CHAPTER 9

Conic Sections

Then

You solved systems of linear equations algebraically and graphically.

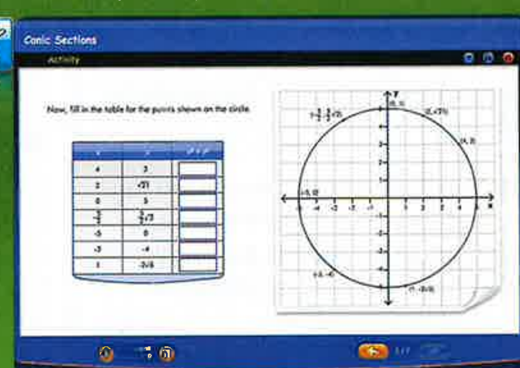
Now

You will:

- Use the Midpoint and Distance Formulas.
- Write and graph equations of parabolas, circles, ellipses, and hyperbolas.
- Identify conic sections.
- Solve systems of quadratic equations and inequalities.

Why?

SPACE Conic sections are evident in many aspects of space. Equations of circles are used to pilot spacecraft and satellites in circular orbits around Earth and the Moon. Planets travel in elliptical paths, not circular ones as previously thought. Comets travel along one branch of a hyperbola, which can help us to predict when they will appear again.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option
Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Solve each equation by completing the square.

1. $x^2 + 8x + 7 = 0$
2. $x^2 + 5x - 6 = 0$
3. $x^2 - 8x + 15 = 0$
4. $x^2 + 2x - 120 = 0$
5. $2x^2 + 7x - 15 = 0$
6. $2x^2 + 3x - 5 = 0$
7. $x^2 - \frac{3}{2}x - \frac{23}{16} = 0$
8. $3x^2 - 4x = 2$

Solve each system of equations by using either substitution or elimination.

9. $y = x + 3$
   $2x - y = -1$
10. $2x - 5y = -18$
    $3x + 4y = 19$
11. $4y + 6x = -6$
    $5y - x = 35$
12. $x = y - 8$
    $4x + 2y = 4$
13. **MONEY** The student council paid \$15 per registration for a conference. They also paid \$10 for T-shirts for a total of \$180. Last year, they spent \$12 per registration and \$9 per T-shirt for a total of \$150 to buy the same number of registrations and T-shirts. Write and solve a system of two equations that represents the number of registrations and T-shirts bought each year.

QuickReview

Example 1

Solve $x^2 + 6x - 16 = 0$ by completing the square.

$x^2 + 6x = 16$

$x^2 + 6x + 9 = 16 + 9$

$(x + 3)^2 = 25$

$x + 3 = \pm 5$

$x + 3 = 5$ or $x + 3 = -5$

$x = 2$ $\quad$ $x = -8$

Example 2

Solve the system of equations algebraically.

$3y = x - 9$

$4x + 5y = 2$

Since x has a coefficient of 1 in the first equation, use the substitution method. First, solve that equation for x.

$3y = x - 9 \quad \rightarrow \quad x = 3y + 9$

$4(3y + 9) + 5y = 2$	Substitute $3y + 9$ for x.
$12y + 36 + 5y = 2$	Distributive Property
$17y = -34$	Combine like terms.
$y = -2$	Divide each side by 17.

To find x, use $y = -2$ in the first equation.

$3(-2) = x - 9$	Substitute -2 for y.
$-6 = x - 9$	Multiply.
$3 = x$	Add 9 to each side.

The solution is $(3, -2)$.

2 Online Option
Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 9. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES® StudyOrganizer

Conic Sections Make this Foldable to help you organize your Chapter 9 notes about conic sections. Begin with eight sheets of grid paper.

1 **Staple** the stack of grid paper along the top to form a booklet.

2 **Cut** seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.

3 **Label** with lesson numbers as shown.

NewVocabulary

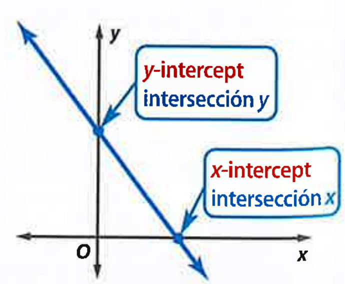
English		Español
parabola	p. 599	parábola
focus	p. 599	foco
directrix	p. 599	directriz
circle	p. 607	círculo
center of a circle	p. 607	centro de un círculo
radius	p. 607	radio
ellipse	p. 615	elipse
foci	p. 615	focos
major axis	p. 615	eje mayor
minor axis	p. 615	eje menor
center of an ellipse	p. 615	centro de una elipse
vertices	p. 615	vértices
co-vertices	p. 615	co-vértices
constant sum	p. 616	suma constante
hyperbola	p. 624	hipérbola
transverse axis	p. 624	eje transversal
conjugate axis	p. 624	eje conjugado
constant difference	p. 627	diferencia constante

ReviewVocabulary

quadratic equation ecuación cuadrática an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

system of equations sistema de ecuaciones a set of equations with the same variables

x- and y-intercepts intersecciones x y y the x- or y-coordinate of the point at which a graph crosses the x- or y-axis



LESSON

9-1 Midpoint and Distance Formulas

Then

- You found the slope of a line passing through two points.

Now

1. Find the midpoint of a segment on the coordinate plane.
2. Find the distance between two points on the coordinate plane.

Why?

- The Zero Milestone in Washington, D.C., was established in 1919. It was intended to serve as the origin for all highway measures with highway markers across the United States displaying the distances from the Zero Milestone.

CCSS Common Core State Standards

Content Standards

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Mathematical Practices

2 Reason abstractly and quantitatively.

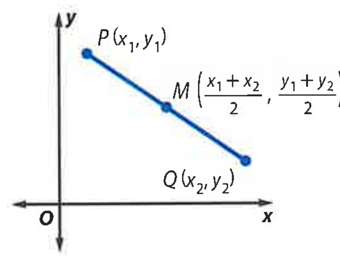
1 The Midpoint Formula

Recall that point M is the midpoint of segment PQ if M is between P and Q and $PM = MQ$. There is a formula for the coordinates of the midpoint of a segment in terms of the coordinates of the endpoints.

KeyConcept Midpoint Formula

Words If a line segment has endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the midpoint of the segment has coordinates $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Model



Example 1 Find a Midpoint

Find the coordinates of M, the midpoint of $\overline{JK}$, for $J(-1, 2)$ and $K(6, 1)$.

Let J be (x_1, y_1) and K be (x_2, y_2).

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint Formula

$= M\left(\frac{-1 + 6}{2}, \frac{2 + 1}{2}\right)$ $(x_1, y_1) = (-1, 2), (x_2, y_2) = (6, 1)$

$= M\left(\frac{5}{2}, \frac{3}{2}\right)$ or $M\left(2\frac{1}{2}, 1\frac{1}{2}\right)$ Simplify.

GuidedPractice

1A. Find the coordinates of the midpoint of $\overline{AB}$ for $A(5, 12)$ and $B(-4, 8)$.

1B. Find the coordinates of the midpoint of $\overline{CD}$ for $C(4, 5)$ and $D(14, 13)$.

2 The Distance Formula

The distance between two points, a and b, on a number line is $|a - b|$ or $|b - a|$. You can use this fact and the Pythagorean Theorem to derive a formula for the distance between two points on a coordinate plane.

U.S. Department of Transportation

StudyTip

Distance In mathematics, just as in real-world situations, distances are always nonnegative.

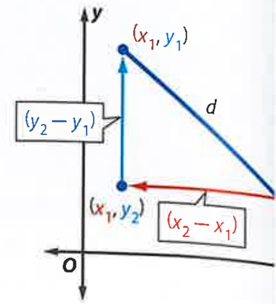
Let d represent the distance between (x_1, y_1) and (x_2, y_2).

$c^2 = a^2 + b^2$ Pythagorean Theorem

$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$ Substitute.

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $|x_2 - x_1|^2 = (x_2 - x_1)^2$, $|y_2 - y_1|^2 = (y_2 - y_1)^2$

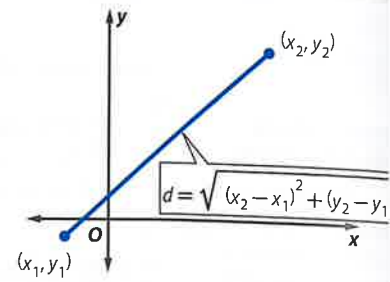
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Find the nonnegative square root of each side.



KeyConcept Distance Formula

Words The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Model

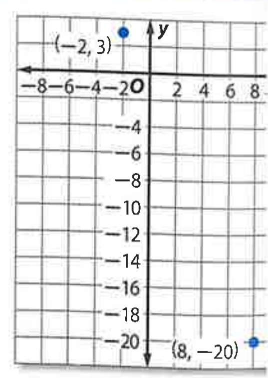


Real-WorldLink

There are more than 700 disc golf courses in the U.S. These courses are permanent installations, usually located in public parks, where players actually "drive" and "putt" with specially-styled discs.

Real-World Example 2 Find the Distance Between Two Points

DISC GOLF **Troy's disc is 20 feet short and 8 feet to the right of the basket. On his first putt, the disc lands 2 feet to the left and 3 feet beyond the basket. If the disc went in a straight line, how far did it go?**



Model the situation. If the basket is at (0, 0), then the location of the disc is (8, −20). The location after the first putt is (−2, 3).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-2 - 8)^2 + [3 - (-20)]^2}$ $(x_1, y_1) = (8, -20)$ and $(x_2, y_2) = (-2, 3)$

$= \sqrt{(-10)^2 + 23^2}$ Simplify.

$= \sqrt{629}$ or about 25

The disc traveled about 25 feet on his first putt.

GuidedPractice

2. Sharon hits a golf ball 12 feet above the hole and 3 feet to the left. Her first putt traveled to 2 feet above the cup and 1 foot to the right. How far did the ball travel on her first putt?

StudyTip

Midpoints The coordinates of the midpoint are the means of the coordinates of the endpoints.

There most likely will be problems involving the Midpoint and Distance Formulas on standardized tests you will have to take.

Standardized Test Example 3 Find the Midpoint Between Coordinates

A coordinate grid is placed over a Florida map. St. Augustine is located at (3, 13), and Rockledge is located at (8, −1). If Port Orange is halfway between St. Augustine and Rockledge, which is closest to the distance in coordinate units from St. Augustine to Port Orange?

A 4.75 **B** 7.43 **C** 14.9 **D** 19

Read the Test Item

The question asks you to find the distance between one city and the midpoint. Find the midpoint, and then use the Distance Formula.

Solve the Test Item

Use the Midpoint Formula to find the coordinates of Port Orange.

$$\text{midpoint} = \left(\frac{3+8}{2}, \frac{13+(-1)}{2}\right) \qquad \text{Midpoint Formula}$$

$$= (5.5, 6) \qquad \text{Simplify.}$$

Use the Distance Formula to find the distance between St. Augustine (3, 13) and Port Orange (5.5, 6).

$$\text{distance} = \sqrt{(3-5.5)^2 + (13-6)^2} \qquad \text{Distance Formula}$$

$$= \sqrt{(-2.5)^2 + 7^2} \qquad \text{Evaluate exponents and add.}$$

$$= \sqrt{55.25} \text{ or about } 7.43 \qquad \text{Simplify.}$$

The answer is B.

Test-TakingTip

Answer Check To check your answer, find the distance between Port Orange and Rockledge. Since Port Orange is at the midpoint, these distances should be equal.

GuidedPractice

3. The coordinates for points A and B are (−4, −5) and (10, −7), respectively. Find the distance between the midpoint of A and B and point B.

F $\sqrt{10}$ units **G** $5\sqrt{10}$ units **H** $\sqrt{50}$ units **J** $10\sqrt{5}$ units

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **CCSS PRECISION** **Find the midpoint of the line segment with endpoints at the given coordinates.**

1. (−4, 7), (3, 9)

2. (8, 2), (−1, −5)

3. (11, 6), (18, 13.5)

4. (−12, −2), (−10.5, −6)

Example 2 **Find the distance between each pair of points with the given coordinates.**

5. (3, −5), (13, −11)

6. (8, 1), (−2, 9)

7. (0.25, 1.75), (3.5, 2.5)

8. (−4.5, 10.75), (−6.25, −7)

Example 3

9. **MULTIPLE CHOICE** The map of a mall is overlaid with a numeric grid. The kiosk for the cell phone store is halfway between The Ice Creamery and the See Clearly eyeglass store. If the ice cream store is at (2, 4) and the eyeglass store is at (78, 46), find the distance the kiosk is from the eyeglass store.

A 43.4 units **B** 47.2 units **C** 62.4 units **D** 94.3 units

Practice and Problem Solving

Extra Practice is on page

Example 1 **Find the midpoint of the line segment with endpoints at the given coordinates.**

10. (20, 3), (15, 5)
11. (−27, 4), (19, −6)
12. (−0.4, 7), (11, −1.6)
13. (5.4, −8), (9.2, 10)
14. (−5.3, −8.6), (−18.7, 1)
15. (−6.4, −8.2), (−9.1, −0.8)

Example 2 **Find the distance between each pair of points with the given coordinates.**

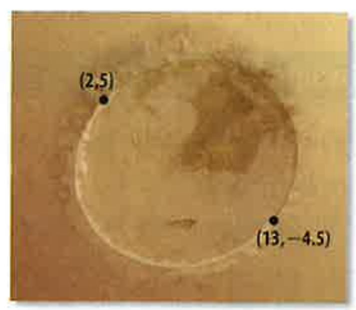
16. (1, 2), (6, 3)
17. (3, −4), (0, 12)
18. (−6, −7), (11, −12)
19. (−10, 8), (−8, −8)
20. (4, 0), (5, −6)
21. (7, 9), (−2, −10)
22. (−4, −5), (15, 17)
23. (14, −20), (−18, 25)

Example 3 **24.** **TRACK AND FIELD** A shot put is thrown from the inside of a circle. A coordinate grid is placed over the shot put circle. The toe board is located at the front of the circle at (−4, 1), and the back of the circle is at (5, 2). If the center of the circle is halfway between these two points, what is the distance from the toe board to the center of the circle?

Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.

25. (−93, 15), (90, −15)
26. (−22, 42), (57, 2)
27. (−70, −87), (59, −14)
28. (−98, 5), (−77, 64)
29. (41, −45), (−25, 75)
30. (90, 60), (−3, −2)
31. (−1.2, 2.5), (0.34, −7)
32. (−7.54, 3.89), (4.04, −0.38)
33. $\left(-\frac{5}{12}, -\frac{1}{3}\right), \left(-\frac{17}{2}, -\frac{5}{3}\right)$
34. $\left(-\frac{5}{4}, -\frac{13}{2}\right), \left(-\frac{4}{3}, -\frac{5}{6}\right)$
35. $\left(-3\sqrt{2}, -4\sqrt{5}\right), \left(-3\sqrt{3}, 9\right)$
36. $\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{2}}{4}\right), \left(\frac{-2\sqrt{3}}{3}, \frac{\sqrt{2}}{4}\right)$

37. **SPACE** Use the labeled points on the outline of the circular crater on Mars to estimate its diameter in kilometers. Assume each unit on the coordinate system is 1 kilometer.



38. CCSS **MODELING** Triangle *ABC* has vertices *A*(2, 1), *B*(−6, 5), and *C*(−2, −3).

a. An isosceles triangle has two sides with equal length. Is triangle *ABC* isosceles? Explain.

b. An equilateral triangle has three sides of equal length. Is triangle *ABC* equilateral? Explain.

c. Triangle *EFG* is formed by joining the midpoints of the sides of triangle *ABC*. What type of triangle is *EFG*? Explain.

d. Describe any relationship between the lengths of the sides of the two triangles.

39. PACKAGE DELIVERY To determine the mileage between cities for their overnight delivery service, a package delivery service superimposes a coordinate grid over the United States. Each side of a grid unit is equal to 0.316 mile. Suppose the locations of two distribution centers are at (132, 428) and (254, 105). Find the actual distance between these locations to the nearest mile.

40. HIKING Orlando wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

a. Use the Distance Formula to determine how far the waterfall is from the campsite.

b. Verify your answer in part **a** by using the Pythagorean Theorem to determine the distance between the campsite and the waterfall.

c. Orlando wants to stop for lunch halfway to the waterfall. If the camp is at the origin, where should he stop?

41. MULTIPLE REPRESENTATIONS Triangle XYZ has vertices $X(4, 9)$, $Y(8, -9)$, and $Z(-6, 5)$.

a. Concrete Draw $\triangle XYZ$ on a coordinate plane.

b. Numerical Find the coordinates of the midpoint of each side of the triangle.

c. Geometric Find the perimeter of $\triangle XYZ$ and the perimeter of the triangle with vertices at the points found in part **b**.

d. Analytical How do the perimeters in part **c** compare?

H.O.T. Problems Use Higher-Order Thinking Skills

42. CHALLENGE Find the coordinates of the point that is three fourths of the way from $P(-1, 12)$ to $Q(5, -10)$.

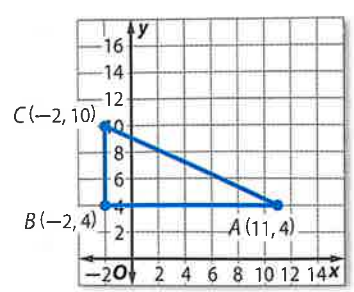
43. REASONING Identify all the points in a plane that are 3 units or less from the point (5, 6). What figure does this make?

44. CCSS ARGUMENTS Triangle ABC is a right triangle.

a. Find the midpoint of the hypotenuse. Call it point Q.

b. Classify $\triangle BQC$ according to the lengths of its sides. Include sufficient evidence to support your conclusion.

c. Classify $\triangle BQA$ according to its angles.



45. OPEN ENDED Plot two points, and find the distance between them. Does it matter which ordered pair is first when using the Distance Formula? Explain.

46. WRITING IN MATH Explain how the Midpoint Formula can be used to approximate the halfway point between two locations on a map.

Standardized Test Practice

47. SHORT RESPONSE You currently earn \$8.10 per hour and your boss gives you a 10% raise. What is your new hourly wage?

48. SAT/ACT A right circular cylinder has a radius of 3 and a height of 5. Which of the following dimensions of a rectangular solid will have a volume closest to that of the cylinder?

A 5, 5, 6
B 5, 6, 6
C 5, 5, 5
D 4, 5, 6
E 3, 5, 9

49. GEOMETRY If the sum of the lengths of the two legs of a right triangle is 49 inches and the hypotenuse is 41 inches, find the longer of the two legs.

F 9 in.
G 40 in.
H 42 in.
J 49 in.

50. Five more than 3 times a number is 17. Find the number.

A 3
B 4
C 5
D 6

Spiral Review

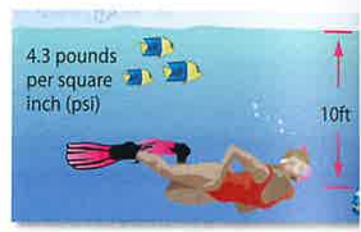
Solve each equation. Check your solutions. (Lesson 8-6)

51. $\frac{12}{v^2 - 16} - \frac{24}{v - 4} = 3$

52. $\frac{w}{w - 1} + w = \frac{4w - 3}{w - 1}$

53. $\frac{4n^2}{n^2 - 9} - \frac{2n}{n + 3} = \frac{3}{n - 3}$

54. SWIMMING When a person swims under water, the pressure in his or her ears varies directly with the depth at which he or she is swimming. (Lesson 8-5)



a. Write a direct variation equation that represents this situation.

b. Find the pressure at 60 feet.

c. It is unsafe for amateur divers to swim where the water pressure is more than 65 pounds per square inch. How deep can an amateur diver safely swim?

d. Make a table showing the number of pounds of pressure at various depths of water. Use the data to draw a graph of pressure versus depth.

Solve each equation or inequality. Round to the nearest ten-thousandth. (Lesson 7-6)

55. $9^{z-4} = 6.28$

56. $8.2^{n-3} = 42.5$

57. $2.1^{t-5} = 9.32$

58. $8^{2n} > 52^{4n+3}$

59. $7^{p+2} \leq 13^{5-p}$

60. $3^{y+2} \geq 8^{3y}$

Solve each equation. (Lesson 6-7)

61. $(6n - 5)^{\frac{1}{3}} + 3 = -2$

62. $(5x + 7)^{\frac{1}{5}} + 3 = 5$

63. $(3x - 2)^{\frac{1}{5}} + 6 = 5$

Skills Review

Write each quadratic equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

64. $y = -x^2 - 4x + 8$

65. $y = x^2 - 6x + 1$

66. $y = -2x^2 + 20x - 35$

LESSON
9-2 Parabolas

Then

- You graphed quadratic functions.

Now

- **1** Write equations of parabolas in standard form.
- **2** Graph parabolas.

Why?

- Satellite dishes can be used to send and receive signals and can be seen attached to residential homes and businesses.

A satellite dish is a type of antenna constructed to receive signals from orbiting satellites. The signals are reflected off of the dish's parabolic surface to a common collection point.

New Vocabulary
parabola
focus
directrix
latus rectum
standard form
general form

Common Core State Standards

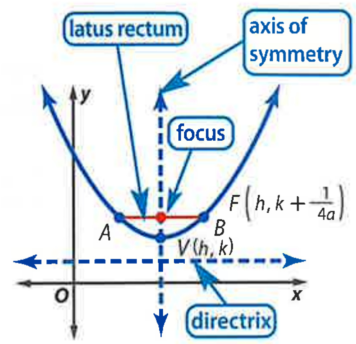
Content Standards
A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Mathematical Practices
1 Make sense of problems and persevere in solving them.

1 Equations of Parabolas

A **parabola** can be defined as the set of all points in a plane that are the same distance from a given point called the **focus** and a given line called the **directrix**.

The line segment through the focus of a parabola and perpendicular to the axis of symmetry is called the **latus rectum**. The endpoints of the latus rectum lie on the parabola.



KeyConcept Equations of Parabolas

Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Vertex	(h, k)	(h, k)
Axis of Symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Length of Latus Rectum	$\left\|\frac{1}{a}\right\|$ units	$\left\|\frac{1}{a}\right\|$ units

The **standard form** of the equation of a parabola with vertex (h, k) and axis of symmetry $x = h$ is $y = a(x - h)^2 + k$.

- If $a > 0$, k is the minimum value of the related function and the parabola opens upward.
- If $a < 0$, k is the maximum value of the related function and the parabola opens downward.

An equation of a parabola in the form $y = ax^2 + bx + c$ is the **general form**. Any equation in general form can be written in standard form. The shape of a parabola and the distance between the focus and directrix depend on the value of a in the equation.

ReviewVocabulary

Completing the Square rewriting a quadratic expression as a perfect square trinomial

Example 1 Analyze the Equation of a Parabola

Write $y = 2x^2 - 12x + 6$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

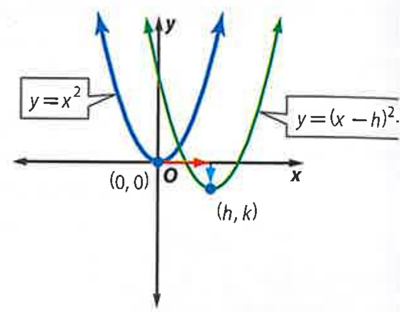
$y = 2x^2 - 12x + 6$	Original equation
$= 2(x^2 - 6x) + 6$	Factor 2 from the x- and x^2-terms.
$= 2(x^2 - 6x + \blacksquare) + 6 - 2(\blacksquare)$	Complete the square on the right side.
$= 2(x^2 - 6x + 9) + 6 - 2(9)$	The 9 added when you complete the square is multiplied by 2.
$= 2(x - 3)^2 - 12$	Factor.

The vertex of this parabola is located at $(3, -12)$, and the equation of the axis of symmetry is $x = 3$. The parabola opens upward.

GuidedPractice

1. Write $y = 4x^2 + 16x + 34$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

2 Graph Parabolas In Chapter 4, you learned that the graph of the quadratic equation $y = a(x - h)^2 + k$ is a transformation of the parent graph of $y = x^2$ translated h units horizontally and k units vertically, and reflected and/or dilated depending on the value of a.



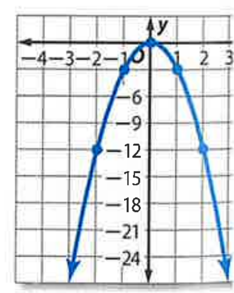
Example 2 Graph Parabolas

Graph each equation.

a. $y = -3x^2$

For this equation, $h = 0$ and $k = 0$. The vertex is at the origin. Since the equation of the axis of symmetry is $x = 0$, substitute some small positive integers for x and find the corresponding y-values.

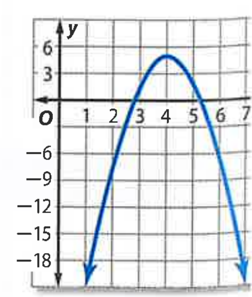
x	y
1	−3
2	−12
3	−27



Since the graph is symmetric about the y-axis, the points at $(-1, -3)$, $(-2, -12)$, and $(-3, -27)$ are also on the parabola. Use all of these points to draw the graph.

b. $y = -3(x - 4)^2 + 5$

The equation is of the form $y = a(x - h)^2 + k$, where $h = 4$ and $k = 5$. The graph of this equation is the graph of $y = -3x^2$ in part **a** translated 4 units to the right and up 5 units. The vertex is now at $(4, 5)$.



GuidedPractice

2A. $y = 2x^2$

2B. $y = 2(x - 1)^2 - 4$

WatchOut!

CCSS Structure Carefully examine the values for h and k before beginning to graph an equation.

- If h is positive, translate the graph h units to the right.
- If h is negative, translate the graph $|h|$ units to the left.
- If k is positive, translate the graph k units up.
- If k is negative, translate the graph $|k|$ units down.

StudyTip

Graphing When graphing these functions, it may be helpful to sketch the graph of the parent function.

Equations of parabolas with vertical axes of symmetry have the parent function $y = x^2$ and are of the form $y = a(x - h)^2 + k$. These are functions. Equations of parabolas with horizontal axes of symmetry are of the form $x = a(y - k)^2 + h$ and are not functions. The parent graph for these equations is $x = y^2$.

Example 3 Graph an Equation in General Form

Graph each equation.

a. $2x - y^2 = 4y + 10$

Step 1 Write the equation in the form $x = a(y - k)^2 + h$.

$2x - y^2 = 4y + 10$	Original equation
$2x = y^2 + 4y + 10$	Add y^2 to each side to isolate the x-term.
$2x = (y^2 + 4y + ■) + 10 - ■$	Complete the square.
$2x = (y^2 + 4y + 4) + 10 - 4$	Add and subtract 4, since $\left(\frac{4}{2}\right)^2 = 4$.
$2x = (y + 2)^2 + 6$	Factor and subtract.
$x = \frac{1}{2}(y + 2)^2 + 3$	$(h, k) = (3, -2)$

Step 2 Use the equation to find information about the graph. Then draw the graph based on the parent graph, $x = y^2$.

vertex: $(3, -2)$

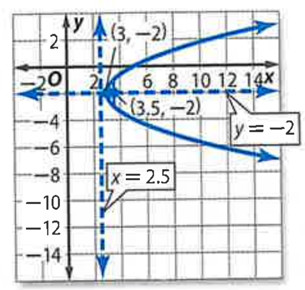
axis of symmetry: $y = -2$

focus: $\left(3 + \frac{1}{4\left(\frac{1}{2}\right)}, -2\right)$ or $(3.5, -2)$

directrix: $x = 3 - \frac{1}{4\left(\frac{1}{2}\right)}$ or 2.5

direction of opening: right, since $a > 0$

length of latus rectum: $\left|\frac{1}{\left(\frac{1}{2}\right)}\right|$ or 2 units



ReadingMath

latus rectum from the Latin *latus*, meaning side, and *rectum*, meaning straight

b. $y + 2x^2 + 32 = -16x - 1$

Step 1 $y + 2x^2 + 32 = -16x - 1$	Original equation
$y = -2x^2 - 16x - 33$	Solve for y.
$y = -2(x^2 + 8x + ■) - 33 - ■$	Complete the square.
$y = -2(x^2 + 8x + 16) - 33 - (-32)$	Add and subtract −32.
$y = -2(x + 4)^2 - 1$	Factor and simplify.

Step 2 vertex: $(-4, -1)$

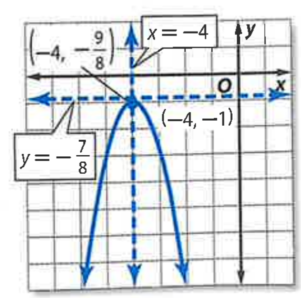
axis of symmetry: $x = -4$

focus: $\left(-4, -\frac{9}{8}\right)$

directrix: $y = -\frac{7}{8}$

length of latus rectum: $\frac{1}{2}$ unit

opens downward



GuidedPractice

3A. $3x - y^2 = 4x + 25$

3B. $y = x^2 + 6x - 4$

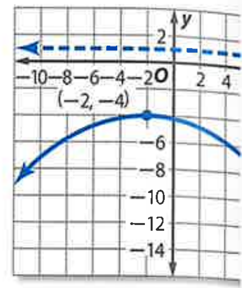
You can use specific information about a parabola to write an equation and draw a gr

Example 4 Write an Equation of a Parabola

Write an equation for a parabola with vertex at (−2, −4) and directrix $y = 1$. Then graph the equation.

The directrix is a horizontal line, so the equation of the parabola is of the form $y = a(x - h)^2 + k$. Find a, h, and k.

- The vertex is at (−2, −4), so $h = -2$ and $k = -4$.
- Use the equation of the directrix to find a.



$y = k - \frac{1}{4a}$	Equation of directrix
$1 = -4 - \frac{1}{4a}$	Replace y with 1 and k with −4.
$5 = -\frac{1}{4a}$	Add 4 to each side.
$20a = -1$	Multiply each side by $4a$.
$a = -\frac{1}{20}$	Divide each side by 20.

So, the equation of the parabola is $y = -\frac{1}{20}(x + 2)^2 - 4$.

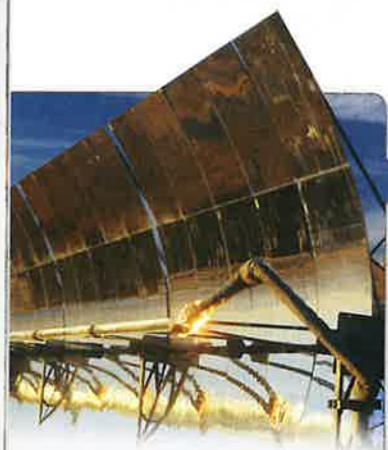
GuidedPractice

Write an equation for each parabola described below. Then graph the equation.

4A. vertex (1, 3), focus (1, 5)

4B. focus (5, 6), directrix $x = -2$

Parabolas are often used in the real world.



Real-WorldLink

In California's Mojave Desert, parabolic mirrors are used to heat oil that flows through tubes placed at the focus. The heated oil is used to produce electricity.

Source: Solel

Real-World Example 5 Write an Equation for a Parabola

ENVIRONMENT **Solar energy may be harnessed by using parabolic mirrors. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of each parabolic mirror at the facility described at the left is 6.25 feet above the vertex. The latus rectum is 25 feet long.**

a. Assume that the focus is at the origin. Write an equation for the parabola form by each mirror.

In order for the mirrors to collect the Sun's energy, the parabola must open upward. Therefore, the vertex must be below the focus.

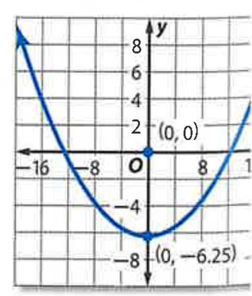
focus: (0, 0)   vertex: (0, −6.25)

The measure of the latus rectum is 25. So $25 = \left|\frac{1}{a}\right|$, and $a = \frac{1}{25}$.

Using the form $y = a(x - h)^2 + k$, an equation for the parabola formed by each mirror is $y = \frac{1}{25}x^2 - 6.25$.

b. Graph the equation.

Now use all of the information to draw a graph.



GuidedPractice

5. Write and graph an equation for a parabolic mirror that has a focus 4.5 feet above the vertex and a latus rectum that is 18 feet long, when the focus is at the origin.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.**

1. $y = 2x^2 - 24x + 40$
2. $y = 3x^2 - 6x - 4$
3. $x = y^2 - 8y - 11$
4. $x + 3y^2 + 12y = 18$

Examples 2–3 **Graph each equation.**

5. $y = (x - 4)^2 - 6$
6. $y = 4(x + 5)^2 + 3$
7. $y = -3x^2 - 4x - 8$
8. $x = 3y^2 - 6y + 9$

Example 4 **Write an equation for each parabola described below. Then graph the equation.**

9. vertex (0, 2), focus (0, 4)
10. vertex (−2, 4), directrix $x = -1$
11. focus (3, 2), directrix $y = 8$
12. vertex (−1, −5), focus (−5, −5)

Example 5

13. **ASTRONOMY** Consider a parabolic mercury mirror like the one described at the beginning of the lesson. The focus is 6 feet above the vertex and the latus rectum is 24 feet long.
    a. Assume that the focus is at the origin. Write an equation for the parabola formed by the parabolic microphone.
    b. Graph the equation.

Practice and Problem Solving

Extra Practice is on page R9.

Example 1 **Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.**

14. $y = x^2 - 8x + 13$
15. $y = 3x^2 + 42x + 149$
16. $y = -6x^2 - 36x - 8$
17. $y = -3x^2 - 9x - 6$
18. $x = \frac{1}{3}y^2 - 3y + 4$
19. $x = \frac{2}{3}y^2 - 4y + 12$

Examples 2–3 **Graph each equation.**

20. $y = \frac{1}{3}x^2$
21. $y = -2x^2$
22. $y = -2(x - 2)^2 + 3$
23. $y = 3(x - 3)^2 - 5$
24. $x = \frac{1}{2}y^2$
25. $4x - y^2 = 2y + 13$

Example 4 **Write an equation for each parabola described below. Then graph the equation.**

26. vertex (0, 1), focus (0, 4)
27. vertex (1, 8), directrix $y = 3$
28. focus (−2, −4), directrix $x = -6$
29. focus (2, 4), directrix $x = 10$
30. vertex (−6, 0), directrix $x = 2$
31. vertex (9, 6), focus (9, 5)

Example 5

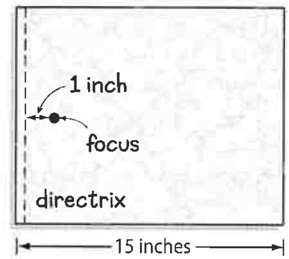
32. **BASEBALL** When a ball is thrown, the path it travels is a parabola. Suppose a baseball is thrown from ground level, reaches a maximum height of 50 feet, and hits the ground 200 feet from where it was thrown. Assuming this situation could be modeled on a coordinate plane with the focus of the parabola at the origin, find the equation of the parabolic path of the ball. Assume the focus is on ground level.

33. CCSS **PERSEVERANCE** Ground antennas and satellites are used to relay signals between the NASA Mission Operations Center and the spacecraft it controls. One such parabolic dish is 146 feet in diameter. Its focus is 48 feet from the vertex.
    a. Sketch two options for the dish, one that opens up and one that opens left.
    b. Write two equations that model the sketches in part **a**.
    c. If you wanted to find the depth of the dish, does it matter which equation you use? Why or why not?

34. **UMBRELLAS** A beach umbrella has an arch in the shape of a parabola that opens downward. The umbrella spans 6 feet across and is $1\frac{1}{2}$ feet high. Write an equation of a parabola to model the arch, assuming that the origin is at the point where the pole and umbrella meet at the vertex of the arch.

35. **AUTOMOBILES** An automobile headlight contains a parabolic reflector. The light coming from the source bounces off the parabolic reflector and shines out the front of the headlight. The equation of the cross section of the reflector is $y = \frac{1}{12}x^2$. How far from the vertex should the filament for the high beams be placed?

36. **MULTIPLE REPRESENTATIONS** Start with a sheet of wax paper that is about 15 inches long and 12 inches wide.



a. **Concrete** Make a line that is perpendicular to the sides of the sheet by folding the sheet near one end. Open up the paper again. This line is the directrix. Mark a point about midway between the sides of the sheet so that the distance from the directrix is about 1 inch. This is the focus.

b. **Concrete** Start with a new sheet of wax paper. Form another outline of a parabola with a focus that is about 3 inches from the directrix.

c. **Concrete** On a new sheet of a wax paper, form a third outline of a parabola with a focus that is about 5 inches from the directrix.

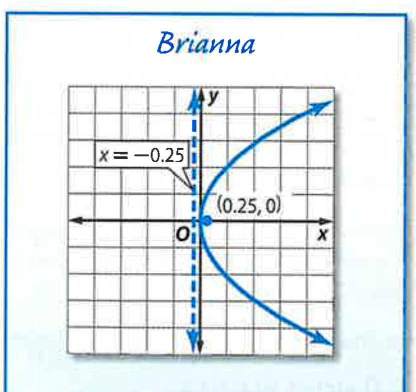
d. **Verbal** Compare the shapes of the three parabolas. How does the distance between the focus and the directrix affect the shape of a parabola?

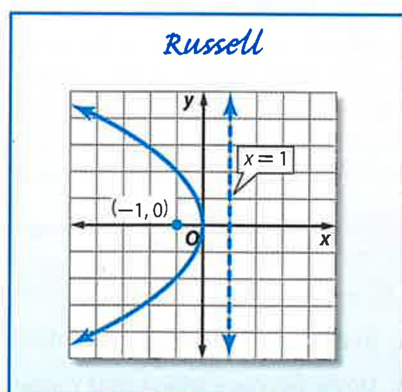
H.O.T. Problems Use Higher-Order Thinking Skills

37. **REASONING** How do you change the equation of the parent function $y = x^2$ to shift the graph to the right?

38. **OPEN ENDED** Two different parabolas have their vertex at $(-3, 1)$ and contain the point with coordinates $(-1, 0)$. Write two possible equations for these parabolas.

39. **CRITIQUE** Brianna and Russell are graphing $\frac{1}{4}y^2 + x = 0$. Is either of them correct? Explain your reasoning.





40. **WRITING IN MATH** Why are parabolic shapes used in the real world?

Standardized Test Practice

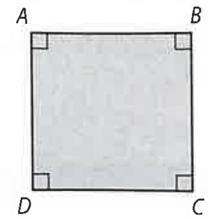
41. A gardener is placing a fence around a 1320-square-foot rectangular garden. He ordered 148 feet of fencing. If he uses all the fencing, what is the length of the longer side of the garden?

A 30 ft
B 34 ft
C 44 ft
D 46 ft

42. SAT/ACT When a number is divided by 5, the result is 7 more than the number. Find the number.

F $-\frac{35}{4}$
G $-\frac{35}{6}$
H $\frac{35}{6}$
J $\frac{28}{4}$
K $\frac{35}{4}$

43. GEOMETRY What is the area of the following square, if the length of $\overline{BD}$ is $2\sqrt{2}$?



A 1
B 2
C 3
D 4

44. SHORT RESPONSE The measure of the smallest angle of a triangle is two thirds the measure of the middle angle. The measure of the middle angle is three sevenths of the measure of the largest angle. Find the largest angle's measure.

Spiral Review

45. GEOMETRY Find the perimeter of a triangle with vertices at (2, 4), (−1, 3), and (1, −3). (Lesson 9-1)

46. WORK A worker can powerwash a wall of a certain size in 5 hours. Another worker can do the same job in 4 hours. If the workers work together, how long would it take to do the job? Determine whether your answer is reasonable. (Lesson 8-6)

Solve each equation or inequality. Round to the nearest ten-thousandth. (Lesson 7-7)

47. $\ln(x + 1) = 1$

48. $\ln(x - 7) = 2$

49. $e^x > 1.6$

50. $e^{5x} \geq 25$

Simplify. (Lesson 6-4)

51. $\sqrt{0.25}$

52. $\sqrt[3]{-0.064}$

53. $\sqrt[4]{z^8}$

54. $-\sqrt[6]{x^6}$

List all of the possible rational zeros of each function. (Lesson 5-8)

55. $h(x) = x^3 + 8x + 6$

56. $p(x) = 3x^3 - 5x^2 - 11x + 3$

57. $h(x) = 9x^6 - 5x^3 + 27$

Skills Review

Simplify each expression.

58. $\sqrt{24}$

59. $\sqrt{45}$

60. $\sqrt{252}$

61. $\sqrt{512}$

EXPLORE 9-3

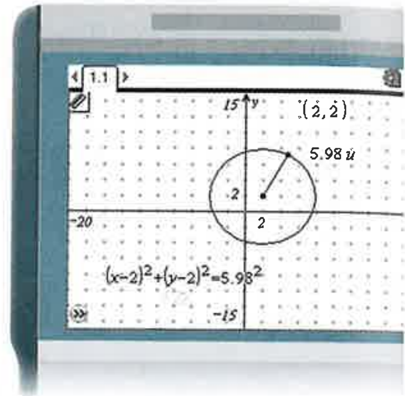
Graphing Technology Lab
Equations of Circles

You can use TI-Nspire technology to examine characteristics of circles and the relationship with an equation of the circle.

Activity

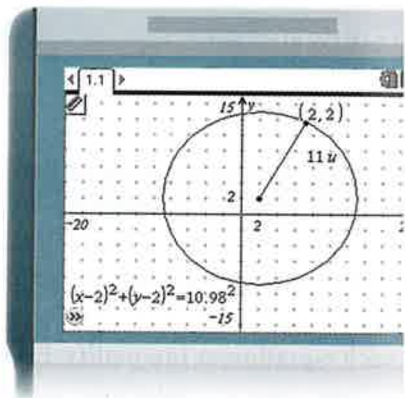
Step 1 Draw a circle.

- Add a new **Graphs** page. Select **Window/Zoom** menu and use the **Windows Setting** tool to adjust the window size as shown. From the **View** menu, select **Show Grid**. Then from the **Shapes** menu, select **Circle**. Place the pointer at the point (2, 2) and press **enter** to set the center of the circle. Move the pointer out, creating a circle like the one shown.
- Use the **Point On** tool from the **Points & Line** menu to place a point on the circle.
- Use the **Segment** tool from the **Points & Line** menu to draw the radius.



Step 2 Add labels.

- From the **Actions** menu, select **Coordinates and Equations.** Use the pointer to select the center of the circle to display its coordinate. Then select the circle to display its equation. Move each display outside the circle.
- Use the **Length** tool from the **Measurement** menu to display the length of the radius.



Step 3 Change the radius.

Move the pointer so that a point on the circle is highlighted, then press and hold the center of the touchpad until it is selected. Examine the equation of the circle. Then move the edge of the circle in. Make note of changes in the equation.

Step 4 Move the center of the circle.

Move the pointer so that the center of the circle is highlighted, then press and hold the center of the touchpad until it is selected. Move the center of the circle. Again, examine the equation of the circle.

Analyze the Results

1. How does moving the edge of the circle in or out affect the equation of the circle?
2. What effect does moving the center of the circle have on the equation?
3. Repeat the activity by placing the center of a circle in Quadrant II. Move the center to each of the other two quadrants. How does the equation change?
4. **MAKE A CONJECTURE** Without graphing, write an equation of each circle.
   - **a.** center: (4, 2), radius: 3
   - **b.** center: (−1, 1), radius: 8
   - **c.** center: (−6, −5), radius: 2.5
   - **d.** center: (h, k), radius: r

LESSON 9-3 Circles

:: Then	:: Now	:: Why?
● You graphed and wrote equations of parabolas.	● **1** Write equations of circles. **2** Graph circles.	● When an object is thrown into water, ripples move out from the center forming concentric circles. If the point where the object entered the water is assigned coordinates, each ripple can be modeled by an equation of a circle.

NewVocabulary

circle
center
radius

Common Core State Standards

Content Standards

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

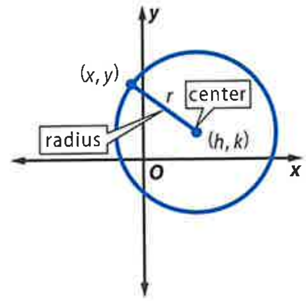
Mathematical Practices

4 Model with mathematics.

1 Equations of Circles

A **circle** is the set of all points in a plane that are equidistant from a given point in the plane, called the **center**. Any segment with endpoints at the center and a point on the circle is a **radius** of the circle.

Assume that (x, y) are the coordinates of a point on the circle at the right. The center is at (h, k), and the radius is r. You can find an equation of the circle by using the Distance Formula.



$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$ — Distance Formula

$\sqrt{(x - h)^2 + (y - k)^2} = r$ — $(x_1, y_1) = (h, k)$, $(x_2, y_2) = (x, y)$, $d = r$

$(x - h)^2 + (y - k)^2 = r^2$ — Square each side.

KeyConcept Equations of Circles

Standard Form of Equation	$x^2 + y^2 = r^2$	$(x - h)^2 + (y - k)^2 = r^2$
Center	$(0, 0)$	(h, k)
Radius	r	r

You can use the standard form of the equation of a circle to write an equation for a circle given the center and the radius or diameter.

Real-World Example 1 Write an Equation Given the Radius

DELIVERY Appliances + More offers free delivery within 35 miles of the store. The Jacksonville store is located 100 miles north and 45 miles east of the corporate office. Write an equation to represent the delivery boundary of the Jacksonville store if the origin of the coordinate system is the corporate office.

Since the corporate office is at (0, 0), the Jacksonville store is at (45, 100). The boundary of the delivery region is the circle centered at (45, 100) with radius 35 miles.

$(x - h)^2 + (y - k)^2 = r^2$ — Equation of a circle

$(x - 45)^2 + (y - 100)^2 = 35^2$ — $(h, k) = (45, 100)$ and $r = 35$

$(x - 45)^2 + (y - 100)^2 = 1225$ — Simplify.

GuidedPractice

1. **WI-FI** A certain wi-fi phone has a range of 30 miles in any direction. If the phone is 4 miles south and 3 miles west of headquarters, write an equation to represent the area within which the phone can operate via the Wi-Fi system.

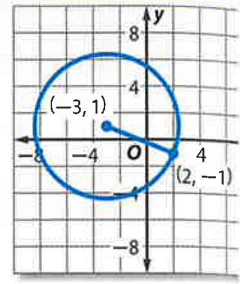
You can write the equation of a circle when you know the location of the center and
point on the circle.

StudyTip

Center-Radius Form Standard form is sometimes referred to as *center-radius form* because the center and radius of the circle are apparent in the equation.

Example 2 Write an Equation from a Graph

Write an equation for the graph.

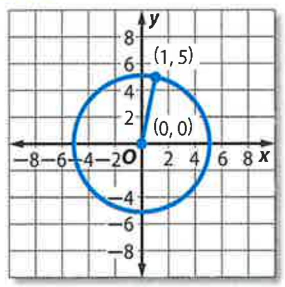


$(x-h)^2+(y-k)^2=r^2$	Standard form
$(2+3)^2+(-1-1)^2=r^2$	$x=2$, $y=-1$, $h=-3$, $k=1$
$(5)^2+(-2)^2=r^2$	Simplify.
$25+4=r^2$	Evaluate the exponents.
$29=r^2$	Add.

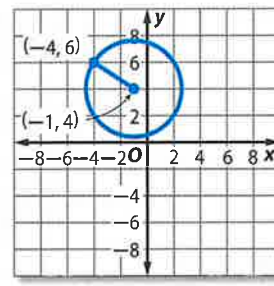
So, the equation of the circle is $(x+3)^2+(y-1)^2=29$.

GuidedPractice

2A.



2B.



You can use the Midpoint and Distance Formulas when you know the endpoints of
radius or diameter of a circle.

Example 3 Write an Equation Given a Diameter

Write an equation for a circle if the endpoints of a diameter are at (7, 6) and (−1,

Step 1 Find the center.

$(h,k)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$	Midpoint Formula
$=\left(\frac{7+(-1)}{2},\frac{6+(-8)}{2}\right)$	$(x_1, y_1)=(7, 6)$, $(x_2, y_2)=(-1, -8)$
$=\left(\frac{6}{2},\frac{-2}{2}\right)$	Add.
$=(3,-1)$	Simplify.

Step 2 Find the radius.

$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance Formula
$=\sqrt{(3-7)^2+(-1-6)^2}$	$(x_1, y_1)=(7, 6)$, $(x_2, y_2)=(3, -1)$
$=\sqrt{(-4)^2+(-7)^2}$	Subtract.
$=\sqrt{65}$	Simplify.

The radius of the circle is $\sqrt{65}$ units, so $r^2=65$. Substitute h, k, and r^2 into the stand
form of the equation of a circle. An equation of the circle is $(x-3)^2+(y+1)^2=65$.

GuidedPractice

3. Write an equation for a circle if the endpoints of a diameter are at (3, −3) and (

StudyTip

Axis of Symmetry Every diameter in a circle is an axis of symmetry. There are infinitely many axes of symmetry in a circle.

2 Graph Circles

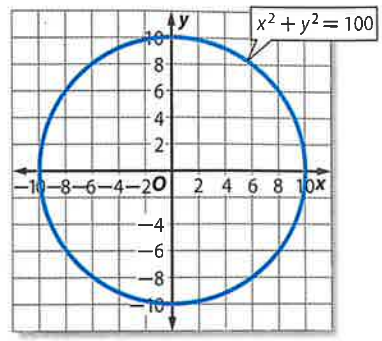
You can use symmetry to help you graph circles.

Example 4 Graph an Equation in Standard Form

Find the center and radius of the circle with equation $x^2 + y^2 = 100$. Then graph the circle.

- The center of the circle is at (0, 0), and the radius is 10.
- The table lists some integer values for x and y that satisfy the equation.
- Because the circle is centered at the origin, it is symmetric about the y-axis. Therefore, the points at (−6, 8), (−8, 6), and (−10, 0) lie on the graph.
- The circle is also symmetric about the x-axis, so the points (−6, −8), (−8, −6), (0, −10), (6, −8), and (8, −6) lie on the graph.
- Plot all of these points and draw the circle that passes through them.

x	y
0	10
6	8
8	6
10	0



GuidedPractice

4. Find the center and radius of the circle with equation $x^2 + y^2 = 81$. Then graph the circle.

Circles with centers that are not (0, 0) can be graphed by using translations. The graph of $(x - h)^2 + (y - k)^2 = r^2$ is the graph of $x^2 + y^2 = r^2$ translated h units horizontally and k units vertically.

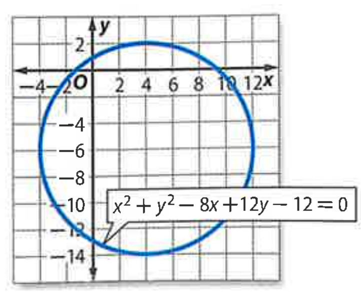
Example 5 Graph an Equation Not in Standard Form

Find the center and radius of the circle with equation $x^2 + y^2 - 8x + 12y - 12 = 0$. Then graph the circle.

Complete the squares.

$$x^2 + y^2 - 8x + 12y - 12 = 0$$
$$x^2 - 8x + \blacksquare + y^2 + 12y + \blacksquare = 12 + \blacksquare + \blacksquare$$
$$x^2 - 8x + 16 + y^2 + 12y + 36 = 12 + 16 + 36$$
$$(x - 4)^2 + (y + 6)^2 = 64$$

The center of the circle is at (4, −6), and the radius is 8. The graph of $(x - 4)^2 + (y + 6)^2 = 64$ is the same as $x^2 + y^2 = 64$ translated 4 units to the right and down 6 units.



GuidedPractice

5. Find the center and radius of the circle with equation $x^2 + y^2 + 4x - 10y - 7 = 0$. Then graph the circle.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

1. **WEATHER** On average, the eye of a tornado is about 200 feet across. Suppose the center of the eye is at the point (72, 39). Write an equation to represent the boundary of the eye.

Write an equation for each circle given the center and radius.

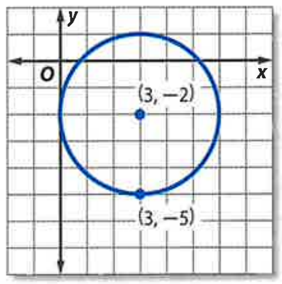
2. center: $(-2, -6)$, $r = 4$ units

3. center: $(1, -5)$, $r = 3$ units

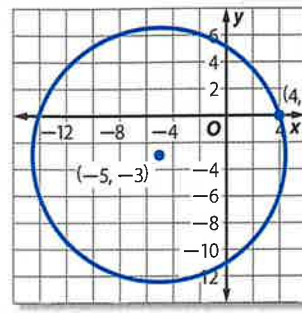
Example 2

Write an equation for each graph.

4.



5.



Example 3

Write an equation for each circle given the endpoints of a diameter.

6. $(-1, -7)$ and $(0, 0)$

7. $(4, -2)$ and $(-4, -6)$

Examples 4–5 **Find the center and radius of each circle. Then graph the circle.**

8. $x^2 + y^2 = 16$

9. $x^2 + (y - 7)^2 = 9$

10. $(x - 4)^2 + (y - 4)^2 = 25$

11. $x^2 + y^2 - 4x + 8y - 5 = 0$

Practice and Problem Solving

Extra Practice is on pag

Example 1

Write an equation for each circle given the center and radius.

12. center: $(4, 9)$, $r = 6$

13. center: $(-3, 1)$, $r = 4$

14. center: $(-7, -3)$, $r = 13$

15. center: $(-2, -1)$, $r = 9$

16. center: $(1, 0)$, $r = \sqrt{15}$

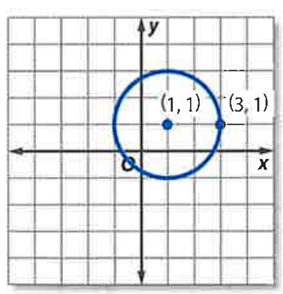
17. center: $(0, -6)$, $r = \sqrt{35}$

18. **CCSS MODELING** The radar for a county airport control tower is located at (5, 10) on a map. It can detect a plane up to 20 miles away. Write an equation for the outer limits of the detection area.

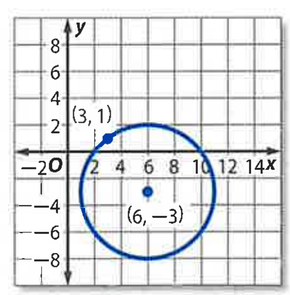
Example 2

Write an equation for each graph.

19.



20.



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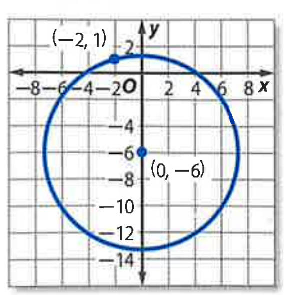

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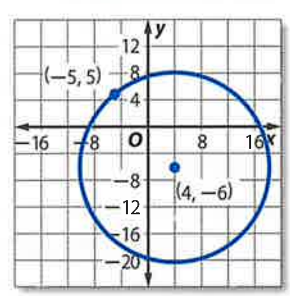


Example 3 **Write an equation for each circle given the endpoints of a diameter.**

23. (2, 1) and (2, −4) **24.** (−4, −10) and (4, −10) **25.** (5, −7) and (−2, −9)

26. (−6, 4) and (4, 8) **27.** (2, −5) and (6, 3) **28.** (18, 11) and (−19, −13)

29. **LAWN CARE** A sprinkler waters a circular section of lawn.

a. Write an equation to represent the boundary of the sprinkler area if the endpoints of a diameter are at (−12, 16) and (12, −16).

b. What is the area of the lawn that the sprinkler waters?

30. **SPACE** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon's surface. Write an equation to model a single circular orbit of the command module if the endpoints of a diameter of the Moon are at (1740, 0) and (−1740, 0). Let the center of the Moon be at the origin of the coordinate system measured in kilometers.

Examples 4–5 **Find the center and radius of each circle. Then graph the circle.**

31. $x^2 + y^2 = 75$

32. $(x - 3)^2 + y^2 = 4$

33. $(x - 1)^2 + (y - 4)^2 = 34$

34. $x^2 + (y - 14)^2 = 144$

35. $(x - 5)^2 + (y + 2)^2 = 16$

36. $x^2 + y^2 = 256$

37. $(x - 4)^2 + y^2 = \frac{8}{9}$

38. $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{16}{25}$

39. $x^2 + y^2 + 4x = 9$

40. $x^2 + y^2 - 6y + 8x = 0$

41. $x^2 + y^2 + 2x + 4y = 9$

42. $x^2 + y^2 - 3x + 8y = 20$

43. $x^2 + y^2 + 6y = -50 - 14x$

44. $x^2 - 18x + 53 = 18y - y^2$

45. $2x^2 + 2y^2 - 4x + 8y = 32$

46. $3x^2 + 3y^2 - 6y + 12x = 24$

e R9.

47. **SPACE** A satellite is in a circular orbit 25,000 miles above Earth.

a. Write an equation for the orbit of this satellite if the origin is at the center of Earth. Use 8000 miles as the diameter of Earth.

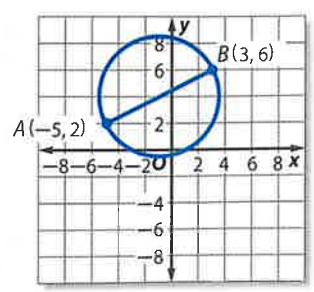
b. Draw a sketch of Earth and the orbit to scale. Label your sketch.

48. **CCSS SENSE-MAKING** Suppose an unobstructed radio station broadcast could travel 120 miles. Assume the station is centered at the origin.

a. Write an equation to represent the boundary of the broadcast area with the origin as the center.

b. If the transmission tower is relocated 40 miles east and 10 miles south of the current location, and an increased signal will transmit signals an additional 80 miles, what is an equation to represent the new broadcast area?

49. **GEOMETRY** Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.



a. Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.

b. Write an equation of the circle concentric with the circle at the right, with radius 2 units less.

c. Graph the circles from parts a and b on the same coordinate plane.

50. **EARTHQUAKES** The Rose Bowl is located about 35 miles west and 40 miles north of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 55 miles from the stadium. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.

CCSS PRECISION **Write an equation for the circle that satisfies each set of conditions.**

51. center $(9, -8)$, passes through $(19, 22)$

52. center $(-\sqrt{15}, 30)$, passes through the origin

53. center at $(8, -9)$, tangent to y-axis

54. center at $(2, 4)$, tangent to x-axis

55. center in the first quadrant; tangent to $x = 5$, the x-axis, and the y-axis

56. center in the second quadrant; tangent to $y = 1$, $y = 5$, and the y-axis

57. **MULTIPLE REPRESENTATIONS** Graph $y = \sqrt{9 - x^2}$ and $y = -\sqrt{9 - x^2}$ on the same graphing calculator screen.

  a. **Verbal** Describe the graph formed by the union of these two graphs.

  b. **Algebraic** Write an equation for the union of the two graphs.

  c. **Verbal** Most graphing calculators cannot graph the equation $x^2 + y^2 = 49$ directly. Describe a way to use a graphing calculator to graph the equation. Then graph the equation.

  d. **Analytical** Solve $(x - 2)^2 + (y + 1)^2 = 4$ for y. Why do you need two equations to graph a circle on a graphing calculator?

  e. **Verbal** Do you think that it is easier to graph the equation in part **d** using graph paper and a pencil or using a graphing calculator? Explain.

Find the center and radius of each circle. Then graph the circle.

58. $x^2 - 12x + 84 = -y^2 + 16y$

59. $4x^2 + 4y^2 + 36y + 5 = 0$

60. $(x + \sqrt{5})^2 + y^2 - 8y = 9$

61. $x^2 + 2\sqrt{7}x + 7 + (y - \sqrt{11})^2 = 11$

H.O.T. Problems Use Higher-Order Thinking Skills

62. **ERROR ANALYSIS** Heather says that $(x - 2)^2 + (y + 3)^2 = 36$ and $(x - 2) + (y + 3) = 6$ are equivalent equations. Carlota says that the equations are *not* equivalent. Is either of them correct? Explain your reasoning.

63. **OPEN ENDED** Consider graphs with equations of the form $(x - 3)^2 + (y - a)^2 = 64$. Assign three different values for a, and graph each equation. Describe all graphs with equations of this form.

64. **REASONING** Explain why the phrase "in a plane" is included in the definition of a circle. What would be defined if the phrase were *not* included?

65. **OPEN ENDED** Concentric circles have the same center, but most often, not the same radius. Write equations of two concentric circles. Then graph the circles.

66. **REASONING** Assume that (x, y) are the coordinates of a point on a circle. The center is at (h, k), and the radius is r. Find an equation of the circle by using the Distance Formula.

67. **WRITING IN MATH** The circle with equation $(x - a)^2 + (y - b)^2 = r^2$ lies in the first quadrant and is tangent to both the x-axis and the y-axis. Sketch the circle. Describe the possible values of a, b, and r. Do the same for a circle in Quadrants II, III, and IV. Discuss the similarities among the circles.

Standardized Test Practice

68. GRIDDED RESPONSE Two circles, both with a radius of 6, have exactly one point in common. If A is a point on one circle and B is a point on the other circle, what is the maximum possible length for the line segment $\overline{AB}$?

69. In the senior class, there are 20% more girls than boys. If there are 180 girls, how many more girls than boys are there among the seniors?

A 30

B 36

C 90

D 144

70. A \$1000 deposit is made at a bank that pays 2% compounded weekly. How much will you have in your account at the end of 10 years?

F \$1200.00

G \$1218.99

H \$1221.36

J \$1224.54

71. The mean of six numbers is 20. If one of the numbers is removed, the average of the remaining numbers is 15. What is the number that was removed?

A 42

B 43

C 45

D 48

Spiral Review

Graph each equation. (Lesson 9-2)

72. $y = -\frac{1}{2}(x - 1)^2 + 4$

73. $4(x - 2) = (y + 3)^2$

74. $(y - 8)^2 = -4(x - 4)$

Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points. (Lesson 9-1)

75. $\left(-3, -\frac{2}{11}\right), \left(5, \frac{9}{11}\right)$

76. $\left(2\sqrt{3}, -5\right), \left(-3\sqrt{3}, 9\right)$

77. $(2.5, 4), (-2.5, 2)$

78. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$. (Lesson 8-5)

79. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$. (Lesson 8-5)

80. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$. (Lesson 8-5)

Evaluate each expression. (Lesson 7-3)

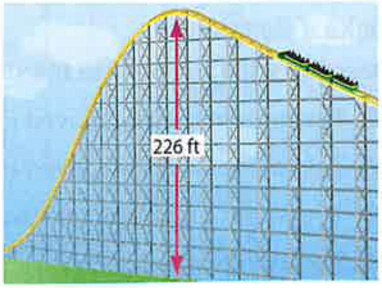
81. $\log_9 243$

82. $\log_2 \frac{1}{32}$

83. $\log_3 \frac{1}{81}$

84. $\log_{10} 0.001$

85. AMUSEMENT PARKS The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$. (Lesson 6-5)



a. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the given formula.

b. What velocity must the coaster have at the top of the hill to achieve a velocity of 125 feet per second at the bottom?

Skills Review

Solve each equation by completing the square.

86. $x^2 + 3x - 18 = 0$

87. $2x^2 - 3x - 3 = 0$

88. $x^2 + 2x + 6 = 0$

EXPLORE

9-4 Algebra Lab
Investigating Ellipses

Follow the steps below to construct a type of conic section.

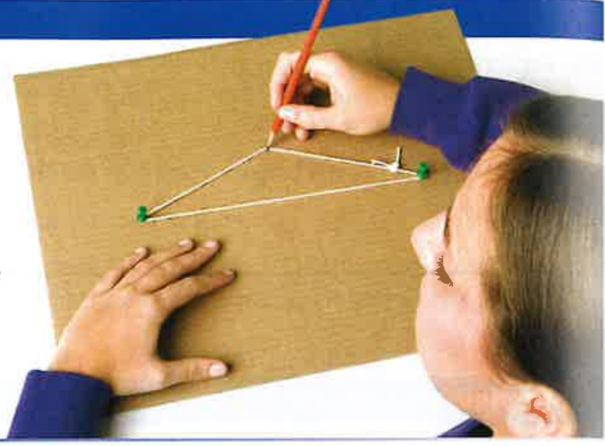
CCSS Common Core State Standards
Mathematical Practices
5 Use appropriate tools strategically.

Activity Make an Ellipse

Step 1 Place two thumbtacks in a piece of cardboard, about 1 foot apart.

Step 2 Tie a knot in a piece of string and loop it around the thumbtacks. Place your pencil in the string.

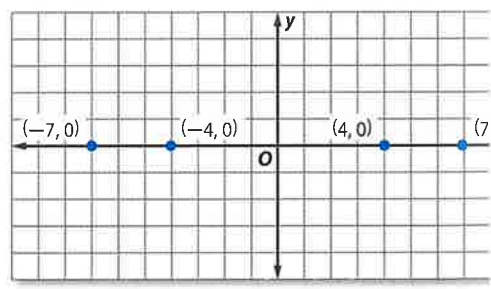
Step 3 Keep the string tight and draw a curve. Continue drawing until you return to your starting point.



The curve you have drawn is called an **ellipse**. The points where the thumbtacks are located are called the **foci** of the ellipse. *Foci* is the plural of *focus*.

Model and Analyze

Place a large piece of grid paper on a piece of cardboard.



1. Place the thumbtacks at (7, 0) and (−7, 0). Choose a string long enough to loop around both thumbtacks. Draw an ellipse.
2. Repeat Exercise 1, but place the thumbtacks at (4, 0) and (−4, 0). Use the same loop of string and draw an ellipse. How does this ellipse compare to the one in Exercise 1?

Place the thumbtacks at each set of points and draw an ellipse. You may change the length of the loop of string if you like.

3. (11, 0), (−11, 0)
4. (3, 0), (−3, 0)
5. (13, 3), (−9, 3)

Make a Conjecture

Describe what happens to the shape of an ellipse when each change is made.

6. The thumbtacks are moved closer together.
7. The thumbtacks are moved farther apart.
8. The length of the loop of string is increased.
9. The thumbtacks are arranged vertically.
10. One thumbtack is removed, and the string is looped around the remaining thumbtack.
11. Pick a point on one of the ellipses you have drawn. Use a ruler to measure the distances from that point to the points where the thumbtacks were located. Add the distances. Repeat for other points on the same ellipse. What relationship do you notice?
12. Could this activity be done with a rubber band instead of a piece of string? Explain.

LESSON 9-4 Ellipses

Then

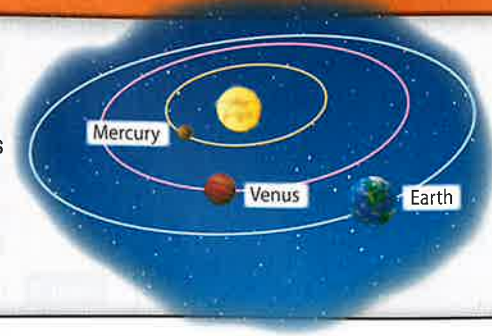
- You graphed and wrote equations for circles.

Now

1. Write equations of ellipses.
2. Graph ellipses.

Why?

- Mercury, like all of the planets of our solar system, does not orbit the Sun in a perfect circular path. At its farthest point, Mercury is about 43 million miles from the Sun. At its closest point, it is only about 28.5 million miles from the Sun. This orbit is in the shape of an ellipse with the Sun at a focus.



NewVocabulary

ellipse
foci
major axis
minor axis
center
vertices
co-vertices
constant sum

Common Core State Standards

Content Standards

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Mathematical Practices

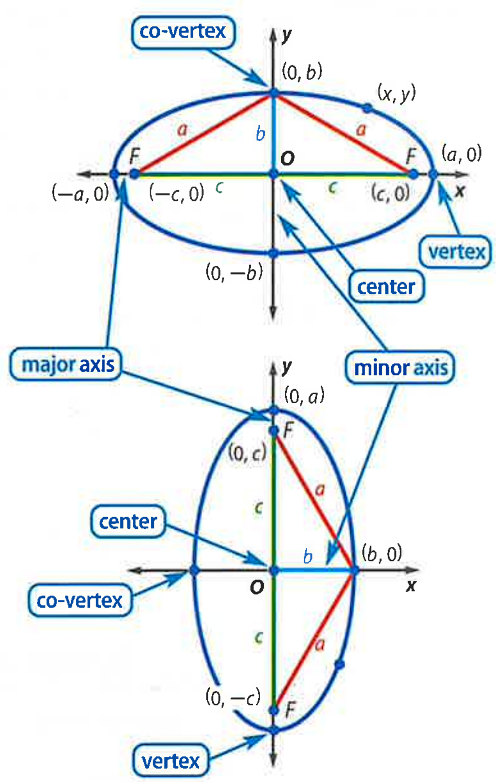
7 Look for and make use of structure.

1 Equations of Ellipses

An **ellipse** is the set of all points in a plane such that the sum of the distances from two fixed points is constant. These two points are called the **foci** of the ellipse.

Every ellipse has two axes of symmetry, the **major axis** and the **minor axis**. The axes are perpendicular at the **center** of the ellipse.

The foci of an ellipse always lie on the major axis. The endpoints of the major axis are the **vertices** of the ellipse and the endpoints of the minor axis are the **co-vertices** of the ellipse.



KeyConcept Equations of Ellipses Centered at the Origin

Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

There are several important relationships among the many parts of an ellipse.

- The length of the major axis, $2a$ units, equals the sum of the distances from the foci to any point on the ellipse.
- The values of a, b, and c are related by the equation $c^2 = a^2 - b^2$.
- The distance from a focus to either co-vertex is a units.

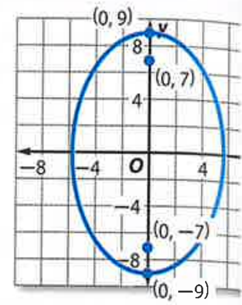
The sum of the distances from the foci to any point on the ellipse, or the constant s
must be greater than the distance between the foci.

StudyTip

Major Axis In standard form, if the x^2-term has the greater denominator, then the major axis is horizontal. If the y^2-term has the greater denominator, then it is vertical.

Example 1 Write an Equation Given Vertices and Foci

Write an equation for the ellipse.



Step 1 Find the center.
The foci are equidistant from the center.
The center is at (0, 0).

Step 2 Find the value of a.
The vertices are (0, 9) and (0, −9), so the length of the major axis is 18.
The value of a is 18 ÷ 2 or 9, and $a^2 = 81$.

Step 3 Find the value of b.
We can use $c^2 = a^2 - b^2$ to find b.
The foci are 7 units from the center, so $c = 7$.

$c^2 = a^2 - b^2$ Equation relating a, b, and c

$49 = 81 - b^2$ $a = 9$ and $c = 7$

$b^2 = 32$ Solve for b^2.

Step 4 Write the equation.
Because the major axis is vertical, a^2 goes with y and b^2 goes with x.
The equation for the ellipse is $\frac{y^2}{81} + \frac{x^2}{32} = 1$.

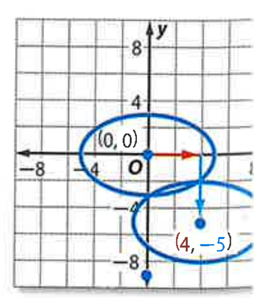
GuidedPractice

1. Write an equation for an ellipse with vertices at (−4, 0) and (4, 0) and foci at (2, 0) and (−2, 0).

Like other graphs, the graph of an ellipse can be translated. When the graph is tran
h units right and k units up, the center of the translation is (h, k). This is equivalent t
replacing x with $x - h$ and replacing y with $y - k$ in the parent function.

KeyConcept Equations of Ellipses Centered at (h, k)

Standard Form	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} =$
Orientation	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-vertices	$(h, k \pm b)$	$(h \pm b, k)$



We can use this information to determine the equations for ellipses. The original ellipse at the right is horizontal and has a major axis of 10 units, so $a = 5$.

The length of the minor axis is 6 units, so $b = 3$.

The ellipse is translated 4 units right and 5 units down. So, the value of h is 4 and the value of k is −5.

The equation for the original ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

The equation for the translation is $\frac{(x-4)^2}{25} + \frac{(y+5)^2}{9} = 1$.

You can also determine the equation for an ellipse if you are given all four vertices.

Example 2 Write an Equation Given the Lengths of the Axes

Write an equation for the ellipse with vertices at (6, −8) and (6, 4) and co-vertices at (3, −2) and (9, −2).

The x-coordinate is the same for both vertices, so the ellipse is vertical.

The center of the ellipse is at $\left(\frac{6+6}{2}, \frac{-8+4}{2}\right)$ or (6, −2).

The length of the major axis is $4 - (-8)$ or 12 units, so $a = 6$.

The length of the minor axis is $9 - 3$ or 6 units, so $b = 3$.

The equation for the ellipse is $\frac{(y+2)^2}{36} + \frac{(x-6)^2}{9} = 1$. $a^2 = 36, b^2 = 9$

GuidedPractice

2. Write an equation for the ellipse with vertices at (−3, 8) and (9, 8) and co-vertices at (3, 12) and (3, 4).

NASA-MSFC

Real-WorldCareer

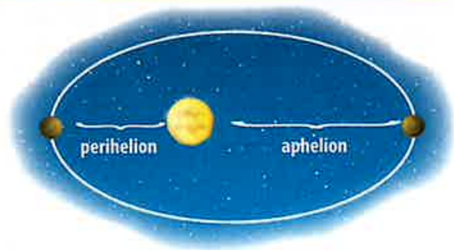
Aerospace Technician
Aerospace technicians work for NASA, helping engineers research and develop virtual reality and verbal communication between humans and computer systems. Although a bachelor's degree is desired, on-the-job training is available.

Source: NASA

Many real-world phenomena can be represented by ellipses.

Real-World Example 3 Write an Equation for an Ellipse

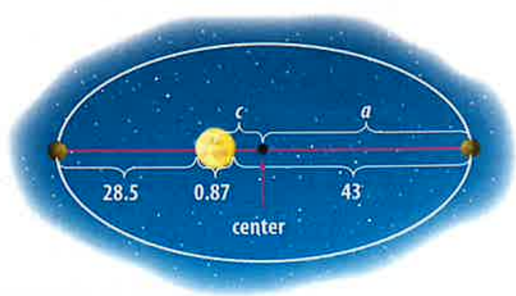
SPACE Refer to the application at the beginning of the lesson. Mercury's greatest distance from the Sun, or *aphelion*, is about 43 million miles. Mercury's closest distance, or *perihelion*, is about 28.5 million miles. The diameter of the Sun is about 870,000 miles. Use this information to determine an equation relating Mercury's elliptical orbit around the Sun in millions of miles.



Understand We need to determine an equation representing Mercury's orbit around the Sun.

Plan Including the diameter of the Sun, the sum of the perihelion and aphelion equals the length on the major axis of the ellipse. We can use this information to determine the values of a, b, and c.

Solve Find the value of a.
The value of a is one half the length of the major axis.
$a = 0.5(43 + 28.5 + 0.87)$ or 36.185



Find the value of c.
The value of c is the distance from the center of the ellipse to the focus.
This distance is equal to a minus the perihelion and the radius of the Sun.
$c = 36.185 - 28.5 - 0.435$ or 7.25

Problem-SolvingTip

CCSS Sense-Making Draw a diagram when the problem situation involves spatial reasoning or geometric figures.

(continued on the next page)

Find the value of b.

$$c^2 = a^2 - b^2$$ Equation relating a, b, and c

$$(7.25)^2 = (36.185)^2 - b^2$$ $c = 7.25$ and $a = 36.185$

$$52.5625 = 1309.3542 - b^2$$ Simplify.

$$b^2 = 1256.828$$ Solve for b^2.

$$b = 35.4518$$ Take the square root of each side.

So, with the center of the orbit at the origin, the equation relating Mercury's orbit around the Sun can be modeled by $\frac{x^2}{1309.3542} + \frac{y^2}{1256.792} = 1$.

Check Use your answer to recalculate a, b, and c. Then determine the aphelion and perihelion based on your answer. Compare to the actual values.

Real-WorldLink

Earth's orbit around the Sun is nearly circular, with only about a 3% difference between perihelion and aphelion.

Source: *The Astronomer*

GuidedPractice

3. SPACE Pluto's distance from the Sun is 2.757 billion miles at perihelion and about 4.583 billion miles at aphelion. Determine an equation relating Pluto's orbit around the Sun in billions of miles with the center of the horizontal ellipse at the origin.

2 Graph Ellipses

When you are given an equation for an ellipse that is not in standard form, you can write it in standard form by completing the square for x and y. Once the equation is in standard form, you can use it to graph the ellipse.

Example 4 Graph an Ellipse

Find the coordinates of the center and foci, and the lengths of the major and m axes of an ellipse with equation $25x^2 + 9y^2 + 250x - 36y + 436 = 0$. Then grap the ellipse.

Step 1 Write in standard form. Complete the square for each variable to write t equation in standard form.

$$25x^2 + 9y^2 + 250x - 36y + 436 = 0$$ Original equation

$$25x^2 + 250x + 9y^2 - 36y = -436$$ Associative Property

$$25(x^2 + 10x) + 9(y^2 - 4y) = -436$$ Distributive Property

$$25(x^2 + 10x + \blacksquare) + 9(y^2 - 4y + \blacksquare) = -436 + 25(\blacksquare) + 9(\blacksquare)$$ Complete the squares

$$25(x^2 + 10x + 25) + 9(y^2 - 4y + 4) = -436 + 25(25) + 9(4)$$ $5^2 = 25$ and $(-2)^2 =$

$$25(x + 5)^2 + 9(y - 2)^2 = 225$$ Write as perfect squar

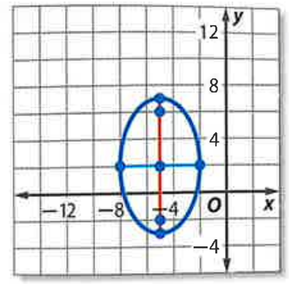
$$\frac{(x+5)^2}{9} + \frac{(y-2)^2}{25} = 1$$ Divide each side by 22

Step 2 Find the center.
$h = -5$ and $k = 2$, so the center of the ellipse is at $(-5, 2)$.

Step 3 Find the lengths of the axes and graph.
The ellipse is vertical.
$a^2 = 25$, so $a = 5$. $b^2 = 9$, so $b = 3$.
The length of the major axis is $2 \cdot 5$ or 10.
The length of the minor axis is $2 \cdot 3$ or 6.
The vertices are at $(-5, 7)$ and $(-5, -3)$.
The co-vertices are at $(-2, 2)$ and $(-8, 2)$.

Step 4 Find the foci.
$c^2 = 25 - 9$ or 16, so $c = 4$.
The foci are at $(-5, 6)$ and $(-5, -2)$.

Step 5 Graph the ellipse.
Draw the ellipse that passes through the vertices and co-vertices.



Guided Practice

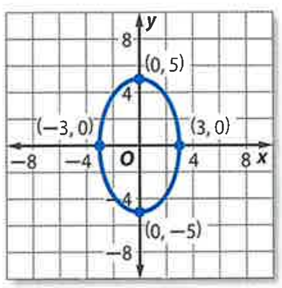
4. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $x^2 + 4y^2 - 2x + 24y + 21 = 0$. Then graph the ellipse.

Check Your Understanding

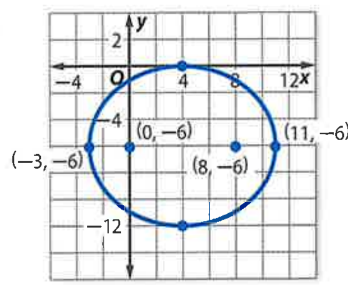
● = Step-by-Step Solutions begin on page R14.

Example 1 **Write an equation for each ellipse.**

1.



2.



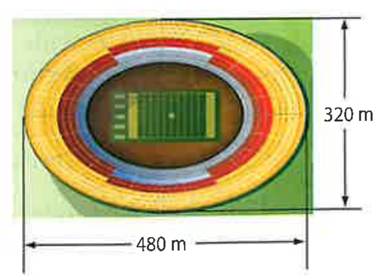
Example 2 **Write an equation for an ellipse that satisfies each set of conditions.**

3. vertices at $(-2, -6)$ and $(-2, 4)$, co-vertices at $(-5, -1)$ and $(1, -1)$

4. vertices at $(-2, 5)$ and $(14, 5)$, co-vertices at $(6, 1)$ and $(6, 9)$

Example 3

5. CCSS **SENSE-MAKING** An architectural firm sent a proposal to a city for building a coliseum, shown at the right.



a. Determine the values of a and b.

b. Assuming that the center is at the origin, write an equation to represent the ellipse.

c. Determine the coordinates of the foci.

6. **SPACE** Earth's orbit is about 91.4 million miles at perihelion and about 94.5 million miles at aphelion. Determine an equation relating Earth's orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

Example 4 **Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.**

7. $\frac{(y+1)^2}{64} + \frac{(x-5)^2}{28} = 1$

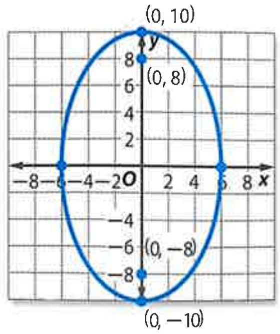
8. $\frac{(x+2)^2}{48} + \frac{(y-1)^2}{20} = 1$

9. $4x^2 + y^2 - 32x - 4y + 52 = 0$

10. $9x^2 + 25y^2 + 72x - 150y + 144 = 0$

Example 1 Write an equation for each ellipse.

11.



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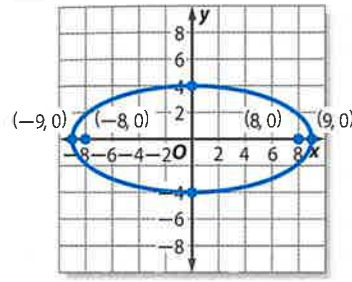

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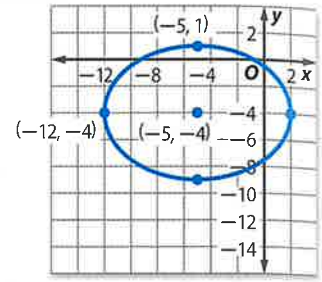

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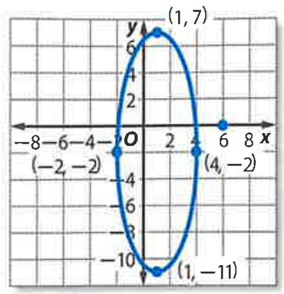

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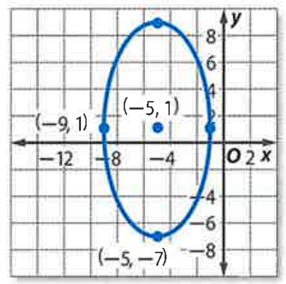

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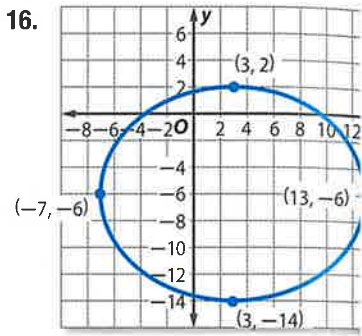


Example 2 Write an equation for an ellipse that satisfies each set of conditions.

17. vertices at $(-6, 4)$ and $(12, 4)$, co-vertices at $(3, 12)$ and $(3, -4)$

18. vertices at $(-1, 11)$ and $(-1, 1)$, co-vertices at $(-4, 6)$ and $(2, 6)$

19. center at $(-2, 6)$, vertex at $(-2, 16)$, co-vertex at $(1, 6)$

20. center at $(3, -4)$, vertex at $(8, -4)$, co-vertex at $(3, -2)$

21. vertices at $(4, 12)$ and $(4, -4)$, co-vertices at $(1, 4)$ and $(7, 4)$

22. vertices at $(-11, 2)$ and $(-1, 2)$, co-vertices at $(-6, 0)$ and $(-6, 4)$

Example 3

23. CCSS **MODELING** The opening of a tunnel in the mountains can be modeled by semiellipses, or halves of ellipses. If the opening is 14.6 meters wide and 8.6 meters high, determine an equation to represent the opening with the center at the origin.

Example 4 Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

24. $\frac{(x-3)^2}{36} + \frac{(y-2)^2}{128} = 1$

25. $\frac{(x+6)^2}{50} + \frac{(y-3)^2}{72} = 1$

26. $\frac{x^2}{27} + \frac{(y-5)^2}{64} = 1$

27. $\frac{(x+4)^2}{16} + \frac{y^2}{75} = 1$

28. $3x^2 + y^2 - 6x - 8y - 5 = 0$

29. $3x^2 + 4y^2 - 18x + 24y + 3 = 0$

30. $7x^2 + y^2 - 56x + 6y + 93 = 0$

31. $3x^2 + 2y^2 + 12x - 20y + 14 = 0$

32. **SPACE** Like the planets, Halley's Comet travels around the Sun in an elliptical orbit. The aphelion is 3282.9 million miles and the perihelion is 54.87 million miles. Determine an equation relating the comet's orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

Write an equation for an ellipse that satisfies each set of conditions.

33. center at $(-5, -2)$, focus at $(-5, 2)$, co-vertex at $(-8, -2)$

34. center at $(4, -3)$, focus at $(9, -3)$, co-vertex at $(4, -5)$

35. foci at $(-2, 8)$ and $(6, 8)$, co-vertex at $(2, 10)$

36. foci at $(4, 4)$ and $(4, 14)$, co-vertex at $(0, 9)$

37. **GOVERNMENT** The Oval Office is located in the West Wing of the White House. It is an elliptical shaped room used as the main office by the President of the United States. The long axis is 10.9 meters long and the short axis is 8.8 meters long. Write an equation to represent the outer walls of the Oval Office. Assume that the center of the room is at the origin.

38. **SOUND** A whispering gallery is an elliptical room in which a faint whisper at one focus cannot be heard by other people in the room, but can easily be heard by someone at the other focus. Suppose an ellipse is 400 feet long and 120 feet wide. What is the distance between the foci?

39. **MULTIPLE REPRESENTATIONS** The *eccentricity* of an ellipse measures how circular the ellipse is.

a. Graphical Graph $\frac{x^2}{81} + \frac{y^2}{36} = 1$ and $\frac{x^2}{81} + \frac{y^2}{9} = 1$ on the same graph.

b. Verbal Describe the difference between the two graphs.

c. Algebraic The eccentricity of an ellipse is $\frac{c}{a}$. Find the eccentricity for each.

d. Analytical Make a conjecture about the relationship between the value of an ellipse's eccentricity and the shape of the ellipse as compared to a circle.

H.O.T. Problems Use Higher-Order Thinking Skills

40. **ERROR ANALYSIS** Serena and Karissa are determining the equation for an ellipse with foci at $(-4, -11)$ and $(-4, 5)$ and co-vertices at $(2, -3)$ and $(-10, -3)$. Is either of them correct? Explain your reasoning.

Serena

$$\frac{(x-4)^2}{64} + \frac{(y+3)^2}{36} = 1$$

Karissa

$$\frac{(x+4)^2}{100} + \frac{(y+3)^2}{36} = 1$$

41. **OPEN ENDED** Write an equation for an ellipse with a focus at the origin.

42. **CHALLENGE** When the values of a and b are equal, an ellipse is a circle. Use this information and your knowledge of ellipses to determine the formula for the area of an ellipse in terms of a and b.

43. **CHALLENGE** Determine an equation for an ellipse with foci at $(2, \sqrt{6})$ and $(2, -\sqrt{6})$ that passes through $(3, \sqrt{6})$.

44. **CCSS ARGUMENTS** What happens to the location of the foci as an ellipse becomes more circular? Explain your reasoning.

45. **REASONING** An ellipse has foci at $(-7, 2)$ and $(18, 2)$. If $(2, 14)$ is a point on the ellipse, show that $(2, -10)$ is also a point on the ellipse.

46. **WRITING IN MATH** Explain why the domain is $\{x \mid -a \le x \le a\}$ and the range is $\{y \mid -b \le y \le b\}$ for an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Standardized Test Practice

47. Multiply.

$$(2 + 3i)(4 + 7i)$$

A $8 + 21i$
B $-13 + 26i$
C $-6 + 10i$
D $13 + 12i$

48. The average lifespan of American women has been tracked, and the model for the data is $y = 0.2t + 73$, where $t = 0$ corresponds to 1960. What is the meaning of the y-intercept?

F In 2007, the average lifespan was 60.
G In 1960, the average lifespan was 58.
H In 1960, the average lifespan was 73.
J The lifespan is increasing 0.2 years every year.

49. GRIDDED RESPONSE If we decrease a number by and then double the result, we get 5 less than t number. What is the number?

50. SAT/ACT The length of a rectangular prism is o inch greater than its width. The height is three times the length. Find the volume of the prism.

A $3x^3 + x^2 + 3x$
B $x^3 + x^2 + x$
C $3x^3 + 6x^2 + 3x$
D $3x^3 + 3x^2 + 3x$
E $3x^3 + 3x^2$

Spiral Review

Write an equation for the circle that satisfies each set of conditions. (Lesson 9-3)

51. center $(8, -9)$, passes through $(21, 22)$

52. center at $(4, 2)$, tangent to x-axis

53. center in the second quadrant; tangent to $y = -1$, $y = 9$, and the y-axis

54. ENERGY A parabolic mirror is used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of a particular mirror is 9.75 feet above the vertex, and the latus rectum is 39 feet long. (Lesson 9-2)

a. Assume that the focus is at the origin. Write an equation for the parabola formed by the mirror.

b. One foot is exactly 0.3048 meter. Rewrite the equation for the mirror in meters.

c. Graph one of the equations for the mirror.

d. Which equation did you choose to graph? Explain why.

Simplify each expression. (Lesson 8-2)

55. $\dfrac{6}{d^2 + 4d + 4} + \dfrac{5}{d + 2}$

56. $\dfrac{a}{a^2 - a - 20} + \dfrac{2}{a + 4}$

57. $\dfrac{x}{x + 1} + \dfrac{3}{x^2 - 4x - 5}$

Solve each equation. (Lesson 7-4)

58. $\log_{10}(x^2 + 1) = 1$

59. $\log_b 64 = 3$

60. $\log_b 121 = 2$

Simplify. (Lesson 5-1)

61. $-5ab^2(-3a^2b + 6a^3b - 3a^4b^4)$

62. $2xy(3xy^3 - 4xy + 2y^4)$

63. $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$

64. $(10x^2 - 3xy + 4y^2) - (3x^2 + 5xy)$

Skills Review

Write an equation of the line passing through each pair of points.

65. $(-2, 5)$ and $(3, 1)$

66. $(7, 1)$ and $(7, 8)$

67. $(-3, 5)$ and $(2, 2)$

CHAPTER 9

Mid-Chapter Quiz

Lessons 9-1 through 9-4

Find the midpoint of the line segment with endpoints at the given coordinates. (Lesson 9-1)

1. (7, 4), (−1, −5)

2. (−2, −9), (−6, 0)

Find the distance between each pair of points with the given coordinates. (Lesson 9-1)

3. (0, 6), (−2, 5)

4. (10, 1), (0, −4)

5. HIKING Carla and Lance left their campsite and hiked 6 miles directly north and then turned and hiked 7 miles east to view a waterfall. (Lesson 9-1)

a. How far is the waterfall from their campsite?

b. Let the campsite be located at the origin on a coordinate grid. At the waterfall they decide to head directly back to the campsite. If they stop halfway between the waterfall and the campsite for lunch, at what coordinates will they stop for lunch?

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola. (Lesson 9-2)

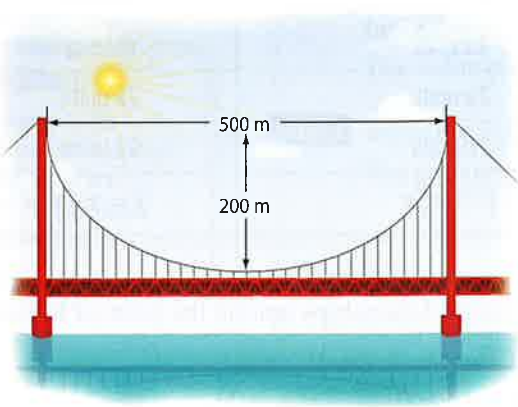
6. $y = 3x^2 - 12x + 21$

7. $x - 2y^2 = 4y + 6$

8. $y = \frac{1}{2}x^2 + 12x - 8$

9. $x = 3y^2 + 5y - 9$

10. BRIDGES Write an equation of a parabola to model the shape of the suspension cable of the bridge shown. Assume that the origin is at the lowest point of the cables. (Lesson 9-2)



Identify the coordinates of the vertex and focus, the equation of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum. (Lesson 9-2)

11. $y = x^2 + 6x + 5$

12. $x = -2y^2 + 4y + 1$

13. Find the center and radius of the circle with equation $(x - 1)^2 + y^2 = 9$. Then graph the circle. (Lesson 9-3)

14. Write an equation for a circle that has center at (3, −2) and passes through (3, 4). (Lesson 9-3)

15. Write an equation for a circle if the endpoints of a diameter are at (8, 31) and (32, 49). (Lesson 9-3)

16. MULTIPLE CHOICE What is the radius of the circle with equation $x^2 + 2x + y^2 + 14y + 34 = 0$? (Lesson 9-3)

A 2

B 4

C 8

D 16

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with the given equation. Then graph the ellipse. (Lesson 9-4)

17. $\frac{(x+4)^2}{16} + \frac{(y-2)^2}{9} = 1$

18. $\frac{(x-1)^2}{20} + \frac{(y+2)^2}{4} = 1$

19. $4y^2 + 9x^2 + 16y - 90x + 205 = 0$

20. MULTIPLE CHOICE Which equation represents an ellipse with endpoints of the major axis at (−4, 10) and (−4, −6) and foci at about (−4, 7.3) and (−4, −3.3)? (Lesson 9-4)

F $\frac{(x-2)^2}{36} + \frac{(y+4)^2}{64} = 1$

G $\frac{(x+4)^2}{64} + \frac{(y-2)^2}{36} = 1$

H $\frac{(y-2)^2}{64} + \frac{(x+4)^2}{36} = 1$

J $\frac{(x-2)^2}{64} + \frac{(y+4)^2}{36} = 1$

LESSON 9-5 Hyperbolas

:: Then	:: Now	:: Why?
● You graphed and analyzed equations of ellipses.	● 1 Write equations of hyperbolas. 2 Graph hyperbolas.	● Because Halley's Comet travels around the Sun in an elliptical path, it reappears in our sky. Other comets pass through our sky only once. Many of these comets travel in paths that resemble hyperbolas.

NewVocabulary

hyperbola
transverse axis
conjugate axis
foci
vertices
co-vertices
constant difference

Common Core State Standards

Content Standards

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Mathematical Practices

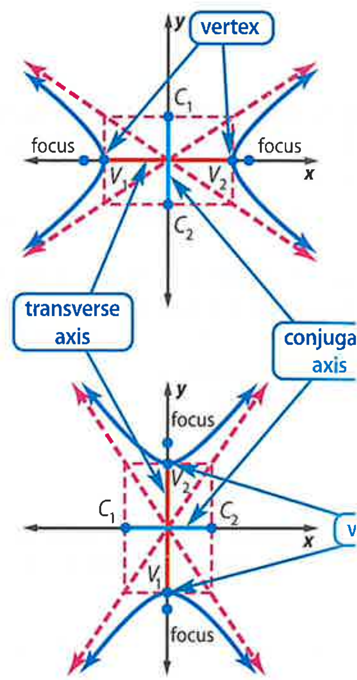
6 Attend to precision.

1 Equations of Hyperbolas Similar to an ellipse, a **hyperbola** is the set of all points in a plane such that the absolute value of the differences of the distances from the foci is constant.

Every hyperbola has two axes of symmetry, the **transverse axis** and the **conjugate axis**. The axes are perpendicular at the center of the hyperbola.

The **foci** of a hyperbola always lie on the transverse axis. The **vertices** are the endpoints of the transverse axis. The **co-vertices** are the endpoints of the conjugate axis.

As a hyperbola recedes from the center, both halves approach asymptotes.



KeyConcept Equations of Hyperbolas Centered at the Origin

Standard Form	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Orientation	horizontal	vertical
Foci	$(\pm c, 0)$	$(0, \pm c)$
Length of Transverse Axis	$2a$ units	$2a$ units
Length of Conjugate Axis	$2b$ units	$2b$ units
Equations of Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

As with ellipses, there are several important relationships among the parts of hyper

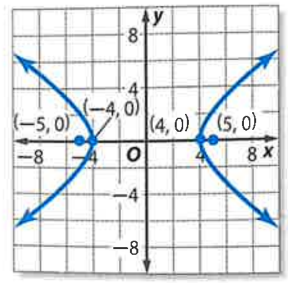
- There are two axes of symmetry.
- The values of a, b, and c are related by the equation $c^2 = a^2 + b^2$.

Math History Link

Hypatia (415–370 B.C.)
Hypatia was a mathematician, scientist, and philosopher in Alexandria, Egypt. She is considered the first woman to write on mathematical topics. Hypatia edited the book *On the Conics of Apollonius*, adding her own problems and examples to clarify the topic for her students. This book developed the ideas of hyperbolas, parabolas, and ellipses.

Example 1 Write an Equation Given Vertices and Foci

Write an equation for the hyperbola shown in the graph.



Step 1 Find the center.

The vertices are equidistant from the center.

The center is at (0, 0).

Step 2 Find the values of a, b, and c.

The value of a is the distance between a vertex and the center, or 4 units.

The value of c is the distance between a focus and the center, or 5 units.

$c^2 = a^2 + b^2$	Equation relating a, b, and c for a hyperbola
$5^2 = 4^2 + b^2$	$c = 5$ and $a = 3$
$9 = b^2$	Subtract 4^2 from each side.

Step 3 Write the equation.

The transverse axis is horizontal, so the equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Guided Practice

1. Write an equation for a hyperbola with vertices at (6, 0) and (−6, 0) and foci at (8, 0) and (−8, 0).

Hyperbolas can also be determined using the equations of their asymptotes.

Example 2 Write an Equation Given Asymptotes

The asymptotes for a vertical hyperbola are $y = \frac{5}{3}x$ and $y = -\frac{5}{3}x$ and the vertices are at (0, 5) and (0, −5). Write the equation for the hyperbola.

Step 1 Find the center.

The vertices are equidistant from the center.

The center of the hyperbola is at (0, 0).

Step 2 Find the values of a and b.

The hyperbola is vertical, so $a = 5$.

From the asymptotes, $b = 3$.

The value of c is not needed.

Step 3 Write the equation.

The equation for the hyperbola is $\frac{y^2}{25} - \frac{x^2}{9} = 1$.

Reading Math

Standard Form In the standard form of a hyperbola, the squared terms are subtracted. For an ellipse, they are added.

Guided Practice

2. The asymptotes for a horizontal hyperbola are $y = \frac{7}{9}x$ and $y = -\frac{7}{9}x$. The vertices are (9, 0) and (−9, 0). Write an equation for the hyperbola.

2 Graphs of Hyperbolas

Hyperbolas can be translated in the same manner as t other conic sections.

KeyConcept Equations of Hyperbolas Centered at (h, k)

Standard Form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} =$
Orientation	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-vertices	$(h, k \pm b)$	$(h \pm b, k)$
Equations of Asymptotes	$y - k = \pm\frac{b}{a}(x - h)$	$y - k = \pm\frac{a}{b}(x - h)$

StudyTip

Calculator You can graph a hyperbola on a graphing calculator by solving for y, and then graphing the two equations on the same screen.

Example 3 Graph a Hyperbola

Graph $\frac{(x-3)^2}{4} - \frac{(y+2)^2}{16} = 1$. Identify the vertices, foci, and asymptotes.

Step 1 Find the center. The center is at (3, −2).

Step 2 Find a, b, and c. From the equation, $a^2 = 4$ and $b^2 = 16$, so $a = 2$ and $b = 4$

$c^2 = a^2 + b^2$	Equation relating a, b, and c for a hyperbola
$c^2 = 2^2 + 4^2$	$a = 2$, $b = 4$
$c^2 = 20$	Simplify.
$c = \sqrt{20}$ or about 4.47	Take the square root of each side.

Step 3 Identify the vertices and foci. The hyperbola is horizontal and the vertice 2 units from the center, so the vertices are at (1, −2) and (5, −2). The foci are about 4.47 units from the center. The foci are at (−1.47, −2) and (7.47, −2).

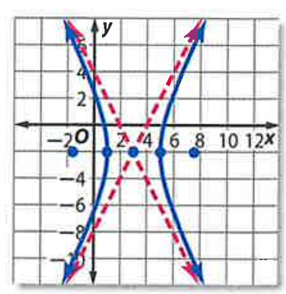
Step 4 Identify the asymptotes.

$y - k = \pm\frac{b}{a}(x - h)$	Equation for asymptotes of a horizontal hyperbola
$y - (-2) = \pm\frac{4}{2}(x - 3)$	$a = 2$, $b = 4$, $h = 3$, and $k = -2$

The equations for the asymptotes are $y = 2x - 8$ and $y = -2x + 4$.

Step 5 Graph the hyperbola. The hyperbola is symmetric about the transverse and conjugate axes. Use this symmetry to plot additional points for the hyperbola.

Use the asymptotes as a guide to draw the hyperbola that passes through the vertices and the other points.

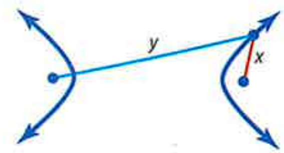


GuidedPractice

3. Graph $\frac{(y-4)^2}{9} - \frac{(x+3)^2}{25} = 1$. Identify the vertices, foci, and asymptotes.

In the equation for any hyperbola, the value of $2a$ represents the **constant difference**. This is the absolute value of the difference between the distances from any point on the hyperbola to the foci of the hyperbola.

Any point on the hyperbola at the right will have the same constant difference, $|y - x|$ or $2a$.



Real-WorldLink

Halley's Comet becomes visible to the unaided eye about every 76 years as it nears the Sun.

Source: NASA

Real-World Example 4 Write an Equation of a Hyperbola

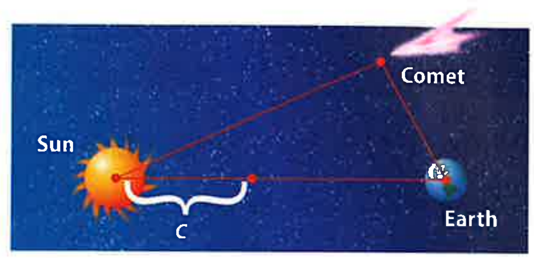
SPACE Earth and the Sun are 146 million kilometers apart. A comet follows a path that is one branch of a hyperbola. Suppose the comet is 30 million miles farther from the Sun than from Earth. Determine the equation of the hyperbola centered at the origin for the path of the comet.

Understand We need to determine the equation for the hyperbola.

Plan Find the center and the values of a and b. Once we have this information, we can determine the equation.

Solve The foci are Earth and the Sun, with the origin between them.

The value of c is $146 \div 2$ or 73.



The difference of the distances from the comet to each body is 30. Therefore, a is $30 \div 2$ or 15 million miles.

$c^2 = a^2 + b^2$ Equation relating a, b, and c for a hyperbola

$73^2 = 15^2 + b^2$ $a = 15$ and $c = 73$

$5104 = b^2$ Simplify.

The equation of the hyperbola is $\frac{x^2}{225} - \frac{y^2}{5104} = 1$.

Since the comet is farther from the Sun, it is located on the branch of the hyperbola near Earth.

Check (21, 70) is a point that satisfies the equation.

The distance between this point and the Sun (−73, 0) is

$\sqrt{[21 - (-73)]^2 + (70 - 0)^2}$ or 117.2 million kilometers.

The distance between this point and Earth (73, 0) is

$\sqrt{(21 - 73)^2 + (70 - 0)^2}$ or 87.2 million kilometers.

The difference between these distances is 30. ✓

StudyTip

Exact Locations A third receiving station is necessary to determine the plane's exact location.

GuidedPractice

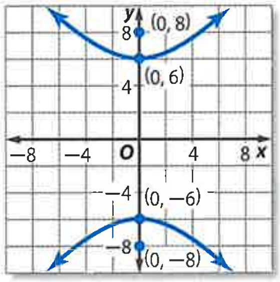
4. **SEARCH AND RESCUE** Two receiving stations that are 150 miles apart receive a signal from a downed airplane. They determine that the airplane is 80 miles farther from station A than from station B. Determine the equation of the hyperbola centered at the origin on which the plane is located.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–2 Write an equation for each hyperbola.

1.



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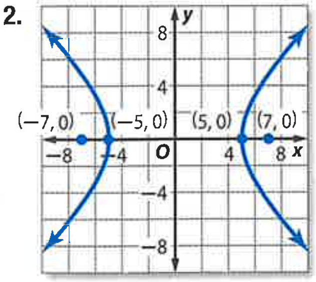

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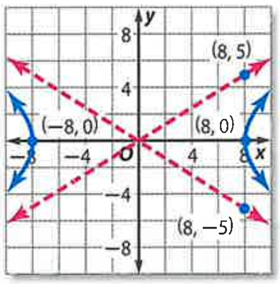

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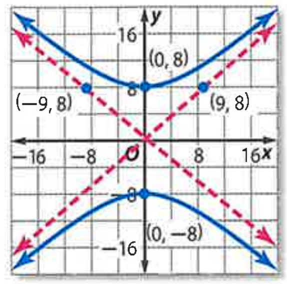


Example 3 **STRUCTURE** Graph each hyperbola. Identify the vertices, foci, and asymptotes.

5. $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1$

6. $\dfrac{y^2}{36} - \dfrac{x^2}{60} = 1$

7. $9y^2 + 18y - 16x^2 + 64x - 199 = 0$

8. $4x^2 + 24x - y^2 + 4y - 4 = 0$

Example 4

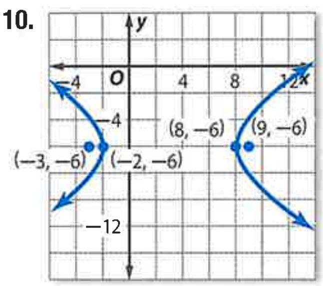
9. **NAVIGATION** A ship determines that the difference of its distances from two stations is 60 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at (−80, 0) and (80, 0).

Practice and Problem Solving

Extra Practice is on pa

Examples 1–2 Write an equation for each hyperbola.

10.



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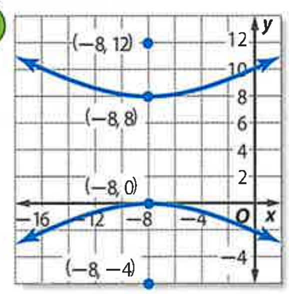

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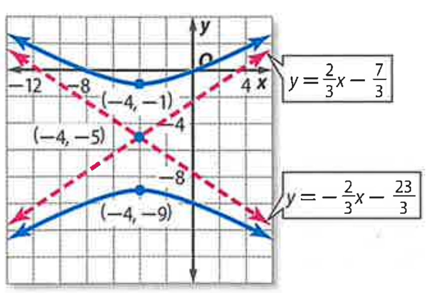

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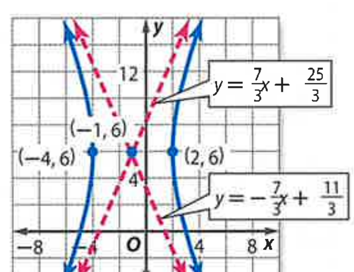


Example 3

Graph each hyperbola. Identify the vertices, foci, and asymptotes.

14. $\frac{x^2}{36} - \frac{y^2}{4} = 1$

15. $\frac{y^2}{9} - \frac{x^2}{49} = 1$

16. $\frac{y^2}{36} - \frac{x^2}{25} = 1$

17. $\frac{x^2}{16} - \frac{y^2}{16} = 1$

18. $\frac{(x-3)^2}{16} - \frac{(y+1)^2}{4} = 1$

19. $\frac{(y+5)^2}{16} - \frac{(x+2)^2}{36} = 1$

20. $9y^2 - 4x^2 - 54y + 32x - 19 = 0$

21. $16x^2 - 9y^2 + 128x + 36y + 76 = 0$

22. $25x^2 - 4y^2 - 100x + 48y - 144 = 0$

23. $81y^2 - 16x^2 - 810y + 96x + 585 = 0$

Example 4

24. **NAVIGATION** A ship determines that the difference of its distances from two stations is 80 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at $(-100, 0)$ and $(100, 0)$.

Determine whether the following equations represent ellipses or hyperbolas.

25. $4x^2 = 5y^2 + 6$

26. $8x^2 - 2x = 8y - 3y^2$

27. $-5x^2 + 4x = 6y + 3y^2$

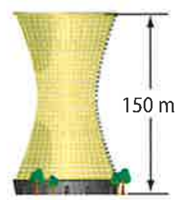
28. $7y - 2x^2 = 6x - 2y^2$

29. $6x - 7x^2 - 5y^2 = 2y$

30. $4x + 6y + 2x^2 = -3y^2$

31. **SPACE** Refer to the application at the beginning of the lesson. With the Sun as a focus and the center at the origin, a certain comet's path follows a branch of a hyperbola. If two of the coordinates of the path are (10, 0) and (30, 100) where the units are in millions of miles, determine the equation of the path.

32. **COOLING** Natural draft cooling towers are shaped like hyperbolas for more efficient cooling of power plants. The hyperbola in the tower at the right can be modeled by $\frac{x^2}{16} - \frac{y^2}{225} = 1$, where the units are in meters. Find the width of the tower at the top and at its narrowest point in the middle.



ge R9.

33. **MULTIPLE REPRESENTATIONS** Consider $xy = 16$.

a. **Tabular** Make a table of values for the equation for $-12 \leq x \leq 12$.

b. **Graphical** Graph the hyperbola represented by the equation.

c. **Logical** Determine and graph the asymptotes for the hyperbola.

d. **Analytical** What special property do you notice about the asymptotes? Hyperbolas that represent this property are called *rectangular hyperbolas*.

e. **Analytical** Without any calculations, what do you think will be the coordinates of the vertices for $xy = 25$? for $xy = 36$?

34. CCSS **MODELING** Two receiving stations that are 250 miles apart receive a signal from a downed airplane. They determine that the airplane is 70 miles farther from station B than from station A. Determine the equation of the horizontal hyperbola centered at the origin on which the plane is located.

35. **WEATHER** Luisa and Karl live exactly 4000 feet apart. While on the phone at their homes, Luisa hears thunder out of her window and Karl hears it 3 seconds later out of his. If sound travels 1100 feet per second, determine the equation for the horizontal hyperbola on which the lightning is located.

36. ARCHITECTURE Large pillars with cross sections in the shape of hyperbolas were popular in ancient Greece. The curves can be modeled by the equation $\frac{x^2}{0.16} - \frac{y^2}{4} = 1$, where the units are in feet. If the pillars are 9 feet tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to the nearest hundredth of a foot.

Write an equation for the hyperbola that satisfies each set of conditions.

37. vertices (−8, 0) and (8, 0), conjugate axis of length 20 units

38. vertices (0, −6) and (0, 6), conjugate axis of length 24 units

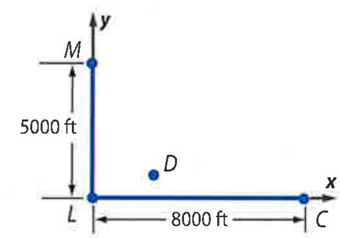
39. vertices (6, −2) and (−2, −2), foci (10, −2) and (−6, −2)

40. vertices (−3, 4) and (−3, −8), foci (−3, 9) and (−3, −13)

41. centered at the origin with a horizontal transverse axis of length 10 units and a conjugate axis of length 4 units

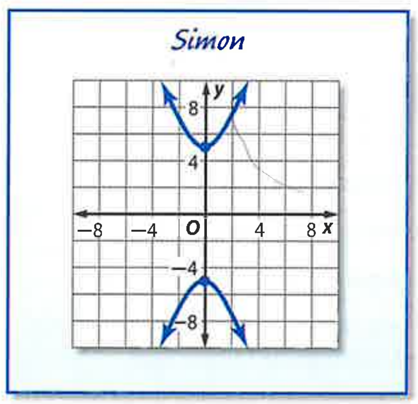
42. centered at the origin with a vertical transverse axis of length 16 units and a conjugate axis of length 12 units

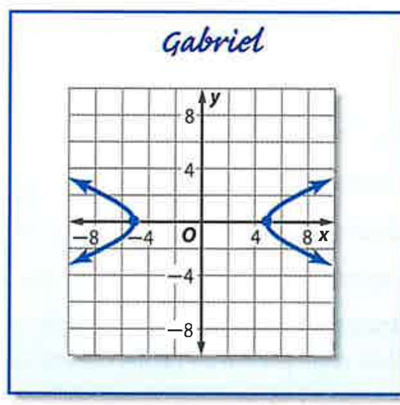
43. TRIANGULATION While looking for their lost dog in the woods, Lae, Meg, and Cesar hear a bark. Meg hears it 2 seconds after Lae and Cesar hears it 3 seconds after Lae. With Lae at the origin, determine the exact location of their dog if sound travels 1100 feet per second.



H.O.T. Problems Use Higher-Order Thinking Skills

44. CCSS CRITIQUE Simon and Gabriel are graphing $\frac{y^2}{25} - \frac{x^2}{4} = 1$. Is either of them correct? Explain your reasoning.





45. CHALLENGE The origin lies on a horizontal hyperbola. The asymptotes for the hyperbola are $y = -x + 1$ and $y = x - 5$. Find the equation for the hyperbola.

46. REASONING What happens to the location of the foci of a hyperbola as the value of a becomes increasingly smaller than the value of b? Explain your reasoning.

47. REASONING Consider $\frac{y^2}{36} - \frac{x^2}{16} = 1$. Describe the change in the shape of the hyperbola and the locations of the vertices and foci if 36 is changed to 9. Explain why this happens.

48. OPEN ENDED Write an equation for a hyperbola with a focus at the origin.

49. WRITING IN MATH Why would you choose a conic section to model a set of data instead of a polynomial function?

Standardized Test Practice

50. You have 6 more dimes than quarters. You have a total of $5.15. How many dimes do you have?

A 13 C 19

B 16 D 25

51. How tall is a tree that is 15 feet shorter than a pole three times as tall as the tree?

F 24.5 ft

G 22.5 ft

H 21.5 ft

J 7.5 ft

52. SHORT RESPONSE A rectangle is 8 feet long and 6 feet wide. If each dimension is increased by the same number of feet, the area of the new rectangle formed is 32 square feet more than the area of the original rectangle. By how many feet was each dimension increased?

53. SAT/ACT When the equation $y = 4x^2 - 5$ is graphed in the coordinate plane, the graph is which of the following?

A line D hyperbola

B circle E parabola

C ellipse

Spiral Review

Write an equation for an ellipse that satisfies each set of conditions. (Lesson 9-4)

54. endpoints of major axis at (2, 2) and (2, −10), endpoints of minor axis at (0, −4) and (4, −4)

55. endpoints of major axis at (0, 10) and (0, −10), foci at (0, 8) and (0, −8)

Find the center and radius of the circle with the given equation. Then graph the circle. (Lesson 9-3)

56. $(x - 3)^2 + y^2 = 16$ **57.** $x^2 + y^2 - 6y - 16 = 0$ **58.** $x^2 + y^2 + 9x - 8y + 4 = 0$

59. BASKETBALL Zonta plays basketball for Centerville High School. So far this season, she has made 6 out of 10 free throws. She is determined to improve her free throw percentage. If she can make x consecutive free throws, her free throw percentage can be determined using $P(x) = \frac{6 + x}{10 + x}$. (Lesson 8-4)

a. Graph the function.

b. What part of the graph is meaningful in the context of the problem?

c. Describe the meaning of the y-intercept.

d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Zonta's shooting percentage.

Solve each equation. (Lesson 7-2)

60. $\left(\frac{1}{7}\right)^{y-3} = 343$ **61.** $10^{x-1} = 100^{2x-3}$ **62.** $36^{2p} = 216^{p-1}$

Graph each inequality. (Lesson 6-3)

63. $y \geq \sqrt{5x - 8}$ **64.** $y \geq \sqrt{x - 3} + 4$ **65.** $y < \sqrt{6x - 2} + 1$

Skills Review

66. Write an equation for a parabola with vertex at the origin that passes through (2, −8)

67. Write an equation for a parabola with vertex at (−3, −4) that opens up and has y-intercept 8.

LESSON 9-6 Identifying Conic Sections

:: Then	:: Now	:: Why?
● You analyzed different conic sections.	● **1** Write equations of conic sections in standard form. **2** Identify conic sections from their equations.	● Parabolas, circles, ellipses, and hyperbolas are called conic sections because they are the cross sections formed when a double cone is sliced by a plane.

Parabola

Circle and Ellipse

H

Common Core State Standards

Content Standards

A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

8 Look for and express regularity in repeated reasoning.

1 Conics in Standard Form The equation for any conic section can be written form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, and C are not all zero. T general form can be converted to the standard forms below by completing the squa

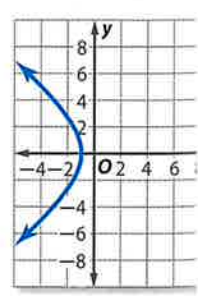
ConceptSummary Standard Forms of Conic Sections

Conic Section	Standard Form of Equation	
Circle	$(x - h)^2 + (y - k) = r^2$	
	Horizontal Axis	**Vertical Axis**
Parabola	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} =$
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} =$

Example 1 Rewrite an Equation of a Conic Section

Write $16x^2 - 25y^2 - 128x - 144 = 0$ in standard form. State whether the graph the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equatio

$16x^2 - 25y^2 - 128x - 144 = 0$	Original equation
$16(x^2 - 8x + ■) - 25y^2 = 144 + 16(■)$	Isolate terms.
$16(x^2 - 8x + 16) - 25y^2 = 144 + 16(16)$	Complete the square.
$16(x - 4)^2 - 25y^2 = 400$	Perfect square
$\frac{(x-4)^2}{25} - \frac{y^2}{16} = 1$	Divide each side by 400.



The graph is a hyperbola with its center at (4, 0).

GuidedPractice

1. Write $4x^2 + y^2 - 16x + 8y - 4 = 0$ in standard form. State whether the graph equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation.

Review Vocabulary

discriminant the expression $b^2 - 4ac$ from the Quadratic Formula

2 Identify Conic Sections

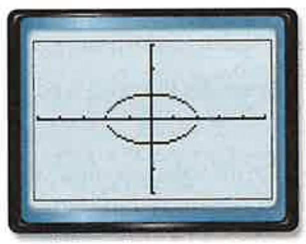
You can determine the type of conic without having to write $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ in standard form. When there is an xy-term ($B \neq 0$), you can use the discriminant to identify the conic. $B^2 - 4AC$ is the discriminant of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

Concept Summary Classify Conics with the Discriminant

Discriminant	Conic Section
$B^2 - 4AC < 0$; $B = 0$ and $A = C$	circle
$B^2 - 4AC < 0$; either $B \neq 0$ or $A \neq C$	ellipse
$B^2 - 4AC = 0$	parabola
$B^2 - 4AC > 0$	hyperbola

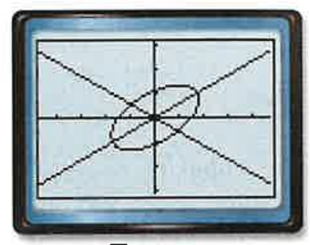
When $B = 0$, the conic will be either vertical or horizontal. When $B \neq 0$, the conic will be neither vertical nor horizontal.

Horizontal Ellipse: $B = 0$



$x^2 + 4y^2 - 4 = 0$

Rotated Ellipse: $B \neq 0$



$7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$

Study Tip

Identifying Conics When there is no xy-term ($B = 0$), use A and C.
Parabola: A or $C = 0$ but not both.
Circle: $A = C$
Ellipse: A and C have the same sign but are not equal.
Hyperbola: A and C have opposite signs.

Example 2 Analyze an Equation of a Conic Section

Without writing in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

a. $y^2 + 4x^2 - 3xy + 4x - 5y - 8 = 0$

$A = 4$, $B = -3$, and $C = 1$

The discriminant is $(-3)^2 - 4(4)(1)$ or -7.

Because the discriminant is less than 0 and $B \neq 0$, the conic is an ellipse.

b. $3x^2 - 6x + 4y - 5y^2 + 2xy - 4 = 0$

$A = 3$, $B = 2$, and $C = -5$

The discriminant is $2^2 - 4(3)(-5)$ or 64.

Because the discriminant is greater than 0, the conic is a hyperbola.

c. $4y^2 - 8x + 6y - 14 = 0$

$A = 0$, $B = 0$, and $C = 4$

The discriminant is $0^2 - 4(0)(4)$ or 0.

Because the discriminant equals 0, the conic is a parabola.

Guided Practice

2A. $8y^2 - 6x^2 + 4xy - 6x + 2y - 4 = 0$

2B. $3xy + 4x^2 - 2y + 9x - 3 = 0$

2C. $3x^2 + 16x - 12y + 2y^2 - 6 = 0$

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.**

1. $x^2 + 4y^2 - 6x + 16y - 11 = 0$

2. $x^2 + y^2 + 12x - 8y + 36 = 0$

3. $9y^2 - 16x^2 - 18y - 64x - 199 = 0$

4. $6y^2 - 24y + 28 - x = 0$

Example 2 **Without writing in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

5. $4x^2 + 6y^2 - 3x - 2y = 12$

6. $5y^2 = 2x + 6y - 8 + 3x^2$

7. $8x^2 + 8y^2 + 16x + 24 = 0$

8. $4x^2 - 6y = 8x + 2$

9. $4x^2 - 3y^2 + 8xy - 12 = 2x + 4y$

10. $5xy - 3x^2 + 6y^2 + 12y = 18$

11. $8x^2 + 12xy + 16y^2 + 4y - 3x = 12$

12. $16xy + 8x^2 + 8y^2 - 18x + 8y = 13$

13. CCSS **MODELING** A military jet performs for an air show. The path of the plane during one maneuver can be modeled by a conic section with equation $24x^2 + 1000y - 31{,}680x - 45{,}600 = 0$, where distances are represented in feet.

a. Identify the shape of the curved path of the jet. Write the equation in standard form.

b. If the jet begins its path upward, or ascent, at $x = 0$, what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?

c. What is the maximum height of the jet?

Practice and Problem Solving

Extra Practice is on pa

Example 1 **Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.**

14. $3x^2 - 2y^2 + 18x + 8y - 35 = 0$

15. $3x^2 + 24x + 4y^2 - 40y + 52 = 0$

16. $x^2 + y^2 = 16 + 6y$

17. $32x + 28 = y - 8x^2$

18. $7x^2 - 8y = 84x - 2y^2 - 176$

19. $x^2 + 8y = 11 + 6x - y^2$

20. $4y^2 = 24y - x - 31$

21. $112y + 64x = 488 + 7y^2 - 8x^2$

22. $28x^2 + 9y^2 - 188 = 56x - 36y$

23. $25x^2 + 384y - 64y^2 + 200x = 1776$

Example 2 **Without writing in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

24. $4x^2 - 5y = 9x - 12$

25. $4x^2 - 12x = 18y - 4y^2$

26. $9x^2 + 12y = 9y^2 + 18y - 16$

27. $18x^2 - 16y = 12x - 4y^2 + 19$

28. $12y^2 - 4xy + 9x^2 = 18x - 124$

29. $5xy + 12x^2 - 16x = 5y + 3y^2 + 18$

30. $19x^2 + 14y = 6x - 19y^2 - 88$

31. $8x^2 + 20xy + 18 = 4y^2 - 12 + 9x$

32. $5x - 12xy + 6x^2 = 8y^2 - 24y - 9$

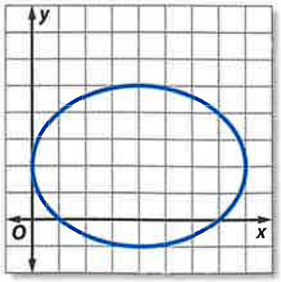
33. $18x - 24y + 324xy = 27x^2 + 3y^2 - 5$

34. **LIGHT** A lamp standing near a wall throws an arc of light in the shape of a conic section. Suppose the edge of the light can be represented by the equation $3y^2 - 2y - 4x^2 + 2x - 8 = 0$. Identify the shape of the edge of the light and graph the equation.

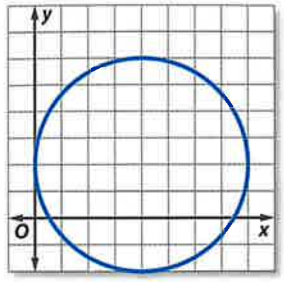
Match each graph with its corresponding equation.

a. $x^2 + y^2 - 8x - 4y = -4$ **b.** $9x^2 - 16y^2 - 72x + 64y = 64$ **c.** $9x^2 + 16y^2 = 72x + 64y - 64$

35.



36.



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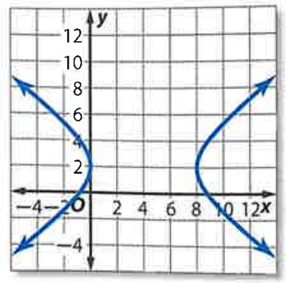


For Exercises 38–41, match each situation with an equation that could be used to represent it.

a. $47.25x^2 - 9y^2 + 18y + 33.525 = 0$ **b.** $25x^2 + 100y^2 - 1900x - 2200y + 45{,}700 = 0$

c. $16x^2 - 90x + y - 0.25 = 0$ **d.** $x^2 + y^2 - 18x - 30y - 14{,}094 = 0$

38. COMPUTERS the boundary of a wireless network with a range of 120 feet

39. FITNESS the oval path of your foot on an exercise machine

40. COMMUNICATIONS the position of a cell phone between two cell towers

41. SPORTS the height of a football above the ground after being kicked

42. CCSS **SENSE-MAKING** The shape of the cables in a suspension bridge is approximately parabolic. If the towers for a planned bridge are 1000 meters apart and the lowest point of the suspension cables is 200 meters below the top of the towers, write the equation in standard form with the origin at the vertex.

43. MULTIPLE REPRESENTATIONS Consider an ellipse with center $(3, -2)$, vertex $M(-1, -2)$, and co-vertex $N(3, -4)$.

a. Analytical Determine the standard form of the equation of the ellipse.

b. Algebraic Convert part **a** to $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ form.

c. Graphical Graph the ellipse.

d. Analytical If the ellipse is rotated such that M is moved to $(3, -6)$, determine the location of N and the angle of rotation.

H.O.T. Problems Use Higher-Order Thinking Skills

44. CHALLENGE When a plane passes through the vertex of a cone, a *degenerate* conic is formed.

a. Determine the type of conic represented by $4x^2 + 8y^2 = 0$.

b. Graph the conic.

c. Describe the difference between this degenerate conic and a standard conic of the same type with $A = 4$ and $B = 8$.

45. REASONING Is the following statement *sometimes*, *always*, or *never* true? Explain.

When a conic is vertical and $A = C$, *it is a circle.*

46. OPEN ENDED Write an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = 9C$, that represents a parabola.

47. WRITING IN MATH Compare and contrast the graphs of the four types of conics and their corresponding equations.

Standardized Test Practice

48. SAT/ACT A class of 25 students took a science test. Ten students had a mean score of 80. The other students had an average score of 60. What is the average score of the whole class?

A 66
B 68
C 70
D 72
E 78

49. Six times a number minus 11 is 43. What is the number?

F 12
G 11
H 10
J 9

50. EXTENDED RESPONSE The amount of water remaining in a storage tank as it is drained car represented by the equation $L = -4t^2 - 10t + 1$ where L represents the number of liters of wa remaining and t represents the number of minu since the drain was opened. How many liters water were in the tank initially? Determine to t nearest tenth of a minute how long it will take the tank to drain completely.

51. Ruben has a square piece of paper with sides 4 inches long. He rolled up the paper to form cylinder. What is the volume of the cylinder?

A $\frac{4}{\pi}$
B $\frac{16}{\pi}$
C 4π
D 16π

Spiral Review

52. ASTRONOMY Suppose a comet's path can be modeled by a branch of the hyperbola with equation $\frac{y^2}{225} - \frac{x^2}{400} = 1$. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola. Then graph the hyperbola. (Lesson 9-5)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse. (Lesson 9-4)

53. $\frac{y^2}{18} + \frac{x^2}{9} = 1$

54. $4x^2 + 8y^2 = 32$

55. $x^2 + 25y^2 - 8x + 100y + 91 =$

Graph each function. (Lesson 8-3)

56. $f(x) = \frac{3}{x}$

57. $f(x) = \frac{-2}{x+5}$

58. $f(x) = \frac{6}{x-2} - 4$

59. SPACE A radioisotope is used as a power source for a satellite. The power output P (in watts) is given by $P = 50e^{-\frac{t}{250}}$, where t is the time in days. (Lesson 7-8)

a. Is the formula for power output an example of exponential *growth* or *decay*? Explain your reasoning.

b. Find the power available after 100 days.

c. Ten watts of power are required to operate the equipment in the satellite. How long can the satellite continue to operate?

Skills Review

Solve each system of equations.

60. $6g - 8h = 50$
$6h = 22 - 4g$

61. $3u + 5v = 6$
$2u - 4v = -7$

62. $10m - 9n = 15$
$5m - 4n = 10$

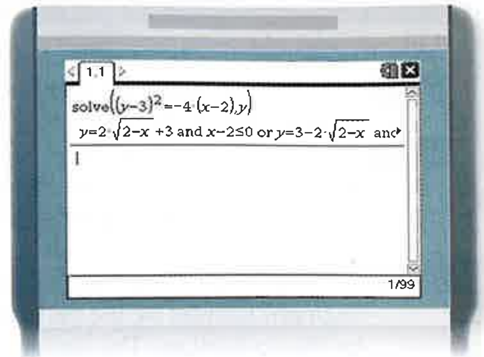
EXTEND

9-6 Graphing Technology Lab
Analyzing Quadratic Relations

You can use TI-Nspire technology to analyze quadratic relations.

Activity 1 Characteristics of a Parabolic Relation

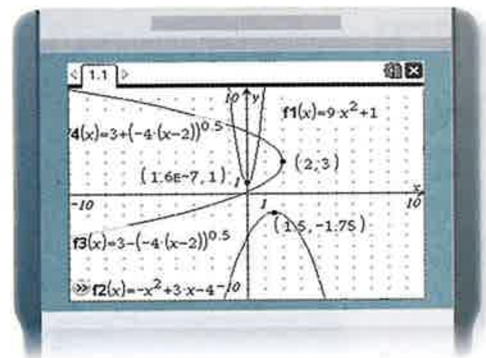
Graph $f(x) = 9x^2 + 1$, $g(x) = -x^2 + 3x - 4$, and $(y - 3)^2 = -4(x - 2)$. Identify the maxima, minima, and axes of symmetry.



Step 1 Add a new **Graphs** and **Calculator** page.

Step 2 Enter $f(x)$ into **f1** and $g(x)$ into **f2**.

Step 3 On the **Calculator** page, use **solve** to solve $(y - 3)^2 = -4(x - 2)$ for y. Hold the **shift** and use ▶ to highlight one equation; then press **ctrl** C. Press **ctrl** ▶ to go to the **Graphs** page; then press **tab** **ctrl** V. Repeat to copy the other equation into **f4**.



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Step 4 Use **Analyze Graph**, **Maximum**, **Minimum**, and **Intersection** to find the coordinates of the extrema. For $f(x)$, the minimum is at (0, 1); for $g(x)$, the maximum is at (1.5, −1.75); for the relation, the vertex is at (2, 3).

Step 5 You can use the coordinates of the extrema to find the equations of the axes of symmetry. For $f(x)$, the equation of the axis of symmetry is $x = 1.5$; for $g(x)$, the equation of the axis of symmetry is $x = 0$; for the relation, the equation of the axis of symmetry is $y = 3$.

You can also use a graphing calculator to determine the equation of a parabola.

Activity 2 Write an Equation for a Parabola

Given that $f(x)$ has zeros at −2 and 4 and $f(x)$ opens downward, write an equation for the parabola.

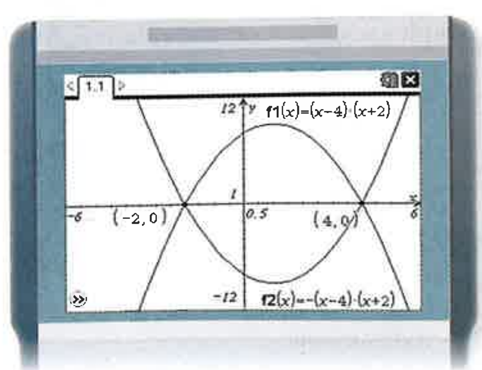
Step 1 Add a new **Graphs** and **Calculator** page.

Step 2 Because the zeros are $x = -2$ and $x = 4$, the factors of the quadratic equation are $(x + 2)$ and $(x - 4)$.

Step 3 Use the **expand** command on the **Calculator** page to multiply $(x + 2)$ and $(x - 4)$. So, $y = x^2 - 2x - 8$.

Step 4 On the **Graphs** page, graph $y = x^2 - 2x - 8$. Verify the roots and direction of opening.

Step 5 The function in the graph opens upward, not downward, so multiply $x^2 - 2x - 8$ by −1. Thus, $y = -x^2 + 2x + 8$. Graph this function.



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An equation for the parabola that has zeros at −2 and 4 and opens downward is $y = -x^2 + 2x + 8$.

(continued on the next page)

Graphing Technology Lab
Analyzing Quadratic Relations *Continued*

You can use a graphing calculator to determine an equation of a quadratic relation.

Activity 3 Write an Equation for an Ellipse

Write an equation for an ellipse that has vertices at (−3, 3) and (−3, −7) and co-vertices at (0, −2) and (−6, −2).

Step 1 Add a new **Graphs** and **Calculator** page.

Step 2 Turn on the grid from **View, Show Axis.** Then use **Points, Points On** to graph the four points. Use **Actions, Coordinates and Equations** to show the coordinates of the points.

Step 3 Use the graph to identify the intersection point of the segment formed by the vertices and the segment formed by the co-vertices. The center is at (−3, −2).

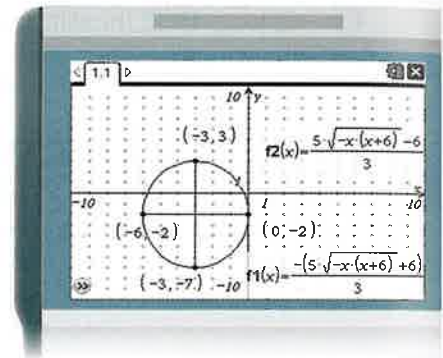
Step 4 Identify other important characteristics. The ellipse is oriented vertically. The length of the major axis is 10, so $a = 5$. The length of the minor axis is 6, so $b = 3$.

Step 5 Write the equation in standard form.

$$\frac{[y-(-2)]^2}{5^2} + \frac{[x-(-3)]^2}{3^2} = 1 \text{ or}$$

$$\frac{(y+2)^2}{25} + \frac{(x+3)^2}{9} = 1$$

Step 6 Check the equation by using **Solve** under the **Algebra** menu on the **Calculator** page to solve for y. Copy and paste the two equations into the **Graph** page to graph the ellipse.



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Exercises

Graph each function and relation. Identify the maxima, minima, and axes of symmetry.

1. $f(x) = -(x-3)^2 + 12$, $g(x) = x^2 + x - 12$, and $(y+5)^2 = -12(x-2)$.

2. $f(x) = -0.25x^2 - 3x - 6$, $g(x) = 2x^2 + 2x + 4$, and $(y+1)^2 = 2(x+6)$.

3. Given that $f(x)$ has zeros at −1 and 3 and $f(x)$ opens upward, write an equation for the parabola.

4. Given that $f(x)$ has zeros at −3 and −1 and $f(x)$ opens downward, write an equation for the parabola.

5. Write an equation for an ellipse that has vertices at (−6, 2) and (−6, −8) and co-vertices at (−3, −3) and (−9, −3).

6. Write an equation for an ellipse that has vertices at (−13, 2) and (1, 2) and co-vertices at (−6, 4) and (−6, 0).

EXPLORE

9-7 Graphing Technology Lab
Linear-Nonlinear Systems

You can use a TI-83/84 Plus application to solve linear-nonlinear systems by using the Y= menu to graph each equation on the same set of axes.

Example Linear-Quadratic System

Solve the system of equations.

$3y - 4x = -7$

$4x^2 + 3y^2 = 91$

Step 1 Solve each equation for y.

$$3y - 4x = -7 \qquad 4x^2 + 3y^2 = 91$$

$$3y = 4x - 7 \qquad 3y^2 = 91 - 4x^2$$

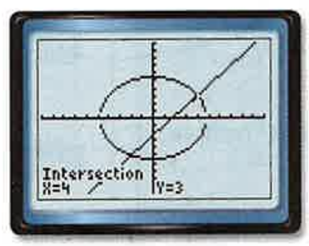
$$y = \frac{4}{3}x - \frac{7}{3} \qquad y = \pm\sqrt{\frac{91 - 4x^2}{3}}$$

Step 2 Enter $y = \frac{4}{3}x - \frac{7}{3}$ as Y1, $y = \sqrt{\frac{91 - 4x^2}{3}}$ as Y2, and $y = -\sqrt{\frac{91 - 4x^2}{3}}$ as Y3. Then graph the equations in a standard viewing window.

KEYSTROKES: [Y=] 4 [÷] 3 [X,T,θ,n] [−] 7 [÷] 3 [ENTER] [2nd] [x^2] [(] 91 [−] 4 [X,T,θ,n] [x^2] [)] [÷] 3 [)] [ENTER] [(−)] [2nd] [x^2] [(] 91 [−] 4 [X,T,θ,n] [x^2] [)] [÷] 3 [)] [ENTER] [ZOOM] 6

Step 3 Find the intersection of $y = \frac{4}{3}x - \frac{7}{3}$ with $y = \sqrt{\frac{91 - 4x^2}{3}}$.

KEYSTROKES: Press [2nd] [TRACE] 5 [ENTER] [ENTER] [ENTER]. The two graphs intersect at (4, 3).

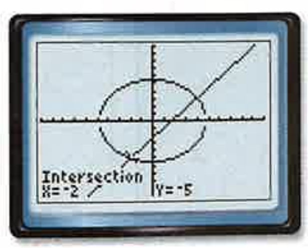


Step 4 Find the intersection of $y = \frac{4}{3}x - \frac{7}{3}$ with $y = -\sqrt{\frac{91 - 4x^2}{3}}$.

KEYSTROKES: Press [2nd] [TRACE] 5 [ENTER] [▼] [ENTER]. Then use [◀] to move the cursor to the second intersection point.

Press [ENTER]. The two graphs intersect at (−2, −5).

The solutions of the system are (4, 3) and (−2, −5).



Exercises

Use a graphing calculator to solve each system of equations.

1. $x^2 + y^2 = 100$
$x + y = 2$

2. $2y - x = 11$
$5x^2 + 2y^2 = 407$

3. $21x + 9y = -36$
$7x^2 + 9y^2 = 1152$

LESSON 9-7 Solving Linear-Nonlinear Systems

Then	Now	Why?
You solved systems of linear equations.	**1** Solve systems of linear and nonlinear equations algebraically and graphically. **2** Solve systems of linear and nonlinear inequalities graphically.	Have you ever wondered how law enforcement agencies can track a cell phone user's location? A person using a cell phone can be located in respect to three cellular towers. The respective coordinates and distances each tower is from the caller are used to pinpoint the caller's location. This is accomplished using a system of quadratic equations.

CCSS Common Core State Standards

Content Standards

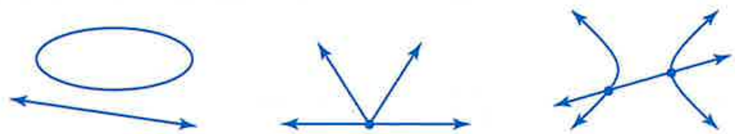
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Mathematical Practices

6 Attend to precision.

1 Systems of Equations

When a system of equations consists of a linear and a nonlinear equation, the system may have zero, one, or two solutions. Some of the possible solutions are shown below.



You can solve linear-quadratic systems by using graphical or algebraic methods.

Example 1 Linear-Quadratic System

Solve the system of equations.
$9x^2 + 25y^2 = 225$ (1)
$10y + 6x = 6$ (2)

Step 1 Solve the linear equation for y.

$10y + 6x = 6$	Equation (2)
$y = -0.6x + 0.6$	Solve for y.

Step 2 Substitute into the quadratic equation and solve for x.

$9x^2 + 25y^2 = 225$	Quadratic equation
$9x^2 + 25(-0.6x + 0.6)^2 = 225$	Substitute $-0.6x + 0.6$ for y.
$9x^2 + 25(0.36x^2 - 0.72x + 0.36) = 225$	Simplify.
$9x^2 + 9x^2 - 18x + 9 = 225$	Distribute.
$18x^2 - 18x - 216 = 0$	Simplify.
$x^2 - x - 12 = 0$	Divide each side by 18.
$(x - 4)(x + 3) = 0$	Factor.
$x = 4 \text{ or } -3$	Zero Product Property

Step 3 Substitute x-values into the linear equation and solve for y.

$y = -0.6x + 0.6$	Equation (2)	$y = -0.6x + 0.6$
$= -0.6(4) + 0.6$	Substitute the x-values	$= -0.6(-3) + 0.6$
$= -1.8$	Simplify.	$= 2.4$

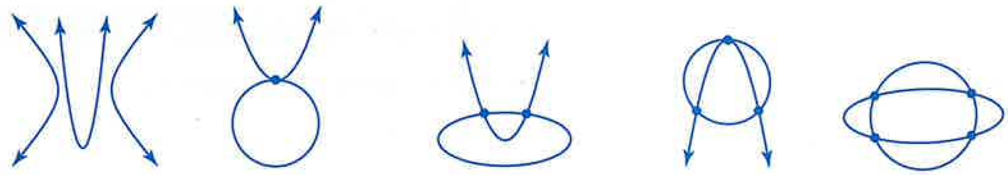
The solutions of the system are $(4, -1.8)$ and $(-3, 2.4)$.

Guided Practice

1A. $3y + x^2 - 4x - 17 = 0$
$3y - 10x + 38 = 0$

1B. $3(y - 4) - 2(x - 3) = -6$
$5x^2 + 2y^2 - 53 = 0$

If a quadratic system contains two conic sections, the system may have anywhere from zero to four solutions. Some graphical representations are shown below.



You can use elimination to solve quadratic-quadratic systems.

Example 2 Quadratic-Quadratic System

Solve the system of equations.

$x^2 + y^2 = 45$ (1)
$y^2 - x^2 = 27$ (2)

$y^2 + x^2 = 45$	Equation (1), Commutative Property
$(+)\ y^2 - x^2 = 27$	Equation (2)
$2y^2 = 72$	Add.
$y^2 = 36$	Divide each side by 2.
$y = \pm 6$	Take the square root of each side.

Substitute 6 and −6 into one of the original equations and solve for x.

$x^2 + y^2 = 45$	Equation (1)	$x^2 + y^2 = 45$
$x^2 + 6^2 = 45$	Substitute for y.	$x^2 + (-6)^2 = 45$
$x^2 = 9$	Subtract 36 from each side.	$x^2 = 9$
$x = \pm 3$	Take the square root of each side.	$x = \pm 3$

The solutions are $(-3, -6)$, $(-3, 6)$, $(3, -6)$, and $(3, 6)$.

StudyTip

CCSS Tools If you use ZSquare on the ZOOM menu, the graph of the first equation will look like a circle.

GuidedPractice

2A. $x^2 + y^2 = 8$
$x^2 + 3y = 10$

2B. $3x^2 + 4y^2 = 48$
$2x^2 - y^2 = -1$

2 Systems of Inequalities

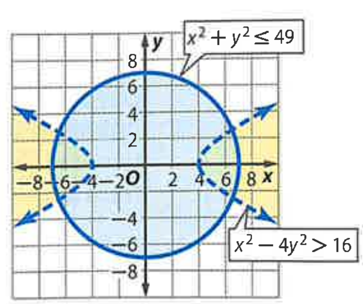
Systems of quadratic inequalities can be solved by graphing.

Example 3 Quadratic Inequalities

Solve the system of inequalities by graphing.

$x^2 + y^2 \leq 49$
$x^2 - 4y^2 > 16$

The intersection of the graphs, shaded green, represents the solution of the system.



CHECK (6, 0) is in the shaded area. Use this point to check your solution.

$x^2 + y^2 \leq 49$	$x^2 - 4y^2 > 16$
$6^2 + 0^2 \stackrel{?}{\leq} 49$	$62 - 4(0)^2 \stackrel{?}{>} 16$
$36 \leq 49$ ✓	$36 > 16$ ✓

GuidedPractice

3A. $5x^2 + 2y^2 \leq 10$
$y \geq x^2 - 2x + 1$

3B. $x^2 - y^2 \leq 8$
$x^2 + y^2 \geq 120$

Systems involving absolute value can also be solved by graphing.

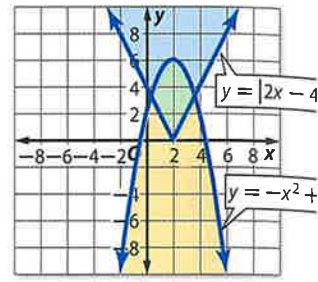
StudyTip

Graphing Calculator Like linear inequalities, systems of quadratic and absolute value inequalities can be checked with a graphing calculator.

Example 4 Quadratics with Absolute Value

Solve the system of inequalities by graphing.

$y \geq |2x - 4|$
$y \leq -x^2 + 4x + 2$



Graph the boundary equations. Then shade appropriately.

The intersection of the graphs, shaded green, represents the solution to the system.

CHECK (2, 4) is in the shaded area. Use the point to check your solution.

$y \geq |2x - 4|$	$y \leq -x^2 + 4x + 2$
$4 \overset{?}{\geq} |2(2) - 4|$	$4 \overset{?}{\leq} -(2)^2 + 4(2) + 2$
$4 \geq 0$ ✓	$4 \leq 6$ ✓

GuidedPractice

4A. $y > |-0.5x + 2|$
$\frac{x^2}{16} + \frac{y^2}{36} \leq 1$

4B. $x^2 + y^2 \leq 49$
$y \geq |x^2 + 1|$

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Examples 1–2 **Solve each system of equations.**

1. $8y = -10x$
$y^2 = 2x^2 - 7$

2. $x^2 + y^2 = 68$
$5y = -3x + 34$

3. $y = 12x - 30$
$4x^2 - 3y = 18$

4. $6y^2 - 27 = 3x$
$6y - x = 13$

5. $x^2 + y^2 = 16$
$x^2 - y^2 = 20$

6. $y^2 - 2x^2 = 8$
$3y^2 + x^2 = 52$

7. $x^2 + 2y = 7$
$y^2 - x^2 = 8$

8. $4y^2 - 3x^2 = 11$
$3y^2 + 2x^2 = 21$

9. **CCSS PERSEVERANCE** Refer to the beginning of the lesson. A person using a cell phone can be located with respect to three cellular towers. In a coordinate system where one unit represents one mile, the location of the caller is determined to be 50 miles from the tower at the origin. The person is also 40 miles from a tower at (0, 30) and 13 miles from a tower at (35, 18). Where is the caller?

Examples 3–4 **Solve each system of inequalities by graphing.**

10. $6x^2 + 9(y - 2)^2 \leq 36$
$x^2 + (y + 3)^2 \leq 25$

11. $16x^2 + 4y^2 \leq 64$
$y \geq -x^2 + 2$

12. $4x^2 - 8y^2 \geq 32$
$y \geq |1.5x| - 8$

13. $x^2 + 8y^2 < 32$
$y < -|x - 2| + 2$

Practice and Problem Solving

Extra Practice is on page R9.

Examples 1–2 **Solve each system of equations.**

14. $3x^2 - 2y^2 = -24$
$2y = -3x$

15. $5x^2 + 4y^2 = 20$
$5y = 7x + 35$

16. $x^2 + 3x = -4y - 2$
$y = -2x + 1$

17. $y = 2x$
$4x^2 - 2y^2 = -36$

18. $2y = x + 10$
$y^2 - 4y = 5x + 10$

19. $9y = 8x - 19$
$8x + 11 = 2y^2 + 5y$

20. $2y^2 + 5x^2 = 26$
$2x^2 - y^2 = 5$

21. $x^2 + y^2 = 16$
$x^2 - 4x + y^2 = 12$

22. $x^2 + y^2 = 8$
$5y^2 = 3x^2$

23. $y^2 - x^2 + 3y = 26$
$x^2 + 2y^2 = 34$

24. $x^2 - y^2 = 25$
$x^2 + y^2 + 7 = 0$

25. $x^2 - 10x + 2y^2 = 47$
$y^2 - 2x^2 = -14$

26. **FIREWORKS** Two fireworks are set off simultaneously but from different altitudes. The height y in feet of one is represented by $y = -16t^2 + 120t + 10$, where t is the time in seconds. The height of the other is represented by $y = -16t^2 + 60t + 310$.

a. After how many seconds are the fireworks the same height?

b. What is that height?

Examples 3–4 CCSS **TOOLS** **Solve each system of inequalities by graphing.**

27. $x^2 + y^2 \geq 36$
$x^2 + 9(y + 6)^2 \leq 36$

28. $-x > y^2$
$4x^2 + 14y^2 \leq 56$

29. $12x^2 - 4y^2 \geq 48$
$16(x - 4)^2 + 25y^2 < 400$

30. $8y^2 - 3x^2 \leq 24$
$2y > x^2 - 8x + 14$

31. $y > x^2 - 6x + 8$
$x \geq y^2 - 6y + 8$

32. $x^2 + y^2 \geq 9$
$25x^2 + 64y^2 \leq 1600$

33. $16(x - 3)^2 + 4y^2 \leq 64$
$y \leq -|x - 2| + 2$

34. $x^2 - 4x + y^2 + 6y \leq 23$
$y > |x - 2| - 6$

35. $2y - 4 \geq |x + 4|$
$12 - 2y > x^2 + 12x + 36$

36. $18y^2 - 3x^2 \leq 54$
$y \geq |2x| - 6$

37. $x^2 + y^2 < 16$
$y \geq |x - 2| + 6$

38. $x^2 < y - 2$
$y \leq |x + 8| - 7$

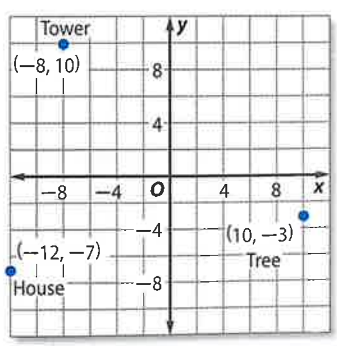
39. **SPACE** Two satellites are placed in orbit about Earth. The equations of the two orbits are $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$ and $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$, where distances are in kilometers and Earth is the center of each curve.

a. Solve each equation for y.

b. Use a graphing calculator to estimate the intersection points of the two orbits.

c. Compare the orbits of the two satellites.

40. **PETS** Taci's dog was missing one day. Fortunately, he was wearing an electronic monitoring device. If the dog is 10 units from the tree, 13 units from the tower, and 20 units from the house, determine the coordinates of his location.



41. **BASEBALL** In 1997, after Mark McGwire hit a home run, the claim was made that the ball would have traveled 538 feet if it had not landed in the stands. The path of the baseball can be modeled by $y = -0.0037x^2 + 1.77x - 1.72$ and the stands can be modeled by $y = \frac{3}{7}x - 128.6$. How far vertically and horizontally from home plate did the ball land in the stands?

42. **ADVERTISING** The corporate logo for an automobile manufacturer is shown at the right. Write a system of three equations to model this logo.

Write a system of equations that satisfies each condition.

43. a circle and an ellipse that intersect at one point

44. a parabola and an ellipse that intersect at two points

45. a hyperbola and a circle that do not intersect

46. an ellipse and a parabola that intersect at three points

47. an ellipse and a hyperbola that intersect at four points

48. **FINANCIAL LITERACY** Prices are often set on an equilibrium curve, where the supply of a certain product equals its corresponding demand by consumers. An economist represents the supply of a product with $y = p^2 + 10p$ and the corresponding demand with $y = -p^2 + 40p$, where p is the price. Determine the equilibrium price.

49. **PAINTBALL** The shape of a paintball field is modeled by $x^2 + 4y^2 = 10{,}000$ in yards where the center is at the origin. The teams are provided with short-range walkie-talkies with a maximum range of 80 yards. Are the teams capable of hearing each other anywhere on the field? Explain your reasoning graphically.

50. **MOVING** Lena is moving to a new city and needs for the location of her new home to satisfy the following conditions.

- It must be less than 10 miles from the office where she will work.
- Because of the terrible smell of the local paper mill, it must be at least 15 miles away from the mill.

If the paper mill is located 9.5 miles east and 6 miles north of Lena's office, write and graph a system of inequalities to represent the area(s) were she should look for a home.

H.O.T. Problems Use Higher-Order Thinking Skills

51. **CHALLENGE** Find all values of k for which the following system of equations has two solutions.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad x^2 + y^2 = k^2$$

52. **CCSS ARGUMENTS** When the vertex of a parabola lies on an ellipse, how many solutions can the quadratic system represented by the two graphs have? Explain your reasoning using graphs.

53. **OPEN ENDED** Write a system of equations, one a hyperbola and the other an ellipse, for which a solution is $(-4, 8)$.

54. **WRITING IN MATH** Explain how sketching the graph of a quadratic system can help you solve it.

Standardized Test Practice

55. SHORT RESPONSE Solve.

$$4x - 3y = 0$$
$$x^2 + y^2 = 25$$

56. You have 16 stamps. Some are postcard stamps that cost \$0.23, and the rest cost \$0.41. If you spent a total of \$5.30 on the stamps, how many postcard stamps do you have?

A 7
B 8
C 9
D 10

57. Ms. Talbot received a promotion and a 7.2% raise. Her new salary is \$53,600 a year. What was her salary before the raise?

F \$50,000
G \$53,600
H \$55,000
J \$57,500

58. SAT/ACT When a number is multiplied by $\frac{2}{3}$, the result is 188. Find the number.

A 292
B 282
C 272
D 262
E $125\frac{1}{3}$

Spiral Review

Match each equation with the situation that it could represent. (Lesson 9-6)

a. $9x^2 + 4y^2 - 36 = 0$
b. $0.004x^2 - x + y - 3 = 0$
c. $x^2 + y^2 - 20x + 30y - 75 = 0$

59. SPORTS the flight of a baseball

60. PHOTOGRAPHY the oval opening in a picture frame

61. GEOGRAPHY the set of all points 20 miles from a landmark

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola. (Lesson 9-5)

62. $\frac{y^2}{16} - \frac{x^2}{25} = 1$

63. $\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$

64. $6y^2 = 2x^2 + 12$

Simplify each expression. (Lesson 8-1)

65. $\frac{12p^2 + 6p - 6}{4(p+1)^2} \div \frac{6p - 3}{2p + 10}$

66. $\frac{x^2 + 6x + 9}{x^2 + 7x + 6} \div \frac{4x + 12}{3x + 3}$

67. $\frac{r^2 + 2r - 8}{r^2 + 4r + 3} \div \frac{r - 2}{3r + 3}$

Graph each function. State the domain and range. (Lesson 7-1)

68. $f(x) = -\left(\frac{1}{5}\right)^x$

69. $y = -2.5(5)^x$

70. $f(x) = 2\left(\frac{1}{3}\right)^x$

Skills Review

Solve each equation or formula for the specified variable.

71. $d = rt$, for r

72. $x = \frac{-b}{2a}$, for a

73. $V = \frac{1}{3}\pi r^2 h$, for h

74. $A = \frac{1}{2}h(a + b)$, for b

CHAPTER 9

Study Guide and Review

Study Guide

KeyConcepts

Midpoint and Distance Formulas (Lesson 9-1)

- $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Parabolas (Lesson 9-2)

- Standard Form: $y = a(x - h)^2 + k$
  $x = a(y - k)^2 + h$

Circles (Lesson 9-3)

- The equation of a circle with center (h, k) and radius r can be written in the form $(x - h)^2 + (y - k)^2 = r^2$.

Ellipses (Lesson 9-4)

- Standard Form: horizontal $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
  vertical $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

Hyperbolas (Lesson 9-5)

- Standard Form: horizontal $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
  vertical $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Solving Linear-Nonlinear Systems (Lesson 9-7)

- Systems of quadratic equations can be solved using substitution and elimination.
- A system of quadratic equations can have zero, one, two, three, or four solutions.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

center (of a circle) (p. 607)
center (of an ellipse) (p. 615)
circle (p. 607)
conjugate axis (p. 624)
constant difference (p. 627)
constant sum (p. 616)
co-vertices (of a hyperbola) (p. 624)
co-vertices (of an ellipse) (p. 615)
directrix (p. 599)
ellipse (p. 615)
foci (of a hyperbola) (p. 624
foci (of an ellipse) (p. 615)
focus (p. 599)
hyperbola (p. 624)
latus rectum (p. 599)
major axis (p. 615)
minor axis (p. 615)
parabola (p. 599)
radius (p. 607)
transverse axis (p. 624)
vertices (of a hyperbola) (p.
vertices (of an ellipse) (p. 6

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replac
underlined term to make a true sentence.

1. The set of all points in a plane that are equidistant from a given point in the plane, called the focus, forms a circle.
2. A(n) ellipse is the set of all points in a plane such that the of the distances from the two fixed points is constant.
3. The endpoints of the major axis of an ellipse are the foci the ellipse.
4. The radius is the distance from the center of a circle to a point on the circle.
5. The line segment with endpoints on a parabola, through focus of the parabola, and perpendicular to the axis of symmetry is called the latus rectum.
6. Every hyperbola has two axes of symmetry, the transvers and the major axis.
7. A directrix is the set of all points in a plane that are equid from a given point in the plane, called the center.
8. A hyperbola is the set of all points in a plane such that th absolute value of the sum of the distances from any poir the hyperbola to two given points is constant.
9. A parabola can be defined as the set of all points in a pla that are the same distance from the focus and a given lir called the directrix.
10. The major axis is the longer of the two axes of symmetry an ellipse.

Lesson-by-Lesson Review

9-1 Midpoint and Distance Formulas

Find the midpoint of the line segment with endpoints at the given coordinates.

11. $(-8, 6), (3, 4)$

12. $(-6, 0), (-1, 4)$

13. $\left(\frac{3}{4}, \frac{2}{3}\right), \left(-\frac{1}{3}, \frac{1}{4}\right)$

14. $(15, 20), (18, 21)$

Find the distance between each pair of points with the given coordinates.

15. $(10, -3), (1, -5)$

16. $(0, 6), (-9, 7)$

17. $\left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{3}{2}, \frac{5}{4}\right)$

18. $(5, -3), (7, -1)$

19. HIKING Marc wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

a. How far away is the waterfall?

b. Marc wants to stop for lunch halfway to the waterfall. Where should he stop?

Example 1

Find the midpoint of a line segment whose endpoints are at (−4, 8) and (10, −1).

Let $(x_1, y_1) = (-4, 8)$ and $(x_2, y_2) = (10, -1)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + 10}{2}, \frac{8 + (-1)}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{7}{2}\right) \text{ or } \left(3, \frac{7}{2}\right)$$

Example 2

Find the distance between $P(5, -3)$ and $Q(-1, 5)$.
Let $(x_1, y_1) = (5, -3)$ and $(x_2, y_2) = (-1, 5)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-1 - 5)^2 + [5 - (-3)]^2}$ Substitute.

$= \sqrt{36 + 64}$ Subtract.

$= \sqrt{100}$ or 10 units Simplify.

9-2 Parabolas

Graph each equation.

20. $y = 3x^2 + 24x - 10$

21. $3y - x^2 = 8x - 11$

22. $x = \frac{1}{2}y^2 - 4y + 3$

23. $x = y^2 - 14y + 25$

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

24. $y = -\frac{1}{2}x^2$

25. $y = 4x^2 - 16x + 9$

26. $x - 6y = y^2 + 4$

27. $x = y^2 + 14y + 20$

28. SPORTS When a football is kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 50 feet, and lands 200 feet away. Assuming the football was kicked at the origin, write an equation of the parabola that models the flight of the football.

Example 3

Write $3y - x^2 = 4x + 7$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

Write the equation in the form $y = a(x - h)^2 + k$ by completing the square.

$3y = x^2 + 4x + 7$ Isolate the terms with x.

$3y = (x^2 + 4x + \blacksquare) + 7 - \blacksquare$ Complete the square.

$3y = (x^2 + 4x + 4) + 7 - 4$ $\left(\frac{4}{2}\right)^2 = 4$

$3y = (x + 2)^2 + 3$ $(x^2 + 4x + 4) = (x + 2)$

$y = \frac{1}{3}(x + 2)^2 + 1$ Divide each side by 3.

Vertex: $(-2, 1)$; axis of symmetry: $x = -2$; direction of opening: upward since $a > 0$.

Chapter 9 Study Guide and Review *Continued*

9-3 Circles

Write an equation for the circle that satisfies each set of conditions.

29. center $(-1, 6)$, radius 3 units

30. endpoints of a diameter $(2, 5)$ and $(0, 0)$

31. endpoints of a diameter $(4, -2)$ and $(-2, -6)$

Find the center and radius of each circle. Then graph the circle.

32. $(x + 5)^2 + y^2 = 9$

33. $(x - 3)^2 + (y + 1)^2 = 25$

34. $(x + 2)^2 + (y - 8)^2 = 1$

35. $x^2 + 4x + y^2 - 2y - 11 = 0$

36. **SOUND** A loudspeaker in a school is located at the point (65, 40). The speaker can be heard in a circle with a radius of 100 feet. Write an equation to represent the possible boundary of the loudspeaker sound.

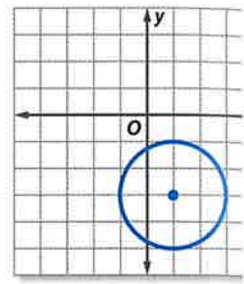
Example 4

Find the center and radius of the circle with equation $x^2 - 2x + y^2 + 6y + 6 = 0$. Then graph the circle.

Complete the squares.

$$x^2 - 2x + y^2 + 6y + 6 = 0$$
$$(x^2 - 2x + \blacksquare) + (y^2 + 6y + \blacksquare) = -6 + \blacksquare + \blacksquare$$
$$(x^2 - 2x + 1) + (y^2 + 6y + 9) = -6 + 1 + 9$$
$$(x - 1)^2 + (y + 3)^2 = 4$$

The center of the circle is at $(1, -3)$ and the radius is 2.



9-4 Ellipses

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

37. $\frac{x^2}{9} + \frac{y^2}{36} = 1$

38. $\frac{y^2}{10} + \frac{x^2}{5} = 1$

39. $\frac{x^2}{36} + \frac{(y - 4)^2}{4} = 1$

40. $27x^2 + 9y^2 = 81$

41. $\frac{(x + 1)^2}{25} + \frac{(y - 2)^2}{16} = 1$

42. $9x^2 + 4y^2 + 54x - 8y + 49 = 0$

43. $9x^2 + 25y^2 - 18x + 50y - 191 = 0$

44. $7x^2 + 3y^2 - 28x - 12y = -19$

45. **LANDSCAPING** The Martins have a garden in their front yard that is shaped like an ellipse. The major axis is 16 feet and the minor axis is 10 feet. Write an equation to model the garden. Assume the origin is at the center of the garden and the major axis is horizontal.

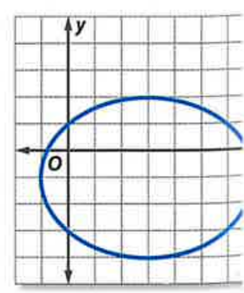
Example 5

Find the coordinates of the center and foci and the lengt of the major and minor axes for the ellipse with equatio $9x^2 + 16y^2 - 54x + 32y - 47 = 0$. Then graph the ell

First, convert to standard form.

$$9x^2 + 16y^2 - 54x + 32y - 47 = 0$$
$$9(x^2 - 6x + \blacksquare) + 16(y^2 + 2y + \blacksquare) = 47 + 9(\blacksquare) + 16$$
$$9(x^2 - 6x + 9) + 16(y^2 + 2y + 1) = 47 + 9(9) + 16$$
$$9(x - 3)^2 + 16(y + 1)^2 = 144$$
$$\frac{(x - 3)^2}{16} + \frac{(y + 1)^2}{9} = 1$$

The center of the ellipse is $(3, -1)$. The ellipse is horizontal. $a^2 = 16$, so $a = 4$. $b^2 = 9$, so $b = 3$. The length of the major axis is $2 \cdot 4$ or 8. The length of the minor axis is $2 \cdot 3$ or 6. To find the foci: $c^2 = 16 - 9$ or 7, so $c = \sqrt{7}$. The foci are $(3 + \sqrt{7}, -1)$ and $(3 - \sqrt{7}, -1)$.



9-5 Hyperbolas

Graph each hyperbola. Identify the vertices, foci, and asymptotes.

46. $\frac{y^2}{9} - \frac{x^2}{4} = 1$

47. $\frac{(x-3)^2}{1} - \frac{(y+2)^2}{4} = 1$

48. $\frac{(y+1)^2}{16} - \frac{(x-4)^2}{9} = 1$

49. $4x^2 - 9y^2 = 36$

50. $9y^2 - x^2 - 4x + 18y + 4 = 0$

51. **MIRRORS** A hyperbolic mirror is shaped like one branch of a hyperbola. It reflects light rays directed at one focus toward the other focus. Suppose a hyperbolic mirror is modeled by the upper branch of the hyperbola $\frac{y^2}{9} - \frac{x^2}{16} = 1$. A light source is located at $(-10, 0)$. Where should the light hit the mirror so that the light will be reflected to $(0, -5)$?

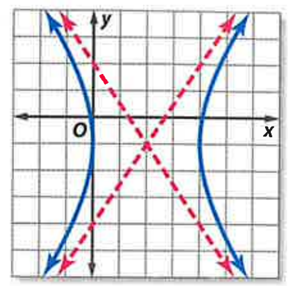
Example 6

Graph $9x^2 - 4y^2 - 36x - 8y - 4 = 0$. Identify the vertices, foci, and asymptotes.

Complete the square.

$$9x^2 - 4y^2 - 36x - 8y - 4 = 0$$
$$9(x^2 - 4x + \blacksquare) - 4(y^2 + 2y + \blacksquare) = 4 + 9(\blacksquare) - 4(\blacksquare)$$
$$9(x^2 - 4x + 4) - 4(y^2 + 2y + 1) = 4 + 9(4) - 4(1)$$
$$9(x-2)^2 - 4(y+1)^2 = 36$$
$$\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$$

The center is at $(2, -1)$. The vertices are at $(0, -1)$ and $(4, -1)$. The foci are at $\left(2 + \sqrt{13}, -1\right)$ and $\left(2 - \sqrt{13}, -1\right)$. The equations of the asymptotes are $y + 1 = \pm\frac{3}{2}(x - 2)$



9-6 Identifying Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph.

52. $3x^2 + 12x - y + 8 = 0$

53. $9x^2 + 16y^2 = 144$

54. $x^2 + y^2 - 8x - 2y + 8 = 0$

55. $-9x^2 + y^2 + 36x - 45 = 0$

Without writing the equation in standard form, state whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*.

56. $7x^2 + 9y^2 = 63$

57. $5y^2 + 2y + 4x - 13x^2 = 81$

58. $x^2 - 8x + 16 = 6y$

59. $x^2 + 4x + y^2 - 285 = 0$

60. **LIGHT** Suppose the edge of a shadow can be represented by the equation $16x^2 + 25y^2 - 32x - 100y - 284 = 0$.

a. What is the shape of the shadow?

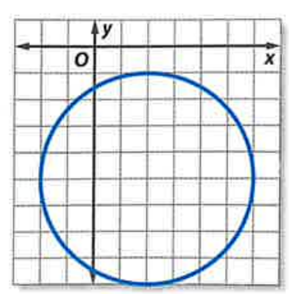
b. Graph the equation.

Example 7

Write $3x^2 + 3y^2 - 12x + 30y + 39 = 0$ in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation.

$$3x^2 + 3y^2 - 12x + 30y + 39 = 0$$
$$3(x^2 - 4x + \blacksquare) + 3(y^2 + 10y + \blacksquare) = -39 + 3(\blacksquare) + 3(\blacksquare)$$
$$3(x^2 - 4x + 4) + 3(y^2 + 10y + 25) = -39 + 3(4) + 3(25)$$
$$3(x-2)^2 + 3(y+5)^2 = 48$$
$$(x-2)^2 + (y+5)^2 = 16$$

In this equation $A = 3$ and $C = 3$. Since A and C are both positive and $A = C$, the graph is a circle. The center is at $(2, -5)$, and the radius is 4.



9-7 Solving Linear-Nonlinear Systems

Solve each system of equations.

61. $x^2 + y^2 = 8$
$x + y = 0$

62. $x - 2y = 2$
$y^2 - x^2 = 2x + 4$

63. $y + x^2 = 4x$
$y + 4x = 16$

64. $3x^2 - y^2 = 11$
$x^2 + 4y^2 = 8$

65. $5x^2 + y^2 = 30$
$9x^2 - y^2 = -16$

66. $\frac{x^2}{30} + \frac{y^2}{6} = 1$
$x = y$

67. **PHYSICAL SCIENCE** Two balls are launched into the air at the same time. The heights they are launched from are different. The height y in feet of one is represented by $y = -16t^2 + 80t + 25$ where t is the time in seconds. The height of the other ball is represented by $y = -16t^2 + 30t + 100$.

a. After how many seconds are the balls at the same height?

b. What is this height?

68. **ARCHITECTURE** An architect is building the front entrance of a building in the shape of a parabola with the equation $y = -\frac{1}{10}(x - 10)^2 + 20$. While the entrance is being built, the construction team puts in two support beams with equations $y = -x + 10$ and $y = x - 10$. Where do the support beams meet the parabola?

Solve each system of inequalities by graphing.

69. $x^2 + y^2 < 64$
$x^2 + 16(y - 3)^2 < 16$

70. $x^2 + y^2 < 49$
$16x^2 - 9y^2 \geq 144$

71. $x + y < 4$
$9x^2 - 4y^2 \geq 36$

72. $x^2 + y^2 < 25$
$4x^2 - 9y^2 < 36$

73. $x^2 + y^2 < 36$
$4x^2 + 9y^2 > 36$

74. $y^2 < x$
$x^2 - 4y^2 < 16$

Example 8

Solve the system of equations.

$x^2 + y^2 = 100$
$3x - y = 10$

Use substitution to solve the system.
First, rewrite $3x - y = 10$ as $y = 3x - 10$.

$$x^2 + y^2 = 100$$
$$x^2 + (3x - 10)^2 = 100$$
$$x^2 + 9x^2 - 60x + 100 = 100$$
$$10x^2 - 60x + 100 = 100$$
$$10x^2 - 60x = 0$$
$$10x(x - 6) = 0$$

$10x = 0$ or $x - 6 = 0$
$x = 0$ $\quad x = 6$

Now solve for y.

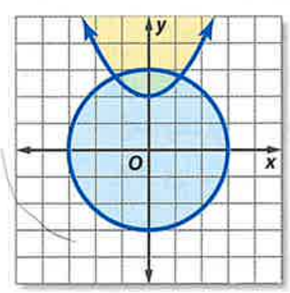
$y = 3x - 10$ $\quad y = 3x - 10$
$= 3(0) - 10$ $\quad = 3(6) - 10$
$= -10$ $\quad = 8$

The solutions of the system are $(0, -10)$ and $(6, 8)$

Example 9

Solve the system of inequalities by graphing.

$x^2 + y^2 \leq 9$
$2y \geq x^2 + 4$



The solution is the green shaded region.

Chapter 9 Practice Test

Find the midpoint of the line segment with endpoints at the given coordinates.

1. $(8, 3), (-4, 9)$
2. $\left(\frac{3}{4}, 0\right), \left(\frac{1}{2}, -1\right)$
3. $(-10, 0), (-2, 6)$

Find the distance between each pair of points with the given coordinates.

4. $(-5, 8), (4, 3)$
5. $\left(\frac{1}{3}, \frac{2}{3}\right), \left(-\frac{5}{6}, -\frac{11}{6}\right)$
6. $(4, -5), (4, 9)$

State whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

7. $y^2 = 64 - x^2$
8. $4x^2 + y^2 = 16$
9. $4x^2 - 9y^2 + 8x + 36y = 68$
10. $\frac{1}{2}x^2 - 3 = y$
11. $y = -2x^2 - 5$
12. $16x^2 + 25y^2 = 400$
13. $x^2 + 6x + y^2 = 16$
14. $\frac{y^2}{4} - \frac{x^2}{16} = 1$
15. $(x + 2)^2 = 3(y - 1)$
16. $4x^2 + 16y^2 + 32x + 63 = 0$

17. **MULTIPLE CHOICE** Which equation represents a hyperbola that has vertices at $(-3, -3)$ and $(5, -3)$ and a conjugate axis of length 6 units?

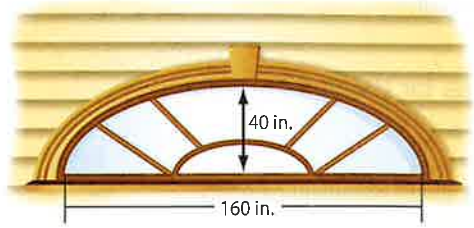
    A $\frac{(y-1)^2}{16} - \frac{(x+3)^2}{9} = 1$

    B $\frac{(x-1)^2}{16} - \frac{(y+3)^2}{9} = 1$

    C $\frac{(y+1)^2}{16} - \frac{(x-3)^2}{9} = 1$

    D $\frac{(x+1)^2}{16} - \frac{(y-3)^2}{9} = 1$

18. **CARPENTRY** Ellis built a window frame shaped like the top half of an ellipse. The window is 40 inches tall at its highest point and 160 inches wide at the bottom. What is the height of the window 20 inches from the center of the base?



Solve each system of equations.

19. $x^2 + y^2 = 100$
    $y = -x - 2$
20. $x^2 + 2y^2 = 11$
    $x + y = 2$
21. $x^2 + y^2 = 34$
    $y^2 - x^2 = 9$

Solve each system of inequalities.

22. $x^2 + y^2 \leq 9$
    $y > -x^2 + 2$
23. $\frac{(x-2)^2}{4} - \frac{(y-4)^2}{9} \geq 1$
    $x - 4y < 8$

24. **MULTIPLE CHOICE** Which is NOT the equation of a parabola?

    F $y = 3x^2 + 5x - 3$

    G $2y + 3x^2 + x - 9 = 0$

    H $x = 3(y + 1)^2$

    J $x^2 + 2y^2 + 6x = 10$

25. **FORESTRY** A forest ranger at an outpost in the Sam Houston National Forest and another ranger at the primary station both heard an explosion. The outpost and the primary station are 6 kilometers apart.

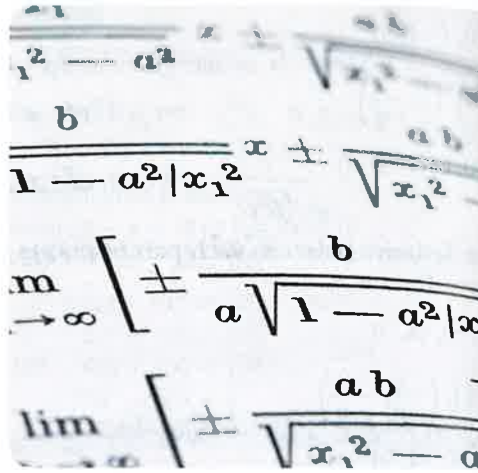
    a. If one ranger heard the explosion 6 seconds before the other, write an equation that describes all the possible locations of the explosion. Place the two ranger stations on the x-axis with the midpoint between the stations at the origin. The transverse axis is horizontal. (*Hint*: The speed of sound is about 0.35 kilometer per second.)

    b. Draw a sketch of the possible locations of the explosion. Include the ranger stations in the drawing.

Preparing for Standardized Tests

Use a Formula

Sometimes it is necessary to use a formula to solve problems on standardized tests. In some cases you may even be given a sheet of formulas that you are permitted to reference while taking the test.



Strategies for Using a Formula

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- Are there any formulas that I can use to help me solve the problem?

Step 2

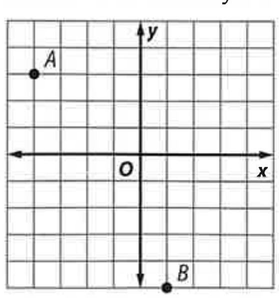
Solve the problem and check your solution.

- Substitute the known quantities that are given in the problem statement into the formula.
- Simplify to solve for the unknown values in the formula.
- Check to make sure your answer makes sense. If time permits, check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

What is the distance between points A and B on the coordinate plane? Round your answer to the nearest tenth if necessary.



Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct but the explanation is incomplete. • The answer is incorrect but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Read the problem statement carefully. You are given the coordinates of two points on a coordinate plane and asked to find the distance between them. To solve this problem, you must use the **Distance Formula**.

Example of a 2-point response:

Use the Distance Formula to find the distance between points $A(-4, 3)$ and $B(1, -5)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[1 - (-4)]^2 + [(-5) - 3]^2}$$

$$= \sqrt{5^2 + (-8)^2}$$

$$= \sqrt{25 + 64}$$

$$= \sqrt{89} \text{ or about } 9.4$$

The distance between points A and B is about 9.4 units.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

Exercises

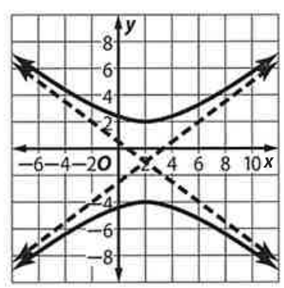
Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. What is the midpoint of segment CD with endpoints $C(5, -12)$ and $D(-9, 4)$?

2. Katrina is making a map of her hometown on a coordinate plane. She plots the school at $S(7, 3)$ and the park at $P(-4, 12)$. If the scale of the map is 1 unit = 250 yards, what is the actual distance between the school and the park? Round to the nearest yard.

3. Mr. Washington is making a concrete table for his backyard. The tabletop will be circular with a diameter of 6 feet and a depth of 6 inches. How much concrete will Mr. Washington need to make the top of the table? Round to the nearest cubic foot.

4. What is the equation, in standard form, of the hyperbola graphed below?



5. If the surface area of a cube is increased by a factor of 9, what is the change in the length of the sides of the cube?

   A The length is 2 times the original length.

   B The length is 3 times the original length.

   C The length is 6 times the original length.

   D The length is 9 times the original length.

Standardized Test Practice

Cumulative, Chapters 1 through 9

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which is the first *incorrect* step in simplifying $\log_3 \frac{3}{48}$?

   Step 1: $\log_3 \frac{3}{48} = \log_3 3 - \log_3 48$
   Step 2: $= 1 - 16$
   Step 3: $= -15$

   A Step 1 **C** Step 3
   B Step 2 **D** Each step is correct.

2. Which is the equation for the parabola that has vertex $(-3, -23)$ and passes through the point $(1, 9)$?

   F $y = x^2 + 10x + 7$
   G $y = x^2 - 6x + 19$
   H $y = 2x^2 + 12x - 5$
   J $y = 2x^2 - 3x + 10$

3. What are the vertices of the ellipse with equation $\frac{(x-3)^2}{36} + \frac{(y-2)^2}{144} = 1$?

   A $(-3, 2)$ and $(9, 2)$
   B $(-2, 3)$ and $(10, 3)$
   C $(3, -10)$ and $(3, 14)$
   D $(2, -11)$ and $(4, 13)$

4. Hooke's Law states that the force needed to keep a spring stretched x units is directly proportional to x. If a force of 40 N is required to maintain a spring stretched to 5 centimeters, what force is needed to keep the spring stretched 14 centimeters?

   F 8 N **H** 112 N
   G 19 N **J** 1600 N

Test-TakingTip

Question 2 You can check your answer by submitting 1 for x and making sure that the y-value is 9.

5. Angela is making a map of her backyard on a coordinate grid. She plots point $G(-4, -6)$ to represent her mom's garden and point $S(3, 7)$ represent the rope swing hanging on an oak tr If the scale of the map is 1 unit = 5 feet, what i the approximate distance between the garden the rope swing?

   A 74 feet **C** 82 feet
   B 79 feet **D** 90 feet

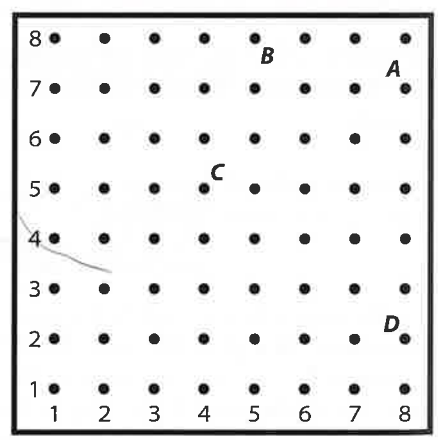
6. If $\sqrt{x+5} + 1 = 4$, what is the value of x?

   F 4 **G** 10 **H** 11 **J** 20

7. The area of the base of a rectangular suitcase measures $3x^2 + 5x - 4$ square units. The heigh the suitcase measures $2x$ units. Which polynor expression represents the volume of the suitca

   A $3x^3 + 5x^2 - 4x$ **C** $6x^3 + 10x^2 - 8x$
   B $6x^2 + 10x - 8$ **D** $3x^3 + 10x^2 - 4$

8. Malina was given this geoboard to model the slope $-\frac{3}{4}$.



   If the peg in the upper right-hand corner repre the origin on a coordinate plane, where could M place a rubber band to represent the given slop

   F from peg *A* to peg *B*
   G from peg *A* to peg *C*
   H from peg *B* to peg *D*
   J from peg *C* to peg *D*

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. A placekicker kicks a ball upward with a velocity of 32 feet per second. Ignoring the height of the kicking tee, how long after the football is kicked does it hit the ground? Use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

10. GRIDDED RESPONSE What is the maximum number of solutions of a system of equations that consists of a circle and a hyperbola?

11. Lupe is preparing boxes of assorted chocolates. Chocolate-covered peanuts cost \$7 per pound. Chocolate-covered caramels cost \$6.50 per pound. The boxes of assorted candies contain five more pounds of peanut candies than caramel candies. If the total amount sold was \$575, how many pounds of each candy were needed to make the boxes?

12. GRIDDED RESPONSE What is the y-coordinate of the midpoint of segment AB with endpoints $A(0.8, 5.32)$ and $B(0.44, 2.2)$?

13. Marc went shopping and bought two shirts, three pairs of pants, one belt, and two pairs of shoes. The following matrix shows the prices for each item respectively.

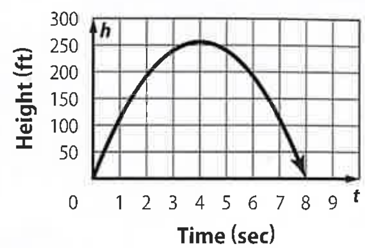
$$\begin{bmatrix} \$20.15 & \$32 & \$15 & \$25.99 \end{bmatrix}$$

Use matrix multiplication to find the total amount of money Marc spent while shopping.

Extended Response

Record your answers on a sheet of paper. Show your work.

14. Clarence graphed the quadratic equation $h(t) = -16t^2 + 128t$ to model the flight of a firework. The parabola shows the height, in feet, of the firework t seconds after it was launched.



a. What is the vertex of the parabola?

b. What does the vertex of the parabola represent?

c. How long is the firework in the air before it lands?

15. The Colonial High School Yearbook Staff is selling yearbooks and chrome picture frames engraved with the year. The number of yearbooks and frames sold to members of each grade is shown in the table.

Sales for Each Class		
Grade	**Yearbooks**	**Frames**
9th	423	256
10th	464	278
11th	546	344
12th	575	497

a. Find the difference in the sales of yearbooks and frames made to the 10th and 11th grade classes.

b. Find the total number of yearbooks and frames sold.

c. A yearbook costs \$48 and a frame costs \$18. Find the sales of yearbooks and frames for each class.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	7-5	9-2	9-4	8-5	9-1	6-7	5-1	2-3	4-2	9-7	3-1	9-1	3-6	9-2	1-3