CHAPTER 10

Radical Functions and Geometry

Then

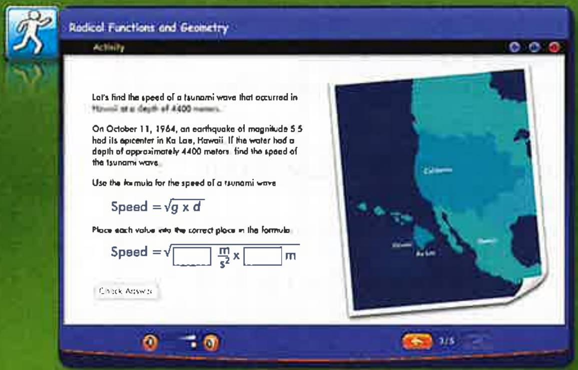
- You solved quadratic and exponential equations.

Now

- In this chapter, you will:
  - Graph and transform radical functions.
  - Simplify, add, subtract, and multiply radical expressions.
  - Solve radical equations.
  - Use the Pythagorean Theorem.
  - Find trigonometric ratios.

Why? ▲

- **OCEANS** Tsunamis, or large waves, are generated by undersea earthquakes. A radical equation can be used to find the speed of a tsunami in meters per second or the depth of the ocean in meters.



Let's find the speed of a tsunami wave that occurred in Hawaii at a depth of 4400 meters.

On October 11, 1964, an earthquake of magnitude 5.5 had its epicenter in Ka Lae, Hawaii. If the water had a depth of approximately 4400 meters, find the speed of the tsunami wave.

Use the formula for the speed of a tsunami wave.

$$\text{Speed} = \sqrt{g \times d}$$

Place each value into the correct place in the formula.

$$\text{Speed} = \sqrt{\boxed{}\ \frac{m}{s^2} \times \boxed{}\ m}$$

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Jean-Pierre Pieuchot/Photodisc/Getty Images

Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 **Textbook Option** Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Find each square root. If necessary, round to the nearest hundredth.

1. $\sqrt{82}$

2. $\sqrt{26}$

3. $\sqrt{15}$

4. $\sqrt{99}$

5. **SANDBOX** Isaac is making a square sandbox with an area of 100 square feet. How long is a side of the sandbox?

QuickReview

Example 1

Find the square root of $\sqrt{50}$. If necessary, round to the nearest hundredth.

$\sqrt{50} = 7.071067812\ldots.$ Use a calculator.

To the nearest hundredth, $\sqrt{50} = 7.07$.

Simplify each expression.

6. $(21x + 15y) - (9x - 4y)$

7. $13x - 5y + 2y$

8. $(10a - 5b) + (6a + 5b)$

9. $6m + 5n + 4 - 3m - 2n + 6$

10. $x + y - 3x - 4y + 2x - 8y$

Example 2

Simplify $3x + 7y - 4x - 8y$.

$3x + 7y - 4x - 8y$

$= (3x - 4x) + (7y - 8y)$ Combine like terms.

$= -x - y$ Simplify.

Solve each equation.

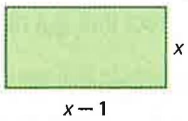
11. $2x^2 - 4x = 0$

12. $6x^2 - 5x - 4 = 0$

13. $x^2 - 7x + 10 = 0$

14. $2x^2 + 7x - 5 = -1$

15. **GEOMETRY** The area of the rectangle is 90 square feet. Find x.



Example 3

Solve $x^2 - 5x + 6 = 0$.

$x^2 - 5x + 6 = 0$ Original equation

$(x - 3)(x - 2) = 0$ Factor.

$x - 3 = 0$ or $x - 2 = 0$ Zero Product Property

$x = 3 \qquad x = 2$ Solve each equation.

2 **Online Option** Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 10. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

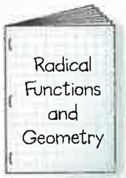
FOLDABLES StudyOrganizer

Radical Functions and Geometry Make this Foldable to help you organize your Chapter 10 notes about radical functions and geometry. Begin with four sheets of grid paper.

1 **Fold** in half along the width.

2 **Staple** along the fold.

3 **Turn** the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.



NewVocabulary

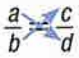
English		Español
radicand	p. 621	radicando
radical function	p. 621	función radicales
conjugate	p. 630	conjugado
radical equations	p. 642	ecuaciones radicales
hypotenuse	p. 648	hipotenusa
legs	p. 648	catetos
converse	p. 649	recíproco
midpoint	p. 654	punto medio
sine	p. 656	seno
cosine	p. 656	coseno
tangent	p. 656	tangente
trigonometry	p. 656	trigonometría
inverse cosine	p. 658	coseno inverso
inverse sine	p. 658	seno inverso
inverse tangent	p. 658	tangente inverse

ReviewVocabulary

FOIL method metodo FOIL to multiply two binomials, find the sum of the products of the First terms, Outer terms, Inner terms, and Last terms

perfect square cuadrado perfecto a number with a square root that is a rational number

proportion proporcion an equation of the form $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$, $d \neq 0$ stating that two ratios are equivalent



$ad = bc$

EXPLORE

10-1

Algebra Lab
Inverse Functions

You have discovered that every nonhorizontal linear function has an inverse function. You have learned how to find the inverse of any function by exchanging the coordinates for a set of ordered pairs. In the following activity, we will exchange coordinates to find the inverse of a quadratic function and determine whether the inverse is a function.

CCSS Common Core State Standards
Content Standards
F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Activity 1 Exchange Coordinates

Find the inverse of $y = x^2$ by exchanging the coordinates. Is the inverse a function?

Step 1 Make a table of values for $y = x^2$ using x from −3 to 3.

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

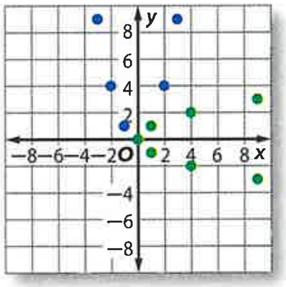
Step 2 Write the coordinates as a set of ordered pairs.

$\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$

Step 3 Exchange the x- and y-coordinates in each ordered pair to form the inverse.

$\{(9, -3), (4, -2), \mathbf{(1, -1)}, (0, 0), \mathbf{(1, 1)}, (4, 2), (9, 3)\}$

Step 4 Examine the set of ordered pairs and determine if it would be a function. This set of ordered pairs would not be a function because each x-value is not paired with a unique y-value. For example, there are two y-values when $x = 1$.



You have also learned how to find the inverse of a linear function algebraically. In the next activity, you will find the inverse of the quadratic function from Activity 1.

Activity 2 Use Algebra

Find the inverse of $y = x^2$ algebraically. Check by graphing the function, its inverse, and the line $y = x$.

Step 1 Find the inverse algebraically.

$y = x^2$	Original function
$x = y^2$	Interchange x and y.
$\sqrt{x} = \sqrt{y^2}$	Take the square root of each side.
$\sqrt{x} = \lvert y \rvert$	Simplify.
$\pm\sqrt{x} = y$	Simplify.

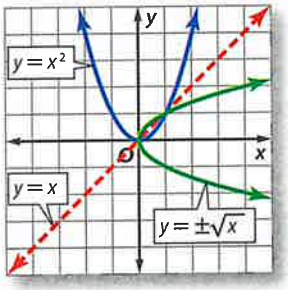
The inverse of $y = x^2$ is $y = \pm\sqrt{x}$.

(continued on the next page)

Inverse Functions *Continued*

Step 2 On a coordinate plane, plot and connect the sets of points from Steps 2 and 3 of Activity 1 with a smooth curve to graph $y = x^2$ and its inverse. Graph the line $y = x$.

Step 3 The graph of $y = \pm\sqrt{x}$ does not pass the vertical line test for a function. The inverse is not a function.



Many functions like $y = x^2$ have inverse relations that are not functions. It is often possible to limit the domains of these functions so that their inverses will be functions.

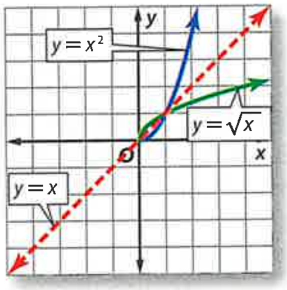
Activity 3 Restricted Domains

Restrict the domain of $y = x^2$ so that its inverse is a function.

Notice from Activity 2 that the graph of $y = x^2$ is symmetric about the y-axis. If we restrict the domain of $y = x^2$ to either $x \geq 0$ or $x \leq 0$, we are left with half of the graph.

For $x \geq 0$, the graph of $y = x^2$ is now the portion of the parabola to the right of the y-axis. Its inverse is its reflection across the line $y = x$, which is the top portion of the graph of $y = \pm\sqrt{x}$.

Since each x-value of this reflection is paired with a unique y-value, the inverse is now a function.



Exercises

Write a set of ordered pairs for the inverse of each function by making a table of values for x from 3 to 3 and exchanging the coordinates. Is the inverse a function?

1. $y = x^2 - 3$

2. $y = (x - 1)^2$

3. $y = 2x^2$

4. $y = 3x^2 - 2$

Find the inverse of each function algebraically. Is the inverse a function?

5. $y = x^2 + 2$

6. $y = (x - 1)^2$

7. $y = (x + 3)^2 - 4$

8. $y = 4x^2 + 2$

Name a restricted domain for each function for which its inverse would be a function.

9. $y = x^2 - 1$

10. $y = (x + 2)^2$

11. $y = (x - 2)^2 + 1$

12. $y = 3x^2 - 1$

LESSON 10-1 Square Root Functions

Then

- You graphed and analyzed linear, exponential, and quadratic functions.

Now

- 1 Graph and analyze dilations of radical functions.
- 2 Graph and analyze reflections and translations of radical functions.

Why?

- Scientists use sounds of whales to track their movements. The distance to a whale can be found by relating time to the speed of sound in water.

The speed of sound in water can be described by the *square root function* $c = \sqrt{\frac{E}{d}}$, where E represents the bulk modulus elasticity of the water and d represents the density of the water.

NewVocabulary

square root function
radical function
radicand

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Mathematical Practices

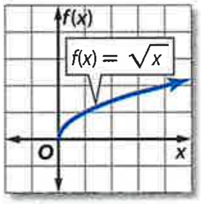
6 Attend to precision.

1 Dilations of Radical Functions

A **square root function** contains the square root of a variable. Square root functions are a type of **radical function**. The expression under the radical sign is called the **radicand**. For a square root to be a real number, the radicand cannot be negative. Values that make the radicand negative are not included in the domain.

KeyConcept Square Root Function

Parent Function:	$f(x) = \sqrt{x}$
Type of Graph:	curve
Domain:	$\{x \mid x \geq 0\}$
Range:	$\{y \mid y \geq 0\}$



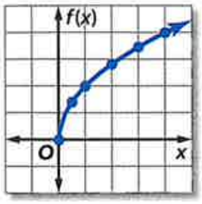
Example 1 Dilation of the Square Root Function

Graph $f(x) = 2\sqrt{x}$. State the domain and range.

Step 1 Make a table.

x	0	0.5	1	2	3	4
$f(x)$	0	≈1.4	2	≈2.8	≈3.5	4

Step 2 Plot points. Draw a smooth curve.



The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 0\}$. Notice that the graph is increasing on the entire domain, the minimum value is 0, and there is no symmetry.

GuidedPractice

1A. $g(x) = 4\sqrt{x}$

1B. $h(x) = 6\sqrt{x}$

Franco Banfi/Photographer's Choice/Getty Images

2 Reflections and Translations of Radical Functions

Recall that when the value of a is negative in the quadratic function $f(x) = ax^2$, the graph of the parent function is reflected across the x-axis.

StudyTip

Graphing Radical Functions Choose perfect squares for x-values that will result in coordinates that are easy to plot.

KeyConcept Graphing $y = a\sqrt{x+h} + k$

Step 1 Draw the graph of $y = a\sqrt{x}$. The graph starts at the origin and passes through $(1, a)$. If $a > 0$, the graph is in quadrant I. If $a < 0$, the graph is reflected across the x-axis and is in quadrant IV.

Step 2 Translate the graph k units up if $k > 0$ and $|k|$ units down if $k < 0$.

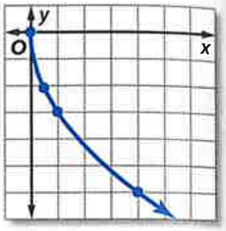
Step 3 Translate the graph h units left if $h > 0$ and $|h|$ units right if $h < 0$.

Example 2 Reflection of the Square Root Function

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table of values. Then plot the points on a coordinate system and draw a smooth curve that connects them.

x	0	0.5	1	4
y	0	≈−2.1	−3	−6



Notice that the graph is in the 4th quadrant. It is obtained by stretching the graph of $y = \sqrt{x}$ vertically and then reflecting across the x-axis. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 0\}$.

GuidedPractice

2A. $y = -2\sqrt{x}$

2B. $y = -4\sqrt{x}$

StudyTip

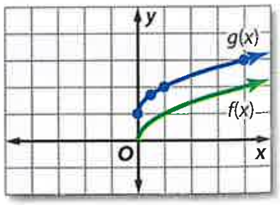
Translating Radical Functions If $h > 0$, a radical function $f(x) = \sqrt{x-h}$ is a horizontal translation h units to the right. $f(x) = \sqrt{x+h}$ is a horizontal translation h units to the left.

Example 3 Translation of the Square Root Function

Graph each function. Compare to the parent graph. State the domain and range.

a. $g(x) = \sqrt{x} + 1$

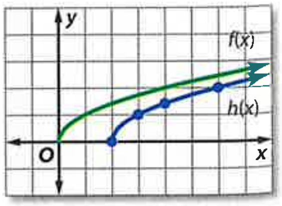
x	0	0.5	1	4	9
y	0	≈1.7	2	3	4



Notice that the values of $g(x)$ are 1 greater than those of $f(x) = \sqrt{x}$. This is a vertical translation 1 unit up from the parent function. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 1\}$.

b. $h(x) = \sqrt{x-2}$

x	2	3	4	6
y	0	1	≈1.4	2



This is a horizontal translation 2 units to the right of the parent function. The domain is $\{x \mid x \geq 2\}$, and the range is $\{y \mid y \geq 0\}$.

GuidedPractice

3A. $g(x) = \sqrt{x} - 4$

3B. $h(x) = \sqrt{x + 3}$

Physical phenomena such as motion can be modeled by radical functions. Often these functions are transformations of the parent square root function.

Real-World Example 4 Analyze a Radical Function

BRIDGES The Golden Gate Bridge is about 67 meters above the water. The velocity v of a freely falling object that has fallen h meters is given by $v = \sqrt{2gh}$, where g is the constant 9.8 meters per second squared. Graph the function. If an object is dropped from the bridge, what is its velocity when it hits the water?

Real-WorldLink

Approximately 39 million cars cross the Golden Gate Bridge in San Francisco each year.

Source: San Francisco Convention and Visitors Bureau

Use a graphing calculator to graph the function. To find the velocity of the object, substitute 67 meters for h.

$v = \sqrt{2gh}$	Original function
$= \sqrt{2(9.8)(67)}$	$g = 9.8$ and $h = 67$
$= \sqrt{1313.2}$	Simplify.
≈ 36.2 m/s	Use a calculator.

The velocity of the object is about 36.2 meters per second after dropping 67 meters.

GuidedPractice

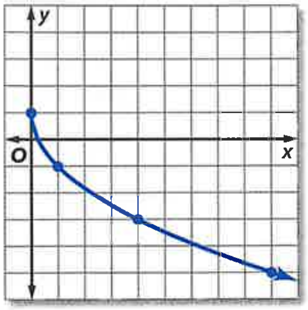
4. Use the graph above to estimate the initial height of an object if it is moving at 20 meters per second when it hits the water.

Transformations such as reflections, translations, and dilations can be combined in one equation.

Example 5 Transformations of the Square Root Function

Graph $y = -2\sqrt{x} + 1$, and compare to the parent graph. State the domain and range.

x	0	1	4	9
y	1	−1	−3	−5



This graph is the result of a vertical stretch of the graph of $y = \sqrt{x}$ followed by a reflection across the x-axis, and then a translation 1 unit up. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 1\}$.

GuidedPractice

5A. $y = \frac{1}{2}\sqrt{x} - 1$

5B. $y = -2\sqrt{x - 1}$

Didier Dorval/Masterfile

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–3 Graph each function. Compare to the parent graph. State the domain and range.

1. $y = 3\sqrt{x}$

2. $y = -5\sqrt{x}$

3. $y = \frac{1}{3}\sqrt{x}$

4. $y = -\frac{1}{2}\sqrt{x}$

5. $y = \sqrt{x} + 3$

6. $y = \sqrt{x} - 2$

7. $y = \sqrt{x + 2}$

8. $y = \sqrt{x - 3}$

Example 4

9. **FREE FALL** The time t, in seconds, that it takes an object to fall a distance d, in feet, is given by the function $t = \frac{1}{4}\sqrt{d}$ (assuming zero air resistance). Graph the function, and state the domain and range.

Example 5 Graph each function, and compare to the parent graph. State the domain and range.

10. $y = \frac{1}{2}\sqrt{x} + 2$

11. $y = -\frac{1}{4}\sqrt{x} - 1$

12. $y = -2\sqrt{x + 1}$

13. $y = 3\sqrt{x - 2}$

Practice and Problem Solving

Extra Practice is on page R10.

Examples 1–3 Graph each function. Compare to the parent graph. State the domain and range.

14. $y = 5\sqrt{x}$

⑮ $y = \frac{1}{2}\sqrt{x}$

16. $y = -\frac{1}{3}\sqrt{x}$

17. $y = 7\sqrt{x}$

18. $y = -\frac{1}{4}\sqrt{x}$

19. $y = -\sqrt{x}$

20. $y = -\frac{1}{5}\sqrt{x}$

21. $y = -7\sqrt{x}$

22. $y = \sqrt{x} + 2$

23. $y = \sqrt{x} + 4$

24. $y = \sqrt{x} - 1$

25. $y = \sqrt{x} - 3$

26. $y = \sqrt{x} + 1.5$

27. $y = \sqrt{x} - 2.5$

28. $y = \sqrt{x + 4}$

29. $y = \sqrt{x - 4}$

30. $y = \sqrt{x + 1}$

31. $y = \sqrt{x - 0.5}$

32. $y = \sqrt{x + 5}$

33. $y = \sqrt{x - 1.5}$

Example 4

34. **GEOMETRY** The perimeter of a square is given by the function $P = 4\sqrt{A}$, where A is the area of the square.

a. Graph the function.

b. Determine the perimeter of a square with an area of 225 m^2.

c. When will the perimeter and the area be the same value?

Example 5 Graph each function, and compare to the parent graph. State the domain and range.

35. $y = -2\sqrt{x} + 2$

36. $y = -3\sqrt{x} - 3$

37. $y = \frac{1}{2}\sqrt{x + 2}$

38. $y = -\sqrt{x - 1}$

39. $y = \frac{1}{4}\sqrt{x - 1} + 2$

40. $y = \frac{1}{2}\sqrt{x - 2} + 1$

41. **ENERGY** An object has kinetic energy when it is in motion. The velocity in meters per second of an object of mass m kilograms with an energy of E joules is given by the function $v = \sqrt{\frac{2E}{m}}$. Use a graphing calculator to graph the function that represents the velocity of a basketball with a mass of 0.6 kilogram.

42. **GEOMETRY** The radius of a circle is given by $r = \sqrt{\frac{A}{\pi}}$, where A is the area of the circle.

a. Graph the function.

b. Use a graphing calculator to determine the radius of a circle that has an area of 27 in^2.

43 **SPEED OF SOUND** The speed of sound in air is determined by the temperature of the air. The speed c in meters per second is given by $c = 331.5\sqrt{1 + \frac{t}{273.15}}$, where t is the temperature of the air in degrees Celsius.

a. Use a graphing calculator to graph the function.

b. How fast does sound travel when the temperature is 55°C?

c. How is the speed of sound affected when the temperature increases to 65°C?

44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the graphs of square root functions and parabolas.

a. **Graphical** Graph $y = x^2$ on a coordinate system.

b. **Algebraic** Write a piecewise-defined function to describe the graph of $y^2 = x$ in each quadrant.

c. **Graphical** On the same coordinate system, graph $y = \sqrt{x}$ and $y = -\sqrt{x}$.

d. **Graphical** On the same coordinate system, graph $y = x$. Plot the points (2, 4), (4, 2), and (1, 1).

e. **Analytical** Compare the graph of the parabola to the graphs of the square root functions.

H.O.T. Problems Use Higher-Order Thinking Skills

CHALLENGE **Determine whether each statement is *true* or *false*. Provide an example or counterexample to support your answer.**

45. Numbers in the domain of a radical function will always be nonnegative.

46. Numbers in the range of a radical function will always be nonnegative.

47. **WRITING IN MATH** Why are there limitations on the domain and range of square root functions?

48. **CCSS** **TOOLS** Write a radical function with a domain of all real numbers greater than or equal to 2 and a range of all real numbers less than or equal to 5.

49. **WHICH DOES NOT BELONG?** Identify the equation that does not belong. Explain.

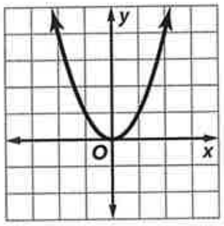
$y = 3\sqrt{x}$	$y = 0.7\sqrt{x}$	$y = \sqrt{x} + 3$	$y = \frac{\sqrt{x}}{6}$

50. **OPEN ENDED** Write a function that is a reflection, translation, and a dilation of the parent graph $y = \sqrt{x}$.

51. **REASONING** If the range of the function $y = a\sqrt{x}$ is $\{y \mid y \leq 0\}$, what can you conclude about the value of a? Explain your reasoning.

52. **WRITING IN MATH** Compare and contrast the graphs of $f(x) = \sqrt{x} + 2$ and $g(x) = \sqrt{x + 2}$.

Standardized Test Practice

53. 

Which function *best* represents the graph?

A $y = x^2$

B $y = 2^x$

C $y = \sqrt{x}$

D $y = x$

54. The statement "$x < 10$ and $3x - 2 \geq 7$" is true when x is equal to what?

F 0

G 2

H 8

J 12

55. Which of the following is the equation of a line parallel to $y = -\frac{1}{2}x + 3$ and passing through $(-2, -1)$?

A $y = \frac{1}{2}x$

B $y = 2x + 3$

C $y = -\frac{1}{2}x + 2$

D $y = -\frac{1}{2}x - 2$

56. SHORT RESPONSE A landscaper needs to mulch 6 rectangular flower beds that are 8 feet by 4 feet and 4 circular flower beds each with a radius of 3 feet. One bag of mulch covers 25 square feet. How many bags of mulch are needed to cover the flower beds?

Spiral Review

Graph each function. (Lesson 9-7)

57. $f(x) = |3x + 2|$

58. $f(x) = \begin{cases} x - 2 \text{ if } x > -1 \\ x + 3 \text{ if } x \leq -1 \end{cases}$

59. $f(x) = [\![x + 1]\!]$

60. $f(x) = \left|\frac{1}{4}x - 1\right|$

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function. (Lesson 9-6)

61. $\{(-2, 5), (-1, 3), (0, 1), (1, -1), (2, -3)\}$

62. $\{(0, 0), (1, 3), (2, 4), (3, 3), (4, 0)\}$

63. $\left\{\left(-2, \frac{1}{4}\right), (0, 1), (1, 2), (2, 4), (3, 8)\right\}$

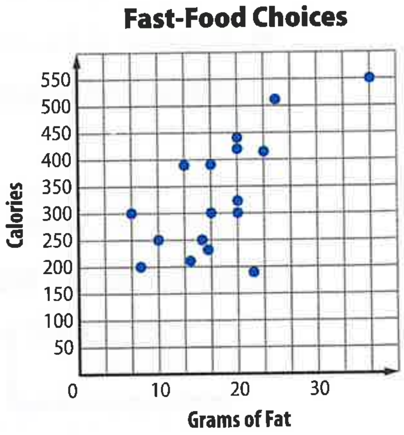
64. $\{(-3, 1), (-2, -5), (-1, -7), (0, -5), (1, 1)\}$

65. HEALTH Aida exercises every day by walking and jogging at least 3 miles. Aida walks at a rate of 4 miles per hour and jogs at a rate of 8 miles per hour. Suppose she has at most one half-hour to exercise today. (Lesson 6-6)

a. Draw a graph showing the possible amounts of time she can spend walking and jogging today.

b. List three possible solutions.

66. NUTRITION Determine whether the graph shows a *positive*, *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation. (Lesson 4-5)



Skills Review

Factor each monomial completely.

67. $28n^3$

68. $-33a^2b$

69. $150rt$

70. $-378nq^2r^2$

71. $225a^3b^2c$

72. $-160x^2y^4$

EXTEND

10-1 Graphing Technology Lab
Graphing Square Root Functions

For a square root to be a real number, the radicand cannot be negative. When graphing a radical function, determine when the radicand would be negative and exclude those values from the domain.

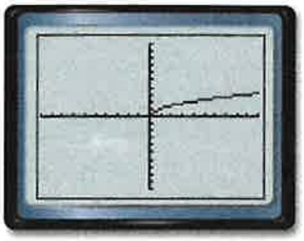
CCSS Common Core State Standards
Content Standards
F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
Mathematical Practices
5 Use appropriate tools strategically.

Activity 1 Parent Function

Graph $y = \sqrt{x}$.

Enter the equation in the **Y=** list, and graph in the standard viewing window.

KEYSTROKES: Y= 2nd [√] X,T,θ,n) ZOOM 6



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1A. Examine the graph. What is the domain of the function?

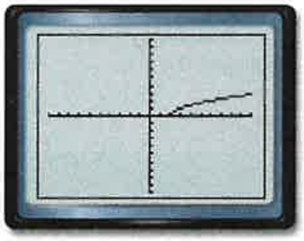
1B. What is the range of the function?

Activity 2 Translation of Parent Function

Graph $y = \sqrt{x - 2}$.

Enter the equation in the **Y=** list, and graph in the standard viewing window.

KEYSTROKES: Y= 2nd [√] X,T,θ,n − 2) ZOOM 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

2A. What are the domain and range of the function?

2B. How does the graph of $y = \sqrt{x - 2}$ compare to the graph of the parent function $y = \sqrt{x}$?

Exercises

Graph each equation, and sketch the graph on your paper. State the domain and range. Describe how the graph differs from that of the parent function $y = \sqrt{x}$.

1. $y = \sqrt{x - 1}$
2. $y = \sqrt{x + 3}$
3. $y = \sqrt{x} - 2$
4. $y = \sqrt{-x}$
5. $y = -\sqrt{x}$
6. $y = \sqrt{2x}$
7. $y = \sqrt{2 - x}$
8. $y = \sqrt{x - 3} + 2$

Solve each equation for y. Does the equation represent a function? Explain your reasoning.

9. $x = y^2$
10. $x^2 + y^2 = 4$
11. $x^2 + y^2 = 2$

Write a function with a graph that translates $y = \sqrt{x}$ in each way.

12. shifted 4 units to the left
13. shifted up 7 units
14. shifted down 6 units
15. shifted 5 units to the right and up 3 units

LESSON

10-2 Simplifying Radical Expressions

Then

- You simplified radicals.

Now

1. Simplify radical expressions by using the Product Property of Square Roots.
2. Simplify radical expressions by using the Quotient Property of Square Roots.

Why?

- The Sunshine Skyway Bridge across Florida's Tampa Bay is supported by 21 steel cables, each 9 inches in diameter.

To find the diameter a steel cable should have to support a given weight, you can use the equation $d = \sqrt{\frac{w}{8}}$, where d is the diameter of the cable in inches and w is the weight in tons.

NewVocabulary
radical expression
rationalizing the denominator
conjugate

Common Core State Standards

Content Standards
A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Mathematical Practices
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

1 Product Property of Square Roots

A **radical expression** contains a radical, such as a square root. Recall the expression under the radical sign is called the radicand. A radicand is in simplest form if the following three conditions are true.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The following property can be used to simplify square roots.

KeyConcept Product Property of Square Roots

Words For any nonnegative real numbers a and b, the square root of ab is equal to the square root of a times the square root of b.

Symbols $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, if $a \geq 0$ and $b \geq 0$

Examples $\sqrt{4 \cdot 9} = \sqrt{36}$ or 6 $\qquad$ $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3$ or 6

PT

Example 1 Simplify Square Roots

Simplify $\sqrt{80}$.

$\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$ Prime factorization of 80

$= \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 2 \cdot 2 \cdot \sqrt{5}$ or $4\sqrt{5}$ Simplify.

GuidedPractice

1A. $\sqrt{54}$

1B. $\sqrt{180}$

Example 2 Multiply Square Roots

Simplify $\sqrt{2} \cdot \sqrt{14}$.

$\sqrt{2} \cdot \sqrt{14} = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{7}$	Product Property of Square Roots
$= \sqrt{2^2} \cdot \sqrt{7}$ or $2\sqrt{7}$	Product Property of Square Roots

GuidedPractice

2A. $\sqrt{5} \cdot \sqrt{10}$

2B. $\sqrt{6} \cdot \sqrt{8}$

Consider the expression $\sqrt{x^2}$. It may seem that $x = \sqrt{x^2}$, but when finding the principal square root of an expression containing variables, you have to be sure that the result is not negative. Consider $x = -3$.

$\sqrt{x^2} \stackrel{?}{=} x$	
$\sqrt{(-3)^2} \stackrel{?}{=} -3$	Replace x with -3.
$\sqrt{9} \stackrel{?}{=} -3$	$(-3)^2 = 9$
$3 \neq -3$	$\sqrt{9} = 3$

Notice in this case, if the right hand side of the equation were $|x|$, the equation would be true. For expressions where the exponent of the variable inside a radical is even and the simplified exponent is odd, you must use absolute value.

$$\sqrt{x^2} = |x| \qquad \sqrt{x^3} = x\sqrt{x} \qquad \sqrt{x^4} = x^2 \qquad \sqrt{x^6} = |x^3|$$

Example 3 Simplify a Square Root with Variables

Simplify $\sqrt{90x^3y^4z^5}$.

$\sqrt{90x^3y^4z^5} = \sqrt{2 \cdot 3^2 \cdot 5 \cdot x^3 \cdot y^4 \cdot z^5}$	Prime factorization
$= \sqrt{2} \cdot \sqrt{3^2} \cdot \sqrt{5} \cdot \sqrt{x^2} \cdot \sqrt{x} \cdot \sqrt{y^4} \cdot \sqrt{z^4} \cdot \sqrt{z}$	Product Property
$= \sqrt{2} \cdot 3 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot z^2 \cdot \sqrt{z}$	Simplify.
$= 3y^2z^2x\sqrt{10xz}$	Simplify.

GuidedPractice

3A. $\sqrt{32r^2k^4t^5}$

3B. $\sqrt{56xy^{10}z^5}$

2 Quotient Property of Square Roots

To divide square roots and simplify radical expressions, you can use the Quotient Property of Square Roots.

ReadingMath

Fractions in the Radicand The expression $\sqrt{\frac{a}{b}}$ is read *the square root of a over b,* or *the square root of the quantity of a over b.*

KeyConcept Quotient Property of Square Roots

Words For any real numbers a and b, where $a \geq 0$ and $b > 0$, the square root of $\frac{a}{b}$ is equal to the square root of a divided by the square root of b.

Symbols $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Eyecon Images/Alamy

You can use the properties of square roots to **rationalize the denominator** of a fraction with a radical. This involves multiplying the numerator and denominator by a factor that eliminates radicals in the denominator.

Test-Taking Tip

CCSS Structure Look at the radicand to see if it can be simplified first. This may make your computations simpler.

Standardized Test Example 4 Rationalize a Denominator

Which expression is equivalent to $\sqrt{\frac{35}{15}}$?

A $\frac{5\sqrt{21}}{15}$ **B** $\frac{\sqrt{21}}{3}$ **C** $\frac{\sqrt{525}}{15}$ **D** $\frac{\sqrt{35}}{15}$

Read the Test Item The radical expression needs to be simplified.

Solve the Test Item

$\sqrt{\frac{35}{15}} = \sqrt{\frac{7}{3}}$	Reduce $\frac{35}{15}$ to $\frac{7}{3}$.
$= \frac{\sqrt{7}}{\sqrt{3}}$	Quotient Property of Square Roots
$= \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$	Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.
$= \frac{\sqrt{21}}{3}$	Product Property of Square Roots

The correct choice is B.

Guided Practice

4. Simplify $\frac{\sqrt{6y}}{\sqrt{12}}$.

F $\frac{\sqrt{y}}{2}$ **G** $\frac{\sqrt{y}}{4}$ **H** $\frac{\sqrt{2y}}{2}$ **J** $\frac{\sqrt{2y}}{4}$

Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates**. For example, $2 + \sqrt{7}$ and $2 - \sqrt{7}$ are conjugates. The product of two conjugates is a rational number and can be found using the pattern for the difference of squares.

Example 5 Use Conjugates to Rationalize a Denominator

Simplify $\frac{3}{5 + \sqrt{2}}$.

$\frac{3}{5+\sqrt{2}} = \frac{3}{5+\sqrt{2}} \cdot \frac{5-\sqrt{2}}{5-\sqrt{2}}$	The conjugate of $5 + \sqrt{2}$ is $5 - \sqrt{2}$.
$= \frac{3(5-\sqrt{2})}{5^2 - (\sqrt{2})^2}$	$(a - b)(a + b) = a^2 - b^2$
$= \frac{15 - 3\sqrt{2}}{25 - 2}$ or $\frac{15 - 3\sqrt{2}}{23}$	$(\sqrt{2})^2 = 2$

Guided Practice Simplify each expression.

5A. $\frac{3}{2 + \sqrt{2}}$

5B. $\frac{7}{3 - \sqrt{7}}$

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–3 **Simplify each expression.**

1. $\sqrt{24}$

2. $3\sqrt{16}$

3. $2\sqrt{25}$

4. $\sqrt{10} \cdot \sqrt{14}$

5. $\sqrt{3} \cdot \sqrt{18}$

6. $3\sqrt{10} \cdot 4\sqrt{10}$

7. $\sqrt{60x^4y^7}$

8. $\sqrt{88m^3p^2r^5}$

9. $\sqrt{99ab^5c^2}$

Example 4 **10. MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{45}{10}}$?

A $\frac{5\sqrt{2}}{10}$

B $\frac{\sqrt{45}}{10}$

C $\frac{\sqrt{50}}{10}$

D $\frac{3\sqrt{2}}{2}$

Example 5 **Simplify each expression.**

11. $\frac{3}{3+\sqrt{5}}$

12. $\frac{5}{2-\sqrt{6}}$

13. $\frac{2}{1-\sqrt{10}}$

14. $\frac{1}{4+\sqrt{12}}$

15. $\frac{4}{6-\sqrt{7}}$

16. $\frac{6}{5+\sqrt{11}}$

Practice and Problem Solving

Extra Practice is on page R10.

Examples 1–3 **Simplify each expression.**

17. $\sqrt{52}$

18. $\sqrt{56}$

19. $\sqrt{72}$

20. $3\sqrt{18}$

21. $\sqrt{243}$

22. $\sqrt{245}$

23. $\sqrt{5} \cdot \sqrt{10}$

24. $\sqrt{10} \cdot \sqrt{20}$

25. $3\sqrt{8} \cdot 2\sqrt{7}$

26. $4\sqrt{2} \cdot 5\sqrt{8}$

27. $3\sqrt{25t^2}$

28. $5\sqrt{81q^5}$

29. $\sqrt{28a^2b^3}$

30. $\sqrt{75qr^3}$

31. $7\sqrt{63m^3p}$

32. $4\sqrt{66g^2h^4}$

33. $\sqrt{2ab^2} \cdot \sqrt{10a^5b}$

34. $\sqrt{4c^3d^3} \cdot \sqrt{8c^3d}$

㉟ ROLLER COASTER Starting from a stationary position, the velocity v of a roller coaster in feet per second at the bottom of a hill can be approximated by $v = \sqrt{64h}$, where h is the height of the hill in feet.

a. Simplify the equation.

b. Determine the velocity of a roller coaster at the bottom of a 134-foot hill.

36. CCSS PRECISION When fighting a fire, the velocity v of water being pumped into the air is modeled by the function $v = \sqrt{2hg}$, where h represents the maximum height of the water and g represents the acceleration due to gravity (32 ft/s^2).

a. Solve the function for h.

b. The Hollowville Fire Department needs a pump that will propel water 80 feet into the air. Will a pump advertised to project water with a velocity of 70 feet per second meet their needs? Explain.

c. The Jackson Fire Department must purchase a pump that will propel water 90 feet into the air. Will a pump that is advertised to project water with a velocity of 77 feet per second meet the fire department's need? Explain.

Examples 4–5 **Simplify each expression.**

37. $\sqrt{\frac{32}{t^4}}$

38. $\sqrt{\frac{27}{m^5}}$

39. $\frac{\sqrt{68ac^3}}{\sqrt{27a^2}}$

40. $\frac{\sqrt{h^3}}{\sqrt{8}}$

41. $\sqrt{\frac{3}{16}} \cdot \sqrt{\frac{9}{5}}$

42. $\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}$

43. $\frac{7}{5+\sqrt{3}}$

44. $\frac{9}{6-\sqrt{8}}$

45. $\frac{3\sqrt{3}}{-2+\sqrt{6}}$

46. $\frac{3}{\sqrt{7}-\sqrt{2}}$

47. $\frac{5}{\sqrt{6}+\sqrt{3}}$

48. $\frac{2\sqrt{5}}{2\sqrt{7}+3\sqrt{3}}$

49. **ELECTRICITY** The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \sqrt{\frac{P}{R}}$, where P is the power in watts and R is the resistance in ohms.

    a. Simplify the formula.

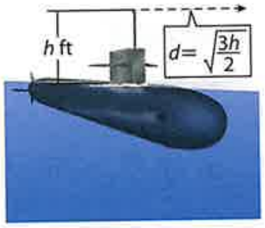
    b. How much current does an appliance use if the power used is 75 watts and the resistance is 5 ohms?

50. **KINETIC ENERGY** The speed v of a ball can be determined by the equation $v = \sqrt{\frac{2k}{m}}$, where k is the kinetic energy and m is the mass of the ball.

    a. Simplify the formula if the mass of the ball is 3 kilograms.

    b. If the ball is traveling 7 meters per second, what is the kinetic energy of the ball in Joules?

51. **SUBMARINES** The greatest distance d in miles that a lookout can see on a clear day is modeled by the formula shown. Determine how high the submarine would have to raise its periscope to see a ship, if the submarine is the given distances away from the ship.



Distance	3	6	9	12	15
Height					

H.O.T. Problems Use Higher-Order Thinking Skills

52. **CCSS STRUCTURE** Explain how to solve $\frac{\sqrt{3}+2}{x} = \frac{\sqrt{3}-1}{\sqrt{3}}$.

53. **CHALLENGE** Simplify each expression.

    a. $\sqrt[3]{27}$ **b.** $\sqrt[3]{40}$ **c.** $\sqrt[3]{750}$

54. **REASONING** Marge takes a number, subtracts 4, multiplies by 4, takes the square root, and takes the reciprocal to get $\frac{1}{2}$. What number did she start with? Write a formula to describe the process.

55. **OPEN ENDED** Write two binomials of the form $a\sqrt{b} + c\sqrt{f}$ and $a\sqrt{b} - c\sqrt{f}$. Then find their product.

56. **CHALLENGE** Use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation $ax^2 + bx + c = 0$. (*Hint*: Begin by completing the square.)

57. **WRITING IN MATH** Summarize how to write a radical expression in simplest form.

Standardized Test Practice

58. Jerry's electric bill is \$23 less than his natural gas bill. The two bills are a total of \$109. Which of the following equations can be used to find the amount of his natural gas bill?

A $g + g = 109$ **C** $g - 23 = 109$

B $23 + 2g = 109$ **D** $2g - 23 = 109$

59. Solve $a^2 - 2a + 1 = 25$.

F $-4, -6$ **H** $-4, 6$

G $4, -6$ **J** $4, 6$

60. The expression $\sqrt{160x^2y^5}$ is equivalent to which of the following?

A $16|x|y^2\sqrt{10y}$ **C** $4|x|y^2\sqrt{10y}$

B $|x|y^2\sqrt{160y}$ **D** $10|x|y^2\sqrt{4y}$

61. **GRIDDED RESPONSE** Miki earns \$10 an hour and 10% commission on sales. If Miki worked 38 hours and had a total sales of \$1275 last week, how much did she make?

Spiral Review

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

62. $y = 2\sqrt{x} - 1$ **63.** $y = \frac{1}{2}\sqrt{x}$ **64.** $y = 2\sqrt{x + 2}$

65. $y = -\sqrt{x + 1}$ **66.** $y = -3\sqrt{x - 3}$ **67.** $y = -2\sqrt{x} + 1$

Determine the domain and range for each function. (Lesson 9-7)

68. $f(x) = |2x - 5|$ **69.** $h(x) = \llbracket x - 1 \rrbracket$ **70.** $g(x) = \begin{cases} -3x + 4 \text{ if } x > 2 \\ x - 1 \text{ if } x \leq 2 \end{cases}$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

71. $x^2 - 25 = 0$ **72.** $r^2 + 25 = 0$ **73.** $4w^2 + 100 = 40w$

74. $2r^2 + r - 14 = 0$ **75.** $5v^2 - 7v = 1$ **76.** $11z^2 - z = 3$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-8)

77. $n^2 - 81$ **78.** $4 - 9a^2$ **79.** $2x^5 - 98x^3$

80. $32x^4 - 2y^4$ **81.** $4t^2 - 27$ **82.** $x^3 - 3x^2 - 9x + 27$

83. **POPULATION** The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2009, its population was 2,261,294. If the trend continues, predict Latvia's population in 2019. (Lesson 7-6)

84. **TOMATOES** There are more than 10,000 varieties of tomatoes. One seed company produces seed packages for 200 varieties of tomatoes. For how many varieties do they not provide seeds? (Lesson 5-1)

Skills Review

Write the prime factorization of each number.

85. 24 **86.** 88 **87.** 180

88. 31 **89.** 60 **90.** 90

EXTEND 10-2

Algebra Lab
Rational and Irrational Numbers

A set is **closed** under an operation if for any numbers in the set, the result of the operation is also in the set. A set may be closed under one operation and not closed under another.

CCSS Common Core State Standards

Content Standards

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Mathematical Practices

7 Look for and make use of structure.

Activity 1 Closure of Rational Numbers and Irrational Numbers

Are the sets of rational and irrational numbers closed under multiplication? under addition?

Step 1 To determine if each set is closed under multiplication, examine several products of two rational factors and then two irrational factors.

Rational: $5 \times 2 = 10$; $-3 \times 4 = -12$; $3.7 \times 0.5 = 1.85$; $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

Irrational: $\pi \times \sqrt{2} = \sqrt{2}\pi$; $\sqrt{3} \times \sqrt{7} = \sqrt{21}$; $\sqrt{5} \times \sqrt{5} = 5$

The product of each pair of rational numbers is rational. However, the products of pairs of irrational numbers are both irrational and rational. Thus, it appears that the set of rational numbers is closed under multiplication, but the set of irrational numbers is not.

Step 2 Repeat this process for addition.

Rational: $3 + 8 = 11$; $-4 + 7 = 3$; $3.7 + 5.82 = 9.52$; $\frac{2}{5} + \frac{1}{4} = \frac{13}{20}$

Irrational: $\sqrt{3} + \pi = \sqrt{3} + \pi$; $3\sqrt{5} + 6\sqrt{5} = 9\sqrt{5}$; $\sqrt{12} + \sqrt{50} = 2\sqrt{3} + 5\sqrt{2}$

The sum of each pair of rational numbers is rational, and the sum of each pair of irrational numbers is irrational. Both sets are closed under addition.

Activity 2 Rational and Irrational Numbers

What kind of numbers are the product and sum of a rational and irrational number?

Step 1 Examine the products of several pairs of rational and irrational numbers.

$3 \times \sqrt{8} = 6\sqrt{2}$; $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$; $1 \times \sqrt{7} = \sqrt{7}$; $0 \times \sqrt{5} = 0$

The product is rational only when the rational factor is 0. The product of each nonzero rational number and irrational number is irrational.

Step 2 Find the sums of several pairs of a rational and irrational number.

$5 + \sqrt{3} = 5 + \sqrt{3}$; $\frac{2}{3} + \sqrt{5} = \frac{2 + 3\sqrt{5}}{3}$; $-4 + \sqrt{6} = -1(4 - \sqrt{6})$

The sum of each rational and irrational number is irrational.

Analyze the Results

1. What kinds of numbers are the difference of two unique rational numbers, two unique irrational numbers, and a rational and an irrational number?
2. Is the quotient of every rational and irrational number always another rational or irrational number? If not, provide a counterexample.
3. **CHALLENGE** Recall that rational numbers are numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Using $\frac{a}{b}$ and $\frac{c}{d}$ show that the sum and product of two rational numbers must always be a rational number.

LESSON

10-3 Operations with Radical Expressions

Then

- You simplified radical expressions.

Now

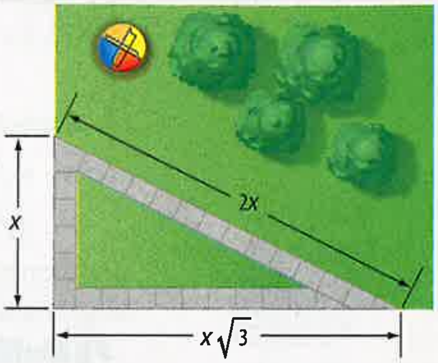
- **1** Add and subtract radical expressions.
- **2** Multiply radical expressions.

Why?

- Conchita is going to run in her neighborhood to get ready for the soccer season. She plans to run the course that she has laid out three times each day.

How far does Conchita have to run to complete the course that she laid out?

How far does she run every day?



Common Core State Standards

Content Standards

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Mathematical Practices

2 Reason abstractly and quantitatively.

1 Add or Subtract Radical Expressions

To add or subtract radical expressions, the radicands must be alike in the same way that monomial terms must be alike to add or subtract.

Monomials	Radical Expressions
$4a + 2a = (4 + 2)a$ $= 6a$	$4\sqrt{5} + 2\sqrt{5} = (4 + 2)\sqrt{5}$ $= 6\sqrt{5}$
$9b - 2b = (9 - 2)b$ $= 7b$	$9\sqrt{3} - 2\sqrt{3} = (9 - 2)\sqrt{3}$ $= 7\sqrt{3}$

Notice that when adding and subtracting radical expressions, the radicand does not change. This is the same as when adding or subtracting monomials.

Example 1 Add and Subtract Expressions with Like Radicands

Simplify each expression.

a. $5\sqrt{2} + 7\sqrt{2} - 6\sqrt{2}$

$5\sqrt{2} + 7\sqrt{2} - 6\sqrt{2} = (5 + 7 - 6)\sqrt{2}$ Distributive Property

$= 6\sqrt{2}$ Simplify.

b. $10\sqrt{7} + 5\sqrt{11} + 4\sqrt{7} - 6\sqrt{11}$

$10\sqrt{7} + 5\sqrt{11} + 4\sqrt{7} - 6\sqrt{11} = (10 + 4)\sqrt{7} + (5 - 6)\sqrt{11}$ Distributive Property

$= 14\sqrt{7} - \sqrt{11}$ Simplify.

GuidedPractice

1A. $3\sqrt{2} - 5\sqrt{2} + 4\sqrt{2}$

1B. $6\sqrt{11} + 2\sqrt{11} - 9\sqrt{11}$

1C. $15\sqrt{3} - 14\sqrt{5} + 6\sqrt{5} - 11\sqrt{3}$

1D. $4\sqrt{3} + 3\sqrt{7} - 6\sqrt{3} + 3\sqrt{7}$

Not all radical expressions have like radicands. Simplifying the expressions may make it possible to have like radicands so that they can be added or subtracted.

StudyTip

Simplify First Simplify each radical term first. Then perform the operations needed.

Example 2 Add and Subtract Expressions with Unlike Radicals

Simplify $2\sqrt{18} + 2\sqrt{32} + \sqrt{72}$.

$2\sqrt{18} + 2\sqrt{32} + \sqrt{72} = 2(\sqrt{3^2} \cdot \sqrt{2}) + 2(\sqrt{4^2} \cdot \sqrt{2}) + (\sqrt{6^2} \cdot \sqrt{2})$ Product Property

$= 2(3\sqrt{2}) + 2(4\sqrt{2}) + (6\sqrt{2})$ Simplify.

$= 6\sqrt{2} + 8\sqrt{2} + 6\sqrt{2}$ Multiply.

$= 20\sqrt{2}$ Simplify.

GuidedPractice

2A. $4\sqrt{54} + 2\sqrt{24}$

2B. $4\sqrt{12} - 6\sqrt{48}$

2C. $3\sqrt{45} + \sqrt{20} - \sqrt{245}$

2D. $\sqrt{24} - \sqrt{54} + \sqrt{96}$

2 Multiply Radical Expressions

Multiplying radical expressions is similar to multiplying monomial algebraic expressions. Let $x \geq 0$.

Monomials	Radical Expressions
$(2x)(3x) = 2 \cdot 3 \cdot x \cdot x$	$(2\sqrt{x})(3\sqrt{x}) = 2 \cdot 3 \cdot \sqrt{x} \cdot \sqrt{x}$
$= 6x^2$	$= 6x$

You can also apply the Distributive Property to radical expressions.

WatchOut!

Multiplying Radicands Make sure that you multiply the radicands when multiplying radical expressions. A common mistake is to add the radicands rather than multiply.

Example 3 Multiply Radical Expressions

Simplify each expression.

a. $3\sqrt{2} \cdot 2\sqrt{6}$

$3\sqrt{2} \cdot 2\sqrt{6} = (3 \cdot 2)\ (\sqrt{2} \cdot \sqrt{6})$ Associative Property

$= 6(\sqrt{12})$ Multiply.

$= 6(2\sqrt{3})$ Simplify.

$= 12\sqrt{3}$ Multiply.

b. $3\sqrt{5}(2\sqrt{5} + 5\sqrt{3})$

$3\sqrt{5}(2\sqrt{5} + 5\sqrt{3}) = (3\sqrt{5} \cdot 2\sqrt{5}) + (3\sqrt{5} \cdot 5\sqrt{3})$ Distributive Property

$= [(3 \cdot 2)(\sqrt{5} \cdot \sqrt{5})] + [(3 \cdot 5)(\sqrt{5} \cdot \sqrt{3})]$ Associative Property

$= [6(\sqrt{25})] + [15(\sqrt{15})]$ Multiply.

$= [6(5)] + [15(\sqrt{15})]$ Simplify.

$= 30 + 15\sqrt{15}$ Multiply.

GuidedPractice

3A. $2\sqrt{6} \cdot 7\sqrt{3}$

3B. $9\sqrt{5} \cdot 11\sqrt{15}$

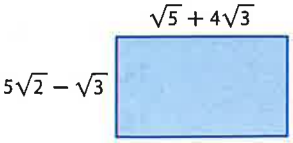
3C. $3\sqrt{2}(4\sqrt{3} + 6\sqrt{2})$

3D. $5\sqrt{3}(3\sqrt{2} - \sqrt{3})$

You can also multiply radical expressions with more than one term in each factor. This is similar to multiplying two algebraic binomials with variables.

Real-World Example 4 Multiply Radical Expressions

GEOMETRY **Find the area of the rectangle in simplest form.**



$$A = (5\sqrt{2} - \sqrt{3})(\sqrt{5} + 4\sqrt{3}) \qquad A = \ell \cdot w$$

$$= \overbrace{(5\sqrt{2})(\sqrt{5})}^{\text{First Terms}} + \overbrace{(5\sqrt{2})(4\sqrt{3})}^{\text{Outer Terms}} + \overbrace{(-\sqrt{3})(\sqrt{5})}^{\text{Inner Terms}} + \overbrace{(\sqrt{3})(4\sqrt{3})}^{\text{Last Terms}}$$

$$= 5\sqrt{10} + 20\sqrt{6} - \sqrt{15} - 4\sqrt{9} \qquad \text{Multiply.}$$

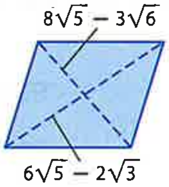
$$= 5\sqrt{10} + 20\sqrt{6} - \sqrt{15} - 12 \qquad \text{Simplify.}$$

ReviewVocabulary

FOIL Method Multiply two binomials by finding the sum of the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

GuidedPractice

4. **GEOMETRY** The area A of a rhombus can be found using the equation $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals. What is the area of the rhombus at the right?



ConceptSummary Operations with Radical Expressions

Operation	Symbols	Example
addition, $b \geq 0$	$a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$ like radicands	$4\sqrt{3} + 6\sqrt{3} = (4 + 6)\sqrt{3}$ $= 10\sqrt{3}$
subtraction, $b \geq 0$	$a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$ like radicands	$12\sqrt{5} - 8\sqrt{5} = (12 - 8)\sqrt{5}$ $= 4\sqrt{5}$
multiplication, $b \geq 0, g \geq 0$	$a\sqrt{b}(f\sqrt{g}) = af\sqrt{bg}$ Radicands do not have to be like radicands.	$3\sqrt{2}(5\sqrt{7}) = (3 \cdot 5)(\sqrt{2 \cdot 7})$ $= 15\sqrt{14}$

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–3 **Simplify each expression.**

1. $3\sqrt{5} + 6\sqrt{5}$

2. $8\sqrt{3} + 5\sqrt{3}$

3. $\sqrt{7} - 6\sqrt{7}$

4. $10\sqrt{2} - 6\sqrt{2}$

5. $4\sqrt{5} + 2\sqrt{20}$

6. $\sqrt{12} - \sqrt{3}$

7. $\sqrt{8} + \sqrt{12} + \sqrt{18}$

8. $\sqrt{27} + 2\sqrt{3} - \sqrt{12}$

9. $9\sqrt{2}(4\sqrt{6})$

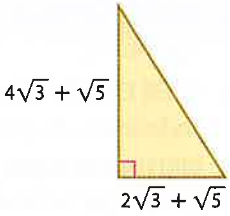
10. $4\sqrt{3}(8\sqrt{3})$

11. $\sqrt{3}(\sqrt{7} + 3\sqrt{2})$

12. $\sqrt{5}(\sqrt{2} + 4\sqrt{2})$

Example 4

13. **GEOMETRY** The area A of a triangle can be found by using the formula $A = \frac{1}{2}bh$, where b represents the base and h is the height. What is the area of the triangle at the right?



Practice and Problem Solving

Extra Practice is on page R10.

Examples 1–3 **Simplify each expression.**

14. $7\sqrt{5} + 4\sqrt{5}$

15. $2\sqrt{6} + 9\sqrt{6}$

16. $3\sqrt{5} - 2\sqrt{20}$

17. $3\sqrt{50} - 3\sqrt{32}$

18. $7\sqrt{3} - 2\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}$

19. $\sqrt{5}(\sqrt{2} + 4\sqrt{2})$

20. $\sqrt{6}(2\sqrt{10} + 3\sqrt{2})$

21. $4\sqrt{5}(3\sqrt{5} + 8\sqrt{2})$

22. $5\sqrt{3}(6\sqrt{10} - 6\sqrt{3})$

23. $(\sqrt{3} - \sqrt{2})(\sqrt{15} + \sqrt{12})$

24. $(3\sqrt{11} + 3\sqrt{15})(3\sqrt{3} - 2\sqrt{2})$

25. $(5\sqrt{2} + 3\sqrt{5})(2\sqrt{10} - 5)$

Example 4 **26. GEOMETRY** Find the perimeter and area of a rectangle with a width of $2\sqrt{7} - 2\sqrt{5}$ and a length of $3\sqrt{7} + 3\sqrt{5}$.

Simplify each expression.

27. $\sqrt{\frac{1}{5}} - \sqrt{5}$

28. $\sqrt{\frac{2}{3}} + \sqrt{6}$

29. $2\sqrt{\frac{1}{2}} + 2\sqrt{2} - \sqrt{8}$

30. $8\sqrt{\frac{5}{4}} + 3\sqrt{20} - 10\sqrt{\frac{1}{5}}$

31. $(3 - \sqrt{5})^2$

32. $(\sqrt{2} + \sqrt{3})^2$

33. ROLLER COASTERS The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$.

a. What velocity must a coaster have at the top of a 225-foot hill to achieve a velocity of 120 feet per second at the bottom?

b. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the formula given.

34. FINANCIAL LITERACY Tadi invests \$225 in a savings account. In two years, Tadi has \$232 in his account. You can use the formula $r = \sqrt{\frac{v_2}{v_0}} - 1$ to find the average annual interest rate r that the account has earned. The initial investment is v_0, and v_2 is the amount in two years. What was the average annual interest rate that Tadi's account earned?

35. ELECTRICITY Electricians can calculate the electrical current in amps A by using the formula $A = \frac{\sqrt{w}}{\sqrt{r}}$, where w is the power in watts and r the resistance in ohms. How much electrical current is running through a microwave oven that has 850 watts of power and 5 ohms of resistance? Write the number of amps in simplest radical form, and then estimate the amount of current to the nearest tenth.

H.O.T. Problems Use Higher-Order Thinking Skills

36. CHALLENGE Determine whether the following statement is *true* or *false*. Provide a proof or counterexample to support your answer.

$$x + y > \sqrt{x^2 + y^2} \text{ when } x > 0 \text{ and } y > 0$$

37. CCSS ARGUMENTS Make a conjecture about the sum of a rational number and an irrational number. Is the sum *rational* or *irrational*? Is the product of a nonzero rational number and an irrational number *rational* or *irrational*? Explain your reasoning.

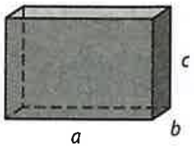
38. OPEN ENDED Write an equation that shows a sum of two radicals with different radicands. Explain how you could combine these terms.

39. WRITING IN MATH Describe step by step how to multiply two radical expressions, each with two terms. Write an example to demonstrate your description.

Standardized Test Practice

40. **SHORT RESPONSE** The population of a town is 13,000 and is increasing by about 250 people per year. This can be represented by the equation $p = 13{,}000 + 250y$, where y is the number of years from now and p represents the population. In how many years will the population of the town be 14,500?

41. **GEOMETRY** Which expression represents the sum of the lengths of the 12 edges on this rectangular solid?



A $2(a + b + c)$
B $3(a + b + c)$
C $4(a + b + c)$
D $12(a + b + c)$

42. Evaluate $\sqrt{n-9}$ and $\sqrt{n} - \sqrt{9}$ for $n = 25$.

F 4; 4 **G** 4; 2
H 2; 4 **J** 2; 2

43. The current I in a simple electrical circuit is given by the formula $I = \frac{V}{R}$, where V is the voltage and R is the resistance of the circuit. If the voltage remains unchanged, what effect will doubling the resistance of the circuit have on the current?

A The current will remain the same.
B The current will double its previous value.
C The current will be half its previous value.
D The current will be two units more than its previous value.

Spiral Review

Simplify. (Lesson 10-2)

44. $\sqrt{18}$ **45.** $\sqrt{24}$ **46.** $\sqrt{60}$

47. $\sqrt{50a^3b^5}$ **48.** $\sqrt{169x^4y^7}$ **49.** $\sqrt{63c^3d^4f^5}$

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

50. $y = 2\sqrt{x}$ **51.** $y = -3\sqrt{x}$ **52.** $y = \sqrt{x+1}$

53. $y = \sqrt{x-4}$ **54.** $y = \sqrt{x} + 3$ **55.** $y = \sqrt{x} - 2$

Factor each trinomial. (Lesson 8-5)

56. $x^2 + 12x + 27$ **57.** $y^2 + 13y + 30$ **58.** $p^2 - 17p + 72$

59. $x^2 + 6x - 7$ **60.** $y^2 - y - 42$ **61.** $-72 + 6w + w^2$

62. **FINANCIAL LITERACY** Determine the value of an investment if \$400 is invested at an interest rate of 7.25% compounded quarterly for 7 years. (Lesson 7-6)

Skills Review

Solve each equation. Round each solution to the nearest tenth, if necessary.

63. $-4c - 1.2 = 0.8$ **64.** $-2.6q - 33.7 = 84.1$ **65.** $0.3m + 4 = 9.6$

66. $-10 - \frac{n}{5} = 6$ **67.** $\frac{-4h - (-5)}{-7} = 13$ **68.** $3.6t + 6 - 2.5t = 8$

EXTEND

10-3 Algebra Lab
Simplifying *n*th Root Expressions

The inverse of raising a number to the *n*th power is finding the **nth root** of a number. The **index** of a radical expression indicates to what root the value under the radicand is being taken. The fourth root of a number is indicated with an index of 4. When simplifying a radical expression in which there is a variable with an exponent in the radicand, divide the exponent by the index.

Common Core State Standards
Content Standards
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

$$13 \div 5 = 2\ \text{R}\ 3 \quad \longrightarrow \quad \boxed{\text{index}} \rightarrow \sqrt[5]{x^{13}} = \overset{\boxed{\text{quotient}}}{x^{2}} \cdot \sqrt[5]{x^{3}} \leftarrow \boxed{\text{remainder}}$$

Example 1 Simplify Expressions

Simplify each expression.

a. $\sqrt[3]{x^7}$

$\sqrt[3]{x^7} = x^2\sqrt[3]{x}$ — $7 \div 3 = 2$ R 1

b. $\sqrt[5]{32x^9}$

$\sqrt[5]{32x^9} = \sqrt[5]{32} \cdot \sqrt[5]{x^9}$ — Multiplication Property

$= 2x\sqrt[5]{x^4}$ — $9 \div 5 = 1$ R 4

The properties of square roots (and *n*th roots) also apply when the radicand contains fractions. Separate the numerator and denominator and then simplify them individually.

Example 2 Simplify Expressions with Fractions

Simplify $\sqrt[3]{\frac{27x^5}{8y^3}}$.

$\sqrt[3]{\frac{27x^5}{8y^3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} \cdot \frac{\sqrt[3]{x^5}}{\sqrt[3]{y^3}}$ — Multiplication Property of Radicals

$= \frac{3}{2} \cdot \frac{x\sqrt[3]{x^2}}{y}$ — Simplify.

$= \frac{3x\sqrt[3]{x^2}}{2y}$ — Multiplication Property of Radicals

The indices *and* the radicands must be alike in order to add or subtract radical expressions.

Example 3 Combine Like Terms

Simplify $8\sqrt[4]{\frac{4}{3}} + \sqrt[4]{\frac{5}{4}} - 3\sqrt[4]{\frac{4}{3}} + \sqrt[3]{\frac{4}{3}}$.

Combine the expressions with identical indices and radicands. Then simplify.

$8\sqrt[4]{\frac{4}{3}} + \sqrt[4]{\frac{5}{4}} - 3\sqrt[4]{\frac{4}{3}} + \sqrt[3]{\frac{4}{3}} = (8-3)\sqrt[4]{\frac{4}{3}} + \sqrt[4]{\frac{5}{4}} + \sqrt[3]{\frac{4}{3}}$ — Associative Property

$= 5\sqrt[4]{\frac{4}{3}} + \sqrt[4]{\frac{5}{4}} + \sqrt[3]{\frac{4}{3}}$ — Simplify.

When multiplying radical expressions, ensure that the indices are the same. Then multiply the radicands and simplify if possible. Once none of the remaining terms can be combined or simplified, the expression is considered simplified.

Example 4 Simplify Expressions with Products

Simplify $5\sqrt[4]{6} \cdot 2\sqrt[4]{12} \cdot \sqrt[3]{10}$.

Multiply the radicands with identical indexes.

$5\sqrt[4]{6} \cdot 2\sqrt[4]{12} \cdot \sqrt[3]{10} = (5 \cdot 2)(\sqrt[4]{6} \cdot \sqrt[4]{12}) \cdot \sqrt[3]{10}$ Associative Property

$= 10 \cdot (\sqrt[4]{6} \cdot \sqrt[4]{12}) \cdot \sqrt[3]{10}$ Multiply.

$= 10\sqrt[4]{72}\sqrt[3]{10}$ Multiply.

The properties of radical expressions still hold when variables are in the radicand.

Example 5 Simplify Expressions with Several Operations

Simplify $6\sqrt[4]{x} \cdot \sqrt[4]{x^3} + 3(\sqrt[3]{x} + 2\sqrt[3]{x})$.

Follow the order of operations and the properties of radical expressions.

$6\sqrt[4]{x} \cdot \sqrt[4]{x^3} + 3(\sqrt[3]{x} + 2\sqrt[3]{x}) = 6\sqrt[4]{x} \cdot \sqrt[4]{x^3} + 3(3\sqrt[3]{x})$ Add like terms.

$= 6\sqrt[4]{x \cdot x^3} + 3(3\sqrt[3]{x})$ Associative Property

$= 6\sqrt[4]{x^4} + 9\sqrt[3]{x}$ Multiply.

$= 6x + 9\sqrt[3]{x}$ Simplify.

Exercises

Simplify each expression.

1. $\sqrt[3]{c^6}$

2. $\sqrt[4]{16d^9}$

3. $\sqrt[3]{9} \cdot \sqrt[3]{6} \cdot \sqrt[3]{3}$

4. $\sqrt[3]{\frac{8a^4}{125b^7}}$

5. $\sqrt[5]{\frac{32x^4}{5y^6z^5}}$

6. $\sqrt[4]{\frac{3}{2}} + 5\sqrt[4]{\frac{3}{2}} - 2\sqrt[4]{\frac{2}{3}}$

7. $3\sqrt[4]{6} \cdot 4\sqrt[3]{6} \cdot 5\sqrt[4]{8}$

8. $3\sqrt[4]{x^2} + 2\sqrt[4]{x} \cdot 4\sqrt[4]{x}$

9. $\sqrt[5]{a} \cdot 2\sqrt[5]{a^3} - 2(\sqrt[5]{a} + 4\sqrt[5]{a})$

10. $\sqrt[4]{\frac{x}{4}} + 5\sqrt[4]{\frac{x}{4}} - 2\sqrt[4]{\frac{2x}{3}}$

11. $\sqrt[4]{\frac{8a^2}{15b^3}} \cdot 3\sqrt[4]{\frac{2a^3}{27b}}$

12. $\sqrt[4]{\frac{16x^3}{81y^5}} + 3\sqrt[4]{\frac{x^3}{y}} + \sqrt[3]{\frac{16x}{y^8}}$

Think About It

13. Provide an example in which two radical expressions with *unlike* radicands can be combined by addition.

14. Provide an example in which two radical expressions with identical indices and with like variables in the radicand *cannot* be combined by addition.

LESSON 10-4 Radical Equations

Then

- You added, subtracted, and multiplied radical expressions.

Now

- **1** Solve radical equations.
- **2** Solve radical equations with extraneous solutions.

Why?

- The waterline length of a sailboat is the length of the line made by the water's edge when the boat is full. A sailboat's hull speed is the fastest speed that it can travel.

You can estimate hull speed h by using the formula $h = 1.34\sqrt{\ell}$, where ℓ is the length of the sailboat's waterline.

David Sanger/The Image Bank/Getty Images

NewVocabulary

radical equations
extraneous solutions

Common Core State Standards

Content Standards

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

1 Radical Equations

Equations that contain variables in the radicand, like $h = 1.34\sqrt{\ell}$, are called **radical equations**. To solve, isolate the desired variable on one side of the equation first. Then square each side of the equation to eliminate the radical.

KeyConcept Power Property of Equality

Words If you square both sides of a true equation, the resulting equation is still true.

Symbols If $a = b$, then $a^2 = b^2$.

Examples If $\sqrt{x} = 4$, then $(\sqrt{x})^2 = 4^2$.

Real-World Example 1 Variable as a Radicand

SAILING Idris and Sebastian are sailing in a friend's sailboat. They measure the hull speed at 9 nautical miles per hour. Find the length of the sailboat's waterline. Round to the nearest foot.

Understand You know how fast the boat will travel and that it relates to the length.

Plan The boat travels at 9 nautical miles per hour. The formula for hull speed is $h = 1.34\sqrt{\ell}$.

Solve

$h = 1.34\sqrt{\ell}$ Formula for hull speed

$9 = 1.34\sqrt{\ell}$ Substitute 9 for h.

$\frac{9}{1.34} = \frac{1.34\sqrt{\ell}}{1.34}$ Divide each side by 1.34.

$6.72 \approx \sqrt{\ell}$ Simplify.

$(6.72)^2 \approx (\sqrt{\ell})^2$ Square each side of the equation.

$45.16 \approx \ell$ Simplify.

The sailboat's waterline length is about 45 feet.

Check Check by substituting the estimate into the original formula.

$h = 1.34\sqrt{\ell}$ Formula for hull speed

$9 \stackrel{?}{=} 1.34\sqrt{45}$ $h = 9$ and $\ell = 45$

$9 \approx 8.98899327$ ✓ Multiply.

GuidedPractice

1. **DRIVING** The equation $v = \sqrt{2.5r}$ represents the maximum velocity that a car can travel safely on an unbanked curve when v is the maximum velocity in miles per hour and r is the radius of the turn in feet. If a road is designed for a maximum speed of 65 miles per hour, what is the radius of the turn?

To solve a radical equation, isolate the radical first. Then square both sides of the equation.

Example 2 Expression as a Radicand

Solve $\sqrt{a+5} + 7 = 12$.

$\sqrt{a+5} + 7 = 12$	Original equation
$\sqrt{a+5} = 5$	Subtract 7 from each side.
$\left(\sqrt{a+5}\right)^2 = 5^2$	Square each side.
$a + 5 = 25$	Simplify.
$a = 20$	Subtract 5 from each side.

WatchOut!

Squaring Each Side Remember that when you square each side of the equation, you must square the entire side of the equation, even if there is more than one term on the side.

GuidedPractice

Solve each equation.

2A. $\sqrt{c-3} - 2 = 4$

2B. $4 + \sqrt{h+1} = 14$

2 Extraneous Solutions

Squaring each side of an equation sometimes produces a solution that is not a solution of the original equation. These are called **extraneous solutions**. Therefore, you must check all solutions in the original equation.

Example 3 Variable on Each Side

Solve $\sqrt{k+1} = k - 1$. Check your solution.

$\sqrt{k+1} = k - 1$	Original equation
$\left(\sqrt{k+1}\right)^2 = (k-1)^2$	Square each side.
$k + 1 = k^2 - 2k + 1$	Simplify.
$0 = k^2 - 3k$	Subtract k and 1 from each side.
$0 = k(k-3)$	Factor.
$k = 0$ or $k - 3 = 0$	Zero Product Property
$k = 3$	Solve.

CHECK	$\sqrt{k+1} = k - 1$	Original equation	$\sqrt{k+1} = k - 1$	Original equation
	$\sqrt{0+1} \stackrel{?}{=} 0 - 1$	$k = 0$	$\sqrt{3+1} \stackrel{?}{=} 3 - 1$	$k = 3$
	$\sqrt{1} \stackrel{?}{=} -1$	Simplify.	$\sqrt{4} \stackrel{?}{=} 2$	Simplify.
	$1 \neq -1$ ✗	False	$2 = 2$ ✓	True

Since 0 does not satisfy the original equation, 3 is the only solution.

StudyTip

Extraneous Solutions When checking solutions for extraneous solutions, we are only interested in principal roots.

GuidedPractice

Solve each equation. Check your solution.

3A. $\sqrt{t+5} = t + 3$

3B. $x - 3 = \sqrt{x-1}$

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 1. **GEOMETRY** The surface area of a basketball is x square inches. What is the radius of the basketball if the formula for the surface area of a sphere is $SA = 4\pi r^2$?

Examples 2–3 **Solve each equation. Check your solution.**

2. $\sqrt{10h} + 1 = 21$
3. $\sqrt{7r + 2} + 3 = 7$
4. $5 + \sqrt{g - 3} = 6$
5. $\sqrt{3x - 5} = x - 5$
6. $\sqrt{2n + 3} = n$
7. $\sqrt{a - 2} + 4 = a$

Practice and Problem Solving

Extra Practice is on page R10.

Example 1 8. **EXERCISE** Suppose the function $S = \pi\sqrt{\frac{9.8\ell}{1.6}}$, where S represents speed in meters per second and ℓ is the leg length of a person in meters, can approximate the maximum speed that a person can run.

a. What is the maximum running speed of a person with a leg length of 1.1 meters to the nearest tenth of a meter?

b. What is the leg length of a person with a running speed of 6.7 meters per second to the nearest tenth of a meter?

c. As leg length increases, does maximum speed increase or decrease? Explain.

Examples 2–3 **Solve each equation. Check your solution.**

⑨ $\sqrt{a} + 11 = 21$
10. $\sqrt{t} - 4 = 7$
11. $\sqrt{n - 3} = 6$
12. $\sqrt{c + 10} = 4$
13. $\sqrt{h - 5} = 2\sqrt{3}$
14. $\sqrt{k + 7} = 3\sqrt{2}$
15. $y = \sqrt{12 - y}$
16. $\sqrt{u + 6} = u$
17. $\sqrt{r + 3} = r - 3$
18. $\sqrt{1 - 2t} = 1 + t$
19. $5\sqrt{a - 3} + 4 = 14$
20. $2\sqrt{x - 11} - 8 = 4$

21. **RIDES** The amount of time t, in seconds, that it takes a simple pendulum to complete a full swing is called the *period*. It is given by $t = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum, in feet.

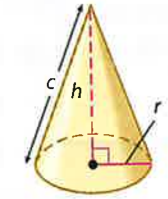
a. The Giant Swing completes a period in about 8 seconds. About how long is the pendulum's arm? Round to the nearest foot.

b. Does increasing the length of the pendulum increase or decrease the period? Explain.

Solve each equation. Check your solution.

22. $\sqrt{6a - 6} = a + 1$
23. $\sqrt{x^2 + 9x + 15} = x + 5$
24. $6\sqrt{\frac{5k}{4}} - 3 = 0$
25. $\sqrt{\frac{5y}{6}} - 10 = 4$
26. $\sqrt{2a^2 - 121} = a$
27. $\sqrt{5x^2 - 9} = 2x$

28. **CCSS REASONING** The formula for the slant height c of a cone is $c = \sqrt{h^2 + r^2}$, where h is the height of the cone and r is the radius of its base. Find the height of the cone if the slant height is 4 units and the radius is 2 units. Round to the nearest tenth.



29. **MULTIPLE REPRESENTATIONS** Consider $\sqrt{2x-7} = x - 7$.

- **a. Graphical** Clear the **Y=** list. Enter the left side of the equation as **Y1** $= \sqrt{2x-7}$. Enter the right side of the equation as **Y2** $= x - 7$. Press GRAPH.
- **b. Graphical** Sketch what is shown on the screen.
- **c. Analytical** Use the **intersect** feature on the **CALC** menu to find the point of intersection.
- **d. Analytical** Solve the radical equation algebraically. How does your solution compare to the solution from the graph?

30. PACKAGING A cylindrical container of chocolate drink mix has a volume of 162 cubic inches. The radius r of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where V is the volume of the container and h is the height.

- **a.** If the radius is 2.5 inches, find the height of the container. Round to the nearest hundredth.
- **b.** If the height of the container is 10 inches, find the radius. Round to the nearest hundredth.

H.O.T. Problems Use Higher-Order Thinking Skills

31. CCSS **CRITIQUE** Jada and Fina solved $\sqrt{6-b} = \sqrt{b+10}$. Is either of them correct? Explain.

Jada

$\sqrt{6-b} = \sqrt{b+10}$

$(\sqrt{6-b})^2 = (\sqrt{b+10})^2$

$6 - b = b + 10$

$-2b = 4$

$b = -2$

Check $\sqrt{6-(-2)} \overset{?}{=} \sqrt{(-2)+10}$

$\sqrt{8} = \sqrt{8}$ ✓

Fina

$\sqrt{6-b} = \sqrt{b+10}$

$(\sqrt{6-b})^2 = (\sqrt{b+10})^2$

$6 - b = b + 10$

$2b = 4$

$b = 2$

Check $\sqrt{6-(2)} \overset{?}{=} \sqrt{(2)+10}$

$\sqrt{4} \neq \sqrt{12}$ ✗

no solution

32. REASONING Which equation has the same solution set as $\sqrt{4} = \sqrt{x+2}$? Explain.

A. $\sqrt{4} = \sqrt{x} + \sqrt{2}$ **B.** $4 = x + 2$ **C.** $2 - \sqrt{2} = \sqrt{x}$

33. REASONING Explain how solving $5 = \sqrt{x} + 1$ is different from solving $5 = \sqrt{x+1}$.

34. OPEN ENDED Write a radical equation with a variable on each side. Then solve the equation.

35. REASONING Is the following equation *sometimes*, *always* or *never* true? Explain.

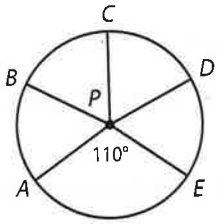
$$\sqrt{(x-2)^2} = x - 2$$

36. CHALLENGE Solve $\sqrt{x+9} = \sqrt{3} + \sqrt{x}$.

37. WRITING IN MATH Write some general rules about how to solve radical equations. Demonstrate your rules by solving a radical equation.

Standardized Test Practice

38. SHORT RESPONSE Zack needs to drill a hole at A, B, C, D, and E on circle P.



If Zack drills holes so that $m\angle APE = 110°$ and the other four angles are congruent, what is $m\angle CPD$?

39. Which expression is undefined when $w = 3$?

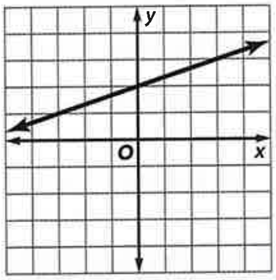
A $\frac{w-3}{w+1}$

C $\frac{w+1}{w^2-3w}$

B $\frac{w^2-3w}{3w}$

D $\frac{3w}{3w^2}$

40. What is the slope of a line that is parallel to the line?



F -3

H $\frac{1}{3}$

G $-\frac{1}{3}$

J 3

41. What are the solutions of $\sqrt{x+3} - 1 = x - 4$?

A $1, 6$

C 1

B $-1, -6$

D 6

Spiral Review

42. ELECTRICITY The voltage V required for a circuit is given by $V = \sqrt{PR}$, where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100-watt light bulb than a 75-watt light bulb if the resistance of both is 110 ohms? (Lesson 10-3)

Simplify each expression. (Lesson 10-2)

43. $\sqrt{6} \cdot \sqrt{8}$

44. $\sqrt{3} \cdot \sqrt{6}$

45. $7\sqrt{3} \cdot 2\sqrt{6}$

46. $\sqrt{\frac{27}{a^2}}$

47. $\sqrt{\frac{5c^5}{4d^5}}$

48. $\frac{\sqrt{9x^3y}}{\sqrt{16x^2y^2}}$

49. PHYSICAL SCIENCE A projectile is shot straight up from ground level. Its height h, in feet, after t seconds is given by $h = 96t - 16t^2$. Find the value(s) of t when h is 96 feet. (Lesson 9-5)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-7)

50. $2x^2 + 7x + 5$

51. $6p^2 + 5p - 6$

52. $5d^2 + 6d - 8$

53. $8k^2 - 19k + 9$

54. $9g^2 - 12g + 4$

55. $2a^2 - 9a - 18$

Determine whether each expression is a monomial. Write *yes* or *no*. Explain. (Lesson 7-1)

56. 12

57. $4x^3$

58. $a - 2b$

59. $4n + 5p$

60. $\frac{x}{y^2}$

61. $\frac{1}{5}abc^{14}$

Skills Review

Simplify.

62. 9^2

63. 10^6

64. 4^5

65. $(8v)^2$

66. $\left(\frac{w^3}{9}\right)^2$

67. $(10y^2)^3$

CHAPTER 10

Mid-Chapter Quiz

Lessons 10-1 through 10-4

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

1. $y = 2\sqrt{x}$

2. $y = -4\sqrt{x}$

3. $y = \frac{1}{2}\sqrt{x}$

4. $y = \sqrt{x} - 3$

5. $y = \sqrt{x - 1}$

6. $y = 2\sqrt{x - 2}$

7. **MULTIPLE CHOICE** The length of the side of a square is given by the function $s = \sqrt{A}$, where A is the area of the square. What is the length of the side of a square that has an area of 121 square inches? (Lesson 10-1)

A 121 inches

B 44 inches

C 11 inches

D 10 inches

Simplify each expression. (Lesson 10-2)

8. $2\sqrt{25}$

9. $\sqrt{12} \cdot \sqrt{8}$

10. $\sqrt{72xy^5z^6}$

11. $\frac{3}{1 + \sqrt{5}}$

12. $\frac{1}{5\sqrt{7}}$

13. **SATELLITES** A satellite is launched into orbit 200 kilometers above Earth. The orbital velocity of a satellite is given by the formula $v = \sqrt{\frac{Gm_E}{r}}$. v is velocity in meters per second, G is a given constant, m_E is the mass of Earth, and r is the radius of the satellite's orbit in meters. (Lesson 10-2)

a. The radius of Earth is 6,380,000 meters. What is the radius of the satellite's orbit in meters?

b. The mass of Earth is 5.97×10^{24} kilograms, and the constant G is 6.67×10^{-11} $\text{N} \cdot \frac{\text{m}^2}{\text{kg}^2}$ where N is in Newtons. Use the formula to find the orbital velocity of the satellite in meters per second.

14. **MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{16}{32}}$? (Lesson 10-2)

F $\frac{1}{2}$

G $\frac{\sqrt{2}}{2}$

H 2

J 4

Simplify each expression. (Lesson 10-3)

15. $3\sqrt{2} + 5\sqrt{2}$

16. $\sqrt{11} - 3\sqrt{11}$

17. $6\sqrt{2} + 4\sqrt{50}$

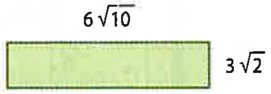
18. $\sqrt{27} - \sqrt{48}$

19. $4\sqrt{3}(2\sqrt{6})$

20. $3\sqrt{20}(2\sqrt{5})$

21. $(\sqrt{5} + \sqrt{7})(\sqrt{20} + \sqrt{3})$

22. **GEOMETRY** Find the area of the rectangle. (Lesson 10-3)



Solve each equation. Check your solution. (Lesson 10-4)

23. $\sqrt{5x} - 1 = 4$

24. $\sqrt{a - 2} = 6$

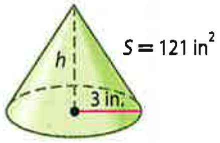
25. $\sqrt{15 - x} = 4$

26. $\sqrt{3x^2 - 32} = x$

27. $\sqrt{2x - 1} = 2x - 7$

28. $\sqrt{x + 1} + 2 = 4$

29. **GEOMETRY** The lateral surface area S of a cone can be found by using the formula $S = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the height of the cone. Find the height of the cone. (Lesson 10-4)



LESSON 10-5 The Pythagorean Theorem

:: Then

- You solved quadratic equations by using the Square Root Property.

:: Now

1. Solve problems by using the Pythagorean Theorem.
2. Determine whether a triangle is a right triangle.

:: Why?

- Televisions are measured along the diagonal of the screen. If the height and width of the screen is known, the Pythagorean Theorem can be used to find the measure of the diagonal.

NewVocabulary

hypotenuse
legs
converse
Pythagorean triple

Common Core State Standards

Mathematical Practices

1 Make sense of problems and persevere in solving them.

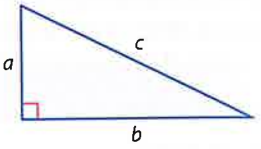
1 The Pythagorean Theorem

1 The Pythagorean Theorem In a right triangle, the side opposite the right angle, called the **hypotenuse**, is always the longest. The other two sides are the **legs**.

KeyConcept The Pythagorean Theorem

Words If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

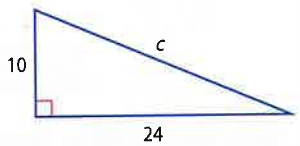
Symbols $c^2 = a^2 + b^2$



Example 1 Find the Length of a Side

Find each missing length. If necessary, round to the nearest hundredth.

a.



$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 10^2 + 24^2$ $a = 10$ and $b = 24$

$c^2 = 100 + 576$ Evaluate squares.

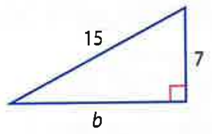
$c^2 = 676$ Simplify.

$c = \pm\sqrt{676}$ Take the square root of each side.

$c = \pm 26$ $(\pm 26)^2 = 676$

Length cannot be negative, so the missing length is 26 units.

b.



$c^2 = a^2 + b^2$ Pythagorean Theorem

$15^2 = 7^2 + b^2$ $a = 7$ and $c = 15$

$225 = 49 + b^2$ Evaluate squares.

$176 = b^2$ Subtract 49 from each side.

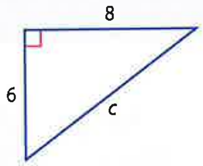
$\pm\sqrt{176} = b$ Take the square root of each side.

$\pm 13.27 \approx b$ Use a calculator to evaluate $\sqrt{176}$.

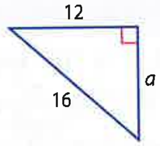
The missing length is 13.27 units.

GuidedPractice

1A.



1B.



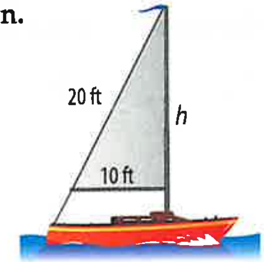
Real-WorldLink

A keelboat is a sailboat with a weighted keel, a vertical fin at the bottom of the boat. Keels are 20 to 30 inches in length.

Source: United States Sailing Association

Real-World Example 2 Find the Length of a Side

SAILING The sail of a keelboat forms a right triangle as shown. Find the height of the sail.



$20^2 = h^2 + 10^2$	Pythagorean Theorem
$400 = h^2 + 100$	Evaluate squares.
$300 = h^2$	Subtract 100 from each side.
$\pm 17.32 \approx h$	Take the square root of each side.
$17.32 \approx h$	Use the positive value.

The sail is approximately 17.32 feet high.

GuidedPractice

2. Suppose the longest side of the sail is 30 feet long and the shortest side is 14 feet long. Find the height of the sail.

2 Right Triangles If you exchange the phrases after *if* and *then* of an if-then statement, the result is the **converse** of the statement. The converse of the Pythagorean Theorem can be used to determine whether a triangle is a right triangle.

KeyConcept Converse of the Pythagorean Theorem

If a triangle has side lengths a, b, and c such that $c^2 = a^2 + b^2$, then the triangle is a right triangle. If $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle.

A **Pythagorean triple** is a group of three counting numbers that satisfy the equation $c^2 = a^2 + b^2$, where c is the greatest number. Examples include (3, 4, 5) and (5, 12, 13). Multiples of Pythagorean triples also satisfy the converse of the Pythagorean Theorem, so (6, 8, 10) is also a Pythagorean triple.

Example 3 Check for Right Triangles

Determine whether 9, 12, and 16 can be the lengths of the sides of a right triangle.

Since the measure of the longest side is 16, let $c = 16$, $a = 9$, and $b = 12$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$16^2 \stackrel{?}{=} 9^2 + 12^2$	$a = 9$, $b = 12$, and $c = 16$
$256 \stackrel{?}{=} 81 + 144$	Evaluate squares.
$256 \neq 225$	Add.

Since $c^2 \neq a^2 + b^2$, segments with these measures cannot form a right triangle.

GuidedPractice

Determine whether each set of measures can be the lengths of the sides of a right triangle.

3A. 30, 40, 50

3B. 6, 12, 18

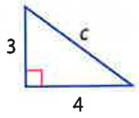
Digital Vision/Getty Images
Pacific Stock/SuperStock

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 **Find each missing length. If necessary, round to the nearest hundredth.**

1.



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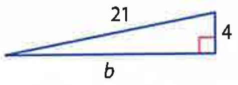

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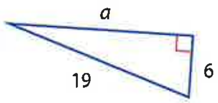

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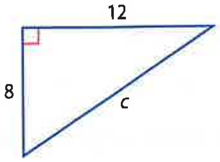
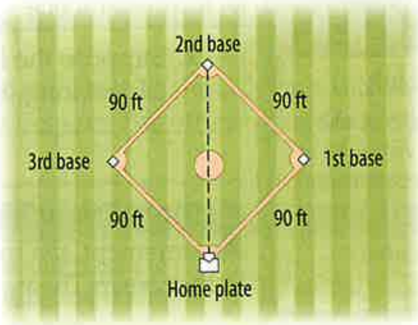


Example 2

5. **BASEBALL** A baseball diamond is a square. The distance between consecutive bases is 90 feet.

a. How far does a catcher have to throw the ball from home plate to second base?

b. How far does a third baseman throw the ball to the first baseman from a point in the baseline 15 feet from third to second base?



c. A base runner going from first to second base is 100 feet from home plate. How far is the runner from second base?

Example 3 **Determine whether each set of measures can be the lengths of the sides of a right triangle.**

6. 8, 12, 16

7. 28, 45, 53

8. 7, 24, 25

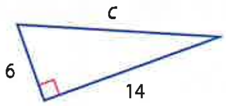
9. 15, 25, 45

Practice and Problem Solving

Extra Practice is on page R10.

Example 1 **Find each missing length. If necessary, round to the nearest hundredth.**

10.



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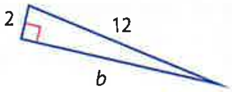

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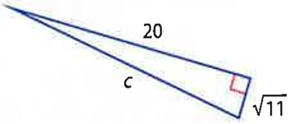

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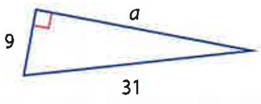

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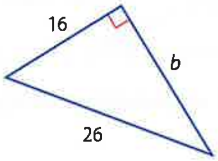

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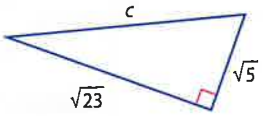

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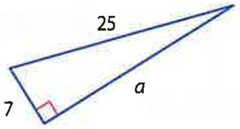
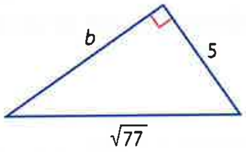
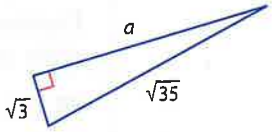


17.



18.



Example 2

⑲ **TELEVISION** Larry is buying an entertainment stand for his television. The diagonal of his television is 42 inches. The space for the television measures 30 inches by 36 inches. Will Larry's television fit? Explain.

Example 3

Determine whether each set of measures can be the lengths of the sides of a right triangle. Then determine whether they form a Pythagorean triple.

20. 9, 40, 41

21. 3, $2\sqrt{10}$, $\sqrt{41}$

22. 4, $\sqrt{26}$, 12

23. $\sqrt{5}$, 7, 14

24. 8, 31.5, 32.5

25. $\sqrt{65}$, $6\sqrt{2}$, $\sqrt{97}$

26. 18, 24, 30

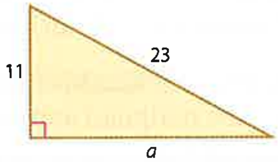
27. 36, 77, 85

28. 17, 33, 98

29. **GEOMETRY** Refer to the triangle at the right.

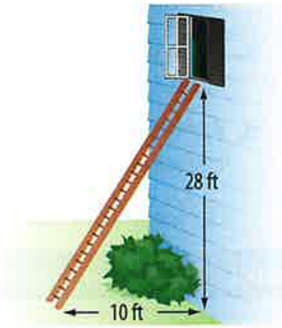
a. What is a?

b. Find the area of the triangle.



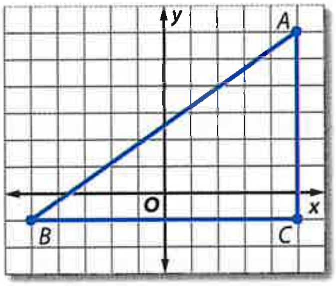
30. **GARDENING** Khaliah wants to plant flowers in a triangular plot. She has three lengths of plastic garden edging that measure 8 feet, 15 feet, and 17 feet. Determine whether these pieces form a right triangle. Explain.

31. **LADDER** Mr. Takeo is locked out of his house. The only open window is on the second floor. There is a bush along the edge of the house, so he places the neighbor's ladder 10 feet from the house. To the nearest foot, what length of ladder does he need to reach the window?

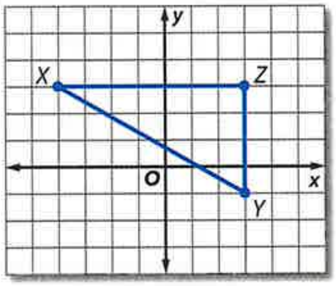


CCSS **TOOLS** **Find the length of the hypotenuse. Round to the nearest hundredth.**

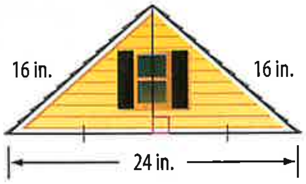
32.



33.



34. **DOLLHOUSE** Alonso is building a dollhouse for his sister's birthday. The roof is 24 inches across and the slanted side is 16 inches long as shown. Find the height of the roof to the nearest tenth of an inch.



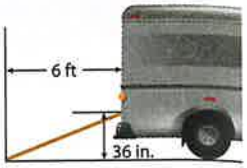
35. **GEOMETRY** Each side of a cube is 5 inches long. Find the length of a diagonal of the cube.

36. **TOWN SQUARES** The largest town square in the world is Tiananmen Square in Beijing, China, covering 98 acres.

a. One square mile is 640 acres. Assuming that Tiananmen Square is a square, how many feet long is a side to the nearest foot?

b. To the nearest foot, what is the diagonal distance across Tiananmen Square?

37. **TRUCKS** Violeta needs to construct a ramp to roll a cart of moving boxes from her garage into the back of her truck. How long does the ramp have to be?



If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

38. $a = x, b = x + 41, c = 85$

39. $a = 8, b = x, c = x + 2$

40. $a = 12, b = x - 2, c = x$

41. $a = x, b = x + 7, c = 97$

42. $a = x - 47, b = x, c = x + 2$

43. $a = x - 32, b = x - 1, c = x$

44. **GEOMETRY** A right triangle has one leg that is 8 inches shorter than the other leg. The hypotenuse is 30 inches long. Find the length of each leg.

45. **MULTIPLE REPRESENTATIONS** In this problem, you will derive a method for finding the midpoint and length of a segment on the coordinate plane.

   a. **Graphical** Use a graph to find the lengths of the segments between (3, 2) and (8, 2) and between (4, 1) and (4, 9). Then find the midpoint of each segment.

   b. **Logical** Use what you learned in part **a** to write expressions for the lengths of the segments between (x_1, y) and (x_2, y) and between (x, y_1) and (x, y_2). What would be the midpoint of each segment?

   c. **Analytical** Based on your results from part **b**, find the midpoint of the segment with endpoints at (x_1, y_1), and (x_2, y_2).

   d. **Analytical** Use the Pythagorean Theorem to write an expression for the distance between (x_1, y_1), and (x_2, y_2).

H.O.T. Problems Use Higher-Order Thinking Skills

46. **ERROR ANALYSIS** Wyatt and Dario are determining whether 36, 77, and 85 form a Pythagorean triple. Is either of them correct? Explain your reasoning.

Wyatt

$36^2 + 77^2 \overset{?}{=} 85^2$

$1296 + 5929 \overset{?}{=} 7225$

$7225 = 7225$

yes

Dario

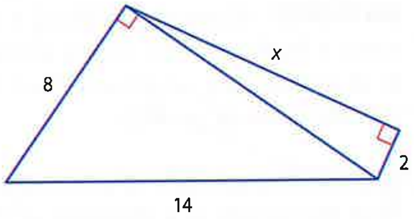
$36^2 + 85^2 \overset{?}{=} 77^2$

$1296 + 7725 \overset{?}{=} 5929$

$9021 \neq 5929$

no

47. **PERSEVERANCE** Find the value of x in the figure.



48. **REASONING** Provide a counterexample that is a specific case to show that the statement is false. *Any two right triangles with the same hypotenuse have the same area.*

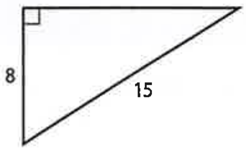
49. **OPEN ENDED** Draw a right triangle that has a hypotenuse of $\sqrt{72}$ units.

50. **WRITING IN MATH** Explain how to determine whether segments in three lengths could form a right triangle.

Standardized Test Practice

51. GEOMETRY Find the missing length.

A −17
B $-\sqrt{161}$
C $\sqrt{161}$
D 17



52. What is a solution of this equation?

$$x + 1 = \sqrt{x + 1}$$

F 0, 3
G 3
H 0
J no solutions

53. SHORT RESPONSE A plumber charges \$40 for the first hour of each house call plus \$8 for each additional half hour. If the plumber works for 4 hours, how much does he charge?

54. Find the next term in the geometric sequence $4, 3, \frac{9}{4}, \frac{27}{16}, \ldots$.

A $\frac{64}{81}$
B $\frac{81}{64}$
C $\frac{4}{3}$
D $\frac{243}{64}$

Spiral Review

Solve each equation. Check your solution. (Lesson 10-4)

55. $\sqrt{x} = 16$
56. $\sqrt{4x} = 64$
57. $\sqrt{10x} = 10$
58. $\sqrt{8x} + 1 = 65$
59. $\sqrt{x + 1} + 2 = 4$
60. $\sqrt{x - 15} = 3 - \sqrt{x}$

Simplify each expression. (Lesson 10-3)

61. $2\sqrt{3} + 5\sqrt{3}$
62. $4\sqrt{5} - 2\sqrt{5}$
63. $6\sqrt{7} + 2\sqrt{28}$
64. $\sqrt{18} - 4\sqrt{2}$
65. $3\sqrt{5} - 5\sqrt{3} + 9\sqrt{5}$
66. $4\sqrt{3} + 6\sqrt{12}$

Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 9-3)

67. $g(x) = x^2 - 8$
68. $h(x) = \frac{1}{4}x^2$
69. $h(x) = -x^2 + 5$
70. $g(x) = (x + 10)^2$
71. $g(x) = -2x^2$
72. $h(x) = -x^2 - \frac{4}{3}$

73. ROCK CLIMBING While rock climbing, Damaris launches a grappling hook from a height of 6 feet with an initial upward velocity of 56 feet per second. The hook just misses the stone ledge that she wants to scale. As it falls, the hook anchors on a ledge 30 feet above the ground. How long was the hook in the air? (Lesson 8-7)

Find each product. (Lesson 8-3)

74. $(b + 8)(b + 2)$
75. $(x - 4)(x - 9)$
76. $(y + 4)(y - 8)$
77. $(p + 2)(p - 10)$
78. $(2w - 5)(w + 7)$
79. $(8d + 3)(5d + 2)$

80. BUSINESS The amount of money spent at West Outlet Mall continues to increase. The total $T(x)$ in millions of dollars can be estimated by the function $T(x) = 12(1.12)^x$, where x is the number of years after it opened in 2005. Find the amount of sales in 2015, 2016, and 2017. (Lesson 7-5)

Skills Review

Solve each proportion.

81. $\frac{x}{5} = \frac{12}{3}$
82. $\frac{12}{x} = \frac{3}{4}$
83. $\frac{5}{4} = \frac{10}{x}$
84. $\frac{3}{5} = \frac{12}{x + 8}$

EXTEND 10-5

Algebra Lab
Distance on the Coordinate Plane

The Pythagorean Theorem can be extended to develop a formula for finding the distance between two points on a coordinate plane.

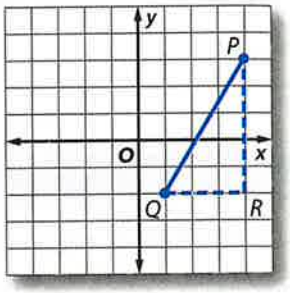
Activity Find Distance

Find the distance between the points at $P(4, 3)$ and $Q(1, -2)$.

Step 1 Graph the points and $\overline{PQ}$.

Step 2 Draw a vertical segment down from P and a horizontal segment to the right from Q. Label the the point of intersection R. Notice that $\triangle PQR$ is a right triangle.

Step 3 Use the Pythagorean Theorem.



$c^2 = a^2 + b^2$ Pythagorean Theorem

$PQ^2 = 3^2 + 5^2$ $a = 3$ and $c = 5$

$PQ^2 = 9 + 25$ Evaluate squares.

$PQ^2 = 34$ Subtract 49 from each side.

$PQ = \pm\sqrt{34}$ Take the square root of each side.

$PQ \approx 5.83$ Use a calculator to evaluate $\sqrt{34}$. Use the positive value.

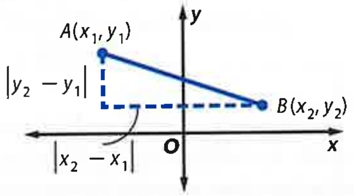
The distance between two points can be generalized as follows.

KeyConcept The Distance Formula

Words The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the following formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Model



Model and Analyze

Find the distance between the points with the given coordinates.

1. $(6, -2), (12, 8)$

2. $(4, 8), (-3, -6)$

3. $(3, 0), (6, -2)$

4. $(-2, -4), (-5, -3)$

5. $(5, 1), (0, 4)$

6. $(-5, 2), (4, -2)$

The Midpoint Formula states that the midpoint M of a line segment with endpoints at (x_1, y_1) and (x_2, y_2) is given by $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Find the coordinates of the midpoint of the segment with the given endpoints.

7. $(5, -10), (5, 8)$

8. $(2, -2), (6, 2)$

9. $(5, 0), (0, 3)$

10. $(-4, 1), (3, -1)$

11. $(3, -17), (2, -8)$

12. $(-2, 2), (4, 10)$

13. The point that is equidistant from both endpoints is the midpoint of a segment. Use the Midpoint Formula to find the midpoint of the segment with endpoints at $(-16, -7)$ and $(-4, -3)$. Then use the Distance Formula to verify that the midpoint you found is correct. (*Hint:* The midpoint must be on the segment.)

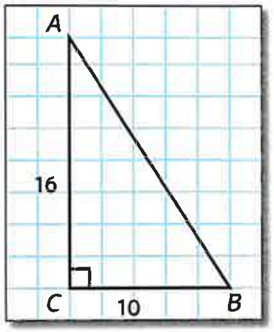
EXPLORE

10-6 Algebra Lab
Investigating Trigonometric Ratios

Recall that *similar triangles* have the same shape, but not necessarily the same size. You can use similar triangles to investigate the ratios of the lengths of sides of right triangles.

Activity Collect the Data

Step 1 Use a ruler and grid paper to draw several right triangles with legs in a ratio of 5:8. Include right triangles with the side lengths listed in the table below and several more right triangles similar to these three. Label the vertices of each triangle as *A*, *B*, and *C*, where *C* is at the right angle, *B* is opposite the longest leg, and *A* is opposite the shortest leg.



Step 2 Copy the table below. Complete the first three columns by measuring the hypotenuse (side $\overline{AB}$) in each right triangle you created and recording its length to the nearest tenth.

Step 3 Calculate and record the ratios in the middle two columns. Round to the nearest hundredth.

Step 4 Use a protractor to carefully measure angles *A* and *B* to the nearest degree in each right triangle. Record the angle measures in the table.

Side Lengths			Ratios		Angle Measures		
side *BC*	side *AC*	side *AB*	$\frac{BC}{AC}$	$\frac{BC}{AB}$	angle *A*	angle *B*	angle *C*
2.5	4						90°
5	8						90°
10	16						90°
							90°
							90°
							90°

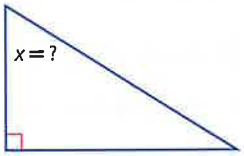
Analyze the Results

1. Examine the measures and ratios in the table. What do you notice? Write a sentence or two to describe any patterns you see.

Make a Conjecture

2. For any right triangle similar to the ones you have drawn here, what will be the value of the ratio of the length of the shortest leg to the length of the longest leg?

3. If you draw a right triangle and calculate the ratio of the length of the shortest leg to the length of the hypotenuse to be approximately 0.53, what will be the measure of the larger acute angle in the right triangle?



LESSON 10-6 Trigonometric Ratios

Then

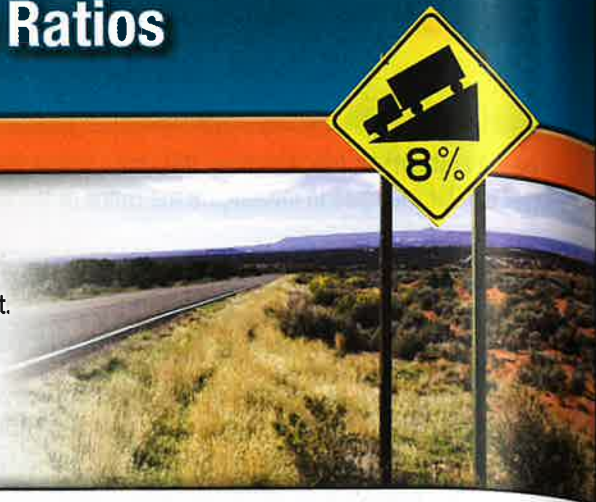
- You used the Pythagorean Theorem

Now

1. Find trigonometric ratios of angles.
2. Use trigonometry to solve triangles.

Why?

- If a road has a percent grade of 8%, this means the road rises or falls 8 feet over a horizontal distance of 100 feet. Trigonometric ratios can be used to determine the angle that the road rises or falls.



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NewVocabulary

trigonometry
trigonometric ratio
sine
cosine
tangent
solving the triangle
inverse sine
inverse cosine
inverse tangent

Common Core State Standards

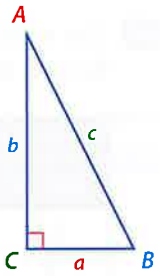
Mathematical Practices

5 Use appropriate tools strategically.

1 Trigonometric Ratios

Trigonometry is the study of relationships among the angles and sides of triangles. A **trigonometric ratio** is a ratio that compares the side lengths of two sides of a right triangle. The three most common trigonometric ratios, **sine**, **cosine**, and **tangent**, are described below.

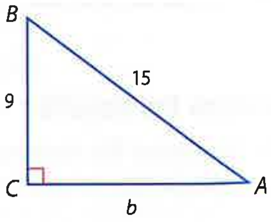
KeyConcept Trigonometric Ratios

Words	Symbols	Model
sine of $\angle A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$	$\sin A = \frac{a}{c}$	
cosine of $\angle A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$	$\cos A = \frac{b}{c}$	
tangent of $\angle A = \frac{\text{leg opposite } \angle A}{\text{Leg adjacent to } \angle A}$	$\tan A = \frac{a}{b}$	

Opposite, adjacent, and hypotenuse are abbreviated *opp*, *adj*, and *hyp*, respectively.

Example 1 Find Sine, Cosine, and Tangent Ratios

Find the values of the three trigonometric ratios for angle A.



Step 1 Use the Pythagorean Theorem to find b.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$9^2 + b^2 = 15^2$ $a = 9$ and $c = 15$

$81 + b^2 = 225$ Simplify.

$b^2 = 144$ Subtract 81 from each side.

$b = 12$ Take the square root of each side.

Step 2 Use the side lengths to write the trigonometric ratios.

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{9}{15} = \frac{3}{5} \quad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{15} = \frac{4}{5} \quad \tan A = \frac{\text{opp}}{\text{adj}} = \frac{9}{12} = \frac{3}{4}$$

GuidedPractice

1. Find the values of the three trigonometric ratios for angle B.

WatchOut!

CCSS Tools Make sure your graphing calculator is in degree mode.

Example 2 Use a Calculator to Evaluate Expressions

Use a calculator to find cos 42° to the nearest ten-thousandth.

KEYSTROKES: COS 42) ENTER

```
cos(42)
      .7431448255
```

Rounded to the nearest ten-thousandth, cos 42° ≈ 0.7431.

Guided Practice

2A. sin 31°

2B. tan 76°

2C. cos 55°

2 Use Trigonometric Ratios

When you find all unknown measures of the sides and angles of a right triangle, you are **solving the triangle**. You can find the missing measures if you know the measure of two sides of the triangle or the measure of one side and the measure of one acute angle.

StudyTip

Remembering Trigonometric Ratios SOH–CAH–TOA can be used to help you remember the ratios for sine, cosine, and tangent. Each letter represents a word.

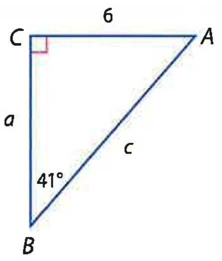
$\sin A = \frac{\text{opp}}{\text{hyp}}$

$\cos A = \frac{\text{adj}}{\text{hyp}}$

$\tan A = \frac{\text{opp}}{\text{adj}}$

Example 3 Solve a Triangle

Solve the right triangle. Round each side length to the nearest tenth.



Step 1 Find the measure of $\angle A$. $180° - (90° + 41°) = 49°$ The measure of $\angle A = 49°$.

Step 2 Find a. Since you are given the measure of the side opposite $\angle B$ and are finding the measure of the side adjacent to $\angle B$, use the tangent ratio.

$\tan 41° = \frac{6}{a}$ Definition of tangent

$a \tan 41° = 6$ Multiply each side by a.

$a = \frac{6}{\tan 41°}$ or about 6.9 Divide each side by tan 41°. Use a calculator.

So the measure of a or $\overline{BC}$ is about 6.9.

Step 3 Find c. Since you are given the measure of the side opposite $\angle B$ and are finding the measure of the hypotenuse, use the sine ratio.

$\sin 41° = \frac{6}{c}$ Definition of sine

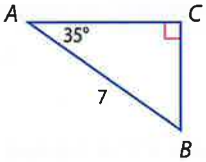
$c \sin 41° = 6$ Multiply each side by c.

$c = \frac{6}{\sin 41°}$ or about 9.1 Divide each side by sin 41°. Use a calculator.

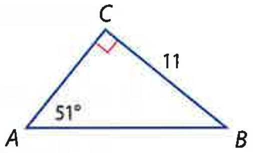
So the measure of c or $\overline{AB}$ is about 9.1.

Guided Practice

3A.



3B.



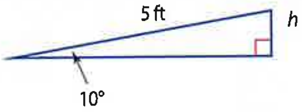
Real-WorldLink

For optimum health, all adults ages 18–65 should get at least 30 minutes of moderately intense activity five days per week.

Source: American Heart Association

Real-World Example 4 Find a Missing Side Length

EXERCISE A trainer sets the incline on a treadmill to 10°. The walking surface of the treadmill is 5 feet long. About how many inches is the end of the treadmill from the floor?



$\sin 10° = \frac{h}{5}$ Definition of sine

$5 \cdot \sin 10° = h$ Multiply each side by 5.

$0.87 \approx h$ Use a calculator.

The value of h is in feet. Multiply 0.87 by 12 to convert feet to inches. The trainer raised the treadmill about 10.4 inches.

GuidedPractice

4. SKATEBOARDING The angle that a skateboarding ramp forms with the ground is 25° and the height of the ramp is 6 feet. Determine the length of the ramp.

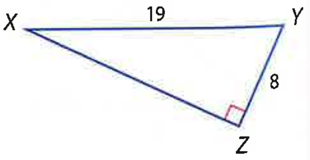
A trigonometric function has a rule given by a trigonometric ratio. If you know the sine, cosine, or tangent of an acute angle, you can use the *inverse* of the trigonometric function to find the measure of the angle.

KeyConcept Inverse Trigonometric Functions

Words	If $\angle A$ is an acute angle and the sine of A is x, then the **inverse sine** of x is the measure of $\angle A$.
Symbols	If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the cosine of A is x, then the **inverse cosine** of x is the measure of $\angle A$.
Symbols	If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the tangent of A is x, then the **inverse tangent** of x is the measure of $\angle A$.
Symbols	If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

Example 5 Find a Missing Angle Measure

Find $m\angle Y$ to the nearest degree.



You know the measure of the side adjacent to $\angle Y$ and the measure of the hypotenuse. Use the cosine ratio.

$\cos Y = \frac{8}{19}$ Definition of cosine

Use a calculator and the [COS^{-1}] function to find the measure of the angle.

KEYSTROKES: 2nd [COS^{-1}] 8 ÷ 19) ENTER 65.098937 So, $m\angle Y = 65°$.

GuidedPractice

5. Find $m\angle X$ to the nearest degree if $XY = 14$ and $YZ = 5$.

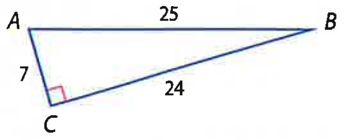
Masterfile

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 **Find the values of the three trigonometric ratios for angle A.**

1.



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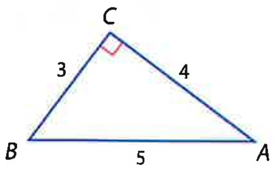

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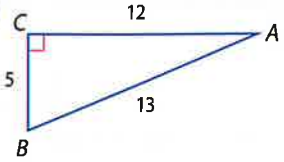
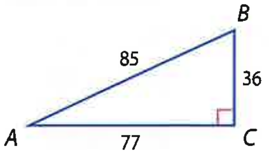


4.



Example 2 **CCSS TOOLS** **Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.**

5. sin 37°

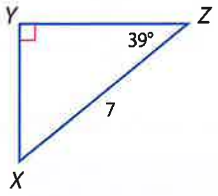
6. cos 23°

7. tan 14°

8. cos 82°

Example 3 **Solve each right triangle. Round each side length to the nearest tenth.**

9.



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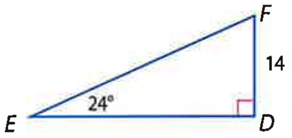

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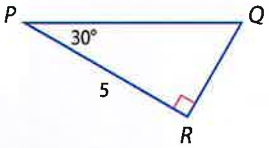
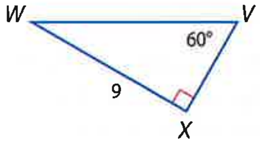


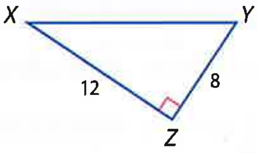
12.



Example 4 **13.** **SNOWBOARDING** A hill used for snowboarding has a vertical drop of 3500 feet. The angle the run makes with the ground is 18°. Estimate the length of r.

Example 5 **Find $m\angle X$ for each right triangle to the nearest degree.**

14.



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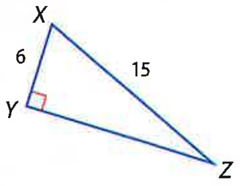

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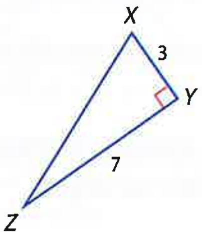

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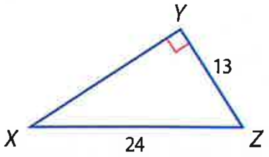
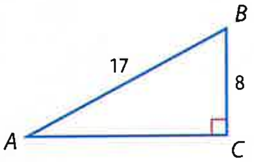


Practice and Problem Solving

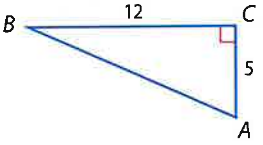
Extra Practice is on page R10.

Example 1 **Find the values of the three trigonometric ratios for angle *B*.**

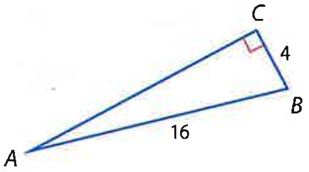
18.



19.



20.



Example 2 **CCSS TOOLS Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.**

21. tan 2°

22. sin 89°

23. cos 44°

24. tan 45°

25. sin 73°

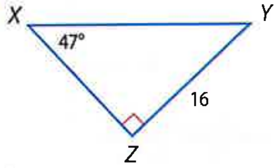
26. cos 90°

27. sin 30°

28. tan 60°

Example 3 **Solve each right triangle. Round each side length to the nearest tenth.**

29



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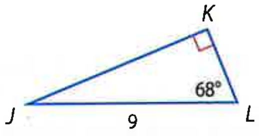

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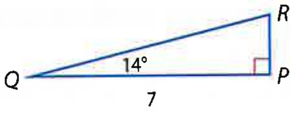

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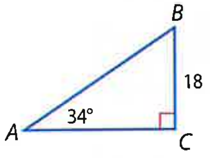

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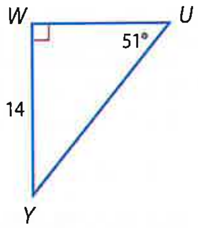

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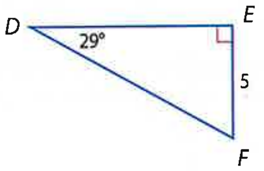
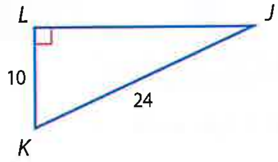


Example 4 **35. ESCALATORS** At a local mall, an escalator is 110 feet long. The angle the escalator makes with the ground is 29°. Find the height reached by the escalator.

Example 5 **Find *m∠J* for each right triangle to the nearest degree.**

36.



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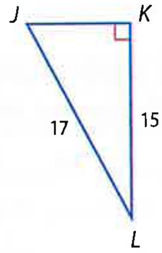

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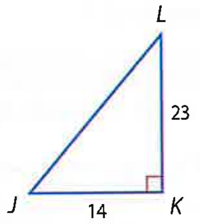

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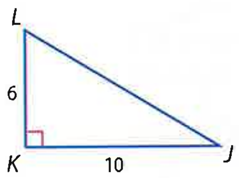

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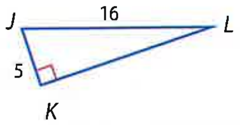

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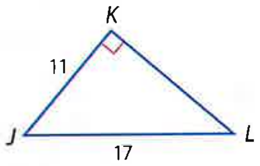


42. MONUMENTS The Lincoln Memorial building measures 204 feet long, 134 feet wide, and 99 feet tall. Chloe is looking at the top of the monument at an angle of 55°. How far away is she standing from the monument?

43 **AIRPLANES** Ella looks down at a city from an airplane window. The airplane is 5000 feet in the air, and she looks down at an angle of 8°. Determine the horizontal distance to the city.

44. **FORESTS** A forest ranger estimates the height of a tree is about 175 feet. If the forest ranger is standing 100 feet from the base of the tree, what is the measure of the angle formed by the ranger and the top of the tree?

Suppose $\angle A$ is an acute angle of right triangle ABC.

45. Find $\sin A$ and $\tan A$ if $\cos A = \frac{3}{4}$.

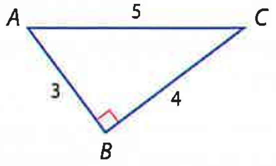
46. Find $\tan A$ and $\cos A$ if $\sin A = \frac{2}{7}$.

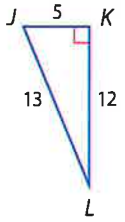
47. Find $\cos A$ and $\tan A$ if $\sin A = \frac{1}{4}$.

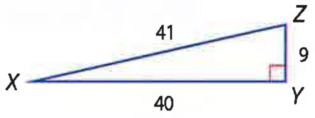
48. Find $\sin A$ and $\cos A$ if $\tan A = \frac{5}{3}$.

49. **SUBMARINES** A submarine descends into the ocean at an angle of 10° below the water line and travels 3 miles diagonally. How far beneath the surface of the water has the submarine reached?

50. **MULTIPLE REPRESENTATIONS** In this problem, you will explore a relationship between the sine and cosine functions.







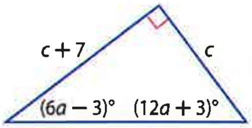
a. Tabular Copy and complete the table using the triangles shown above.

Triangle	Trigonometric Ratios		$\sin^2$	$\cos^2$	$\sin^2 + \cos^2 =$
ABC	$\sin A =$	$\cos A =$	$\sin^2 A =$	$\cos^2 A =$	
	$\sin C =$	$\cos C =$	$\sin^2 C =$	$\cos^2 C =$	
JKL	$\sin J =$	$\cos J =$	$\sin^2 J =$	$\cos^2 J =$	
	$\sin L =$	$\cos L =$	$\sin^2 L =$	$\cos^2 L =$	
XYZ	$\sin X =$	$\cos X =$	$\sin^2 X =$	$\cos^2 X =$	
	$\sin Z =$	$\cos Z =$	$\sin^2 Z =$	$\cos^2 Z =$	

b. Verbal Make a conjecture about the sum of the squares of the sine and cosine functions of an acute angle in a right triangle.

H.O.T. Problems Use Higher-Order Thinking Skills

51. **CHALLENGE** Find a and c in the triangle shown.



52. **REASONING** Use the definitions of the sine and cosine ratios to define the tangent ratio.

53. **WRITING IN MATH** How can triangles be used to solve problems?

54. **ARGUMENTS** The sine and cosine of an acute angle in a right triangle are equal. What can you conclude about the triangle?

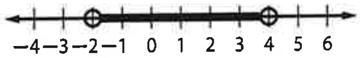
55. **WRITING IN MATH** Explain how to use trigonometric ratios to find the missing length of a side of a right triangle given the measure of one acute angle and the length of one side.

Standardized Test Practice

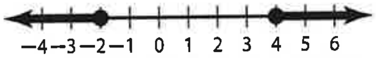
56. Which graph below represents the solution set for $-2 \le x \le 4$?

A

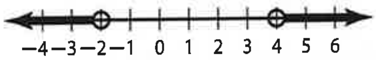
B



C



D



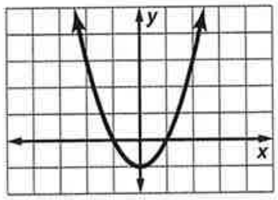
57. PROBABILITY Suppose one chip is chosen from a bin with the chips shown. To the nearest tenth, what is the probability that a green chip is chosen?

Color	Number
yellow	7
blue	9
orange	3
green	5
red	6

F 0.2 **H** 0.6

G 0.5 **J** 0.8

58. In the graph, for what value(s) of x is $y = 0$?



A 0 **C** 1

B −1 **D** 1 and −1

59. EXTENDED RESPONSE A 16-foot ladder is placed against the side of a house so that the bottom of the ladder is 8 feet from the base of the house.

a. If the bottom of the ladder is moved closer to the base of the house, does the height reached by the ladder increase or decrease?

b. What conclusion can you make about the distance between the bottom of the ladder and the base of the house and the height reached by the ladder?

c. How high does the ladder reach if the ladder is 3 feet from the base of the house?

Spiral Review

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth. (Lesson 10-5)

60. $a = 16, b = 63, c = ?$

61. $b = 3, a = \sqrt{112}, c = ?$

62. $c = 14, a = 9, b = ?$

63. $a = 6, b = 3, c = ?$

64. $b = \sqrt{77}, c = 12, a = ?$

65. $a = 4, b = \sqrt{11}, c = ?$

66. AVIATION The relationship between a plane's length L in feet and the pounds P its wings can lift is described by $L = \sqrt{kP}$, where k is the constant of proportionality. Find k for this plane to the nearest hundredth. (Lesson 10-4)

67. FINANCIAL LITERACY A salesperson is paid $32,000 a year plus 5% of the amount in sales made. What is the amount of sales needed to have an annual income greater than $45,000? (Lesson 5-3)

Skills Review

Solve each proportion.

68. $\frac{8}{9} = \frac{6}{z}$

69. $\frac{p}{6} = \frac{4}{3}$

70. $\frac{0.3}{r} = \frac{0.9}{1.7}$

71. $\frac{0.6}{1.1} = \frac{y}{8.47}$

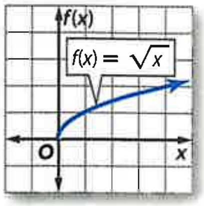
10 Study Guide and Review

Study Guide

KeyConcepts

Square Root Functions (Lesson 10-1)

- A square root function contains the square root of a variable.
- The parent function of the family of square root functions is $f(x) = \sqrt{x}$.



Simplifying Radical Expressions (Lesson 10-2)

- A radical expression is in simplest form when
  - no radicands have perfect square factors other than 1,
  - no radicals contain fractions,
  - and no radicals appear in the denominator of a fraction.

Operations with Radical Expressions and Equations (Lessons 10-3 and 10-4)

- Radical expressions with like radicals can be added or subtracted.
- Use the FOIL method to multiply radical expressions.

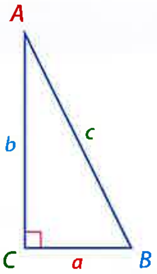
Pythagorean Theorem and Trigonometric Ratios (Lessons 10-5 and 10-6)

Pythagorean Theorem $c^2 = a^2 + b^2$

$\sin A = \frac{a}{c}$

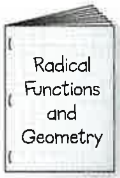
$\cos A = \frac{b}{c}$

$\tan A = \frac{a}{b}$



FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

conjugate (p. 630)
converse (p. 649)
cosine (p. 656)
Distance Formula (p. 654)
extraneous solution (p. 653)
hypotenuse (p. 648)
inverse cosine (p. 658)
inverse sine (p. 658)
inverse tangent (p. 658)
legs (p. 648)
midpoint (p. 654)
Pythagorean triple (p. 649)
radical equation (p. 642)
radical expression (p. 628)
radical function (p. 621)
radicand (p. 621)
rationalizing the denominator (p. 630)
sine (p. 656)
solving the triangle (p. 657)
square root function (p. 621)
tangent (p. 656)
trigonometric ratio (p. 656)
trigonometry (p. 656)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word, phrase, expression, or number to make a true sentence.

1. A triangle with sides having measures of 3, 4, and 6 is a right triangle.
2. The expressions $12\sqrt{4}$ and $\sqrt{288}$ are equivalent.
3. The expressions $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are conjugates.
4. In the expression $-5\sqrt{2}$, the radicand is 2.
5. The shortest side of a right triangle is the hypotenuse.
6. The cosine of an angle is found by dividing the measure of the side opposite the angle by the hypotenuse.
7. The domain of the function $y = \sqrt{x}$ is $\{x | x \leq 0\}$.
8. After the first step in solving $\sqrt{2x + 4} = x + 5$, you would have $2x + 4 = x^2 + 10x + 25$
9. The converse of the Pythagorean Theorem is true.
10. The range of the function $y = \sqrt{x}$ is $\{y |> 0\}$.

Lesson-by-Lesson Review

10-1 Square Root Functions

Graph each function. Compare to the parent graph. State the domain and range.

11. $y = \sqrt{x} - 3$

12. $y = \sqrt{x} + 2$

13. $y = -5\sqrt{x}$

14. $y = \sqrt{x} - 6$

15. $y = \sqrt{x - 1}$

16. $y = \sqrt{x} + 5$

17. **GEOMETRY** The function $s = \sqrt{A}$ can be used to find the length of a side of a square given its area. Use this function to determine the length of a side of a square with an area of 90 square inches. Round to the nearest tenth if necessary.

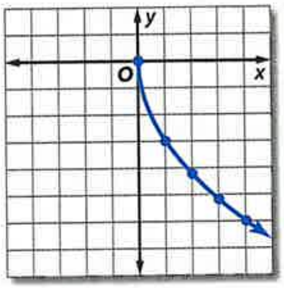
Example 1

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table. Choose nonnegative values for x.

x	0	1	2	3	4
y	0	−3	≈−4.2	≈−5.2	−6

Plot points and draw a smooth curve.



The graph of $y = \sqrt{x}$ is stretched vertically and is reflected across the x-axis.

The domain is $\{x | x \geq 0\}$.

The range is $\{y | y \leq 0\}$.

10-2 Simplifying Radical Expressions

Simplify.

18. $\sqrt{36x^2y^7}$

19. $\sqrt{20ab^3}$

20. $\sqrt{3} \cdot \sqrt{6}$

21. $2\sqrt{3} \cdot 3\sqrt{12}$

22. $(4 - \sqrt{5})^2$

23. $(1 + \sqrt{2})^2$

24. $\sqrt{\frac{50}{a^2}}$

25. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{3}{4}}$

26. $\frac{3}{2 - \sqrt{5}}$

27. $\frac{5}{\sqrt{7} + 6}$

28. **WEATHER** To estimate how long a thunderstorm will last, use $t = \sqrt{\frac{d^3}{216}}$, where t is the time in hours and d is the diameter of the storm in miles. A storm is 10 miles in diameter. How long will it last?

Example 2

Simplify $\frac{2}{4 + \sqrt{3}}$.

$$\frac{2}{4 + \sqrt{3}} \qquad \text{Original expression}$$

$$= \frac{2}{4 + \sqrt{3}} \cdot \frac{4 - \sqrt{3}}{4 - \sqrt{3}} \qquad \text{Rationalize the denominator.}$$

$$= \frac{2(4) - 2\sqrt{3}}{4^2 - (\sqrt{3})^2} \qquad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{8 - 2\sqrt{3}}{16 - 3} \qquad (\sqrt{3})^2 = 3$$

$$= \frac{8 - 2\sqrt{3}}{13} \qquad \text{Simplify.}$$

10-3 Operations with Radical Expressions

Simplify each expression.

29. $\sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3}$

30. $2\sqrt{6} - \sqrt{48}$

31. $4\sqrt{3x} - 3\sqrt{3x} + 3\sqrt{3x}$

32. $\sqrt{50} + \sqrt{75}$

33. $\sqrt{2}(5 + 3\sqrt{3})$

34. $(2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6})$

35. $(6\sqrt{5} + 2)(4\sqrt{2} + \sqrt{3})$

36. **MOTION** The velocity of a dropped object when it hits the ground can be found using $v = \sqrt{2gd}$, where v is the velocity in feet per second, g is the acceleration due to gravity, and d is the distance in feet the object drops. Find the speed of a penny when it hits the ground, after being dropped from 984 feet. Use 32 feet per second squared for g.

Example 3

Simplify $2\sqrt{6} - \sqrt{24}$.

$2\sqrt{6} - \sqrt{24} = 2\sqrt{6} - \sqrt{4 \cdot 6}$ Product Property

$= 2\sqrt{6} - 2\sqrt{6}$ Simplify.

$= 0$ Simplify.

Example 4

Simplify $(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})$.

$$(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})$$

$$= (\sqrt{3})(\sqrt{3}) + (\sqrt{3})(2\sqrt{2}) + (-\sqrt{2})(\sqrt{3}) + (\sqrt{2})(2\sqrt{2})$$

$$= 3 + 2\sqrt{6} - \sqrt{6} + 4$$

$$= 7 + \sqrt{6}$$

10-4 Radical Equations

Solve each equation. Check your solution.

37. $10 + 2\sqrt{x} = 0$

38. $\sqrt{5 - 4x} - 6 = 7$

39. $\sqrt{a + 4} = 6$

40. $\sqrt{3x} = 2$

41. $\sqrt{x + 4} = x - 8$

42. $\sqrt{3x - 14} + x = 6$

43. **FREE FALL** Assuming no air resistance, the time t in seconds that it takes an object to fall h feet can be determined by $t = \frac{\sqrt{h}}{4}$. If a skydiver jumps from an airplane and free falls for 10 seconds before opening the parachute, how many feet does she free fall?

Example 5

Solve $\sqrt{7x + 4} - 18 = 5$.

$\sqrt{7x + 4} - 18 = 5$ Original equation

$\sqrt{7x + 4} = 23$ Add 18 to each side.

$(\sqrt{7x + 4})^2 = 23^2$ Square each side.

$7x + 4 = 529$ Simplify.

$7x = 525$ Subtract 4 from each side.

$x = 75$ Divide each side by 7.

CHECK $\sqrt{7x + 4} - 18 = 5$ Original equation

$\sqrt{7(75) + 4} - 18 \stackrel{?}{=} 5$ $x = 75$

$\sqrt{525 + 4} - 18 \stackrel{?}{=} 5$ Multiply.

$\sqrt{529} - 18 \stackrel{?}{=} 5$ Add.

$23 - 18 \stackrel{?}{=} 5$ Simplify.

$5 = 5$ ✓ True.

CHAPTER 10 Study Guide and Review *Continued*

10-5 The Pythagorean Theorem

Determine whether each set of measures can be the lengths of the sides of a right triangle.

44. 6, 8, 10

45. 3, 4, 5

46. 12, 16, 21

47. 10, 12, 15

48. 2, 3, 4

49. 7, 24, 25

50. 5, 12, 13

51. 15, 19, 23

52. **LADDER** A ladder is leaning on a building. The base of the ladder is 10 feet from the building, and the ladder reaches up 15 feet on the building. How long is the ladder?

Example 6

Determine whether the set of measures 12, 16, and 20 can be the lengths of the sides of a right triangle.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$12^2 + 16^2 \stackrel{?}{=} 20^2$ $a = 12$, $b = 16$, and $c = 20$

$144 + 256 \stackrel{?}{=} 400$ Multiply.

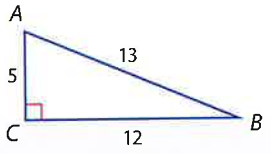
$400 = 400$ ✓ Add.

The measures can be the lengths of the sides of a right triangle.

10-6 Trigonometric Ratios

Find the values of the three trigonometric ratios for angle *A*.

53.



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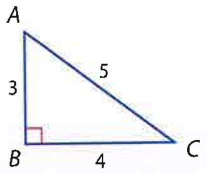
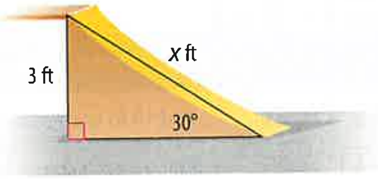
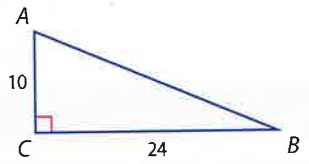


55. **RAMPS** How long is the ramp?



Example 7

Find the values of the three trigonometric ratios for angle *A*.



Find the hypotenuse: $c^2 = 10^2 + 24^2$, so $c = 26$.

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{24}{26} = \frac{12}{13}$$

$$\cos A = \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}} = \frac{10}{26} = \frac{5}{13}$$

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A} = \frac{24}{10} = \frac{12}{5}$$

CHAPTER 10 Practice Test

Graph each function, and compare to the parent graph. State the domain and range.

1. $y = -\sqrt{x}$

2. $y = \frac{1}{4}\sqrt{x}$

3. $y = \sqrt{x} + 5$

4. $y = \sqrt{x + 4}$

5. **MULTIPLE CHOICE** The length of the side of a square is given by the function $s = \sqrt{A}$, where A is the area of the square. What is the perimeter of a square that has an area of 64 square inches?

A 64 inches

B 8 inches

C 32 inches

D 16 inches

Simplify each expression.

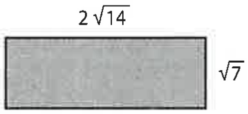
6. $5\sqrt{36}$

7. $\dfrac{3}{1 - \sqrt{2}}$

8. $2\sqrt{3} + 7\sqrt{3}$

9. $3\sqrt{6}(5\sqrt{2})$

10. **MULTIPLE CHOICE** Find the area of the rectangle.



F $7\sqrt{2}$

G 14

H $14\sqrt{2}$

J $98\sqrt{2}$

Solve each equation. Check your solution.

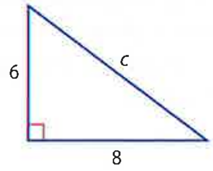
11. $\sqrt{10x} = 20$

12. $\sqrt{4x - 3} = 6 - x$

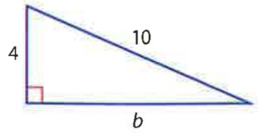
13. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of about 162 in^3. The radius of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where r is the radius and h is the height. If the height is 8.25 inches, find the radius of the container.

Find each missing length. If necessary, round to the nearest tenth.

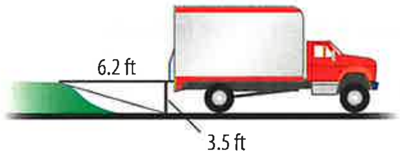
14.



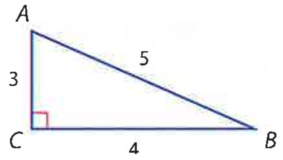
15.



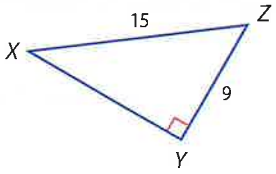
16. **DELIVERY** Ben and Amado are delivering a freezer. The bank in front of the house is the same height as the back of the truck. They set up their ramp as shown. What is the length of the slanted part of the ramp to the nearest foot?



17. Find the values of the three trigonometric ratios for angle A.



18. Find $m\angle X$ to the nearest degree.



19. **LIGHTHOUSE** How tall is the lighthouse?

CHAPTER 10 Preparing for Standardized Tests

Draw a Picture

Sometimes it is easier to visualize how to solve a problem if you draw a picture first. You can sketch your picture on scrap paper or in your test booklet (if allowed). Be careful not make any marks on your answer sheet other than your answers.

Strategies for Drawing a Picture

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What is the unknown quantity for which I need to solve?

Step 2

Sketch and label your picture.

- Draw your picture as clearly and accurately as possible.
- Label the picture carefully. Be sure to include all of the information given in the problem statement.

Step 3

Solve the problem.

- Use your picture to help you model the problem situation with an equation. Then solve the equation.
- Check your answer to make sure it is reasonable.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

An 18-foot ladder is leaning against a building. For stability, the base of the ladder must be 36 inches away from the wall. How far up the wall does the ladder reach?

Jupiterimages/Polka Dot/Alamy

Read the problem statement carefully. You know the height of the ladder leaning against the building and you know that the base of the ladder must be 36 inches away from the wall. You need to find how far up the wall the ladder reaches.

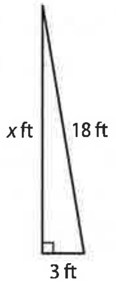
Scoring Rubric	
Criteria	Score
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Example of a 2-point response:

First convert all measurements to feet.

36 inches = 3 feet

Use a right triangle to find how high the ladder reaches. Draw and label a triangle to represent the situation.



You know the measures of a leg and the hypotenuse, and need to know the length of the other leg. So you can use the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$18^2 = 3^2 + b^2$$
$$324 = 9 + b^2$$
$$315 = b^2$$
$$\pm 315 = b$$
$$17.7 \approx b$$

The ladder reaches about 17.7 feet or about 17 feet 9 inches.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. A building casts a 15-foot shadow, while a billboard casts a 4.5-foot shadow. If the billboard is 26 feet high, what is the height of the building? Round to the nearest tenth if necessary.

2. A space shuttle is directed toward the Moon, but drifts 1.2° from its intended course. The distance from Earth to the Moon is about 240,000 miles. If the pilot doesn't get the shuttle back on course, how far will the shuttle have drifted from its intended landing position?

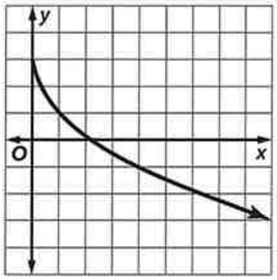
CHAPTER 10 Standardized Test Practice

Cumulative, Chapters 1 through 10

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or a sheet of paper.

1. What is the equation of the square root function graphed below?



A $y = -2\sqrt{x} + 1$

B $y = -2\sqrt{x} + 3$

C $y = 2\sqrt{x} + 3$

D $y = 2\sqrt{x} + 1$

2. Simplify $\frac{1}{4+\sqrt{2}}$.

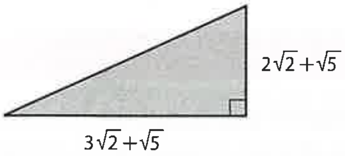
F $\frac{4+\sqrt{2}}{14}$

G $\frac{2-\sqrt{2}}{7}$

H $\frac{4-\sqrt{2}}{14}$

J $\frac{2+\sqrt{2}}{7}$

3. What is the area of the triangle below?



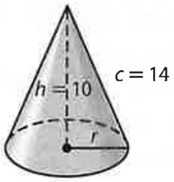
A $3\sqrt{2} + 10\sqrt{5}$

B $17 + 5\sqrt{10}$

C $12\sqrt{2} + 8\sqrt{5}$

D $8.5 + 2.5\sqrt{10}$

4. The formula for the slant height c of a cone is $c = \sqrt{h^2 + r^2}$, where h is the height of the cone and r is the radius of its base. What is the radius of the cone below? Round to the nearest tenth.



F 4.9

G 6.3

H 9.8

J 10.2

5. Which of the following sets of measures could not be the sides of a right triangle?

A (12, 16, 24)

B (10, 24, 26)

C (24, 45, 51)

D (18, 24, 30)

6. Which of the following is an equation of the line perpendicular to $4x - 2y = 6$ and passing through $(4, -4)$?

F $y = -\frac{3}{4}x + 3$

G $y = -\frac{3}{4}x - 1$

H $y = -\frac{1}{2}x - 4$

J $y = -\frac{1}{2}x - 2$

7. The scale on a map shows that 1.5 centimeters is equivalent to 40 miles. If the distance on the map between two cities is 8 centimeters, about how many miles apart are the cities?

A 178 miles

B 213 miles

C 224 miles

D 275 miles

Test-Taking Tip

Question 4 Substitute for c and h in the formula. Then solve for r.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** How many times does the graph of $y = x^2 - 4x + 10$ cross the x-axis?

9. Factor $2x^4 - 32$ completely.

10. **GRIDDED RESPONSE** In football, a field goal is worth 3 points, and the extra point after a touchdown is worth 1 point. During the 2006 season, John Kasay of the Carolina Panthers scored a total of 100 points for his team by making a total of 52 field goals and extra points. How many field goals did he make?

11. Shannon bought a satellite radio and a subscription to satellite radio. What is the total cost for his first year of service?

Item	Cost
radio	$39.99
subscription	$11.99 per month

12. **GRIDDED RESPONSE** The distance required for a car to stop is directly proportional to the square of its velocity. If a car can stop in 242 meters at 22 kilometers per hour, how many meters are needed to stop at 30 kilometers per hour?

13. The highest point in Kentucky is at an elevation of 4145 feet above sea level. The lowest point in the state is at an elevation of 257 feet above sea level. Write an inequality to describe the possible elevations in Kentucky.

14. Simplify the expression below. Show your work.

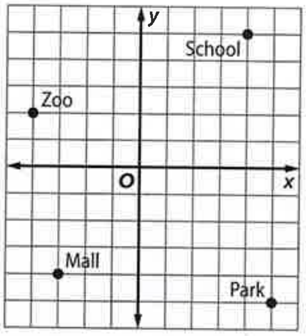
$$\left(\frac{-2r^{-2}q^5t^2}{5r^4q^2t^{-3}}\right)^{-2}$$

15. **GRIDDED RESPONSE** For the first home basketball game, 652 tickets were sold for a total revenue of $5216. If each ticket costs the same, how much is the cost per ticket? State your answer in dollars.

Extended Response

Record your answers on a sheet of paper. Show your work.

16. Karen is making a map of her hometown using a coordinate grid. The scale of her map is 1 unit = 2.5 miles.



a. Use the Pythagorean Theorem to find the actual distance between Karen's school and the park. Round to the nearest tenth of a mile if necessary.

b. Suppose Karen's house is located at (0.5, 0.5). What is farthest from her house, the zoo, the park, the school, or the mall?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Go to Lesson...	10-1	10-2	10-3	10-4	10-5	4-4	2-6	9-5	8-8	6-4	1-4	3-6	5-4	7-2	2-2	10-5