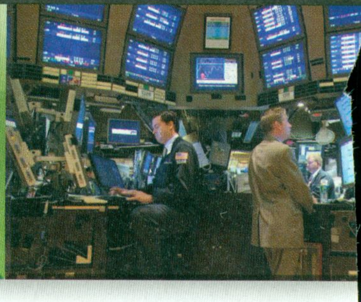
CHAPTER 9 Quadratic Functions and Equations



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CHAPTER 9

Quadratic Functions and Equations

Then

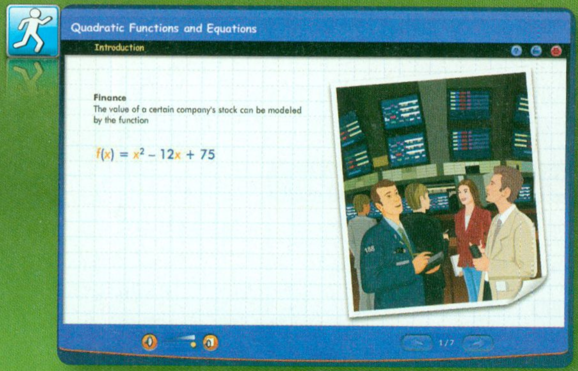
- You solved quadratic equations by factoring and by using the Square Root Property.

Now

- In this chapter, you will:
  - Solve quadratic equations by graphing, completing the square, and using the Quadratic Formula.
  - Analyze functions with successive differences and ratios.
  - Identify and graph special functions.

Why? ▲

- **FINANCE** The value of a certain company's stock can be modeled by the function $f(x) = x^2 - 12x + 75$. By graphing this quadratic function, we can make an educated guess as to how the stock will perform in the near future.



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Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Use a table of values to graph each equation.

1. $y = x + 3$
2. $y = 2x + 2$
3. $y = -2x - 3$
4. $y = 0.5x - 1$
5. $4x - 3y = 12$
6. $3y = 6 + 9x$
7. **SAVINGS** Jack has \$100 to buy a game system. He plans to save \$10 each week. Graph an equation to show the total amount T Jack will have in w weeks.

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no.* If so, factor it.

8. $a^2 + 12a + 36$
9. $x^2 + 5x + 25$
10. $x^2 - 12x + 32$
11. $x^2 + 20x + 100$
12. $4x^2 + 28x + 49$
13. $k^2 - 16k + 64$
14. $a^2 - 22a + 121$
15. $5t^2 - 12t + 25$

Evaluate each expression if $a = -2$, $b = -1$, $c = 0$, and $d = 2.5$.

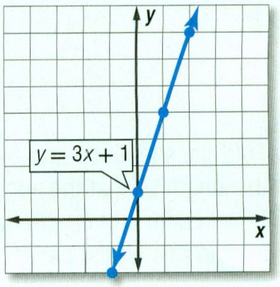
16. $|a - 3|$
17. $|2a + 1|$
18. $|4 - b|$
19. $\left|\frac{1}{2}b - 2\right|$
20. $|12 - 4c|$
21. $|2c - 3| + 1$
22. $|4d - 6|$
23. $|3d - 2| - 8$

QuickReview

Example 1

Use a table of values to graph $y = 3x + 1$.

x	$y = 3x + 1$	y
−1	3(−1) + 1	−2
0	3(0) + 1	1
1	3(1) + 1	4
2	3(2) + 1	7



Example 2

Determine whether $x^2 - 10x + 25$ is a perfect square trinomial. Write *yes* or *no.* If so, factor it.

1. Is the first term a perfect square? yes
2. Is the last term a perfect square? yes
3. Is the middle term equal to $-2(1x)(5)$? yes

$x^2 - 10x + 25 = (x - 5)^2$

Example 3

Evaluate $|2x + 1| - 7$ if $x = -1$.

$\|2x + 1\| - 7 = \|2(-1) + 1\| - 7$	$x = -1$
$= \|-2 + 1\| - 7$	Multiply.
$= \|-1\| - 7$	Add.
$= 1 - 7$	$\|-1\| = 1$
$= -6$	Subtract.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

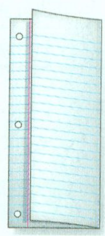
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 9. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

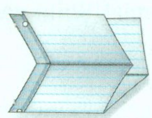
FOLDABLES® Study Organizer

Quadratic Functions and Equations Make this Foldable to help you organize your Chapter 9 notes about quadratic functions. Begin with a sheet of notebook paper.

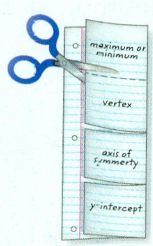
1 **Fold** the sheet of paper along the length so that the edge of the paper aligns with the margin rule on the paper.



2 **Fold** the sheet twice widthwise to form four sections.



3 **Unfold** the sheet, and cut along the folds on the front flap only.



4 **Label** each section as shown.

New Vocabulary

English		Español
quadratic function	p. 543	función cuadrática
parabola	p. 543	parábola
axis of symmetry	p. 543	eje de simetría
vertex	p. 543	vértice
minimum	p. 543	mínimo
maximum	p. 543	máximo
double root	p. 556	doble raíz
transformation	p. 564	transformación
completing the square	p. 574	completar el cuadrado
Quadratic Formula	p. 583	Formula cuadrática
discriminant	p. 586	discriminante
step function	p. 598	función etapa
greatest integer function	p. 598	función del máximo entero
absolute value function	p. 599	función del valor absoluto

Review Vocabulary

domain dominio all the possible values of the independent variable, x

leading coefficient coeficiente delantero the coefficient of the first term of a polynomial written in standard form

range rango all the possible values of the dependent variable, y

In the function represented by the table, the domain is {0, 2, 4, 6}, and the range is {3, 5, 7, 9}.

x	y
0	3
2	5
4	7
6	9

LESSON 9-1 Graphing Quadratic Functions

:: Then	:: Now	:: Why?
● You graphed linear and exponential functions.	● **1** Analyze the characteristics of graphs of quadratic functions. **2** Graph quadratic functions.	● The Innovention Fountain in Epcot's Futureworld in Orlando, Florida, is an elaborate display of water, light, and music. The sprayers shoot water in shapes that can be modeled by quadratic equations. You can use graphs of these equations to show the path of the water.

NewVocabulary
quadratic function
standard form
parabola
axis of symmetry
vertex
minimum
maximum

CCSS Common Core State Standards

Content Standards
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices
2 Reason abstractly and quantitatively.

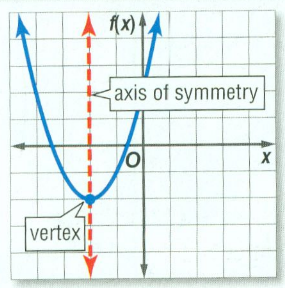
1 Characteristics of Quadratic Functions

Quadratic functions are nonlinear and can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. This form is called the **standard form** of a quadratic function.

The shape of the graph of a quadratic function is called a **parabola**. Parabolas are symmetric about a central line called the **axis of symmetry**. The axis of symmetry intersects a parabola at only one point, called the **vertex**.

KeyConcept Quadratic Functions

Parent Function:	$f(x) = x^2$
Standard Form:	$f(x) = ax^2 + bx + c$
Type of Graph:	parabola
Axis of Symmetry:	$x = -\frac{b}{2a}$
y-intercept:	c

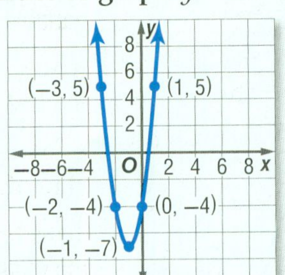


When $a > 0$, the graph of $y = ax^2 + bx + c$ opens upward. The lowest point on the graph is the **minimum**. When $a < 0$, the graph opens downward. The highest point is the **maximum**. The maximum or minimum is the vertex.

Example 1 Graph a Parabola

Use a table of values to graph $y = 3x^2 + 6x - 4$. State the domain and range.

x	y
1	5
0	−4
−1	−7
−2	−4
−3	5



Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity. The domain is all real numbers. The range is $\{y \mid y \geq -7\}$, because −7 is the minimum.

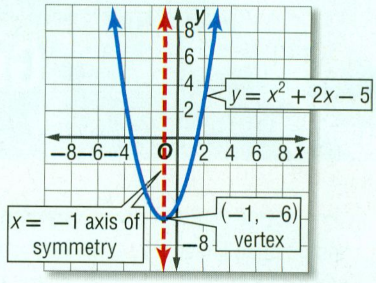
GuidedPractice

1. Use a table of values to graph $y = x^2 + 3$. State the domain and range.

Bob Turner/Art Directors & Trips Photo/age fotostock

Recall that figures with symmetry are those in which each half of the figure matches exactly.

A parabola is symmetric about the axis of symmetry. Every point on the parabola to the left of the axis of symmetry has a corresponding point on the other half. The function is increasing on one side of the axis of symmetry and decreasing on the other side.

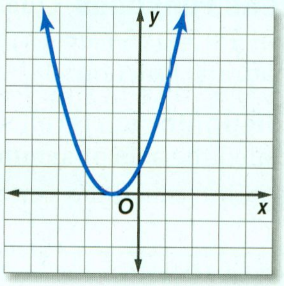


When identifying characteristics from a graph, it is often easiest to locate the vertex first. It is either the maximum or minimum point of the graph.

Example 2 Identify Characteristics from Graphs

Find the vertex, the equation of the axis of symmetry, and the *y*-intercept of each graph.

a.

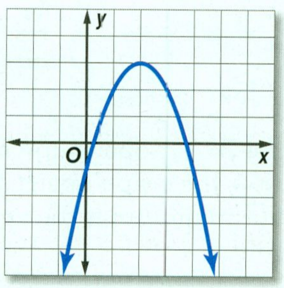


Step 1 Find the vertex.
Because the parabola opens upward, the vertex is located at the minimum point of the parabola. It is located at $(-1, 0)$.

Step 2 Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at $x = -1$.

Step 3 Find the y-intercept.
The y-intercept is the point where the graph intersects the y-axis. It is located at $(0, 1)$, so the y-intercept is 1.

b.



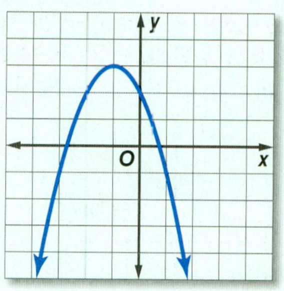
Step 1 Find the vertex.
The parabola opens downward, so the vertex is located at its maximum point, $(2, 3)$.

Step 2 Find the axis of symmetry.
The axis of symmetry is located at $x = 2$.

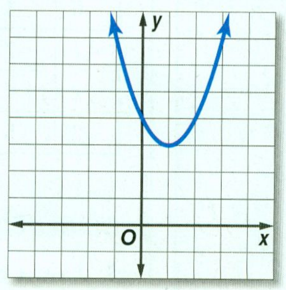
Step 3 Find the y-intercept.
The y-intercept is where the parabola crosses the y-axis. It is located at $(0, -1)$, so the y-intercept is -1.

GuidedPractice

2A.



2B.



Example 3 Identify Characteristics from Functions

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

a. $y = 2x^2 + 4x - 3$

$x = -\frac{b}{2a}$ — Formula for the equation of the axis of symmetry

$x = -\frac{4}{2 \cdot 2}$ — $a = 2$ and $b = 4$

$x = -1$ — Simplify.

The equation for the axis of symmetry is $x = -1$.

To find the vertex, use the value you found for the axis of symmetry as the x-coordinate of the vertex. Find the y-coordinate using the original equation.

$y = 2x^2 + 4x - 3$ — Original equation

$= 2(-1)^2 + 4(-1) - 3$ — $x = -1$

$= -5$ — Simplify.

The vertex is at $(-1, -5)$.

The y-intercept always occurs at $(0, c)$. So, the y-intercept is -3.

b. $y = -x^2 + 6x + 4$

$x = -\frac{b}{2a}$ — Formula for the equation of the axis of symmetry

$x = -\frac{6}{2(-1)}$ — $a = -1$ and $b = 6$

$x = 3$ — Simplify.

The equation of the axis of symmetry is $x = 3$.

$y = -x^2 + 6x + 4$ — Original equation

$= -(3)^2 + 6(3) + 4$ — $x = 3$

$= 13$ — Simplify.

The vertex is at $(3, 13)$.

The y-intercept is 4.

StudyTip

Function Characteristics When identifying characteristics of a function, it is often easiest to locate the axis of symmetry first.

StudyTip

y-intercept The y-coordinate of the y-intercept is also the constant term (c) of the quadratic function in standard form.

GuidedPractice

3A. $y = -3x^2 + 6x - 5$

3B. $y = 2x^2 + 2x + 2$

Next you will learn how to identify whether the vertex is a maximum or a minimum.

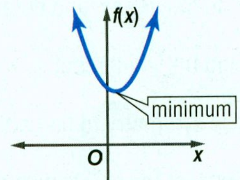
KeyConcept Maximum and Minimum Values

Words The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$:

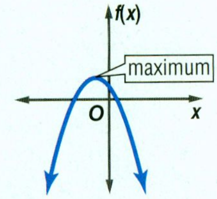
- opens upward and has a minimum value when $a > 0$, and
- opens downward and has a maximum value when $a < 0$.
- The range of a quadratic function is all real numbers greater than or equal to the minimum, or all real numbers less than or equal to the maximum.

Examples

a is positive.



a is negative.



Example 4 Maximum and Minimum Values

Consider $f(x) = -2x^2 - 4x + 6$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = -2x^2 - 4x + 6$, $a = -2$, $b = -4$, and $c = 6$.

Because a is negative the graph opens down, so the function has a maximum value.

b. State the maximum or minimum value of the function.

The maximum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{4}{2(-2)}$ or -1.

$f(x) = -2x^2 - 4x + 6$	Original function
$f(-1) = -2(-1)^2 - 4(-1) + 6$	$x = -1$
$f(-1) = 8$	Simplify.

The maximum value is 8.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $\{y \mid y \leq 8\}$.

WatchOut!

Minimum and Maximum Values Don't forget to find both coordinates of the vertex (x, y). The minimum or maximum value is the y-coordinate.

ReviewVocabulary

Domain and Range The domain is the set of all of the possible values of the independent variable x. The range is the set of all of the possible values of the dependent variable y.

GuidedPractice

Consider $g(x) = 2x^2 - 4x - 1$.

4A. Determine whether the function has a *maximum* or *minimum* value.

4B. State the maximum or minimum value.

4C. State the domain and range of the function.

2 Graph Quadratic Functions

You have learned how to find several important characteristics of quadratic functions.

KeyConcept Graph Quadratic Functions

Step 1 Find the equation of the axis of symmetry.

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

Step 3 Find the y-intercept.

Step 4 Use symmetry to find additional points on the graph, if necessary.

Step 5 Connect the points with a smooth curve.

StudyTip

Symmetry and Points When locating points that are on opposite sides of the axis of symmetry, not only are the points equidistant from the axis of symmetry, they are also equidistant from the vertex.

Example 5 Graph Quadratic Functions

Graph $f(x) = x^2 + 4x + 3$.

Step 1 Find the equation of the axis of symmetry.

$x = \frac{-b}{2a}$ — Formula for the equation of the axis of symmetry

$x = \frac{-4}{2 \cdot 1}$ or -2 — $a = 1$ and $b = 4$

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

$f(x) = x^2 + 4x + 3$ — Original equation

$= (-2)^2 + 4(-2) + 3$ — $x = -2$

$= -1$ — Simplify.

The vertex lies at $(-2, -1)$. Because a is positive the graph opens up, and the vertex is a minimum.

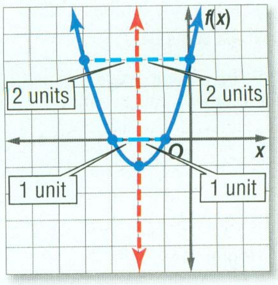
Step 3 Find the y-intercept.

$f(x) = x^2 + 4x + 3$ — Original equation

$= (0)^2 + 4(0) + 3$ — $x = 0$

$= 3$ — The y-intercept is 3.

Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same y-value.



Step 5 Connect the points with a smooth curve.

GuidedPractice **Graph each function.**

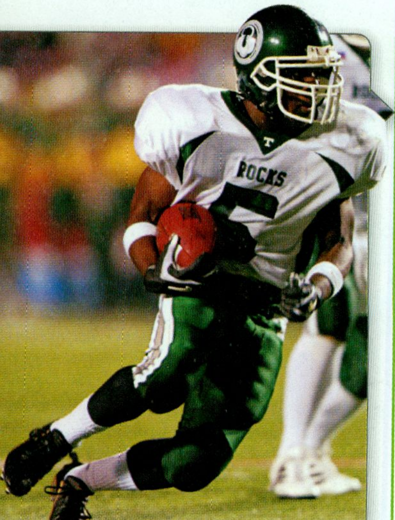
5A. $f(x) = -2x^2 + 2x - 1$

5B. $f(x) = 3x^2 - 6x + 2$

There are general differences between linear, exponential, and quadratic functions.

	Linear Functions	Exponential Functions	Quadratic Functions
Equation	$y = mx + b$	$y = ab^x$, $a \neq 0$, $b > 0$, $b \neq 1$	$y = ax^2 + bx + c$, $a \neq 0$
Degree	1	x	2
Graph	line	curve	parabola
Increasing / Decreasing	$m > 0$: y is increasing on the entire domain. $m < 0$: y is decreasing on the entire domain.	$a > 0$, $b > 1$ or $a < 0$, $0 < b < 1$: y is increasing on the entire domain. $a > 0$, $0 < b < 1$ or $a < 0$, $b > 1$: y is decreasing on the entire domain.	$a > 0$: y is decreasing to the left of the axis of symmetry and increasing on the right. $a < 0$: y is increasing to the left of the axis of symmetry and decreasing on the right.
End Behavior	$m > 0$: as x increases, y increases; as x decreases, y decreases $m < 0$: as x increases, y decreases; as x decreases, y increases	$b > 1$: as x decreases, y approaches 0; $a > 0$, as x increases, y increases; $a < 0$, as x increases, y decreases. $0 < b < 1$: as x increases, y approaches 0; $a > 0$, as x decreases, y increases; $a < 0$, as x decreases, y decreases.	$a > 0$: as x increases, y increases; as x decreases, y increases. $a < 0$: as x increases, y decreases; as x decreases, y decreases

You have used what you know about quadratic functions, parabolas, and symmetry to create graphs. You can analyze these graphs to solve real-world problems.



Real-WorldLink

About 1 in 17 high school seniors playing football will go on to play football at an NCAA school.

Source: National Collegiate Athletic Association

Real-World Example 6 Use a Graph of a Quadratic Function

SCHOOL SPIRIT The cheerleaders at Lake High School launch T-shirts into the crowd every time the Lakers score a touchdown. The height of the T-shirt can be modeled by the function $h(x) = -16x^2 + 48x + 6$, where $h(x)$ represents the height in feet of the T-shirt after x seconds.

a. Graph the function.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

$x = -\frac{48}{2(-16)}$ or $\frac{3}{2}$ $a = -16$ and $b = 48$

The equation of the axis of symmetry is $x = \frac{3}{2}$. Thus, the x-coordinate for the vertex is $\frac{3}{2}$.

$y = -16x^2 + 48x + 6$ Original equation

$= -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 6$ $x = \frac{3}{2}$

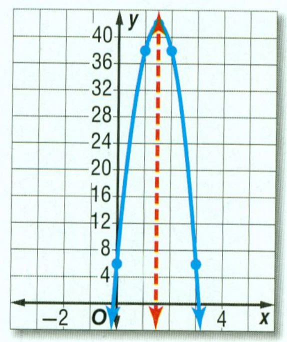
$= -16\left(\frac{9}{4}\right) + 48\left(\frac{3}{2}\right) + 6$ $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$= -36 + 72 + 6$ or 42 Simplify.

The vertex is at $\left(\frac{3}{2}, 42\right)$.

Let's find another point. Choose an x-value of 0 and substitute. Our new point is at (0, 6). The point paired with it on the other side of the axis of symmetry is (3, 6).

Repeat this and choose an x-value of 1 to get (1, 38) and its corresponding point (2, 38). Connect these points and create a smooth curve.



b. At what height was the T-shirt launched?

The T-shirt is launched when time equals 0, or at the y-intercept.

So, the T-shirt was launched 6 feet from the ground.

c. What is the maximum height of the T-shirt? When was the maximum height reached?

The maximum height of the T-shirt occurs at the vertex.

So the T-shirt reaches a maximum height of 42 feet. The time was $\frac{3}{2}$ or 1.5 seconds after launch.

GuidedPractice

6. TRACK Emilio is competing in the javelin throw. The height of the javelin can be modeled by the equation $y = -16x^2 + 64x + 6$, where y represents the height in feet of the javelin after x seconds.

A. Graph the path of the javelin.

B. At what height is the javelin thrown?

C. What is the maximum height of the javelin?

Andy Lyons/Getty Images Sport/Getty Images

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Use a table of values to graph each equation. State the domain and range.**

1. $y = 2x^2 + 4x - 6$

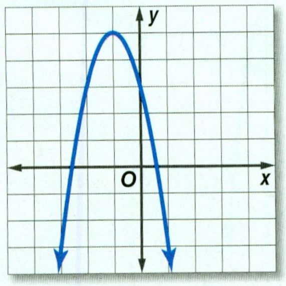
2. $y = x^2 + 2x - 1$

3. $y = x^2 - 6x - 3$

4. $y = 3x^2 - 6x - 5$

Example 2 **Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.**

5.



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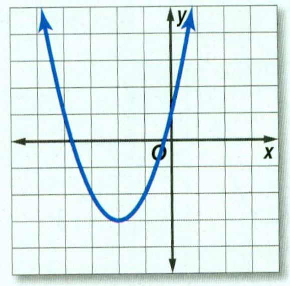

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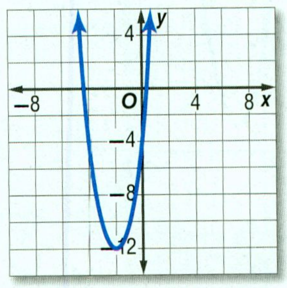

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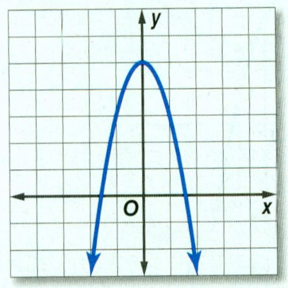

Example 3 **Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each function.**

9. $y = -3x^2 + 6x - 1$

10. $y = -x^2 + 2x + 1$

11. $y = x^2 - 4x + 5$

12. $y = 4x^2 - 8x + 9$

Example 4 **Consider each function.**

a. **Determine whether the function has *maximum* or *minimum* value.**

b. **State the maximum or minimum value.**

c. **What are the domain and range of the function?**

13 $y = -x^2 + 4x - 3$

14. $y = -x^2 - 2x + 2$

15. $y = -3x^2 + 6x + 3$

16. $y = -2x^2 + 8x - 6$

Example 5 **Graph each function.**

17. $f(x) = -3x^2 + 6x + 3$

18. $f(x) = -2x^2 + 4x + 1$

19. $f(x) = 2x^2 - 8x - 4$

20. $f(x) = 3x^2 - 6x - 1$

Example 6 **21.** **CCSS REASONING** A juggler is tossing a ball into the air. The height of the ball in feet can be modeled by the equation $y = -16x^2 + 16x + 5$, where y represents the height of the ball at x seconds.

a. Graph this equation.

b. At what height is the ball thrown?

c. What is the maximum height of the ball?

Practice and Problem Solving

Extra Practice is on page R9.

Example 1 Use a table of values to graph each equation. State the domain and range.

22. $y = x^2 + 4x + 6$ **23.** $y = 2x^2 + 4x + 7$ **24.** $y = 2x^2 - 8x - 5$

25. $y = 3x^2 + 12x + 5$ **26.** $y = 3x^2 - 6x - 2$ **27.** $y = x^2 - 2x - 1$

Example 2 Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

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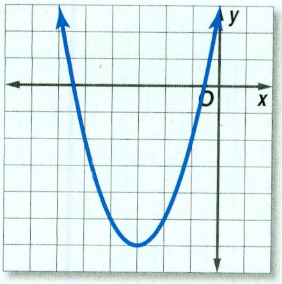

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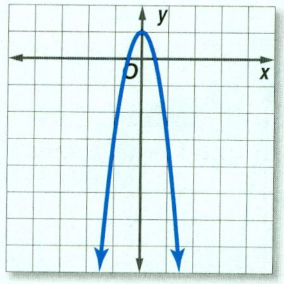

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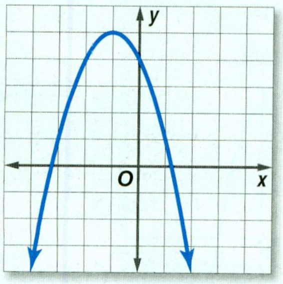

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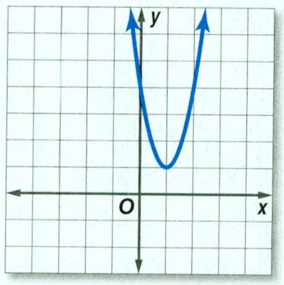

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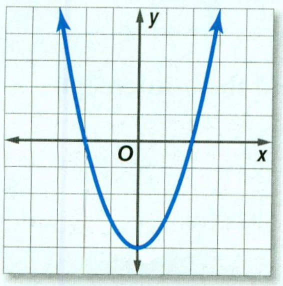

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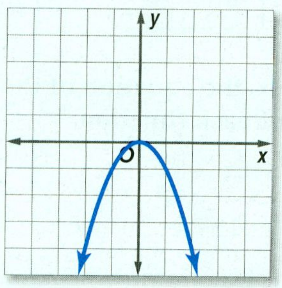


Example 3 Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

34. $y = x^2 + 8x + 10$ **35.** $y = 2x^2 + 12x + 10$ **36.** $y = -3x^2 - 6x + 7$

37. $y = -x^2 - 6x - 5$ **38.** $y = 5x^2 + 20x + 10$ **39.** $y = 7x^2 - 28x + 14$

40. $y = 2x^2 - 12x + 6$ **41.** $y = -3x^2 + 6x - 18$ **42.** $y = -x^2 + 10x - 13$

Example 4 Consider each function.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

43. $y = -2x^2 - 8x + 1$ **44.** $y = x^2 + 4x - 5$ **45.** $y = 3x^2 + 18x - 21$

46. $y = -2x^2 - 16x + 18$ **47.** $y = -x^2 - 14x - 16$ **48.** $y = 4x^2 + 40x + 44$

49. $y = -x^2 - 6x - 5$ **50.** $y = 2x^2 + 4x + 6$ **51.** $y = -3x^2 - 12x - 9$

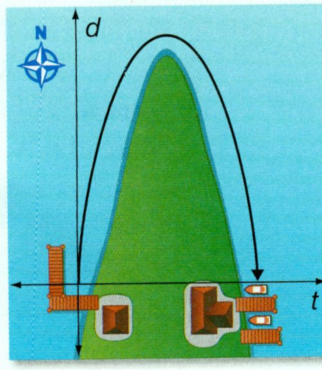
Example 5 Graph each function.

52. $y = -3x^2 + 6x - 4$ **53.** $y = -2x^2 - 4x - 3$ **54.** $y = -2x^2 - 8x + 2$

55. $y = x^2 + 6x - 6$ **56.** $y = x^2 - 2x + 2$ **57.** $y = 3x^2 - 12x + 5$

Example 6

58. **BOATING** Miranda has her boat docked on the west side of Casper Point. She is boating over to the Casper Marina. The distance traveled by Miranda over time can be modeled by the equation $d = -16t^2 + 66t$, where d is the number of feet she travels in t minutes.



a. Graph this equation.

b. What is the maximum number of feet north that she traveled?

c. How long did it take her to reach Casper Marina?

GRAPHING CALCULATOR **Graph each equation. Use the TRACE feature to find the vertex on the graph. Round to the nearest thousandth if necessary.**

59. $y = 4x^2 + 10x + 6$

60. $y = 8x^2 - 8x + 8$

61. $y = -5x^2 - 3x - 8$

62. $y = -7x^2 + 12x - 10$

63. **GOLF** The average amateur golfer can hit a ball with an initial velocity of 31.3 meters per second. The height can be modeled by the equation $h = -4.9t^2 + 31.3t$, where h is the height of the ball, in meters, after t seconds.

a. Graph this equation. What do the portions of the graph where $h > 0$ represent in the context of the situation? What does the end behavior of the graph represent?

b. At what height is the ball hit?

c. What is the maximum height of the ball?

d. How long did it take for the ball to hit the ground?

e. State a reasonable range and domain for this situation.

64. **FUNDRAISING** The marching band is selling poinsettias to buy new uniforms. Last year the band charged \$5 each, and they sold 150. They want to increase the price this year, and they expect to lose 10 sales for each \$1 increase. The sales revenue R, in dollars, generated by selling the poinsettias is predicted by the function $R = (5 + p)(150 - 10p)$, where p is the number of \$1 price increases.

a. Write the function in standard form.

b. Find the maximum value of the function.

c. At what price should the poinsettias be sold to generate the most sales revenue? Explain your reasoning.

65. **FOOTBALL** A football is kicked up from ground level at an initial upward velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the football after t seconds.

a. What is the height of the ball after one second?

b. When is the ball 126 feet high?

c. When is the height of the ball 0 feet? What do these points represent in the context of the situation?

66. CCSS **STRUCTURE** Let $f(x) = x^2 - 9$.

a. What is the domain of $f(x)$?

b. What is the range of $f(x)$?

c. For what values of x is $f(x)$ negative?

d. When x is a real number, what are the domain and range of $f(x) = \sqrt{x^2 - 9}$?

67. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate solving quadratic equations using tables.

a. Algebraic Determine the related function for each equation. Copy and complete the first two columns of the table below.

Equation	Related Function	Zeros	*y*-Values
$x^2 - x = 12$			
$x^2 + 8x = 9$			
$x^2 = 14x - 24$			
$x^2 + 16x = -28$			

b. Graphical Graph each related function with a graphing calculator.

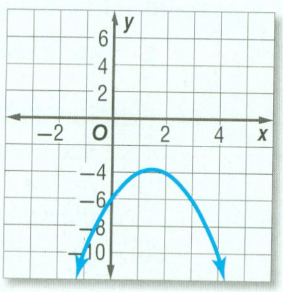
c. Analytical The number of zeros is equal to the degree of the related function. Use the table feature on your calculator to determine the zeros of each related function. Record the zeros in the table above. Also record the values of the function one unit less than and one unit more than each zero.

d. Verbal Examine the function values for x-values just before and just after a zero. What happens to the sign of the function value before and after a zero?

H.O.T. Problems Use Higher-Order Thinking Skills

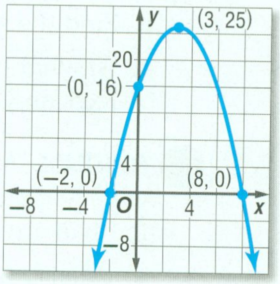
68. OPEN ENDED Write a quadratic function for which the graph has an axis of symmetry of $x = -\frac{3}{8}$. Summarize your steps.

69. ERROR ANALYSIS Jade thinks that the parabolas represented by the graph and the description have the same axis of symmetry. Chase disagrees. Who is correct? Explain your reasoning.



a parabola that opens downward, passing through (0, 6) and having a vertex at (2, 2)

70. CHALLENGE Using the axis of symmetry and one x-intercept, write an equation for the graph shown.



71. CCSS STRUCTURE The graph of a quadratic function has a vertex at (2, 0). One point on the graph is (5, 9). Find another point on the graph. Explain how you found it.

72. OPEN ENDED Describe a real-world situation that involves a quadratic equation. Explain what the vertex represents.

73. REASONING Provide a counterexample that is a specific case to show that the following statement is false. *The vertex of a parabola is always the minimum of the graph.*

74. WRITING IN MATH Use tables and graphs to compare and contrast an exponential function $f(x) = ab^x + c$, where $a \neq 0$, $b > 0$, and $b \neq 1$, a quadratic function $g(x) = ax^2 + c$, and a linear function $h(x) = ax + c$. Include intercepts, portions of the graph where the functions are increasing, decreasing, positive, or negative, relative maxima and minima, symmetries, and end behavior. Which function eventually exceeds the others?

Standardized Test Practice

75. Which of the following is an equation for the line that passes through (2, −5) and is perpendicular to $2x + 4y = 8$?

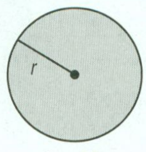
A $y = 2x + 10$

B $y = -\frac{1}{2}x - 4$

C $y = 2x - 9$

D $y = -2x - 1$

76. GEOMETRY The area of the circle is 36π square units. If the radius is doubled, what is the area of the new circle?



$A = 36\pi$

F 1296π units^2

G 144π units^2

H 72π units^2

J 9π units^2

77. What is the range of the function $f(x) = -4x^2 - \frac{1}{2}$?

A $\left\{\text{all integers less than or equal to } \frac{1}{2}\right\}$

B {all nonnegative integers}

C {all real numbers}

D $\left\{\text{all real numbers less than or equal to } -\frac{1}{2}\right\}$

78. SHORT RESPONSE Dylan delivers newspapers for extra money. He starts delivering the newspapers at 3:15 P.M. and finishes at 5:05 P.M. How long does it take Dylan to complete his route?

Spiral Review

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it. (Lesson 8-9)

79. $4x^2 + 4x + 1$

80. $4x^2 - 20x + 25$

81. $9x^2 + 8x + 16$

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*. (Lesson 8-8)

82. $n^2 - 16$

83. $x^2 + 25$

84. $9 - 4a^2$

Find each product. (Lesson 8-3)

85. $(b - 7)(b + 3)$

86. $(c - 6)(c - 5)$

87. $(2x - 1)(x + 9)$

88. MULTIPLE BIRTHS The number of quadruplet births Q in the United States in recent years can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where t represents the number of years since 1992. For what values of t does this model no longer allow for realistic predictions? Explain your reasoning. (Lesson 8-1)

Use elimination to solve each system of equations. (Lesson 6-4)

89. $2x + y = 5$
$3x - 2y = 4$

90. $4x - 3y = 12$
$x + 2y = 14$

91. $2x - 3y = 2$
$5x + 4y = 28$

92. HEALTH About 20% of the time you sleep is spent in rapid eye movement (REM), which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep? (Lesson 5-4)

Skills Review

Find the x-intercept of the graph of each equation.

93. $x + 2y = 10$

94. $2x - 3y = 12$

95. $3x - y = -18$

EXTEND 9-1

Algebra Lab
Rate of Change of a Quadratic Function

CCSS Common Core State Standards
Content Standards
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

A model rocket is launched from the ground with an upward velocity of 144 feet per second. The function $y = -16x^2 + 144x$ models the height y of the rocket in feet after x seconds. Using this function, we can investigate the rate of change of a quadratic function.

Activity

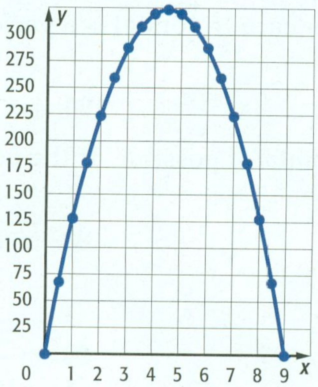
Step 1 Copy the table below.

x	0	0.5	1.0	1.5	…	9.0
y	0					
Rate of Change	–					

Step 2 Find the value of y for each value of x from 0 through 9.

Step 3 Graph the ordered pairs (x, y) on grid paper. Connect the points with a smooth curve. Notice that the function *increases* when $0 < x < 4.5$ and *decreases* when $4.5 < x < 9$.

Step 4 Recall that the *rate of change* is the change in y divided by the change in x. Find the rate of change for each half second interval of x and y.



Exercises

Use the quadratic function $y = x^2$.

1. Make a table, similar to the one in the Activity, for the function using $x = -4, -3, -2, -1, 0, 1, 2, 3,$ and 4. Find the values of y for each x-value.

2. Graph the ordered pairs on grid paper. Connect the points with a smooth curve. Describe where the function is increasing and where it is decreasing.

3. Find the rate of change for each column starting with $x = -3$. Compare the rates of change when the function is increasing and when it is decreasing.

4. **CHALLENGE** If an object is dropped from 100 feet in the air and air resistance is ignored, the object will fall at a rate that can be modeled by the function $f(x) = -16x^2 + 100$, where $f(x)$ represents the object's height in feet after x seconds. Make a table like that in Exercise 1, selecting appropriate values for x. Fill in the x-values, the y-values, and rates of change. Compare the rates of change. Describe any patterns that you see.

Ron Brown/age fotostock

LESSON 9-2

Solving Quadratic Equations by Graphing

:: Then

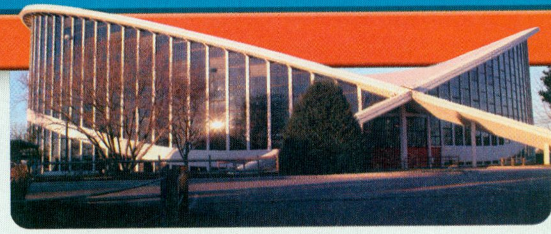
- You solved quadratic equations by factoring.

:: Now

1. Solve quadratic equations by graphing.
2. Estimate solutions of quadratic equations by graphing.

:: Why?

- Dorton Arena in Raleigh, North Carolina, has a shape created by two intersecting parabolas. The shape of one of the parabolas can be modeled by $y = -x^2 + 127x$, where x is the width of the parabola in feet, and y is the length in feet. The x-intercepts of the graph of this function can be used to find the distance between the points where the parabola meets the ground.



N.C. Department of Agriculture and Consumer Services

NewVocabulary
double root

Common Core State Standards

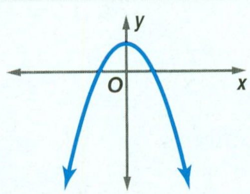
Content Standards
A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
6 Attend to precision.

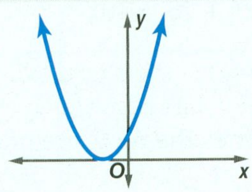
1 Solve by Graphing

A quadratic equation can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. To write a quadratic function as an equation, replace y or $f(x)$ with 0. Recall that the solutions or roots of an equation can be identified by finding the x-intercepts of the related graph. Quadratic equations may have two, one, or no real solutions. Quadratic equations with solutions that are not real numbers lead us to extend the number system to allow for solutions of these equations. These numbers are called *complex numbers*. You will study complex numbers in Algebra 2.

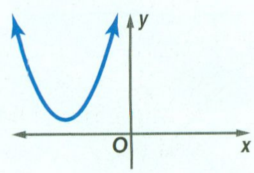
KeyConcept Solutions of Quadratic Equations



two unique real solutions



one unique real solution



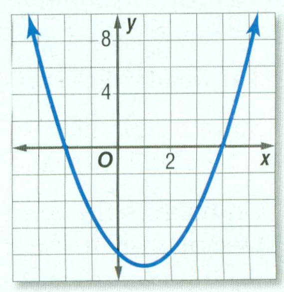
no real solutions

Example 1 Two Roots

Solve $x^2 - 2x - 8 = 0$ by graphing.

Graph the related function $f(x) = x^2 - 2x - 8$.

The x-intercepts of the graph appear to be at -2 and 4, so the solutions are -2 and 4.



CHECK Check each solution in the original equation.

$x^2 - 2x - 8 = 0$	Original equation	$x^2 - 2x - 8 = 0$
$(-2)^2 - 2(-2) - 8 \stackrel{?}{=} 0$	$x = -2$ or $x = 4$	$(4)^2 - 2(4) - 8 \stackrel{?}{=} 0$
$0 = 0$ ✓	Simplify.	$0 = 0$ ✓

GuidedPractice Solve each equation by graphing.

1A. $-x^2 - 3x + 18 = 0$

1B. $x^2 - 4x + 3 = 0$

The solutions in Example 1 were two distinct numbers. Sometimes the two roots are the same number, called a **double root**.

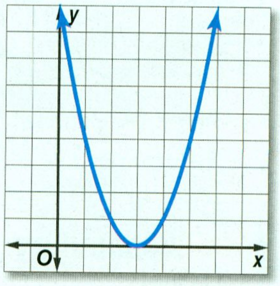
Example 2 Double Root

Solve $x^2 - 6x = -9$ by graphing.

Step 1 Rewrite the equation in standard form.

$$x^2 - 6x = -9 \quad \text{Original equation}$$
$$x^2 - 6x + 9 = 0 \quad \text{Add 9 to each side.}$$

Step 2 Graph the related function $f(x) = x^2 - 6x + 9$.



Step 3 Locate the x-intercepts of the graph. Notice that the vertex of the parabola is the only x-intercept. Therefore, there is only one solution, 3.

CHECK Solve by factoring.

$$x^2 - 6x + 9 = 0 \quad \text{Original equation}$$
$$(x - 3)(x - 3) = 0 \quad \text{Factor.}$$
$$x - 3 = 0 \text{ or } x - 3 = 0 \quad \text{Zero Product Property}$$
$$x = 3 \qquad x = 3 \quad \text{Add 3 to each side.}$$

The only solution is 3.

WatchOut!

CCSS Precision Solutions found from the graph of an equation may appear to be exact. Check them in the original equation to be sure.

GuidedPractice

Solve each equation by graphing.

2A. $x^2 + 25 = 10x$

2B. $x^2 = -8x - 16$

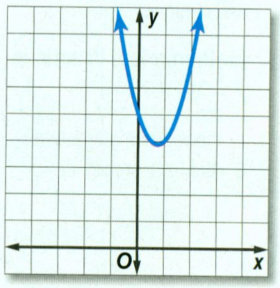
Sometimes the roots are not real numbers.

Example 3 No Real Roots

Solve $2x^2 - 3x + 5 = 0$ by graphing.

Step 1 Rewrite the equation in standard form.
This equation is written in standard form.

Step 2 Graph the related function $f(x) = 2x^2 - 3x + 5$.



Step 3 Locate the x-intercepts of the graph. This graph has no x-intercepts. Therefore, this equation has no real number solutions. The solution set is $\varnothing$.

CHECK Solve by factoring.

There are no factors of 10 that have a sum of -3, so the expression is not factorable. Thus, the equation has no real number solutions.

GuidedPractice

Solve each equation by graphing.

3A. $-x^2 - 3x = 5$

3B. $-2x^2 - 8 = 6x$

2 Estimate Solutions

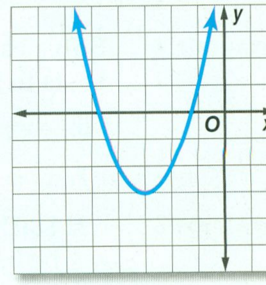
The real roots found thus far have been integers. However, the roots of quadratic equations are usually not integers. In these cases, use estimation to approximate the roots of the equation.

Example 4 Approximate Roots with a Table

Study Tip

Location of Zeros Since quadratic functions are continuous, there must be a zero between two x-values for which the corresponding y-values have opposite signs.

Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.



Graph the related function $f(x) = x^2 + 6x + 6$.

The x-intercepts are located between -5 and -4 and between -2 and -1.

Make a table using an increment of 0.1 for the x-values located between -5 and -4 and between -2 and -1.

Look for a change in the signs of the function values. The function value that is closest to zero is the best approximation for a zero of the function.

x	−4.9	−4.8	−4.7	−4.6	−4.5	−4.4	−4.3	−4.2	−4.1
y	0.61	0.24	−0.11	−0.44	−0.75	−1.04	−1.31	−1.56	−1.79

x	−1.9	−1.8	−1.7	−1.6	−1.5	−1.4	−1.3	−1.2	−1.1
y	−1.79	−1.56	−1.31	−1.04	−0.75	−0.44	−0.11	0.24	0.61

For each table, the function value that is closest to zero when the sign changes is -0.11. Thus, the roots are approximately -4.7 and -1.3.

Guided Practice

4. Solve $2x^2 + 6x - 3 = 0$ by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

Approximating the x-intercepts of graphs is helpful for real-world applications.

Real-World Example 5 Approximate Roots with a Calculator

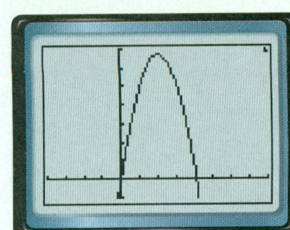
moodboard/Alamy

Real-World Link

The game of soccer, called "football" outside of North America, began in 1863 in Britain when the Football Association was founded. Soccer is played on every continent of the world.

Source: Sports Know How

SOCCER A goalie kicks a soccer ball with an upward velocity of 65 feet per second, and her foot meets the ball 1 foot off the ground. The quadratic function $h = -16t^2 + 65t + 1$ represents the height of the ball h in feet after t seconds. Approximately how long is the ball in the air?



[−4, 7] scl: 1 by [−10, 70] scl: 10

You need to find the roots of the equation $-16t^2 + 65t + 1 = 0$. Use a graphing calculator to graph the related function $f(x) = -16t^2 + 65t + 1$.

The positive x-intercept of the graph is approximately 4. Therefore, the ball is in the air for approximately 4 seconds.

Guided Practice

5. If the goalie kicks the soccer ball with an upward velocity of 55 feet per second and his foot meets the ball 2 feet off the ground, approximately how long is the ball in the air?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Examples 1–3 **Solve each equation by graphing.**

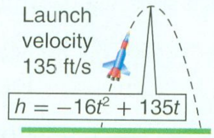
1. $x^2 + 3x - 10 = 0$
2. $2x^2 - 8x = 0$
3. $x^2 + 4x = -4$
4. $x^2 + 12 = -8x$

Example 4 **Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.**

5. $-x^2 - 5x + 1 = 0$
6. $-9 = x^2$
7. $x^2 = 25$
8. $x^2 - 8x = -9$

Example 5

9. **SCIENCE FAIR** Ricky built a model rocket. Its flight can be modeled by the equation shown, where h is the height of the rocket in feet after t seconds. About how long was Ricky's rocket in the air?



Practice and Problem Solving

Extra Practice is on page R9.

Examples 1–3 **Solve each equation by graphing.**

10. $x^2 + 7x + 14 = 0$
11. $x^2 + 2x - 24 = 0$
12. $x^2 - 16x + 64 = 0$
13. $x^2 - 5x + 12 = 0$
14. $x^2 + 14x = -49$
15. $x^2 = 2x - 1$
16. $x^2 - 10x = -16$
17. $-2x^2 - 8x = 13$
18. $2x^2 - 16x = -30$
19. $2x^2 = -24x - 72$
20. $-3x^2 + 2x = 15$
21. $x^2 = -2x + 80$

Example 4 **Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.**

22. $x^2 + 2x - 9 = 0$
23. $x^2 - 4x = 20$
24. $x^2 + 3x = 18$
25. $2x^2 - 9x = -8$
26. $3x^2 = -2x + 7$
27. $5x = 25 - x^2$

Example 5

28. **SOFTBALL** Sofia hits a softball straight up. The equation $h = -16t^2 + 90t$ models the height h, in feet, of the ball after t seconds. How long is the ball in the air?

29. **RIDES** A skyrocket roller coaster takes riders straight up and then returns straight down. The equation $h = -16t^2 + 185t$ models the height h, in feet, of the coaster after t seconds. How long is it until the coaster returns to the bottom?

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each zero.

30. $y = x^2 - 8x + 16$
31. $y = x^2 + 3x + 4$
32. $y = x^2 + 2x - 24$
33. $y = x^2 + 12x + 32$

34. **NUMBER THEORY** Use a quadratic equation to find two numbers that have a sum of 9 and a product of 20.

35. **NUMBER THEORY** Use a quadratic equation to find two numbers that have a sum of 1 and a product of −12.

36. CCSS **MODELING** The height of a golf ball in the air can be modeled by the equation $h = -16t^2 + 60t + 3$, where h is the height in feet of the ball after t seconds.
    a. How long was the ball in the air?
    b. What is the ball's maximum height?
    c. When will the ball reach its maximum height?

37. **SNOWBOARDING** Stefanie is in a snowboarding competition. The equation $h = -16t^2 + 30t + 10$ models Stefanie's height h, in feet, in the air after t seconds.

a. How long is Stefanie in the air?

b. When will Stefanie reach a height of 15 feet?

c. To earn bonus points in the competition, you must reach a height of 20 feet. Will Stefanie earn bonus points?

38. **MULTIPLE REPRESENTATIONS** In this problem, you will explore how to further interpret the relationship between quadratic functions and graphs.

a. **Graphical** Graph $y = x^2$.

b. **Analytical** Name the vertex and two other points on the graph.

c. **Graphical** Graph $y = x^2 + 2$, $y = x^2 + 4$, and $y = x^2 + 6$ on the same coordinate plane as the previous graph.

d. **Analytical** Name the vertex and two points from each of these graphs that have the same x-coordinates as the first graph.

e. **Analytical** What conclusion can you draw from this?

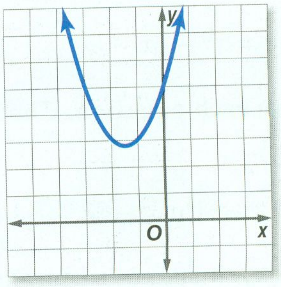
GRAPHING CALCULATOR Solve each equation by graphing.

39. $x^3 - 3x^2 - 6x + 8 = 0$

40. $x^3 - 8x^2 + 15x = 0$

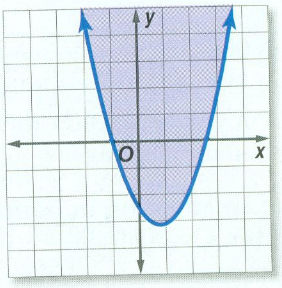
H.O.T. Problems Use Higher-Order Thinking Skills

41. **CCSS CRITIQUE** Iku and Zachary are finding the number of real zeros of the function graphed at the right. Iku says that the function has no real zeros because there are no x-intercepts. Zachary says that the function has one real zero because the graph has a y-intercept. Is either of them correct? Explain your reasoning.



42. **OPEN ENDED** Describe a real-world situation in which a thrown object travels in the air. Write an equation that models the height of the object with respect to time, and determine how long the object travels in the air.

43. **REASONING** The graph shown is that of a *quadratic inequality*. Analyze the graph, and determine whether the y-value of a solution of the inequality is *sometimes*, *always*, or *never* greater than 2. Explain.



44. **CHALLENGE** Write a quadratic equation that has the roots described.

a. one double root

b. one rational (nonintegral) root and one integral root

c. two distinct integral roots that are additive opposites.

45. **CHALLENGE** Find the roots of $x^2 = 2.25$ without using a calculator. Explain your strategy.

46. **WRITING IN MATH** Explain how to approximate the roots of a quadratic equation when the roots are not integers.

Standardized Test Practice

47. Adrahan earned 50 out of 80 points on a test. What percentage did Adrahan score on the test?

A 62.5%

B 16%

C 6.25%

D 1.6%

48. Ernesto needs to loosen a bolt. He needs a wrench that is smaller than a $\frac{7}{8}$-inch wrench, but larger than a $\frac{3}{4}$-inch wrench. Which of the following sizes should Ernesto use?

F $\frac{3}{8}$ inch

G $\frac{5}{8}$ inch

H $\frac{13}{16}$ inch

J $\frac{15}{16}$ inch

49. EXTENDED RESPONSE Two boats leave a dock. One boat travels 4 miles east and then 5 miles north. The second boat travels 12 miles south and 9 miles west. Draw a diagram that represents the paths traveled by the boats. How far apart are the boats in miles?

50. The formula $s = \frac{1}{2}at^2$ represents the distance s in meters that a free-falling object will fall near a planet or the Moon in a given time t in seconds. Solve the formula for a, the acceleration due to gravity.

A $a = \frac{1}{2}t^2 - s$

B $a = 2s - t^2$

C $a = s - \frac{1}{2}t^2$

D $a = \frac{2s}{t^2}$

Spiral Review

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 9-1)

51. $y = 3x^2$

52. $y = -4x^2 - 5$

53. $y = -x^2 + 4x - 7$

54. $y = x^2 - 6x - 8$

55. $y = 3x^2 + 2x + 1$

56. $y = -4x^2 - 8x + 5$

Solve each equation. Check the solutions. (Lesson 8-9)

57. $2x^2 = 32$

58. $(x - 4)^2 = 25$

59. $4x^2 - 4x + 1 = 16$

60. $2x^2 + 16x = -32$

61. $(x + 3)^2 = 5$

62. $4x^2 - 12x = -9$

Find each sum or difference. (Lesson 8-1)

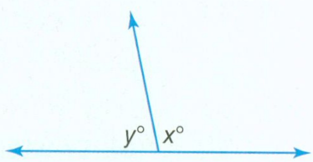
63. $(3n^2 - 3) + (4 + 4n^2)$

64. $(2d^2 - 7d - 3) - (4d^2 + 7)$

65. $(2b^3 - 4b^2 + 4) - (3b^4 + 5b^2 - 9)$

66. $(8 - 4h^2 + 6h^4) + (5h^2 - 3 + 2h^3)$

67. GEOMETRY Supplementary angles are two angles with measures that have a sum of 180°. For the supplementary angles in the figure, the measure of the larger angle is 24° greater than the measure of the smaller angle. Write and solve a system of equations to find these measures. (Lesson 6-5)



Write an equation in point-slope form for the line that passes through each point with the given slope. (Lesson 4-3)

68. $(2, 5), m = 3$

69. $(-3, 6), m = -7$

70. $(-1, -2), m = -\frac{1}{2}$

Skills Review

Graph each function.

71. $y = x^2 + 5$

72. $y = x^2 - 8$

73. $y = 2x^2 - 7$

74. $y = -x^2 + 2$

75. $y = -0.5x^2 - 3$

76. $y = (-x)^2 + 1$

EXTEND 9-2

Graphing Technology Lab
Quadratic Inequalities

Recall that the graph of a linear inequality consists of the boundary and the shaded half plane. The solution set of the inequality lies in the shaded region of the graph. Graphing quadratic inequalities is similar to graphing linear inequalities.

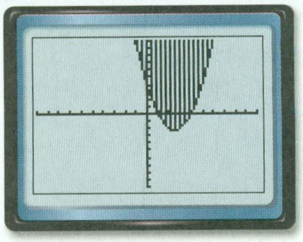
Activity 1 Shade Inside a Parabola

Graph $y \geq x^2 - 5x + 4$ in a standard viewing window.

First, clear all functions from the Y= list.

To graph $y \geq x^2 - 5x + 4$, enter the equation in the Y= list. Then use the left arrow to select =. Press [ENTER] until shading above the line is selected.

KEYSTROKES: [◀] [◀] [ENTER] [ENTER] [▶] [▶] [X,T,θ,n] [x^2] [−] 5 [X,T,θ,n] [+] 4 [ZOOM] 6



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All ordered pairs for which y is *greater than or equal* to $x^2 - 5x + 4$ lie *above or on* the line and are solutions.

A similar procedure will be used to graph an inequality in which the shading is outside of the parabola.

Activity 2 Shade Outside a Parabola

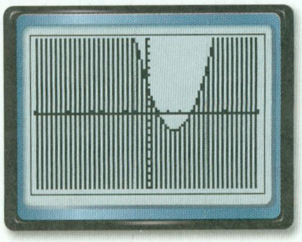
Graph $y - 4 \leq x^2 - 5x$ in a standard viewing window.

First, clear the graph that is displayed.

KEYSTROKES: [Y=] [CLEAR]

Then rewrite $y - 4 \leq x^2 - 5x$ as $y \leq x^2 - 5x + 4$, and graph it.

KEYSTROKES: [◀] [◀] [ENTER] [ENTER] [ENTER] [▶] [▶] [X,T,θ,n] [x^2] [−] 5 [X,T,θ,n] [+] 4 [GRAPH]



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All ordered pairs for which y is *less than or equal* to $x^2 - 5x + 4$ lie *below or on* the line and are solutions.

Exercises

1. Compare and contrast the two graphs shown above.
2. Graph $y - 2x + 6 \geq 5x^2$ in the standard viewing window. Name three solutions of the inequality.
3. Graph $y - 6x \leq -x^2 - 3$ in the standard viewing window. Name three solutions of the inequality.

EXPLORE 9-3

Graphing Technology Lab
Family of Quadratic Functions

You have studied the effects of changing parameters in the equations of linear and exponential functions. You can use a graphing calculator to analyze how changing the parameters of the equation of a quadratic function affects the graphs in the family of quadratic functions.

CCSS Common Core State Standards
Content Standards
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

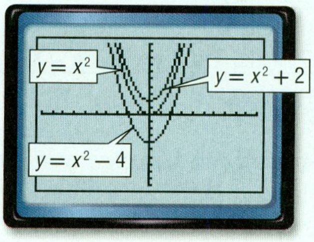
PT

Activity 1 Change k in $y = a(x - h)^2 + k$

Graph the set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

$y = x^2$, $y = x^2 + 2$, $y = x^2 - 4$

Enter the equations in the **Y =** list and graph in the standard viewing window. Use the **ZOOM** feature to investigate the key features of the graphs.



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The graphs have the same shape, and all open up. The vertex of each graph is on the y-axis, which is the axis of symmetry.

However, the graphs have different vertical positions. The graph of $y = x^2 + 2$ is shifted up 2 units. The graph of $y = x^2 - 4$ is shifted down 4 units.

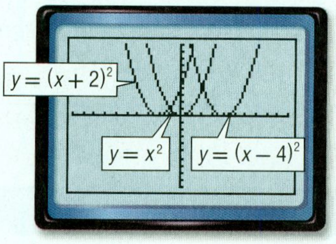
Changing the value of h in $y = a(x - h)^2 + k$ affects the graphs in a different way than changing k.

Activity 2 Change h in $y = a(x - h)^2 + k$

Graph the set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

$y = x^2$, $y = (x + 2)^2$, $y = (x - 4)^2$

The graphs have the same shape, and all open up. The vertex of each graph is on the x-axis.



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However, the graphs have different horizontal positions. Each has a different axis of symmetry. The graph of $y = (x + 2)^2$ is shifted to the left 2 units. The graph of $y = (x - 4)^2$ is shifted to the right 4 units.

It appears that changing the values of h and k in $y = a(x - h)^2 + k$ moves the graph vertically or horizontally. How does changing the value of a affect the graphs?

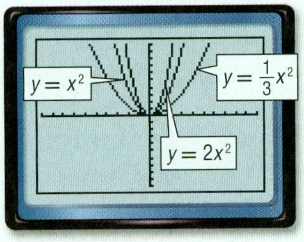
Activity 3 Change a in $y = a(x - h)^2 + k$

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

a. $y = x^2$, $y = 2x^2$, $y = \frac{1}{3}x^2$

The graphs have the same vertex, they have the same axis of symmetry, and all open up.

However, the graphs have different widths. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$. The graph of $y = \frac{1}{3}x^2$ is wider than the graph of $y = x^2$.



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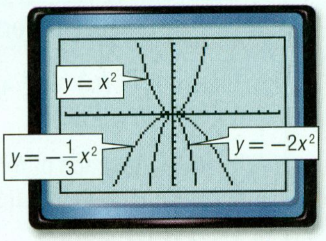
b. $y = x^2$, $y = -\frac{1}{3}x^2$, $y = -2x^2$

The graphs have the same vertex and the same axis of symmetry.

However, the graphs of $y = -\frac{1}{3}x^2$ and $y = -2x^2$ open down. Also the graph of $y = -2x^2$ is narrower than the graph of $y = x^2$. The graph of $y = -\frac{1}{3}x^2$ is wider than the graph of $y = x^2$.



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Model and Analyze

How does each parameter affect the graph of $y = a(x - h)^2 + k$? Give examples.

1. k

2. h

3. a

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs.

4. $y = x^2$, $y = x^2 + 3$

5. $y = \frac{1}{2}x^2$, $y = 3x^2$

6. $y = x^2$, $y = (x - 5)^2$

7. $y = 3x^2$, $y = -3x^2$

8. $y = x^2$, $y = -4x^2$

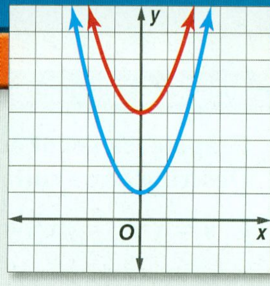
9. $y = x^2 - 1$, $y = x^2 + 2$

10. $y = \frac{1}{2}x^2 + 3$, $y = -2x^2$

11. $y = x^2 - 4$, $y = (x - 4)^2$

LESSON 9-3

Transformations of Quadratic Functions



Then

- You graphed quadratic functions by using the vertex and axis of symmetry.

Now

1. Apply translations to quadratic functions.
2. Apply dilations and reflections to quadratic functions.

Why?

- The graphs of the parabolas shown at the right are the same size and shape, but notice that the vertex of the red parabola is higher on the *y*-axis than the vertex of the blue parabola. Shifting a parabola up and down is an example of a transformation.

NewVocabulary

transformation
translation
dilation
reflection
vertex form

Common Core State Standards

Content Standards

A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

8 Look for and express regularity in repeated reasoning.

1 Translations

A **transformation** changes the position or size of a figure. One transformation, a **translation**, moves a figure up, down, left, or right. When a constant k is added to or subtracted from the parent function, the graph of the resulting function $f(x) \pm k$ is the graph of the parent function translated up or down.

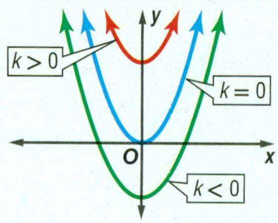
The parent function of the family of quadratics is $f(x) = x^2$. All other quadratic functions have graphs that are transformations of the graph of $f(x) = x^2$.

KeyConcept Vertical Translations

The graph of $f(x) = x^2 + k$ is the graph of $f(x) = x^2$ translated vertically.

If $k > 0$, the graph of $f(x) = x^2$ is translated $|k|$ units **up**.

If $k < 0$, the graph of $f(x) = x^2$ is translated $|k|$ units **down**.



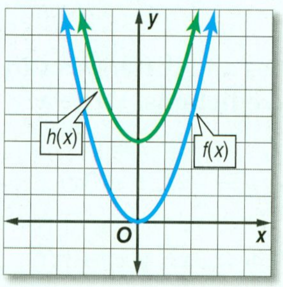
Example 1 Describe and Graph Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $h(x) = x^2 + 3$

$k = 3$ and $3 > 0$

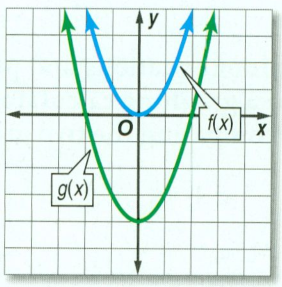
$h(x)$ is a translation of the graph of $f(x) = x^2$ up 3 units.



b. $g(x) = x^2 - 4$

$k = -4$ and $-4 < 0$

$g(x)$ is a translation of the graph of $f(x) = x^2$ down 4 units.



GuidedPractice

1A. $f(x) = x^2 - 7$ **1B.** $g(x) = 5 + x^2$ **1C.** $h(x) = -5 + x^2$ **1D.** $f(x) = x^2 + 1$

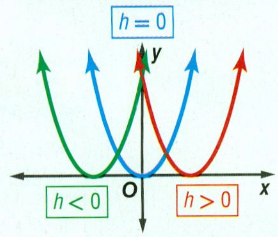
A quadratic graph can be translated horizontally by subtracting an h term from x.

KeyConcept Horizontal Translations

The graph of $g(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.

If $h > 0$, the graph of $f(x) = x^2$ is translated h units to the **right**.

If $h < 0$, the graph of $f(x) = x^2$ is translated $|h|$ units to the **left**.



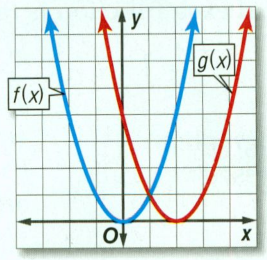
Example 2 Horizontal Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $g(x) = (x - 2)^2$

$k = 0, h = 2$ and $2 > 0$

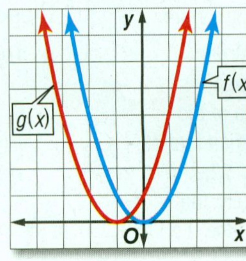
$g(x)$ is a translation of the graph of $f(x) = x^2$ to the right 2 units.



b. $g(x) = (x + 1)^2$

$k = 0, h = -1$ and $-1 < 0$

$g(x)$ is a translation of the graph of $f(x) = x^2$ to the left 1 unit.



GuidedPractice

2A. $g(x) = (x - 3)^2$

2B. $g(x) = (x + 2)^2$

A quadratic graph can be translated both horizontally and vertically.

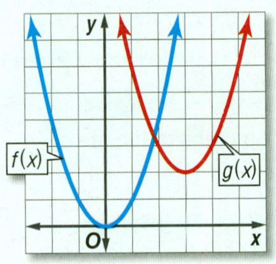
Example 3 Horizontal and Vertical Translations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $g(x) = (x - 3)^2 + 2$

$k = 2, h = 3$ and $3 > 0$

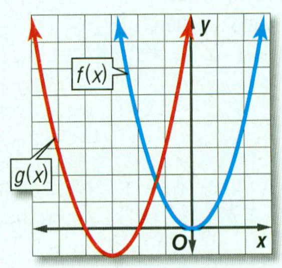
$g(x)$ is a translation of the graph of $f(x) = x^2$ to the right 3 units and up 2 units.



b. $g(x) = (x + 3)^2 - 1$

$k = -1, h = -3$ and $-3 < 0$

$g(x)$ is a translation of the graph of $f(x) = x^2$ to the left 3 units and down 1 unit.



GuidedPractice

3A. $g(x) = (x + 2)^2 + 3$

3B. $g(x) = (x - 4)^2 - 4$

2 Dilations and Reflections

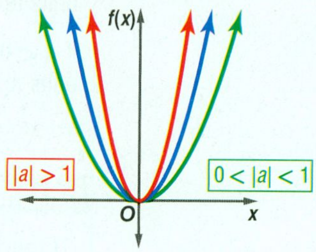
Another type of transformation is a dilation. A **dilation** makes the graph narrower than the parent graph or wider than the parent graph. When the parent function $f(x) = x^2$ is multiplied by a constant a, the graph of the resulting function $f(x) = ax^2$ is either stretched or compressed vertically.

KeyConcept Dilations

The graph of $g(x) = ax^2$ is the graph of $f(x) = x^2$ stretched or compressed vertically.

If $|a| > 1$, the graph of $f(x) = x^2$ is stretched vertically.

If $0 < |a| < 1$, the graph of $f(x) = x^2$ is compressed vertically.



StudyTip

CCSS Sense-Making When the graph of a quadratic function is stretched vertically, the shape of the graph is narrower than that of the parent function. When it is compressed vertically, the graph is wider than the parent function.

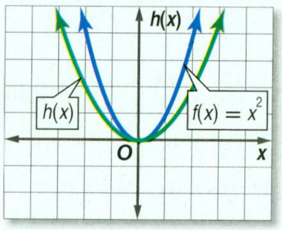
Example 4 Describe and Graph Dilations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

a. $h(x) = \frac{1}{2}x^2$

$a = \frac{1}{2}$ and $0 < \frac{1}{2} < 1$

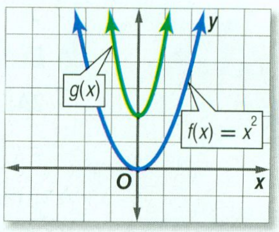
$h(x)$ is a dilation of the graph of $f(x) = x^2$ that is compressed vertically.



b. $g(x) = 3x^2 + 2$

$a = 3$ and $3 > 1$, $k = 2$ and $2 > 0$

$g(x)$ is a dilation of the graph of $f(x) = x^2$ that is stretched vertically and translated up 2 units.



GuidedPractice

4A. $j(x) = 2x^2$

4B. $h(x) = 5x^2 - 2$

4C. $g(x) = \frac{1}{3}x^2 + 2$

StudyTip

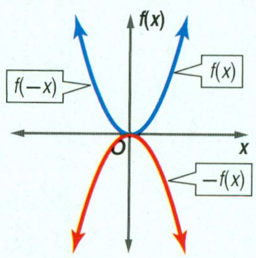
Reflection A reflection of $f(x) = x^2$ across the y-axis results in the same function, because $f(-x) = (-x)^2 = x^2$.

A **reflection** flips a figure across a line. When $f(x) = x^2$ or the variable x is multiplied by -1, the graph is reflected across the x- or y-axis.

KeyConcept Reflections

The graph of $-f(x)$ is the reflection of the graph of $f(x) = x^2$ across the x-axis.

The graph of $f(-x)$ is the reflection of the graph of $f(x) = x^2$ across the y-axis.

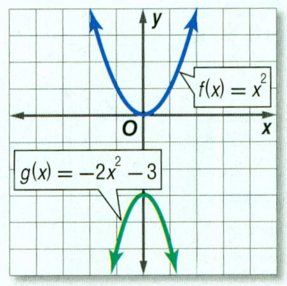


Example 5 Describe and Graph Transformations

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

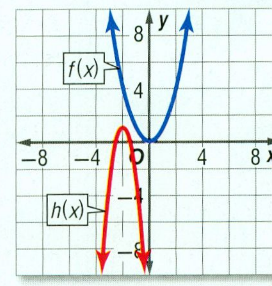
a. $g(x) = -2x^2 - 3$

- $a = -2$, $-2 < 0$, and $|-2| > 1$, so there is a reflection across the x-axis and the graph is vertically stretched.
- $k = -3$ and $-3 < 0$, so there is a translation down 3 units.



b. $h(x) = -4(x + 2)^2 + 1$

- $a = -4$, $-4 < 0$, and $|-4| > 1$, so there is a reflection across the x-axis and the graph is vertically stretched.
- $h = -2$ and $-2 < 0$, so there is a translation 2 units to the left.
- $k = 1$ and $1 > 0$, so there is a translation up 1 unit.



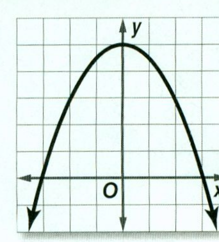
GuidedPractice

5A. $h(x) = 2(-x)^2 - 9$ **5B.** $g(x) = \frac{1}{5}x^2 + 3$ **5C.** $j(x) = -2(x - 1)^2 - 2$

You can use what you know about the characteristics of graphs of quadratic equations to match an equation with a graph.

Standardized Test Example 6 Identify an Equation for a Graph

Which is an equation for the function shown in the graph?



A $y = \frac{1}{2}x^2 - 5$ **C** $y = -\frac{1}{2}x^2 + 5$

B $y = -2x^2 - 5$ **D** $y = 2x^2 + 5$

Read the Test Item

You are given a graph. You need to find its equation.

Solve the Test Item

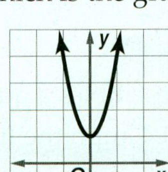
The graph opens downward, so the graph of $y = x^2$ has been reflected across the x-axis. The leading coefficient should be negative, so eliminate choices A and D.

The parabola is translated up 5 units, so $k = 5$. Look at the equations. Only choices C and D have $k = 5$. The answer is C.

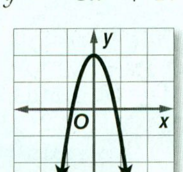
GuidedPractice

6. Which is the graph of $y = -3x^2 + 1$?

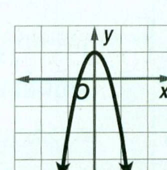
F



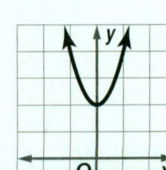
G



H



J



A quadratic function written in the form $f(x) = a(x - h)^2 + k$ is said to be in **vertex form**. Transformations of the parent graph are easily found from an equation in vertex form.

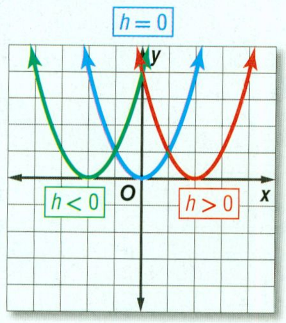
ConceptSummary Transformations of Quadratic Functions

$$f(x) = a(x - h)^2 + k$$

h, Horizontal Translation

h units to the right if h is positive

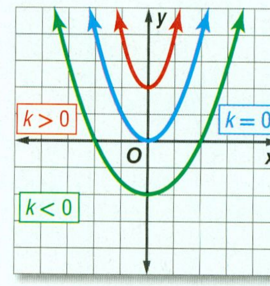
$|h|$ units to the left if h is negative



k, Vertical Translation

k units up if k is positive

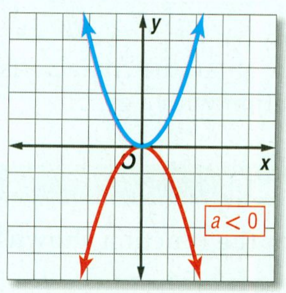
$|k|$ units down if k is negative



a, Reflection

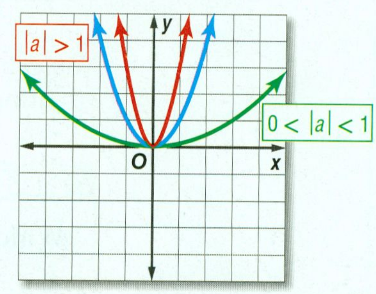
If $a > 0$, the graph opens up.

If $a < 0$, the graph opens down.



a, Dilation

If $|a| > 1$, the graph is stretched vertically. If $0 < |a| < 1$, the graph is compressed vertically.

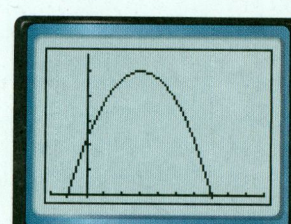


Real-World Example 7 Transformations with a Calculator

FIREWORKS During a firework show, the height h in meters of a specific rocket after t seconds can be modeled by $h(t) = -4.6(t - 3)^2 + 75$. Graph the function. How is it related to the graph of $f(x) = x^2$?

Four separate transformations are occurring.

The negative sign of the coefficient of x^2 causes a reflection across the x-axis. A dilation occurs, which compresses the graph vertically. There are also translations up 75 units and to the right 3 units.



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GuidedPractice

7. MONUMENTS The St. Louis Arch resembles a quadratic and can be modeled by $h(x) = -\frac{2}{315}x^2 + 630$. Graph the function. How is it related to the graph of $f(x) = x^2$?

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–5, 7

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 - 11$
2. $h(x) = \frac{1}{2}(x - 2)^2$
3. $h(x) = -x^2 + 8$
4. $g(x) = x^2 + 6$
5. $g(x) = -4(x + 3)^2$
6. $h(x) = -x^2 - 2$

Example 6

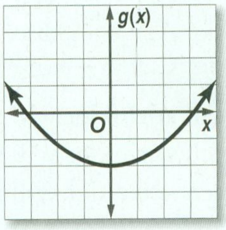
7. **MULTIPLE CHOICE** Which is an equation for the function shown in the graph?

A $g(x) = \frac{1}{5}x^2 + 2$

B $g(x) = -5x^2 - 2$

C $g(x) = \frac{1}{5}x^2 - 2$

D $g(x) = -\frac{1}{5}x^2 - 2$



Practice and Problem Solving

Extra Practice is on page R9.

Examples 1–5, 7

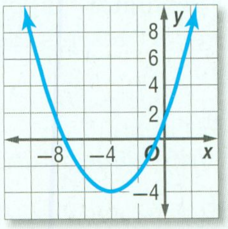
Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = -10 + x^2$
9. $h(x) = -7 - x^2$
10. $g(x) = 2(x - 3)^2 + 8$
11. $h(x) = 6 + \frac{2}{3}x^2$
12. $g(x) = -5 - \frac{4}{3}x^2$
13. $h(x) = 3 + \frac{5}{2}x^2$
14. $g(x) = 0.25x^2 - 1.1$
15. $h(x) = 1.35(x + 1)^2 + 2.6$
16. $g(x) = \frac{3}{4}x^2 + \frac{5}{6}$
17. $h(x) = 1.01x^2 - 6.5$

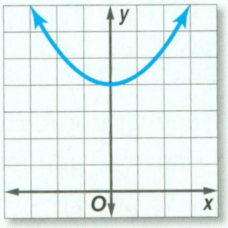
Example 6

Match each equation to its graph.

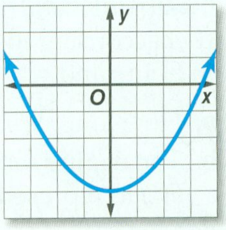
A



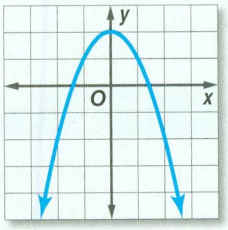
B



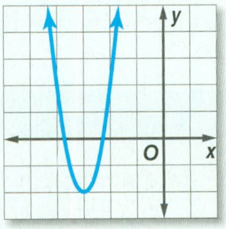
C



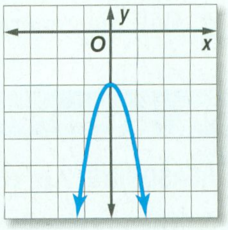
D



E



F



18. $y = \frac{1}{3}x^2 - 4$
19. $y = \frac{1}{3}(x + 4)^2 - 4$
20. $y = \frac{1}{3}x^2 + 4$
21. $y = -3x^2 - 2$
22. $y = -x^2 + 2$
23. $y = (2x + 6)^2 + 2$

24. **SQUIRRELS** A squirrel 12 feet above the ground drops an acorn from a tree. The function $h = -16t^2 + 12$ models the height of the acorn above the ground in feet after t seconds. Graph the function, and compare this graph to the graph of its parent function.

CCSS REGULARITY List the functions in order from the most stretched vertically to the least stretched vertically graph.

25. $g(x) = 2x^2, h(x) = \frac{1}{2}x^2$
26. $g(x) = -3x^2, h(x) = \frac{2}{3}x^2$
27. $g(x) = -4x^2, h(x) = 6x^2, f(x) = 0.3x^2$
28. $g(x) = -x^2, h(x) = \frac{5}{3}x^2, f(x) = -4.5x^2$

29. **ROCKS** A rock drops from a cliff 20,000 inches above the ground. At the same time, another rock drops from a cliff 30,000 inches above the ground.

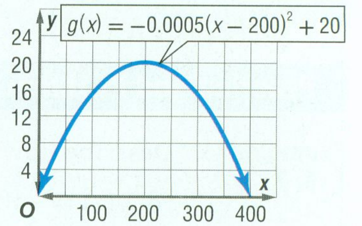
a. Write two functions that model the heights h of the rocks after t seconds.
b. Which rock will reach the ground first?

30. **SPRINKLERS** The path of water from a sprinkler can be modeled by quadratic functions. The following functions model paths for three different sprinklers.

Sprinkler A: $y = -0.35x^2 + 3.5$ Sprinkler B: $y = -0.21x^2 + 1.7$
Sprinkler C: $y = -0.08x^2 + 2.4$

a. Which sprinkler will send water the farthest? Explain.
b. Which sprinkler will send water the highest? Explain.
c. Which sprinkler will produce the narrowest path? Explain.

31. **GOLF** The path of a drive can be modeled by a quadratic function where $g(x)$ is the vertical distance in yards of the ball from the ground and x is the horizontal distance in yards.



a. How can you obtain $g(x)$ from the graph of $f(x) = x^2$.
b. A second golfer hits a ball from the red tee, which is 30 yards closer to the hole. What function $h(x)$ can be used to describe the second golfer's shot?

Describe the transformations to obtain the graph of $g(x)$ from the graph of $f(x)$.

32. $f(x) = x^2 + 3$
$g(x) = x^2 - 2$

33. $f(x) = x^2 - 4$
$g(x) = (x - 2)^2 + 7$

34. $f(x) = -6x^2$
$g(x) = -3x^2$

35. **COMBINING FUNCTIONS** An engineer created a self-refueling generator that burns fuel according to the function $g(t) = -t^2 + 10t + 200$, where t represents the time in hours and $g(t)$ represents the number of gallons remaining.

a. How long will it take for the generator to run out of fuel?
b. The engine self-refuels at a rate of 40 gallons per hour. Write a linear function $h(t)$ to represent the refueling of the generator.
c. Find $T(t) = g(t) + h(t)$. What does this new function represent?
d. Will the generator run out of fuel? If so, when?

H.O.T. Problems Use Higher-Order Thinking Skills

36. **REASONING** Are the following statements *sometimes*, *always*, or *never* true? Explain.

a. The graph of $y = x^2 + k$ has its vertex at the origin.
b. The graphs of $y = ax^2$ and its reflection over the x-axis are the same width.
c. The graph of $y = x^2 + k$, where $k \geq 0$, and the graph of a quadratic with vertex at $(0, -3)$ have the same maximum or minimum point.

37. **CHALLENGE** Write a function of the form $y = ax^2 + k$ with a graph that passes through the points $(-2, 3)$ and $(4, 15)$.

38. CCSS **ARGUMENTS** Determine whether all quadratic functions that are reflected across the y-axis produce the same graph. Explain your answer.

39. **OPEN ENDED** Write a quadratic function that opens downward and is wider than the parent graph.

40. **WRITING IN MATH** Describe how the values of a and k affect the graphical and tabular representations for the functions $y = ax^2$, $y = x^2 + k$, and $y = ax^2 + k$.

Standardized Test Practice

41. SHORT RESPONSE A tutor charges a flat fee of \$55 and \$30 for each hour of work. Write a function that represents the total charge C, in terms of the number of hours h worked.

42. Which *best* describes the graph of $y = 2x^2$?

A a line with a y-intercept of (0, 2) and an x-intercept at the origin

B a parabola with a minimum point at (0, 0) and that is twice as wide as the graph of $y = x^2$ when $y = 2$

C a parabola with a maximum point at (0, 0) and that is half as wide as the graph of $y = x^2$ when $y = 2$

D a parabola with a minimum point at (0, 0) and that is half as wide as the graph of $y = x^2$ when $y = 2$

43. Candace is 5 feet tall. If 1 inch is about 2.54 centimeters, how tall is Candace to the nearest centimeter?

F 13 cm **H** 123 cm

G 26 cm **J** 152 cm

44. While in England, Imani spent 49.60 British pounds on a pair of jeans. If this is equivalent to \$100 in U.S. currency, how many British pounds would Imani have spent on a sweater that cost \$60?

A 2976 pounds

B 29.76 pounds

C 19.84 pounds

D 8.26 pounds

Spiral Review

Solve each equation by graphing. (Lesson 9-2)

45. $x^2 + 6 = 0$

46. $x^2 - 10x = -24$

47. $x^2 + 5x + 4 = 0$

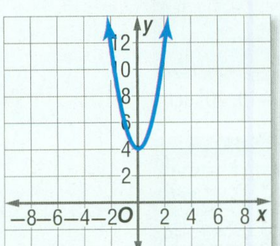
48. $2x^2 - x = 3$

49. $2x^2 - x = 15$

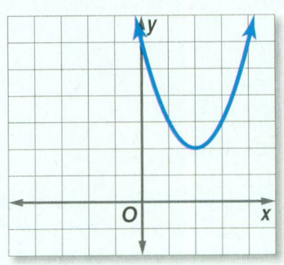
50. $12x^2 = -11x + 15$

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph. (Lesson 9-1)

51.



52.



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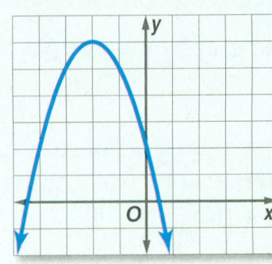


54. CLASS TRIP Mr. Wong's American History class will take taxis from their hotel in Washington, D.C., to the Lincoln Memorial. The fare is \$2.75 for the first mile and \$1.25 for each additional mile. If the distance is m miles and t taxis are needed, write an expression for the cost to transport the group. (Lesson 8-2)

Solve each inequality. Check your solution. (Lesson 5-3)

55. $-3t + 6 \leq -3$

56. $59 > -5 - 8f$

57. $-2 - \frac{d}{5} < 23$

Skills Review

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

58. $16x^2 - 24x + 9$

59. $9x^2 + 6x + 1$

60. $25x^2 - 60x + 36$

61. $x^2 - 8x + 81$

62. $36x^2 - 84x + 49$

63. $4x^2 - 3x + 9$

EXTEND 9-3

Graphing Technology Lab
Systems of Linear and Quadratic Equations

You can use a graphing calculator to solve systems involving linear and quadratic equations.

CCSS Common Core State Standards
Content Standards
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

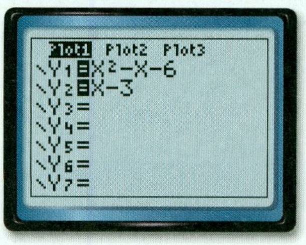
Activity 1 Solve a System of Equations Graphically

Use a graphing calculator to solve the system of equations.

$y = x^2 - x - 6$
$y = x - 3$

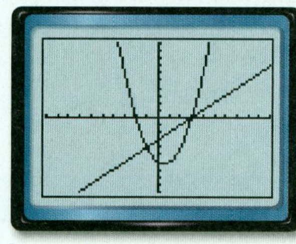
Step 1 Enter each equation in the Y= list.

KEYSTROKES: [X,T,θ,n] [x^2] [–] [X,T,θ,n] [–] 6 [ENTER] [X,T,θ,n] [–] 3



Step 2 Graph the system. **KEYSTROKES:** [GRAPH]

The graphs intersect at two points. So, there are two solutions.



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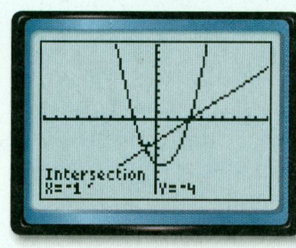
Step 3 Find the intersection on the left by using the **CALC** menu.

KEYSTROKES: [2nd] [CALC] 5 [ENTER] [ENTER]

Use the arrow keys to move the cursor close to the intersection on the left. Press [ENTER] again.

The graphs intersect at (−1, −4).



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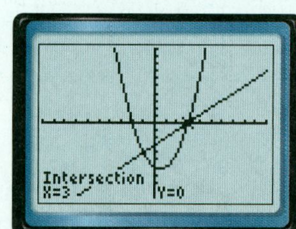
Step 4 Repeat Step 3 but move the cursor to the other intersection. The graphs intersect at (3, 0).



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You can use a graphing calculator to verify solutions of systems found algebraically.

Activity 2 Check Solutions Graphically

Solve the system of equations algebraically. Use a graphing calculator to check your solutions.

$y = 2x - 6$
$y = x^2 - 8x + 19$

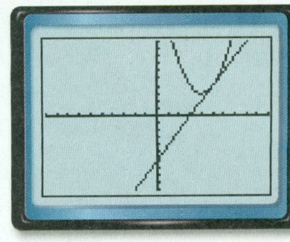
Step 1 Set the expressions equal to each other, and solve for x.

$x^2 - 8x + 19 = 2x - 6$	Substitute $x^2 - 8x + 19$ for y.
$x^2 - 10x + 25 = 0$	Simplify.
$(x - 5)^2 = 0$	Factor.
$x = 5$	Solve for x.

Step 2 Substitute the x-value into either equation to find the y-value: $y = 2(5) - 6$ or 4.

Step 3 Graph the system and find the point(s) of intersection as in Activity 1.

The graphs intersect at (5, 4). Thus, the solution of the system of equations is (5, 4).



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You can solve a quadratic equation graphically by writing each side of the equation as a separate function. The x-coordinate of the point(s) of intersection will be the solution of the equation, since at that point(s) the original equations are true.

Activity 3 Use a System to Solve an Equation

Use a system of equations to solve $x^2 - 3x + 1 = \frac{11}{4}x - 6$.

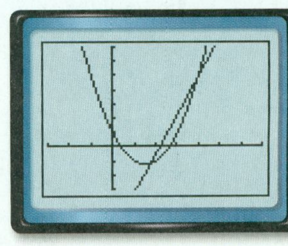
Step 1 Write as a system of equations. $y = x^2 - 3x + 1$

$y = \frac{11}{4}x - 6$

Step 2 Enter the equations into the graphing calculator, and graph them.

Step 3 Use the **CALC** menu to find the two points of intersection.

The graphs intersect at (1.75, −1.1875) and (4, 5). Thus, the solutions of $x^2 - 3x + 1 = \frac{11}{4}x - 6$ are 1.75 and 4.



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Exercises

Use a graphing calculator to solve each system of equations.

1. $y = x^2$
$y = 2x$

2. $y = -2x^2 + 7x - 2$
$y = 3 - 4x$

3. $y = -x^2 + 4$
$y = \frac{1}{2}x + 5$

Solve each system of equations algebraically. Use a graphing calculator to check your solutions.

4. $y = x^2 + 7x + 12$
$y = 2x + 8$

5. $y = x^2 - x - 20$
$y = 3x + 12$

6. $y = 3x^2 - x - 2$
$y = -2x + 2$

Use a system of equations to solve each equation.

7. $x^2 = -2x - 1$

8. $\frac{1}{2}x^2 - 4 = 3x + 4$

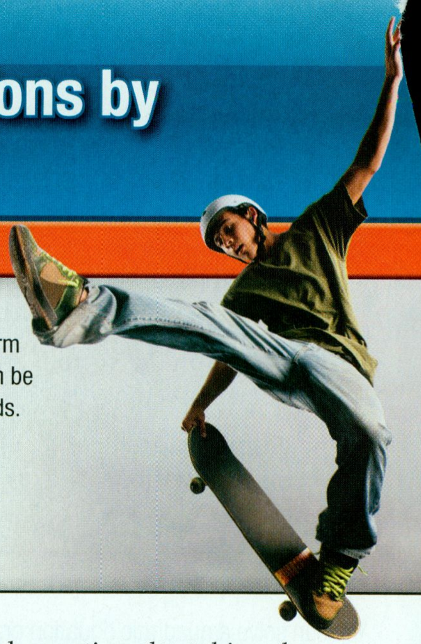
9. $x^2 + 5x + 5 = -x - 8$

CHALLENGE **Use a graphing calculator to solve other types of systems.**

10. $y = x^2 + 3x - 5$
$y = -x^2$

11. $y = \frac{3}{4}x$
$x^2 + y^2 = 1$ (*Hint:* Enter as two functions, $y = \sqrt{1 - x^2}$ and $y = -\sqrt{1 - x^2}$.)

LESSON 9-4 Solving Quadratic Equations by Completing the Square



Then

- You solved quadratic equations by using the square root property.

Now

1. Complete the square to write perfect square trinomials.
2. Solve quadratic equations by completing the square.

Why?

- In competitions, skateboarders may launch themselves from a half pipe into the air to perform tricks. The equation $h = -16t^2 + 20t + 12$ can be used to model their height, in feet, after t seconds.

  To find how long a skateboarder is in the air if he is 25 feet above the half pipe, you can solve $25 = -16t^2 + 20t + 12$ by using a method called completing the square.

NewVocabulary
completing the square

Common Core State Standards

Content Standards

A.REI.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Mathematical Practices

4 Model with mathematics.

1 Complete the Square

You have previously solved equations by taking the square root of each side. This method worked only because the expression on the left-hand side was a perfect square. In perfect square trinomials in which the leading coefficient is 1, there is a relationship between the **coefficient of the *x*-term** and the **constant term**.

$$(x + 5)^2 = x^2 + 2(5)(x) + 5^2$$
$$= x^2 + 10x + 25$$

Notice that $\left(\frac{10}{2}\right)^2 = 25$. To get the constant term, divide the coefficient of the x-term by 2 and square the result. Any quadratic expression in the form $x^2 + bx$ can be made into a perfect square by using a method called **completing the square**.

KeyConcept Completing the Square

Words To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

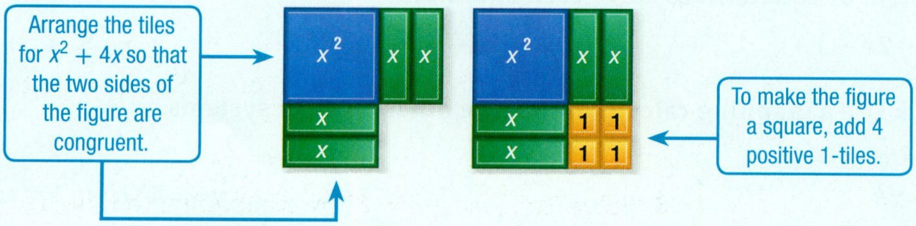
Step 3 Add the result of Step 2 to $x^2 + bx$.

Symbols $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Example 1 Complete the Square

Find the value of c that makes $x^2 + 4x + c$ a perfect square trinomial.

Method 1 Use algebra tiles.



Erik Isakson/Blend Images/age fotostock

StudyTip

Algorithms An algorithm is a series of steps for carrying out a procedure or solving a problem.

Method 2 Use complete the square algorithm.

Step 1 Find $\frac{1}{2}$ of 4. $\frac{4}{2} = 2$

Step 2 Square the result in Step 1. $2^2 = 4$

Step 3 Add the result of Step 2 to $x^2 + 4x$. $x^2 + 4x + 4$

Thus, $c = 4$. Notice that $x^2 + 4x + 4 = (x + 2)^2$.

GuidedPractice

1. Find the value of c that makes $r^2 - 8r + c$ a perfect square trinomial.

2 Solve Equations by Completing the Square

You can complete the square to solve quadratic equations. First, you must isolate the x^2- and bx-terms.

Example 2 Solve an Equation by Completing the Square

PT

Solve $x^2 - 6x + 12 = 19$ by completing the square.

$x^2 - 6x + 12 = 19$	Original equation
$x^2 - 6x = 7$	Subtract 12 from each side.
$x^2 - 6x + 9 = 7 + 9$	Since $\left(\frac{-6}{2}\right)^2 = 9$, add 9 to each side.
$(x - 3)^2 = 16$	Factor $x^2 - 6x + 9$.
$x - 3 = \pm 4$	Take the square root of each side.
$x = 3 \pm 4$	Add 3 to each side.
$x = 3 + 4$ or $x = 3 - 4$	Separate the solutions.
$= 7$ $\quad$ $= -1$	The solutions are 7 and -1.

GuidedPractice

2. Solve $x^2 - 12x + 3 = 8$ by completing the square.

To solve a quadratic equation in which the leading coefficient is not 1, divide each term by the coefficient. Then isolate the x^2- and x-terms and complete the square.

Example 3 Equation with $a \neq 1$

PT

WatchOut!

Leading Coefficient Remember that the leading coefficient has to be 1 before you can complete the square.

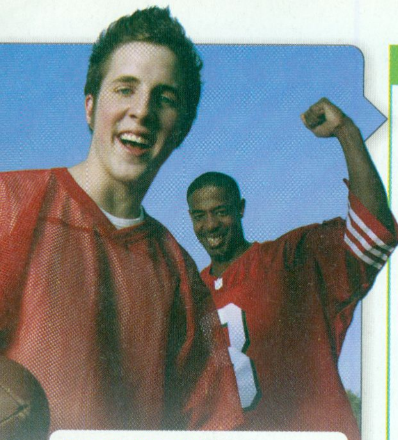
Solve $-2x^2 + 8x - 18 = 0$ by completing the square.

$-2x^2 + 8x - 18 = 0$	Original equation
$\frac{-2x^2 + 8x - 18}{-2} = \frac{0}{-2}$	Divide each side by -2.
$x^2 - 4x + 9 = 0$	Simplify.
$x^2 - 4x = -9$	Subtract 9 from each side.
$x^2 - 4x + 4 = -9 + 4$	Since $\left(\frac{-4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = -5$	Factor $x^2 - 4x + 4$.

No real number has a negative square. So, this equation has no real solutions.

GuidedPractice

3. Solve $3x^2 - 9x - 3 = 21$ by completing the square.



Real-World Link

The oldest public high school rivalry takes place between Wellesley High School and Needham Heights High School in Massachusetts. The first football game between them took place on Thanksgiving morning in 1882 in Needham.

Source: USA Football

Real-World Example 4 Use a Graph of a Quadratic Function

JERSEYS The senior class at Bay High School buys jerseys to wear to the football games. The cost of the jerseys can be modeled by the equation $C = 0.1x^2 + 2.4x + 25$, where C is the amount it costs to buy x jerseys. How many jerseys can they purchase for \$430?

The seniors have \$430, so set the equation equal to 430 and complete the square.

$0.1x^2 + 2.4x + 25 = 430$	Original equation
$\frac{0.1x^2 + 2.4x + 25}{0.1} = \frac{430}{0.1}$	Divide each side by 0.1.
$x^2 + 24x + 250 = 4300$	Simplify.
$x^2 + 24x + 250 - 250 = 4300 - 250$	Subtract 250 from each side.
$x^2 + 24x = 4050$	Simplify.
$x^2 + 24x + 144 = 4050 + 144$	Since $\left(\frac{24}{2}\right)^2 = 144$, add 144 to each side.
$x^2 + 24x + 144 = 4194$	Simplify.
$(x + 12)^2 = 4194$	Factor $x^2 + 24x + 144$.
$x + 12 = \pm\sqrt{4194}$	Take the square root of each side.
$x = -12 \pm\sqrt{4194}$	Subtract 12 from each side.

Use a calculator to approximate each value of x.

$x = -12 + \sqrt{4194}$	or	$x = -12 - \sqrt{4194}$	Separate the solutions.
≈ 52.8		≈ -76.8	Evaluate.

Since you cannot buy a negative number of jerseys, the negative solution is not reasonable. The seniors can afford to buy 52 jerseys.

Guided Practice

4. If the senior class were able to raise \$620, how many jerseys could they buy?

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 **Find the value of c that makes each trinomial a perfect square.**

1. $x^2 - 18x + c$

2. $x^2 + 22x + c$

3. $x^2 + 9x + c$

4. $x^2 - 7x + c$

Examples 2–3 **Solve each equation by completing the square. Round to the nearest tenth if necessary.**

5. $x^2 + 4x = 6$

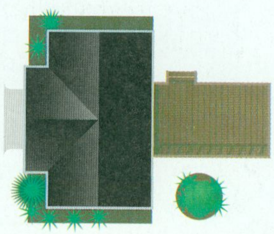
6. $x^2 - 8x = -9$

7. $4x^2 + 9x - 1 = 0$

8. $-2x^2 + 10x + 22 = 4$

Example 4

9. CCSS **MODELING** Collin is building a deck on the back of his family's house. He has enough lumber for the deck to be 144 square feet. The length should be 10 feet more than its width. What should the dimensions of the deck be?



DreamPictures/Photodisc/Getty Images

Practice and Problem Solving

Extra Practice is on page R9.

Example 1 **Find the value of c that makes each trinomial a perfect square.**

10. $x^2 + 26x + c$
11. $x^2 - 24x + c$
12. $x^2 - 19x + c$
13. $x^2 + 17x + c$
14. $x^2 + 5x + c$
15. $x^2 - 13x + c$
16. $x^2 - 22x + c$
17. $x^2 - 15x + c$
18. $x^2 + 24x + c$

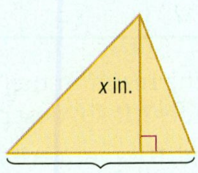
Examples 2–3 **Solve each equation by completing the square. Round to the nearest tenth if necessary.**

19. $x^2 + 6x - 16 = 0$
20. $x^2 - 2x - 14 = 0$
21. $x^2 - 8x - 1 = 8$
22. $x^2 + 3x + 21 = 22$
23. $x^2 - 11x + 3 = 5$
24. $5x^2 - 10x = 23$
25. $2x^2 - 2x + 7 = 5$
26. $3x^2 + 12x + 81 = 15$
27. $4x^2 + 6x = 12$
28. $4x^2 + 5 = 10x$
29. $-2x^2 + 10x = -14$
30. $-3x^2 - 12 = 14x$

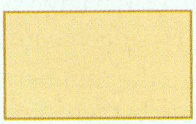
Example 4 **31. FINANCIAL LITERACY** The price p in dollars for a particular stock can be modeled by the quadratic equation $p = 3.5t - 0.05t^2$, where t represents the number of days after the stock is purchased. When is the stock worth \$60?

GEOMETRY **Find the value of x for each figure. Round to the nearest tenth if necessary.**

32. $A = 45 \text{ in}^2$



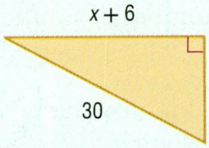
33. $A = 110 \text{ ft}^2$



34. NUMBER THEORY The product of two consecutive even integers is 224. Find the integers.

35. CCSS PRECISION The product of two consecutive negative odd integers is 483. Find the integers.

36. GEOMETRY Find the area of the triangle below.



Solve each equation by completing the square. Round to the nearest tenth if necessary.

37. $0.2x^2 - 0.2x - 0.4 = 0$
38. $0.5x^2 = 2x - 0.3$
39. $2x^2 - \frac{11}{5}x = -\frac{3}{10}$
40. $\frac{2}{3}x^2 - \frac{4}{3}x = \frac{5}{6}$
41. $\frac{1}{4}x^2 + 2x = \frac{3}{8}$
42. $\frac{2}{5}x^2 + 2x = \frac{1}{5}$

43. **ASTRONOMY** The height of an object t seconds after it is dropped is given by the equation $h = -\frac{1}{2}gt^2 + h_0$, where h_0 is the initial height and g is the acceleration due to gravity. The acceleration due to gravity near the surface of Mars is 3.73 m/s^2, while on Earth it is 9.8 m/s^2. Suppose an object is dropped from an initial height of 120 meters above the surface of each planet.

a. On which planet would the object reach the ground first?

b. How long would it take the object to reach the ground on each planet? Round each answer to the nearest tenth.

c. Do the times that it takes the object to reach the ground seem reasonable? Explain your reasoning.

44. Find all values of c that make $x^2 + cx + 100$ a perfect square trinomial.

45. Find all values of c that make $x^2 + cx + 225$ a perfect square trinomial.

46. **PAINTING** Before she begins painting a picture, Donna stretches her canvas over a wood frame. The frame has a length of 60 inches and a width of 4 inches. She has enough canvas to cover 480 square inches. Donna decides to increase the dimensions of the frame. If the increase in the length is 10 times the increase in the width, what will the dimensions of the frame be?

47. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a property of quadratic equations.

a. Tabular Copy the table shown and complete the second column.

b. Algebraic Set each trinomial equal to zero, and solve the equation by completing the square. Complete the last column of the table with the number of roots of each equation.

Trinomial	$b^2 - 4ac$	Number of Roots
$x^2 - 8x + 16$	0	1
$2x^2 - 11x + 3$		
$3x^2 + 6x + 9$		
$x^2 - 2x + 7$		
$x^2 + 10x + 25$		
$x^2 + 3x - 12$		

c. Verbal Compare the number of roots of each equation to the result in the $b^2 - 4ac$ column. Is there a relationship between these values? If so, describe it.

d. Analytical Predict how many solutions $2x^2 - 9x + 15 = 0$ will have. Verify your prediction by solving the equation.

H.O.T. Problems Use Higher-Order Thinking Skills

48. **CCSS PERSEVERANCE** Given $y = ax^2 + bx + c$ with $a \neq 0$, derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form $y = a(x - h)^2 + k$.

49. **REASONING** Determine the number of solutions $x^2 + bx = c$ has if $c < -\left(\frac{b}{2}\right)^2$. Explain.

50. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

$n^2 - n + \frac{1}{4}$	$n^2 + n + \frac{1}{4}$	$n^2 - \frac{2}{3}n + \frac{1}{9}$	$n^2 + \frac{1}{3}n + \frac{1}{9}$

51. **OPEN ENDED** Write a quadratic equation for which the only solution is 4.

52. **WRITING IN MATH** Compare and contrast the following strategies for solving $x^2 - 5x - 7 = 0$: completing the square, graphing, and factoring.

Standardized Test Practice

53. The length of a rectangle is 3 times its width. The area of the rectangle is 75 square feet. Find the length of the rectangle in feet.

A 25　　B 15　　C 10　　D 5

54. PROBABILITY At a festival, winners of a game draw a token for a prize. There is one token for each prize. The prizes include 9 movie passes, 8 stuffed animals, 5 hats, 10 jump ropes, and 4 glow necklaces. What is the probability that the first person to draw a token will win a movie pass?

F $\frac{1}{36}$　　G $\frac{1}{9}$　　H $\frac{9}{61}$　　J $\frac{1}{4}$

55. GRIDDED RESPONSE The population of a town can be modeled by $P = 22{,}000 + 125t$, where P represents the population and t represents the number of years from 2000. How many years after 2000 will the population be 26,000?

56. Percy delivers pizzas for Pizza King. He is paid \$6 an hour plus \$2.50 for each pizza he delivers. Percy earned \$280 last week. If he worked a total of 30 hours, how many pizzas did he deliver?

A 250 pizzas
B 184 pizzas
C 40 pizzas
D 34 pizzas

Spiral Review

Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 9-3)

57. $g(x) = -12 + x^2$　　**58.** $h(x) = (x + 2)^2$　　**59.** $g(x) = 2x^2 + 5$

60. $h(x) = \frac{2}{3}(x - 6)^2$　　**61.** $g(x) = 6 + \frac{4}{3}x^2$　　**62.** $h(x) = -1 - \frac{3}{2}x^2$

63. RIDES A popular amusement park ride whisks riders to the top of a 250-foot tower and drops them. A function for the height of a rider is $h = -16t^2 + 250$, where h is the height and t is the time in seconds. The ride stops the descent of the rider 40 feet above the ground. Write an equation that models the drop of the rider. How long does it take to fall from 250 feet to 40 feet? (Lesson 9-2)

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

64. $\frac{a^6}{a^3}$　　**65.** $\frac{4^7}{4^5}$　　**66.** $\frac{c^3d^4}{cd^7}$

67. $\left(\frac{4h^{-2}g}{2g^5}\right)^0$　　**68.** $\frac{5q^{-2}t^6}{10q^2t^{-4}}$　　**69.** $b^3(m^{-3})(b^{-6})$

Solve each open sentence. (Lesson 5-5)

70. $|y - 2| > 7$　　**71.** $|z + 5| < 3$　　**72.** $|2b + 7| \le -6$

73. $|3 - 2y| \ge 8$　　**74.** $|9 - 4m| < -1$　　**75.** $|5c - 2| \le 13$

Skills Review

Evaluate $\sqrt{b^2 - 4ac}$ for each set of values. Round to the nearest tenth if necessary.

76. $a = 2, b = -5, c = 2$　　**77.** $a = 1, b = 12, c = 11$　　**78.** $a = -9, b = 10, c = -1$

79. $a = 1, b = 7, c = -3$　　**80.** $a = 2, b = -4, c = -6$　　**81.** $a = 3, b = 1, c = 2$

EXTEND 9-4

Algebra Lab
Finding the Maximum or Minimum Value

In Lesson 9-3, we learned about the vertex form of the equation of a quadratic function. You will now learn how to write equations in vertex form and use them to identify key characteristics of the graphs of quadratic functions.

CCSS Common Core State Standards
Content Standards
A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Activity 1 Find a Minimum

Write $y = x^2 + 4x - 10$ in vertex form. Identify the axis of symmetry, extrema, and zeros. Then graph the function.

Step 1 Complete the square to write the function in vertex form.

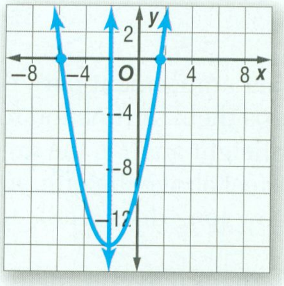
$y = x^2 + 4x - 10$	Original function
$y + 10 = x^2 + 4x$	Add 10 to each side.
$y + 10 + 4 = x^2 + 4x + 4$	Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$y + 14 = (x + 2)^2$	Factor $x^2 + 4x + 4$.
$y = (x + 2)^2 - 14$	Subtract 14 from each side to write in vertex form.

Step 2 Identify the axis of symmetry and extrema based on the equation in vertex form. The vertex is at (h, k) or $(-2, -14)$. Since there is no negative sign before the x^2-term, the parabola opens up and has a minimum at $(-2, -14)$. The equation of the axis of symmetry is $x = -2$.

Step 3 Solve for x to find the zeros.

$(x + 2)^2 - 14 = 0$	Vertex form, $y = 0$
$(x + 2)^2 = 14$	Add 14 to each side.
$x + 2 = \pm\sqrt{14}$	Take square root of each side.
$x \approx -5.74$ or 1.74	Subtract 2 from each side.

The zeros are approximately -5.74 and 1.74.



Step 4 Use the key features to graph the function.

There may be a negative coefficient before the quadratic term. When this is the case, the parabola will open down and have a maximum.

Activity 2 Find a Maximum

Write $y = -x^2 + 6x - 5$ in vertex form. Identify the axis of symmetry, extrema, and zeros. Then graph the function.

Step 1 Complete the square to write the equation of the function in vertex form.

$y = -x^2 + 6x - 5$	Original function
$y + 5 = -x^2 + 6x$	Add 5 to each side.
$y + 5 = -(x^2 - 6x)$	Factor out −1.
$y + 5 - 9 = -(x^2 - 6x + 9)$	Since $\left(\frac{6}{2}\right)^2 = 9$, add −9 to each side.
$y - 4 = -(x - 3)^2$	Factor $x^2 - 6x + 9$.
$y = -(x - 3)^2 + 4$	Add 4 to each side to write in vertex form.

Step 2 Identify the axis of symmetry and extrema based on the equation in vertex form. The vertex is at (h, k) or (3, 4). Since there is a negative sign before the x^2-term, the parabola opens down and has a maximum at (3, 4). The equation of the axis of symmetry is $x = 3$.

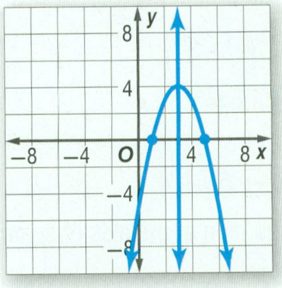
Step 3 Solve for x to find the zeros.

$0 = -(x - 3)^2 + 4$ — Vertex form, $y = 0$

$(x - 3)^2 = 4$ — Add $(x - 3)^2$ to each side.

$x - 3 = \pm 2$ — Take the square root of each side.

$x = 5$ or 1 — Add 3 to each.



Step 4 Use the key features to graph the function.

Analyze the Results

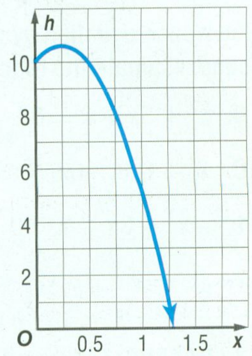
1. Why do you need to complete the square to write the equation of a quadratic function in vertex form?

Write each function in vertex form. Identify the axis of symmetry, extrema, and zeros. Then graph the function.

2. $y = x^2 + 6x$
3. $y = x^2 - 8x + 6$
4. $y = x^2 + 2x - 12$
5. $y = x^2 + 6x + 8$
6. $y = x^2 - 4x + 3$
7. $y = x^2 - 2.4x - 2.2$
8. $y = -4x^2 + 16x - 11$
9. $y = 3x^2 - 12x + 5$
10. $y = -x^2 + 6x - 5$

Activity 3 Use Extrema in the Real World

DIVING **Alexis jumps from a diving platform upward and outward before diving into the pool. The function $h = -9.8t^2 + 4.9t + 10$, where h is the height of the diver in meters above the pool after t seconds approximates Alexis's dive. Graph the function, then find the maximum height that she reaches and the equation of the axis of symmetry.**



Step 1 Graph the function.

Step 2 Complete the square to write the eqution of the function in vertex form.

$h = -9.8t^2 + 4.9t + 10$

$h = -9.8(t - 0.25)^2 + 10.6125$

Step 3 The vertex is at (0.25, 10.6125), so the maximum height is 10.6125 meters. The equation of the axis of symmetry is $x = 0.25$.

Exercise

11. **SOFTBALL** Jenna throws a ball in the air. The function $h = -16t^2 + 40t + 5$, where h is the height in feet and t represents the time in seconds, approximates Jenna's throw. Graph the function, then find the maximum height of the ball and the equation of the axis of symmetry. When does the ball hit the ground?

CHAPTER 9

Mid-Chapter Quiz

Lessons 9-1 through 9-4

Use a table of values to graph each equation. State the domain and range. (Lesson 9-1)

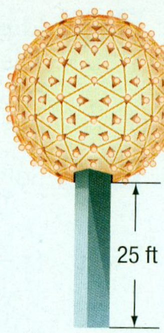
1. $y = x^2 + 3x + 1$
2. $y = 2x^2 - 4x + 3$
3. $y = -x^2 - 3x - 3$
4. $y = -3x^2 - x + 1$

Consider $y = x^2 - 5x + 4$. (Lesson 9-1)

5. Write the equation of the axis of symmetry.
6. Find the coordinates of the vertex. Is it a maximum or minimum point?
7. Graph the function.
8. **SOCCER** A soccer ball is kicked from ground level with an initial upward velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the ball after t seconds. (Lesson 9-1)
   a. What is the height of the ball after one second?
   b. How many seconds will it take for the ball to reach its maximum height?
   c. When is the height of the ball 0 feet? What do these points represent in this situation?

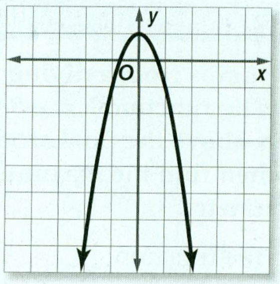
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth. (Lesson 9-2)

9. $x^2 + 5x + 6 = 0$
10. $x^2 + 8 = -6x$
11. $-x^2 + 3x - 1 = 0$
12. $x^2 = 12$
13. **BASEBALL** Juan hits a baseball. The equation $h = -16t^2 + 120t$ models the height h, in feet, of the ball after t seconds. How long is the ball in the air? (Lesson 9-2)
14. **CONSTRUCTION** Christopher is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 feet. This is represented by the equation $h = -16t^2 + 14$, where h is the height in feet and t is the time in seconds. Describe how the graph is related to $h = t^2$. (Lesson 9-3)
15. **PARTIES** Della's parents are throwing a Sweet 16 party for her. At 10:00, a ball will slide 25 feet down a pole and light up. A function that models the drop is $h = -t^2 + 5t + 25$, where h is height in feet of the ball after t seconds. How many seconds will it take for the ball to reach the bottom of the pole? (Lesson 9-2)



Describe how the graph of each function is related to the graph of $f(x) = x^2$. (Lesson 9-3)

16. $g(x) = x^2 + 3$
17. $h(x) = 2x^2$
18. $g(x) = x^2 - 6$
19. $h(x) = \frac{1}{5}x^2$
20. $g(x) = -x^2 + 1$
21. $h(x) = -\frac{5}{8}x^2$
22. **MULTIPLE CHOICE** Which is an equation for the function shown in the graph? (Lesson 9-3)



A $y = -2x^2$

B $y = 2x^2 + 1$

C $y = x^2 - 1$

D $y = -2x^2 + 1$

Solve each equation by completing the square. Round to the nearest tenth. (Lesson 9-4)

23. $x^2 + 4x + 2 = 0$
24. $x^2 - 2x - 10 = 0$
25. $2x^2 + 4x - 5 = 7$

LESSON 9-5 Solving Quadratic Equations by Using the Quadratic Formula

Then

- You solved quadratic equations by completing the square.

Now

- 1 Solve quadratic equations by using the Quadratic Formula.
- 2 Use the discriminant to determine the number of solutions of a quadratic equation.

Why?

- For adult women, the normal systolic blood pressure P in millimeters of mercury (mm Hg) can be modeled by $P = 0.01a^2 + 0.05a + 107$, where a is age in years. This equation can be used to approximate the age of a woman with a certain systolic blood pressure. However, it would be difficult to solve by factoring, graphing, or completing the square.

NewVocabulary
Quadratic Formula
discriminant

Common Core State Standards

Content Standards
A.REI.4 Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Mathematical Practices
6 Attend to precision.

1 Quadratic Formula

Completing the square of the quadratic equation $ax^2 + bx + c = 0$ produces a formula that allows you to find the solutions of *any* quadratic equation. This formula is called the **Quadratic Formula**.

KeyConcept The Quadratic Formula

The solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You will derive this formula in Lesson 10-2.

PT

Example 1 Use the Quadratic Formula

Solve $x^2 - 12x = -20$ by using the Quadratic Formula.

Step 1 Rewrite the equation in standard form.

$x^2 - 12x = -20$ Original equation

$x^2 - 12x + 20 = 0$ Add 20 to each side.

Step 2 Apply the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(20)}}{2(1)}$ $a = 1$, $b = -12$, and $c = 20$

$= \frac{12 \pm \sqrt{144 - 80}}{2}$ Multiply.

$= \frac{12 \pm \sqrt{64}}{2}$ or $\frac{12 \pm 8}{2}$ Subtract and take the square root.

$x = \frac{12 - 8}{2}$ or $x = \frac{12 + 8}{2}$ Separate the solutions.

$= 2$ $= 10$ The solutions are 2 and 10.

GuidedPractice

1. Solve $2x^2 + 9x = 18$ by using the Quadratic Formula.

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The solutions of quadratic equations are not always integers.

PT

Example 2 Use the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

a. $3x^2 + 5x - 12 = 0$

For this equation, $a = 3$, $b = 5$, and $c = -12$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-12)}}{2(3)}$$ $a = 3$, $b = 5$, and $c = -12$

$$= \frac{-5 \pm \sqrt{25 + 144}}{6}$$ Multiply.

$$= \frac{-5 \pm \sqrt{169}}{6} \text{ or } \frac{-5 \pm 13}{6}$$ Add and simplify.

$$x = \frac{-5 - 13}{6} \text{ or } x = \frac{-5 + 13}{6}$$ Separate the solutions.

$$= -3 \qquad = \frac{4}{3}$$ Simplify.

The solutions are -3 and $\frac{4}{3}$.

b. $10x^2 - 5x = 25$

Step 1 Rewrite the equation in standard form.

$$10x^2 - 5x = 25$$ Original equation

$$10x^2 - 5x - 25 = 0$$ Subtract 25 from each side.

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(10)(-25)}}{2(10)}$$ $a = 10$, $b = -5$, and $c = -25$

$$= \frac{5 \pm \sqrt{25 + 1000}}{20}$$ Multiply.

$$= \frac{5 \pm \sqrt{1025}}{20}$$ Add.

$$= \frac{5 - \sqrt{1025}}{20} \text{ or } \frac{5 + \sqrt{1025}}{20}$$ Separate the solutions.

$$\approx -1.4 \qquad \approx 1.9$$ Simplify.

The solutions are about -1.4 and 1.9.

StudyTip

CCSS Precision In Example 2, the number $\sqrt{1025}$ is irrational, so the calculator can only give you an approximation of its value. So, the exact answer in Example 2 is $\frac{5 \pm \sqrt{1025}}{20}$. The numbers -1.4 and 1.9 are approximations.

GuidedPractice

2A. $4x^2 - 24x + 35 = 0$

2B. $3x^2 - 2x - 9 = 0$

You can solve quadratic equations by using many different methods. No one way is always best.

WatchOut!

Solutions No matter what method is used to solve a quadratic equation, all of the methods should produce the same solution(s).

Example 3 Solve Quadratic Equations Using Different Methods

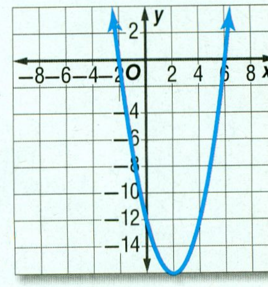
Solve $x^2 - 4x = 12$.

Method 1 Graphing

Rewrite the equation in standard form.

$x^2 - 4x = 12$ — Original equation

$x^2 - 4x - 12 = 0$ — Subtract 12 from each side.



Graph the related function $f(x) = x^2 - 4x - 12$.
Locate the x-intercepts of the graph.

The solutions are -2 and 6.

Method 2 Factoring

$x^2 - 4x = 12$	Original equation
$x^2 - 4x - 12 = 0$	Subtract 12 from each side.
$(x - 6)(x + 2) = 0$	Factor.
$x - 6 = 0$ or $x + 2 = 0$	Zero Product Property
$x = 6 \qquad x = -2$	Solve for x.

Method 3 Completing the Square

The equation is in the correct form to complete the square, since the leading coefficient is 1 and the x^2 and x terms are isolated.

$x^2 - 4x = 12$	Original equation
$x^2 - 4x + 4 = 12 + 4$	Since $\left(\frac{-4}{2}\right)^2 = 4$, add 4 to each side.
$(x - 2)^2 = 16$	Factor $x^2 - 4x + 4$.
$x - 2 = \pm 4$	Take the square root of each side.
$x = 2 \pm 4$	Add 2 to each side.
$x = 2 + 4$ or $x = 2 - 4$	Separate the solutions.
$= 6 \qquad = -2$	Simplify.

Method 4 Quadratic Formula

From Method 1, the standard form of the equation is $x^2 - 4x - 12 = 0$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$	$a = 1$, $b = -4$, and $c = -12$
$= \dfrac{4 \pm \sqrt{16 + 48}}{2}$	Multiply.
$= \dfrac{4 \pm \sqrt{64}}{2}$ or $\dfrac{4 \pm 8}{2}$	Add and simplify.
$x = \dfrac{4 - 8}{2}$ or $x = \dfrac{4 + 8}{2}$	Separate the solutions.
$= -2 \qquad = 6$	Simplify.

GuidedPractice

Solve each equation.

3A. $2x^2 - 17x + 8 = 0$

3B. $4x^2 - 4x - 11 = 0$

ConceptSummary Solving Quadratic Equations

Method	When to Use
Factoring	Use when the constant term is 0 or if the factors are easily determined. Not all equations are factorable.
Graphing	Use when an approximate solution is sufficient.
Using Square Roots	Use when an equation can be written in the form $x^2 = n$. Can only be used if the equation has no x-term.
Completing the Square	Can be used for any equation $ax^2 + bx + c = 0$, but is simplest to apply when b is even and $a = 1$.
Quadratic Formula	Can be used for any equation $ax^2 + bx + c = 0$.

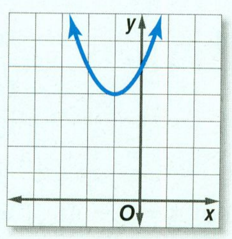
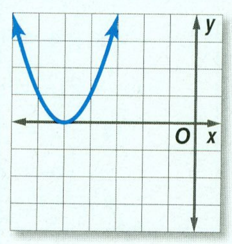
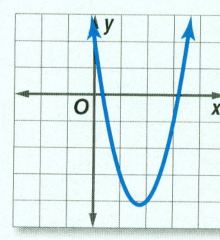
2 The Discriminant

In the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real solutions of a quadratic equation.

StudyTip

Discriminant Recall that when the left side of the standard form of an equation is a perfect square trinomial, there is only one solution. Therefore, the discriminant of a perfect square trinomial will always be zero.

KeyConcept Using the Discriminant

Equation	$x^2 + 2x + 5 = 0$	$x^2 + 10x + 25 = 0$	$2x^2 - 7x + 2 = 0$
Discriminant	$b^2 - 4ac = -16$ negative	$b^2 - 4ac = 0$ zero	$b^2 - 4ac = 33$ positive
Graph of Related Function	 0 x-intercepts	 1 x-intercept	 2 x-intercepts
Real Solutions	0	1	2

Example 4 Use the Discriminant

State the value of the discriminant of $4x^2 + 5x = -3$. Then determine the number of real solutions of the equation.

Step 1 Rewrite in standard form. $4x^2 - 5x = -3 \rightarrow 4x^2 - 5x + 3 = 0$

Step 2 Find the discriminant.

$$b^2 - 4ac = (-5)^2 - 4(4)(3) \quad a = 4, b = -5, \text{ and } c = 3$$

$$= -23 \quad \text{Simplify.}$$

Since the discriminant is negative, the equation has no real solutions.

GuidedPractice

4A. $2x^2 + 11x + 15 = 0$

4B. $9x^2 - 30x + 25 = 0$

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–2 Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 2x - 15 = 0$
2. $x^2 - 10x + 16 = 0$
3. $x^2 - 8x = -10$
4. $x^2 + 3x = 12$
5. $10x^2 - 31x + 15 = 0$
6. $5x^2 + 5 = -13x$

Example 3 Solve each equation. State which method you used.

7. $2x^2 + 11x - 6 = 0$
8. $2x^2 - 3x - 6 = 0$
9. $9x^2 = 25$
10. $x^2 - 9x = -19$

Example 4 State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

11. $x^2 - 9x + 21 = 0$
12. $2x^2 - 11x + 10 = 0$
13. $9x^2 + 24x = -16$
14. $3x^2 - x = 8$
15. **TRAMPOLINE** Eva is jumping on a trampoline. Her height h in feet can be modeled by the equation $h = -16t^2 + 2.4t + 6$, where t is time in seconds. Use the discriminant to determine if Eva will ever reach a height of 20 feet. Explain.

Practice and Problem Solving

Extra Practice is on page R9.

Examples 1–2 Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

16. $4x^2 + 5x - 6 = 0$
17. $x^2 + 16 = 0$
18. $6x^2 - 12x + 1 = 0$
19. $5x^2 - 8x = 6$
20. $2x^2 - 5x = -7$
21. $5x^2 + 21x = -18$
22. $81x^2 = 9$
23. $8x^2 + 12x = 8$
24. $4x^2 = -16x - 16$
25. $10x^2 = -7x + 6$
26. $-3x^2 = 8x - 12$
27. $2x^2 = 12x - 18$
28. **AMUSEMENT PARKS** The Demon Drop at Cedar Point in Ohio takes riders to the top of a tower and drops them 60 feet. A function that approximates this ride is $h = -16t^2 + 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?

Example 3 Solve each equation. State which method you used.

29. $2x^2 - 8x = 12$
30. $3x^2 - 24x = -36$
31. $x^2 - 3x = 10$
32. $4x^2 + 100 = 0$
33. $x^2 = -7x - 5$
34. $12 - 12x = -3x^2$

Example 4 State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

35. $0.2x^2 - 1.5x + 2.9 = 0$
36. $2x^2 - 5x + 20 = 0$
37. $x^2 - \frac{4}{5}x = 3$
38. $0.5x^2 - 2x = -2$
39. $2.25x^2 - 3x = -1$
40. $2x^2 = \frac{5}{2}x + \frac{3}{2}$
41. **CCSS MODELING** The percent of U.S. households with high-speed Internet h can be estimated by $h = -0.2n^2 + 7.2n + 1.5$, where n is the number of years since 1990.
    a. Use the Quadratic Formula to determine when 20% of the population will have high-speed Internet.
    b. Is a quadratic equation a good model for this information? Explain.

42. **TRAFFIC** The equation $d = 0.05v^2 + 1.1v$ models the distance d in feet it takes a car traveling at a speed of v miles per hour to come to a complete stop. The speed limit on some highways is 65 miles per hour. If Hannah's car stopped after 250 feet, was she speeding? Explain your reasoning.

Without graphing, determine the number of x-intercepts of the graph of the related function for each equation.

43. $4.25x + 3 = -3x^2$

44. $x^2 + \frac{2}{25} = \frac{3}{5}x$

45. $0.25x^2 + x = -1$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

46. $-2x^2 - 7x = -1.5$

47. $2.3x^2 - 1.4x = 6.8$

48. $x^2 - 2x = 5$

49. **POSTER** Bartolo is making a poster for the dance. He wants to cover three fourths of the area with text.

a. Write an equation for the area of the section with text.

b. Solve the equation by using the Quadratic Formula.

c. What should be the margins of the poster?

50. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate writing a quadratic equation with given roots. If p is a root of $0 = ax^2 + bx + c$, then $(x - p)$ is a factor of $ax^2 + bx + c$.

a. **Tabular** Copy and complete the first two columns of the table.

b. **Algebraic** Multiply the factors to write each equation with integral coefficients. Use the equations to complete the last column of the table. Write each equation.

c. **Analytical** How could you write an equation with three roots? Test your conjecture by writing an equation with roots 1, 2, and 3. Is the equation quadratic? Explain.

Roots	Factors	Equation
2, 5	$(x-2), (x-5)$	$(x-2)(x-5) = 0$ $x^2 - 7x + 10 = 0$
1, 9		
−1, 3		
0, 6		
$\frac{1}{2}$, 7		
$-\frac{2}{3}$, 4		

H.O.T. Problems Use Higher-Order Thinking Skills

51. **CHALLENGE** Find all values of k such that $2x^2 - 3x + 5k = 0$ has two solutions.

52. **REASONING** Use factoring techniques to determine the number of real zeros of $f(x) = x^2 - 8x + 16$. Compare this method to using the discriminant.

CCSS **STRUCTURE Determine whether there are *two*, *one*, or *no* real solutions of each equation.**

53. The graph of the related quadratic function does not have an x-intercept.

54. The graph of the related quadratic function is tangent to the x-axis.

55. The graph of the related quadratic function intersects the x-axis twice.

56. Both a and b are greater than 0 and c is less than 0 in a quadratic equation.

57. **WRITING IN MATH** Why can the discriminant be used to confirm the number of real solutions of a quadratic equation?

58. **WRITING IN MATH** Describe the advantages and disadvantages of each method of solving quadratic equations. Which method do you prefer, and why?

Standardized Test Practice

59. If n is an even integer, which expression represents the product of three consecutive even integers?

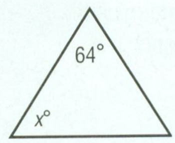
A $n(n + 1)(n + 2)$

B $(n + 1)(n + 2)(n + 3)$

C $3n + 2$

D $n(n + 2)(n + 4)$

60. SHORT RESPONSE The triangle shown is an isosceles triangle. What is the value of x?



61. Which statement best describes the graph of $x = 5$?

F It is parallel to the x-axis.

G It is parallel to the y-axis.

H It passes through the point (2, 5).

J It has a y-intercept of 5.

62. What are the solutions of the quadratic equation $6h^2 + 6h = 72$?

A 3 or −4

B −3 or 4

C no solution

D 12 or −48

Spiral Review

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-4)

63. $6x^2 - 17x + 12 = 0$

64. $x^2 - 9x = -12$

65. $4x^2 = 20x - 25$

Describe the transformations needed to obtain the graph of $g(x)$ from the graph of $f(x)$. (Lesson 9-3)

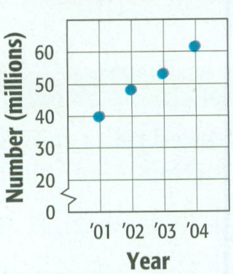
66. $f(x) = 4x^2$
$g(x) = 2x^2$

67. $f(x) = x^2 + 5$
$g(x) = x^2 - 1$

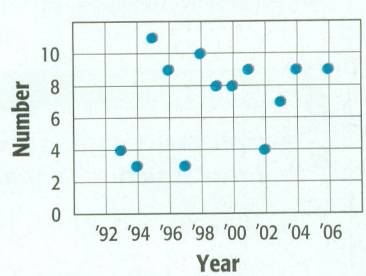
68. $f(x) = x^2 - 6$
$g(x) = x^2 + 3$

Determine whether each graph shows a *positive*, a *negative*, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation. (Lesson 4-5)

69. **Electronic Tax Returns**



70. **Atlantic Hurricanes**



71. ENTERTAINMENT Coach Washington wants to take her softball team out for pizza and soft drinks after the last game of the season. A large pizza costs \$12 and a pitcher of a soft drink costs \$3. She does not want to spend more than \$60. Write an inequality that represents this situation and graph the solution set. (Lesson 5-6)

Skills Review

Determine whether each sequence is *arithmetic*, *geometric*, or neither. Explain.

72. 20, 25, 30, …

73. 1000, 950, 900, …

74. 200, 350, 650, …

75. 6, 18, 54, …

76. 2, 4, 16, …

77. 8, −4, 2 …

LESSON 9-6 Analyzing Functions with Successive Differences

Then

- You graphed linear, quadratic, and exponential functions.

Now

1. Identify linear, quadratic, and exponential functions from given data.
2. Write equations that model data.

Why?

- Every year the golf team sells candy to raise money for charity. By knowing what type of function models the sales of the candy, they can determine the best price of the candy.

CCSS Common Core State Standards

Content Standards

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

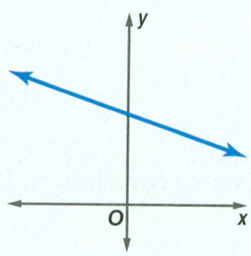
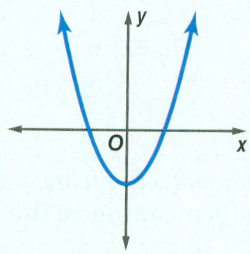
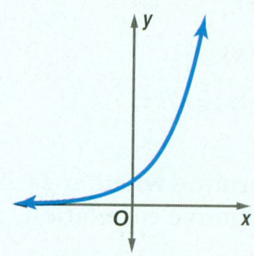
Mathematical Practices

7 Look for and make use of structure.

1 Identify Functions

You can use linear functions, quadratic functions, and exponential functions to model data. The general forms of the equations and a graph of each function type are listed below.

ConceptSummary Linear and Nonlinear Functions

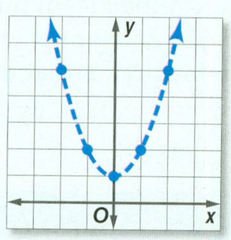
Linear Function	Quadratic Function	Exponential Function
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$, when $b > 0$
		

Example 1 Choose a Model Using Graphs

Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

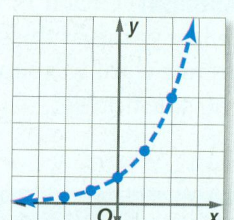
a. $\{(-2, 5), (-1, 2), (0, 1), (1, 2), (2, 5)\}$

The ordered pairs appear to represent a quadratic function.



b. $\left\{\left(-2, \frac{1}{4}\right), \left(-1, \frac{1}{2}\right), (0, 1), (1, 2), (2, 4)\right\}$

The ordered pairs appear to represent an exponential function.



GuidedPractice

1A. $(-2, -3), (-1, -1), (0, 1), (1, 3)$

1B. $(-1, 0.25), (0, 1), (1, 4), (2, 16)$

Elan Fleisher/Look/age fotostock

Another way to determine which model best describes data is to use patterns. The differences of successive y-values are called *first differences*. The differences of successive first differences are called *second differences*.

- If the differences of successive y-values are all equal, the data represent a linear function.
- If the second differences are all equal, but the first differences are not equal, the data represent a quadratic function.
- If the ratios of successive y-values are all equal and $r \neq 1$, the data represent an exponential function.

Watch Out!

***x*-Values** Before you check for successive differences or ratios, make sure the x-values are increasing by the same amount.

PT

Example 2 Choose a Model Using Differences or Ratios

Look for a pattern in each table of values to determine which kind of model best describes the data.

a.

x	−2	−1	0	1	2
y	−8	−3	2	7	12

First differences: −8 −3 2 7 12

5 5 5 5

Since the first differences are all equal, the table of values represents a linear function.

b.

x	−1	0	1	2	3
y	8	4	2	1	0.5

First differences: 8 4 2 1 0.5

−4 −2 −1 −0.5

The first differences are not all equal. So, the table of values does not represent a linear function. Find the second differences and compare.

First differences: −4 −2 −1 −0.5

Second differences: 2 1 0.5

The second differences are not all equal. So, the table of values does not represent a quadratic function. Find the ratios of the y-values and compare.

8 4 2 1 0.5

Ratios: $\frac{4}{8} = \frac{1}{2}$ $\frac{2}{4} = \frac{1}{2}$ $\frac{1}{2}$ $\frac{0.5}{1} = \frac{1}{2}$

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.

Guided Practice

2A.

x	−3	−2	−1	0	1
y	−3	−7	−9	−9	−7

2B.

x	−2	−1	0	1	2
y	−18	−13	−8	−3	2

2 Write Equations Once you find the model that best describes the data, you can write an equation for the function. For a quadratic function in this lesson, the equation will have the form $y = ax^2$.

Example 3 Write an Equation

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

Step 1 Determine which model fits the data.

	32	18	8	2	0
First differences:	−14	−10	−6	−2	
Second differences:	4	4	4		

Since the second differences are equal, a quadratic function models the data.

Step 2 Write an equation for the function that models the data.
The equation has the form $y = ax^2$. Find the value of a by choosing one of the ordered pairs from the table of values. Let's use $(-1, 2)$.

$y = ax^2$ Equation for quadratic function

$2 = a(-1)^2$ $x = -1$ and $y = 2$

$2 = a$ An equation that models the data is $y = 2x^2$.

WatchOut!

Finding a In Example 3, the point (0, 0) cannot be used to find the value of a. You will have to divide each side by 0, giving you an undefined value for a.

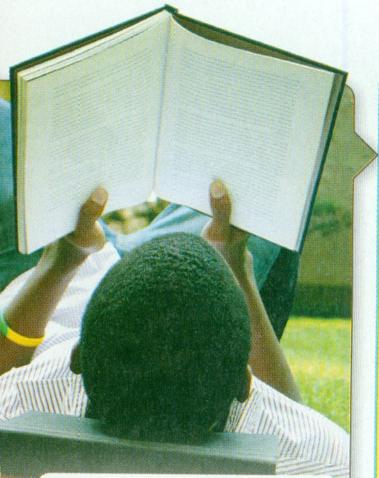
GuidedPractice

3A.

x	−2	−1	0	1	2
y	11	7	3	−1	−5

3B.

x	−3	−2	−1	0	1
y	0.375	0.75	1.5	3	6



Real-WorldLink

A poll by the National Education Association found that 87% of all teens polled found reading relaxing, 85% viewed reading as rewarding, and 79% found reading exciting.

Source: *American Demographics*

Real-World Example 4 Write an Equation for a Real-World Situation

BOOK CLUB The table shows the number of book club members for four consecutive years. Determine which model best represents the data. Then write a function that models the data.

Understand We need to find a model for the data, and then write a function.

Time (years)	0	1	2	3	4
Members	5	10	20	40	80

Plan Find a pattern using successive differences or ratios. Then use the general form of the equation to write a function.

Solve The constant ratio is 2. This is the value of the base. An exponential function of the form $y = ab^x$ models the data.

$y = ab^x$ Equation for exponential function

$5 = a(2)^0$ $x = 0$, $y = 5$, and $b = 2$

$5 = a$ The equation that models the data is $y = 5 \cdot 2^x$.

Check You used (0, 5) to write the function. Verify that every other ordered pair satisfies the equation.

GuidedPractice

4. ADVERTISING The table shows the cost of placing an ad in a newspaper. Determine a model that best represents the data and write a function that models the data.

No. of Lines	5	6	7	8
Total Cost ($)	14.50	16.60	18.70	20.80

DK Stock/David Deas/Collection Mix: Subjects/Getty Images

Check Your Understanding

= Step-by-Step Solutions begin on page R13.

Example 1 Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. (−2, 8), (−1, 5), (0, 2), (1, −1)

2. (−3, 7), (−2, 3), (−1, 1), (0, 1), (1, 3)

3. (−3, 8), (−2, 4), (−1, 2), (0, 1), (1, 0.5)

4. (0, 2), (1, 2.5), (2, 3), (3, 3.5)

Example 2 Look for a pattern in each table of values to determine which kind of model best describes the data.

5.

x	0	1	2	3	4
y	5	8	17	32	53

6.

x	−3	−2	−1	0
y	−6.75	−7.5	−8.25	−9

7.

x	−1	0	1	2	3
y	3	6	12	24	48

8.

x	3	4	5	6	7
y	−1.5	0	2.5	6	10.5

Example 3 Determine which kind of model best describes the data. Then write an equation for the function that models the data.

9.

x	−1	0	1	2	3
y	1	3	9	27	81

10.

x	−5	−4	−3	−2	−1
y	125	80	45	20	5

11.

x	−3	−2	−1	0	1
y	1	1.5	2	2.5	3

12.

x	−1	0	1	2
y	−1.25	−1	−0.75	−0.5

Example 4 **13. PLANTS** The table shows the height of a plant for four consecutive weeks. Determine which kind of function best models the height. Then write a function that models the data.

Week	0	1	2	3	4
Height (in.)	3	3.5	4	4.5	5

Practice and Problem Solving

Extra Practice is on page R9.

Example 1 Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

14. (−1, 1), (0, −2), (1, −3), (2, −2), (3, 1)

15. (1, 2.75), (2, 2.5), (3, 2.25), (4, 2)

16. (−3, 0.25), (−2, 0.5), (−1, 1), (0, 2)

17. (−3, −11), (−2, −5), (−1, −3), (0, −5)

18. (−2, 6), (−1, 1), (0, −4), (1, −9)

19. (−1, 8), (0, 2), (1, 0.5), (2, 0.125)

Examples 2–3 Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

20.

x	−3	−2	−1	0
y	−8.8	−8.6	−8.4	−8.2

21.

x	−2	−1	0	1	2
y	10	2.5	0	2.5	10

22.

x	−1	0	1	2	3
y	0.75	3	12	48	192

23.

x	−2	−1	0	1	2
y	0.008	0.04	0.2	1	5

24.

x	0	1	2	3	4
y	0	4.2	16.8	37.8	67.2

25.

x	−3	−2	−1	0	1
y	14.75	9.75	4.75	−0.25	−5.25

Example 4 **26. WEB SITES** A company tracked the number of visitors to its Web site over 4 days. Determine which kind of model best represents the number of visitors to the Web site with respect to time. Then write a function that models the data.

Day	0	1	2	3	4
Visitors (in thousands)	0	0.9	3.6	8.1	14.4

27. CALLING The cost of an international call depends on the length of the call. The table shows the cost for up to 6 minutes.

Length of call (min)	1	2	3	4	5	6
Cost ($)	0.12	0.24	0.36	0.48	0.60	0.72

a. Graph the data and determine which kind of function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine how much a 10-minute call would cost.

28. DEPRECIATION The value of a car depreciates over time. The table shows the value of a car over a period of time.

Year	0	1	2	3	4
Value ($)	18,500	15,910	13,682.60	11,767.04	10,119.65

a. Determine which kind of function best models the data.

b. Write an equation for the function that models the data.

c. Use your equation to determine how much the car is worth after 7 years.

29. BACTERIA A scientist estimates that a bacteria culture with an initial population of 12 will triple every hour.

a. Make a table to show the bacteria population for the first 4 hours.

b. Which kind of model best represents the data?

c. Write a function that models the data.

d. How many bacteria will there be after 8 hours?

30. PRINTING A printing company charges the fees shown to print flyers. Write a function that models the total cost of the flyers, and determine how much 30 flyers would cost.

Quick 2 U Printing
Set Up Fee $25
15¢ each flyer

H.O.T. Problems Use Higher-Order Thinking Skills

31. CHALLENGE Write a function that has constant second differences, first differences that are not constant, a y-intercept of -5, and passes through the point at $(2, 3)$.

32. CCSS ARGUMENTS What type of function will have constant third differences but not constant second differences? Explain.

33. OPEN ENDED Write a linear function that has a constant first difference of 4.

34. PROOF Write a paragraph proof to show that linear functions grow by equal differences over equal intervals, and exponential functions grow by equal factors over equal intervals. (*Hint:* Let $y = ax$ represent a linear function and let $y = a^x$ represent an exponential function.)

35. WRITING IN MATH How can you determine whether a given set of data should be modeled by a *linear* function, a *quadratic* function, or an *exponential* function?

Standardized Test Practice

36. SHORT RESPONSE Write an equation that models the data in the table.

x	0	1	2	3	4
y	3	6	12	24	48

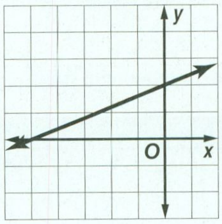
37. What is the equation of the line below?

A $y = \frac{2}{5}x + 2$

B $y = \frac{2}{5}x - 2$

C $y = \frac{5}{2}x + 2$

D $y = \frac{5}{2}x - 2$



38. The point $(r, -4)$ lies on a line with an equation of $2x + 3y = -8$. Find the value of r.

F -10

G 0

H 2

J 8

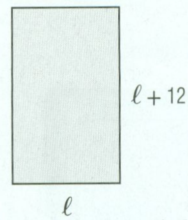
39. GEOMETRY The rectangle has an area of 220 square feet. Find the length ℓ.

A 8 feet

B 10 feet

C 22 feet

D 34 feet



Spiral Review

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

40. $6x^2 - 3x - 30 = 0$

41. $4x^2 + 18x = 10$

42. $2x^2 + 6x = 7$

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary. (Lesson 9-4)

43. $x^2 = 25$

44. $x^2 + 6x + 9 = 16$

45. $x^2 - 14x + 49 = 15$

46. INVESTMENTS Joey's investment of $2500 has been decreasing in value at a rate of 1.5% each year. What will his investment be worth in 5 years? (Lesson 7-7)

Write an equation for the nth term of each geometric sequence, and find the seventh term of each sequence. (Lesson 7-6)

47. $1, 2, 4, 8, \ldots$

48. $-20, -10, -5, \ldots$

49. $4, -12, 36, \ldots$

50. $99, -33, 11, \ldots$

51. $22, 44, 88, \ldots$

52. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \ldots$

53. CANOE RENTAL To rent a canoe, you must pay a daily rate plus $10 per hour. Ilia and her friends rented a canoe for 3 hours and paid $45. Write a linear equation for the cost C of renting the canoe for h hours, and determine how much it cost to rent the canoe for 8 hours. (Lesson 4-2)

Determine whether each equation is a linear equation. If so, write the equation in standard form. (Lesson 3-1)

54. $3x = 5y$

55. $6 - y = 2x$

56. $6xy + 3x = 4$

57. $y + 5 = 0$

58. $7y = 2x + 5x$

59. $y = 4x^2 - 1$

Skills Review

Evaluate each expression if $x = -3$, $y = -1$, and $z = 4$.

60. $|x - 4|$

61. $|2y + 1|$

62. $|4 - z|$

63. $\left|\frac{1}{2}x + 2\right|$

64. $|12 - 4z|$

65. $|2y - 3| - 6$

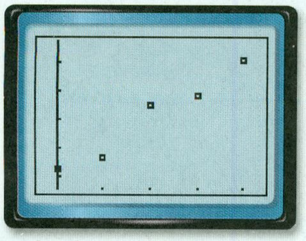
EXTEND

9-6 Graphing Technology Lab: Curve Fitting

If there is a constant increase or decrease in data values, there is a linear trend. If the values are increasing or decreasing more and more rapidly, there may be a quadratic or exponential trend.

CCSS Common Core State Standards
Content Standards
F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Linear Trend



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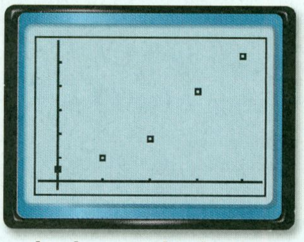
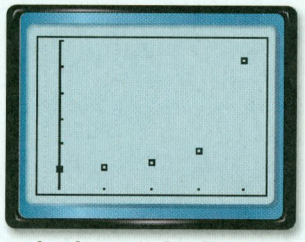
Quadratic Trend



[0, 5] scl: 1 by [0, 6] scl: 1

Exponential Trend



[0, 5] scl: 1 by [0, 6] scl: 1

With a graphing calculator, you can find the appropriate regression equation.

Activity

CHARTER AIRLINE The table shows the average monthly number of flights made each year by a charter airline that was founded in 2000.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Flights	17	20	24	28	33	38	44	50

Step 1 Make a scatter plot.

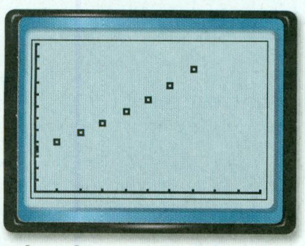
- Enter the number of years since 2000 in **L1** and the number of flights in **L2**.

KEYSTROKES: *Review entering a list on page 255.*

- Use **STAT PLOT** to graph the scatter plot.

KEYSTROKES: *Review statistical plots on page 256.*

Use ZOOM 9 to graph.



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From the scatter plot we can see that the data may have either a quadratic trend or an exponential trend.

Step 2 Find the regression equation.

We will check both trends by examining their regression equations.

- Select **DiagnosticOn** from the **CATALOG**.
- Select **QuadReg** on the **STAT** menu.

KEYSTROKES: STAT ▶ 5 ENTER ENTER

The equation is in the form $y = ax^2 + bx + c$.

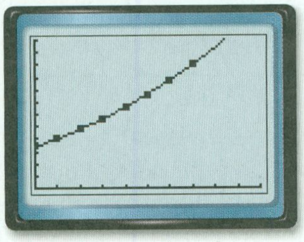
The equation is about $y = 0.25x^2 + 3x + 17$.

R^2 is the **coefficient of determination**. The closer R^2 is to 1, the better the model. To acquire the exponential equation select **ExpReg** on the **STAT** menu. To choose a quadratic or exponential model, fit both and use the one with the R^2 value closer to 1.

Step 3 Graph the quadratic regression equation.

- Copy the equation to the **Y=** list and graph.

KEYSTROKES: [Y=] [VARS] 5 [▶] [▶] 1 [ZOOM] 9

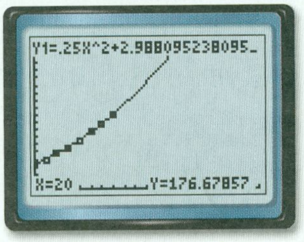


[0, 10] scl: 1 by [0, 60] scl: 5

Step 4 Predict using the equation.

If this trend continues, we can use the graph of our equation to predict the monthly number of flights the airline will make in a specific year. Let's check the year 2020. First adjust the window.

KEYSTROKES: [2nd] [CALC] 1 At $x =$ enter 20 [ENTER].



[0, 25] scl: 1 by [0, 200] scl: 5

There will be approximately 177 flights per month if this trend continues.

Exercises

Plot each set of data points. Determine whether to use a *linear, quadratic* or *exponential* regression equation. State the coefficient of determination.

1.

x	y
1	30
2	40
3	50
4	55
5	50
6	40

2.

x	y
0.0	12.1
0.1	9.6
0.2	6.3
0.3	5.5
0.4	4.8
0.5	1.9

3.

x	y
0	1.1
2	3.3
4	2.9
6	5.6
8	11.9
10	19.8

4.

x	y
1	1.67
5	2.59
9	4.37
13	6.12
17	5.48
21	3.12

5. BAKING **Alyssa baked a cake and is waiting for it to cool so she can ice it. The table shows the temperature of the cake every 5 minutes after Alyssa took it out of the oven.**

Time (min)	Temperature (°F)
0	350
5	244
10	178
15	137
20	112
25	96
30	89

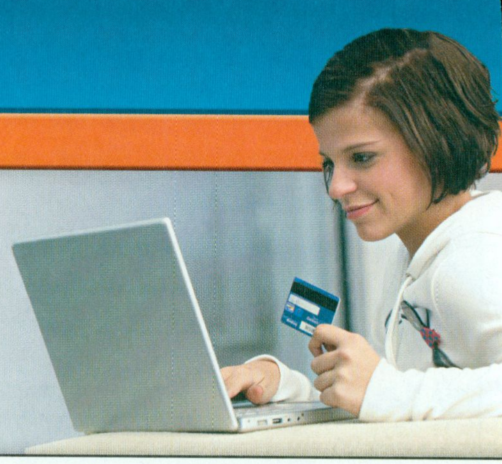
a. Make a scatter plot of the data.

b. Which regression equation has an R^2 value closest to 1? Is this the equation that best fits the context of the problem? Explain your reasoning.

c. Find an appropriate regression equation, and state the coefficient of determination. What is the domain and range?

d. Alyssa will ice the cake when it reaches room temperature (70°F). Use the regression equation to predict when she can ice her cake.

LESSON 9-7 Special Functions



:: Then	:: Now	:: Why?
● You identified and graphed linear, exponential, and quadratic functions.	● 1 Identify and graph step functions. 2 Identify and graph absolute value and piecewise-defined functions.	● Kim is ordering books online. The site charges for shipping based on the amount of the order. If the order is less than \$10, shipping costs \$3. If the order is at least \$10 but less than \$20, it will cost \$5 to ship it.

NewVocabulary
step function
piecewise-linear function
greatest integer function
absolute value function
piecewise-defined function

Common Core State Standards

Content Standards

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Mathematical Practices

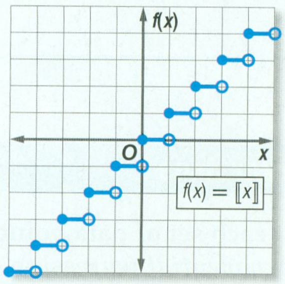
4 Model with mathematics.

1 Step Functions

The graph of a **step function** is a series of line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function**. One example of a step function is the **greatest integer function**, written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x. For example, $[\![6.8]\!] = 6$ because 6 is the greatest integer that is not greater than 6.8.

KeyConcept Greatest Integer Function

Parent function:	$f(x) = [\![x]\!]$
Type of graph:	disjointed line segments
Domain:	all real numbers
Range:	all integers

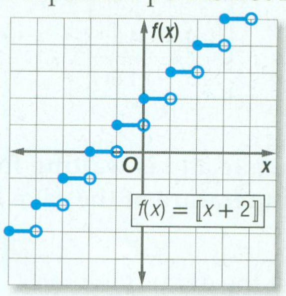


Example 1 Greatest Integer Function

Graph $f(x) = [\![x + 2]\!]$. State the domain and range.

First, make a table. Select a few values between integers. On the graph, dots represent included points. Circles represent points not included.

x	$x + 2$	$[\![x + 2]\!]$
0	2	2
0.25	2.25	2
0.5	2.5	2
1	3	3
1.25	3.25	3
1.5	3.5	3
2	4	4
2.25	4.25	4



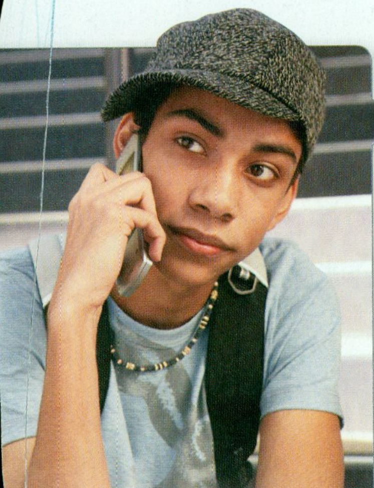
Note that this is the graph of $f(x) = [\![x]\!]$ shifted 2 units to the left.

Because the dots and circles overlap, the domain is all real numbers. The range is all integers. Notice that the graph has no symmetry and no maximum or minimum values. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ decreases.

GuidedPractice

1. Graph $g(x) = 2[\![x]\!]$. State the domain and range.

Step functions can be used to represent many real-world situations involving money.



Real-WorldLink

North Americans are the most likely to have cell phones with 93.2% of the population owning phones.

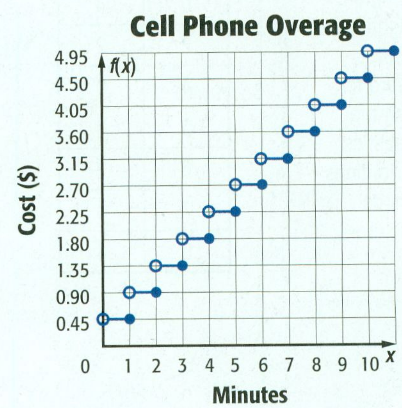
Source: IT Facts

Real-World Example 2 Step Function

CELL PHONE PLANS Cell phone companies charge by the minute, not by the second. A cell phone company charges $0.45 per minute or any fraction thereof for exceeding the number of minutes allotted on each plan. Draw a graph that represents this situation.

The total cost for the extra minutes will be a multiple of $0.45, and the graph will be a step function. If the time is greater than 0 but less than or equal to 1 minute, the charge will be $0.45. If the time is greater than 2 but is less than or equal to 3 minutes, you will be charged for 3 minutes or $1.35.

x	$f(x)$
$0 < x \leq 1$	0.45
$1 < x \leq 2$	0.90
$2 < x \leq 3$	1.35
$3 < x \leq 4$	1.80
$4 < x \leq 5$	2.25
$5 < x \leq 6$	2.70
$6 < x \leq 7$	3.15



GuidedPractice

2. **PARKING** A garage charges $4 for the first hour and $1 for each additional hour. Draw a graph that represents this situation.

ReviewVocabulary

absolute value the distance a number is from zero on a number line; written $|n|$

2 Absolute Value Functions

Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value parent function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

KeyConcept Absolute Value Function

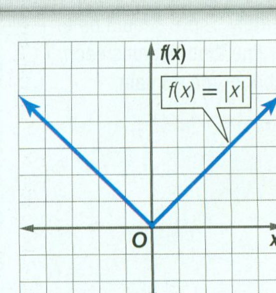
Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

Type of graph: V-shaped

Domain: all real numbers

Range: all nonnegative real numbers



The absolute value function is called a **piecewise-defined function** because it is defined using two or more expressions.

Yellow Dog Productions/Photodisc/Getty Images

Math HistoryLink

Florence Nightingale David (1909–1993) A renowned statistician born in Ivington, England, Florence Nightingale David received the first Elizabeth L. Scott Award for her "efforts in opening the door to women in statistics; … for research contributions to … statistical methods …; and her spirit as a lecturer and as a role model."

Example 3 Absolute Value Function

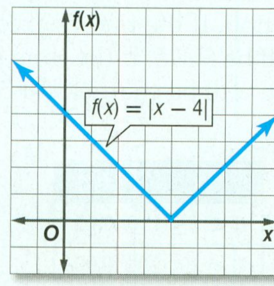
Graph $f(x) = |x - 4|$. State the domain and range.

Since $f(x)$ cannot be negative, the minimum point of the graph is where $f(x) = 0$.

$f(x) = \|x - 4\|$	Original function
$0 = x - 4$	Replace $f(x)$ with 0 and $\|x - 4\|$ with $x - 4$.
$4 = x$	Add 4 to each side.

Next make a table of values. Include values for $x > 4$ and $x < 4$.

$f(x) = \|x - 4\|$	
x	$f(x)$
−2	6
0	4
2	2
4	0
5	1
6	2
7	3
8	4



The domain is all real numbers. The range is all real numbers greater than or equal to 0. Note that this is the graph of $f(x) = |x|$ shifted 4 units to the right. Notice that the graph is symmetric about the line $x = 4$, and the minimum value of the function is 0 at $x = 4$. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ increases.

GuidedPractice

3. Graph $f(x) = |2x + 1|$. State the domain and range.

Not all piecewise-defined functions are absolute value functions. Step functions are also piecewise-defined functions. In fact, all piecewise-linear functions are piecewise-defined.

StudyTip

Piecewise Functions To graph a piecewise-defined function, graph each "piece" separately. There should be a dot or line that contains each member of the domain.

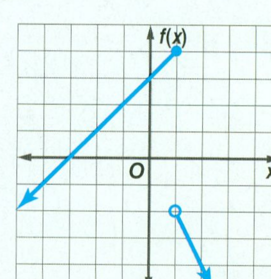
Example 4 Piecewise-Defined Function

Graph $f(x) = \begin{cases} -2x \text{ if } x > 1 \\ x + 3 \text{ if } x \leq 1 \end{cases}$. State the domain and range.

Graph the first expression. Create a table of values for when $x > 1$, $f(x) = -2x$ and draw the graph. Since x is not equal to 1, place a circle at $(1, -2)$.

Next, graph the second expression. Create a table of values for when $x \leq 1$, $f(x) = x + 3$ and draw the graph. If $x = 1$, then $f(x) = 4$; place a dot at $(1, 4)$.

The domain is all real numbers. The range is $y \leq 4$.



GuidedPractice

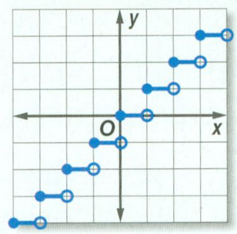
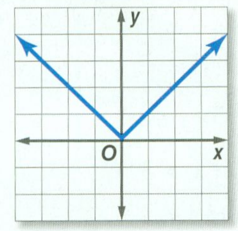
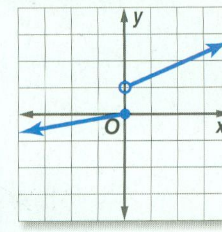
4. Graph $f(x) = \begin{cases} 2x + 1 \text{ if } x > 0 \\ 3 \text{ if } x \leq 0 \end{cases}$. State the domain and range.

Courtesy of American Statistical Association

StudyTip

Nonlinear Functions Like exponential and quadratic functions, the greatest integer function, absolute value function, and piecewise defined functions are nonlinear functions.

ConceptSummary Special Functions

Step Function	Absolute Value Function	Piecewise-Defined Function
		

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Example 1 Graph each function. State the domain and range.

1. $f(x) = \frac{1}{2}[\![x]\!]$

2. $g(x) = -[\![x]\!]$

3. $h(x) = [\![2x]\!]$

Example 2

4. **SHIPPING** Elan is ordering a gift for his dad online. The table shows the shipping rates. Graph the step function.

Order Total ($)	Shipping Cost ($)
0–15	3.99
15.01–30	5.99
30.01–50	6.99
50.01–75	7.99
75.01–100	8.99
Over $100	9.99

Examples 3–4 Graph each function. State the domain and range.

5. $f(x) = |x - 3|$

6. $g(x) = |2x + 4|$

7. $f(x) = \begin{cases} 2x - 1 \text{ if } x > -1 \\ -x \text{ if } x \leq -1 \end{cases}$

8. $g(x) = \begin{cases} -3x - 2 \text{ if } x > -2 \\ -x + 1 \text{ if } x \leq -2 \end{cases}$

Practice and Problem Solving

Extra Practice is on page R9.

Example 1 Graph each function. State the domain and range.

9. $f(x) = 3[\![x]\!]$

10. $f(x) = [\![-x]\!]$

11. $g(x) = -2[\![x]\!]$

12. $g(x) = [\![x]\!] + 3$

13. $h(x) = [\![x]\!] - 1$

14. $h(x) = \frac{1}{2}[\![x]\!] + 1$

Example 2

15. **CAB FARES** Lauren wants to take a taxi from a hotel to a friend's house. The rate is $3 plus $1.50 per mile after the first mile. Every fraction of a mile is rounded up to the next mile.
   a. Draw a graph to represent the cost of using a taxi cab.
   b. What is the cost if the trip is 8.5 miles long?

16. **CCSS MODELING** The United States Postal Service increases the rate of postage periodically. The table shows the cost to mail a letter weighing 1 ounce or less from 1995 through 2009. Draw a step graph to represent the data.

Year	1995	1999	2001	2002	2006	2007	2008	2009
Cost ($)	0.32	0.33	0.34	0.37	0.39	0.41	0.42	0.44

Examples 3–4 **Graph each function. State the domain and range.**

17. $f(x) = |2x - 1|$

18. $f(x) = |x + 5|$

19. $g(x) = |-3x - 5|$

20. $g(x) = |-x - 3|$

21. $f(x) = \left|\frac{1}{2}x - 2\right|$

22. $f(x) = \left|\frac{1}{3}x + 2\right|$

23. $g(x) = |x + 2| + 3$

24. $g(x) = |2x - 3| + 1$

25. $f(x) = \begin{cases} \frac{1}{2}x - 1 \text{ if } x > 3 \\ -2x + 3 \text{ if } x \leq 3 \end{cases}$

26. $f(x) = \begin{cases} 2x - 5 \text{ if } x > 1 \\ 4x - 3 \text{ if } x \leq 1 \end{cases}$

27. $f(x) = \begin{cases} 2x + 3 \text{ if } x \geq -3 \\ -\frac{1}{3}x + 1 \text{ if } x < -3 \end{cases}$

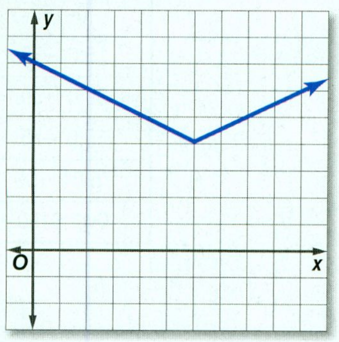
28. $f(x) = \begin{cases} 3x + 4 \text{ if } x \geq 1 \\ x + 3 \text{ if } x < 1 \end{cases}$

29. $f(x) = \begin{cases} 3x + 2 \text{ if } x > -1 \\ -\frac{1}{2}x - 3 \text{ if } x \leq -1 \end{cases}$

30. $f(x) = \begin{cases} 2x + 1 \text{ if } x < -2 \\ -3x - 1 \text{ if } x \geq -2 \end{cases}$

Determine the domain and range of each function.

31



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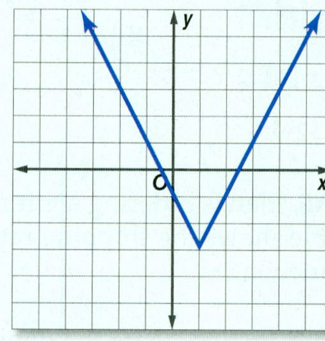

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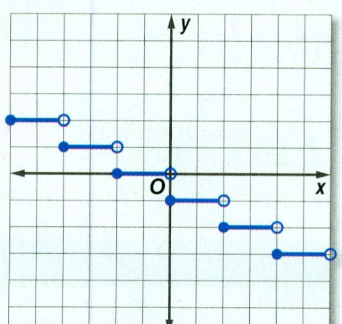

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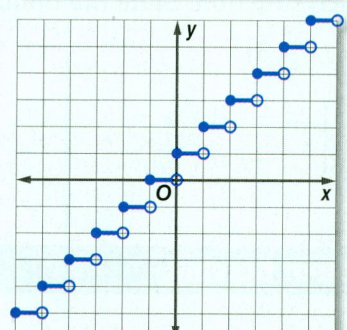

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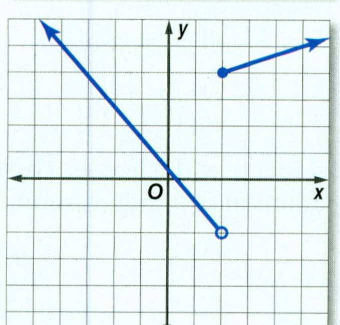

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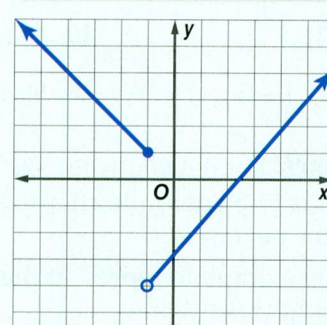


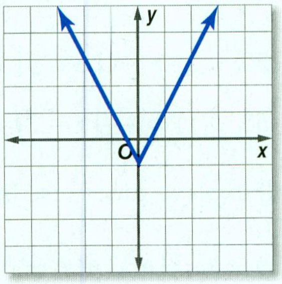
37. **BOATING** According to Boat Minnesota, the maximum number of people that can safely ride in a boat is determined by the boat's length and width. The table shows some guidelines for the length of a boat that is 6 feet wide. Graph this relation.

Length of Boat (ft)	18–19	20–22	23–24
Number of People	7	8	9

For Exercises 38–41, match each graph to one of the following equations.

A	B	C	D
$y = 2x - 1$	$y = [\![2x]\!] - 1$	$y = \lvert 2x \rvert - 1$	$y = \begin{cases} 2x + 1 \text{ if } x > 0 \\ -2x + 1 \text{ if } x \leq 0 \end{cases}$

38.



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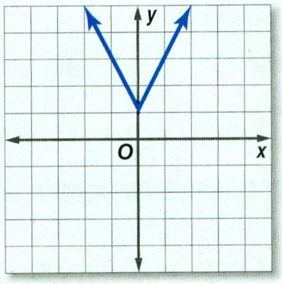

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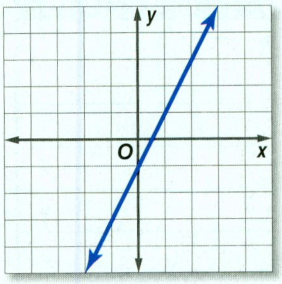

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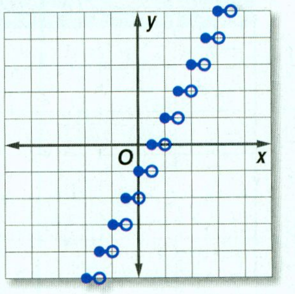


42. CAR LEASE As part of Marcus' leasing agreement, he will be charged \$0.20 per mile for each mile over 12,000. Any fraction of a mile is rounded up to the next mile. Make a step graph to represent the cost of going over the mileage.

43. BASEBALL A baseball team is ordering T-shirts with the team logo on the front and the players' names on the back. A graphic design store charges \$10 to set up the artwork plus \$10 per shirt, \$4 each for the team logo, and \$2 to print the last name for an order of 10 shirts or less. For orders of 11–20 shirts, a 5% discount is given. For orders of more than 20 shirts, a 10% discount is given.

a. Organize the information into a table. Include a column showing the total order price for each size order.

b. Write an equation representing the total price for an order of x shirts.

c. Graph the piecewise relation.

44. Consider the function $f(x) = \lvert 2x + 3 \rvert$.

a. Make a table of values where x is all integers from -5 to 5, inclusive.

b. Plot the points on a coordinate grid.

c. Graph the function.

45. Consider the function $f(x) = \lvert 2x \rvert + 3$.

a. Make a table of values where x is all integers from -5 to 5, inclusive.

b. Plot the points on a coordinate grid.

c. Graph the function.

d. Describe how this graph is different from the graph in Exercise 44.

46. **DANCE** A local studio must have at least 5 students enrolled in a class, or else the class will be canceled. Once 10 students are enrolled, a second class is started. Draw a graph for this situation.

47. **THEATERS** A certain theater will not have a show unless it has sold 50 tickets for that show. Once the capacity of 250 seats are sold, the theater begins selling tickets for the next show. Draw a graph that describes this situation.

Graph each function.

48. $f(x) = \frac{1}{2}|x| + 2$

49. $g(x) = \frac{1}{3}|x| + 4$

50. $h(x) = -2|x - 3| + 2$

51. $f(x) = -4|x + 2| - 3$

52. $g(x) = -\frac{2}{3}|x + 6| - 1$

53. $h(x) = -\frac{3}{4}|x - 8| + 1$

54. **MULTIPLE REPRESENTATIONS** In this problem, you will explore piecewise linear functions.

a. **Tabular** Copy and complete the table of values for $f(x) = |[\![x]\!]|$ and $g(x) = [\![|x|]\!]$.

x	$[\![x]\!]$	$f(x) = \lvert[\![x]\!]\rvert$	$\lvert x\rvert$	$g(x) = [\![\lvert x\rvert]\!]$
−3	−3	3	3	3
−2.5				
−2				
0				
0.5				
1				
1.5				

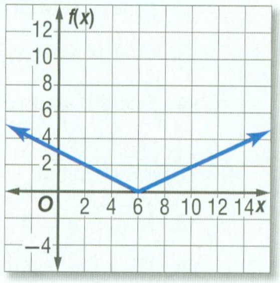
b. **Graphical** Graph each function on a coordinate plane.

c. **Analytical** Compare and contrast the graphs of $f(x)$ and $g(x)$.

H.O.T. Problems Use Higher-Order Thinking Skills

55. **REASONING** Does the piecewise relation $y = \begin{cases} -2x + 4 \text{ if } x \geq 2 \\ -\frac{1}{2}x - 1 \text{ if } x \leq 4 \end{cases}$ represent a function? Why or why not?

CCSS SENSE-MAKING **Refer to the graph.**



56. Write an absolute value function that represents the graph.

57. Write a piecewise function to represent the graph.

58. What are the domain and range?

59. **WRITING IN MATH** Compare and contrast the graphs of absolute value, step, and piecewise-defined functions with the graphs of quadratic and exponential functions. Discuss the domains, ranges, maxima, minima, and symmetry.

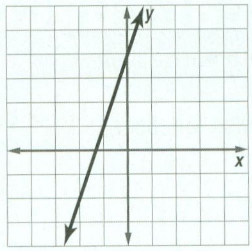
60. **CHALLENGE** A bicyclist travels up and down a hill with a vertical cross section that can be modeled by $y = -\frac{1}{4}|x - 400| + 100$, where x and y are measured in feet.

a. If $0 \leq x \leq 800$, find the slope for the uphill portion of the trip and downhill portion of the trip.

b. Graph this function. What are the domain and range?

Standardized Test Practice

61. Which equation represents a line that is perpendicular to the graph and passes through the point at (2, 0)?



A $y = 3x - 6$

B $y = -3x + 6$

C $y = -\frac{1}{3}x + \frac{2}{3}$

D $y = \frac{1}{3}x - \frac{2}{3}$

62. A giant tortoise travels at a rate of 0.17 mile per hour. Which equation models the time t it would take the giant tortoise to travel 0.8 mile?

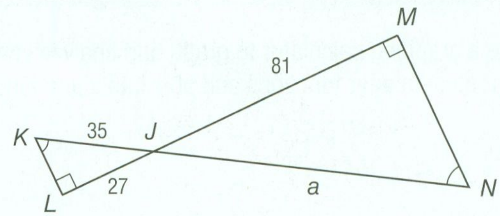
F $t = \frac{0.8}{0.17}$

H $t = \frac{0.17}{0.8}$

G $t = (0.17)(0.8)$

J $0.8 = \frac{0.17}{t}$

63. GEOMETRY If $\triangle JKL$ is similar to $\triangle JNM$ what is the value a?



A 62.5

B 105

C 125

D 155.5

64. GRIDDED RESPONSE What is the difference in the value of $2.1(x + 3.2)$, when $x = 5$ and when $x = 3$?

Spiral Review

Look for a pattern in each table of values to determine which model best describes the data. (Lesson 9-6)

65.

x	0	1	2	3	4
y	1	3	5	7	9

66.

x	−2	−1	0	1	2
y	5	2	1	2	5

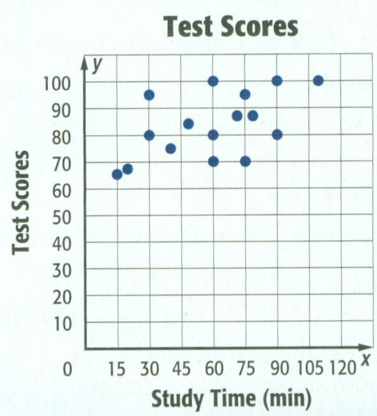
67.

x	−1	0	1	2	3
y	1	2	4	8	16

68.

x	5	6	7	8	9
y	−2.5	−1.5	1.5	6.5	13.5

69. TESTS Determine whether the graph at the right shows a *positive*, *negative*, or *no* correlation. If there is a correlation, describe its meaning. (Lesson 4-5)



Suppose y varies directly as x. (Lesson 3-4)

70. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

71. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

72. If $y = 2.6$ when $x = 0.25$, find y when $x = 1.125$.

73. If $y = 6$ when $x = 0.6$, find x when $y = 12$.

Skills Review

Evaluate each expression. If necessary, round to the nearest hundredth.

74. $\sqrt{9}$

75. $\sqrt{12}$

76. $\sqrt{4.5}$

77. $3\sqrt{16}$

78. $2\sqrt{10}$

79. $\sqrt{5} - 2$

Graphing Technology Lab
Piecewise-Linear Functions

You can use a graphing calculator to graph and analyze various piecewise functions, including greatest integer functions and absolute value functions.

CCSS Common Core State Standards
Content Standards
F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

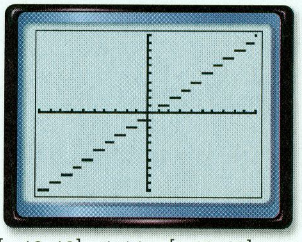
Activity 1 Greatest Integer Functions

Graph $f(x) = [\![x]\!]$ in the standard viewing window.

The calculator may need to be changed to dot mode for the function to graph correctly. Press MODE then use the arrow and ENTER keys to select **DOT**.

Enter the equation in the **Y=** list. Then graph the equation.

KEYSTROKES: Y= MATH ▶ 5 X,T,θ,n) ZOOM 6



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1A. How does the graph of $f(x) = [\![x]\!]$ compare to the graph of $f(x) = x$?

1B. What are the domain and range of the function $f(x) = [\![x]\!]$? Explain.

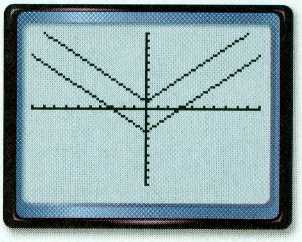
The graphs of piecewise functions are affected by changes in parameters.

Activity 2 Absolute Value Functions

Graph $y = |x| - 3$ and $y = |x| + 1$ in the standard viewing window.

Enter the equations in the **Y=** list. Then graph.

KEYSTROKES: Y= MATH ▶ 1 X,T,θ,n) − 3 ENTER MATH ▶ 1 X,T,θ,n) + 1 ZOOM 6



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2A. Compare and contrast the graphs to the graph of $y = |x|$.

2B. How does the value of k affect the graph of $y = |x| + k$?

Analyze the Results

1. A parking garage charges $4 for every hour or fraction of an hour. Is this situation modeled by a *linear* function or a *step* function? Explain your reasoning.
2. A maintenance technician is testing an elevator system. The technician starts the elevator at the fifth floor. It is sent to the ground floor, then back to the fifth floor. Assume the elevator travels at a constant rate. Should the height of the elevator be modeled by a step function or an absolute value function? Explain.

Because the points on a graph are solutions of its equation, the x-coordinates of points where $y = f(x)$ and $y = g(x)$ intersect are solutions of $f(x) = g(x)$. For example, the solution of $5x - 2 = |x|$ is the intersection of the graphs of $y = 5x - 2$ and $y = |x|$. Write each equation as a system of equations, and then use a graph or a table to solve it.

3. $5x - 2 = |x|$
4. $2|x - 2| = x - 1$
5. $|4x + 2| = -|x| + 3$

Study Guide and Review

Study Guide

KeyConcepts

Graphing Quadratic Functions (Lesson 9-1)

- A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.
- The axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Solving Quadratic Equations (Lessons 9-2, 9-4, and 9-5)

- Quadratic equations can be solved by graphing. The solutions are the x-intercepts or zeros of the related quadratic function.
- Quadratic equations can be solved by completing the square. To complete the square for $x^2 + bx$, find $\frac{1}{2}$ of b, square this result, and then add the result to $x^2 + bx$.
- Quadratic equations can be solved by using the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Transformations of Quadratic Functions (Lesson 9-3)

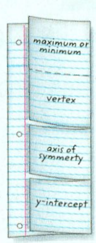
- $f(x) = x^2 + c$ translates the graph up or down.
- $f(x) = ax^2$ compresses or expands the graph vertically.

Special Functions (Lesson 9-7)

- The greatest integer function is written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x.
- The absolute value function is written as $f(x) = |x|$, where $f(x)$ is the distance from x to 0 on a number line.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

absolute value function (p. 599)
axis of symmetry (p. 543)
completing the square (p. 574)
dilation (p. 566)
discriminant (p. 586)
double root (p. 556)
greatest integer function (p. 598)
maximum (p. 543)
minimum (p. 543)
parabola (p. 543)
piecewise-defined function (p. 599)
piecewise-linear function (p. 598)
Quadratic Formula (p. 583)
quadratic function (p. 543)
reflection (p. 566)
standard form (p. 543)
step function (p. 598)
transformation (p. 564)
translation (p. 564)
vertex (p. 543)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The axis of symmetry of a quadratic function can be found by using the equation $x = -\frac{b}{2a}$.
2. The vertex is the maximum or minimum point of a parabola.
3. The graph of a quadratic function is a straight line.
4. The graph of a quadratic function has a maximum if the coefficient of the x^2 is positive.
5. A quadratic equation with a graph that has two x-intercepts has one real root.
6. The expression $b^2 - 4ac$ is called the discriminant.
7. A function that is defined differently for different parts of its domain is called a piecewise-defined function.
8. The range of the greatest integer function is the set of all real numbers.
9. The solutions of a quadratic equation are called roots.
10. The graph of the parent function is translated down to form the graph of $f(x) = x^2 + 5$.

Lesson-by-Lesson Review

9-1 Graphing Quadratic Functions

Consider each equation.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

11. $y = x^2 - 4x + 4$

12. $y = -x^2 + 3x$

13. $y = x^2 - 2x - 3$

14. $y = -x^2 + 2$.

15. BASEBALL A baseball is thrown with an upward velocity of 32 feet per second. The equation $h = -16t^2 + 32t$ gives the height of the ball t seconds after it is thrown.

a. Determine whether the function has a *maximum* or *minimum* value.

b. State the maximum or minimum value.

c. State a reasonable domain and range for this situation.

Example 1

Consider $f(x) = x^2 + 6x + 5$.

a. Determine whether the function has a *maximum* or *minimum* value.

For $f(x) = x^2 + 6x + 5$, $a = 1$, $b = 6$, and $c = 5$.

Because a is positive, the graph opens up, so the function has a minimum value.

b. State the *maximum* or *minimum* value of the function.

The minimum value is the y-coordinate of the vertex.

The x-coordinate of the vertex is $\frac{-b}{2a}$ or $\frac{-6}{2(1)}$ or -3.

$f(x) = x^2 + 6x + 5$ — Original function

$f(-3) = (-3)^2 + 6(-3) + 5$ — $x = -3$

$f(-3) = -4$ — Simplify.

The minimum value is -4.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or $\{y \mid y \geq -4\}$.

9-2 Solving Quadratic Equations by Graphing

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

16. $x^2 - 3x - 4 = 0$

17. $-x^2 + 6x - 9 = 0$

18. $x^2 - x - 12 = 0$

19. $x^2 + 4x - 3 = 0$

20. $x^2 - 10x = -21$

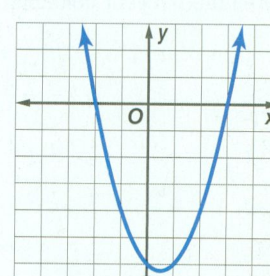
21. $6x^2 - 13x = 15$

22. NUMBER THEORY Find two numbers that have a sum of 2 and a product of -15.

Example 2

Solve $x^2 - x - 6 = 0$ by graphing.

Graph the related function $f(x) = x^2 - x - 6$.



The x-intercepts of the graph appear to be at -2 and 3, so the solutions are -2 and 3.

9-3 Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

23. $f(x) = x^2 + 8$

24. $f(x) = x^2 - 3$

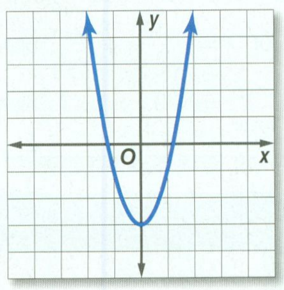
25. $f(x) = 2x^2$

26. $f(x) = 4x^2 - 18$

27. $f(x) = \frac{1}{3}x^2$

28. $f(x) = \frac{1}{4}x^2$

29. Write an equation for the function shown in the graph.



30. PHYSICS A ball is dropped off a cliff that is 100 feet high. The function $h = -16t^2 + 100$ models the height h of the ball after t seconds. Compare the graph of this function to the graph of $h = t^2$.

Example 3

Describe how the graph of $f(x) = x^2 - 2$ is related to the graph of $f(x) = x^2$.

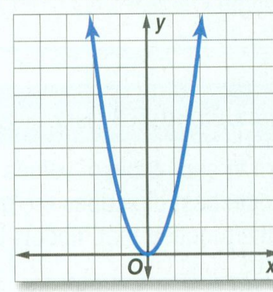
The graph of $f(x) = x^2 + c$ represents a translation up or down of the parent graph.

Since $c = -2$, the translation is down.

So, the graph is translated down 2 units from the parent function.

Example 4

Write an equation for the function shown in the graph.



Since the graph opens upward, the leading coefficient must be positive. The parabola has not been translated up or down, so $c = 0$. Since the graph is stretched vertically, it must be of the form of $f(x) = ax^2$ where $a > 1$. The equation for the function is $y = 2x^2$.

9-4 Solving Quadratic Equations by Completing the Square

Solve each equation by completing the square. Round to the nearest tenth if necessary.

31. $x^2 + 6x + 9 = 16$

32. $-a^2 - 10a + 25 = 25$

33. $y^2 - 8y + 16 = 36$

34. $y^2 - 6y + 2 = 0$

35. $n^2 - 7n = 5$

36. $-3x^2 + 4 = 0$

37. NUMBER THEORY Find two numbers that have a sum of -2 and a product of -48.

Example 5

Solve $x^2 - 16x + 32 = 0$ by completing the square. Round to the nearest tenth if necessary.

Isolate the x^2- and x-terms. Then complete the square and solve.

$x^2 - 16x + 32 = 0$	Original equation
$x^2 - 16x = -32$	Isolate the x^2- and x-terms.
$x^2 - 16x + 64 = -32 + 64$	Complete the square.
$(x - 8)^2 = 32$	Factor.
$x - 8 = \pm\sqrt{32}$	Take the square root.
$x = 8 \pm\sqrt{32}$	Add 8 to each side.
$x = 8 \pm 4\sqrt{2}$	Simplify.

The solutions are about 2.3 and 13.7.

Study Guide and Review *Continued*

9-5 Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

38. $x^2 - 8x = 20$

39. $21x^2 + 5x - 7 = 0$

40. $d^2 - 5d + 6 = 0$

41. $2f^2 + 7f - 15 = 0$

42. $2h^2 + 8h + 3 = 3$

43. $4x^2 + 4x = 15$

44. **GEOMETRY** The area of a square can be quadrupled by increasing the side length and width by 4 inches. What is the side length?

Example 6

Solve $x^2 + 10x + 9 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-10 \pm \sqrt{10^2 - 4(1)(9)}}{2(1)}$$ $a = 1, b = 10, c = 9$

$$= \frac{-10 \pm \sqrt{64}}{2}$$ Simplify.

$$x = \frac{-10 + 8}{2} \quad \text{or} \quad x = \frac{-10 - 8}{2}$$ Separate the solutions.

$$= -1 \qquad\qquad = -9$$ Simplify.

9-6 Analyzing Functions with Successive Differences

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

45.

x	0	1	2	3	4
y	0	3	12	27	48

46.

x	0	1	2	3	4
y	1	2	4	8	16

47.

x	0	1	2	3	4
y	0	−1	−4	−9	−16

Example 7

Determine the model that best describes the data. Then write an equation for the function that models the data.

x	0	1	2	3	4
y	3	4	5	6	7

Step 1 First differences: 3 4 5 6 7 → 1 1 1 1

A linear function models the data.

Step 2 The slope is 1 and the y-intercept is 3, so the equation is $y = x + 3$.

9-7 Special Functions

Graph each function. State the domain and range.

48. $f(x) = [\![x]\!]$

49. $f(x) = [\![2x]\!]$

50. $f(x) = |x|$

51. $f(x) = |2x - 2|$

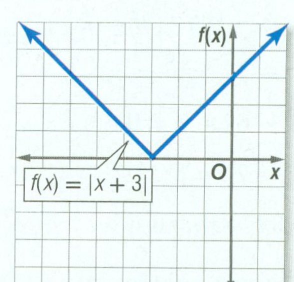
52. $f(x) = \begin{cases} x - 2 \text{ if } x < 1 \\ 3x \text{ if } x \geq 1 \end{cases}$

53. $f(x) = \begin{cases} 2x - 3 \text{ if } x \leq 2 \\ x + 1 \text{ if } x > 2 \end{cases}$

Example 8

Graph $f(x) = |x + 3|$. State the domain and range.

x	f(x)
−5	2
−4	1
−3	0
−2	1
−1	2



The domain is all real numbers, and the range is $f(x) \geq 0$.

Practice Test

Use a table of values to graph the following functions. State the domain and range.

1. $y = x^2 + 2x + 5$

2. $y = 2x^2 - 3x + 1$

Consider $y = x^2 - 7x + 6$.

3. Determine whether the function has a *maximum* or *minimum* value.

4. State the maximum or minimum value.

5. What are the domain and range?

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

6. $x^2 + 7x + 10 = 0$

7. $x^2 - 5 = -3x$

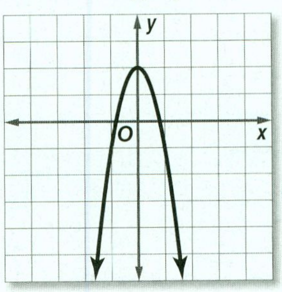
Describe how the graph of each function is related to the graph of $f(x) = x^2$.

8. $g(x) = x^2 - 5$

9. $g(x) = -3x^2$

10. $h(x) = \frac{1}{2}x^2 + 4$

11. MULTIPLE CHOICE Which is an equation for the function shown in the graph?



A $y = -3x^2$

B $y = 3x^2 + 1$

C $y = x^2 + 2$

D $y = -3x^2 + 2$

Solve each equation by completing the square.

12. $x^2 + 2x + 5 = 0$

13. $x^2 - x - 6 = 0$

14. $2x^2 - 36 = -6x$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

15. $x^2 - x - 30 = 0$

16. $x^2 - 10x = -15$

17. $2x^2 + x - 15 = 0$

18. BASEBALL Elias hits a baseball into the air. The equation $h = -16t^2 + 60t + 3$ models the height h in feet of the ball after t seconds. How long is the ball in the air?

19. Graph $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$. Determine whether the ordered pairs represent a *linear function*, a *quadratic function*, or an *exponential function*.

20. Look for a pattern in the table to determine which kind of model best describes the data.

x	0	1	2	3	4
y	1	3	5	7	9

21. CAR CLUB The table shows the number of car club members for four consecutive years after it began.

Time (years)	0	1	2	3	4
Members	10	20	40	80	160

a. Determine which model best represents the data.

b. Write a function that models the data.

c. Predict the number of car club members after 6 years.

Graph each function.

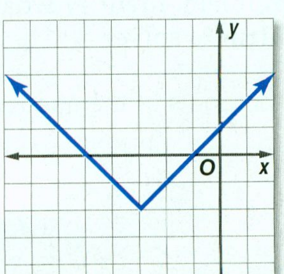
22. $f(x) = |x - 1|$

23. $f(x) = -|2x|$

24. $f(x) = [\![x]\!]$

25. $f(x) = \begin{cases} 2x - 1 \text{ if } x < 2 \\ x - 3 \text{ if } x \geq 2 \end{cases}$

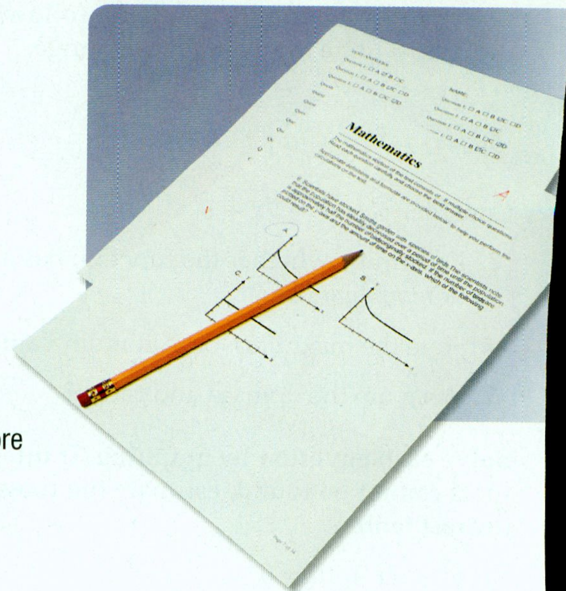
26. Determine the domain and range of the function graphed below.



Preparing for Standardized Tests

Use a Formula

A *formula* is an equation that shows a relationship among certain quantities. Many standardized test problems will require using a formula to solve them.



Strategies for Using a Formula

Step 1

Become familiar with common formulas and their uses. You may or may not be given access to a formula sheet to use during the test.

- **If given a formula sheet,** be sure to practice with the formulas on it before taking the test so you know how to apply them.
- **If not given a formula sheet,** study and practice with common formulas such as perimeter, area, and volume formulas, the Distance Formula, the Pythagorean Theorem, the Midpoint Formula, the Quadratic Formula, and others.

Step 2

Choose a formula and solve.

- **Ask Yourself:** What quantities are given in the problem statement?
- **Ask Yourself:** What quantities am I looking for?
- **Ask Yourself:** Is there a formula I know that relates these quantities?
- **Write:** Write the formula out that you have chosen each time.
- **Solve:** Substitute known quantities into the formula and solve for the unknown quantity.
- **Check:** Check your answer if time permits.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Find the exact roots of the quadratic equation $-2x^2 + 6x + 5 = 0$.

A $\frac{3 \pm \sqrt{17}}{4}$

B $\frac{4 \pm \sqrt{17}}{3}$

C $\frac{3 \pm \sqrt{19}}{2}$

D $\frac{3 \pm \sqrt{19}}{4}$

Stockbyte/PunchStock

Read the problem carefully. You are given a quadratic equation and asked to find the exact roots of the equation. Use the **Quadratic Formula** to find the roots.

$-2x^2 + 6x + 5 = 0$ — Original equation

$a = -2, b = 6, c = 5$ — Identify the coefficients of the equation.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$= \dfrac{-(6) \pm \sqrt{(6)^2 - 4(-2)(5)}}{2(-2)}$ — $a = -2$, $b = 6$, and $c = 5$

$= \dfrac{-6 \pm \sqrt{36 - (-40)}}{-4}$ — Simplify.

$= \dfrac{-6 \pm \sqrt{76}}{-4}$ — Subtract.

$= \dfrac{-6 \pm 2\sqrt{19}}{-4}$ — $\sqrt{76} = \sqrt{4 \cdot 19}$ or $2\sqrt{19}$.

$= \dfrac{-2(3 \pm \sqrt{19})}{-2(2)}$ — Factor out −2 from the numerator and denominator.

$= \dfrac{3 \pm \sqrt{19}}{2}$ — Simplify.

The roots of the equation are $\dfrac{3 + \sqrt{19}}{2}$ and $\dfrac{3 - \sqrt{19}}{2}$. The correct answer is C.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Find the exact roots of the quadratic equation $x^2 + 5x - 12 = 0$.

   A $\dfrac{-5 \pm \sqrt{73}}{2}$ C $\dfrac{-3 \pm \sqrt{73}}{4}$

   B $\dfrac{4 \pm \sqrt{61}}{3}$ D $\dfrac{-1 \pm \sqrt{61}}{2}$

2. The area of a triangle in which the length of the base is 4 centimeters greater than twice the height is 80 square centimeters. What is the length of the base of the triangle?

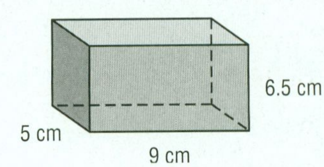
   F −10

   G 8

   H 16

   J 20

3. Find the volume of the figure below.



   A 18.5 cm^3 C 272 cm^3

   B 91 cm^3 D 292.5 cm^3

4. Myron is traveling 263.5 miles at an average rate of 62 miles per hour. How long will it take Myron to complete his trip?

   F 4 h 10 min

   G 4 h 15 min

   H 5 h 10 min

   J 5 h 25 min

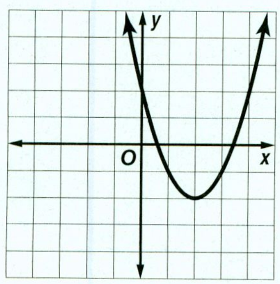
CHAPTER 9 Standardized Test Practice

Cumulative, Chapters 1 through 9

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the vertex of the parabola graphed below?



A $(2, 0)$

B $(0, 2)$

C $(-2, 2)$

D $(2, -2)$

2. Write an equation in slope-intercept form with a slope of $\frac{9}{10}$ and y-intercept of 3.

F $y = 3x + \frac{9}{10}$

G $y = \frac{9}{10}x + 3$

H $y = \frac{9}{10}x - 3$

J $y = 3x - \frac{9}{10}$

3. Use the Quadratic Formula to find the exact solutions of the equation $2x^2 - 6x + 3 = 0$.

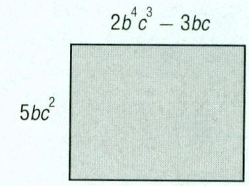
A $\frac{3 \pm \sqrt{3}}{2}$

B $\frac{3 \pm \sqrt{2}}{4}$

C $\frac{2 \pm \sqrt{5}}{3}$

D $\frac{5 \pm \sqrt{2}}{2}$

4. Write an expression for the area of the rectangle below.



F $10b^5c^5 - 3bc$

G $10b^5c^5 - 15b^2c^3$

H $2b^5c^5 - 3b^2c^3$

J $10b^4c^6 - 15bc^2$

5. Solve the quadratic equation below by graphing.

$$x^2 - 2x - 15 = 0$$

A $-1, 4$

B $-3, 5$

C $3, -5$

D $\varnothing$

6. Jason is playing games at a family fun center. So far he has won 38 prize tickets. How many more tickets would he need to win to place him in the gold prize category?

Number of Tickets	Prize Category
1–20	bronze
21–40	silver
41–60	gold
61–80	platinum

F $2 \leq t \leq 22$

G $3 \leq t \leq 22$

H $1 \leq t \leq 20$

J $3 \leq t \leq 20$

Test-Taking Tip

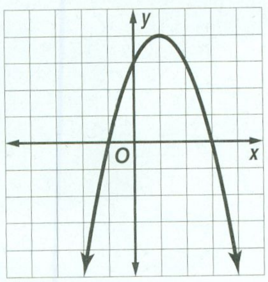
Question 5 If permitted, you can use a graphing calculator to quickly graph an equation and find its roots.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. **GRIDDED RESPONSE** Misty purchased a car several years ago for \$21,459. The value of the car depreciated at a rate of 15% annually. What was the value of the car after 5 years? Round your answer to the nearest whole dollar.

8. Use the graph of the quadratic equation shown below to answer each question.



   a. What is the vertex?

   b. What is the y-intercept?

   c. What is the axis of symmetry?

   d. What are the roots of the corresponding quadratic equation?

9. The cost of 5 notebooks and 3 pens is \$9.75. The cost of 4 notebooks and 6 pens is \$10.50.

   a. Write a system of equations to model the situation.

   b. Solve the system of equations. How much does each item cost?

10. The table shows the total cost of renting a canoe for n hours.

Number of Hours (n)	Rental Cost (C)
1	\$15
2	\$20
3	\$25
4	\$30

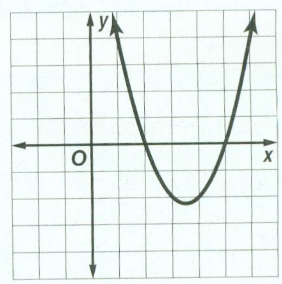
   a. Write a function to represent the situation.

   b. How much would it cost to rent the canoe for 7 hours?

Extended Response

Record your answers on a sheet of paper. Show your work.

11. Use the equation and its graph to answer each question.



   a. Factor $x^2 - 7x + 10$.

   b. What are the solutions of $x^2 - 7x + 10 = 0$?

   c. What do you notice about the graph of the quadratic equation and where it crosses the x-axis? How do these values compare to the solutions of $x^2 - 7x + 10 = 0$? Explain.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson...	9-1	4-2	9-5	8-2	9-2	5-1	7-6	9-1	6-4	3-5	8-6