12}{3} = 16y + 24$

49. $\frac{15x+20}{4} = \frac{3y+6}{5}$

50. FUNDRAISING The Freshman Class Student Council wanted to raise money by giving car washes. The students spent \$10 on supplies and charged \$2 per car wash.

a. Write an equation to model the situation.

b. Graph the equation.

c. How much money did they earn after 20 car washes?

d. How many car washes are needed for them to earn \$100?

51. MULTIPLE REPRESENTATIONS Consider the following linear functions.

$f(x) = -2x + 4 \qquad g(x) = 6 \qquad h(x) = \frac{1}{3}x + 5$

a. Graphical Graph the linear functions on separate graphs.

b. Tabular Use the graphs to complete the table.

Function	One-to-One	Onto
$f(x) = -2x + 4$		
$g(x) = 6$		
$h(x) = \frac{1}{3}x + 5$		

c. Verbal Are all linear functions one-to-one and/or onto? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

52. CHALLENGE Write a function with an x-intercept of $(a, 0)$ and a y-intercept of $(0, b)$.

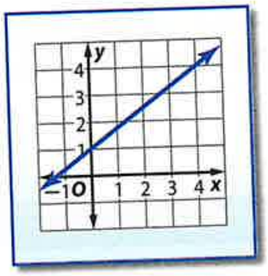
53. OPEN ENDED Write an equation of a line with an x-intercept of 3.

54. REASONING Determine whether an equation of the form $x = a$, where a is a constant, is *sometimes*, *always*, or *never* a function. Explain your reasoning.

55. CCSS ARGUMENTS Of the four equations shown, identify the one that does not belong. Explain your reasoning.

$y = 2x + 3$

x	y
0	4
1	2
2	0
3	−2



$y = 2xy$

56. WRITING IN MATH Consider the graph of the relationship between hours worked and earnings.

a. When would this graph represent a linear relationship? Explain your reasoning.

b. Provide another example of a linear relationship in a real-world situation.

Standardized Test Practice

57. Tom bought n DVDs for a total cost of $15n - 2$ dollars. Which expression represents the cost of each DVD?

A $n(15n - 2)$

B $n + (15n - 2)$

C $(15n - 2) \div n; n \neq 0$

D $(15n - 2) - n$

58. SHORT RESPONSE What is the complete solution of the equation?

$$|9 - 3x| = 18$$

59. NUMBER THEORY If a, b, c, and d are consecutive odd integers and $a < b < c < d$, how much greater is $c + d$ than $a + b$?

F 2

G 4

H 6

J 8

60. SAT/ACT Which function is linear?

A $f(x) = x^2$

B $g(x) = \sqrt{x - 1}$

C $f(x) = \sqrt{9 - x^2}$

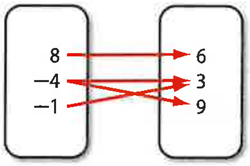
D $g(x) = \frac{2.7}{x}$

E $f(x) = 2x$

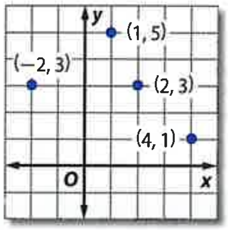
Spiral Review

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*. (Lesson 2-1)

61.



62.



63.

x	y
−4	−2
−3	−1
−3	−1
7	9

64. SHOPPING Claudio is shopping for a new television. The average price of the televisions he likes is \$800, and the actual prices differ from the average by up to \$350. Write and solve an absolute value inequality to determine the price range of the televisions. (Lesson 1-6)

Evaluate each expression if $a = -6$, $b = 5$, and $c = 3.6$. (Lesson 1-1)

65. $\frac{6a - 3c}{2ab}$

66. $\frac{a + 7b}{4bc}$

67. $\frac{b - c}{a + c}$

68. FOOD Brandi can order a small, medium, or large pizza with pepperoni, mushrooms, or sausage. How many different one-topping pizzas can she order? (Lesson 0-4)

Skills Review

Evaluate each expression.

69. $\frac{12 - 8}{4 - (-2)}$

70. $\frac{5 - 9}{-3 - (-6)}$

71. $\frac{-2 - 8}{3 - (-5)}$

72. $\frac{-2 - (-6)}{-1 - (-8)}$

73. $\frac{-7 - (-11)}{-3 - 9}$

74. $\frac{-1 - 8}{7 - (-3)}$

75. $\frac{-12 - (-3)}{-6 - (-5)}$

76. $\frac{4 - 3}{2 - 5}$

EXTEND

2-2

Algebra Lab

Roots of Equations and Zeros of Functions

The *solution* of an equation is called the *root* of the equation.

Common Core State Standards
Content Standards
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Example Determine Roots

Find the root of $0 = 5x - 10$.

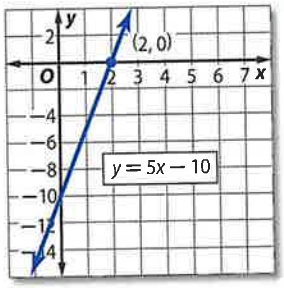
$0 = 5x - 10$	Original equation
$10 = 5x$	Add 10 to each side.
$2 = x$	Divide each side by 5.

The root of the equation is 2.

You can also find the root of an equation by finding the *zero* of its related function. Values of x for which $f(x) = 0$ are called *zeros* of the function f.

Linear Equation	Related Linear Function
$0 = 5x - 10$	$f(x) = 5x - 10$ or $y = 5x - 10$

The zero of a function is the *x-intercept* of its graph. Since the graph of $y = 5x - 10$ intersects the x-axis at 2, the zero of the function is 2.

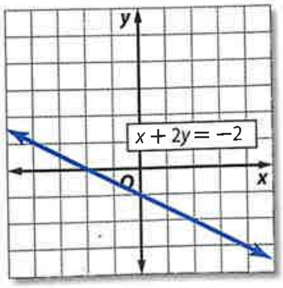


Exercises

1. Use $0 = 4x + 10$ and $f(x) = 4x + 10$ to distinguish among roots, solutions, and zeros.

2. Relate solutions of equations and x-intercepts of graphs.

Determine whether each statement is *true* or *false*. Explain your reasoning.



3. The function graphed at the right has two zeros, −2 and −1.

4. The root of $6x + 9 = 0$ is −1.5.

5. $f(0)$ is a zero of the function $f(x) = -\frac{2}{3}x + 12$.

6. **FUNDRAISERS** The function $y = 2x - 150$ represents the money raised y when the Boosters sell x soft drinks at a basketball game. Find the zero and describe what it means in the context of this situation. Make a connection between the zero of the function and the root of $0 = 2x - 150$.

LESSON 2-3

Rate of Change and Slope

Then

- You graphed linear relations.

Now

1. Find rate of change.
2. Determine the slope of a line.

Why?

- The table shows the total distance a car traveled over various time intervals. The distance formula, $rt = d$ or $r = \frac{d}{t}$, relates time and distance.

Time (h)	Distance (mi)
1	68
2.5	170
3	204
4.5	306
5	340

NewVocabulary

rate of change
slope

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Mathematical Practices

8 Look for and express regularity in repeated reasoning.

1 Rate of Change

Rate of change is a ratio that compares how much one quantity changes, on average, relative to the change in another quantity. If x is the independent variable and y is the dependent variable, then rate of change $= \frac{\text{change in } y}{\text{change in } x}$. This is sometimes referred to as $\frac{\Delta y}{\Delta x}$.

Real-World Example 1 Constant Rate of Change

PT

CHEMISTRY The table shows the temperature of a solution after it has been removed from a heat source. Find the rate of change in temperature for the solution.

Time (min)	Temperature (°C)
0	143.6
2	139.4
5	133.1
8	126.8
12	118.4

Use the ordered pairs (2, 139.4) and (5, 133.1).

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{change in temperature}}{\text{change in time}}$$

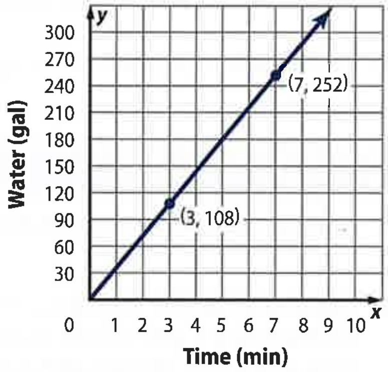
$$= \frac{133.1 - 139.4}{5 - 2}$$

$$= \frac{-6.3}{3} \text{ or } -2.1$$

The rate of change is −2.1. This means that the temperature is decreasing by 2.1°C each minute.

GuidedPractice

1. **RECREATION** The graph at the right shows the number of gallons of water in a swimming pool as it is being filled. At what rate is the pool being filled?



$\frac{y_2 - y_1}{x_2 - x_1}$

time = X

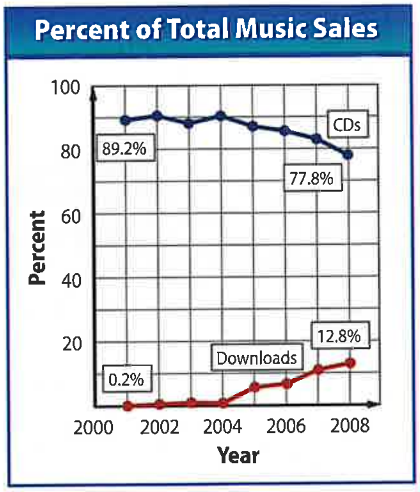
StudyTip

Independent Quantities Rates of change often include a measure of *time* as the independent variable.

Up to this point, you have used rates of change that are constant. Many real-world situations involve rates of change that are not constant. These situations are often described using an average rate of change over a specified interval.

Real-World Example 2 Average Rate of Change

MUSIC **Refer to the graph at the right. Find the average rate of change of the percent of total music sales for both CDs and downloads from 2001 to 2008. Compare the rates.**



Source: Recording Industry Association of America

CDs:

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{change in percent}}{\text{change in time}}$$

$$= \frac{77.8 - 89.2}{2008 - 2001}$$

$$= \frac{-11.4}{7} \text{ or } -1.63$$

Downloads:

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\text{change in percent}}{\text{change in time}}$$

$$= \frac{12.8 - 0.2}{2008 - 2001}$$

$$= \frac{12.6}{7} \text{ or } 1.8$$

The percent of CD music sales declined at an average rate of 1.63% per year, while the percent of downloaded music sales increased at an average rate of 1.8% per year.

GuidedPractice

2. **EDUCATION** In 2002, 23,142 students applied to State College, and 34,689 students applied to Central University. In 2010, 29,563 students applied to State College, and 36,107 applied to Central University. Determine the average rate of change in applicants for both schools from 2002 to 2010.

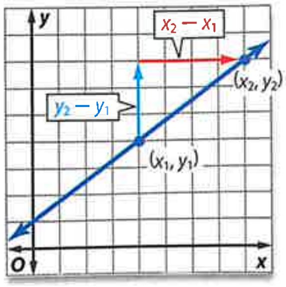
Real-WorldLink

As of April 2007, 36% of teens preferred to purchase CDs and 64% preferred to download music online.

Source: CNN

2 Slope

The **slope** of a line is the ratio of the change in the y-coordinates to the corresponding change in the x-coordinates. The slope of a line is the same as its rate of change.



Suppose a line passes through points at (x_1, y_1) and (x_2, y_2).

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Glen Allison/Photodisc/Getty Images
Rubberball/Getty Images

StudyTip

Slope The formula for slope is often remembered as *rise over run*, where the rise is the difference in *y*-coordinates and the run is the difference in *x*-coordinates.

KeyConcept Slope of a Line

Words The slope of a line is the ratio of the change in *y*-coordinates to the change in *x*-coordinates.

Symbols The slope m of a line passing through (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Example 3 Find Slope Using Coordinates

Find the slope of the line that passes through (−4, 3) and (2, 5).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \frac{5 - 3}{2 - (-4)}$ $(x_1, y_1) = (-4, 3), (x_2, y_2) = (2, 5)$

$= \frac{2}{6}$ or $\frac{1}{3}$ Simplify.

GuidedPractice

Find the slope of the line that passes through each pair of points.

3A. (1, −3) and (3, 5)

3B. (−8, 11) and (24, −9)

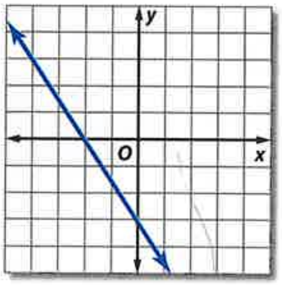
StudyTip

Slope is Constant The slope of a line is the same, no matter what two points on the line are used.

You can choose any two points from the graph of a line to find the slope.

Example 4 Find Slope Using a Graph

Find the slope of the line shown at the right.



The line passes through (−2, 0) and (0, −3).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

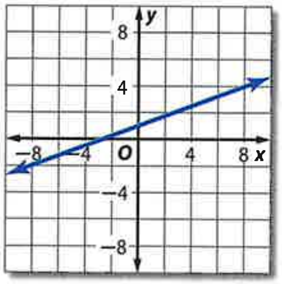
$= \frac{-3 - 0}{0 - (-2)}$ $(x_1, y_1) = (-2, 0), (x_2, y_2) = (0, -3)$

$= \frac{-3}{2}$ or $-\frac{3}{2}$ Simplify.

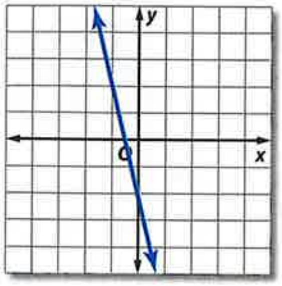
GuidedPractice

Find the slope of each line.

4A.



4B.



Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **CCSS REGULARITY** **Find the rate of change for each set of data.**

1.

Time (min)	2	4	6	8	10
Distance (ft)	12	24	36	48	60

2.

Time (sec)	5	10	15	20	25
Volume (cm^3)	16	32	48	64	80

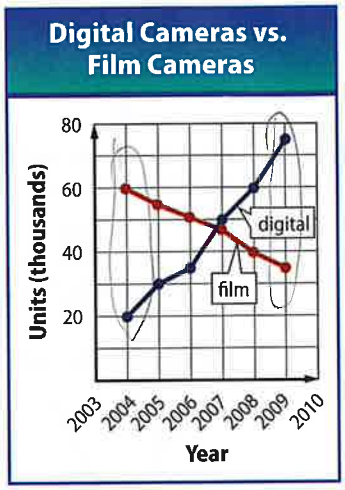
Example 2

3. CAMERAS The graph shows the number of digital still cameras and film cameras sold by Yellow Camera Stores in recent years.

a. Find the average rate of change of the number of digital cameras sold from 2004 to 2009.

b. Find the average rate of change of the number of film cameras sold from 2004 to 2009.

c. What do the signs of each rate of change represent?



Example 3 **Find the slope of the line that passes through each pair of points.**

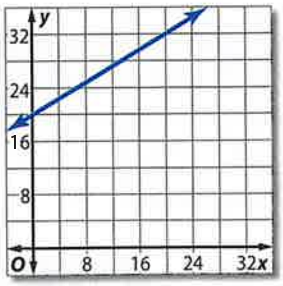
4. (3, 2), (8, 12)

5. (−1, 4), (3, −8)

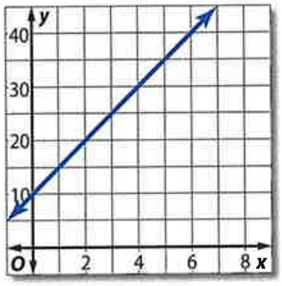
6. (−2, −5), (−7, 10)

Example 4 **Determine the rate of change of each graph.**

7.



8.



Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Find the rate of change for each set of data.**

9.

Time (day)	3	6	9	12	15
Height (mm)	20	40	60	80	100

10.

Weight (lb)	11	22	33	44	55
Cost ($)	8	16	24	32	40

Example 2

11. HEALTH The table below shows Lisa's temperature during an illness over a 3-day period.

Day	Monday		Tuesday		Wednesday	
Time	8:00 A.M.	8:00 P.M.	8:00 A.M.	8:00 P.M.	8:00 A.M.	8:00 P.M.
Temp (°F)	100.5	102.3	103.1	100.7	99.9	98.6

a. What was the average rate of change in Lisa's temperature from 8:00 A.M. on Monday to 8:00 P.M. on Monday?

b. What was the average rate of change in Lisa's temperature from 8:00 A.M. on Tuesday to 8:00 P.M. on Wednesday? Is your answer reasonable? What does the sign of the rate mean?

c. During which 12-hour period was the average rate of change in Lisa's temperature the greatest?

Example 3

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

12. $(-2, 11), (5, 6)$

13. $(-9, -11), (6, 3)$

14. $(-1.5, 3.5), (4.5, 6)$

15. $(-4.5, 9.5), (-1, 2.5)$

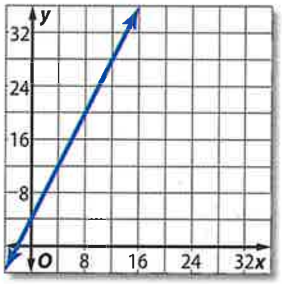
16. $(-8, -0.5), (-4, 5)$

17. $(-6, -2), (-1.5, 5.5)$

Example 4

Determine the rate of change of each graph.

18.



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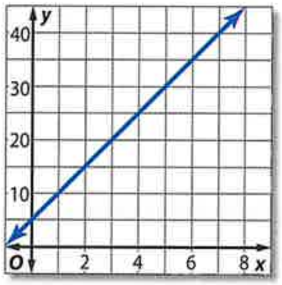

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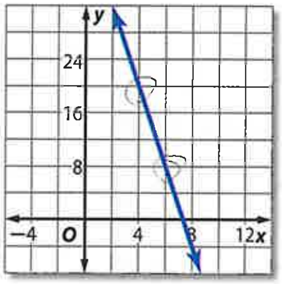
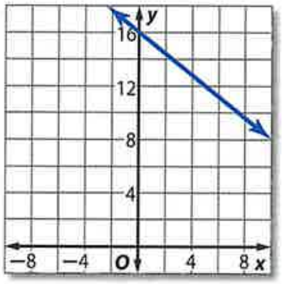


21.



22. **REASONING** The table shows your height on a water slide at various time intervals.

a. Graph the height versus the time on the water slide.

b. Find the average rate of change in a rider's height between 1 and 3 seconds.

c. Find the average rate of change in a rider's height between 0 and 5 seconds.

d. What is another word for *rate of change* in this situation?

Time (s)	Height (ft)
0	120
1	90
2	60
3	30
4	0
5	0

Determine the rate of change for each equation.

23. $6y = 8x - 40$

24. $-2y - 16x = 41$

25. $12x - 4y + 5 = 18$

26. $20x + 85y = 120$

27. $\frac{3}{2}x - \frac{5}{4}y = 15$

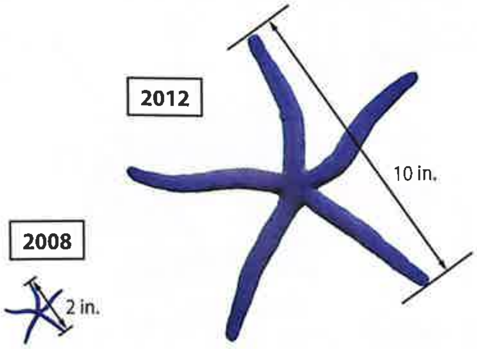
28. $\frac{1}{6}y + \frac{3}{8}x = 24$

(29) **WASHINGTON MONUMENT** The Washington Monument is 555 feet $5\frac{1}{8}$ inches tall and weighs 90,854 tons. The monument is topped by an aluminum square pyramid. The sides of the pyramid's base measure 5.6 inches, and the pyramid is 8.9 inches tall. Estimate the slope that a face of the pyramid makes with its base.

30. MARINE LIFE The illustrations show the growth of a starfish over time.

a. Find the average rate of change in the measure over time.

b. Predict the size of the starfish in 2014.



Find the value of r so that the line that passes through each pair of points has the given slope.

31. $(6, r), (3, 3), m = 2$

32. $(8, 1), (5, r), m = \frac{1}{3}$

33. $(10, r), (4, -3), m = \frac{4}{3}$

34. $(8, -2), (r, -6), m = -4$

35. MULTIPLE REPRESENTATIONS In this problem, you will explore the rate of change for the function $f(x) = x^2$.

a. Graphical Graph $f(x) = x^2$.

b. Tabular Copy and complete the table. To complete the slope row, find the slope of the line containing two consecutive points such as $(-4, 16)$ and $(-3, 9)$. The first one is completed for you.

x	−4	−3	−2	−1	0	1	2	3	4
$f(x)$	16	9							
slope		−7							

c. Verbal Describe what happens to the rate of change for $f(x) = x^2$ as x increases.

H.O.T. Problems Use Higher-Order Thinking Skills

36. CRITIQUE Patty and Tim are asked to find the slope of the line passing through the points (4, 3) and (7, 9). Is either of them correct? Explain.

Patty	Tim
$m = \frac{9-3}{7-4}$ $= \frac{6}{3}$ or 2	$m = \frac{7-4}{9-3}$ $= \frac{3}{6}$ or $\frac{1}{2}$

37. CHALLENGE The graph of a line passes through the points (2, 3) and (5, 8). Explain how you would find the y-coordinate of the point $(11, y)$ on the same line. Then find y.

38. WRITING IN MATH In what ways can change be represented mathematically?

39. REASONING Determine whether the statement *A line has a slope that is a real number* is *sometimes*, *always*, or *never* true. Explain your reasoning.

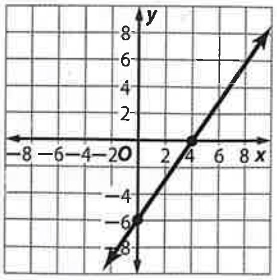
40. WRITING IN MATH Describe the process of finding the rate of change for each.

a. a table of values **b.** a graph **c.** an equation

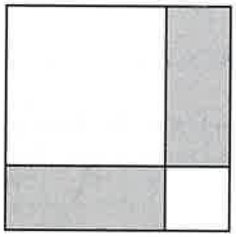
Comstock Images/PictureQuest

Standardized Test Practice

41. GRIDDED RESPONSE What is the slope of the line shown in the graph?



42. SAT/ACT In the figure below, the large square contains two smaller squares. If the areas of the two smaller squares are 4 and 25, what is the sum of the perimeters of the two shaded rectangles?



A 14

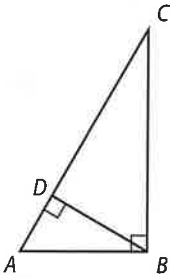
B 20

C 24

D 28

E 49

43. GEOMETRY In △*ABC* shown, $AC = 16$ and $m\angle DAB = 60°$. What is the measure of $\overline{BD}$?



F $9\sqrt{2}$

G 9

H $4\sqrt{3}$

J 4

44. The table shows the cost of bananas depending on the amount purchased. Which conclusion can be made based on information in the table?

Cost of Bananas	
Number of Pounds	Cost ($)
5	1.45
20	4.60
50	10.50
100	19.00

A The cost of 10 pounds of bananas would be more than $4.

B The cost of 200 pounds of bananas would be at most $38.

C The cost of bananas is always more than $0.20 per pound.

D The cost of bananas is always less than $0.28 per pound.

Spiral Review

State whether each equation or function is a linear function. Write *yes* or *no*. Explain. (Lesson 2-2)

45. $6y - 8x = 19$

46. $4x^2 = 2y - 9$

47. $18 = 2xy + 6$

Evaluate each function. (Lesson 2-1)

48. $f(-9)$ if $f(x) = -7x + 8$

49. $g(-4)$ if $g(x) = -3x^2 + 2$

50. $h(12)$ if $h(x) = 4x^2 - 10x$

51. RACING There are 8 contestants in a 400-meter race. In how many different ways can the top three runners finish? (Lesson 0-4)

Determine the quadrant of the coordinate plane where each point is located. (Lesson 0-1)

52. $(-4, -8)$

53. $(-2, 6)$

54. $(3, -1)$

Skills Review

Solve each equation.

55. $8 = 4m - 6$

56. $-6 = 3(8) + b$

57. $-2 = -3x + 5$

LESSON 2-4 Writing Linear Equations

::Then

- You determined slopes of lines.

::Now

- **1** Write an equation of a line given the slope and a point on the line.

2 Write an equation of a line parallel or perpendicular to a given line.

::Why?

- Medical insurance companies often require their customers to make a co-payment for every doctor's office visit in addition to an annual insurance premium.

If an insurance company charges \$2280 annually and requires a copayment of \$35 per doctor's office visit, then the linear equation $y = 35x + 2280$ represents the total annual cost y for x doctor's office visits.

Corbis

NewVocabulary

slope-intercept form
point-slope form
parallel
perpendicular

Common Core State Standards

Content Standards

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

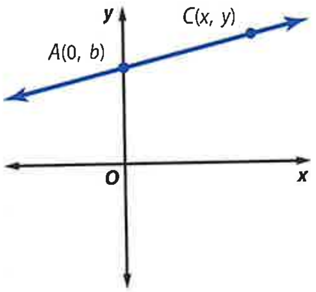
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Forms of Equations

Consider the line through $A(0, b)$ and $C(x, y)$. Notice that b is the y-intercept. You can use these two points to find the slope of $\overleftrightarrow{AC}$.



$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

$$= \frac{y - b}{x - 0} \quad (x_1, y_1) = (0, b), (x_2, y_2) = (x, y)$$

$$= \frac{y - b}{x} \quad \text{Simplify.}$$

Now solve the equation for y.

$$mx = y - b \quad \text{Multiply each side by } x.$$

$$mx + b = y \quad \text{Add } b \text{ to each side.}$$

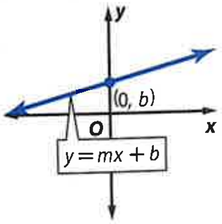
$$y = mx + b \quad \text{Symmetric Property of Equality}$$

Equations written in this format are in **slope-intercept form**.

KeyConcept Slope-Intercept Form

Words The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

Model



Symbols $y = mx + b$

slope → m; b ← y-intercept

If you are given the slope and y-intercept of a line, you can find an equation of the line by substituting the values of m and b into the slope-intercept form.

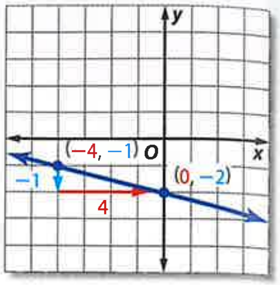
Sometimes it is necessary to calculate the slope before you can write an equation.

WatchOut!

CCSS Reasoning The equation of a vertical line cannot be written in slope-intercept form because its slope is undefined.

Example 1 Write an Equation in Slope-Intercept Form

Write an equation in slope-intercept form for the line.



The graph intersects the y-axis at −2. So $b = -2$.

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{-2-(-1)}{0-(-4)}$	$(x_1, y_1) = (-4, -1), (x_2, y_2) = (0, -2)$
$= \frac{-1}{4}$ or $-\frac{1}{4}$	Simplify.

Step 2 Substitute the values into the slope-intercept equation.

$y = mx + b$	Slope-intercept form
$y = -\frac{1}{4}x - 2$	$m = -\frac{1}{4}, b = -2$

StudyTip

Check Your Results You can check that your equation satisfies the conditions by graphing it.

GuidedPractice

Write an equation in slope-intercept form for the line described.

1A. slope $\frac{4}{3}$, passes through (0, 4)

1B. passes through (0, −6) and (−4, 10)

If you know the slope of a line and the coordinates of a point on the line, you can use the **point-slope form** to find an equation of the line.

KeyConcept Point-Slope Form

Words The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

Symbols

slope

$$y - y_1 = m(x - x_1)$$

coordinates of a point on the line

Example 2 Write an Equation Given Slope and One Point

Write an equation of the line through (6, −2) with a slope of −4.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = -4(x - 6)$	$(x_1, y_1) = (6, -2), m = -4$
$y + 2 = -4x + 24$	Simplify.
$y = -4x + 22$	Subtract 2 from each side.

GuidedPractice

Write an equation in slope-intercept form for the line described.

2A. passes through (2, 3); $m = \frac{1}{2}$

2B. passes through (−2, −1); $m = -3$

You can use any two points on a line to write an equation.

Standardized Test Example 3 Write an Equation Given Two Points

Which is an equation of the line that passes through $(-2, 7)$ and $(3, -3)$?

A $y = -\frac{1}{2}x - \frac{3}{2}$

B $y = -2x + 3$

C $y = \frac{1}{2}x + 8$

D $y = 2x + 11$

Test-Taking Tip

Definitions Be certain to review key vocabulary, such as *y*-intercept, so that you understand what is being asked in a question.

Read the Test Item

You are given the coordinates of two points on the line.

Solve the Test Item

Step 1 Find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$= \frac{-3 - 7}{3 - (-2)}$$ $(x_1, y_1) = (-2, 7)$, $(x_2, y_2) = (3, -3)$

$$= -\frac{10}{5}$$ Subtract.

$$= -2$$ Divide.

Step 2 Write an equation. Use either ordered pair for (x_1, y_1).

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - (-3) = -2(x - 3)$$ $(x_1, y_1) = (3, -3)$ and $m = -2$

$$y + 3 = -2x + 6$$ Simplify.

$$y = -2x + 3$$ Subtract 3 from each side.

The answer is B.

Guided Practice

3. Which is an equation of the line that passes through $(4, -9)$ and $(2, -4)$?

F $y = -\frac{5}{2}x + 1$

G $y = -\frac{5}{2}x - 1$

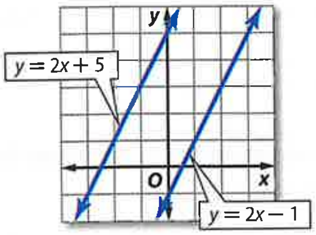
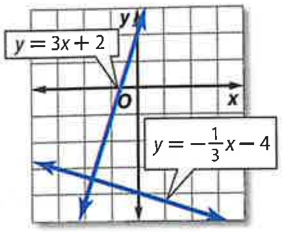
H $y = -\frac{2}{5}x + \frac{37}{5}$

J $y = -\frac{2}{5}x - \frac{37}{5}$

2 Parallel and Perpendicular Lines

Slopes can help you determine whether two lines are parallel or perpendicular.

Key Concept Parallel and Perpendicular Lines

Parallel Lines	Perpendicular Lines
Two nonvertical lines are **parallel** if and only if they have the same slope. All vertical lines are parallel.  $y = 2x + 5$ and $y = 2x - 1$	Two nonvertical lines are **perpendicular** if and only if the product of the slopes is −1. Vertical lines and horizontal lines are perpendicular.  $y = 3x + 2$ and $y = -\frac{1}{3}x - 4$

Example 4 Write an Equation of a Parallel or Perpendicular Line

Write an equation in slope-intercept form for the line that passes through (5, −6) and is perpendicular to the line with equation $y = -\frac{3}{2}x + 7$.

The slope of the given line is $-\frac{3}{2}$. Because the slopes of perpendicular lines are opposite reciprocals, the slope of the line perpendicular to the given line is $\frac{2}{3}$.

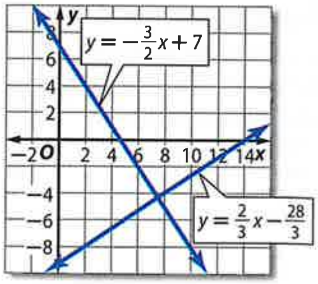
Use the point-slope form and the ordered pair (5, −6).

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-6) = \frac{2}{3}(x - 5)$ $(x_1, y_1) = (5, -6)$ and $m = \frac{2}{3}$

$y + 6 = \frac{2}{3}x - \frac{10}{3}$ Distributive Property

$y = \frac{2}{3}x - \frac{28}{3}$ Subtract 6 from each side and simplify.



CHECK Graph both equations to verify the solution.

GuidedPractice

4. Write an equation in slope-intercept form for the line that passes through (3, 7) and is parallel to the line with equation $y = \frac{3}{4}x - 5$.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **Write an equation in slope-intercept form for the line described.**

1. slope 1.5, passes through (0, 5)

2. passes through (−2, 3) and (0, 1)

Example 2

3. passes through (3, 5); $m = -2$

4. passes through (−8, −2); $m = \frac{5}{2}$

Example 3

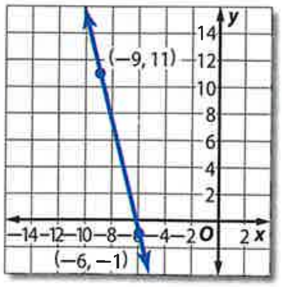
5. **MULTIPLE CHOICE** Which is an equation of the line?

A $y = -4x - 25$

B $y = -\frac{2}{3}x - 5$

C $y = \frac{4}{5}x + \frac{29}{25}$

D $y = 6x + 35$



Example 4 **PERSEVERANCE** **Write an equation in slope-intercept form for the line that satisfies each set of conditions.**

6. passes through (−9, −3), perpendicular to $y = -\frac{5}{3}x - 8$

7 passes through (4, −10), parallel to $y = \frac{7}{8}x - 3$

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 **Write an equation in slope-intercept form for the line described.**

8. slope 3, passes through (0, −2)

9. slope $-\frac{1}{2}$, passes through (0, 5)

10. slope $-\frac{6}{5}$, passes through (0, 8)

11. slope $\frac{9}{2}$, passes through $\left(0, -\frac{13}{2}\right)$

Example 2

12. slope −2, passes through (−3, 14)

13. slope 4, passes through (6, 9)

14. slope $\frac{3}{5}$, passes through (−6, −8)

15. slope $-\frac{1}{4}$, passes through (12, −4)

16. **PART-TIME JOB** Each week, Carmen earns a base pay of \$15 plus \$0.17 for every pamphlet that she delivers. Write an equation that can be used to find how much Carmen earns each week. How much will she earn the week that she delivers 300 pamphlets?

Example 3 **Write an equation of the line passing through each pair of points.**

17. (−2, −6), (4, 6)

18. (−8, −5), (−3, 10)

19. (−4, 12), (−2, −4)

20. (4.6, 3.4), (2.2, 2.8)

21. (5.5, 0.6), (1.1, 2.8)

22. (−25, −16), (−29, 12)

Example 4 **CCSS PERSEVERANCE** **Write an equation in slope-intercept form for the line that satisfies each set of conditions.**

23. passes through (4, 2), perpendicular to $y = -2x + 3$

24. passes through (−6, −6), parallel to $y = \frac{4}{3}x + 8$

25. passes through (12, 0), parallel to $y = -\frac{1}{2}x - 3$

26. passes through (10, 2), perpendicular to $y = 4x + 6$

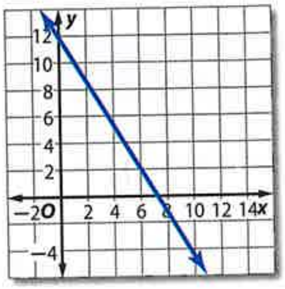
27. **FINANCIAL LITERACY** Julio buys a used car for \$5900. Monthly expenses for the car—which include insurance, maintenance, and gas—average \$180 per month. Write an equation that represents the total cost of buying and owning the car for x months.

28. **DELI** The sales of a sandwich store increased approximately linearly from \$52,000 to \$116,000 during the first five years of business. Write an equation that models the sales y after x years. Determine what the sales will be at the end of 12 years if the pattern continues.

29. **WHALES** In 2009, it was estimated that there were 300 northern right whales in existence. The population of northern right whales is expected to decline by at least 25 whales each generation. Write an equation that represents the number of northern right whales that will be in existence in x generations.

Write an equation in slope-intercept form for each graph.

30.



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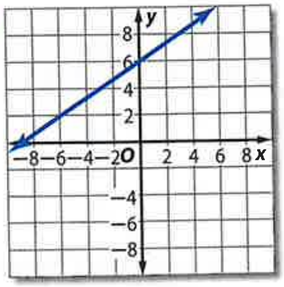
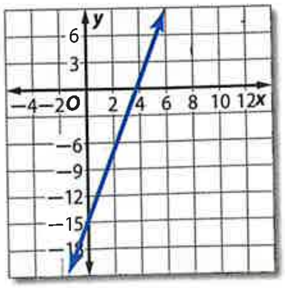


32.



33. **ROSES** Brad wants to send his girlfriend Kelli a dozen roses. He visits two stores. For what distance do the two stores charge the same amount to deliver a dozen roses?

Full Bloom	Flowers R US
Dozen roses \$30 Delivery: \$3 per mile	Dozen roses \$40 Delivery: \$2 per mile

34. **TYPING** The equation $y = 55(23 - x)$ can be used to model the number of words y you have left to type after x minutes.
   - **a.** Write this equation in slope-intercept form.
   - **b.** Identify the slope and y-intercept.
   - **c.** Find the number of words you have left to type after 20 minutes.

35. **RECRUITING** As an army recruiter, Ms. Cooper is paid a daily salary plus commission. When she recruits 10 people, she earns \$100. When she recruits 14 people, she earns \$120.
   - **a.** Write a linear equation to model this situation.
   - **b.** What is Ms. Cooper's daily salary?
   - **c.** How much would Ms. Cooper earn in a day if she recruits 20 people?

36. **CCSS MODELING** Refer to the table at the right.
   - **a.** Write and graph the linear equation that gives the distance y in kilometers in terms of the number x in miles.
   - **b.** What distance in kilometers corresponds to 20 miles?
   - **c.** What number is the same in kilometers and miles? Explain your reasoning.

Miles	Kilometers
100	161
50	80.5

H.O.T. Problems Use Higher-Order Thinking Skills

37. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

The quadrilateral formed by any two parallel lines and two lines perpendicular to those lines is a square.

38. **CHALLENGE** Given $\square ABCD$ with vertices $A(a, b)$, $B(c - a, d)$, $C(c + a, d)$, and $D(c, b)$, write an equation of a line perpendicular to diagonal $\overline{BD}$ that contains A.

39. **REASONING** Write $y = ax + b$ in point-slope form.

40. **OPEN ENDED** Write the equations of two parallel lines with negative slopes.

41. **REASONING** Write an equation in point-slope form of a line with an x-intercept of c and y-intercept of d.

42. **WRITING IN MATH** Why do we represent linear equations in more than one form?

Standardized Test Practice

43. The total cost c in dollars to go to a water park and ride n water rides is given by the equation

$$c = 15 + 3n.$$

If the total cost was $33, how many water rides were ridden?

A 6 **B** 7 **C** 8 **D** 9

44. SHORT RESPONSE To raise money, the service club bought 1000 candy bars for $0.60 each. If the club sells all of the candy bars for $1 each, what will be their total profit?

45. PROBABILITY A fair six-sided die is tossed. What is the probability that a number less than 3 will show on the face of the die?

F $\frac{1}{6}$ **G** $\frac{1}{3}$ **H** $\frac{1}{2}$ **J** $\frac{2}{3}$

46. SAT/ACT What is an equation of the line through $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$?

A $y = -2x - \frac{1}{2}$

B $y = -3x$

C $y = 2x - 5$

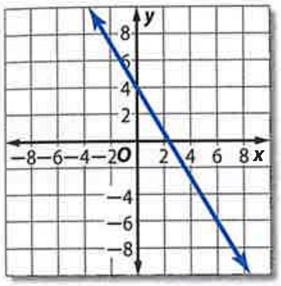
D $y = \frac{1}{2}x + 1$

E $y = -2x - \frac{5}{2}$

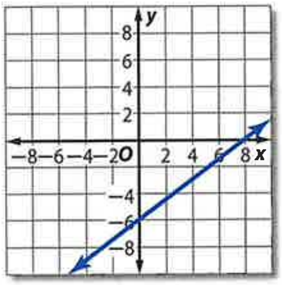
Spiral Review

Determine the rate of change of each graph. (Lesson 2-3)

47.



48.



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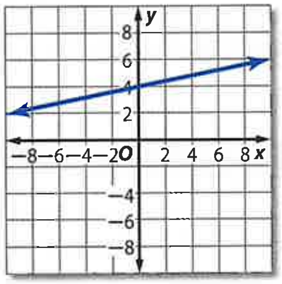


50. RECREATION Scott is currently on page 210 of an epic novel that is 980 pages long. He plans to read 30 pages per day until he finishes the novel. Write and solve a linear equation to determine how many days it will take Scott to complete the novel. (Lesson 2-2)

Solve each inequality. (Lesson 1-5)

51. $-6x - 4 \leq 12 - 2x$

52. $\frac{x + 2}{5} > -3x + 1$

53. $\frac{5x + 3}{3} \geq \frac{4x - 2}{5}$

Determine if the triangles with the following lengths are right triangles. (Lesson 0-8)

54. 5, 12, 13

55. 36, 48, 60

56. 7, 23, 25

Multiply. (Lesson 0-2)

57. $(4c - 6)(2c + 5)$

58. $(-3b + 2)(b + 3)$

59. $(2a - 5)(-3a - 4)$

Skills Review

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

60. (4, 8), (−2, −6)

61. (−6, 3), (−2, 9)

62. (−4, −1), (−8, −8)

63. (12, 4), (42, 10)

64. (10.5, −3), (18, −8)

65. (3.5, −2.5), (−1, −2)

Todd Gipstein/National Geographic/Getty Images

EXTEND 2-4

Graphing Technology Lab
Direct Variation

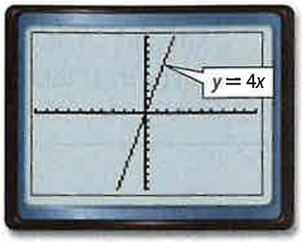
An equation of a direct variation is a special case of a linear equation. A **direct variation** can be expressed in the form $y = kx$. This means that y is a multiple of x. The k in this equation is a constant and is called the **constant of variation**.

CCSS Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Notice that the graph of $y = 4x$ is a straight line through the origin. An equation of a direct variation is a special case of an equation written in slope-intercept form, $y = mx + b$. When $m = k$ and $b = 0$, $y = mx + b$ becomes $y = kx$. So the slope of a direct variation equation is its constant of variation.



[−10, 10] scl: 1 by [−10, 10] scl: 1

To express a direct variation, we say that y varies directly as x. In other words, as x increases, y increases or decreases at a constant rate.

KeyConcept Direct Variation

y varies directly as x if there is some nonzero constant k such that $y = kx$.
k is called the *constant of variation.*

Activity

GOLD The karat rating r of a gold object varies directly as the percentage p of gold in the object. A 14-karat ring is 58.25% gold.

a. Write and graph a direct variation equation relating r and p.

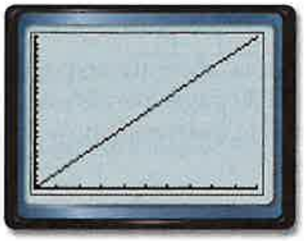
Use the point (0.5825, 14) to find the constant of variation.

$y = kx$ — Direct variation equation

$14 = k(0.5825)$ — $x = 0.5825$, $y = 14$

$24.03 \approx k$ — Divide each side by 0.5825.

The direct variation equation is $r = 24.03p$.



[0, 1] scl: 0.1 by [0, 24] scl: 1

b. Find the karat rating of a ring that is 75% gold.

Use the calculator to find the karat rating.

KEYSTROKES: 2nd [CALC] 0.75 ENTER **18.0225**

The karat rating of a ring that is 75% gold is 18 karats.

MENTAL CHECK $75\% \text{ of } 24 = \frac{3}{4} \text{ of } 24$ Think $75\% = \frac{3}{4}$.

$= 18$ ✓ Think $\frac{3}{4}$ of 24 is 18.

Exercises

1. **SWIMMING** When you swim under water, the pressure on your ears varies directly with the depth at which you are swimming. If you are swimming in 8 feet of water, the pressure on your ears is 3.44 pounds per square inch. Write and graph a direct variation equation relating pressure and depth. Then find the pressure at a depth of 65 feet.

2. Graph the direct variation equations $y = -4x$, $y = -2x$, $y = 4x$, and $y = 2x$. Compare and contrast the graphs of the equations.

CHAPTER 2

Mid-Chapter Quiz

Lessons 2-1 through 2-4

1. State the domain and range of the relation $\{(-3, 2), (4, 1), (0, 3), (5, -2), (2, 7)\}$. Then determine whether the relation is a function. (Lesson 2-1)

2. Graph $y = 2x - 3$ and determine whether the equation is a *function*, is *one-to-one, onto, both*, or *neither*. State whether it is *discrete* or *continuous*. (Lesson 2-1)

Given $f(x) = 3x^3 - 2x + 7$, find each value. (Lesson 2-1)

3. $f(-2)$	**4.** $f(2y)$	**5.** $f(1.4)$

6. State whether $f(x) = 2x^2 - 9$ is a linear function. Explain. (Lesson 2-2)

7. MULTIPLE CHOICE The daily pricing for renting a mid-sized car is given by the function $f(x) = 0.35x + 49$, where $f(x)$ is the total rental price for a car driven x miles. Find the rental cost for a car driven 250 miles. (Lesson 2-2)

A \$84

B \$112.50

C \$136.50

D \$215

Write each equation in standard form. Identify *A*, *B*, and *C*. (Lesson 2-2)

8. $y = -6x + 5$	**9.** $y = 10x$
10. $-\frac{5}{8}x = 2y + 11$	**11.** $0.5x = 3$

Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation using the intercepts. (Lesson 2-2)

12. $4x - 3y + 12 = 0$

13. $10 - x = 2y$

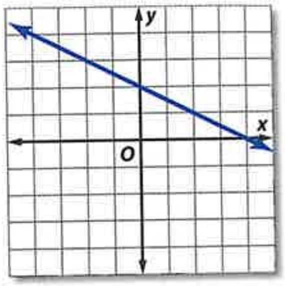
14. SPEED The table shows the distance traveled by a car after each time given in minutes. Find the rate of change in distance for the car. (Lesson 2-3)

Time (min)	Distance (mi)
15	20
30	40
45	60
60	80
75	100

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form. (Lesson 2-3)

15. $(-2, 6), (1, 15)$	**16.** $(3, 5), (7, 15)$
17. $(4, 8), (4, -3)$	**18.** $(-2.5, 4), (1.5, -2)$

19. Find the slope of the line shown. (Lesson 2-3)



Write an equation for the line that satisfies each set of conditions. (Lesson 2-4)

20. slope $\frac{2}{3}$, passes through $(3, -4)$

21. slope -2.5, passes through $(1, 2)$

Write an equation of the line through each set of points. (Lesson 2-4)

22. $(-2, 3), (4, 1)$	**23.** $(4.2, 3.6), (1.8, -1.2)$

24. MULTIPLE CHOICE Each week, Jaya earns \$32 plus \$0.25 for each newspaper she delivers. Write an equation that can be used to determine how much Jaya earns each week. How much will she earn during a week in which she delivers 240 papers? (Lesson 2-4)

F \$75

G \$92

H \$148

J \$212

25. PART-TIME JOB Jesse is a pizza delivery driver. Each day his employer gives him \$20 plus \$0.50 for every pizza that he delivers. (Lesson 2-4)

a. Write an equation that can be used to determine how much Jesse earns each day if he delivers x pizzas.

b. How much will he earn the day he delivers 20 pizzas?

LESSON 2-5

Scatter Plots and Lines of Regression

Then

- You wrote linear equations.

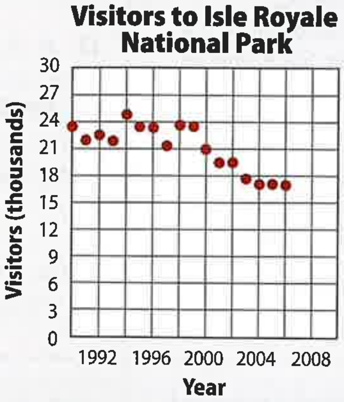
Now

- **1** Use scatter plots and prediction equations.

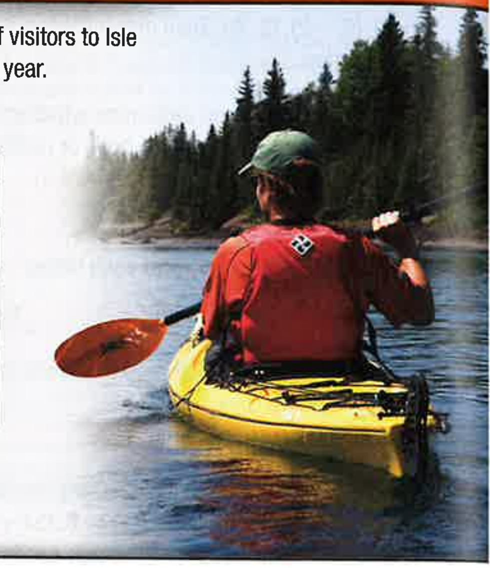
2 Model data using lines of regression.

Why?

- The scatter plot shows the number of visitors to Isle Royale National Park in Michigan per year.



Source: National Park Service



NewVocabulary

bivariate data
scatter plot
dot plot
positive correlation
negative correlation
line of fit
prediction equation
regression line
correlation coefficient

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Mathematical Practices

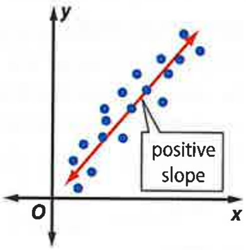
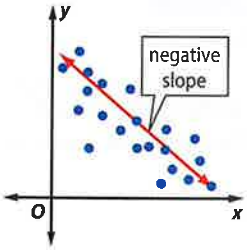
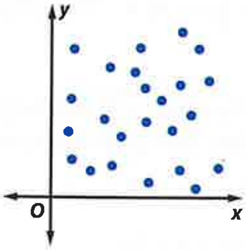
4 Model with mathematics.
5 Use appropriate tools strategically.

1 Scatter Plots and Prediction Equations

Data with two variables, such as year and number of visitors, are called **bivariate data**. A set of bivariate data graphed as ordered pairs in a coordinate plane is called a **scatter plot** or **dot plot**.

A scatter plot can show whether there is a positive, negative, or no correlation between two variables. Correlations are usually described as *strong* or *weak*. In a strong correlation, the points of the scatterplot are closer to the graph of a line than the points representing a weak correlation.

KeyConcept Scatter Plots

positive correlation	negative correlation	no correlation
		
Strong Positive Correlation	Weak Negative Correlation	No Relative Correlation
The slope of the line is positive and the points are close to the line.	The slope of the line is negative and the points are not close to the line.	There is no obvious pattern of increase or decrease for the given data.

When you find a line that closely approximates a set of data, you are finding a **line of fit** for the data. An equation of such a line is often called a **prediction equation** because it can be used to predict one of the variables given the other variable.

To find a line of fit and a prediction equation for a set of data, select two points that appear to represent the data well. This is a matter of personal judgment, so your line and prediction equation may differ from those of others.

Real-WorldLink

According to the National Technology Scan, about 70% of U.S. households have computers connected to the Internet.

Source: *The National Technology Scan*

Real-World Example 1 Use a Scatter Plot and Prediction Equation

TECHNOLOGY **The table shows the percent of U.S. households with Internet access.**

Year	1997	2000	2001	2003	2007
Percent	18.0	41.5	50.4	54.7	61.7

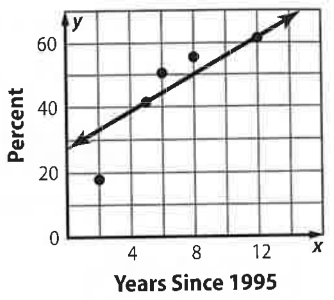
Source: U.S. Census Bureau

a. **Make a scatter plot and a line of fit, and describe the correlation. Let x be the number of years since 1995.**

Graph the data as ordered pairs with the number of years since 1995 on the horizontal axis and the percent of households on the vertical axis.

The points (5, 41.5) and (12, 61.7) appear to represent the data well. Draw a line through these two points. The data show a strong positive correlation.

Percent of Households with Internet Access



b. **Use two ordered pairs to write a prediction equation.**

Find an equation of the line through (5, 41.5) and (12, 61.7).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \frac{61.7 - 41.5}{12 - 5}$ Substitute.

≈ 2.89 Simplify.

$y - y_1 = m(x - x_1)$ Point-Slope form

$y - 41.5 \approx 2.89(x - 5)$ Substitute.

$y - 41.5 \approx 2.89x - 14.45$ Distributive Property

$y \approx 2.89x + 27.05$ Simplify.

One prediction equation is $y = 2.89x + 27.05$.

c. **Predict the percent of households with Internet access in 2020.**

The year 2020 is 25 years after 1995, so find y when $x = 25$.

$y \approx 2.89x + 27.05$ Prediction equation

$\approx 2.89(25) + 27.05$ $x = 25$

≈ 99.3 Simplify.

The model predicts that 99.3% of U.S. households will have Internet access in 2020.

d. **How accurate does your prediction appear to be?**

Except for the outlier at (2, 18.0), the line fits the data well, so the prediction value should be fairly accurate.

ReviewVocabulary

outlier a data point that does not appear to belong to the rest of the set.

GuidedPractice

1. **HOUSING** The table shows the mean selling price of new, privately-owned, single-family homes for six consecutive years.

Year	0	1	2	3	4	5
Price($1000)	154.5	166.4	181.9	207.0	228.7	273.5

A. Make a scatter plot and a line of fit, and describe the correlation.

B. Write a prediction equation.

C. Predict the selling price of a new home for year 8.

D. How accurate does your prediction appear to be?

Per Breiehagen/The Image Bank/Getty Images
Jose Luis Pelaez Inc/Blend Images/Getty Images

2 Lines of Regression

Lines of Regression Another method for writing a line of fit is to use a line of regression. A **regression line** is determined through complex calculations to ensure that the distance of all data points to the line of fit are at a minimum. Most graphing calculators and spreadsheets can perform these calculations easily.

The **correlation coefficient r**, $-1 \leq r \leq 1$, is a measure that shows how well data are modeled by a linear equation.

- When r is close to -1, the data have a negative correlation.
- When $r = 0$, the data have no correlation.
- When r is close to 1, the data have a positive correlation.

Real-World Example 2 Regression Line

The table shows the life expectancy for people born in the United States.

Year of Birth	1980	1983	1990	1995	2000	2006
Life Expectancy (yr)	73.7	74.6	75.4	75.8	76.8	77.7

Source: U.S. CDC

Use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the equation to predict the life expectancy of a person born in 2025.

Step 1 Make a scatter plot.

- Enter the years of birth in **L1** and the ages in **L2**.

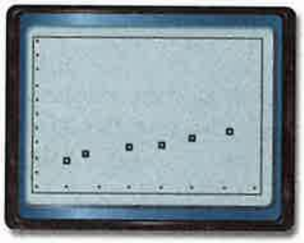
KEYSTROKES: STAT ENTER 1980 ENTER 1983 ENTER 1990 ENTER ...

- Set the viewing window to fit the data.

KEYSTROKES: WINDOW 1975 ENTER 2010 ENTER 5 ENTER 70 ENTER 90 ENTER 2

- Use **STAT PLOT** to graph the scatter plot.

KEYSTROKES: 2nd [STAT PLOT] ENTER ENTER GRAPH



[1975, 2010] scl: 5 by [70, 90] scl: 2

Step 2 Find the equation of the line of regression.

- Find the regression equation by selecting **LinReg(*ax* + *b*)** on the **STAT CALC** menu.

KEYSTROKES: STAT ▶ 4 ENTER

```
LinReg
 y=ax+b
 a=.1441018767
 b=⁻211.4323056
 r²=.98230512
 r=.9911130713
```

The regression equation is about $y = 0.14x - 211.43$. The slope indicates that the life expectancy increases at a rate of about 0.14 per year. The correlation coefficient r is about 0.99, which is very close to 1. So, the data fit the regression line very well.

Step 3 Graph the regression equation.

- Copy the equation to the **Y=** list and graph.

KEYSTROKES: Y= VARS 5 ▶ ▶ 1 GRAPH

Notice that the regression line comes close to all of the data points. As the correlation coefficient indicated, the line fits the data very well.

[1975, 2010] scl: 5 by [70, 90] scl: 2

StudyTip

CCSS Modeling An alternate method for creating a model of the data is to let x represent the number of years since the first year in the data set. This simplifies the math involved in finding a function to model the data. But the rate of change and fit of the regression equation are the same. Only the y-intercept of the equation is changed.

ReadingMath

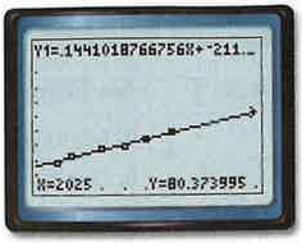
Predictions
When you are predicting an x-value greater than or less than any in the data set, the process is known as **extrapolation**.
When you are predicting an x-value between the least and greatest in the data set, the process is known as **interpolation**.

Step 4 Predict using the function.

- Find y when $x = 2025$. Use **VALUE** on the **CALC** menu. Reset the window size to accommodate the x-value of 2025.

KEYSTROKES: 2nd [CALC] 1 2025 ENTER

According to the function, the life expectancy of a person born in 2025 will be about 80.4 years.



```
Y1=.1441018766756X+⁻211..
X=2025 .  .  .Y=80.373995 .
```

[1975, 2025] scl: 5 by [70, 90] scl: 2

GuidedPractice

2. MUSIC The table at the right shows the percent of sales that were made in music stores in the United States for the period 1999–2008. Use a graphing calculator to make a scatter plot of the data. Find and graph a line of regression. Then use the function to predict the percent of sales made in a music store in 2018.

Music Store Sales

Year	Sales (percent)
1999	44.5
2000	42.4
2001	42.5
2002	36.8
2003	33.2
2004	32.5
2005	39.4
2006	35.4
2007	31.1
2008	30.0

Source: Recording Industry Association of America

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1

① **OCEANS** The table shows the temperature in the ocean at various depths.

Depth (in meters)	0	300	500	1000	2000	2500
Temp (°C)	22	20	13	7	6	?

Source: NDAA

a. Make a scatter plot and a line of fit, and describe the correlation.

b. Use two ordered pairs to write a prediction equation.

c. Use your prediction equation to predict the missing value.

Example 2

2. **CCSS TOOLS** The table shows the median income of families in North Carolina by family size in a recent year. Use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the equation to predict the median income of a North Carolina family of 9.

Family Size	Income ($)
1	33,265
2	44,625
3	50,528
4	59,481

Source: U.S. Department of Justice

Practice and Problem Solving

Extra Practice is on page R2.

Example 1

For Exercises 3–6, complete parts *a–c*.

a. Make a scatter plot and a line of fit, and describe the correlation.

b. Use two ordered pairs to write a prediction equation.

c. Use your prediction equation to predict the missing value.

3. COMPACT DISC SALES The table shows the number of CDs sold in recent years at Jerome's House of Music. Let x be the number of years since 2000.

Year	2004	2005	2006	2007	2008	2017
Number of CDs sold	49,300	47,280	43,450	40,125	35,792	?

4. BASKETBALL The table shows the number of field goals and assists for some of the members of the Miami Heat in a recent NBA season.

Field Goals	472	353	278	283	238	265	186	162	144
Assists	384	97	81	79	18	130	94	95	?

Source: NBA

5 ICE CREAM The table shows the amount of ice cream Sunee's Homemade Ice Creams sold for eight months. Let $x = 1$ for January.

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept
Gallons sold	37	44	72	80	105	110	119	131	?

6. DRAMA CLUB The table shows the total revenue of all of Central High School's plays in recent school years. Let x be the number of years since 2003.

School Year	2005	2006	2007	2008	2009	2016
Revenue ($)	603	666	643	721	771	?

Example 2

7. SALES The table shows the sales of Chayton's Computers. Let x be the number of years since 2002 and use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the function to predict the sales in 2018.

Year	Sales ($ thousands)
2004	640
2005	715
2006	791
2007	852
2008	910
2009	944

8. CCSS TOOLS The table shows the number of employees of a small company. Let x be the number of years since 2000 and use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the function to predict the number of employees in 2025.

Year	Number of Employees
2002	4
2003	7
2004	11
2005	14
2006	20
2007	23

9. **BASEBALL** The table at the right shows the total attendance and wins for the Cleveland Indians in some recent years.

a. Make a scatter plot of the years and attendance.

b. If a linear shape is apparent, find a regression equation to summarize the trend.

c. Interpret the slope of the regression line in the context of the data.

d. Is the relationship between years and attendance stronger than the one between wins and attendance? Explain.

Year	Wins	Attendance
2009	65	1,776,904
2008	81	2,182,087
2007	96	2,275,916
2006	78	1,997,936
2005	93	2,014,220
2004	80	1,814,401
2003	68	1,730,001

10. **CLASS SIZE** The table at the right shows the relationship between the number of students in a mathematics class and the average grade for each class.

a. Make a scatter plot and find a regression equation for the data. Then graph the regression line.

b. What is the correlation coefficient of the data?

c. Describe the correlation. How accurate is the regression equation?

Class Size	Class Average
16	81.2
19	80.5
24	82.5
25	79.9
27	78.6
29	79.3
32	77.7

11. **CCSS TOOLS** Jocelyn is analyzing the sales of her company. The table at the right shows the total sales for each of six years.

a. Find a regression equation and correlation coefficient for the data. Let x be the year.

b. Use the regression equation to predict the 2020 sales.

c. Remove the outlier from the data set and find a new regression equation and correlation coefficient.

d. Use the new regression equation to predict the sales in 2020.

e. Compare the correlation coefficients for the two equations. Which function fits the data better? Which prediction should Jocelyn expect to be more accurate?

Year	Sales ($ millions)
2003	31.2
2004	34.6
2005	18.6
2006	37.7
2007	41.3
2008	45.1

H.O.T. Problems Use Higher-Order Thinking Skills

12. **REASONING** What is the relevance of the correlation coefficient of a linear regression line? Explain your reasoning.

13. **CHALLENGE** If statements a and b have a positive correlation, b and c have a negative correlation, and c and d have a positive correlation, what can you determine about the correlation between statements a and d? Explain your reasoning.

14. **OPEN ENDED** Provide real-world quantities that represent each of the following.

a. positive correlation **b.** negative correlation **c.** no correlation

15. **CHALLENGE** Draw a scatter plot for the following data set.

x	1.0	1.5	2.0	2.8	3.2	4.0	4.8	5.8
y	3.5	4.7	5.1	6.8	7.1	7.5	8.8	10.3

Which of the following best represents the correlation coefficient r for the data? Justify your answer.

a. 0.99 **b.** −0.98 **c.** 0.62 **d.** 0.08

16. **WRITING IN MATH** What are the strengths and weaknesses of using a regression equation to approximate data?

Standardized Test Practice

17. **SHORT RESPONSE** What is the value of the expression below?

$$17 - 3[-1 + 2(7 - 4)]$$

18. Anna took brownies to a club meeting. She gave half of her brownies to Selena. Selena gave a third of her brownies to Randall. Randall gave a fourth of his brownies to Trina. If Trina has 3 brownies, how many brownies did Anna bring to the meeting?

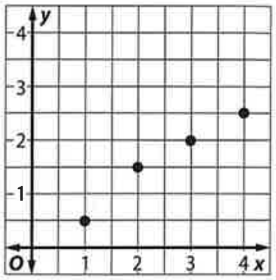
   A 12
   B 36
   C 72
   D 144

19. **GEOMETRY** Which is always true?

   F A parallelogram is a square.
   G A parallelogram is a rectangle.
   H A quadrilateral is a trapezoid.
   J A square is a rectangle.

20. **SAT/ACT** Which line best fits the data in the graph?

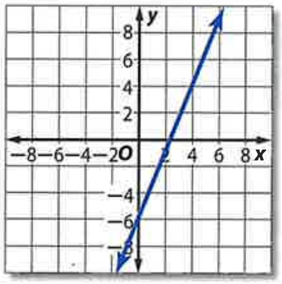
   A $y = x$
   B $y = -0.5x + 4$
   C $y = -0.5x - 4$
   D $y = 0.5x + 0.5$
   E $y = 1.5x - 1.5$



Spiral Review

Write an equation in slope-intercept form for each graph. (Lesson 2-4)

21.



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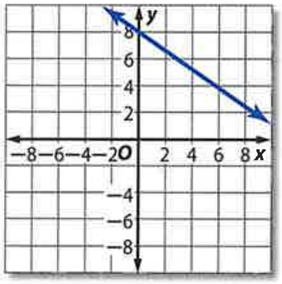

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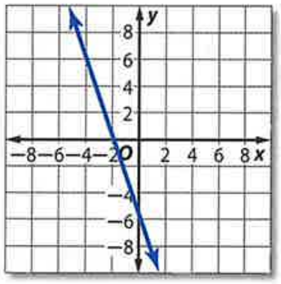


Find the rate of change for each set of data. (Lesson 2-3)

24.

Time (day)	3	6	9	12	15
Height (mm)	12	24	36	48	60

25.

Time (h)	2	4	6	8
Distance (mi)	35	70	105	140

26.

Time (s)	12	16	20	24	28
Volume (cm^3)	45	60	75	90	105

27.

Force (*N*)	32	40	48	56	64
Work (*J*)	48	60	72	84	96

28. **RECREATION** Ramona estimates that she will need 50 tennis balls for every player that signs up for the tennis club and at least 150 more just in case. Write an inequality to express the situation. (Lesson 1-5)

29. **DODGEBALL** Six teams played in a dodgeball tournament. In how many ways can the top three teams finish? (Lesson 0-4)

Skills Review

Solve each equation.

30. $-4|x - 2| = -12$

31. $|3x + 4| = 21$

32. $2|4x - 1| + 3 = 9$

EXTEND

2-5

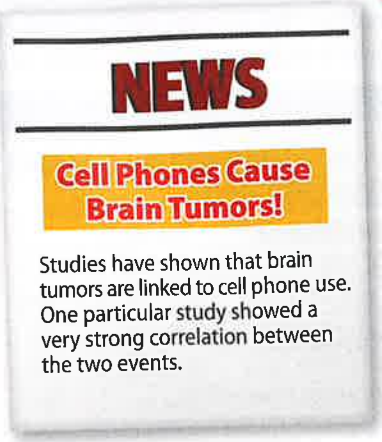
Algebra Lab
Correlation and Causation

CCSS Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

You have learned that the correlation coefficient measures how well an equation fits a set of data. A correlation coefficient close to 1 or −1 indicates a high correlation. However, this does not imply causation. When there is **causation**, one data set is the direct cause of the other data set. When there is a correlation between two sets of data, the data sets are related.



NEWS

Cell Phones Cause Brain Tumors!

Studies have shown that brain tumors are linked to cell phone use. One particular study showed a very strong correlation between the two events.

The news clip to the right uses correlation to imply causation. While a study may have found a high correlation between these two variables, this does not mean that cell phone use causes brain tumors. Other variables, such as an individual's family history, diet, and environment could also have an effect on the formation of brain tumors.

A **lurking variable** affects the relationship between two other variables, but is not included in the study that compares them.

EXAMPLE 1

Determine whether the following correlations possibly show causation. Write *yes* or *no*. If not, identify other lurking variables.

a. Studies have shown that students are less energetic after they eat lunch.

No; students could have had gym class before lunch, or the class after lunch could be one that they do not find interesting.

b. If Kevin lifts weights, he will make the football team.

No; Kevin may need to maintain a certain grade-point average to make the team. He also needs to be talented enough to make the team.

c. When the Sun is visible, we have daylight.

Yes; no other variables cause daylight.

Exercises

Determine whether the following correlations possibly show causation. Write *yes* or *no*. If not, identify other lurking variables.

1. If Allison studies, she will get an A.

2. When Lisa exercises, she is in a better mood.

3. The gun control laws have reduced violent crime.

4. Smoking causes lung cancer.

5. If we have a Level 2 snow emergency, we do not have school.

6. Reading more increases one's intelligence.

Think About It

7. What do you think must be done to show that a correlation between two variables actually shows causation?

(continued on the next page)

Algebra Lab
Correlation and Causation *Continued*

Statistics are often provided that show a correlation between two events and causation is implied, but not confirmed. The best method to find evidence that *x* causes *y* is to actually perform multiple experiments with large sample sizes. When an experiment is not available, the only option is to determine if causation is *possible*.

EXAMPLE 2

The table shows IQ scores for ten high school students and their corresponding grade-point averages.

IQ	GPA	IQ	GPA
106	3.4	118	3.9
110	3.6	107	3.3
109	3.5	111	3.6
98	2.9	109	3.7
115	3.8	102	3.1

a. Calculate the correlation coefficient.
The value of r is about 0.96.

b. Describe the correlation.
There is a strong positive correlation.

c. Is there causation? Explain your reasoning.
There is no causation. Many other factors determine grade-point averages, including effort, study habits, knowledge of the material in the particular classes, and extracurricular activities.

EXAMPLE 3

The table shows the approximate boiling temperatures of water at different altitudes.

Altitude (ft)	Temp. (°F)
0	212
984	210
2000	208
3000	206
5000	203
7500	198
10,000	194
20,000	178
26,000	168

a. Calculate the correlation coefficient.
The value of r is about -0.99.

b. Describe the correlation.
There is a strong negative correlation.

c. Is there causation? Explain your reasoning.
There appears to be causation. Altitude is a direct cause for the boiling temperature of water to decrease.

Exercises

Calculate the correlation coefficient and describe the correlation. Determine if causation is possible. Explain your reasoning.

8.

Car Value ($)	Miles per Gallon	Car Value ($)	Miles per Gallon
14,000	55	25,000	33
16,000	48	29,000	29
18,500	37	32,000	19
20,000	35	35,000	26
22,500	29	40,000	33

9.

Temp. (°F)	Ice Cream Sales ($)	Temp. (°F)	Ice Cream Sales ($)
76	850	85	905
78	885	86	1005
80	875	87	1060
82	920	88	1105
84	955	89	1165

10.

Year	Sales via Internet ($)	Year	Sales via Internet ($)
2001	1923	2006	7067
2002	2806	2007	8568
2003	3467	2008	9991
2004	4848	2009	10,587
2005	5943	2010	12,695

11.

Number of Rings from Pith to Bark	Age of Tree (yr.)	Number of Rings from Pith to Bark	Age of Tree (yr.)
5	5	30	30
10	10	40	40
15	15	50	50
20	20	60	60

LESSON

2-6 Special Functions

Then

- You modeled data using lines of regression.

Now

- **1** Write and graph piecewise-defined functions.
- **2** Write and graph step and absolute value functions.

Why?

- The table shows a recent federal income tax rate schedule. The amount of federal income tax an individual is required to pay is a function of income.

Federal Tax Rate Schedule – Filing Single

If taxable income is over	But not over	The tax is:
\$0	\$7,825	10% of the amount over \$0
\$7,825	\$31,850	\$782.50 plus 15% of the amount over \$7,825
\$31,850	\$77,100	\$4,386.25 plus 25% of the amount over \$31,850
\$77,100	\$160,850	\$15,698.75 plus 28% of the amount over \$77,100
\$160,850	\$349,700	\$39,148.75 plus 33% of the amount over \$160,850
\$349,700	no limit	\$101,469.25 plus 35% of the amount over \$349,700

Source: Internal Revenue Service

NewVocabulary

piecewise-defined function
piecewise-linear function
step function
greatest integer function
absolute value function

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Piecewise-Defined Functions

The function relating income and tax is not a linear function because each interval, or piece, of the function is defined by a different expression. A function that is written using two or more expressions is called a **piecewise-defined function.** On the graph of a piecewise-defined function, a dot indicates that the point is included in the graph. A circle indicates that the point is not included in the graph.

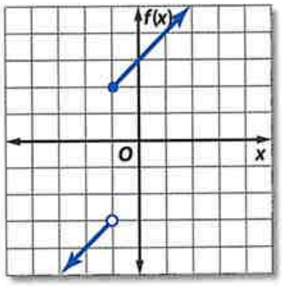
Example 1 Piecewise-Defined Function

Graph $f(x) = \begin{cases} x - 2 \text{ if } x < -1 \\ x + 3 \text{ if } x \geq -1 \end{cases}$. Identify the domain and range.

Step 1 Graph $f(x) = x - 2$ for $x < -1$.

$$\begin{aligned} f(x) &= x - 2 \\ &= (-1) - 2 \\ &= -3 \end{aligned}$$

Because -1 does not satisfy the inequality, begin with a circle at $(-1, -3)$.



Step 2 Graph $f(x) = x + 3$ for $x \geq -1$.

$$\begin{aligned} f(x) &= x + 3 \\ &= (-1) + 3 \\ &= 2 \end{aligned}$$

Because -1 satisfies the inequality, begin with a dot at $(-1, 2)$.

The function is defined for all values of x, so the domain is all real numbers.

The $f(x)$-coordinates of points on the graph are all real numbers less than -3 and all real numbers greater than or equal to 2, so the range is $\{f(x) | f(x) < -3 \text{ or } f(x) \geq 2\}$.

GuidedPractice

1. Graph $f(x) = \begin{cases} x + 2 \text{ if } x < 0 \\ x \quad\quad \text{if } x \geq 0 \end{cases}$. Identify the domain and range.

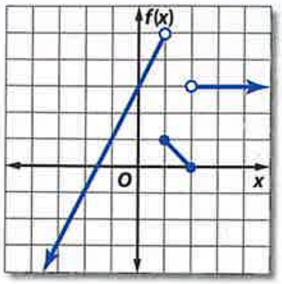
Piecewise-defined functions are often defined by several linear functions.

StudyTip

Graphs of Piecewise-Defined Functions The graphs of different parts of a piecewise-defined function may or may not connect. A graph may also stop at a given x-value and then begin at a different y-value for the same value of x.

Example 2 Write a Piecewise-Defined Function

Write the piecewise-defined function shown in the graph.



Examine and write a function for each portion of the graph.

The left portion of the graph is the graph of $f(x) = 2x + 3$. There is a circle at (1, 5), so the linear function is defined for $\{x|x < 1\}$.

The center portion of the graph is the graph of $f(x) = -x + 2$. There are dots at (1, 1) and (2, 0), so the linear function is defined for $\{x|1 \leq x \leq 2\}$.

The right portion of the graph is the constant function $f(x) = 3$. There is a circle at (2, 3), so the constant function is defined for $\{x|x > 2\}$.

Write the piecewise-defined function.

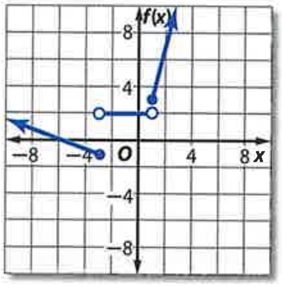
$$f(x) = \begin{cases} 2x + 3 \text{ if } x < 1 \\ -x + 2 \text{ if } 1 \leq x \leq 2 \\ 3 \text{ if } x > 2 \end{cases}$$

CHECK The graph shows a portion of a line with positive slope for $x < 1$. The graph has negative slope for $1 \leq x \leq 2$ and constant slope for $x > 2$. The function is reasonable for the graph.

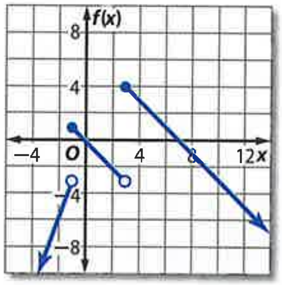
GuidedPractice

Write the piecewise-defined function shown in each graph.

2A.



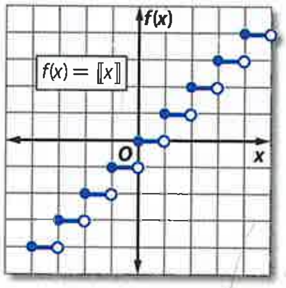
2B.



2 Step Functions and Absolute Value Functions

Step Functions and Absolute Value Functions Unlike a piecewise-defined function, a **piecewise-linear function** contains a single expression. A common piecewise-linear function is the step function. The graph of a **step function** consists of line segments.

The **greatest integer function**, written $f(x) = [\![x]\!]$, is one kind of step function. The symbol $[\![x]\!]$ means *the greatest integer less than or equal to x*. For example, $[\![3.25]\!] = 3$ and $[\![-4.6]\!] = -5$.



StudyTip

Greatest Integer Function Notice that the domain of this step function is all real numbers and the range is all integers.

Real-World Example 3 Use a Step Function

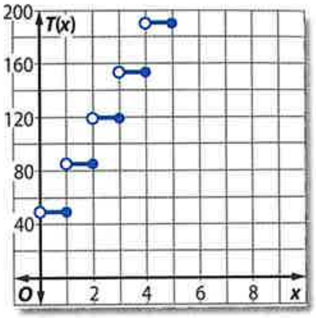
BUSINESS **An automotive repair center charges $50 for any part of the first hour of labor, and $35 for any part of each additional hour. Draw a graph that represents this situation.**

Understand The total labor charge is $50 for the first hour plus $35 for each additional fraction of an hour, so the graph will be a step function.

Plan If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor charge is $50. If the time is greater than 1 hour but less than 2 hours, then the labor charge is $85, and so on.

Solve Use the pattern of times and costs to make a table, where x is the number of hours of labor and $T(x)$ is the total labor charge. Then graph.

x	$T(x)$
$0 < x \leq 1$	$50
$1 < x \leq 2$	$85
$2 < x \leq 3$	$120
$3 < x \leq 4$	$155
$4 < x \leq 5$	$190



Check Since the repair center rounds any fraction of an hour up to the next whole number, each segment of the graph has a circle at the left endpoint and a dot at the right endpoint.

GuidedPractice

3. **RECYCLING** A recycling company pays $5 for every full box of newspaper. They do not give any money for partial boxes. Draw a graph that shows the amount of money $P(x)$ for the number of boxes x brought to the recycling center.

Math HistoryLink

Karl Weierstrass

(1815–1897) At the wishes of his father, Weierstrass studied law, economics, and finance at the University of Bonn, but then dropped out to study his true interest, mathematics, at the University of Münster. In an 1841 essay, Weierstrass first used | | to denote absolute value.

Photo: The Granger Collection

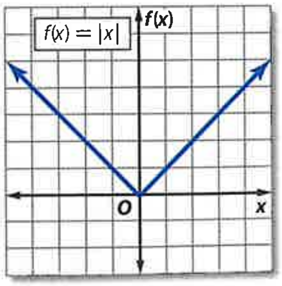
Andrew Ward/Life File/Getty Images

Another piecewise-linear function is the absolute value function. An **absolute value function** is a function that contains an algebraic expression within absolute value symbols.

KeyConcept Parent Function of Absolute Value Functions

Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$



Type of graph:	V-shaped
Domain:	all real numbers
Range:	all nonnegative real numbers
Intercepts:	$x = 0$, $f(x) = 0$
Not defined:	$f(x) < 0$

Example 4 Absolute Value Functions

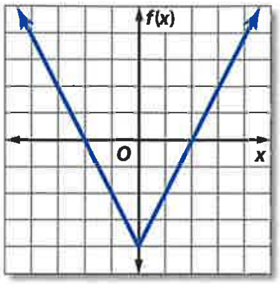
Graph $f(x) = |2x| - 4$. Identify the domain and range.

Create a table of values.

x	$\lvert 2x \rvert - 4$
−3	2
−2	0
−1	−2
0	−4
1	−2
2	0
3	2

Graph the points and connect them.

The domain is the set of all real numbers. The range is $\{f(x) \mid f(x) \geq -4\}$.



GuidedPractice

Graph each function. Identify the domain and range.

4A. $f(x) = |x - 2|$

4B. $f(x) = -|x| + 1$

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

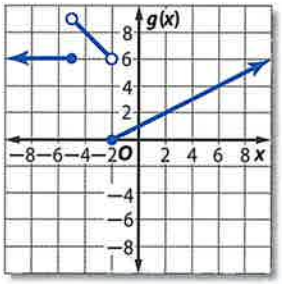
Example 1 **Graph each function. Identify the domain and range.**

1. $g(x) = \begin{cases} -3 \text{ if } x \leq -4 \\ x \text{ if } -4 < x < 2 \\ -x + 6 \text{ if } x \geq 2 \end{cases}$

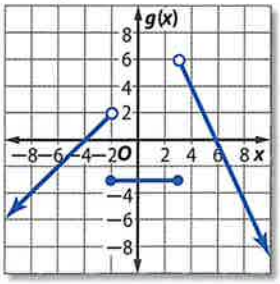
2. $f(x) = \begin{cases} 8 \text{ if } x \leq -1 \\ 2x \text{ if } -1 < x < 4 \\ -4 - x \text{ if } x \geq 4 \end{cases}$

Example 2 **Write the piecewise-defined function shown in each graph.**

3.



4.



Example 3 **5.** **CCSS REASONING** Springfield High School's theater can hold 250 students. The drama club is performing a play in the theater. Draw a graph of a step function that shows the relationship between the number of tickets sold x and the minimum number of plays y that the drama club must perform.

Graph each function. Identify the domain and range.

6. $g(x) = -2[\![x]\!]$

7. $h(x) = [\![x - 5]\!]$

Example 4 **Graph each function. Identify the domain and range.**

8. $g(x) = |-3x|$

9. $f(x) = 2|x|$

10. $h(x) = |x + 4|$

11. $s(x) = |-2x| + 6$

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 Graph each function. Identify the domain and range.

12. $f(x) = \begin{cases} -3x \text{ if } x \leq -4 \\ x \text{ if } 0 < x \leq 3 \\ 8 \text{ if } x > 3 \end{cases}$

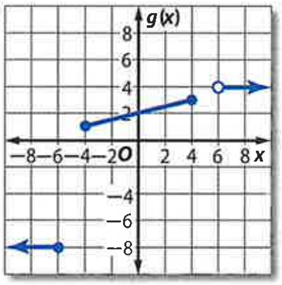
13. $f(x) = \begin{cases} 2x \text{ if } x \leq -6 \\ 5 \text{ if } -6 < x \leq 2 \\ -2x + 1 \text{ if } x > 4 \end{cases}$

14. $g(x) = \begin{cases} 2x + 2 \text{ if } x < -6 \\ x \text{ if } -6 \leq x \leq 2 \\ -3 \text{ if } x > 2 \end{cases}$

15. $g(x) = \begin{cases} -2 \text{ if } x < -4 \\ x - 3 \text{ if } -1 \leq x \leq 5 \\ 2x - 15 \text{ if } x > 7 \end{cases}$

Example 2 Write the piecewise-defined function shown in each graph.

16.



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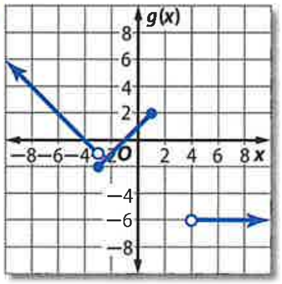

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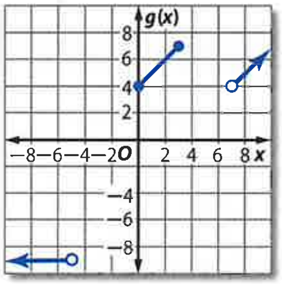

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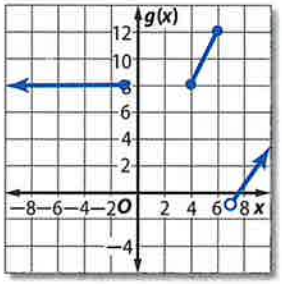


Example 3 Graph each function. Identify the domain and range.

20. $f(x) = \llbracket x \rrbracket - 6$

21. $h(x) = \llbracket 3x \rrbracket - 8$

22. $f(x) = \llbracket 3x + 2 \rrbracket$

23. $g(x) = 2\llbracket 0.5x + 4 \rrbracket$

Example 4 Graph each function. Identify the domain and range.

24. $f(x) = |x - 5|$

25. $g(x) = |x + 2|$

26. $h(x) = |2x| - 8$

27. $k(x) = |-3x| + 3$

28. $f(x) = 2|x - 4| + 6$

29. $h(x) = -3|0.5x + 1| - 2$

30. **GIVING** Patrick is donating money and volunteering his time to an organization that restores homes for the needy. His employer will match his monetary donations up to $100.

a. Identify the type of function that models the total amount of money received by the charity when Patrick donates x dollars.

b. Write and graph a function for the situation.

31. **CCSS SENSE-MAKING** A car's speedometer reads 60 miles an hour.

a. Write an absolute value function for the difference between the car's actual speed a and the reading on the speedometer.

b. What is an appropriate domain for the function? Explain your reasoning.

c. Use the domain to graph the function.

32\. **RECREATION** The charge for renting a bicycle from a rental shop for different amounts of time is shown at the right.

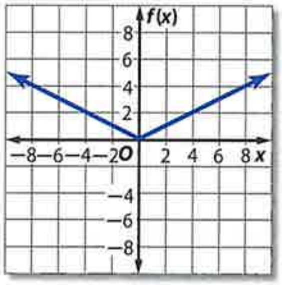
a. Identify the type of function that models this situation.

b. Write and graph a function for the situation.

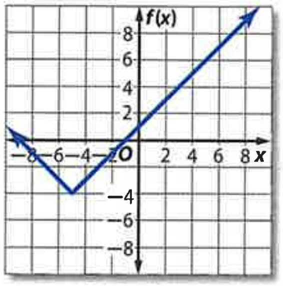
Time	Price
$\frac{1}{2}$ hour	\$6
1 hour	\$10
2 hours	\$16
Daily	\$24

Use each graph to write the absolute value function.

33\.



34\.



Graph each function. Identify the domain and range.

35\. $f(x) = \llbracket |0.5x| \rrbracket$

36\. $g(x) = |\llbracket 2x \rrbracket|$

37\. $g(x) = \begin{cases} \llbracket x \rrbracket \text{ if } x < -4 \\ x + 1 \text{ if } -4 \leq x \leq 5 \\ -|x| \text{ if } x > 3 \end{cases}$

38\. $h(x) = \begin{cases} -|x| \text{ if } x < -6 \\ |x| \text{ if } -6 \leq x \leq 2 \\ |-x| \text{ if } x > 2 \end{cases}$

39\. **MULTIPLE REPRESENTATIONS** Consider the following absolute value functions.

$$f(x) = |x| - 4 \qquad g(x) = |3x|$$

a. Tabular Use a graphing calculator to create a table of $f(x)$ and $g(x)$ values for $x = -4$ to $x = 4$.

b. Graphical Graph the functions on separate graphs.

c. Numerical Determine the slope between each two consecutive points in the table.

d. Verbal Describe how the slopes of the two sections of an absolute value graph are related.

H.O.T. Problems Use Higher-Order Thinking Skills

40\. **OPEN ENDED** Write an absolute value relation in which the domain is all nonnegative numbers and the range is all real numbers.

41\. **CHALLENGE** Graph $|y| = 2|x + 3| - 5$.

42\. **CCSS ARGUMENTS** Find a counterexample to the statement and explain your reasoning.

In order to find the greatest integer function of x when x is not an integer, round x to the nearest integer.

43\. **OPEN ENDED** Write an absolute value function in which $f(5) = -3$.

44\. **WRITING IN MATH** Explain how piecewise functions can be used to accurately represent real-world problems.

Standardized Test Practice

45. SHORT RESPONSE What expression gives the nth term of the linear pattern defined by the table?

2	4	6	8	n
7	13	19	25	?

46. Solve: $5(x + 4) = x + 4$

Step 1: $5x + 20 = x + 4$

Step 2: $4x + 20 = 4$

Step 3: $4x = 24$

Step 4: $x = 6$

Which is the first *incorrect* step in the solution shown above?

A Step 4
B Step 3
C Step 2
D Step 1

47. NUMBER THEORY Twelve consecutive integers are arranged in order from least to greatest. If the sum of the first six integers is 381, what is the sum of the last six integers?

F 345
G 381
H 387
J 417

48. SAT/ACT For which function does $f\left(-\frac{1}{2}\right) \neq -1$?

A $f(x) = 2x$
B $f(x) = |-2x|$
C $f(x) = \llbracket x \rrbracket$
D $f(x) = \llbracket 2x \rrbracket$
E $f(x) = -|2x|$

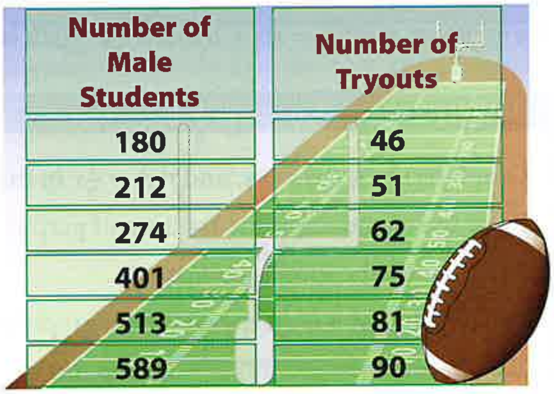
Spiral Review

49. FOOTBALL The table shows the relationship between the total number of male students per school and the number of students who tried out for the football team. (Lesson 2-5)

a. Find a regression equation for the data.

b. Determine the correlation coefficient.

c. Predict how many students will try out for football at a school with 800 male students.



Number of Male Students	Number of Tryouts
180	46
212	51
274	62
401	75
513	81
589	90

Write an equation in slope-intercept form for the line described. (Lesson 2-4)

50. passes through $(-3, -6)$, perpendicular to $y = -2x + 1$

51. passes through $(4, 0)$, parallel to $3x + 2y = 6$

52. passes through the origin, perpendicular to $4x - 3y = 12$

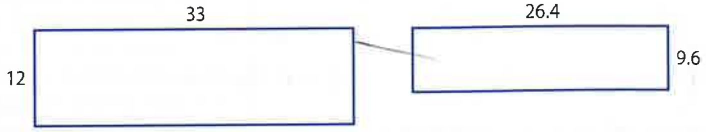
Find each value if $f(x) = -4x + 6$, $g(x) = -x^2$, and $h(x) = -2x^2 - 6x + 9$. (Lesson 2-1)

53. $f(2c)$

54. $g(a + 1)$

55. $h(6)$

56. Determine whether the figures below are similar. (Lesson 0-7)



Skills Review

Graph each equation.

57. $y = -0.25x + 8$

58. $y = \frac{4}{3}x + 2$

59. $8x + 4y = 32$

Ben Blankenburg/Corbis

EXPLORE 2-7

Graphing Technology Lab
Families of Lines

The parent function of the family of linear functions is $f(x) = x$. You can use a graphing calculator to investigate how changing the parameters m and b in $f(x) = mx + b$ affects the graphs as compared to the parent function.

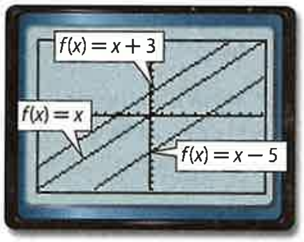
CCSS Common Core State Standards
Content Standards
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Activity 1 b in $f(x) = mx + b$

Graph $f(x) = x$, $f(x) = x + 3$, and $f(x) = x - 5$ in the standard viewing window.

Enter the equations in the **Y=** list as **Y1**, **Y2**, and **Y3**. Then graph the equations.

KEYSTROKES: $\boxed{\text{Y=}}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\text{ENTER}}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{+}$ 3
$\boxed{\text{ENTER}}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{-}$ 5 $\boxed{\text{ENTER}}$ $\boxed{\text{ZOOM}}$ 6



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1A. Compare and contrast the graphs.

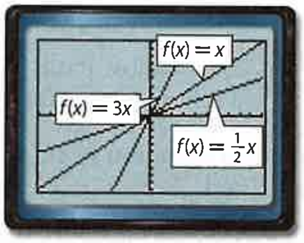
1B. How would you obtain the graphs of $f(x) = x + 3$ and $f(x) = x - 5$ from the graph of $f(x) = x$?

The parameter m in $f(x) = mx + b$ affects the graphs in a different way than b.

Activity 2 m in $f(x) = mx + b$

Graph $f(x) = x$, $f(x) = 3x$, and $f(x) = \frac{1}{2}x$ in the standard viewing window.

Enter the equations in the **Y=** list and graph.



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2A. How do the graphs compare?

2B. Which graph is steepest? Which graph is the least steep?

2C. Graph $f(x) = -x$, $f(x) = -3x$, and $f(x) = -\frac{1}{2}x$ in the standard viewing window. How do these graphs compare?

Analyze the Results

Graph each set of equations on the same screen. Describe the similarities or differences among the graphs.

1. $f(x) = 3x$
$f(x) = 3x + 1$
$f(x) = 3x - 2$

2. $f(x) = x + 2$
$f(x) = 5x + 2$
$f(x) = \frac{1}{2}x + 2$

3. $f(x) = x - 3$
$f(x) = 2x - 3$
$f(x) = 0.75x - 3$

4. What do the graphs of equations of the form $f(x) = mx + b$ have in common?

5. What are the domain and range of functions of the form $f(x) = mx + b$, where $m \neq 0$?

6. How do the values of b and m affect the graph of $f(x) = mx + b$ as compared to the parent function $f(x) = x$?

7. Summarize your results. How can knowing about the effects of m and b help you sketch the graph of a function?

LESSON 2-7 Parent Functions and Transformations

Then

- You analyzed and used relations and functions.

Now

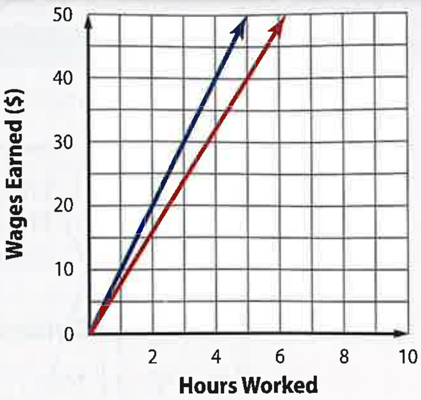
- **1** Identify and use parent functions.

  2 Describe transformations of functions.

Why?

- Nick makes $8 an hour working at a pizza shop. The red line represents his wages. If he also delivers the pizzas, he is paid $2 more per hour. The blue line represents Nick's wages when he delivers the pizzas.

These graphs are examples of transformations.



NewVocabulary

family of graphs
parent graph
parent function
constant function
identity function
quadratic function
translation
reflection
line of reflection
dilation

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Mathematical Practices

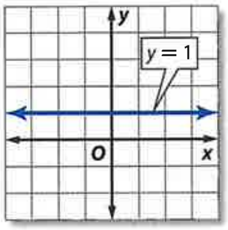
6 Attend to precision.

1 Parent Graphs

A **family of graphs** is a group of graphs that display one or more similar characteristics. The **parent graph**, which is the graph of the **parent function**, is the simplest of the graphs in a family. This is the graph that is transformed to create other members in a family of graphs.

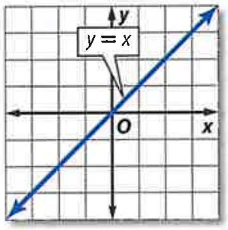
KeyConcept Parent Functions

Constant Function



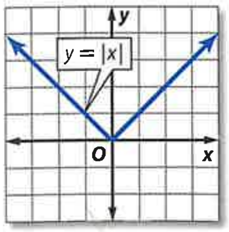
The general equation of a **constant function** is $f(x) = a$, where a is any number. The domain is all real numbers, and the range consists of a single real number a.

Identity Function



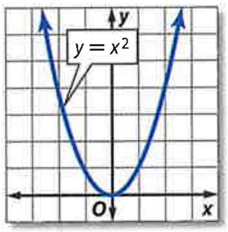
The **identity function** $f(x) = x$ passes through all points with coordinates (a, a). It is the parent function of most linear functions. Its domain and range are all real numbers.

Absolute Value Function



Recall that the parent function of absolute value functions is $f(x) = |x|$. The domain of $f(x) = |x|$ is the set of real numbers, and the range is the set of real numbers greater than or equal to 0.

Quadratic Function

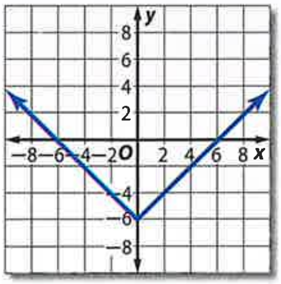


The parent function of **quadratic functions** is $f(x) = x^2$. The domain of $f(x) = x^2$ is the set of real numbers, and the range is the set of real numbers greater than or equal to 0.

Example 1 Identify a Function Given the Graph

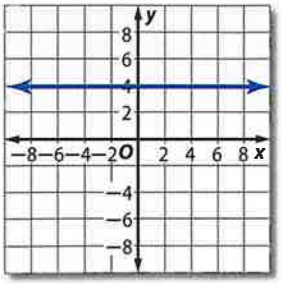
Identify the type of function represented by each graph.

a.



The graph is in the shape of a V. The graph represents an absolute value function.

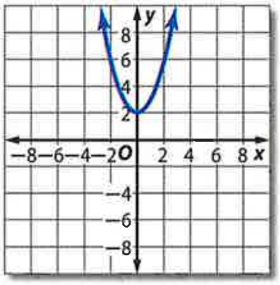
b.



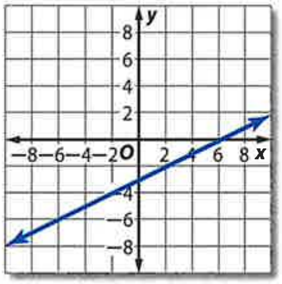
The graph is a horizontal line that crosses the y-axis at 4. The graph represents a constant function.

GuidedPractice

1A.



1B.



2 Transformations

Transformations Transformations of a parent graph may appear in a different location, flip over an axis, or appear to have been stretched or compressed. The transformed graph may resemble the parent graph, or it may not.

A **translation** moves a figure up, down, left, or right.

- When a constant k is added to or subtracted from a parent function, the result $f(x) \pm k$ is a translation of the graph up or down.
- When a constant h is added to or subtracted from x before evaluating a parent function, the result, $f(x \pm h)$, is a translation left or right.

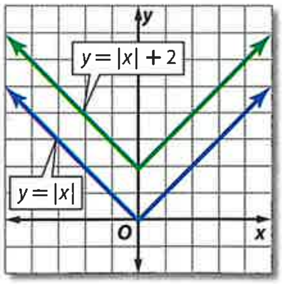
ReadingMath

translation A translation is also called a *slide*, a *shift*, or a *glide*.

Example 2 Describe and Graph Translations

Describe the translation in $y = |x| + 2$. Then graph the function.

The graph of $y = |x| + 2$ is a translation of the graph of $y = |x|$ up 2 units.



GuidedPractice

Describe the translation in each function. Then graph the function.

2A. $y = |x + 3|$

2B. $y = x^2 - 4$

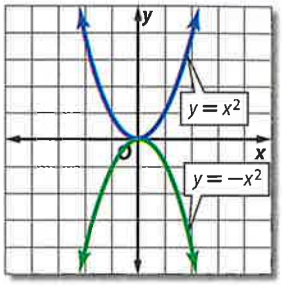
A **reflection** flips a figure over a line called the **line of reflection**.

- When a parent function is multiplied by -1, the result $-f(x)$ is a reflection of the graph in the x-axis.
- When only the variable is multiplied by -1, the result $f(-x)$ is a reflection of the graph in the y-axis.

Example 3 Describe and Graph Reflections

Describe the reflection in $y = -x^2$. Then graph the function.

The graph of $y = -x^2$ is a reflection of the graph of $y = x^2$ in the x-axis.



GuidedPractice

Describe the reflection in each function. Then graph the function.

3A. $y = -|x|$

3B. $y = -x$

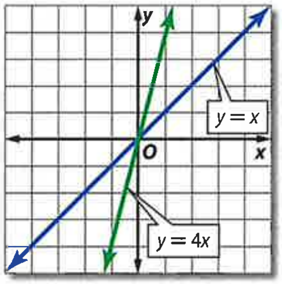
A **dilation** shrinks or enlarges a figure proportionally. When the variable in a linear parent function is multiplied by a nonzero number, the slope of the graph changes.

- When a nonlinear parent function is multiplied by a nonzero number, the function is stretched or compressed vertically.
- Coefficients greater than 1 cause the graph to be stretched vertically, and coefficients between 0 and 1 cause the graph to be compressed vertically.

Example 4 Describe and Graph Dilations

Describe the dilation in $y = 4x$. Then graph the function.

The graph of $y = 4x$ is a dilation of the graph of $y = x$. The slope of the graph of $y = 4x$ is steeper than that of the graph of $y = x$.



GuidedPractice

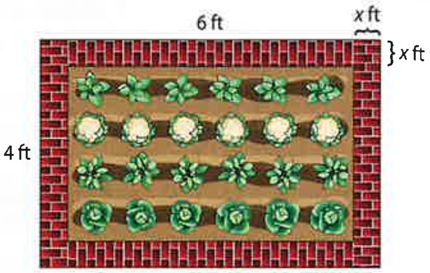
Describe the dilation in each function. Then graph the function.

4A. $y = 2x^2$

4B. $y = \left|\frac{1}{3}x\right|$

Real-World Example 5 Identify Transformations

LANDSCAPING Ethan is going to add a brick walkway around the perimeter of his vegetable garden. The area of the walkway can be represented by the function $f(x) = 4(x + 2.5)^2 - 25$. Describe the transformations in the function. Then graph the function.



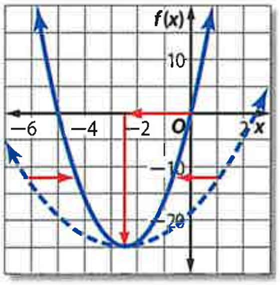
The graph of $f(x) = 4(x + 2.5)^2 - 25$ is a combination of transformations of the parent graph $f(x) = x^2$. Determine how each transformation affects the parent graph.

$f(x) = 4(x + 2.5)^2 - 25$

$+ 2.5$ translates $f(x) = x^2$ left 2.5 units.

$- 25$ translates $f(x) = x^2$ down 25 units.

4 stretches $f(x) = x^2$ vertically.



GuidedPractice

5. **SCIENCE** The function $C(x) = \frac{5}{9}(x - 32)$ can be used to determine the temperature in degrees Celsius when given the temperature in degrees Fahrenheit. Describe the transformations in the function. Then graph the function.

Real-WorldLink

The average American homeowner spends over \$350 each year on indoor and outdoor plants.

Source: American Nursery & Landscape Association

The table summarizes the changes to the parent graph under different transformations.

StudyTip

CCSS Regularity

Ask yourself these questions to help you identify transformations.

1. What type of function is it?
2. Does the graph open up or down?
3. Does the vertex lie on an axis?

ConceptSummary Transformations of Functions

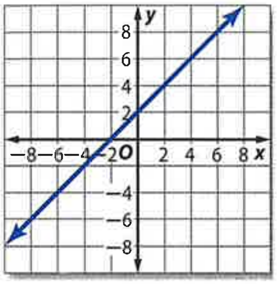
Transformation	Change to Parent Graph
Translation	
$f(x + h), h > 0$	Translates graph h units left.
$f(x - h), h > 0$	Translates graph h units right.
$f(x) + k, k > 0$	Translates graph k units up.
$f(x) - k, k > 0$	Translates graph k units down.
Reflection	
$-f(x)$	Reflects graph in the x-axis.
$f(-x)$	Reflects graph in the y-axis.
Dilation	
$a \cdot f(x), \lvert a \rvert > 1$	Stretches graph vertically.
$a \cdot f(x), 0 < \lvert a \rvert < 1$	Compresses graph vertically
$f(bx), \lvert b \rvert > 1$	Compresses graph horizontally.
$f(bx), 0 < \lvert b \rvert < 1$	Stretches graph horizontally.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 Identify the type of function represented by each graph.

1.



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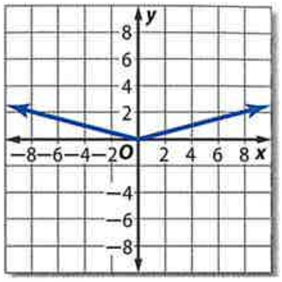


Example 2 **CCSS SENSE-MAKING** Describe the translation in each function. Then graph the function.

3. $y = x^2 - 4$

4. $y = |x + 1|$

Example 3 Describe the reflection in each function. Then graph the function.

5. $y = -|x|$

6. $y = (-x)^2$

Example 4 Describe the dilation in each function. Then graph the function.

7. $y = \frac{3}{5}x$

8. $y = 3x^2$

Example 5

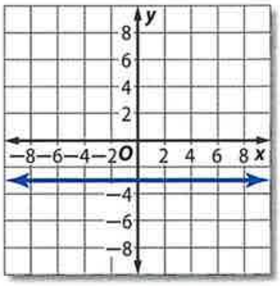
9. **FOOD** The manager of a coffee shop is randomly checking coffee drinks prepared by employees to ensure that the correct amount of coffee is in each cup. Each 12-ounce drink should contain half coffee and half steamed milk. The amount of coffee by which each drink varies can be represented by $f(x) = \frac{1}{2}|x - 12|$. Describe the transformations in the function. Then graph the function.

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 Identify the type of function represented by each graph.

10.



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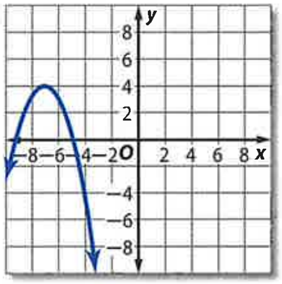

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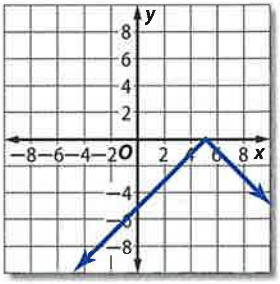

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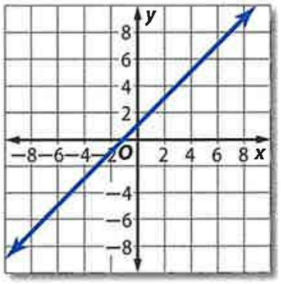

Ariel Skelley/Photographer's Choice/Getty Images

Example 2 **Describe the translation in each function. Then graph the function.**

14. $y = x^2 + 4$
15. $y = |x| - 3$
16. $y = x - 1$
17. $y = x + 2$
18. $y = (x - 5)^2$
19. $y = |x + 6|$

Example 3 **Describe the reflection in each function. Then graph the function.**

20. $y = -x$
21. $y = -x^2$
22. $y = (-x)^2$
23. $y = |-x|$
24. $y = -|x|$
25. $y = (-x)$

Example 4 **Describe the dilation in each function. Then graph the function.**

26. $y = (3x)^2$
27. $y = 6x$
28. $y = 4|x|$
29. $y = |2x|$
30. $y = \frac{2}{3}x$
31. $y = \frac{1}{2}x^2$

Example 5 **32.** **CCSS SENSE-MAKING** A non-impact workout can burn up to 7.5 Calories per minute. The equation to represent how many Calories a person burns after m minutes of the workout is $C(m) = 7.5m$. Identify the transformation in the function. Then graph the function.

Write an equation for each function.

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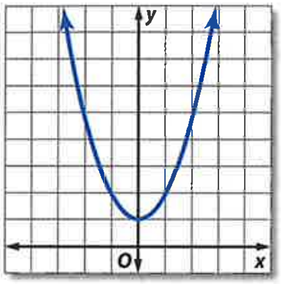

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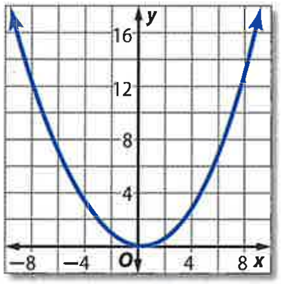

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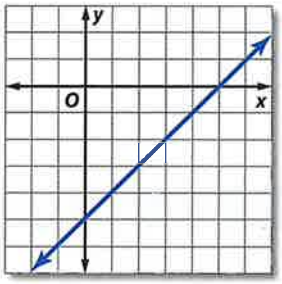

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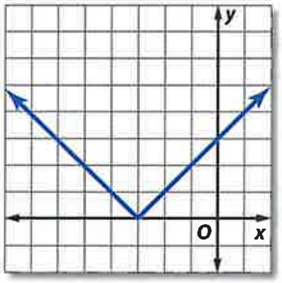

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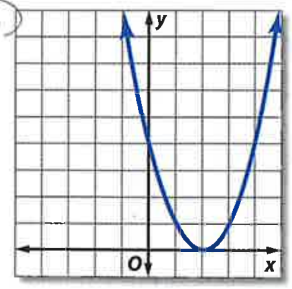

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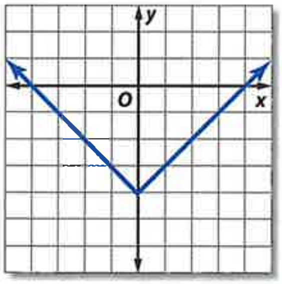
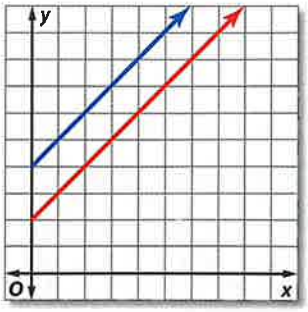


39 **BUSINESS** The graph of the cost of producing x widgets is represented by the blue line in the graph. After hiring a consultant, the cost of producing x widgets is represented by the red line in the graph. Write the equations of both lines and describe the transformation from the blue line to the red line.



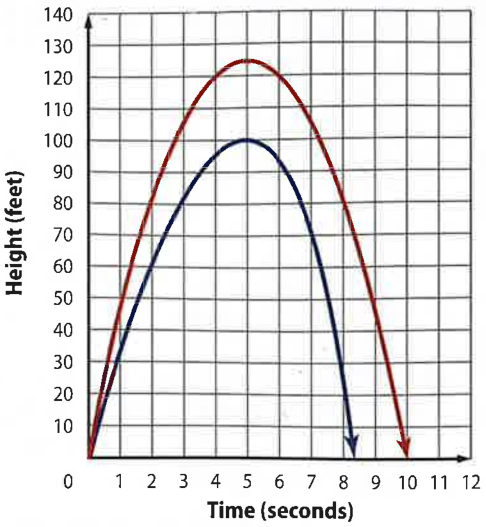
40. **ROCKETRY** Kenji launched a toy rocket from ground level. The height $h(t)$ of Kenji's rocket after t seconds is shown in blue. Emily believed that her rocket could fly higher and longer than Kenji's. The flight of Emily's rocket is shown in red.

a. Identify the type of function shown.

b. How much longer than Kenji's rocket did Emily's rocket stay in the air?

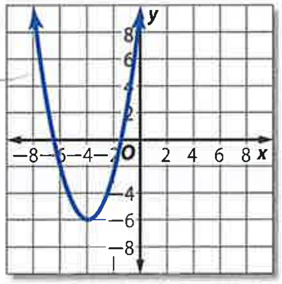
c. How much higher than Kenji's rocket did Emily's rocket go?

d. Describe the type of transformation between the two graphs.



Write an equation for each function.

41



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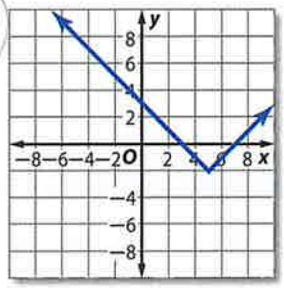
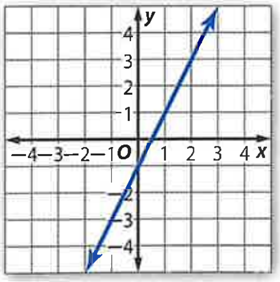


H.O.T. Problems Use Higher-Order Thinking Skills

43. **CHALLENGE** Explain why performing a horizontal translation followed by a vertical translation ends up being the same transformation as performing a vertical translation followed by a horizontal translation.

44. **CCSS CRITIQUE** Kimi thinks that the graph and table below are representations of the same linear relation. Carla disagrees. Who is correct? Explain your reasoning.



x	y
0	−1
1	1
2	3
3	5

45. **OPEN ENDED** Draw a figure in Quadrant II. Use any of the transformations you learned in this lesson to move your figure to Quadrant IV. Describe your transformation.

46. **REASONING** Study the parent graphs at the beginning of this lesson. Select a parent graph with positive y-values at its leftmost points and positive y-values at its rightmost points.

47. **WRITING IN MATH** Explain why the reflection of the graph of $f(x) = x^2$ in the y-axis is the same as the graph of $f(x) = x^2$. Is this true for all reflections of quadratic equations? If not, describe a case when it is false.

Standardized Test Practice

48. What is the solution set of the inequality?

$$6 - |x + 7| \leq -2$$

A $\{x \mid -15 \leq x \leq 1\}$

B $\{x \mid x \leq -1 \text{ or } x \geq 3\}$

C $\{x \mid -1 \leq x \leq 3\}$

D $\{x \mid x \leq -15 \text{ or } x \geq 1\}$

49. GEOMETRY The measures of two angles of a triangle are x and $4x$. Which of these expressions represents the measure of the third angle?

F $180 + x + 4x$

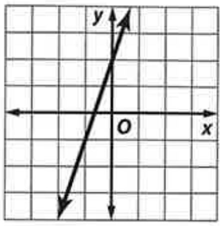
G $180 - x - 4x$

H $180 - x + 4x$

J $180 + x - 4x$

50. GRIDDED RESPONSE Find the value of x that makes $\frac{1}{2} = \frac{x - 2}{x + 2}$ true.

51. SAT/ACT Which could be the equation for the graph?



A $y = 3x + 2$

B $y = 3x - 2$

C $y = -3x + 2$

D $y = -\frac{1}{3}x + 2$

E $y = \frac{1}{3}x + 2$

Spiral Review

Graph each function. Identify the domain and range. (Lesson 2-6)

52. $f(x) = |x - 3|$

53. $h(x) = [\![x]\!] - 5$

54. $f(x) = \begin{cases} -2x \text{ if } x \leq -2 \\ x \text{ if } -2 < x \leq 1 \\ 4 \text{ if } x > 1 \end{cases}$

55. ATTENDANCE The table shows the annual attendance to West High School's Summer Celebration. (Lesson 2-5)

a. Find a regression equation for the data.

b. Determine the correlation coefficient.

c. Predict how many people will attend the Summer Celebration in 2014.

Year	Attendance
2008	61
2009	83
2010	85
2011	92
2012	97
2013	106

Solve each inequality. (Lesson 1-6)

56. $-12 \leq 2x + 4 \leq 8$

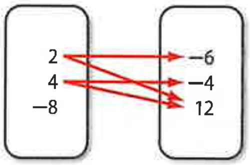
57. $-4 < -3y + 2 < 11$

58. $|x - 3| > 7$

59. CARS Loren is buying her first car. She is considering 4 different models and 5 different colors. How many different cars could she buy? (Lesson 0-4)

Determine if each relation is a function. (Lesson 0-1)

60.



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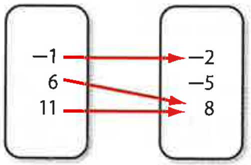

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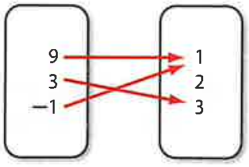


Skills Review

Evaluate each expression if $x = -4$ and $y = 6$.

63. $4x - 8y + 12$

64. $5y + 3x - 8$

65. $-12x + 10y - 24$

LESSON 2-8

Graphing Linear and Absolute Value Inequalities

Then

- You described transformations of functions.

Now

1. Graph linear inequalities.
2. Graph absolute value inequalities.

Why?

- Randy is planning to treat his lacrosse team to a pizza party after the championship game, but he does not want to spend more than $200.

Randy can use the inequality $11p + 2.25d \leq 200$, where p represents the number of pizzas and d represents the number of soft drinks, to check whether certain combinations of pizzas and drinks will fall within his budget.

NewVocabulary

linear inequality
boundary

Common Core State Standards

Content Standards

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

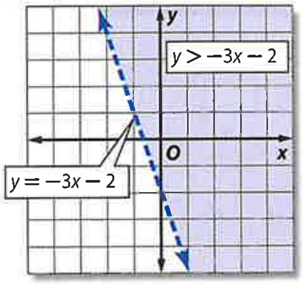
Mathematical Practices

1 Make sense of problems and persevere in solving them.

1 Graph Linear Inequalities

A **linear inequality** resembles a linear equation, but with an inequality symbol instead of an equals symbol. For example, $y > -3x - 2$ is a linear inequality and $y = -3x - 2$ is the related linear equation.

The graph of the inequality $y > -3x - 2$ is shown at the right as a shaded region. Every point in the shaded region satisfies the inequality. The graph of $y = -3x - 2$ is the **boundary** of the region. It is drawn as a dashed line to show that points on the line do not satisfy the inequality. If the symbol was $\leq$ or $\geq$, then points on the boundary would satisfy the inequality, so the boundary would be drawn as a solid line.

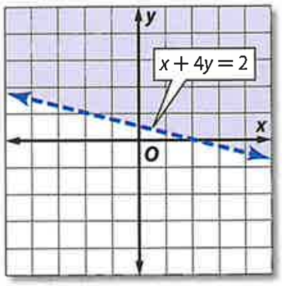


Example 1 Dashed Boundary

Graph $x + 4y > 2$.

Step 1 The boundary of the graph is the graph of $x + 4y = 2$. Since the inequality symbol is $>$, the boundary will be dashed.

Step 2 Test the point (0, 0) because it is not on the boundary.



$x + 4y > 2$ Original inequality
$0 + 4(0) \overset{?}{>} 2$ $(x, y) = (0, 0)$
$0 > 2$ ✗ False

The region that does *not* contain (0, 0) is shaded.

CHECK The graph indicates that (0, 3) is a solution.

$x + 4y > 2$ Original inequality
$0 + 4(3) \overset{?}{>} 2$ $(x, y) = (0, 3)$
$12 > 2$ ✓ True

The solution checks.

GuidedPractice

1A. Graph $3x + \frac{1}{2}y < 2$.

1B. Graph $-x + 2y > 4$.

Real-WorldCareer

Recreation Workers
Recreation workers plan, organize, and manage recreational activities to meet the needs of a variety of people. Educational requirements range from a high school diploma to a graduate degree.

Real-World Example 2 Solid Boundary

RECREATION A recreation center offers various 30-minute and 60-minute art classes. The recreation director has allotted up to 20 hours per week for art classes.

a. Write an inequality to represent the number of classes that can be offered per week. Graph the inequality.

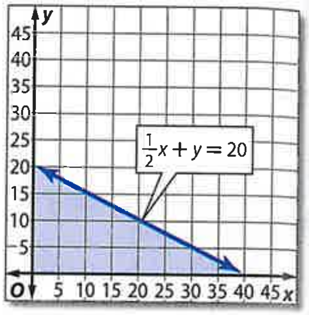
Let x represent the number of 30-minute or $\frac{1}{2}$-hour art classes, and let y represent the number of 60-minute or 1-hour art classes. Because the sum can equal the maximum, the inequality symbol is $\leq$, and the boundary is solid. The inequality is $\frac{1}{2}x + y \leq 20$.

Step 1 Graph the boundary $\frac{1}{2}x + y = 20$.

Step 2 Test the point (0, 0).

$\frac{1}{2}x + y \leq 20$	Original inequality
$\frac{1}{2}(0) + (0) \overset{?}{\leq} 20$	$(x, y) = (0, 0)$
$0 \leq 20$ ✓	True

The region that contains (0, 0) is shaded.



b. Can the recreation director schedule 25 of the 30-minute classes and 15 of the 60-minute classes during a given week? Explain your reasoning.

The point (25, 15) lies outside the shaded region, so it does not satisfy the inequality. Thus, the recreation director cannot schedule 25 30-minute and 15 60-minute classes.

GuidedPractice

2. Manuel has $15 to spend at the county fair. The fair costs $5 for admission, $0.75 for each ride ticket, and $0.25 for each game ticket. Write an inequality, and draw a graph that represents the number of r ride and g game tickets that Manuel can buy.

2 Graph Absolute Value Inequalities

Graphing absolute value inequalities is similar to graphing linear inequalities. First you graph the absolute value equation. Then you determine whether the boundary is dashed or solid and which region should be shaded.

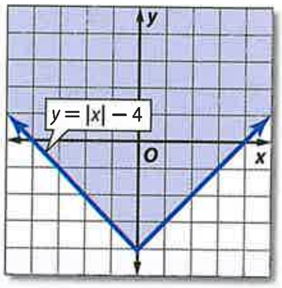
Example 3 Absolute Value Inequality

Graph $y \geq |x| - 4$.

Since the inequality symbol is $\geq$, the boundary is solid. Graph the equation. Then test (0, 0).

$y \geq |x| - 4$	Original inequality
$0 \overset{?}{\geq} |0| - 4$	$(x, y) = (0, 0)$
$0 \geq -4$ ✓	True

The region that includes (0, 0) is shaded.



GuidedPractice

3A. Graph $y \leq 2|x| + 3$.

3B. Graph $y \geq 3|x + 1|$.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 Graph each inequality.

1. $y \leq 4$

2. $x \geq -6$

3. $x + 4y \leq 2$

4. $3x + y > -8$

Example 2

5. **CCSS MODELING** Gregg needs to buy gas and oil for his car. Gas costs $3.45 a gallon, and oil costs $2.41 a quart. He has $50 to spend.

- **a.** Write an inequality to represent the situation, where g is the number of gallons of gas he buys and q is the number of quarts of oil.
- **b.** Graph the inequality.
- **c.** Can Gregg buy 10 gallons of gasoline and 8 quarts of oil? Explain.

Example 3 Graph each inequality.

6. $y \geq |x + 3|$

7. $y - 6 < |x|$

Practice and Problem Solving

Extra Practice is on page R2.

Example 1 Graph each inequality.

8. $x + 2y > 6$

9. $y \geq -3x - 2$

10. $2y + 3 \leq 11$

11. $4x - 3y > 12$

12. $6x + 4y \leq -24$

13. $y \geq \frac{3}{4}x + 6$

Example 2

14. **COLLEGE** April's guidance counselor says that she needs a combined score of at least 1700 on her college entrance exams to be eligible for the college of her choice. The highest possible score is 2400. There are 1200 possible points on the math portion and 1200 on the verbal portion.

- **a.** The inequality $x + y \geq 1700$ represents this situation, where x is the verbal score and y is the math score. Graph this inequality.
- **b.** Refer to your graph. If she scores a 680 on the math portion of the test and 910 on the verbal portion of the test, will April be eligible for the college of her choice?

Example 3 Graph each inequality.

15. $y > |3x|$

16. $y + 4 \leq |x - 2|$

17. $y - 6 < |-2x|$

18. $y + 8 < 2\left|\frac{2}{3}x + 6\right|$

19. $2y > |4x - 5|$

20. $-y \leq |3x - 4|$

21. **SCHOOL DANCE** Carlos estimates that he will need to earn at least $700 to take his girlfriend to the prom. Carlos works two jobs as shown in the table.

- **a.** Write an inequality to represent this situation.
- **b.** Graph the inequality.
- **c.** Will he make enough money if he works 50 hours at each job?

Job	Pay
Main St. Deli	$8 per hour
Babysitting	$6 per hour

Graph each inequality.

22. $y \geq |-2x - 6|$

23. $y \leq |x - 3| + 4$

24. $y - 3 > -2|x + 4|$

25. $|y| > |x|$

26. $|x - y| > 5$

27. $|x + 3y| \geq -2$

Pauline St. Denis/Fancy/Corbis

28. **CCSS MODELING** Mei is making necklaces and bracelets to sell at a craft show. She has enough beads to make 50 pieces. Let x represent the number of bracelets and y represent the number of necklaces.

- **a.** Write an inequality that shows the possible number of necklaces and bracelets Mei can make.
- **b.** Graph the inequality.
- **c.** Give three possible solutions for the number of necklaces and bracelets that can be made.

㉙ **GIFT CARDS** Susan received a gift card from an electronics store for $400. She wants to spend the money on DVDs, which cost $20 each, and CDs, which cost $15 each.

- **a.** Let d equal the number of DVDs, and let c equal the number of CDs. Write an inequality that shows the possible combinations of DVDs and CDs that Susan can purchase.
- **b.** Graph the inequality.
- **c.** Give three possible solutions for the number of DVDs and CDs she can buy.

Graph each inequality.

30. $y \geq \llbracket x \rrbracket$

31. $y < \llbracket x + 2 \rrbracket$

32. $y \geq |\llbracket x \rrbracket|$

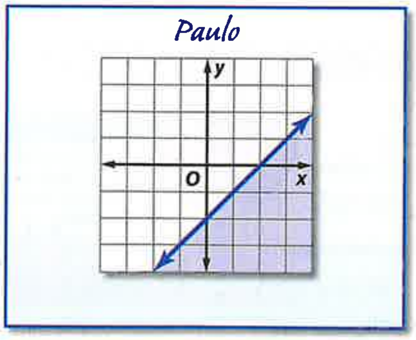
H.O.T. Problems Use Higher-Order Thinking Skills

33. **OPEN ENDED** Create an absolute value inequality in which none of the possible solutions fall in the second or third quadrant.

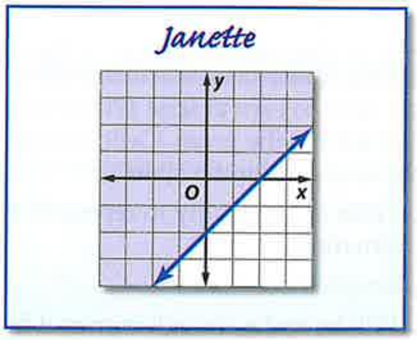
34. **CHALLENGE** Graph the following inequality.

$$g(x) > \begin{cases} |x+1| \text{ if } x \leq -4 \\ -|x| \text{ if } -4 < x < 2 \\ |x-4| \text{ if } x \geq 2 \end{cases}$$

35. **ERROR ANALYSIS** Paulo and Janette are graphing $x - y \geq 2$. Is either of them correct? Explain your reasoning.



Paulo



Janette

36. **REASONING** When will it be possible to shade two different areas when graphing a linear absolute value inequality? Explain your reasoning.

37. **WRITING IN MATH** Describe a situation in which there are no solutions to an absolute value inequality. Explain your reasoning.

Standardized Test Practice

38. EXTENDED RESPONSE Craig scored 85%, 96%, 79%, and 81% on his first four math tests. He hopes to score high enough on the final test to earn a 90% average. If the final test is worth twice as much as one of the other tests, determine if Craig can earn a 90% average. If so, what score does Craig need to get on the final test to accomplish this? Explain how you found your answer.

39. Which of the following sets of numbers represents an infinite set?

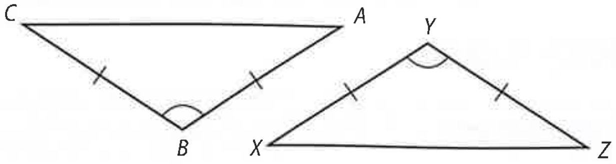
A {2, 4, 6}

B {whole numbers between −50 and 50}

C {integers}

D $\left\{\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$

40. SHORT RESPONSE Which theorem of congruence should be used to prove $\triangle ABC \cong \triangle XYZ$?



41. SAT/ACT For which function is the range $\{f(x) \mid f(x) \leq 0\}$?

F $f(x) = -x$

G $f(x) = [\![x]\!]$

H $f(x) = [\![-x]\!]$

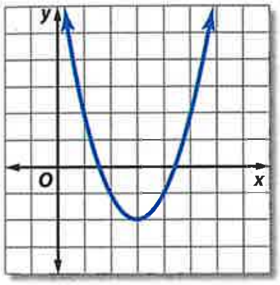
J $f(x) = |x|$

K $f(x) = -|x|$

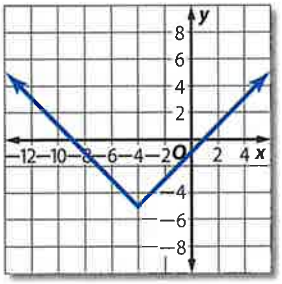
Spiral Review

Write an equation for each graph. (Lesson 2-7)

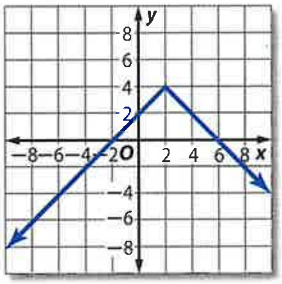
42.



43.



44.



Graph each function. (Lesson 2-6)

45. $f(x) = \begin{cases} x \text{ if } x < 1 \\ 3 \text{ if } 1 \leq x \leq 3 \\ -2x \text{ if } x > 3 \end{cases}$

46. $f(x) = \begin{cases} x + 3 \text{ if } x < -2 \\ 2x \text{ if } -2 \leq x \leq 2 \\ -3x \text{ if } x > 2 \end{cases}$

47. $f(x) = \begin{cases} -2x \text{ if } x \leq -2 \\ x + 1 \text{ if } 0 < x \leq 6 \\ x - 5 \text{ if } x > 6 \end{cases}$

Write each equation in standard form. Identify *A*, *B*, and *C*. (Lesson 2-2)

48. $-6y = 8x - 3$

49. $12y + x = -3y + 5x - 6$

50. $\frac{x+3}{4} + \frac{y-1}{2} = 3$

51. TENNIS Sixteen players signed up for tennis lessons. The instructor plans to use 50 tennis balls for every player and have 200 extra. How many tennis balls are needed for the lessons? (Lesson 1-3)

Multiply. (Lesson 0-2)

52. $(3x - 4)(2x + 1)$

53. $(6x + 5)(-x - 3)$

54. $(5x + 2)(-2x + 3)$

Skills Review

Graph each linear equation.

55. $y = 2x - 8$

56. $y = -\frac{3}{4}x + 2$

57. $3y - 4x = 24$

CHAPTER 2

Study Guide and Review

Study Guide

KeyConcepts

Relations and Functions (Lesson 2-1)

- A function is a relation where each member of the domain is paired with exactly one member of the range.

Linear Equations and Slope (Lessons 2-2 to 2-4)

- Standard Form: $Ax + By = C$, where A, B, and C are integers whose greatest common factor is 1, $A \geq 0$, and A and B are not both zero.
- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Scatter Plots and Lines of Regression (Lesson 2-5)

- A prediction equation can be used to predict the value of one of the variables given the value of the other variable.
- A line of regression can be used to model data.

Special Functions and Parent Functions (Lessons 2-6 and 2-7)

- A piecewise-defined function is made up of two or more expressions.
- Translations, reflections, and dilations of a parent graph form a family of graphs.

Graphing Linear and Absolute Value Inequalities (Lesson 2-8)

- You can graph an inequality by following these steps.

  Step 1 Determine whether the boundary is solid or dashed. Graph the boundary.

  Step 2 Choose a point not on the boundary and test it in the inequality.

  Step 3 If a true inequality results, shade the region containing your test point. If a false inequality results, shade the other region.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

- absolute value function (p. 103)
- bivariate data (p. 92)
- continuous relation (p. 62)
- correlation coefficient (p. 94)
- dependent variable (p. 64)
- dilation (p. 111)
- direct variation (p. 90)
- discrete relation (p. 62)
- family of graphs (p. 109)
- greatest integer function (p. 102)
- independent variable (p. 64)
- linear equation (p. 69)
- linear function (p. 69)
- linear inequality (p. 117)
- line of fit (p. 92)
- negative correlation (p. 92)
- nonlinear relation (p. 69)
- parent function (p. 109)
- piecewise-defined function (p. 101)
- point-slope form (p. 84)
- positive correlation (p. 92)
- prediction equation (p. 92)
- quadratic function (p. 109)
- rate of change (p. 76)
- reflection (p. 111)
- regression line (p. 94)
- scatter plot (p. 92)
- slope (p. 77)
- slope-intercept form (p. 83)
- standard form (p. 70)
- step function (p. 102)
- translation (p. 110)
- vertical line test (p. 62)

VocabularyCheck

Choose the correct term to complete each sentence.

1. A function is (discrete, one-to-one) if each element of the domain is paired to exactly one unique element of the range.
2. The (domain, range) of a relation is the set of all first coordinates from the ordered pairs which determine the relation.
3. The (constant, identity) function is a linear function described by $f(x) = x$.
4. If you are given the coordinates of two points on a line, you can use the (slope-intercept, point-slope) form to find the equation of the line that passes through them.
5. A set of bivariate data graphed as ordered pairs in a coordinate plane is called a (scatter plot, line of fit).
6. A function that is written using two or more expressions is called a (linear, piecewise-defined) function.

Lesson-by-Lesson Review

2-1 Relations and Functions

State the domain and range of each relation. Then determine whether the relation is a function. If it is a function, determine if it is *one-to-one, onto, both,* or *neither.*

7. $\{(1, 2), (3, 4), (5, 6), (7, 8)\}$

8. $\{(-3, 0), (0, 2), (2, 4), (4, 5), (5, 2)\}$

9. $\{(-4, 1), (3, 3), (1, 1), (-2, 5), (3, -4)\}$

10. $\{(7, -4), (5, -2), (3, 0), (1, 2), (-1, 4)\}$

Find each value if $f(x) = -3x + 2$.

11. $f(4)$

12. $f(-3)$

13. $f(0)$

14. $f(y)$

15. $f(-a)$

16. $f(2w)$

17. **BOWLING** A bowling alley charges \$2.50 for shoe rental and \$3.25 per game bowled. The amount a bowler is charged can be expressed as $y = 2.50 + 3.25x$, when $x \geq 1$, and is an integer. Find the domain and range. Then determine whether the equation is a function. Is the equation discrete or continuous?

Example 1

State the domain and range of the relation $\{(-4, 3), (-1, 0), (-2, 4), (3, -1), (2, 6)\}$. Then determine whether the relation is a function. If it is a function, determine if it is *one-to-one, onto, both,* or *neither.*

Domain: $\{-4, -2, -1, 2, 3\}$
Range: $\{-1, 0, 3, 4, 6\}$

Each element of the domain is paired with one element of the range, so the relation is a function. The function is both because each element of the domain is paired with a unique element of the range and each element of the range is paired with a unique element of the domain.

Example 2

Find $f(-2)$ if $f(x) = 4x - 3$.

$f(-2) = 4(-2) - 3$ Substitute -3 for x.

$= -8 - 3$ Multiply.

$= -11$ Simplify.

2-2 Linear Relations and Functions

State whether each function is a linear function. Write *yes* or *no.* Explain.

18. $3x + 4y = 12$

19. $x^2 + y^2 = 4$

20. $y = x^3 - 6$

21. $y = 6x - 19$

22. $f(x) = -2x + 9$

23. $\frac{1}{x} + 3y = -5$

Write each equation in standard form. Identify *A*, *B*, and *C*.

24. $2x + 5y = 10$

25. $y = 12x$

26. $-4y = 3x - 24$

27. $4x = 8y - 12$

28. **TRAVEL** The distance the Green family traveled during their family vacation is given by the equation $y = 65x$, where x represents the number of hours spent driving. How far does the Green family travel in 8 hours?

Example 3

State whether $f(x) = 3x^2$ is a linear function. Write *yes* or *no.* Explain.

No, because the expression includes a variable raised to the second power.

Example 4

Write the equation $y = -5x + 8$ in standard form. Identify *A*, *B*, and *C*.

$y = -5x + 8$ Original equation

$5x + y = 8$ Add $5x$ to each side.

$A = 5$, $B = 1$, and $C = 8$

CHAPTER 2

Study Guide and Review *Continued*

2-3 Rate of Change and Slope

29. **RETAIL** The table shows the number of DVDs sold each week at the Super Movie Store. Find the average rate of change of the number of DVDs sold from week 2 to week 5.

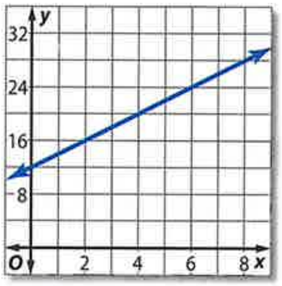
Week	1	2	3	4	5
DVDs Sold	76	58	94	83	112

Find the slope of the line that passes through each pair of points.

30. (2, 5), (6, −3)

31. (8, 2), (2, 8)

32. Determine the rate of change of the graph.



Example 5

Find the slope of the line that passes through each pair of points.

a. (−2, 9), (1, 4)

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{4 - 9}{1 - (-2)}$	$(x_1, y_1) = (-2, 9), (x_2, y_2) = (1, 4)$
$= -\frac{5}{3}$	Simplify.

b. (−3, 6), (4, 6)

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{6 - 6}{4 - (-3)}$	$(x_1, y_1) = (-3, 6), (x_2, y_2) = (4, 6)$
$= \frac{0}{7}$ or 0	Simplify.

2-4 Writing Linear Equations

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

33. slope −2, passes through (−3, −5)

34. slope $\frac{2}{3}$, passes through (4, −1)

35. passes through (−2, 4) and (0, 8)

36. passes through (3, 5) and (−1, 5)

Write an equation of the line passing through each pair of points.

37. (6, 1), (4, 9)

38. (−4, 2), (6, 8)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

39. through (1, 2), parallel to $y = 4x - 3$

40. through (−3, 5), perpendicular to $y = \frac{2}{3}x - 8$

41. **PETS** Drew paid a $250 fee when he adopted a puppy. The average monthly cost of feeding and caring for the puppy is $32. Write an equation that represents the total cost of adopting and caring for the puppy for *x* months.

Example 6

Write an equation of the line through (−2, 5) and (0, −9).

Find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{-9 - 5}{0 - (-2)}$	$(x_1, y_1) = (-2, 5)$, $(x_2, y_2) = (0, -9)$
$= \frac{-14}{2}$ or −7	Simplify.

Write an equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 5 = -7(x - (-2))$	Substitute.
$y - 5 = -7(x + 2)$	Simplify.
$y - 5 = -7x - 14$	Distributive Property
$y = -7x - 9$	Add 5 to each side.

The equation is $y = -7x - 9$.

2-5 Scatter Plots and Lines of Regression

Make a scatter plot and a line of fit and describe the correlation for each set of data. Then, use two ordered pairs to write a prediction equation.

42. **HEATING** The table shows the monthly heating cost for a large home.

Month	Sep	Oct	Nov	Dec	Jan	Feb
Bill ($)	72	114	164	198	224	185

43. **AMUSEMENT PARK** The table shows the annual attendance in thousands at an amusement park during the last 5 years.

Year	1	2	3	4	5
People (thousands)	44	42	39	31	24

Example 7

SCHOOL ENROLLMENT The table shows the number of students each year at a school.

Year	'07	'08	'09	'10	'11	'12
Students	125	116	142	154	146	175

Use (2007, 125) and (2012, 175) to find a prediction equation.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{175 - 125}{2012 - 2007}$	Substitution
$= \frac{50}{5}$ or 10	Simplify.
$y - y_1 = m(x - x_1)$	Point-slope form
$y - 125 = 10(x - 2007)$	Substitution
$y - 125 = 10x - 20{,}070$	Distributive Property
$y = 10x - 20{,}195$	Add 125 to each side.

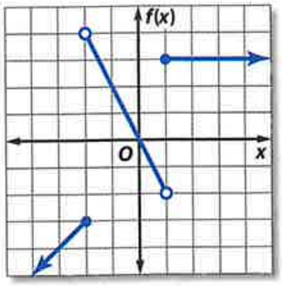
2-6 Special Functions

Graph each function. Identify the domain and range.

44. $f(x) = \begin{cases} -2x \text{ if } x \leq -1 \\ x + 1 \text{ if } -1 < x < 3 \\ x \text{ if } x \geq 3 \end{cases}$

45. $f(x) = \begin{cases} -3 \text{ if } x < -1 \\ 4x - 3 \text{ if } -1 \leq x \leq 3 \\ x \text{ if } x > 3 \end{cases}$

46. Write the piecewise-defined function shown in the graph.



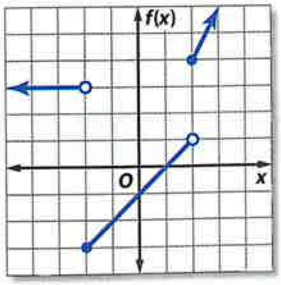
Graph each function. Identify the domain and range.

47. $f(x) = [\![x]\!] + 2$

48. $f(x) = [\![x + 3]\!]$

Example 8

Write the piecewise-defined function shown in the graph.



The left portion of the graph is the graph of $f(x) = 3$. There is a circle at $(-2, 3)$, so the linear function is defined for $x < -2$.

The center portion of the graph is the graph of $f(x) = x - 1$. There is a dot at $(-2, -3)$ and a circle at $(2, 1)$, so the linear function is defined for $-2 \leq x < 2$.

The right portion of the graph is the graph of $f(x) = 2x$. There is a dot at $(2, 4)$, so the linear function is defined for $x \geq 2$.

$$f(x) = \begin{cases} 3 \text{ if } x < -2 \\ x - 1 \text{ if } -2 \leq x < 2 \\ 2x \text{ if } x \geq 2 \end{cases}$$

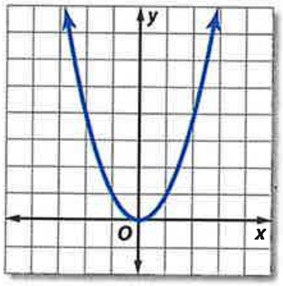
CHAPTER 2

Study Guide and Review *Continued*

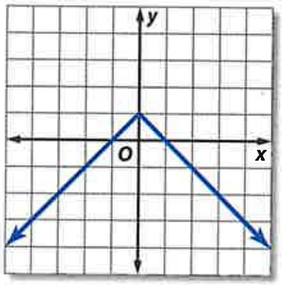
2-7 Parent Functions and Transformations

Identify the type of function represented by each graph.

49.



50.



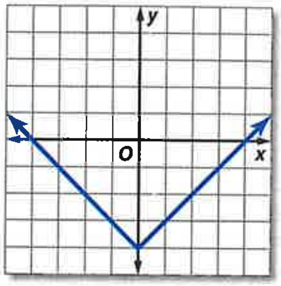
51. Describe the translation in $y = x^2 - 3$.

52. Describe the reflection in $y = -x^2$.

53. **CONSTRUCTION** A large arch is being constructed at the entrance of a new city hall building. The shape of the arch resembles the graph of the function $f(x) = -0.025x^2 + 3.64x - 0.038$. Describe the shape of the arch.

Example 9

Identify the type of function represented by the graph.



The graph is in the shape of a V. The graph represents an absolute value function.

Example 10

Describe the translation in $y = |x + 6|$.

The graph of $y = |x + 6|$ is a translation of the graph of $y = |x|$ shifted left 6 units.

2-8 Graphing Linear and Absolute Value Inequalities

Graph each inequality.

54. $x - 3y < 6$

55. $y \geq 2x + 1$

56. $2x + 4y \leq 12$

57. $y < -3x - 5$

58. $y > |2x|$

59. $y \geq |2x - 2|$

60. $y + 3 < |x + 1|$

61. $2y \leq |x - 3|$

62. **BOOKS** Spencer has saved $96 for a trip to his favorite bookstore. Each paperback book costs $8 and each hardback book costs $12. Write and graph an inequality that shows the number of paperback books and hardback books Spencer can purchase.

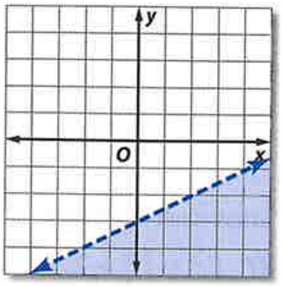
Example 11

Graph $x - 2y > 6$.

Since the inequality symbol is $>$, the graph of the boundary line should be dashed. Graph $x - 2y = 6$.

Test $x - 2y > 6$ at (0, 0).

$$x - 2y > 6$$
$$0 - 2(0) \overset{?}{>} 6$$
$$0 > 6 \text{ ✗}$$



CHAPTER 2 Practice Test

1. State the domain and range of the relation shown in the table. Then determine if it is a function. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.

x	y
−2	3
4	−1
3	2
6	3

Find each value if $f(x) = -2x + 3$.

2. $f(-4)$

3. $f(3y)$

4. Write $2y = -6x + 4$ in standard form. Identify A, B and C.

5. Find the x-intercept and the y-intercept for $3x - 4y = -24$.

6. **MULTIPLE CHOICE** The cost of producing x pumpkin pies at a small bakery is given by $C(x) = 49 + 1.75x$. Find the cost of producing 25 pies.

A $74.00

B $81.50

C $92.75

D $108.25

Find the slope of the line that passes through each pair of points.

7. (1, 6), (3, 10)

8. (−2, 7), (3, −1)

9. **MULTIPLE CHOICE** Find the equation of the line that passes through (0, −3) and (4, 1).

F $y = -x + 3$

G $y = -x - 3$

H $y = x - 3$

J $y = x + 3$

10. Write an equation in slope-intercept form for the line that has slope −2 and passes through the point (3, −4).

11. Write an equation of the line that passes through the points (2, −4) and (1, 6).

12. Write an equation in slope-intercept form for the line that passes through (−3, 5) and is parallel to $y = -6x + 1$.

13. **EMERGENCY ROOM** A hospital tracks the number of emergency room visits during the fall and winter months.

Month	Oct	Nov	Dec	Jan	Feb
Visits	124	163	155	171	192

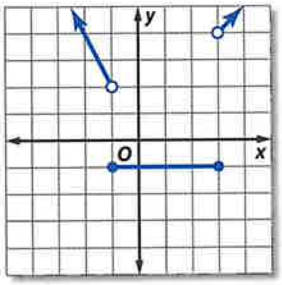
a. Make a scatter plot and describe the correlation.

b. Use two ordered pairs to write a prediction equation.

c. Use your prediction equation to predict the number of emergency room visits for March.

14. Graph $f(x) = \begin{cases} -x \text{ if } x < -2 \\ x + 2 \text{ if } -2 \le x \le 2 \\ 5 \text{ if } x > 2 \end{cases}$.

15. Write the piecewise-defined function shown.



16. Identify the domain and range of $y = [\![x]\!] + 2$.

17. Describe the translation to $y = x^2 + 5$.

18. Describe the reflection in $y = -|x|$.

Graph each inequality.

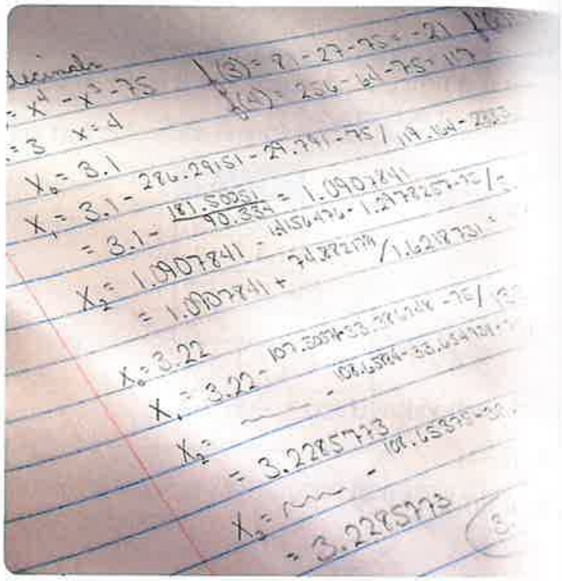
19. $y \ge 4x - 1$

20. $2x + 6y < -12$

CHAPTER 2

Preparing for Standardized Tests

Reading Math Problems



The first step to solving any math problem is to read the problem. When reading a math problem to get the information you need to solve, it is helpful to use special reading strategies.

Strategies for Reading Math Problems

Step 1

Read the problem quickly to gain a general understanding of it.

- **Ask yourself:** "What do I know?" "What do I need to find out?"
- **Think:** "Is there enough information to solve the problem? Is there extra information?"
- **Highlight:** If you are allowed to write in your test booklet, underline or highlight important information. Cross out any information you do not need.

Step 2

Reread the problem to identify relevant facts.

- **Analyze:** Determine how the facts are related.
- **Key Words:** Look for key words to solve the problem.
- **Vocabulary:** Identify mathematical terms. Think about the concepts and how they are related.
- **Plan:** Make a plan to solve the problem.
- **Estimate:** Quickly estimate the answer.

Step 3

Identify any obvious wrong answers.

- **Eliminate:** Eliminate any choices that are very different from your estimate.
- **Units of Measure:** Identify choices that are possible answers based on the units of measure in the question. For example, if the question asks for area, eliminate all the answer choices that are not in square units.

Step 4

Look back after solving the problem.

Check: Make sure you have answered the question.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Sandy heated a solution over a burner and then removed it from the heat source. The temperature of the solution decreased linearly as it cooled. The temperatures after 0, 2, 5, and 9 minutes are shown in the table. What is the rate of change in the temperature of the solution as it cools?

Time (min)	Temperature (°C)
0	133.2
2	130.4
5	126.2
9	120.6

A −1.4 degrees per minute

B −0.8 degrees per minute

C 0.8 degrees per minute

D 1.4 degrees per minute

Read the problem carefully. There is extra information in the problem. To determine the slope, you only need information from two points on the linear function. Use two of the points to find the slope.

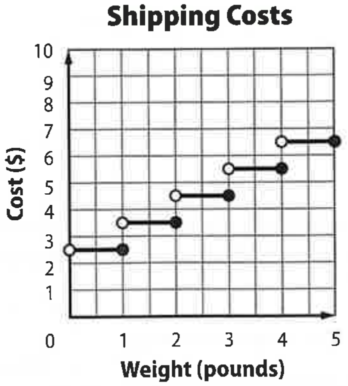
$$m = \frac{130.4 - 133.2}{2 - 0} = -1.4$$

The correct answer is A.

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The graph shows the cost of shipping packages. How much would it cost to ship a package that weighs 2 pounds 8 ounces?



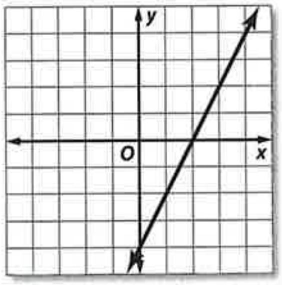
A $3.50

B $4.50

C $5.00

D $5.50

2. What is the slope of the line shown in the graph?



F −2

G $-\frac{1}{2}$

H $\frac{1}{2}$

J 2

Corbis

CHAPTER 2

Standardized Test Practice

Cumulative, Chapters 1 through 2

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the domain of the relation shown below?

x	y
−3	4
1	−1
2	0
6	−3

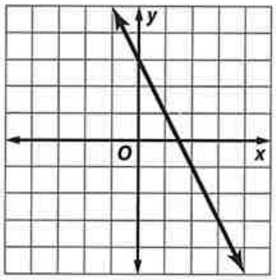
A {0, 1, 2, 4, 6}

B {−3, −1, 0, 4}

C {−3, 1, 2, 6}

D {−3, −1}

2. What is the slope of the line?



F −2

G $-\frac{1}{2}$

H $\frac{1}{2}$

J 2

3. The Robinson family bought their home in 1998 for $152,400. When they sold it in 2010, the value was $174,900. What was the annual rate of change in the value of the home?

A $1225

B $1875

C $22,500

D $27,275

Test-Taking Tip

Question 2 Since the graph slopes downward from left to right, you know the slope is negative. So, answer choices H and J can be eliminated.

4. Carmen works for an electronics retailer. She earns a weekly salary of $450 plus a commission of 4.5% on her weekly sales. Write a linear equation for Carmen's weekly earnings E if she has d dollars in sales.

F $E = (450 + 4.5)d$

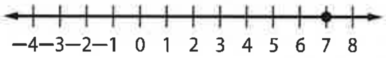
G $E = (450 + 0.045)d$

H $E = 450 + 0.045d$

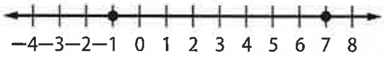
J $E = 450 + 4.5d$

5. Which of the graphs represents the solution set for $|x - 3| - 4 = 0$?

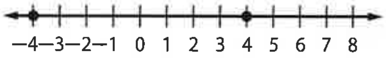
A



B



C



D

6. Karissa has $10 per month to spend text messaging on her cell phone. The phone company charges $4.95 for the first 100 messages and $0.10 for each additional message. How many text messages can Karissa afford to send and receive each month?

F 50

G 100

H 150

J 151

7. Given $y = 2.24x + 16.45$, which statement best describes the effect of decreasing the y-intercept by 20.25?

A The x-intercept increases.

B The y-intercept increases.

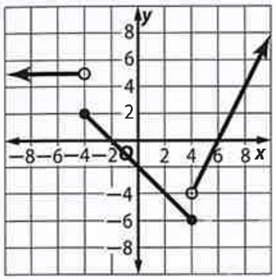
C The new line has a greater rate of change.

D The new line is perpendicular to the original.

Short Response/Gridded Response

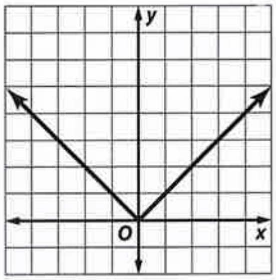
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Write an equation for the piecewise-defined function shown in the graph below.



9. **GRIDDED RESPONSE** Evaluate the piecewise-defined function shown in Exercise 8 for $x = -3$.

10. The graph of the absolute value parent function is shown below.



a. What is the equation of this parent function?

b. What equation would result in the graph of the parent function being reflected over the x-axis and shifted up 2 units?

c. What equation would result in the graph of the parent function being shifted left 3 units and down 1 unit?

Extended Response

Record your answers on a sheet of paper. Show your work.

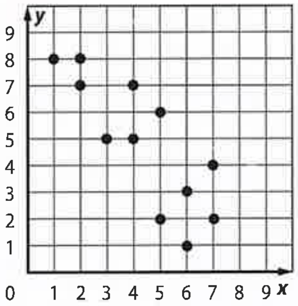
11. The soccer team is having a bake sale this week to raise money for the program. For each cookie sold, the profit is \$0.45, and for each brownie sold, the profit is \$0.50.

a. The team hopes to earn \$150 in profits from the bake sale. Let x represent the number of cookies sold and y the number of brownies sold. Write an inequality to model the situation.

b. Graph the inequality.

c. If the team sells 180 cookies and 160 brownies this week, will they meet their goal? Explain.

12. Use the scatter plot to answer each question.



a. What type of correlation is shown by the data in the plot?

b. Determine a regression line for the data.

c. Use your regression line to predict the value of y when $x = 12$.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	2-1	2-3	2-3	2-4	1-4	1-3	2-7	2-6	2-6	2-7	2-8	2-5